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Assignment 1 - Linear Regression

In this assigment you will be coding for a Linear Regression task hands-on. (10 Points)

The notebook uses some popular libraries. If your environment is missing any of these libraries, you can install them using the following `pip` commands:

```
!pip install matplotlib seaborn scikit-learn
```

```
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
from sklearn.datasets import fetch_california_housing
import pandas as pd
from pandas.plotting import scatter_matrix
from scipy import stats
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import StandardScaler
```

```
#make sizes bigger for readability
import matplotlib.pyplot as plt
plt.rcParams.update({'font.size': 17})
plt.rcParams["figure.figsize"] = (12,12)
```

Load and Explore Data

```
# Load the California Housing dataset
housing = fetch_california_housing()
# Convert the dataset into a DataFrame
df = pd.DataFrame(housing.data, columns=housing.feature_names)
df['MedHouseVal'] = housing.target  # Add the target (median house value)
```

Number of Instances:

```
20640
```

Number of Attributes:

```
8 numeric, predictive attributes and the target
```

Attribute Information:

```
MedInc median income in block group

HouseAge median house age in block group

AveRooms average number of rooms per household
```

AveBedrms average number of bedrooms per household

Population block group population

AveOccup average number of household members

Latitude block group latitude

Longitude block group longitude

```
1 display(df)
```

	MedInc	HouseAge	AveRooms	AveBedrms	Population	AveOccup	Latitude	Longitude	MedHouseVal
0	8.3252	41.0	6.984127	1.023810	322.0	2.555556	37.88	-122.23	4.526
1	8.3014	21.0	6.238137	0.971880	2401.0	2.109842	37.86	-122.22	3.585
2	7.2574	52.0	8.288136	1.073446	496.0	2.802260	37.85	-122.24	3.521
3	5.6431	52.0	5.817352	1.073059	558.0	2.547945	37.85	-122.25	3.413
4	3.8462	52.0	6.281853	1.081081	565.0	2.181467	37.85	-122.25	3.422
...	...	...	...	...	...	...	...	...	...
20635	1.5603	25.0	5.045455	1.133333	845.0	2.560606	39.48	-121.09	0.781
20636	2.5568	18.0	6.114035	1.315789	356.0	3.122807	39.49	-121.21	0.771
20637	1.7000	17.0	5.205543	1.120092	1007.0	2.325635	39.43	-121.22	0.923
20638	1.8672	18.0	5.329513	1.171920	741.0	2.123209	39.43	-121.32	0.847
20639	2.3886	16.0	5.254717	1.162264	1387.0	2.616981	39.37	-121.24	0.894

20640 rows × 9 columns

Next steps: Generate code with df | View recommended plots | New interactive sheet

```
1 #Explore data for missingness
2 print(df.info())
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 20640 entries, 0 to 20639
Data columns (total 9 columns):
 #   Column       Non-Null Count  Dtype  
---  ------       --------------  -----  
 0   MedInc       20640 non-null  float64
 1   HouseAge     20640 non-null  float64
 2   AveRooms     20640 non-null  float64
 3   AveBedrms    20640 non-null  float64
 4   Population   20640 non-null  float64
 5   AveOccup     20640 non-null  float64
 6   Latitude     20640 non-null  float64
 7   Longitude    20640 non-null  float64
 8   MedHouseVal  20640 non-null  float64
dtypes: float64(9)
memory usage: 1.4 MB
None
```

```
1 #Check statistics of the data
2 print(df.describe())
```

