MACHINE LEARNING & COMPUTATIONAL INTELLIGENCE

Single Layer Perceptron



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In Task 1 of Machine Learning I use a dataset from https://archive.ics.uci.edu/ml/machine-learning-databases/iris/iris.data. In the dataset, the first column is x1, the second column is x2, the third column is x3, the fourth column is x4, and the last column is the labeling of the class. To classify 'Iris-versicolor, I made class '0', while for 'Iris-virginica' I made it a class '1'. I took the first 100 data out of 150 data contained in iris.data

A single layer perceptron (SLP) is a feed-forward network based on a threshold transfer function. SLP is the simplest type of artificial neural networks and can only classify linearly separable cases with a binary target (1, 0).

	Α	В	С	D	Е
1	x1	x2	х3	x4	type
2	7.0	3.2	4.7	1.4	Iris-versicolor
3	6.4	3.2	4.5	1.5	Iris-versicolor
4	6.9	3.1	4.9	1.5	Iris-versicolor
5	5.5	2.3	4.0	1.3	Iris-versicolor
6	6.5	2.8	4.6	1.5	Iris-versicolor
7	5.7	2.8	4.5	1.3	Iris-versicolor
8	6.3	3.3	4.7	1.6	Iris-versicolor
9	4.9	2.4	3.3	1.0	Iris-versicolor
10	6.6	2.9	4.6	1.3	Iris-versicolor
11	5.2	2.7	3.9	1.4	Iris-versicolor
12	5.0	2.0	3.5	1.0	Iris-versicolor
13	5.9	3.0	4.2	1.5	Iris-versicolor
14	6.0	2.2	4.0	1.0	Iris-versicolor
15	6.1	2.9	4.7	1.4	Iris-versicolor
16	5.6	2.9	3.6	1.3	Iris-versicolor
17	6.7	3.1	4.4	1.4	Iris-versicolor
18	5.6	3.0	4.5	1.5	Iris-versicolor
19	5.8	2.7	4.1	1.0	Iris-versicolor
20	6.2	2.2	4.5	1.5	Iris-versicolor

Figure 1. Samples of iris.data

K-Fold Cross-Validation is a procedure in Machine Learning that is used to evaluate Machine Learning Models on a limited amount of data. This procedure uses the K parameter which represents the number of groups formed from the data. The steps in this procedure are as follows:

- 1. Divide the dataset into K number of groups
- 2. In each group, determine the majority of data as training data and part of it others as data validation
- 3. Evaluate the training data model and validation for each group

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Sigmoid Function

Epoch = 100

Alpha or learning rate = 0.01

H(x, theta, bias) = theta1*x1 + theta2*x2 + theta3*x3 + theta4*x4 + bias

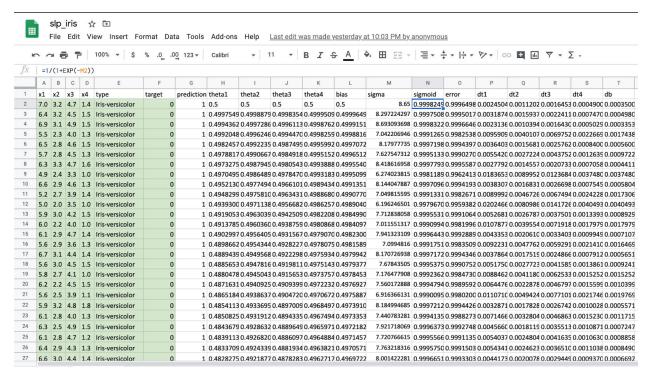


Figure 2. The manual calculations of linear classification with Google Sheet

(source: https://bit.ly/slp_csv)

