Module 2
Noise and Fourier Representation of Signal and System

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Objectives

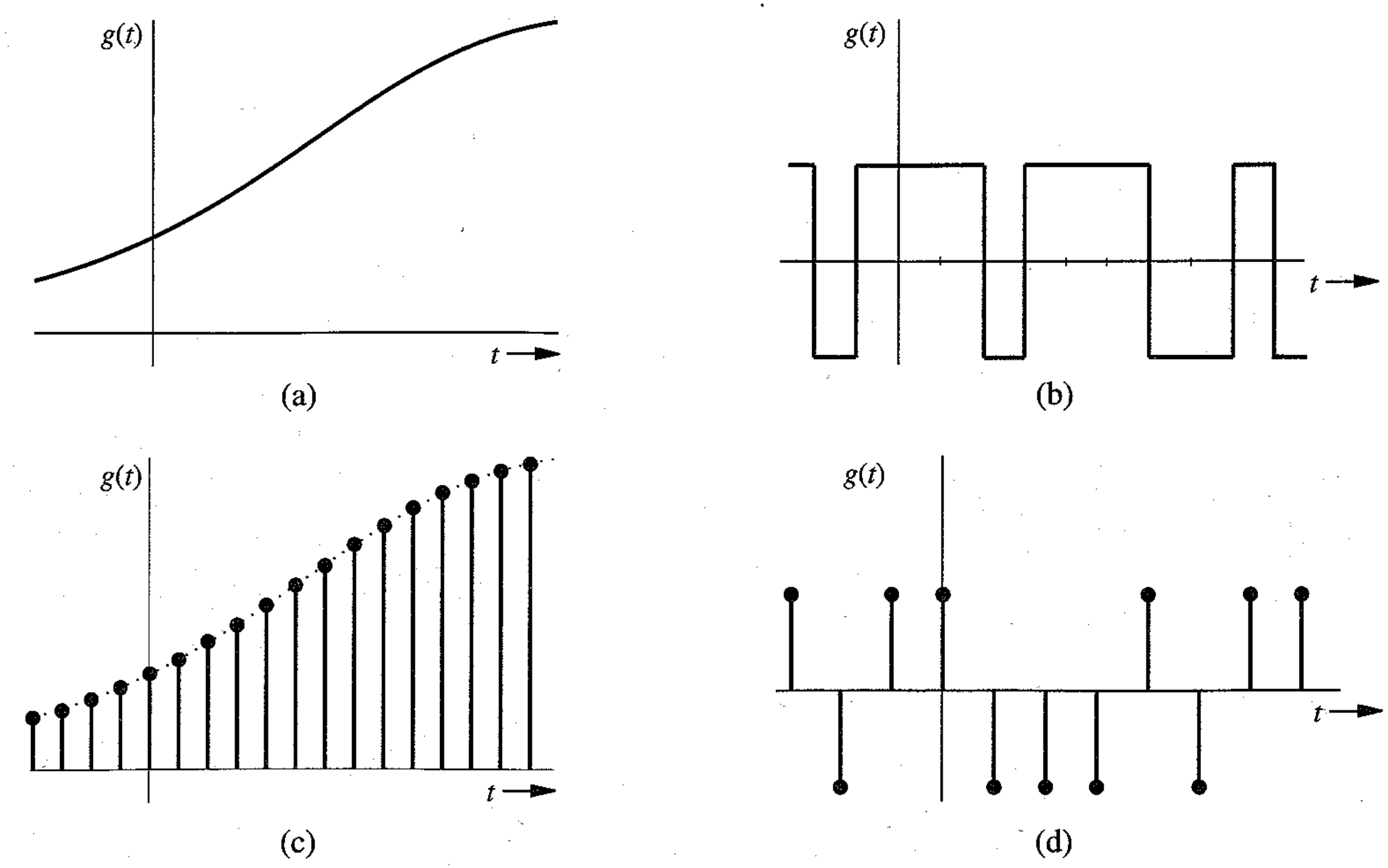
- **Basics of signal representation & analysis**
- **Fourier Transform**
- **Noise parameters**

Classification of Signals

Basically Signals can be classified as:

- Continuous time and Discrete time
- Analog and digital signals
- Periodic and Aperiodic signals
- Energy and Power signals
- Deterministic and Probabilistic signals

Signals Examples

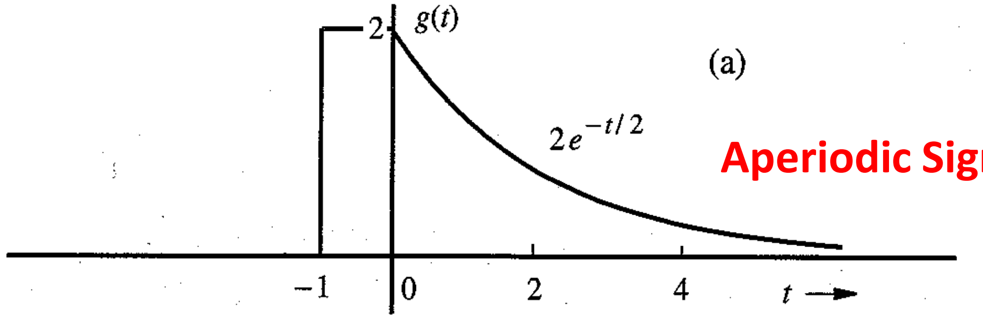


Examples of signals. (a) Analog, continuous time. (b) Digital, continuous time. (c) Analog, discrete time. (d) Digital, discrete time.

Periodic and Aperiodic Signals

A signal g(t) is said to be periodic if for some positive constant T_0

$$g(t) = g(t+m\,T_0) \quad \text{for all t.}$$

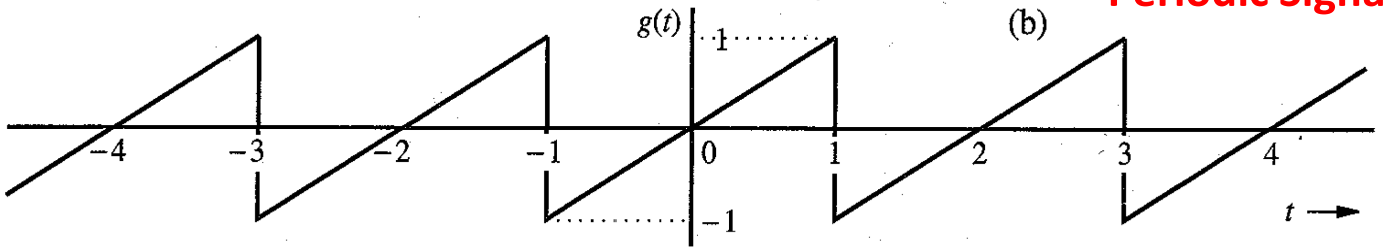


Aperiodic Signal

The smallest value of T_0, that satisfies the periodicity condition is the period of g(t)!

T_0 = 2…

Periodic Signal



Two Properties of periodic signal➔

1. ***Must start at -∞ and continue forever….***
2. ***g(t) can be generated by periodic extension of any segment of g(t) of duration T_0 period…***

Energy and Power Signal

The signals which have finite energy are called ***energy signals***. The nonperiodic signals like exponential signals will have constant energy and so nonperiodic signals are energy signals.

The signals which have finite average power are called ***power signals***. The periodic signals like sinusoidal and complex exponential signals will have constant power and so periodic signals are power signals.

