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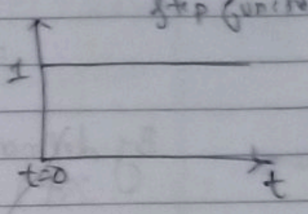
ASSIGNMENT - 2

1) Evaluate fourier transform of unit step, Delta & gate function

→ (i) Step function:

The step signal is defined as



$x(t) = 1 \quad t \geq 0$
$\quad = 0 \quad t < 0$

from definition of fourier transform

$X(f) = \int_{-\infty} x(t)\, e^{-j\omega t}\, dt$

$= \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t}\, dt$

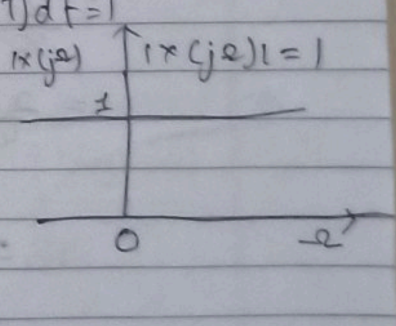
$= \int_{0}^{\infty} 1 \cdot e^{-j\omega t}\, dt$

$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_0^{\infty}$

$= 0 - \frac{(-e^0)}{j\omega} = \frac{1}{j\omega}$

fourier transform of unit step is $\frac{1}{j\omega}$

(ii) $x(t) = \delta(t) = \infty : t = 0$ & $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$
$\quad = 0 \; ; \; t \neq 0$



By defination of fourier transform

$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$

$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\Omega t}\, dt$

$= 1 \times e^{-j\Omega t} \Big|_{t=0}$ —— $\because \delta(t)$ exists only for $t = 0$

$= 1 \times e^{0}$

$= 1$

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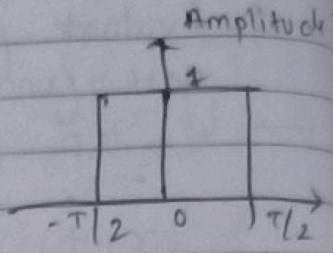
iii) Gate function is defined as

$$x(t) = 0 \quad t < -T/2$$
$$= 1 \quad -T/2 < t < T/2$$
$$= 0 \quad t > T/2$$



Q.

By defination of fourier transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{T/2} 1 \; e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]^{T/2}$$

$$= \frac{1}{-j\omega} \left\{ e^{-j\omega T/2} - e^{j\omega T/2} \right\}$$

$$= \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\omega}$$

$$= \frac{2}{\omega} \left\{ \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j2} \right\} \quad \ldots \{\text{multiplying \& dividing by } 2$$

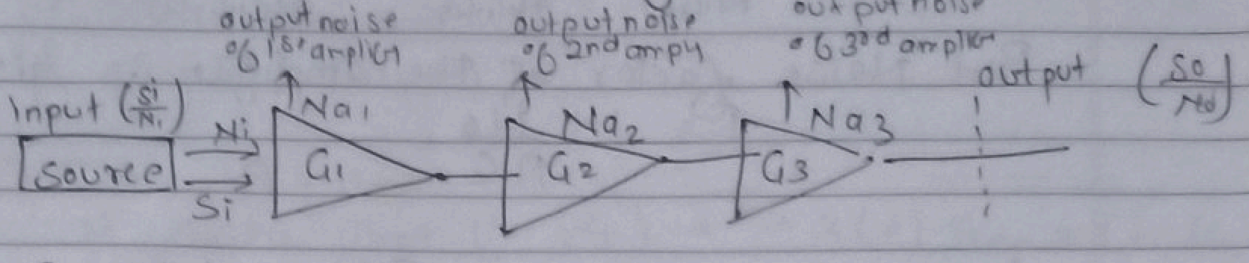
$$= \frac{2}{\omega} \sin\left(\frac{\omega T}{2}\right)$$

$$= \frac{2}{\omega} \cdot \frac{T}{2} \frac{\sin(\omega T/2)}{T/2} \quad \text{— \{multiplying \& div by } T/2$$

$$= T \cdot \frac{\sin(\omega T/2)}{\omega t/2}$$

$$= T \operatorname{sinc}\left(\frac{\omega t}{2}\right) \quad - \left\{ \operatorname{sinc} x = \frac{\sin x}{x} \right.$$

