# The Goldbach Conjecture

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## 1 Task

Goldbach's conjecture states that every positive even number greater than 2 is the sum of 2 prime numbers. Write a program that prompts the user for a number, verifies it's valid and then finds two primes which sum to make the user's number. Add an option to find all pairs of primes for a given number.

## 2 Implementation

The code in main.cpp prompts the user for a number. I have set up a class called Goldbach which stores this number. The method 'valid' returns true is the number the user has input is even and more than 2. Once a valid input has been entered, either a single pair of primes or all pairs of primes that sum to the input number is returned in the console.

Let $n$ denote the input. My approach to this problem was as follows:

- Generate all the primes $p$ such that $2 \leq p \leq \frac{n}{2}$ and store these in a vector.
- Now for each prime $p$ that we have generated, check if $n - p$ is a prime. If so we have found a pair. Add $p$ to the output vector.
- Either stop after one pair has been found or continue depending on what the user has input.

The code for the implementation is in the method 'GoldbachConjecture' in Goldbach.cpp. There is an input integer 'flag' which tells the program whether to stop once it has found one such pair. The output vector is a vector of primes, where each element $p$ represents a pair $(p, n - p)$ where $p$ and $n - p$ are both prime.

The helper function 'IsPrime' takes an integer $x$ and a vector of primes less than $x$ to determine whether $x$ is a prime or not. As all positive integers more than 1 can be written as a product of primes, we only need to check the potential prime divisors of $x$, which are provided in the input vector, to determine whether $x$ is prime. Furthermore

if $x$ has a prime divisor $y$ such that $y > 1$ and $y \neq x$, then we must have a prime divisor $z$ such that $z \leq \sqrt{x}$, so we only have to check the prime divisors less than or equal to $\sqrt{x}$.

When checking if $n - p$ is prime, we note that $\sqrt{n} \leq \frac{n}{2}$ for $n \geq 4$, so we do not need to generate a new list of primes, as we only accept $n \geq 4$.

The provided files are: main.cpp, Goldbach.h, Goldbach.cpp.