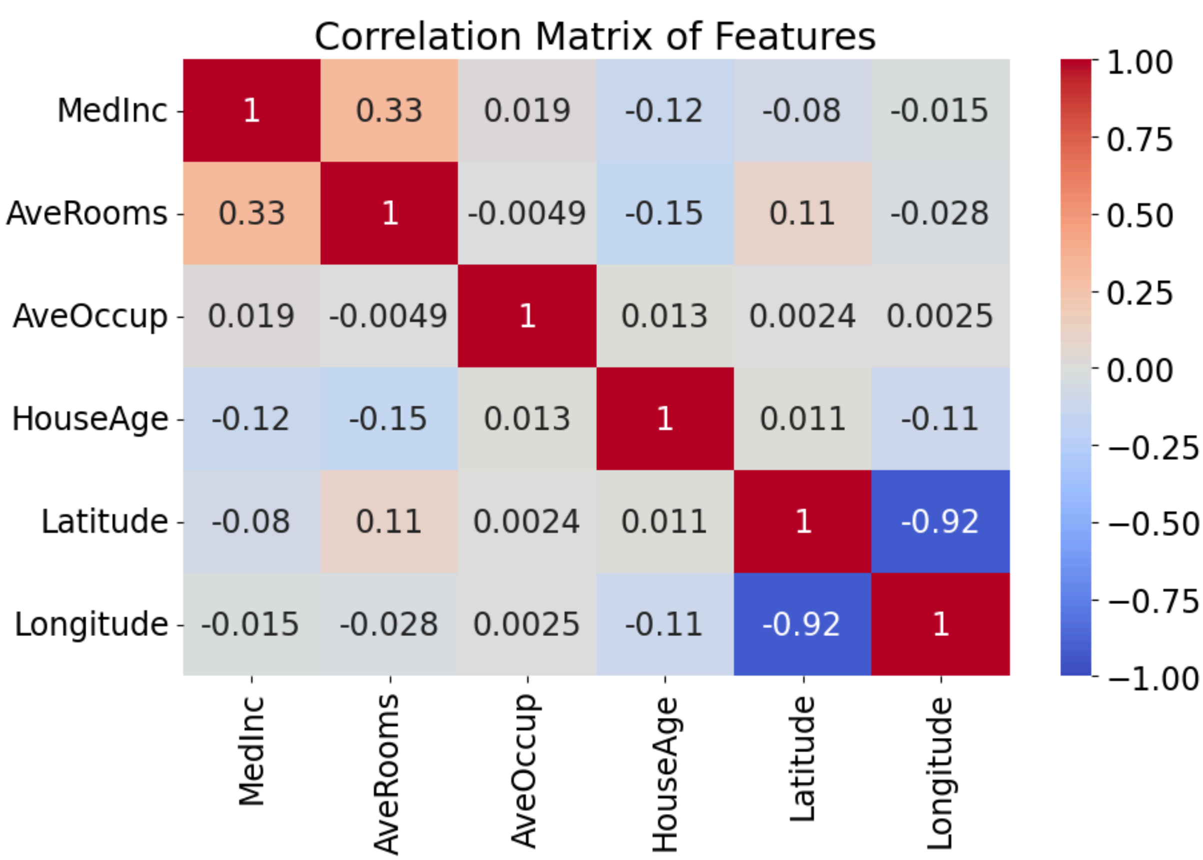
```
             MedInc      HouseAge      AveRooms     AveBedrms    Population  \
count  20640.000000  20640.000000  20640.000000  20640.000000  20640.000000   
mean       3.870671     28.639486      5.429000      1.096675   1425.476744   
std        1.899822     12.585558      2.474173      0.473911   1132.462122   
min        0.499900      1.000000      0.846154      0.333333      3.000000   
25%        2.563400     18.000000      4.440716      1.006079    787.000000   
50%        3.534800     29.000000      5.229129      1.048780   1166.000000   
75%        4.743250     37.000000      6.052381      1.099526   1725.000000   
max       15.000100     52.000000    141.909091     34.066667  35682.000000   

           AveOccup      Latitude     Longitude   MedHouseVal  
count  20640.000000  20640.000000  20640.000000  20640.000000  
mean       3.070655     35.631861   -119.569704      2.068558  
std       10.386050      2.135952      2.003532      1.153956  
min        0.692308     32.540000   -124.350000      0.149990  
```

```
25%        2.429741     33.930000   -121.800000      1.196000
50%        2.818116     34.260000   -118.490000      1.797000
75%        3.282261     37.710000   -118.010000      2.647250
max     1243.333333     41.950000   -114.310000      5.000010
```

```python
# Display the first few rows
print(df.head())
```

```
   MedInc  HouseAge  AveRooms  AveBedrms  Population  AveOccup  Latitude  \
0  8.3252      41.0  6.984127   1.023810       322.0  2.555556     37.88
1  8.3014      21.0  6.238137   0.971880      2401.0  2.109842     37.86
2  7.2574      52.0  8.288136   1.073446       496.0  2.802260     37.85
3  5.6431      52.0  5.817352   1.073059       558.0  2.547945     37.85
4  3.8462      52.0  6.281853   1.081081       565.0  2.181467     37.85

   Longitude  MedHouseVal
0    -122.23        4.526
1    -122.22        3.585
2    -122.24        3.521
3    -122.25        3.413
4    -122.25        3.422
```

