Assignment #2

Elements of Machine Learning

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Problem 3 (Linear & Quadratic Discriminate Analysis)

0.1 a)

Let's first define the datapoints for each of the two classes.

Table 1: Data for x_1, x_2, and their respective classes.

x_1	x_2	**Class**
1	1	0
2	1	0
3	2	0
2	3	0
1	3	0
7	1	1
5	2	1
6	4	1
4	5	1
6	5	1

First, let's calculate the mean of each class. The mean of the first class μ_0 would be:

$$\mu_0 = \frac{1}{5}\sum_{i=1}^{5} X_i = \frac{1}{5}\left(\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}3\\2\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}1\\3\end{bmatrix}\right) = \begin{bmatrix}1.8\\2\end{bmatrix} \tag{1}$$

Similarly, the mean of the second class μ_1 would be:

$$\mu_1 = \frac{1}{5}\sum_{i=6}^{10} X_i = \frac{1}{5}\left(\begin{bmatrix}7\\1\end{bmatrix} + \begin{bmatrix}5\\2\end{bmatrix} + \begin{bmatrix}6\\4\end{bmatrix} + \begin{bmatrix}4\\5\end{bmatrix} + \begin{bmatrix}6\\5\end{bmatrix}\right) = \begin{bmatrix}5.6\\3.4\end{bmatrix} \tag{2}$$

The covariance matrix for two predictors x_1 and x_2 and a class k is defined as follows:

$$\Sigma_k = \begin{bmatrix} var(x_1^k) & cov(x_1^k, x_2^k) \\ cov(x_1^k, x_2^k) & var(x_2^k) \end{bmatrix} \tag{3}$$

First, let's calculate the covariance matrix for class $k = 0$. The covariance matrix for class $k = 0$ would be:

$$\Sigma_0 = \begin{bmatrix} \mathbb{E}[(x_1^0)^2] - \mathbb{E}[x_1^0]^2 & \mathbb{E}[x_1^0 x_2^0] - \mathbb{E}[x_1^0]\mathbb{E}[x_2^0] \\ \mathbb{E}[x_1^0 x_2^0] - \mathbb{E}[x_1^0]\mathbb{E}[x_2^0] & \mathbb{E}[(x_2^0)^2] - \mathbb{E}[x_2^0]^2 \end{bmatrix} \tag{4}$$

Calculating the expectations and substituting the values, we get:

$$\Sigma_0 = \begin{bmatrix} 0.7 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

Following a similiar aproach, we can calculate the covariance matrix for class $k = 1$. The reuslting covariance matrix would be:

$$\Sigma_1 = \begin{bmatrix} 1.3 & -1.05 \\ -1.05 & 3.3 \end{bmatrix} \tag{6}$$

NOTE: Fix the computations for the covariance matrices.

0.2 b)

In order to determin in which class the new data point is going to be classified, we need to calculate the discriminant function for each class. The discriminant function for class k is defined as follows:

$$\delta_k(x) = x^T \Sigma_k^{-1} \mu_k - \frac{1}{2} \mu_k \Sigma_k^{-1} \mu_k + \log(\pi_k) \tag{7}$$