Monte Carlo Methods Spring 2025
Homework 02 - Pseudo Random Number Generators

Due: Tuesday, Feb 04, 2024, 11:59 PM

1. (20 points) The normal distribution has the pdf

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{1}$$

The χ^2 distribution with 1 degree of freedom can be defined as the square of a standard normal random variable. That is, if $Z \sim N(0,1)$, then $X = Z^2 \sim \chi^2(1)$.

   (a) Find the pdf of the χ^2 distribution with 1 degree of freedom by transforming (1).

   (b) Compute the critical value of the χ^2 function for 1 degree of freedom at the 95% confidence level. For this problem you are only allowed to use the Python function `scipy.stats.chi2.ppf` which computes the inverse of the cdf of the normal distribution.

   (c) Suppose you toss a coin 100 times and record the number of heads N. Suppose your null hypothesis is that the coin is fair. You want to test this hypothesis at the 95% confidence level using the χ^2 goodness of fit test. For what values of N would you fail to reject the null hypothesis?

2. (20 points) Jupyter Notebook on Canvas.