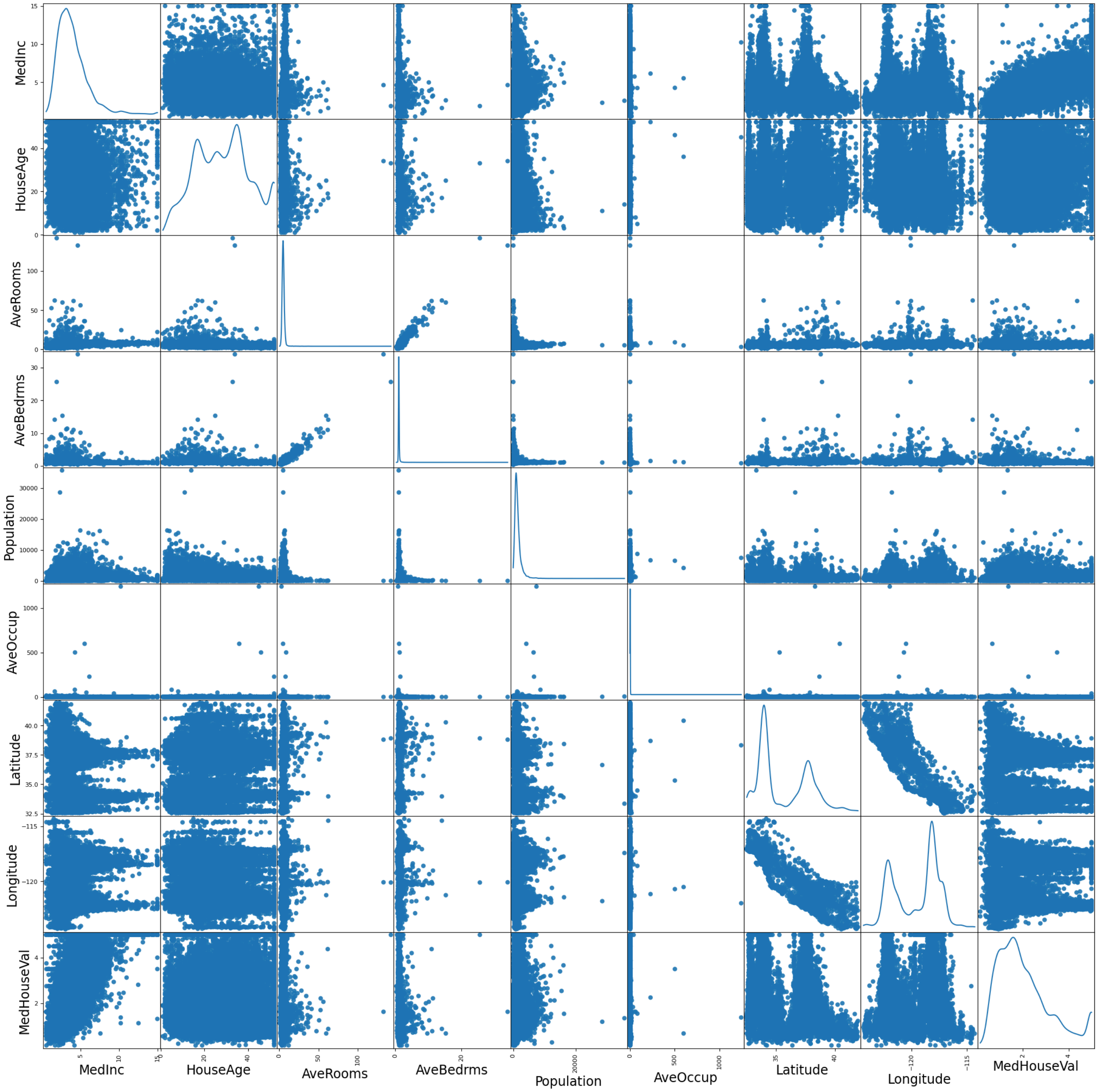
```python
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from statsmodels.stats.outliers_influence import variance_inflation_factor

# Select multiple features for the correlation check
X_all = df[['MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude', 'Longitude']]

# Calculate correlation matrix
corr_matrix = X_all.corr()

# Visualize the correlation matrix
plt.figure(figsize=(10, 6))
sns.heatmap(corr_matrix, annot=True, cmap='coolwarm', vmin=-1, vmax=1)
plt.title('Correlation Matrix of Features')
plt.show()

# Note that correlation between Latitude and Longitude is coming from geographical location of California
```




```python
#display scatter_matrix also
```

```
fig = plt.figure()
scatter_matrix(df,figsize =(25,25),alpha=0.9,diagonal="kde",marker="o");
```

```
<Figure size 1200x1200 with 0 Axes>
```



