

# Monte Carlo Methods Spring 2025

## Homework 06 - Markov Chains

Due: Tuesday, Mar 04, 2024, 11:59 PM

### Q.2b updated.

Textbook = Rubinstein, R. Y. and Kroese, D. P. (2017), Simulation and the Monte Carlo Method (3rd edition), Wiley, ISBN: 978-1-118-63216-1.

1. (10 points) Textbook 1.27, Pg. 44.
2. (15 points) Let  $G$  a graph. Consider the Markov chain  $X_\bullet$  for the random walk on  $G$ .
  - (a) Show that  $X_\bullet$  is irreducible if and only if  $G$  is connected.
  - (b) Suppose  $X_\bullet$  is irreducible. Show that  $X_\bullet$  is aperiodic if and only if  $G$  is non-bipartite.

You can assume the following theorem: A connected graph with at least 2 vertices is non-bipartite if and only if it contains an odd cycle starting at any vertex, where a cycle is a path on the graph that starts and ends at the same vertex and does not repeat any edge or vertex.

- (c) Show that the distribution

$$\Pi(x) = \frac{\deg(x)}{2|E|}$$

is a stationary distribution of  $X_\bullet$ , where  $\deg(x)$  is the degree of the vertex  $x$  and  $|E|$  is the number of edges in the graph.

You can assume the following theorem: The sum of the degrees of the vertices in a graph is equal to twice the number of edges in the graph.

3. (25 points) Jupyter Notebook on Canvas.