

Monte Carlo Methods (Spring 2025)

Homework 10: Metropolis–Hastings

Due: Tuesday, April 8, 2025, 11:59 PM

1. (5 points) Suppose we want to find $\mathbb{P}(X > 2)$, where X is a standard normal random variable. Using the crude Monte Carlo estimator, how many samples do we need to generate so that the width of the relative 95%-confidence interval is 0.01?

2. (15 points)

- (a) Let $g(x)$ and $h(x)$ be integrable functions on $[a, b]$, and assume that $g(x)$ is strictly increasing and $h(x)$ is strictly decreasing. Show that:

$$\frac{1}{b-a} \int_a^b g(x)h(x) dx \leq \left(\frac{1}{b-a} \int_a^b g(x) dx \right) \left(\frac{1}{b-a} \int_a^b h(x) dx \right).$$

For this, consider the “difference function”

$$F(x, y) = (g(x) - g(y))(h(x) - h(y))$$

and integrate it over the region $(x, y) \in [a, b] \times [a, b]$.

- (b) Let $f(x)$ be an increasing function on $[a, b]$ and let $X \sim U(a, b)$. Show that the random variables $f(X)$ and $f(a + b - X)$ are negatively correlated.
3. (30 points) Complete the Jupyter Notebook assignment on Canvas.