Relevant Metrics

1. Residual Sum of Squares (RSS)

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Where:

$$y_i \text{ is the actual value}$$
$$\hat{y}_i \text{ is the predicted value}$$
$$n \text{ is the number of observations}$$

---

2. Residual Standard Error (RSE)

$$RSE = \sqrt{\frac{RSS}{n-p-1}}$$

Where:

$$RSS \text{ is the Residual Sum of Squares}$$
$$n \text{ is the number of observations}$$
$$p \text{ is the number of predictors (excluding the intercept)}$$

---

3. t-statistic

$$t = \frac{\hat{\beta}_j}{SE(\hat{\beta}_j)}$$

Where:

$$\hat{\beta}_j \text{ is the estimated coefficient for predictor } j$$
$$SE(\hat{\beta}_j) \text{ is the standard error of the estimated coefficient for predictor } j$$

---

4. p-value

$$p = 2 \cdot (1 - T(|t|, df))$$

Where:

$$t \text{ is the t-statistic}$$
$$df \text{ is the degrees of freedom, calculated as } n-p-1$$
$$T \text{ is the CDF of the t-distribution}$$

Task 1: Fill the missing parts (#TODO) of metric computations (1 Point Each, 3 Points)

```
def compute_rss(y, y_pred):
    """
```

```
    Compute Residual Sum of Squares (RSS)
    y: array of true target values
    y_pred: array of predicted target values
    """
    # diff = y - y_pred
    # return diff.T @ diff
    return np.sum((y - y_pred)**2)

def compute_rse(y, y_pred, n, p):
    """
    Compute Residual Standard Error (RSE)
    y: array of true target values
    y_pred: array of predicted target values
    n: number of observations
    p: number of predictors
    """
    rss = compute_rss(y, y_pred)

    return np.sqrt(rss / (n - p - 1))

def compute_pvalue(X, y, y_pred):
    '''
    Compute p-values for the coefficients of a linear regression model.

    X: array of features
    y: array of true target values
    y_pred: array of predicted target values
    return: p-values for each feature
    '''
    n, p = X.shape  # Number of observations (n) and number of predictors (p)

    # Compute RSS and RSE
    rss = compute_rss(y, y_pred)
    rse = compute_rse(y, y_pred, n, p)

    # # Add intercept (constant term) to the design matrix X
    X = np.c_[np.ones(n), X]

    # Calculate (X^T X)^-1
    XTX_inv = np.linalg.inv(np.dot(X.T, X))

    # Compute standard error (SE) for each coefficient
    se = np.sqrt(np.diagonal(rse ** 2 * XTX_inv))

    # Fit the model to compute the coefficients (betas)
    beta_hat = np.linalg.lstsq(X, y, rcond=None)[0]

    # Compute t-statistics for each coefficient
    t_stats = beta_hat / se

    degrees_of_freedom = n - p - 1

    # Compute p-values
    p_values = 2 * (1 - stats.t.cdf(np.abs(t_stats), df=degrees_of_freedom))

    return p_values

