UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

SECL1013

DISCREET STRUCTURE

ASSIGNMENT 2- CHAPTER 2

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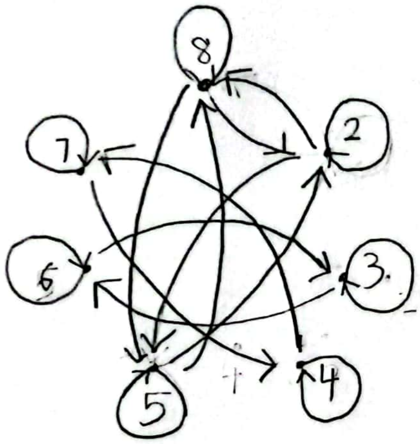
1) $x - y = 3n$, $n = -2, -1, 0, 1, 2$

x	2	2	2	3	3	4	4	5	5	5	6	6	7	7	8	8	8
y	2	5	8	3	6	4	7	2	5	8	3	6	4	7	2	5	8

$R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3)$
$(6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$



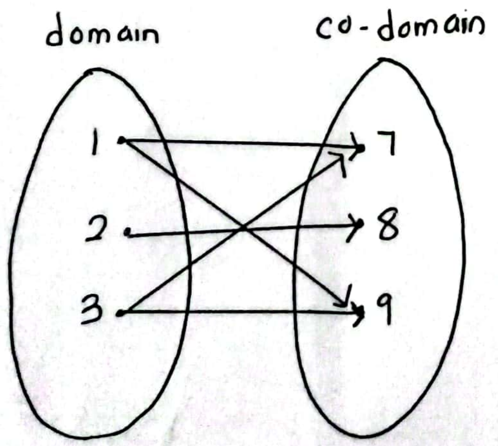
This is equivalance relation because reflexive, symmetric and transitive relation are exist.

2)

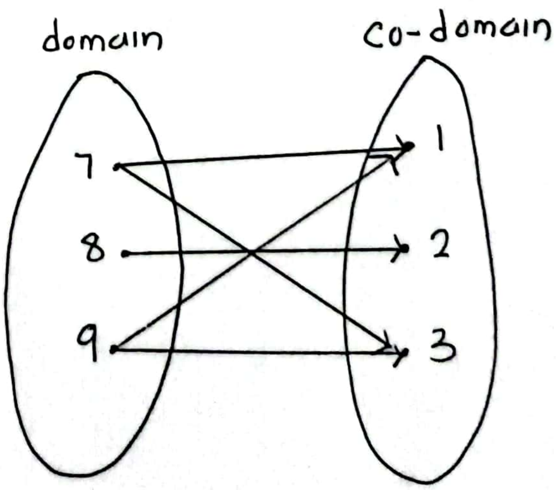
a. $R: \{(1,9), (1,7), (2,8), (3,7), (3,9)\}$

$R^{-1} \{(9,1), (7,1), (8,2), (7,3), (9,3)\}$

b.



R



R^{-1}

c. $aR^{-1}b \longleftrightarrow b+a$ even

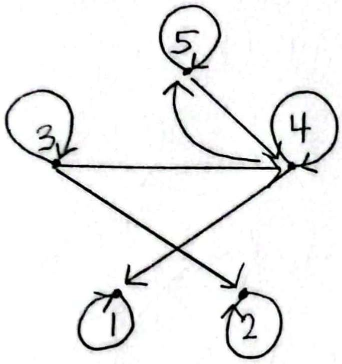
3.

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & & & & & \\ 2 & & & & & \\ 3 & & & & & \\ 4 & & & & & \\ 5 & & & & & \end{array}$$

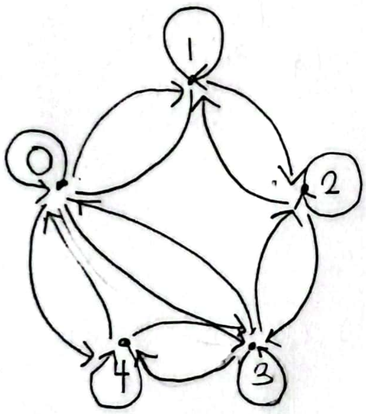
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

R{ (1,1), (2,2), (3,2), (3,3), (3,4), (4,1), (4,4), (4,5), (5,4), (5,5)}

	1	2	3	4	5
In degree	2	2	1	3	2
Out degree	1	1	3	3	2



4.



