

Q.1.
Ans.:- the time complexity of the given code is $O(n^2)$ and we can improve it by using the formula $(n*(n+1))/2$.

Q.2.

Ans. $T(2) = 3T(1) + 12(2)$
 $\Rightarrow 3[3T(0) + 12(1)] + 24$
 $= 3[3 \times 5 + 12] + 24 \Rightarrow 3[15 + 12] + 24$
 $\Rightarrow 3[27] + 24 \Rightarrow 105$

Q.3.

Ans. $T(n) = T(n-1) + c$ — (1)

$$T(n-1) = T(n-2) + c$$

$$T(n) = T(n-2) + 2c$$
 — (2)

$$T(n-2) = T(n-2-1) + c$$
$$= T(n-3) + c$$

$$T(n) = T(n-3) + 3c$$
 — (3)

$$T(n) = T(n-k) + kc$$

$$n-k = 1$$

$$n = k+1$$

$$k = n-1$$

$$T(n) = T(n-(n-1)) + (n-1)c$$

$$\Rightarrow T(1) + nc - c$$

Q.4.
Ans.

$$T(n) = 16 T\left(\frac{n}{4}\right) + n^2 \log n$$

$$a=16, b=4, k=2, p=1$$

$$b^k = 4^2 \geq 16$$

$$\underline{a = b^k}$$

$$p > -1$$

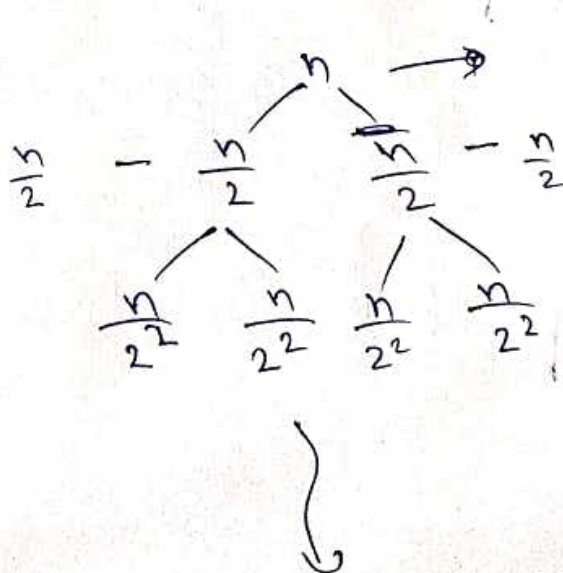
$$T(n) = \Theta(n^{\log_4 16} \log^{1+1} n)$$

$$[\because \log_4 16 = 2]$$

$$\Rightarrow \Theta(n^2 \log^2 n)$$

Q.5.
Ans.

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$



$$\frac{n}{2} + \frac{n}{2} = n$$

$$\downarrow$$

$$(n \log n)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = \log_2 2^k$$

$$\Rightarrow \log n = k \log_2 2$$

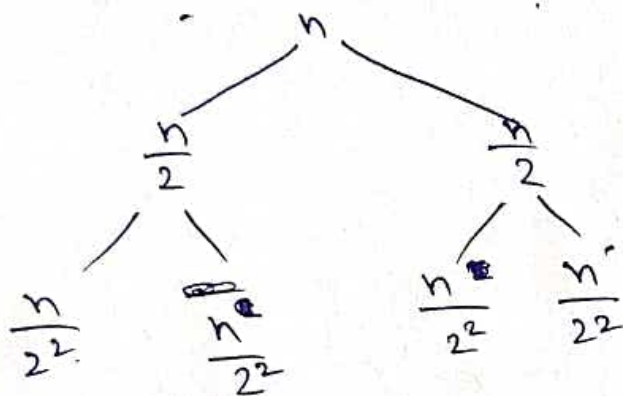
$$\Rightarrow \boxed{\log n = k}$$

$$n + n + n + \dots \rightarrow nk$$

$$n(\log n)$$

Q.6.
Ans

$$T(n) = 2 + \left(\frac{n}{2}\right) + k$$



$$\frac{n}{2^k}$$

k Constant

$$O(\log n)$$

$$\begin{array}{l} - k \\ - k \\ - k \\ \vdots \\ k \log n \end{array}$$