The ***energy*** E of a continuous time signal x(t) is defined as,

$$\boxed{\text{Energy, } E = \underset{T \to \infty}{Lt} \int_{-T}^{T} |x(t)|^2 \, dt \text{ in joules}}$$

The average ***power*** of a continuous time signal x(t) is defined as,

$$\boxed{\text{Power, } P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \text{ in watts}}$$

Energy and Power Signal

For periodic signals, the average power over one period will be same as average power over an infinite interval.

$$\boxed{\therefore \text{For periodic signals, power, } P = \frac{1}{T}\int_0^T |x(t)|^2 \, dt}$$

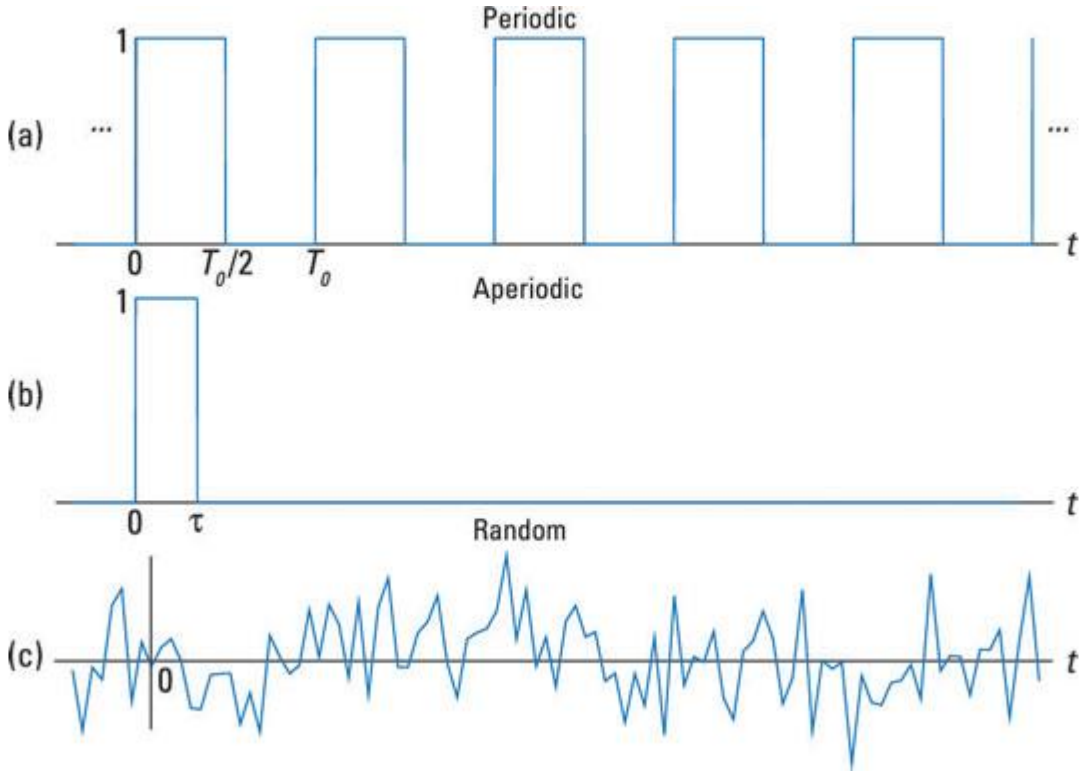
For energy signals, the energy will be finite (or constant) and average power will be zero. For power signals the average power is finite (or constant) and energy will be infinite.

i.e., For energy signal, E is constant (i.e., $0 < E < \infty$) and $P = 0$.

For power signal, P is constant (i.e., $0 < P < \infty$) and $E = \infty$.

Deterministic And Random Signal

Deterministic Signal: A signal whose physical description is known completely in either a mathematical form or a graphical form …



Random Signal: If a signal is known only in terms of probabilistic description, such as mean value, mean squared value…then the signal is random in characteristic…!!

Standard continuous Time signals

2.2 Standard Continuous Time Signals

1. Impulse signal /delta signal

The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A. Mathematically, impulse signal is defined as,

$$\text{Impulse Signal, } \delta(t) = \infty ;\ t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = A$$
$$= 0 ;\ t \neq 0$$

The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, unit impulse signal is defined as,

$$\text{Unit Impulse Signal, } \delta(t) = \infty ;\ t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0 ;\ t \neq 0$$

Fig 2.1 : *Impulse signal (or Unit Impulse signal).*

Standard continuous Time signals

2. Step signal

The step signal is defined as,

$$x(t) = A \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit step signal is defined as,

$$x(t) = u(t) = 1 \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

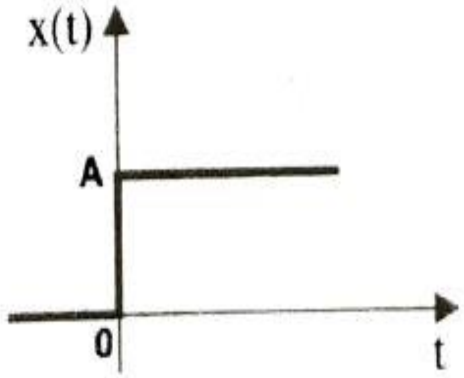


Fig 2.2 : Step signal.

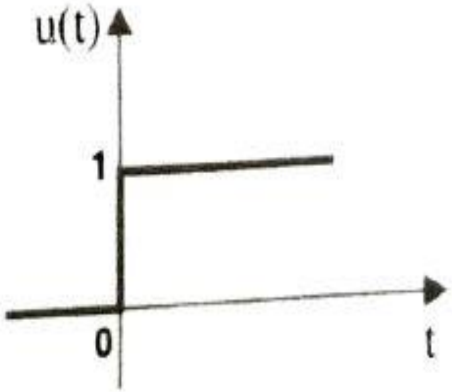


Fig 2.3 : Unit step signal.

Standard continuous Time signals

3. *Ramp signal*

The ramp signal is defined as,

$$x(t) = At \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit ramp signal is defined as,

$$x(t) = t \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

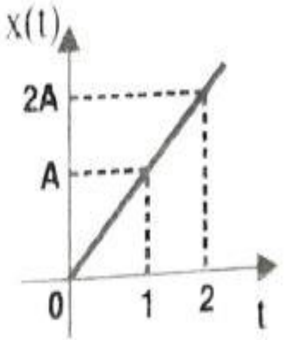


Fig 2.4 : *Ramp signal.*

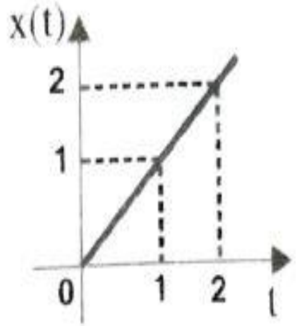


Fig 2.5 : *Unit ramp signal.*

Standard continuous Time signals

4. Parabolic signal

The parabolic signal is defined as,

$$x(t) = \frac{At^2}{2} \quad ; \quad \text{for } t \geq 0$$
$$= 0 \quad ; \quad t < 0$$

The unit parabolic signal is defined as,

$$x(t) = \frac{t^2}{2} \quad ; \quad \text{for } t \geq 0$$
$$= 0 \quad ; \quad t < 0$$

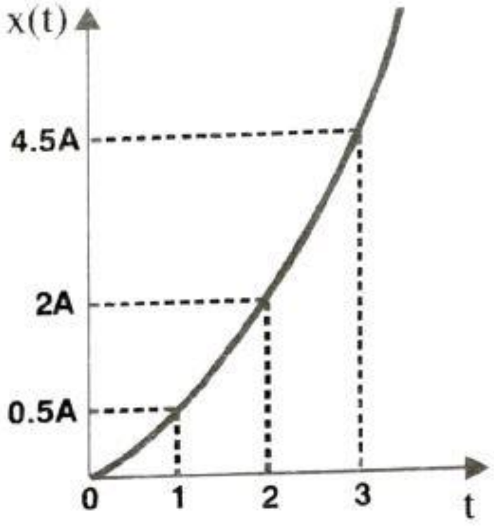


Fig 2.6 : Parabolic signal.

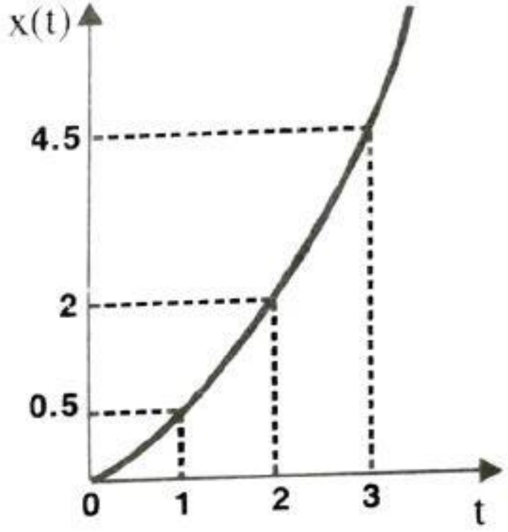


Fig 2.7 : Unit parabolic signal.