R is reflexive because every point has loop.

R is symmetric because there are both ways.

R is not transitive because (1,2), (2,3) $\in$ R but (1,3) $\notin$ R

5.

$R\{(1,3),(2,6),(3,9),(4,12)\}$

$(1,3)$	$(2,6)$	$(3,9)$	$(4,12)$
$3(1)-3=0$	$3(2)-6=0$	$3(3)-9=0$	$3(4)-12=0$
$0=0$	$0=0$	$0=0$	$0=0$

a. R is not reflexive because no loop

b. R is not symmetric because no both ways

c. R is not transitive because $(a,b) \in R$ but $(b,c) \notin R$ and $(a,c) \notin R$

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6. a) RS

$$\begin{array}{ccccc} R & & S & & RS \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \times & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} & = & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

b) SR

$$\begin{array}{ccccc} S & & R & & SR \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} & \times & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} & = & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Q2. Function

7) - A function is a relation in which each element in the domain has only a single result.
- A relation is when there are multiple mappings in domain and co-domain.

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8) i) It is a function because every element in domain has only one value.
ii) It is a function because every element in domain has only one value.
iii) Not a function because, 2 has multiple values, 3 and 4 has no value assigned.
iv) Not a function because, 5 was not assigned to any value.
v) Not a function because, 2 and 4 has multiple values and 3 and 5 was not assigned any value.

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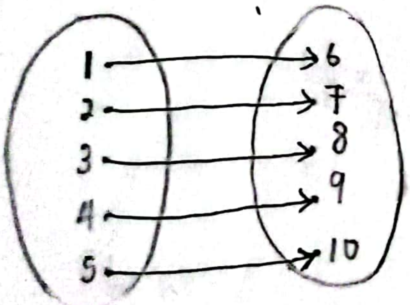
9) Since x is less than 6, $x = \{1, 2, 3, 4, 5\}$
$y = \{6, 7, 8, 9, 10\}$

$y = x + 5$



domain : $\{1, 2, 3, 4, 5\}$

Range : $\{6, 7, 8, 9, 10\}$

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10) v) $f(x) = 1 - 2x$

$f(x_1) = f(x_2)$

$1 - 2x_1 = 1 - 2x_2$

$\cancel{+}2x_1 = \cancel{+}2x_2$

$2x_1 = 2x_2$

$x_1 = x_2$

$\therefore$ One-to-one

$y = 1 - 2x$

$x = \frac{1-y}{2}$

$\therefore$ x is a real number for any value of y.
Thus, it is onto.

$\therefore$ It is bijective because it is one-to-one and onto.

vi) $f: R \to R$, $f(x) = 5x^2 - 1$

$f(x_1) = f(x_2)$

$5x_1^2 - 1 = 5x_2^2 - 1$

$5x_1^2 = 5x_2^2$

$x_1^2 = x_2^2$

$x_1 = \pm\sqrt{x_2^2}$

$x_1 = \pm x_2$

$\therefore$ not one-to-one

$y = 5x^2 - 1$

$y + 1 = 5x^2$

$x^2 = \frac{y+1}{5}$

$x = \sqrt{\frac{y+1}{5}}$

$\therefore$ x is not a real number when y is a negative value.

· Not onto for negative values.

$\therefore$ it is not bijective because it is not one-to-one and not onto.

vii) $f: R \to R$, $f(x) = x^4$

$f(x_1) = f(x_2)$

$x_1^4 = x_2^4$

$\sqrt{x_1^4} = \sqrt{x_2^4}$

$x_1^2 = x_2^2$

$x_1 = \sqrt{x_2^2}$

$x_1 = \pm x_2$

$\therefore$ not one-to-one

$y = x^4$

$x = \sqrt[4]{y}$

$\therefore$ x is not a real number when y is a negative value

Thus, it is not onto for negative values.

$\therefore$ It is not bijective because it is not one-to-one and not onto.

viii) $f: R \to R$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\cancel{x_1x_2} - 3x_1 - 2x_2 \cancel{+6} = \cancel{x_1x_2} - 3x_2 - 2x_1 \cancel{+6}$

$-3x_1 - 2x_2 = -3x_2 - 2x_1$

$3x_1 + 2x_2 = 3x_2 + 2x_1$

$3x_1 - 2x_1 = 3x_2 - 2x_2$

$x_1 = x_2$

$\therefore$ One-to-one function.