```

Linear Regression with single predictor

```
# Select features and target
X = df[['AveRooms']]
#z-normalize the data for each column
X = (X - X.mean()) / X.std()
y = df['MedHouseVal']

# Split the data into training and testing sets (80% training, 20% testing) with a fixing seed that ensures same split every
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)

independent_scaler = StandardScaler()
X_train = independent_scaler.fit_transform(X_train)
X_test = independent_scaler.transform(X_test)

# Create a linear regression model
```

```
model_1 = LinearRegression()

# Train the model
model_1.fit(X_train, y_train)

# Get the coefficients
print(f"Intercept (β0): {model_1.intercept_}")
print(f"Coefficients (β1, β2): {model_1.coef_}")

#Compute RSS for training data
y_pred = model_1.predict(X_train)

# Compute RSS
rss = compute_rss(y_train, y_pred)

# Calculate R-squared
r_squared_all = model_1.score(X_train, y_train)

# Compute the p-value
p_value = compute_pvalue(X_train, y_train, y_pred)

# Display the coefficients and p-values in a DataFrame
coefficients = np.concatenate([[model_1.intercept_], model_1.coef_])
p_values = np.concatenate([ p_value])

display(pd.DataFrame(pd.DataFrame({'features': ['intercept'] + list(X.columns), 'coefficients': coefficients, 'p-values': p_v
print(f"RSS (test data): {rss}")
print(f"R-squared (test data): {r_squared_all}")
```

```
Intercept (β0): 2.071946937378876
Coefficients (β1, β2): [0.18323882]
```

	features	coefficients	p-values
0	intercept	2.071947	0.0
1	AveRooms	0.183239	0.0

```
RSS (test data): 21518.467257765213
R-squared (test data): 0.025117453148833846
```

Task 2: Use 'MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude', 'Longitude' as predictors. (2 Points)

```
X_all = df[['MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude', 'Longitude']]
y = df['MedHouseVal']

# Split the data into training and testing sets (80% training, 20% testing) with a fixing seed that ensures same split every
X_train_all, X_test_all, y_train_all, y_test_all = train_test_split(X_all, y, test_size=0.2, random_state=42)
independent_scaler = StandardScaler()
X_train_all = independent_scaler.fit_transform(X_train_all)
X_test_all = independent_scaler.transform(X_test_all)

# Fit the linear regression model
model_2 = LinearRegression()
model_2.fit(X_train_all, y_train_all)

# Predictions on the test set
y_pred_all = model_2.predict(X_test_all)

#Code this part
rss = compute_rss(y_test_all, y_pred_all)

# Calculate R-squared
r_squared_all = model_2.score(X_test_all, y_test_all)

# Compute the p-value
p_value = compute_pvalue(X_test_all, y_test_all, y_pred_all)

# Display the coefficients and p-values in a DataFrame
coefficients = np.concatenate([[model_2.intercept_], model_2.coef_])
p_values = np.concatenate([ p_value])

# pd.DataFrame({'features': ['intercept'] + list(X_all.columns), 'coefficients': coefficients, 'p-values': p_values})
display(pd.DataFrame({'features': ['intercept'] + list(X_all.columns), 'coefficients': coefficients, 'p-values': p_values}))
print(f"RSS (test data): {rss}")
print(f"R-squared (test data): {r_squared_all}")