With trials

Theta 1 = 0.5

Theta2 = 0.5

Theta3 = 0.5

Theta4 = 0.5

Bias = 0.5

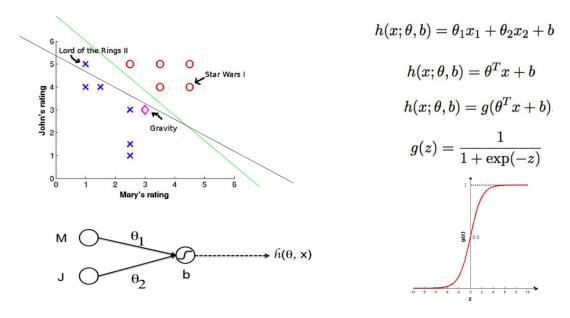


Figure 3. Mathematical formula for sigma and sigmoid

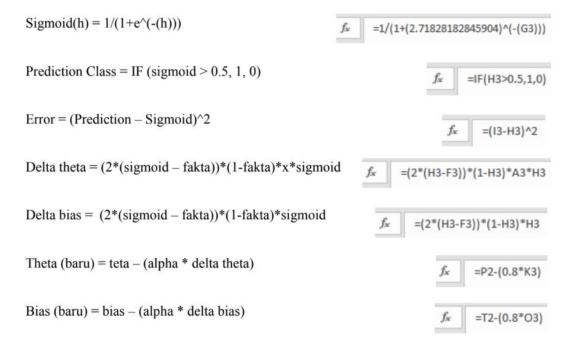


Figure 4. Formula on Google Sheet (source https://bit.ly/slp_csv)

Error Function

Error = $(0 - \text{sigmoid})^2 = (0 - 0.9998249038)^2 = 0.9996498383$

Gradient Descent Algorithm

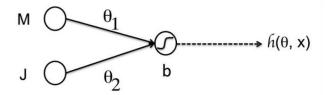
Error calculation is really crucial to update the weight. I minimize error value by finding the best weight for every input or in a mathematical way is known as "Stochastic Gradient Descent (SGD)".

- new_weight = weight learning_rate * (dError/dWeight)
- dError/dWeight = (dError/dSigmoid) * (dSigmoid/dSigma) * (dSigma/dWeight)
- dError/dWeight = -2 * (Target-Sigmoid) * Sigmoid * (1-Sigmoid) * Input
- dError/dWeight1 = (-2*(0.99)*0.99*(1-0.99)*7.0 = 0.0024
- dError/dWeight2 = (-2*(0.99)*0.99*(1-0.99)*3.2 = 0.0011
- dError/dWeight3 = (-2*(0.99)*0.99*(1-0.99)*4.7 = 0.0016
- dError/dWeight4 = (-2*(0.99)*0.99*(1-0.99)*1.4 = 0.0004
- dError/dBias = (-2*(0.99)*0.99*(1-0.99) = 0.00035

By using SGD, I get new_weight for every input;

- new_weight1 = 0.5 0.1 * 0.0024 = 0.4997549512
- new_weight2 = 0.5 0.1 * 0.0011 = 0.4998879777
- new_weight3 = 0.5 0.1 * 0.0016 = 0.4998354672
- $new_weight4 = 0.5 0.1 * 0.0004 = 0.4999509902$
- bias = 0.5 0.1 * 0.00035 = 0.499964993





$$\begin{array}{l} \Delta\theta_1 = \frac{\partial}{\partial\theta_1} \bigg(h(x^{(i)};\theta,b) - y^{(i)} \bigg)^2 \\ \theta_2 = \theta_2 - \alpha \Delta \theta_2 \\ b = b - \alpha \Delta b \end{array}$$

$$= 2 \bigg(h(x^{(i)};\theta,b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} h(x^{(i)};\theta,b) \\ = 2 \bigg(g(\theta^T x^{(i)} + b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} g(\theta^T x^{(i)} + b) \end{array}$$

$$\begin{split} \Delta\theta_1 &= \frac{\partial}{\partial\theta_1} \bigg(h(x^{(i)};\theta,b) - y^{(i)} \bigg)^2 \\ &= 2 \bigg(h(x^{(i)};\theta,b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} h(x^{(i)};\theta,b) & \text{M} & \underbrace{\theta_1} \\ &= 2 \bigg(g(\theta^T x^{(i)} + b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} g(\theta^T x^{(i)} + b) & \text{J} & \underbrace{\theta_2} \end{split}$$