Standard continuous Time signals

5. *Unit pulse signal* */ Rectangular signal /Gate signal*

The unit pulse signal is defined as,

$$x(t) = \Pi(t) = u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)$$

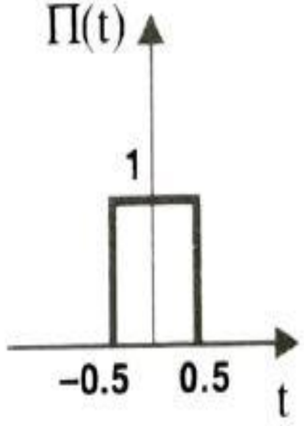


Fig 2.8 : Unit pulse signal.

OR

$$x(t) = 1 \qquad -\frac{T}{2} < t < +\frac{T}{2}$$

Standard continuous Time signals

Sinusoidal signal

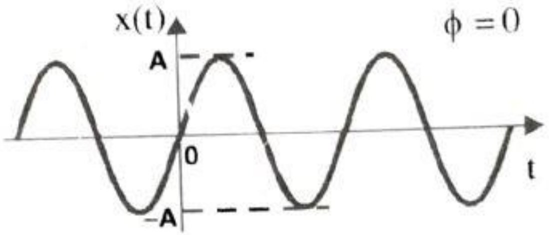
The sinusoidal signal is defined as,

$$x(t) = A\sin(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec



Standard continuous Time signals

Signum signal

The Signum signal is defined as the sign of the independent variable t. Therefore, the Signum signal is expressed as,

$$x(t) = \operatorname{sgn}(t) = \begin{cases} 1 & ; \ t > 0 \\ 0 & ; \ t = 0 \\ -1 & ; \ t < 0 \end{cases}$$

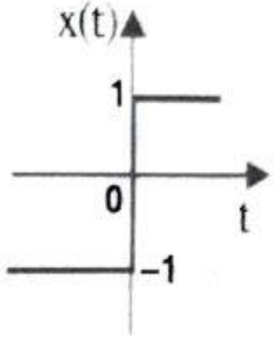


Fig 2.16 : *Signum signal.*

Standard continuous Time signals

Sinc signal

The Sinc signal is defined as,

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t} \; ; \quad -\infty < t < \infty$$

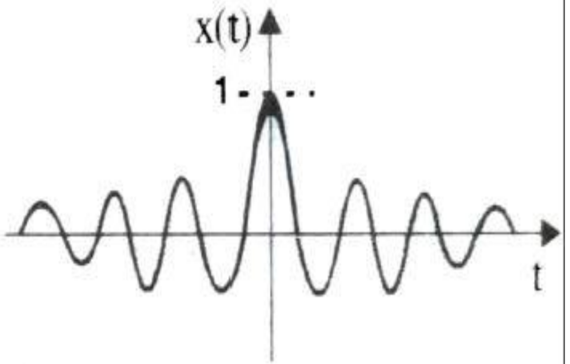


Fig 2.17 : *Sinc signal.*

Fourier Transform

Definition of Fourier Transform

Let, $x(t)$ = Continuous time signal

$X(f)$ = $X(j\Omega)$ = Fourier transform of $x(t)$

The Fourier transform of continuous time signal, $x(t)$ is defined as,

$$X(f) = X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt$$

Also, $X(j\Omega)$ is denoted as $\mathcal{F}\{x(t)\}$ where "$\mathcal{F}$" is the symbol used to denote the Fourier transform operation.

$$\boxed{\therefore\ \mathcal{F}\{x(t)\} = X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt} \quad .....(4.35)$$

Fourier Transform

Definition of Inverse Fourier Transform

Definition of Inverse Fourier Transform

The ***inverse Fourier transform*** of $X(j\Omega)$ is defined as,

$$\boxed{x(t) = \mathcal{F}^{-1}\{X(j\Omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega} \quad .....(4.36)$$

The signals $x(t)$ and $X(j\Omega)$ are called ***Fourier transform pair*** and can be expressed as shown below,

$$\boxed{x(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\Longleftrightarrow}} X(j\Omega)}$$

Fourier Transform of some standard signal

Unit impulse or delta function:

The impulse signal is defined as,

$$x(t) = \delta(t) = \infty \quad ; \quad t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$
$$= 0 \quad ; \quad t \neq 0$$

By definition of Fourier transform,

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\Omega t}\, dt$$

$$= 1 \times e^{-j\Omega t}\Big|_{t=0} = 1 \times e^{0} = 1$$

$\boxed{\delta(t) \text{ exists only for } t = 0}$

$$\therefore \boxed{\mathcal{F}\{x(t)\} = 1}$$

Fourier Transform of some standard signal

The plot of impulse signal and its magnitude spectrum are shown in fig 4.18 and fig 4.19 respectively.

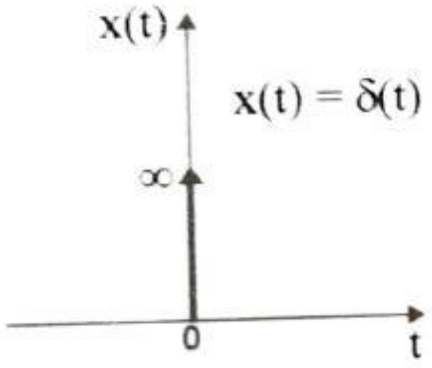


Fig 4.18 : *Impulse signal.*

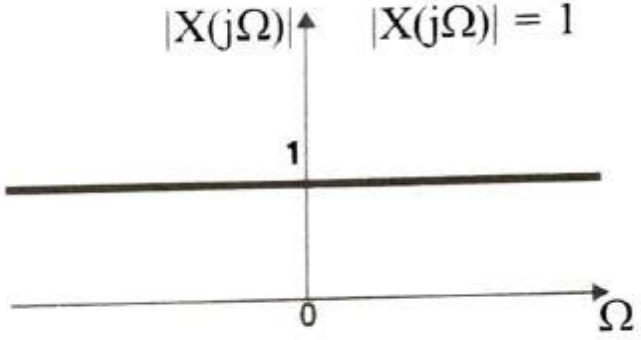
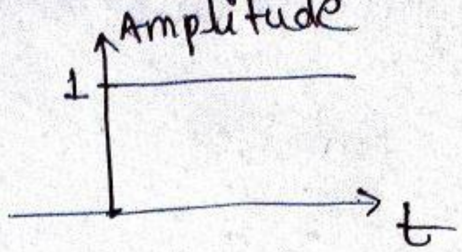


Fig 4.19 : *Magnitude spectrum of impulse signal.*

Fourier Transform of some standard signal

The step signal is defined as

$$x(t) = 1 \quad t \geq 0$$
$$\quad = 0 \quad t < 0$$



from def. of F.T.

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} 1 \cdot e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_0^{\infty}$$

$$= 0 - \left(\frac{-e^{0}}{j\omega} \right) = \frac{1}{j\omega}$$

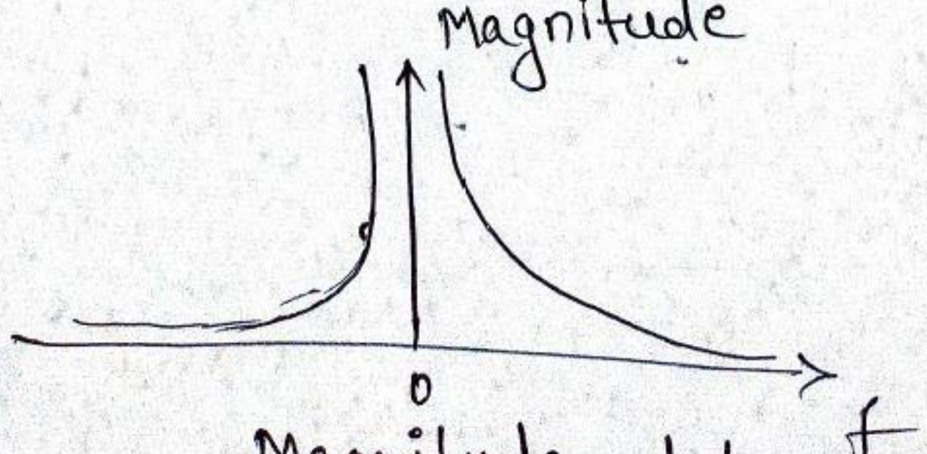
Fourier Transform of some standard signal

Magnitude plot

$$|X(f)| = \frac{1}{\sqrt{j^2 \omega^2}} = \frac{1}{\omega^2} = \frac{1}{4\pi^2 f^2}$$

$f = 0, \ |X(f)| = \infty$

$f = \infty \ |X(f)| = 0$



Magnitude plot.