```

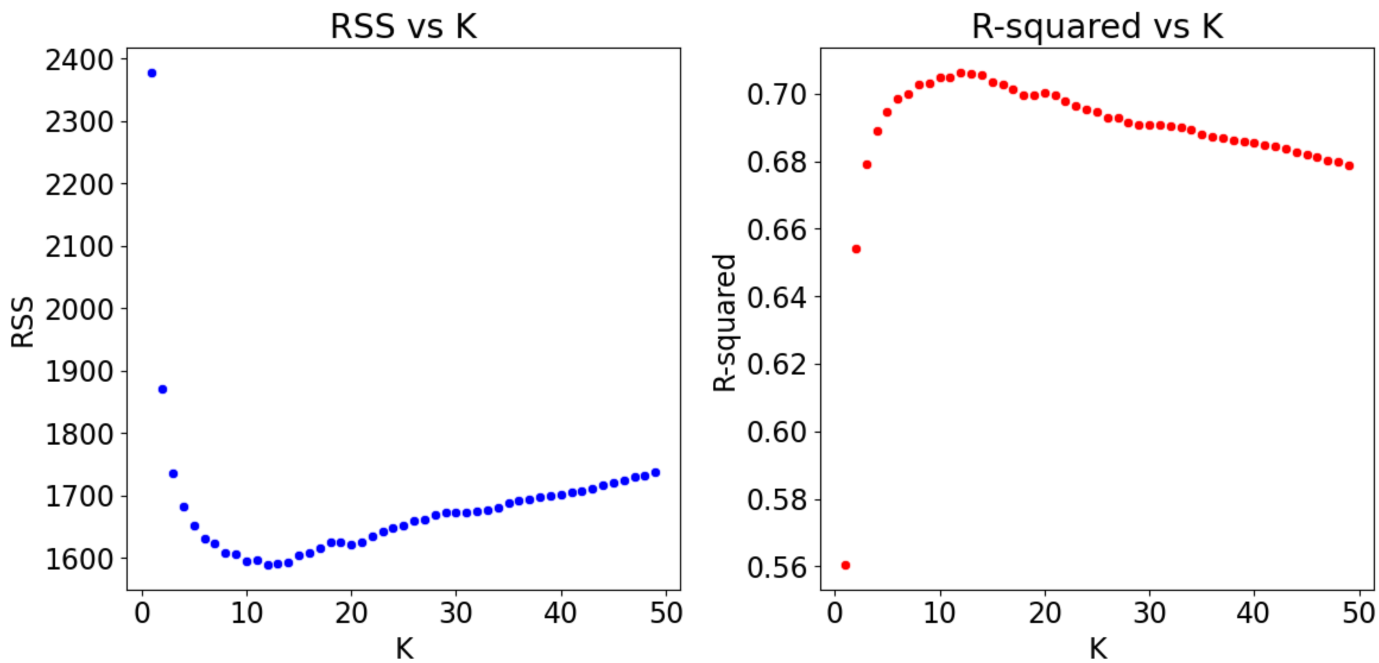
	features	coefficients	p-values
0	intercept	2.071947	0.000000
1	MedInc	0.708366	0.000000
2	AveRooms	0.045937	0.010799
3	AveOccup	-0.037746	0.000000
4	HouseAge	0.124500	0.000000
5	Latitude	-0.977368	0.000000
6	Longitude	-0.931079	0.000000

```
RSS (test data): 2259.450956262675
R-squared (test data): 0.5823077951522642
```