$$\begin{split} \frac{\partial}{\partial \theta_1} g(\theta^T x^{(i)} + b) &= \frac{\partial g(\theta^T x^{(i)} + b)}{\partial (\theta^T x^{(i)} + b)} \frac{\partial (\theta^T x^{(i)} + b)}{\partial \theta_1} & \frac{\partial g}{\partial z} = [1 - g(z)]g(z) \\ &= [1 - g(\theta^T x^{(i)} + b)]g(\theta^T x^{(i)} + b) \frac{\partial (\theta_1 x_1^{(i)} + \theta_2 x_2^{(i)} + b)}{\partial \theta_1} \\ &= [1 - g(\theta^T x^{(i)} + b)]g(\theta^T x^{(i)} + b)x_1^{(i)} \end{split}$$

$$\begin{split} \Delta\theta_1 &= \frac{\partial}{\partial\theta_1} \bigg(h(x^{(i)};\theta,b) - y^{(i)} \bigg)^2 \\ &= 2 \bigg(h(x^{(i)};\theta,b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} h(x^{(i)};\theta,b) & \Delta\theta_1 = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) x_1^{(i)} \\ &= 2 \bigg(g(\theta^T x^{(i)} + b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} g(\theta^T x^{(i)} + b) & \Delta\theta_2 = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) x_2^{(i)} \\ &= 2 \bigg(g(\theta^T x^{(i)} + b) - y^{(i)} \bigg) \frac{\partial}{\partial\theta_1} g(\theta^T x^{(i)} + b) & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)} + b) - y^{(i)}] [1 - g(\theta^T x^{(i)} + b)] g(\theta^T x^{(i)} + b) \\ & \Delta b = 2 [g(\theta^T x^{(i)}$$

 $= [1 - g(\theta^T x^{(i)} + b)]g(\theta^T x^{(i)} + b)x_1^{(i)}$

Figure 5. Mathematical explanation for SGD

Train the Neural Network - SLP

In machine learning, epoch = iteration for training process. I just combine the training data with the same training dataset + use the recent weight and bias that I got before from the previous epoch. The full journey is at https://bit.ly/slp_csv

- Train the Neural Network SLP algorithm on the training data with learning rate : 0.001 and maximum iteration : 100
- Find theta that minimize the cost function based on Figure 4.
- Test the linear classifier on the test data, and calculate the accuracy
- Plot the cost/error function plot

Implementation

Full version:

https://github.com/dhifaaans/slp_iris/blob/master/linearclassifier_nadhifasofia_KKPMDD.ipvnb

Binary-Classification using Linear Classification

Using The part of Iris data (2 classes and two features, choosen by yourself => 100 data). http://archive.ics.uci.edu/ml/datasets/iris

1. Load the Iris data from scikit learn and divide the data into two parts: training and test data with ration 80:20. Make sure that the class within each parts of the data is balance. (score: 0.5)

```
In [2]: #initialize the data
        cols = ['x1','x2','x3','x4','class','binary','t1','t2','t3','t4','bia
        df_training = pd.read_csv('iris.data', header=None, names=cols)
        #to choose 100 first data
        df_training.head(100)
        df_training = df_training.head(100)
        #to generate 2 classes, i chose these
        for i in range(100):
            if df_training.at[i,'class']=='Iris-versicolor':
            df_training.at[i,'binary'] = 0
elif df_training.at[i,'class'] == 'Iris-virginica':
                 df_training.at[i, 'binary'] = 1
        #divide data into k-fold, 100 epoch
        df_k1 = df_training.iloc[0:80]
        df_{epoch_1} = df_k1
        for i in range(0,99):
            df_k1 = df_k1.append(df_epoch_1,ignore_index=True, sort=False)
        df_k1_validation = df_training.iloc[80:100]
        df_k1_validation = df_k1_validation.reset_index(drop=True)
```

