Algorithm 1 Particle Filter with Resampling

Require: Observations $\{y_1, y_2, \ldots, y_T\}$, number of particles N

Require: Transition model $p(x_t \mid x_{t-1})$, emission model $p(y_t \mid x_t)$, initial distribution $p(x_1)$

Initialize:
for $i = 1$ to N **do**
    Set $x_0^{(i)} \leftarrow x_0$
    Set weight $w_1^{(i)} \leftarrow 1/N$
end for
for $t = 1$ to T **do**
    for $i = 1$ to N **do**
        Sample $x_t^{(i)} \sim p(x_t \mid x_{t-1}^{(i)})$
        Compute weight: $w_t^{(i)} \leftarrow w_{t-1}^{(i)} \cdot p(y_t \mid x_t^{(i)})$
    end for
    Normalize weights: $w_t^{(i)} \leftarrow \frac{w_t^{(i)}}{\sum_{j=1}^{N} w_t^{(j)}}$
end for
Return: Particles $\{x_t^{(i)}\}$ and weights $w_t^{(i)}$ for all t

Algorithm 2 Particle Filter with Resampling

Require: Observations $\{y_1, y_2, \ldots, y_T\}$, number of particles N

Require: Transition model $p(x_t \mid x_{t-1})$, emission model $p(y_t \mid x_t)$, initial distribution $p(x_1)$

Initialize:
for $i = 1$ to N **do**
    Set $x_0^{(i)} \leftarrow x_0$
    Set weight $w_1^{(i)} \leftarrow 1/N$
end for
for $t = 1$ to T **do**
    for $i = 1$ to N **do**
        Sample $x_t^{(i)} \sim p(x_t \mid x_{t-1}^{(i)})$
        Compute weight: $w_t^{(i)} \leftarrow w_{t-1}^{(i)} \cdot p(y_t \mid x_t^{(i)})$
    end for
    Normalize weights: $w_t^{(i)} \leftarrow \frac{w_t^{(i)}}{\sum_{j=1}^{N} w_t^{(j)}}$
    Compute effective sample size:

$$N_{\text{eff}} \leftarrow \frac{1}{\sum_{i=1}^{N} (w_t^{(i)})^2}$$

    if $N_{\text{eff}} < N_{\text{threshold}}$ **then**
        Resample particles $\{x_t^{(i)}\}$ with probabilities $w_t^{(i)}$
        Reset weights: $w_t^{(i)} \leftarrow \frac{1}{N}$
    end if
end for
Return: Particles $\{x_t^{(i)}\}$ and weights $w_t^{(i)}$ for all t

Algorithm 3 Sequential Importance Sampling- Particle Filter with general proposal

Require: Observations $\{y_1, y_2, \ldots, y_T\}$, number of particles N
Require: Transition model $p(x_t \mid x_{t-1})$, emission model $p(y_t \mid x_t)$, initial distribution $p(x_1)$
Require: Proposal distribution $q(x_t \mid x_{t-1}, y_t)$
Initialize:
for $i = 1$ to N **do**
    Set $x_0^{(i)} \leftarrow x_0$
    Set weight $w_1^{(i)} \leftarrow 1/N$
end for
for $t = 1$ to T **do**
    for $i = 1$ to N **do**
        Sample $x_t^{(i)} \sim q(x_t \mid x_{t-1}^{(i)}, y_t)$
        Compute weight:

$$w_t^{(i)} \leftarrow w_{t-1}^{(i)} \cdot \frac{p(y_t \mid x_t^{(i)}) \cdot p(x_t^{(i)} \mid x_{t-1}^{(i)})}{q(x_t^{(i)} \mid x_{t-1}^{(i)}, y_t)}$$

    end for
    Normalize weights: $w_t^{(i)} \leftarrow \frac{w_t^{(i)}}{\sum_{j=1}^{N} w_t^{(j)}}$
    Compute effective sample size:

$$N_{\text{eff}} \leftarrow \frac{1}{\sum_{i=1}^{N} (w_t^{(i)})^2}$$

    if $N_{\text{eff}} < N_{\text{threshold}}$ **then**
        Resample particles $\{x_t^{(i)}\}$ with probabilities $w_t^{(i)}$
        Reset weights: $w_t^{(i)} \leftarrow \frac{1}{N}$
    end if
end for
Return: Particles $\{x_t^{(i)}\}$ and weights $w_t^{(i)}$ for all t