Task 3: Try model performance on different K values by using the code below, observe the effect of very large K values which one would you pick? (3 Points)

```
from sklearn.neighbors import KNeighborsRegressor
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error, r2_score

# ADDED CODE FOR VISUALIZATION PURPOSES
MAX_K = 50
results = []

X_all = df[['MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude', 'Longitude']]
X_all = (X_all - X_all.mean()) / X_all.std()
y = df['MedHouseVal']

# Split the data into training and testing sets (80% training, 20% testing) with a fixing seed that ensures same split every
X_train_all, X_test_all, y_train_all, y_test_all = train_test_split(X_all, y, test_size=0.2, random_state=42)
independent_scaler = StandardScaler()
X_train_all = independent_scaler.fit_transform(X_train_all)
X_test_all = independent_scaler.transform(X_test_all)

# WE ADDED A LOOP HERE SO THAT IT IS EASIER TO TRY DIFFERENT k VALUES

for k in range(1, MAX_K):
  #Fit the KNN model (you can tune 'n_neighbors' for optimal performance)
  knn_model = KNeighborsRegressor(n_neighbors=k)
  knn_model.fit(X_train_all, y_train_all)

  #Make predictions on the test set
  y_pred_knn = knn_model.predict(X_test_all)

  #Compute RSS and R-squared
  rss_knn = compute_rss(y_test, y_pred_knn)
  r2_knn = r2_score(y_test_all, y_pred_knn)

  # COMMENTED OUT JUST SO THAT THE GRAPHS WOULD BE MORE VISIBLE
  # print(f"KNN Model RSS: {rss_knn}")
  # print(f"KNN Model R-squared: {r2_knn}")

  results.append((k, rss_knn, r2_knn))

# CONVERT THE RESULTS TO A DATAFRAME FOR BETTER VISUALIZATION
results_df = pd.DataFrame(results, columns=['K', 'RSS', 'R-squared'])
fig, axes = plt.subplots(1, 2, figsize=(12, 6), sharex=True)

sns.scatterplot(x=results_df['K'], y=results_df['RSS'], color='blue', ax=axes[0])
axes[0].set_title("RSS vs K")
axes[0].set_ylabel("RSS")

sns.scatterplot(x=results_df['K'], y=results_df['R-squared'], color='red', ax=axes[1])
axes[1].set_title("R-squared vs K")
axes[1].set_ylabel("R-squared")

for ax in axes:
    ax.set_xlabel("K")

plt.tight_layout()
```

```
plt.show()

# PRINT THE BEST VALUE OF K
print(f"\nThe optimal K is: {results_df.loc[results_df['RSS'].idxmin()]['K']}")
```




```
The optimal K is: 12.0
```

The RSS and R-squared values for various values of k for k-NN are listed below. Both from the table and the generated graphs, we can see that $k = 12$ is the optimal solution to our problem.

K	RSS	R-squared
1	2377.7160159583	0.560444787502643
4	1682.3242720372937	0.6889980140934615
8	1608.2738669443797	0.702687303027747
12	1588.7586109921492	0.7062949804877341
14	1592.3531727871673	0.7056304737245043
18	1625.2111634761068	0.6995561986712271