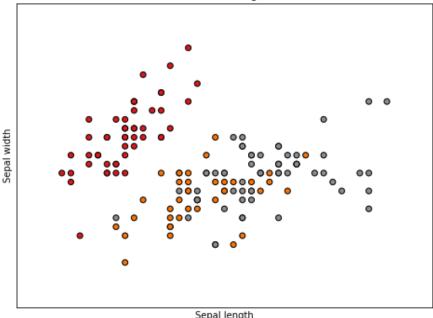
Iris Dataset Visualization

2. Using scatter plot (matplotlib), visualise the Iris data (training part only) (score: 0.5)

```
In [3]: # import some data to play with
        from sklearn import datasets
        iris = datasets.load_iris()
        X = iris.data[:, :2] # we only take the first two features.
        y = iris.target
        x_{min}, x_{max} = X[:, 0].min() - .5, X[:, 0].max() + .5

y_{min}, y_{max} = X[:, 1].min() - .5, X[:, 1].max() + .5
        plt.figure(2, figsize=(8, 6))
        plt.clf()
        # Plot the training points
        plt.xlabel('Sepal length')
        plt.ylabel('Sepal width')
        plt.title('Scatter Plot for Setosa vs Virginica vs Versicolor')
        plt.xlim(x_min, x_max)
        plt.ylim(y_min, y_max)
        plt.grid(True)
        plt.xticks(())
        plt.yticks(())
Out[3]: ([], <a list of 0 Text yticklabel objects>)
```

Scatter Plot for Setosa vs Virginica vs Versicolor



Sigmoid Function

3. Define a python function for sigmoid function (score:0.5)

By way of example, we have data that is saved into our model; you can see it thru https://bit.ly/slp_csv

```
input1 * weight1 = 7.0 * 0.5 = 3.5
input2 * weight2 = 3.2 * 0.5 = 1.6
input3 * weight3 = 4.7 * 0.5 = 2.35
input4 * weight4 = 1.4 * 0.5 = 0.7
bias = 0.5
sigma(input * weight) + bias = (3.5+1.6+2.35+0.7) + 0.5 = 8.65
```

```
• sigmoid = 1/(1 + (exp(-8.65))) = 0.9998249038
```

```
In [6]: def sigmoid(ha):
    return (1/(1+math.exp(-1*ha)))
```

Error Function

4. Define a python function of the cost function (score:0.5)

```
Error = (0 - sigmoid)^2 = (0 - 0.9998249038) ^ 2 = 0.9996498383
```

```
In [8]: def local_error(a,b):
    return math.fabs((a-b))
```

Gradient Descent Algorithm

5. Define a python function for the Gradient Descent algorithm (score: 0.5)

Error calculation is really crucial to update the weight. I minimize error value by finding the best weight for every input or in mathematical way is known as "Stochastic Gradient Descent (SGD)".

- new_weight = weight learning_rate * (dError/dWeight)
- dError/dWeight = (dError/dSigmoid) * (dSigmoid/dSigma) * (dSigma/dWeight)
- dError/dWeight = -2 * (Target-Sigmoid) * Sigmoid * (1-Sigmoid) * Input

```
    dError/dWeight1 = (-2*(0.99) * 0.99 * (1-0.99) * 7.0 = 0.0024
```

- dError/dWeight2 = (-2*(0.99) * 0.99 * (1-0.99) * 3.2 = 0.0011
- dError/dWeight3 = (-2*(0.99) * 0.99 * (1-0.99) * 4.7 = 0.0016
- dError/dWeight4 = (-2*(0.99) * 0.99 * (1-0.99) * 1.4 = 0.0004
- dError/dBias = (-2*(0.99) * 0.99 * (1-0.99) = 0.00035

By using SGD, I get new_weight for every input;

```
new_weight1 = 0.5 - 0.1 * 0.0024 = 0.4997549512
```

- new_weight2 = 0.5 0.1 * 0.0011 = 0.4998879777
- new_weight3 = 0.5 0.1 * 0.0016 = 0.4998354672
- new_weight4 = 0.5 0.1 * 0.0004 = 0.4999509902
- bias = 0.5 0.1 * 0.00035 = 0.499964993

```
In [10]: def sgd(g, y, x):
    return (2*(g-y)*(1-g)*g*x)
```