$y = \frac{x-2}{x-3}$

$y(x-3) = x - 2$

$xy - 3y = x - 2$

$xy - x = -2 + 3y$

$x(y-1) = -2 + 3y$

$x = \frac{3y-2}{y-1}$

$\therefore$ x is a real number for any value of y. Thus, it is onto.

$\therefore$ It is bijective because it is one-to-one and

11) ix) $f(g(x))$.

$f(x) = 3x - 1$; $g(x) = x^2 - 1$

$= fg(x)$

$= f[x^2 - 1]$

$= 3(x^2 - 1) - 1$

$= 3x^2 - 3 - 1$

$fg(x) = 3x^2 - 4$

$fg(x) = 3x^2 - 4$

$x = 0 : 3(0)^2 - 4$

$= -4$

$x = 1 : 3(1)^2 - 4$

$= -1$

$x = 2 : 3(2)^2 - 4$

$= 3(4) - 4$

$= 8$

$x = 3 : 3(3)^2 - 4$

$= 3(9) - 4$

$= 23$

$x = \{0, 1, 2, 3\}$

$fg(x) = \{-4, -1, 8, 23\}$

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x) $f(x) = x^2$

$g(x) = 5x - 6$

$= fg(x)$

$= f[5x - 6]$

$= (5x - 6)^2$

$fg(x) = 25x^2 - 60x + 36$

$fg(x) = 25x^2 - 60x + 36$

$x = 0 : 25(0)^2 - 60(0) + 36$

$= 36$

$x = 1 : 25(1)^2 - 60(1) + 36$

$= 1$

$x = 2 : 25(2)^2 - 60(2) + 36$

$= 16$

$x = 3 : 25(3)^2 - 60(3) + 36$

$= 81$

$x = \{0, 1, 2, 3\}$

$fg(x) = \{36, 1, 16, 81\}$

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xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

$= fg(x)$

$= f[x^3 + 1]$

$= (x^3 + 1) - 1$

$fg(x) = x^3$

$fg(x) = x^3$

$x = 0 : (0)^3$

$= 0$

$x = 1 : (1)^3$

$= 1$

$x = 2 : (2)^3$

$= 8$

$x = 3 : (3)^3$

$= 27$

$x = \{0, 1, 2, 3\}$

$fg(x) = \{0, 1, 8, 27\}$

Recurrence Relation

2) (xii) $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0 = 1$ and $a_1 = 6$

$a_2 = 6a_1 - 9a_0$
$a_2 = 6(6) - 9(1)$
$a_2 = 36 - 9$
$= 27$

$a_3 = 6a_2 - 9a_1$
$a_3 = 6(27) - 9(6)$
$= 108$

$a_4 = 6a_3 - 9a_2$
$= 6(108) - 9(27)$
$= 405$

$\therefore 1, 6, 27, 108, 405, \ldots$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; $a_0 = 2$, $a_1 = 5$ & $a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_3 - 11a_2 + 6a_1$
$= 6(47) - 11(15) + 6(5)$
$= 147$

$a_5 = 6a_4 - 11a_3 + 6a_2$
$= 6(147) - 11(47) + 6(15)$
$= 455$

$\therefore 2, 5, 15, 47, 147, 455, \ldots$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$
$= -3(-1) - 3(-2) + 1$
$= 10$

$a_4 = -3a_3 - 3a_2 + a_1$
$= -3(10) - 3(-1) + (-2)$
$= -29$

$a_5 = -3a_4 - 3a_3 + a_2$
$= -3(-29) - 3(10) + (-1)$
$= 56.$

$\therefore 1, -2, -1, 10, -29, 56, \ldots$

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3) $a_{n+1} = 5a_n - 3$; $a_1 = k$

i) $a_4 = 5a_3 - 3$ $a_1 = k.$

$a_2 = 5a_1 - 3$
$a_2 = 5k - 3$

$a_3 = 5a_2 - 3$
$= 5(5k-3) - 3$
$= 25k - 15 - 3$
$= 25k - 18$

$a_4 = 5a_3 - 3$
$= 5(25k - 18) - 3$
$= 125k - 90 - 3$
$= 125k - 93$

ii) $a_4 = 7$

$125k - 93 = 7$
$125k = 100$
$k = \frac{100}{125}.$

$\boxed{k = \frac{4}{5}}$