Q.2 Explain friss Transmission formula



Cascaded amplifier stage

- Let the input signal power be S_i & noise power be N_i. Similary let output signal power be S_0 and Noise power be N_0
- Hence input $SNR = \frac{S_i}{N_i}$
- Now signal power $S_0 = S_i \cdot G_1 G_2 \cdot G_3$ —— ①
- where G_1, G_2 & G_3 are gain of amplifier of stage 1, 2, 3 and 3 respectively
- Now output noise power is given by,

$$N_0 = N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3} \quad \text{—— ②}$$

- from equation ① & ② we get,

output $(SNR) = [SNR]_0 = \frac{S_0}{N_0}$

$$\therefore \frac{S_0}{N_0} = \frac{S_i G_1 G_2 G_3}{N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3}} \quad \text{—— ③}$$

Now total noise factor is given as

$$F_{total} = \frac{S_i/N_i}{S_0/N_0}$$

$$= \frac{S_i/N_i}{\frac{S_i G_1 G_2 G_3}{N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3}}}$$

$$= 1 + \frac{Na_1}{N_i G_1} + \frac{Na_2}{N_i G_1 G_2} + \frac{Na_3}{N_i G_1 G_2 G_3} \quad \text{——— (4)}$$

But Noise factor of the amplifier is given a

$$F = 1 + \frac{Na}{N_i G}$$

Substituting in equation (4) we get,

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

This is friss formula for 3 stages

- Friis's formula is used to calculate the total noice factor of Cascade of stages, each with its own noise factor and power. The total noise factor can be used to calculate total noise figure. The total noise factor is given as for n^{a} stages

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots \frac{F_n}{G_1 G}$$

Where F_i & G_i are the noise factor and avo power gain respectively of the ith stage, B magnitude are expressed as ratios not in decibles

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Sundaram

Q3 State and prove following properties of fourier transform
1. Time shifting
2. frequency shifting

1) Time Shifting
It states that if $x(t) \xleftrightarrow{F} X(f)$ then $x(t-td)$
$\xleftrightarrow{F} e^{-j2\pi f td} X(f)$
here, the signal $x(t-td)$ is a time shifted signal

proof: Defination of fourier transform is
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$
$$= \int_{-\infty}^{\infty} x(t-td) e^{-j\omega t} dt \quad \text{—} \; 1$$

put $t - td = \tau$
$t = \tau + td$
$dt = d\tau$
Substituting in equation 1
$$X(f) = F(x(t-td)) = \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f(td-} d\tau$$
$$= e^{-j2\pi f td} \int_{-\infty}^{\infty} x(\tau) e \cdot d\tau$$
$$= e^{-j2\pi f td} X(f)$$
Hence proved

Thus shift in time correspond to a rotation in the frequency domain

2) frequency shifting property
it state that if $x(t) \xleftrightarrow{F} x(f)$ then $x(t)$
$e^{j2\pi f_0 t} \xleftrightarrow{F} x(f-f_0)$
Here $(f-f_0)$ is a frequency shifted property
proof: Defination of Inverse fourier transform

$$x(t) = \int_{-\infty}^{\infty} x(f)\, e^{j\omega t}\, df$$

$$= \int_{-\infty}^{\infty} x(f-f_0)\, e^{j2\pi f t}\, df$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{j2\pi (f_0+\tau) t}\, d\tau$$

where

$f-f_0 = \tau$
$\therefore f = f_0 + \tau$
$\therefore df = d\tau$

Hence proved

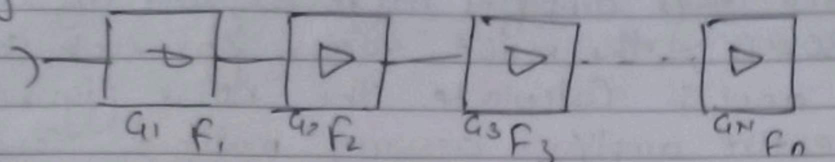
Q4) Define Signal to noise ratio - Appy the Concept to explain the effect of cascade connection on a Signal to Noise ratio

