**Abstract**

The problem of solution transfer between meshes arises frequently in computational physics, e.g. in Lagrangian methods where remeshing occurs. The interpolation process must be conservative, i.e. it must conserve physical properties, such as mass. We extend previous works — which described the solution transfer process for straight sided unstructured meshes — by considering high-order isoparametric meshes with curved elements. The implementation is highly reliant on accurate computational geometry routines for evaluating points on and intersecting Bézier curves and triangles.

*Keywords*: Remapping, Curved Meshes, Lagrangian, Solution Transfer, Numerical analysis

# Contents



# 1 Introduction

The first part is a general-purpose tool for computational physics problems. The tool enables solution transfer across two curved meshes. Since the tool requires a significant amount of computational geometry, the second half focuses on computational geometry. In particular, it considers cases where the geometric methods used have seriously degraded accuracy due to ill-conditioning.

In computational physics, the problem of solution transfer between meshes occurs in several applications. For example, by allowing the underyling computational domain to change during a simulation, computational effort can be focused dynamically to resolve sensitive features of a numerical solution. Mesh adaptivity (see, for example, [BR78, PVMZ87, PUdOG01]), this in-flight change in the mesh, requires translating the numerical solution from the old mesh to the new, i.e. solution transfer. As another example, Lagrangian or particle-based methods treat each node in the mesh as a particle and so with each timestep the mesh travels *with* the fluid (see, for example, [HAC74]). However, over (typically limited) time the mesh becomes distorted and suffers a loss in element quality which causes catastrophic loss in the accuracy of computation. To overcome this, the domain must be remeshed or rezoned and the solution must be transferred (remapped) onto the new mesh configuration.

When pointwise interpolation is used to transfer a solution, quantities with physical meaning (e.g. mass, concentration, energy) may not be conserved. To address this, there have been many explorations (for example, [JH04, FPP+09, FM11]) of *conservative interpolation* (typically using Galerkin or $L_2$-minimizing methods). In this work, the author introduces a conservative interpolation method for solution transfer between high-order meshes. These high-order meshes are typically curved, but not necessarily all elements or at all timesteps.

The existing work on solution transfer has considered straight sided meshes, which use shape functions that have degree $p = 1$ to represent solutions on each element or so-called superparametric elements (i.e. a linear mesh with degree $p > 1$ shape functions on a regular grid of points). However, both to allow for greater geometric flexibility and for high order of convergence, this work will consider the case of curved isoparametric[1] meshes. Allowing curved geometries is useful since many practical problems involve geometries that change over time, such as flapping flight or fluid-structure interactions. In addition, high-order CFD methods ([WFA+13]) have the ability to produce highly accurate solutions with low dissipation and low dispersion error.

[1]I.e. the degree of the discrete field on the mesh is same as the degree of the shape functions that determine the mesh.

### 1.1 Overview

This work is organized as follows. Section 2 establishes common notation and reviews basic results relevant to the topics at hand. Section 3 is an in-depth discussion of the computational geometry methods needed to implement to enable solution transfer. Section 4 describes the solution transfer process and gives results of some numerical experiments confirming the rate of convergence.

## 2 Preliminaries

Placeholder.

## 3 Bézier Intersection Problems

Placeholder.

## 4 Solution Transfer

Placeholder.

## References

[BR78] I. Babuška and W. C. Rheinboldt. Error Estimates for Adaptive Finite Element Computations. *SIAM Journal on Numerical Analysis*, 15(4):736–754, 1978.

[FM11] P.E. Farrell and J.R. Maddison. Conservative interpolation between volume meshes by local Galerkin projection. *Computer Methods in Applied Mechanics and Engineering*, 200(1-4):89–100, Jan 2011.

[FPP+09] P.E. Farrell, M.D. Piggott, C.C. Pain, G.J. Gorman, and C.R. Wilson. Conservative interpolation between unstructured meshes via supermesh construction. *Computer Methods in Applied Mechanics and Engineering*, 198(33-36):2632–2642, Jul 2009.

[HAC74] C.W Hirt, A.A Amsden, and J.L Cook. An arbitrary Lagrangian-Eulerian computing method for all flow speeds. *Journal of Computational Physics*, 14(3):227–253, Mar 1974.

[JH04] Xiangmin Jiao and Michael T. Heath. Common-refinement-based data transfer between non-matching meshes in multiphysics simulations. *International Journal for Numerical Methods in Engineering*, 61(14):2402–2427, 2004.

[PUdOG01] C.C. Pain, A.P. Umpleby, C.R.E. de Oliveira, and A.J.H. Goddard. Tetrahedral mesh optimisation and adaptivity for steady-state and transient finite element calculations. *Computer Methods in Applied Mechanics and Engineering*, 190(29-30):3771–3796, Apr 2001.

[PVMZ87] J Peraire, M Vahdati, K Morgan, and O.C Zienkiewicz. Adaptive remeshing for compressible flow computations. *Journal of Computational Physics*, 72(2):449–466, Oct 1987.

[WFA+13] Z.J. Wang, Krzysztof Fidkowski, Rémi Abgrall, Francesco Bassi, Doru Caraeni, Andrew Cary, Herman Deconinck, Ralf Hartmann, Koen Hillewaert, H.T. Huynh, Norbert Kroll, Georg May, Per-Olof Persson, Bram van Leer, and Miguel Visbal. High-order CFD methods: current status and perspective. *International Journal for Numerical Methods in Fluids*, 72(8):811–845, Jan 2013.