3) Claim: For all $n \geq 7$, $3^n < n!$

Base Case: $(n=7)$

$3^7 < 7!$

$2187 < 5040$ ✓

So we know $3^7 < 7!$

Inductive Step: (n)

Choose $k \in \mathbb{N}$ with $k \geq 7$ and assume $3^k < k!$

goal: $3^{k+1} < (k+1)!$

$$\begin{aligned} 3^{k+1} &= 3 \cdot 3^k \\ &< 3 \cdot k! \quad (IH) \\ &< 8 \cdot k! \\ &< (k+1) \cdot k! \quad (\text{since } k+1 \geq 7+1 = 8) \\ &= (k+1)! \end{aligned}$$

4) ~~the~~ Claim: For all natural numbers $n \geq 1$,

$$\sum_{i=1}^{n} 2i = n(n+1)$$

Pf:

Base Case: $(n=1)$

$$\sum_{i=1}^{1} 2(1) = 2, \text{ and } 1(1+1) = 2$$

So therefore $\sum_{i=1}^{1} 2i = 1(1+1)$.

Inductive Step:

Assume that for some positive integer k, $\sum_{i=1}^{k} 2i = k(k+1)$

goal: $(k+1)(k+1+1)$
$(k+1)(k+2)$

$$\sum_{i=1}^{k+1} 2i = \sum_{i=1}^{k} 2i + 2(k+1)$$

$= k(k+1) + 2(k+1)$ (IH)

$= (k+2)(k+1)$

$= (k+1)(k+2)$

$= (k+1)((k+1)+1)$

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5) Claim: For all $n \in \mathbb{N}$, $n^2 - 3n$ is even.

Base Case: $(n=0)$

For n^2-3n to be even, there must exist an integer x such that $n^2-3n=2x$.

$$0^2-3(0)=0=2(0).$$

Therefore, when $n=0$, n^2-3n is even.

Induction Step:

Choose a $k \in \mathbb{N}$ with $k \geq 0$ and assume $n^2-3n=2$.

$$\begin{aligned}(n+1)^2-3(n+1) &= n^2+2n+1-3n-3 \\ &= n^2-3n+2n-2 \\ &= 2k+2n-2 \quad (IH) \\ &= 2(k+n-2)\end{aligned}$$

Therefor n^2-3n is even.