Fourier Transform of some standard signal

The plot of unit step signal and its magnitude spectrum are shown in fig 4.28 and fig 4.29 respectively.

$$x(t) = u(t) = 1 \quad ; \quad t \geq 0$$
$$= 0 \quad ; \quad t < 0$$

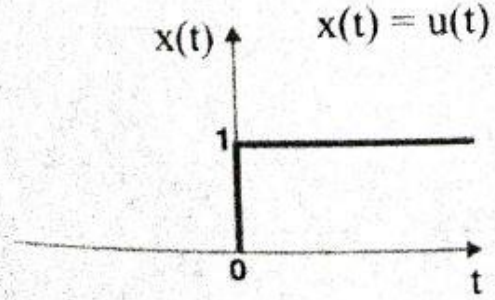


Fig 4.28 : *Unit step signal.*

$$|X(j\Omega)| = \pi\delta(\Omega) + \frac{1}{\Omega}$$

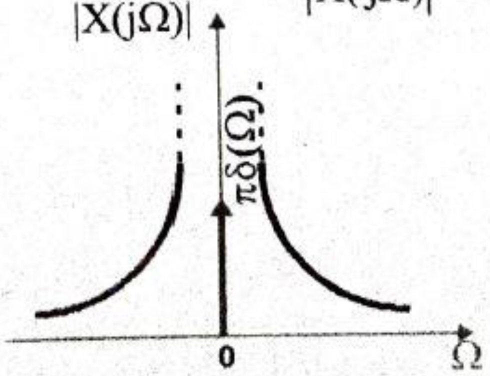


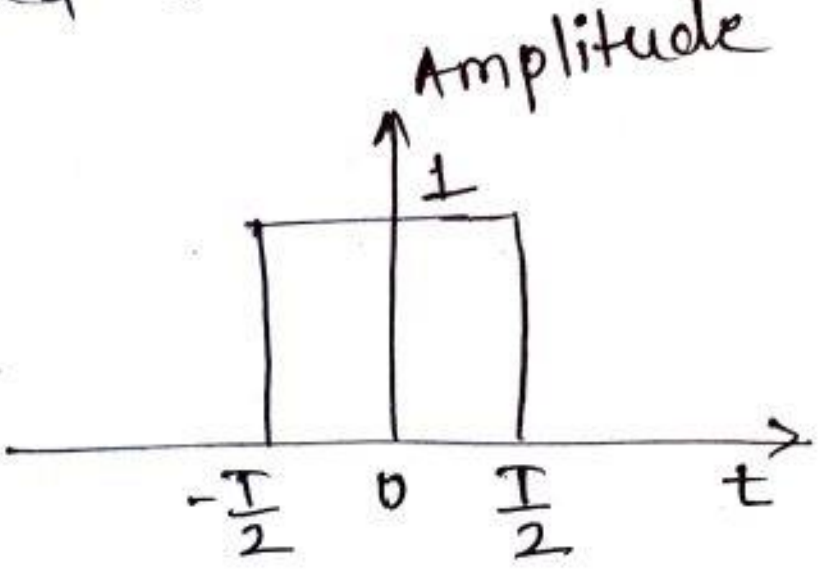
Fig 4.29 : *Magnitude spectrum of unit step signal.*

Fourier Transform of some standard signal

Fourier transform of rectangular pulse or gate function.

Gate function is defined as

$$x(t) = \begin{cases} 0 & t < -\frac{T}{2} \\ 1 & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases}$$



Fourier Transform of some standard signal

From Defⁿ. of F.T.

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} 1 .\, e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{T/2} 1\, e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-T/2}^{T/2}$$

$$= \frac{1}{-j\omega} \left[e^{-j\omega T/2} - e^{j\omega T/2} \right]$$

$$= \frac{e^{j\omega T/2} - e^{-j\omega \frac{T}{2}}}{j\omega}$$

$$= \frac{2}{\omega} \left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j2} \right] \quad \ldots \text{ M. \& D. by 2}$$

Fourier Transform of some standard signal

$$= \frac{2}{\omega} \sin\left(\frac{\omega T}{2}\right)$$

M. & D. by $T/2$

$$= \frac{\cancel{2}}{\omega} \cdot \frac{T}{\cancel{2}} \; \frac{\sin\left(\frac{\omega T}{2}\right)}{T/2}$$

$$= T \cdot \frac{\sin(\omega T/2)}{\omega T/2} \quad \ldots \qquad \operatorname{sinc} x = \frac{\sin x}{x}$$

$$= T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$$

Fourier Transform of some standard signal

Magnitude plot of rectangular / gate function

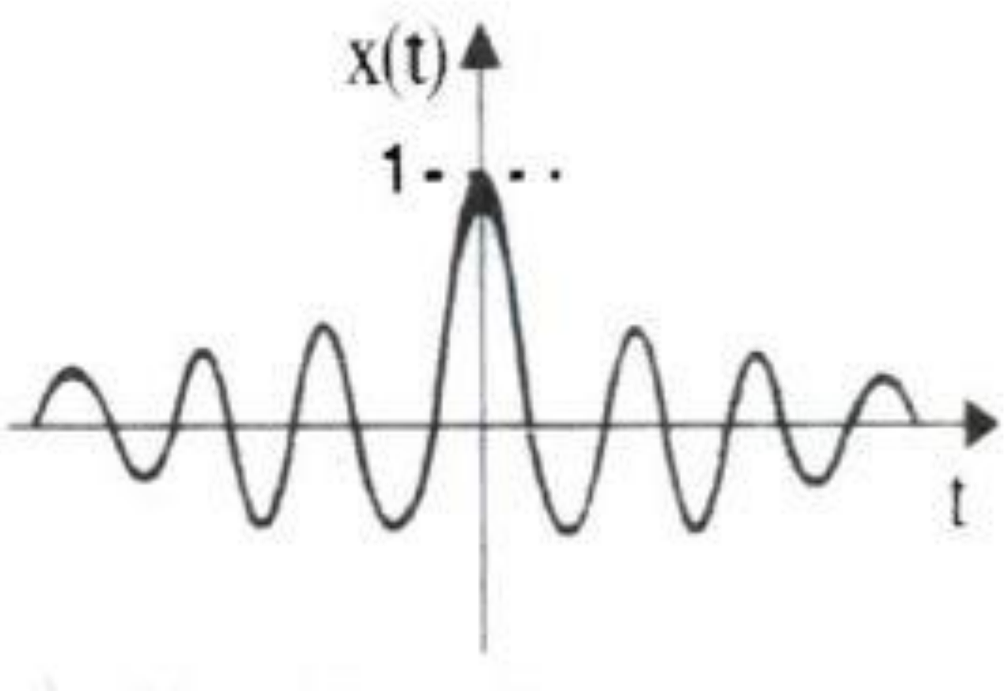


Table 4.3 : Summary of Properties of Fourier Transform

Let, $\mathcal{F}\{x(t)\} = X(j\Omega)$; $\mathcal{F}\{x_1(t)\} = X_1(j\Omega)$; $\mathcal{F}\{x_2(t)\} = X_2(j\Omega)$

Property	Time domain signal	Frequency domain signal
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(j\Omega) + a_2 X_2(j\Omega)$
Time shifting	$x(t - t_0)$	$e^{-j\Omega t_0} X(j\Omega)$
Time scaling	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{j\Omega}{a}\right)$
Time reversal	$x(-t)$	$X(-j\Omega)$
Conjugation	$x^*(t)$	$X^*(-j\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(j(\Omega - \Omega_0))$
Time differentiation	$\frac{d}{dt} x(t)$	$j\Omega X(j\Omega)$
Time integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{X(j\Omega)}{j\Omega} = \pi X(0) \delta(\Omega)$

Properties of Fourier Transform

① Time shifting property

If $x(t) \xleftrightarrow{F} X(f)$

then $x(t-t_d) \xleftrightarrow{F} e^{-j2\pi f t_d} X(f)$

Proof: Def. of F.T. is

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(t-t_d) e^{-j\omega t} dt \quad \text{—} ①$$

put $t - t_d = \tau$

$t = \tau + t_d$

$dt = d\tau$

Properties of Fourier Transform

substituting in eq^n (1),

$$X(f) = F[x(t-t_d)] = \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f(t_d+\tau)} d\tau$$

$$= e^{-j2\pi f t_d} \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f \tau} d\tau$$

$$= e^{-j2\pi f t_d} X(f) \quad \text{--- Proved.}$$

Thus shift in time corresponds to a phase rotation in the frequency domain.