Train the Neural Network - SLP

6. Train your NN-SLP algorithm on the training data, set learning rate: 0.001 and maximum iteration: 100 (score: 0.5)

In machine learning, epoch = iteration for training process. I just combine the training data with the same training dataset + use the recent weight and bias that I got before from previous epoch. The full journey is at https://bit.ly/slp_csv

7. Find thetha(s) that minimize the cost function, and plot the decision boundary using matplotlib. (score: 1.0)

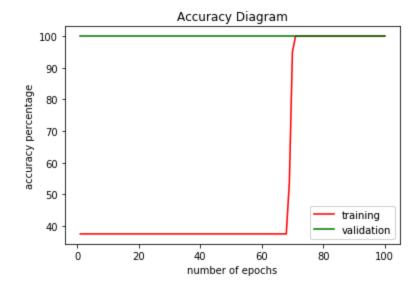
Testing Process

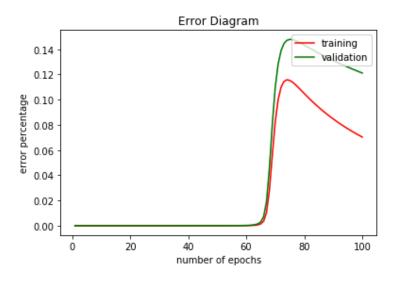
8. Test your Linear Classifier on the test data, and calculate the accuracy (score: 0.5)

```
In [15]:
                      for i in range(1,df validating.shape[0]):
                           df validating.at[i,'t1'] = df validating.at[i-1,'t1']+lea
                           df_validating.at[i,'t2'] = df_validating.at[i-1,'t2']+lea
                           df_validating.at[i,'t3'] = df_validating.at[i-1,'t3']+lea
                           df_validating.at[i,'t4'] = df_validating.at[i-1,'t4']+lea
                           df_validating.at[i,'bias'] = df_validating.at[i-1,'bias']
                           df_validating.at[i,'target'] = df_validating.at[i,'x1']*d¹
                           df_validating.at[i,'sigmoid']= 1/(1+math.exp(-df_validat:
                           if df_validating.at[i,'sigmoid']>0.5:
                                df_validating.at[i, 'prediction'] = 1
                           else:
                                df_validating.at[i,'prediction']= 0
                           df_validating.at[i,'error']= math.pow(df_validating.at[i,
df_validating.at[i,'dt1']= 2*df_validating.at[i,'x1']*(df_validating.at[i,'x1']*(df_validating.at[i,'x2']*(df_validating.at[i,'x2']*(df_validating.at[i,'x3']*(df_validating.at[i,'x4']*(df_validating.at[i,'x4']*(df_validating.at[i,'dbias']= 2*(df_validating.at[i,'bina)
                           if df_validating.at[i,'prediction'] == df_validating.at[:
                                correctvalidate = correctvalidate+1
                      ploterrorvalidate.append(df_validating.at[19,'error'] )
                      #print("correct training: ",correcttraining, "correct valida"
                      plotcorrecttrain.append(100*correcttraining/80)
                      plotcorrectvalidate.append(100*correctvalidate/19)
                      plotcounter.append(counter/80)
                      correcttraining = correctvalidate = 0
           print("K-1 Learning Rate: ",learningrate," Epochs:100")
plotaccuracy.plot(plotcounter, plotcorrecttrain, color="red",label="1")
           plotaccuracy.plot(plotcounter, plotcorrectvalidate, color = "green",
           plotaccuracy.legend(loc = "lower right")
           nlotaccuracy set title("Accuracy Diagram")
```

Error Function Plot

9. Plot your cost function using matplotlib (cost function vs iteration) (score: 0.5)



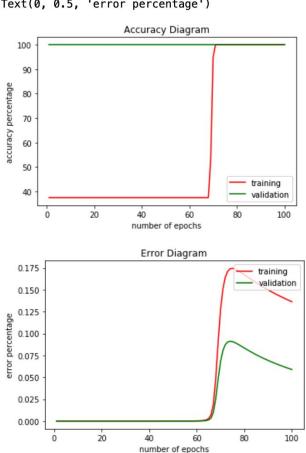


ACCURACY FOR EVERY K-FOLD (K=1:5)

