Monte Carlo Methods Spring 2025
Homework 07 - Gibbs Sampling in 2D

Due: Tuesday, Mar 11, 2024, 11:59 PM

In class, we worked through the Gibbs sampler for the truncated exponential distribution. In this homework, you will extend those calculations and implement the Gibbs sampler yourself.

1. (30 points) Consider two random variables X and Y with the joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} c\exp(-\lambda xy), & \text{for } (x, y) \in [0, A] \times [0, B], \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

   Here, c is a normalizing constant ensuring the total probability sums to 1. For running the Gibbs sampler, you do not need to determine c explicitly.

   To simplify notation, we will omit subscripts from the distributions where appropriate, so $f(x, y) = f_{X,Y}(x, y)$, $f(x) = f_X(x)$, $F(x) = F_X(x)$, $F(x \mid y_0) = F_{X|Y}(x \mid Y = y_0)$, etc.

   Complete the following steps:

   (a) Derive the marginal distributions $f(x)$ and $f(y)$.

   (b) Compute the conditional distributions $f(x \mid y_0)$ and $f(y \mid x_0)$. These will be truncated exponential distributions in the variables x and y, respectively.

   (c) Determine the cumulative distribution functions $F(x \mid y_0)$ and $F(y \mid x_0)$, treating them as functions of x and y, respectively.

   (d) Outline inverse transform sampling algorithms to generate samples from $f(x \mid y_0)$ and $f(y \mid x_0)$.

   (e) Write a Gibbs sampling algorithm to generate samples from the joint pdf in Equation (1). Provide all the details. The only source of randomness should be a uniform random number generator $U \sim U(0, 1)$.

2. (20 points) Jupyter Notebook on Canvas.