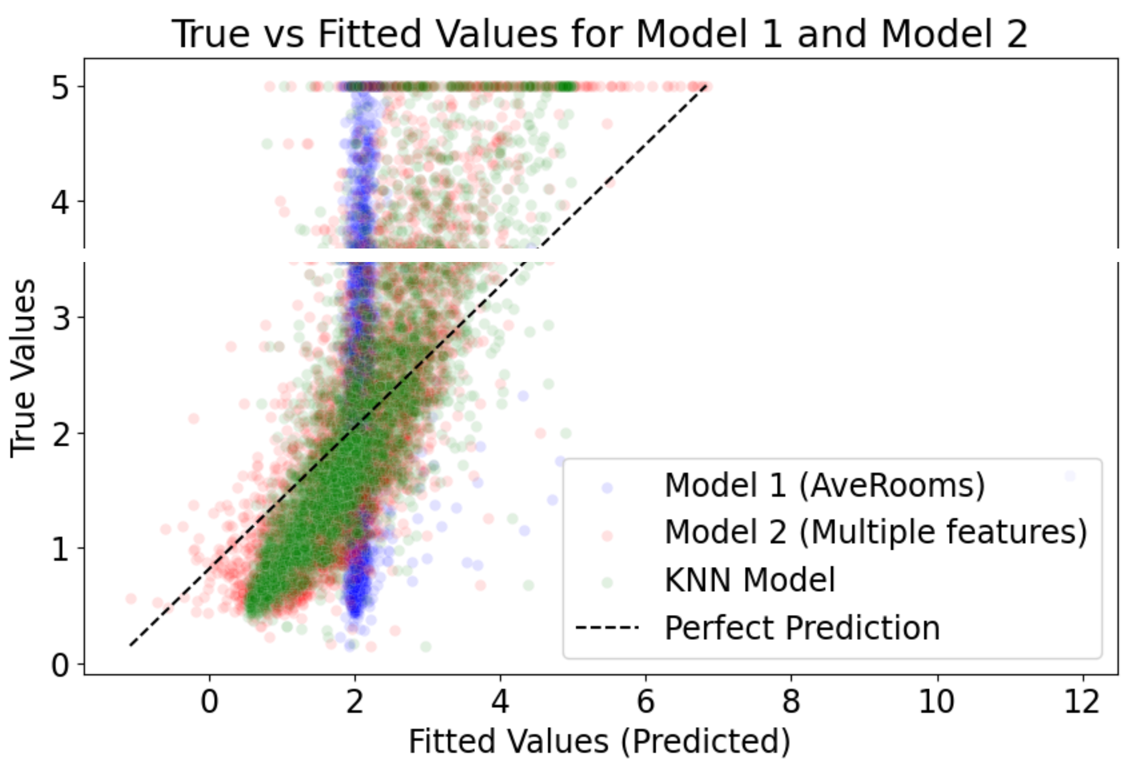
Task 4: Comment on R-squared and RSS values (1 Point)

We can see that the performance of the model on the test set increases as we increase k up to a certain point, $k = 12$, then again it starts to decrease. For values that are smaller than our chosen k, the model captures details and noise in the training data. Therefore the model overfits the training dataset and fails to generalize to unseen data. On the other hand, as k increases beyond our optimal value, the model begins to average over more neighbors, smoothing out the data patterns. This causes the model to underfit the data and as a result, fail to generalize to the test set. This means a higher RSS and lower R^2 value.

Visualize results

```
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.metrics import mean_squared_error


# Make predictions on the test set for the first model (only AveRooms)
y_pred = model_1.predict(X_test)

# Make predictions on the test set for the second model (multiple features)
y_pred_all = model_2.predict(X_test_all)
```

```python

# Make predictions using the KNN model
y_pred_knn = knn_model.predict(X_test_all)  # Use scaled features for KNN

plt.figure(figsize=(10, 6))

# Model 1: True vs Fitted (only AveRooms)
sns.scatterplot(x=y_pred, y=y_test, color='blue', label='Model 1 (AveRooms)', alpha=0.1)

# Model 2: True vs Fitted (multiple features)
sns.scatterplot(x=y_pred_all, y=y_test_all, color='red', label='Model 2 (Multiple features)', alpha=0.1)

# KNN Model: True vs Fitted
sns.scatterplot(x=y_pred_knn, y=y_test_all, color='green', label='KNN Model', alpha=0.1)

# Add perfect prediction line
plt.plot([min(y_pred_all), max(y_pred_all)], [min(y_test_all), max(y_test_all)], color='black', linestyle='--', label='Perf

# Labels and title
plt.xlabel('Fitted Values (Predicted)')
plt.ylabel('True Values')
plt.title('True vs Fitted Values for Model 1 and Model 2')
plt.legend()
plt.show()
```



Task 5: Compute residuals (1 Point)

```python
### 2. Residuals vs Fitted

# Compute residuals
residuals_model_1 = y_test - y_pred
```