Properties of Fourier Transform

② Frequency shifting property:

If $x(t) \xleftrightarrow{F} X(f)$ then

$$x(t)\, e^{j2\pi f_0 t} \xleftrightarrow{F} X(f-f_0)$$

Proof: Def. of Inverse F. T.

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j\omega t}\, df$$

$$= \int_{-\infty}^{\infty} X(f-f_0)\, e^{j2\pi f t}\, df$$

$$= \int_{-\infty}^{\infty} X(\tau)\, e^{j2\pi (f_0+\tau) t}\, d\tau$$

where $f - f_0 = \tau$

$f = f_0 + \tau$

$df = d\tau$

NOISE

- Electrical disturbances interfere with signals, producing noise.
- Noise may be defined, in electrical terms, as any unwanted introduction of energy tending to interfere with the proper reception and reproduction of transmitted signals

Noise

- External
- Internal

NOISE (Book: ECS by George Kennedy)

EXTERNAL NOISE

- Atmospheric Noise
- Extraterrestrial Noise
  - Solar Noise
  - Cosmic Noise
- Industrial Noise

INTERNAL NOISE

- Thermal Agitation Noise (White/Johnson)
- Shot Noise
- Transit-Time Noise

EXTERNAL NOISE

Atmospheric Noise

- ✓ This noise gets generated within Earth’s atmosphere. It is commonly called static electricity.
- ✓ Common source is lightning.
- ✓ Static electricity is often in the form of impulses that spread energy throughout a wide range of frequencies.
- ✓ The magnitude of this energy is inversely proportional to its frequencies. At frequencies above 30 MHz, atmospheric noise is relatively insignificant.

EXTERNAL NOISE

Extra-terrestrial Noise

There are almost as many types of space noise as there are sources !!!

We divide into two sub groups

1. **Solar : Surface temperature is over 6000 degree Celsius..**

Two Types:

- In normal " quiet'' conditions It radiates over a very broad frequency spectrum which includes the frequencies we use for communication.
- Undergoes cycles of peak activity from which electrical disturbances erupt, such as corona flares and sunspots

2. Cosmic (Thermal/black-body): Due to distant stars radiates RF noise like sun!

Characteristic : It is distributed fairly uniformly over the entire sky..

SPACE NOISE FREQUENCIES

- Space noise is observable at frequencies in the range from about 8 MHz to somewhat above 1.43 gigahertz (1.43 GHz)

- Man-made noise is the strongest component over the range of about 20 to 120 MHz

EXTERNAL NOISE

Industrial Noise

Lies between the frequencies of 1 to 600 MHz (in urban, suburban and other industrial areas)

Characteristic: This noise obey the general principle **that received noise increases as the receiver bandwidth is increased**

INTERNAL NOISE

- Noise created by any of the active or passive devices found in receivers.

Characteristics:

- Are described statistically
- Random noise power is proportional to the bandwidth over which it is measured.

INTERNAL NOISE: THERMAL AGITATION NOISE

➢ Thermal Agitation Noise :

- The noise generated in a resistance or the resistive component is random and is referred to as thermal, agitation noise, White or Johnson noise.

- It is due to the rapid and random motion of the molecules (atoms and electrons) inside the component itself.

- The kinetic energy of these particles becomes approximately zero {i.e., their motion ceases) at the temperature of absolute zero, which is 0 degree K and is very nearly equals - 273°C.

- It becomes apparent that the noise generated by a resistor is proportional to its absolute temperature, in addition to being proportional to the bandwidth over which the noise is to be measured.

INTERNAL NOISE

Therefore

$P_n \propto T\Delta f = kT\Delta f$ - - - - - - - - - - - (15)

where k, = Boltzmann's constant = 1.38×10^{-23} J(joules)/K the appropriate proportionality constant in this case

T = absolute temperature, K = 273 + °C

Δf = bandwidth of interest

P_n = maximum noise power output of a resistor

$\propto$ = varies directly

THERMAL NOISE PROBLEM (Internal Noise)

If the resistor is operating at 27°C and the bandwidth of interest is 2 MHz, then what is the maximum noise power output of a resistor?

Solution

$$P_n = k.T.\Delta f = 1.38 \times 10^{-23} \times 300 \times 2 \times 10^6$$

$$P_n = 1.38 \times 10^{-17} \times 600 = 0.138 \times 0.6 \times 10^{-12}$$

$$P_n = 0.0828 \times 10^{-12} \text{ Watts}$$

K=1.380 649. 10^-23 J / K
Boltzmann constant

INTERNAL NOISE (rms noise voltage associated with a resistor)

If an ordinary resistor at the standard temperature of 17°C (290 K) is not connected to any voltage source, there may even be quite a large voltage across it.

Since it is random and therefore has a finite rms value but no dc component, only the alternating current (ac) meter will register a reading.

This noise voltage is caused by the random movement of electrons within the resistor, which constitutes a current.

INTERNAL NOISE (rms noise voltage associated with a resistor)

$$P_n = \frac{V^2}{R_L} = \frac{V_n^2}{4R}$$

$$V_n^2 = 4RP_n = 4RkT\,\Delta f$$

$$V_n = \sqrt{4kT\,\Delta f\,R} \quad \text{- . - . - . - . - . -} \quad (16)$$

$PL=Vn^2\{RL/(R+RL)^2\}$

$RL = R;$

$PL=Vn^2\{R/(R+R)^2\}$

$PL=Vn^2\{R/4(R)^2\}$

$PL=Vn^2\{1/4R\}$

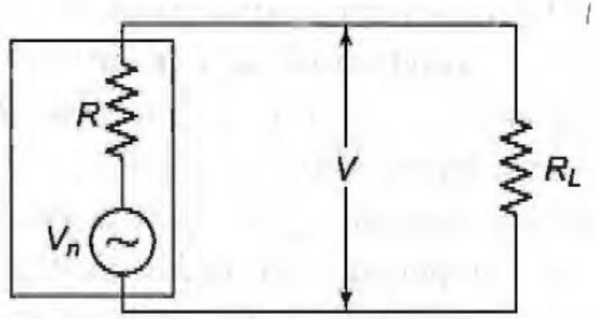


Fig. 2.1 *Resistance noise generator.*

INTERNAL NOISE (rms noise voltage associated with a resistor)

An amplifier operating over the frequency range from 18 to 20 MHz has a 10-kilohm (10-kΩ) input resistor. What is the rms noise voltage at the input to this amplifier if the ambient temperature is 27°C?

Solution

$$V_n = \sqrt{4kT\Delta f R}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times (27 + 273) \times (20 - 18) \times 10^6 \times 10^4}$$

$$= \sqrt{4 \times 1.38 \times 3 \times 2 \times 10^{-11}} = 1.82 \times 10^{-5}$$

$$= 18.2 \text{ microvolts } (18.2 \mu V)$$

As we can see from this example, it would be futile to expect this amplifier to handle signals unless they were considerably larger than 18.2 μV. A low voltage fed to this amplifier would be masked by the noise and lost.

INTERNAL NOISE : Shot Noise

- Shot noise occurs in all amplifying devices and virtually in all active devices.
- It is caused by random variations in the arrival of electrons (or holes) at the output electrode of an amplifying device and appears as a randomly varying noise current superimposed on the output.
- When amplified, it is supposed to sound as though a shower of lead shot were falling on a metal sheet. Hence the name shot noise.
- Shot noise behaves in a similar manner to thermal agitation noise, apart from the fact that it has a different source.

INTERNAL NOISE : Shot Noise

- Solution: The most convenient method of dealing with shot noise is to find the value or formula for an equivalent input-noise resistor

- The value of the equivalent shot-noise resistance R_{eq} of a device is generally quoted in the manufacturer’s specifications.

INTERNAL NOISE: Transit-Time Noise (Active Device)

- If the time taken by an electron to travel from the emitter to the collector of a transistor becomes significant to the period of the signal being amplified, i.e., at frequencies in the upper VHF (30-300 MHz) range and beyond, the so-called transit-time effect takes place....

- The minute currents induced in the input of the device by random fluctuations in the output current become of great importance at VHF and above frequencies and create random noise (frequency distortion).