K=1, Learning Rate = 0.001, Epoch = 100

input k-fold index: 1
input learning rate: 0.001
K-1 Learning Rate: 0.001 Epochs:100

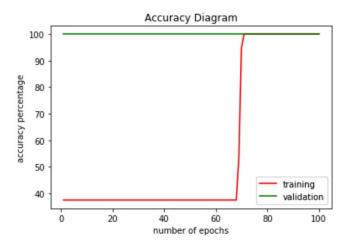
Out[4]: Text(0, 0.5, 'error percentage')

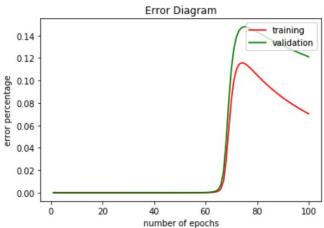


K=2, Learning Rate = 0.001, Epoch = 100

input k-fold index: 2
input learning rate: 0.001
K-1 Learning Rate: 0.001 Epochs:100

Out[5]: Text(0, 0.5, 'error percentage')

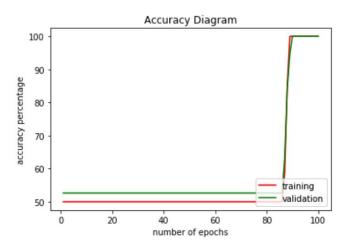


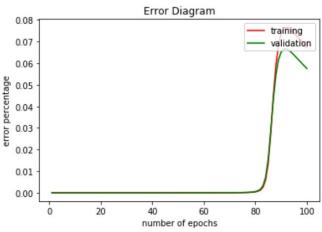


K=3, Learning Rate = 0.001, Epoch = 100

input k-fold index: 3
input learning rate: 0.001
K-1 Learning Rate: 0.001 Epochs:100

Out[6]: Text(0, 0.5, 'error percentage')

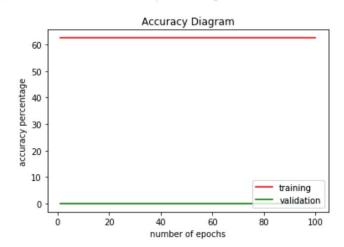


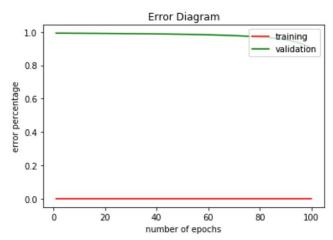


K=4, Learning Rate = 0.001, Epoch = 100

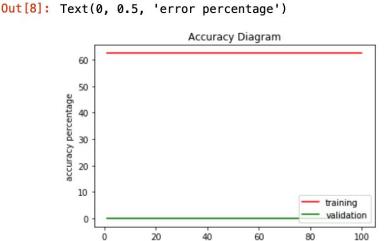
input k-fold index: 4
input learning rate: 0.001
K-1 Learning Rate: 0.001 Epochs:100

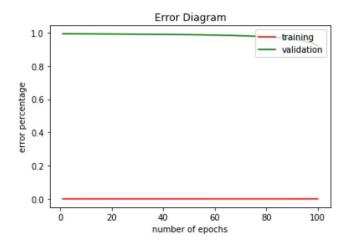
Out[7]: Text(0, 0.5, 'error percentage')





K=5, Learning Rate = 0.001, Epoch = 100
 input k-fold index: 5
 input learning rate: 0.001
 K-1 Learning Rate: 0.001 Epochs:100





number of epochs

SUMMARY

From the algorithm, we get the best performance on K=1 and K=2 with well accuracy and error improvements. On K=1, we get steady 100% accuracy on validation, 100% accuracy after 70+ epochs for the training process. After that, the error lowers from 0.075% -> 0.050% after 70+ epochs on validation. In addition, on K=2, we still get steady 100% accuracy like the K1 gets and error reduction from 0.14% -> 0.12% after 70+ epochs.