→ i) signal to Noise ratio is defined as the ra[tio] of signal power to noise power at the s[ame] point. It is represented as.

$$\frac{S}{N} = \frac{P_S}{P_N} = \frac{V_S^2/R}{V^2_N/R} = \frac{V_S^2}{V_N^2}$$

$$\frac{S}{N}(dB) = 10 \log \frac{P_S}{P_N} = 10 \log \frac{V_S^2}{V^2_N} = 20l$$

ii]



A multistage amplifier circuit is often used to achieve the required gain. in this scheme the amplifier is a series of seperate amplifier module or stages connected in series. Each amplification stage, like any active RF component has its own noise level, which is characterize by a noise figure. The main contribution to the noise of the amplifier stage is its thermal noise, the source of which is the chaos movement of the electric charge carrier, the intensity of which depends on the temperature of the amplifying elements of the stage, The noise figure as defined as the value of the signal to noise ration at the input of the cascade divided by the signal to the noise ration and it output

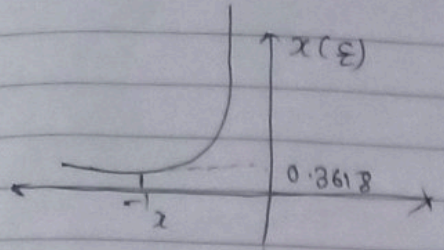
The noise from the first stage will be amplified in the second stage and the noise from the second stage will be added to this stage. the formula for determing the noise figure F of two-stage amplier is as follows

$$F = F_1 + (F_2 - 1) / G_1$$

for more no of amplifier

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

Q6)



pulse shown can be represented as $x(t) = e^{\alpha t}$ $t \le 0$
$= 0$ $t > 0$

It can also be represented as

$$x(t) = e^{\alpha t} u(-t)$$

$$x(f) = \int_{-\infty}^{\infty} e^{\alpha t} u(-t) e^{j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{\alpha t} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{(\alpha - j\omega)t}\, dt$$

$$= \frac{1}{\alpha - j2\pi f} \left[e^{(\alpha - j2\pi f)t} \right]_{-\infty}^{0}$$

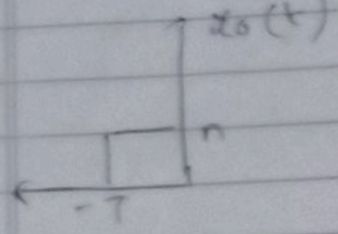
$$= \frac{1}{\alpha - j2\pi f} \left[e^{0} - e^{-\infty} \right]$$

$$= \frac{1}{\alpha - 2\pi j f} (1 - 0) = \frac{1}{\alpha - 2j\pi f}$$

$$\therefore e^{\alpha t} u(-t) \xleftrightarrow{F} \frac{1}{\alpha - 2\pi j f}$$

Q7)

Q7



This advanced Version of the set'd rectangle pulse with an advance of T/2 Hence using time Shifting property

$X_2(f) = e^{-j2\pi f t_d} x(f)$

$= e^{-j2\pi f (T/2)}$ AT $gain e(ft)$

$= AT\ e^{j2\pi f t}$ AT $gain c(ft)$

$= At\ e^{j\pi f t}$ $gain\ c(ft)$

Given

stage	Power gain	Noise
1	10	2
2	20	4
3	30	5

Given

Fo

$G_1 = 10\ dB = 10 \log_{10}(10) = 1(10) = 10$

$G_2 = 20\ dB = 10 \log_{10}(20) = 1.301(10) = 13.01$

$G_3 = 30\ dB = 10 \log_{10}(30) = 1.477(10) = 14.7$

Noise factor is

$F_1 = 2$, $F_2 = 4$, $F_3 = 5$

overall factor ~~given~~ gain = $G_1 G_2 G_3 = 10(13.01)(14.77)$

$= 1921.577$

overall Noise figure is

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{F_3 + 1}{G_1 G_2}$$

$$F = 2 + \frac{(4-1)}{10} + \frac{(5-1)}{13.01}$$

$F = 2.3307$

$\therefore$ Overall Noise figure $F_{dB} = 10 \log_{10}(2.3307)$

$= \underline{\underline{3.67}}$