INTERNAL NOISE: Transit-Time Noise (Active Device)

- Once this high-frequency noise makes its presence felt, it goes on increasing with frequency at a rate that soon approaches 6 decibels (6 dB) per octave

- The result of all this is that it **is preferable to measure noise at such high frequencies, instead of trying to calculate an input equivalent noise resistance for it.**

NOISE CALCULATIONS :Addition of Noise due to Several Sources

Let's assume there are two sources of thermal agitation noise generators in series: $V_{n1} = \sqrt{4kT\Delta f\, R_1}$ and

$V_{n2} = \sqrt{4kT\Delta f\, R_2}$.

The sum of two such rms voltages in series is given by the square root of the sum of their squares, so that we have

$$V_{n,tot} = \sqrt{V_{n1}^2 + V_{n2}^2} = \sqrt{4kT\,\Delta f R_1 + 4kT\,\Delta f\, R_2}$$
$$= \sqrt{4kT\Delta f\,(R_1 + R_2)} = \sqrt{4kT\Delta f\, R_{tot}} \qquad (17)$$

where $R_{tot} = R_1 + R_2 + \cdots$

NOISE CALCULATIONS :Addition of Noise due to Several Sources

Calculate the noise voltage at the input of a television RF amplifier, using a device that has a 200-ohm (200-Ω) equivalent noise resistance and a 300-Ω input resistor. The bandwidth of the amplifier is 6 MHz, and the temperature is 17°C.

Solution

$$V_{n,\text{tot}} = \sqrt{4kT\,\Delta f\, R_{\text{tot}}}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times (17 + 273) \times 6 \times 10^{6} \times (300 + 200)}$$

$$= \sqrt{4 \times 1.38 \times 2.9 \times 6 \times 5 \times 10^{-13}} = \sqrt{48 \times 10^{-12}}$$

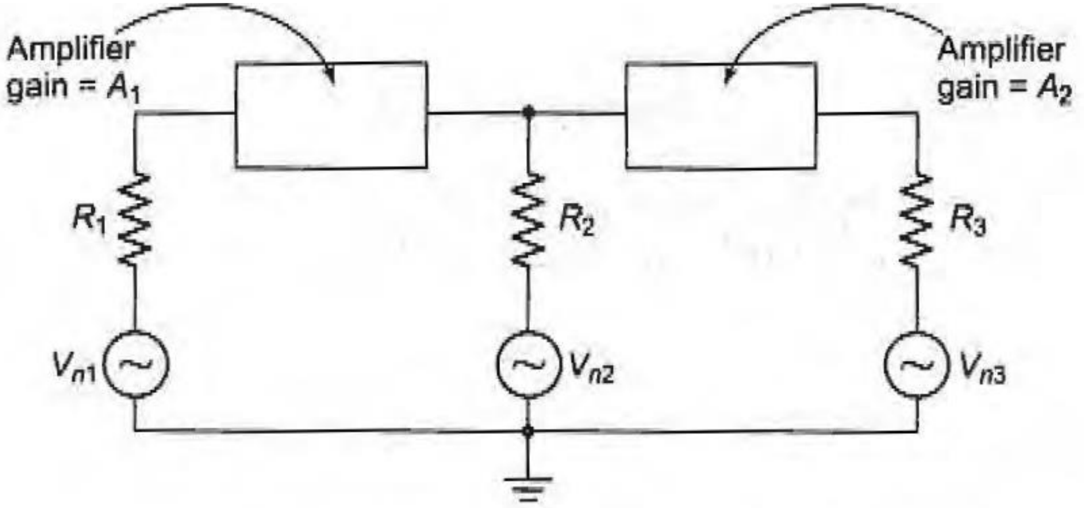
$$= 6.93 \times 10^{-6} = 6.93\,\mu\text{V}$$

To calculate the noise voltage due to several resistors in parallel, find the total resistance by standard methods, and then substitute this resistance into Equation **(17)** as before. This means that the total noise voltage is less than that due to any of the individual resistors, but, as shown in Equation **(15)**, the noise power remains constant.

Addition of Noise due to Several Amplifiers in Cascade

Figure below shows a number of amplifying stages in cascade, each having a resistance at its input and output.

The first such stage is very often an RF amplifier, while the second is a mixer. The problem is to find their combined effect on the receiver noise



Noise of several amplifying stages in cascade.

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

Addition of Noise due to Several Amplifiers in Cascade

Q1 *The first stage of a two-stage amplifier has a voltage gain of 10, a 600-Ω input resistor, a 1600-(noise resistance and a 27-kΩ output resistor. For the second stage, these values are 25. 81 kΩ, 1 megaohm (1 MΩ), respectively. Calculate the equivalent input-noise resistance of this two-stage*

Solution

$$R_1 = 600 + 1600 = 2200\ \Omega$$

$$R_2 = \frac{27 \times 81}{27 + 81} + 10 = 20.2 + 10 = 30.2\ \text{k}\Omega$$

$$R_3 = 1\ \text{M}\Omega \qquad \text{(as given)}$$

$$R_{eq} = 2200 + \frac{30.200}{10^2} + \frac{1{,}000{,}000}{10^2 \times 25^2} = 2200 + 302 + 16$$

$$= 2518\ \Omega$$

Note that the 1-MΩ output resistor has the same noise effect as a 16-Ω resistor at the input.

NOISE PARAMETERS

Signal-to-Noise Ratio

The calculation of the equivalent noise resistance of an amplifier, receiver or device may have one of two purposes or sometimes both.

- The first purpose is comparison of two kinds of equipment in evaluating their performance.
- The second is comparison of noise and signal at the same point to ensure that the noise is not excessive.

In the second instance, and also when equivalent noise resistance is difficult to obtain, the signal-to-noise ratio (SIN) is very often used.

It is defined as the ratio of signal power to noise power at the same point. ….

SIGNAL TO NOISE RATIO

$$\frac{S}{N} = \frac{P_S}{P_N} = \frac{V_S^2/_R}{V_N^2/_R} = \frac{V_S^2}{V_N^2} \quad \text{- - - - - - - - - -} \quad (18)$$

$$\frac{S}{N}\,(\text{dB}) = 10 \log \frac{P_S}{P_N} = 10 \log \left(\frac{V_S^2}{V_N^2}\right) = 20 \log \frac{V_S}{V_N} \quad \text{- - - - - - - - - -} \quad (19)$$

Equation 18, is a simplification that applies whenever the resistance across which the noise is developed is the same as the resistance across which signal is developed, and this is almost invariable

An effort is naturally made to keep the signal-to-noise ratio as high as practicable under a given set of conditions.

NOISE FACTOR (F)

It is a figure of Merit used to indicate how much the signal to noise ratio deteriorates as a signal passes through a circuit or series of circuits.

Definition: Ratio of input signal to noise power ratio to the output signal to noise power ratio.

$$F = \frac{\text{input signal to noise power ratio}}{\text{output signal to noise power ratio}}$$

NOISE FIGURE

It is defined as noise factor stated in dB. It is used to indicate quality of quality of a receiver.

$$NF(dB) = 10\log F$$

NOISE FIGURE

Instead of equivalent noise resistance, a quantity known as noise figure, sometimes called noise factor, is defined and used.

The noise figure F is defined as the ratio of the signal-to-noise power supplied to the input terminals of a receiver or amplifier to the signal-to-noise power supplied to the output or load resistor. Thus

$$F = \frac{\text{input } S/N}{\text{output } S/N} \qquad (19)$$

It can be seen immediately that a practical receiver will generate some noise, and the S/N will deteriorate as one moves toward the output.

Consequently, in a practical receiver, the output S/N will be lower than the input value, and so the noise figure will exceed 1.

However, the noise figure will be 1 for an ideal receiver, which introduces no noise of its own.

NOISE FIGURE

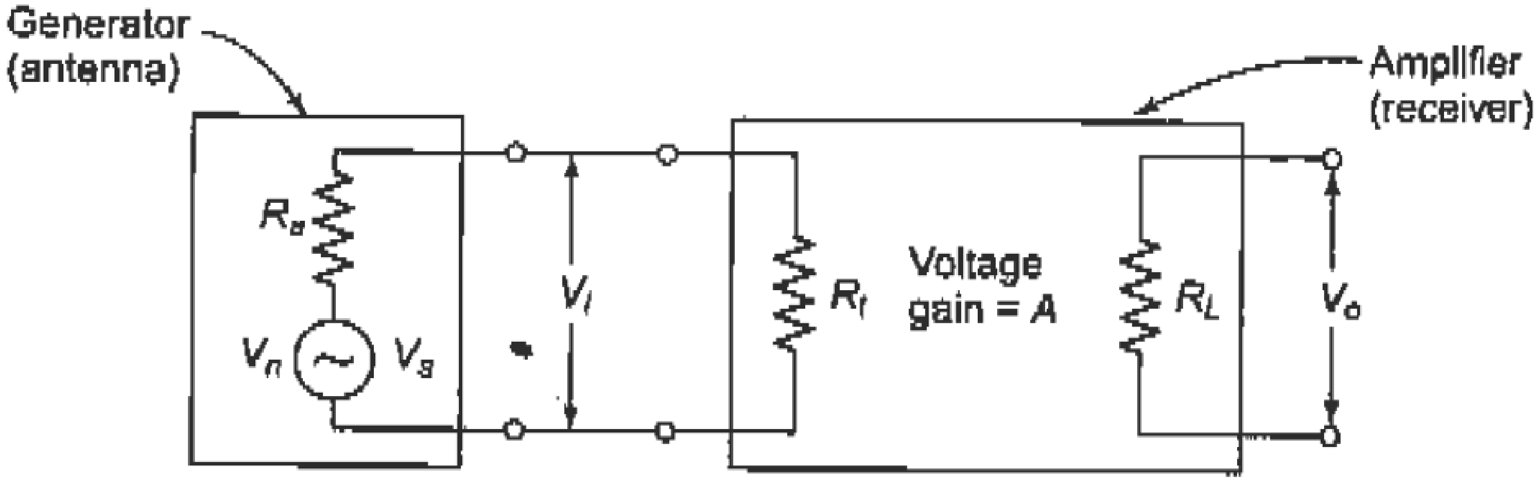
We have the alterative definition of noise figure, which states that

$$F = \frac{\text{S/N of an ideal system}}{\text{S/N at the output of the receiver or amplifier under test}} \quad \text{- . - . - . - . - . - . - . - .} \quad (20)$$

Both working at the same temperature over the same bandwidth and fed from the same source

The noise figure may be expressed an actual ratio or in decibels

NOISE FIGURE CALCULATION FROM EQUIVALENT NOISE RESISTANCE



Block diagram for noise figure calculation.

$$R'_{eq} = R_{eq} - R_t \quad \text{(21)}$$

$$F = 1 + \frac{R'_{eq}(R_a + R_t)}{R_a R_t} \quad \text{(23)}$$

The total equivalent noise resistance for this receiver will now be

$$R = R'_{eq} + \frac{R_a R_t}{R_a + R_t} \quad \text{(22)}$$

NOISE FIGURE CALCULATION FROM EQUIVALENT NOISE RESISTANCE

if the noise is to be a minimum for any given value of the antenna

so that R_t must be much larger than R_a.

Under Mismatched Condition $$F = 1 + \frac{R'_{eq}}{R_a} \qquad \text{(24)}$$

Under Matched Condition

$(R_t = R_a)$ $$F = 1 + \frac{R'_{eq}(R_a + R_t)}{R_a R_t} \qquad \text{(25)}$$

NOISE FIGURE CALCULATION FROM EQUIVALENT NOISE RESISTANCE

Calculate the noise figure of the amplifier of Question1 *if it is driven by a generator whose output impedance is 50 Ω. (Note that this constitutes a large enough mismatch.)*

Solution

$$R'_{eq} = R_{eq} - R_t = 2518 - 600 = 1918\ \Omega$$

$$F = 1 + \frac{R'_{eq}}{R_a} = 1 + 38.4$$

$$= 39.4 \qquad (= 15.84\ \text{dB})$$

Note that if an "equivalent noise resistance" is given without any other comment in connection with noise figure calculations, it may be assumed to be R'_{eq}.

EQUIVALENT NOISE TEMPERATURE

The concept of noise figure, although frequently used, is not always the most convenient measure of noise, particularly in dealing with UHF and microwave low-noise antennas, receivers or devices.

Definition: The temperature at which the noisy resistor has to be maintained so that by connecting this resistor to the input of a noiseless version of the system, it will produce the same amount of noise power at the system output as that produced by the actual system.

$$T_{eq} = T(F-1)$$

T_{eq} is the equivalent noise temperature (Kelin)

T is environmental temperature (Ref value 300° K

or

$$F \text{ is } 1 + \frac{T_{eq}}{T}$$

NOISE TEMPERATURE

$$P_t = kT\,\Delta f$$

$$= P_1 + P_2 = kT_1\,\Delta f + kT_2\,\Delta f$$

$$kT_t\,\Delta f = kT_1\,\Delta f + kT_2\,\Delta f \qquad (23)$$

$$T_t = T_1 + T_2$$

where P_1 and P_2 = two individual noise powers (e.g., received by the antenna and generated by the antenna, respectively) and P_t is their sum

T_1 and T_2 = the individual noise temperatures

T_t = the "total" noise temperature

Another advantage of the use of noise temperature for low noise levels is that it shows a greater variation for any given noise-level change than does the noise figure, so changes are easier to grasp in their true perspective.

NOISE TEMPERATURE (Problem)

A receiver connected to an antenna whose resistance is 50 Ω has an equivalent noise resistance of 30 Ω. Calculate the receiver's noise figure in decibels and its equivalent noise temperature.

Solution

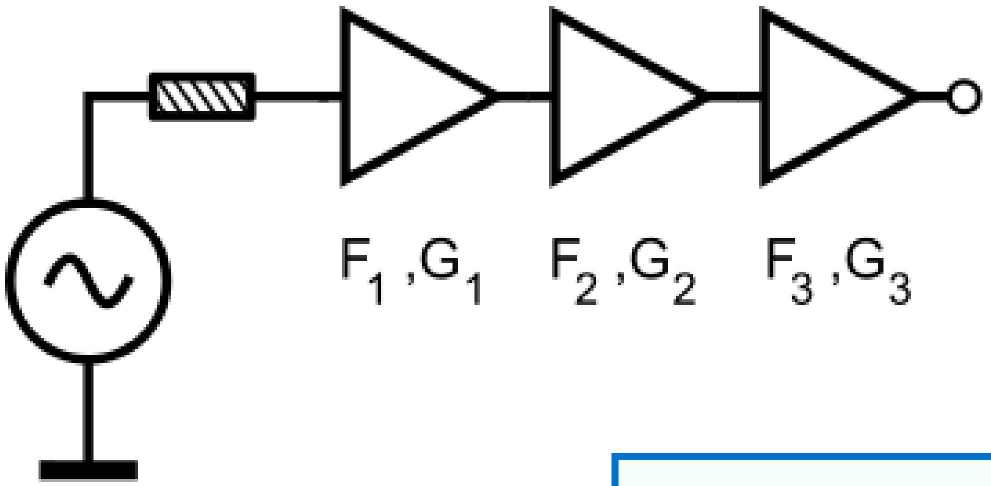
$$F = 1 + \frac{R_{eq}}{R_a} = 1 + \frac{30}{50} = 1 + 0.6 = 1.6$$

$$= 10 \log 1.6 = 10 \times 0.204 = 2.04\,\text{dB}$$

$$T_{eq} = T_0(F - 1) = 290(1.6 - 1) = 290 \times 0.6$$

$$= 174\text{K}$$

FRIIS Formula



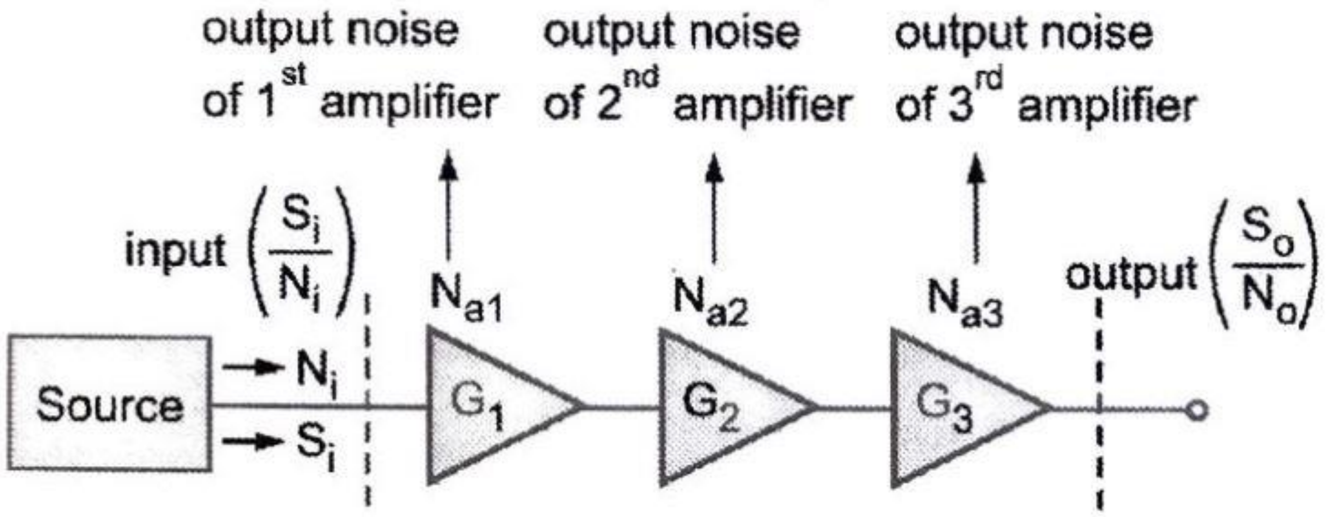
Friis's formula is used to calculate the total noise factor of a cascade of stages, each with its own noise factor and power. The total noise factor can then be used to calculate the total noise figure. The total noise factor is given as

$$F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

where F_i and G_i are the noise factor and available power gain, respectively, of the ith stage, and n is the number of stages. Both magnitudes are expressed as ratios, not in decibels.

Derivation

– ***Refer Fig. 2.6.2***. It shows three cascaded amplifiers.



(4B7)**Fig. 2.6.2 : Cascaded amplifier stages**

- Let the input signal power be S_i and noise power be N_i. similarly let output signal power be S_o and noise power be N_o.
- Hence input SNR $= \frac{S_i}{N_i}$.
- Now signal power $S_o = S_i \cdot G_1 G_2 G_3$ ...(2.6.5)
- Where G_1, G_2 and G_3 are gains of amplifiers of stages 1, 2 and 3 respectively.
- Now output noise power is given by,

$$N_o = N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3} \quad \ldots(2.6.6)$$

- From Equations (2.6.5) and (2.6.6) we get,

$$\text{Output (SNR)} = [\text{SNR}]_o = \frac{S_o}{N_o}$$

$$\therefore \frac{S_o}{N_o} = \frac{S_i G_1 G_2 G_3}{N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3}} \quad \ldots(2.6.7)$$

Now total noise factor is given as,

$$F_{total} = \frac{S_i/N_i}{S_o/N_o}$$

$$= \frac{\frac{S_i}{N_i}}{\frac{S_i G_1 G_2 G_3}{N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3}}}$$

$$= \quad 1 + \frac{N_{a1}}{N_i G_1} + \frac{N_{a2}}{N_i G_1 G_2} + \frac{N_{a3}}{N_i G_1 G_2 G_3} \quad \ldots(2.6.8)$$

But noise factor of the amplifier is given as

$$F \quad = \quad 1 + \frac{N_o}{N_i G}$$

Substituting in Equation (2.6.8) we get,

$$F_{total} \quad = \quad F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \Leftarrow \text{Friis formula}$$

For 3 cascaded ampr stages, each with noise fig of 3dB & power gain of 10dB determine total noise fig.

$$F_T = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1 A_2}$$

$$3dB = 10 \log F_1$$

$$\frac{3}{10} = \log F_1 \quad \therefore \quad F_1 = \text{Antilog}\,(0.3) = 1.995$$

& power gain = 10dB

$$= \text{Antilog}\left(\frac{10}{10}\right) = \underline{\underline{10}}$$

$$F_T = 1.995 + \frac{1.995 - 1}{10} + \frac{1.995 - 1}{10 \times 10}$$

$$f_T = 1.995 + \frac{1.995 - 1}{10} + \frac{1.995 - 1}{10 \times 10}$$

$$f_T = 2.10445$$

$$f_T(dB) = 10 \log(2.10445) = \underline{\underline{3.23 dB}}$$

Analog Communications Questions and Answers – Noise in AM

1. Which one of the following noise becomes of great importance at high frequencies?

a) flicker noise
b) shot noise
c) impulse noise
d) transit-time noise

2. Which one of the following statement is false?

a) High Frequency mixers are generally noisier
b) Voltage of impulse noise is independent of bandwidth
c) Thermal noise is not dependent on frequency
d) Flicker noise occurs at low frequency

Analog Communications Questions and Answers – Noise in AM

Q3. Which of broad classifications of noise are most difficult to treat?

a) noise generated in the receiver
b) noise generated in the transmitter
c) external noise
d) internal noise

Q4.What points must be important to remember, when we deal with random noise calculations?

a) all calculations are based on peak to peak values
b) calculations are based on quantised values
c) calculations are based on average values
d) calculations are based on RMS values

Analog Communications Questions and Answers – Noise in AM

Q5. Which of the following statement is true?

a) Random noise power is inversely proportional to bandwidth
b) Flicker noise occurs at high frequency
c) Noise mixers are caused by inadequate image frequency rejection
d) A random voltage across a resistance cannot be calculated

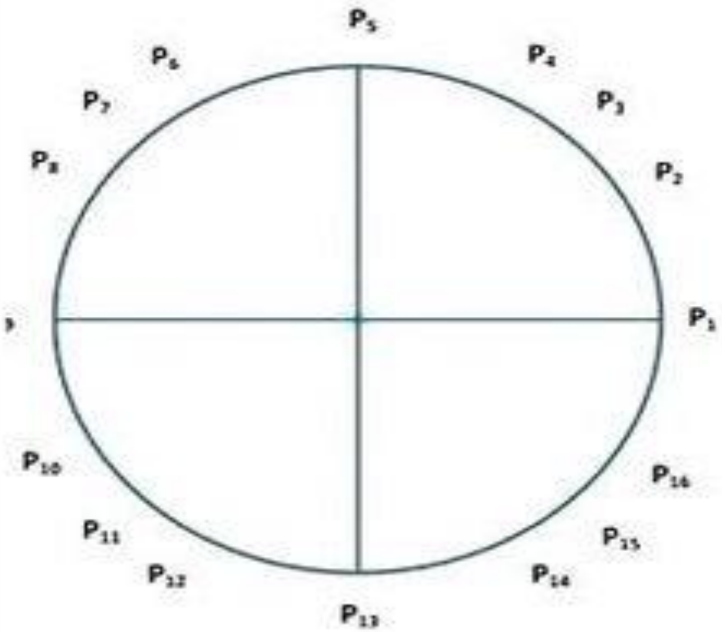
Q6. Which of the following statement is false?

a) Modulation is used to reduce the bandwidth
b) Modulation is used to separate different transmissions
c) Modulation is used to allow the use of practical antennas
d) Modulation is used to ensure that wave is transmitted over long distances

Trigonometric Table

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.

Trigonometric Table



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Math Analysis

Unit Circle and Trigonometric Functions

Point	Degree	Radian	Sin ?	Cos ?	Tan ?	Csc ?	Sec ?	Cot ?
P_1	0, 360	0, 2π	0	1	0	Und.	1	Und.
P_2	30	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$	$\sqrt{3}$
P_3	45	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\sqrt{2}$	$\sqrt{2}$	1
P_4	60	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2\sqrt{3}/3$	2	$\sqrt{3}/3$
P_5	90	$\pi/2$	1	0	Und.	1	Und.	0
P_6	120	$2\pi/3$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$	$2\sqrt{3}/3$	-2	$-\sqrt{3}/3$
P_7	135	$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
P_8	150	$5\pi/6$	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/3$	2	$-2\sqrt{3}/3$	$-\sqrt{3}$
P_9	180	π	0	-1	0	Und.	-1	Und.
P_{10}	210	$7\pi/6$	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}/3$	-2	$-2\sqrt{3}/3$	$\sqrt{3}$
P_{11}	225	$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
P_{12}	240	$4\pi/3$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$-2\sqrt{3}/3$	-2	$\sqrt{3}/3$
P_{13}	270	$3\pi/2$	-1	0	Und.	-1	Und.	0
P_{14}	300	$5\pi/3$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$	$-2\sqrt{3}/3$	2	$-\sqrt{3}/3$
P_{15}	315	$7\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
P_{16}	330	$11\pi/6$	$-1/2$	$\sqrt{3}/2$	$-\sqrt{3}/3$	-2	$2\sqrt{3}/3$	$-\sqrt{3}$

THANK YOU !