This report has been prepared in accordance with the honor code of Brown University. The report and MATLAB code are my own work.

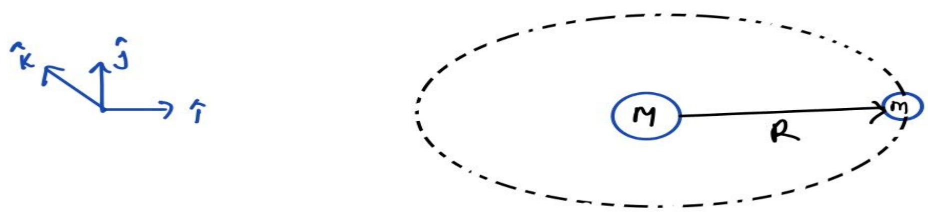
LUNAR IMPACT PROJECT 2020

Abstract
In this mission, a probe is launched from earth to impact the moon with a prescribed velocity. The ejecta plumes are monitored from earth. I am required to calculate the date when we can launch the satellite with critical velocity that will be able to hit the moon. One can imagine a rocket fuel that maintains that velocity of constant magnitude into the direction of its motion. For that we are required to find the appropriate points that can hit the moon and check which point might be a better shot. Along with that we also need to make sure our orbit equations are correct within reason by calculation of other constant quantities in the system that are conserved. In this project, energy and angular momentum has been an appropriate check for that. It is inspiring to see how we are able to determine the path of impact to the moon just by using Newton's laws of gravitation. Thus, for a simpler system like this it is essential to check conserved quantities to be more confident about the results.
Another interesting remark is that most of the numbers derived from the calculations such as time to impact and velocity required can make intuitive sense. However, I could not fully understand my velocity impact result which was around 5 km/s. I cannot judge if that velocity is enough or we want to hit the moon with faster velocity.

Introduction

The mission of this project is to figure out how much velocity might be needed to push a satellite from its GTO orbit such that it will impact the moon. The major problem with that is the orbits of the moon and our satellite are inclined at an angle, meaning they are not coplanar. However, at some special points like apogee where the distance of elliptical orbit from the earth is longest and perigee where it is the shortest. Essentially, it is preferable to attempt to hit the moon at one of these points where the moon’s orbit crosses the satellite’s orbital plane.
After finding the orbit trajectory of both the satellite and the moon, I was able to find those two ‘special’ points. I calculated different times where the collision was possible while finding how much ‘velocity’ satellite should have to hit the moon in one of the smaller points. All this enabled me to find the velocity and date which the satellite should be fired to hit the moon in the shortest time.
MATLAB was used to solve the equations of motion which are derived in the report by using Newton’s equations. The values obtained from different plots and what they signify is also written in the results.

Free Body Diagram:



We know,
Magnitude of the force is $F_g = -GMm/R^2$, where M,m are masses, R is the distance between their center and G is the Gravitational constant.

$F_{net} = ma = -(GMm/R^2)$

$or,\ F_{net} =-(GMm/R^3)*(xi + yj + zk)$

This general equation is used to derive the motion of both the satellite and the moon.
The energy calculations for the satellite or the moon is given as:

$E/m = -GM/R + 1/2(v)^2$

This should be conversed for a closed system.
Additionally,
Angular momentum is also a good check for the trajectory. The expression can be obtained by crossing the position vector with its momentum vector. In closed system, its magnitude is always conserved.

$L/m = R \times v$

Assumptions:
Sun’s gravity is neglected and the Earth is the reference point of observation(earth’s motion around the sun is neglected). Any gravitational forces caused by the satellite's mass has not been considered.

Data and Calculations
To solve the system of differential equations in the MATLAB , one has to specify the initial conditions of motion. Hence, the initial conditions for position and velocity is required to solve the equations of motion in MATLAB.

For the Arianne GTO orbit satellite, the information is given in phase representation for Perigee. The initial conditions for Apogee can be found in part by speaculation and one part by solving for the conservation of angular momentum. Since, perigee and apogee are in the opposite side of the orbit, the positional vector of apogee is the opposite in direction to perigee.

Similarly, sum of the angular momentum about the earth being origin gives us:

$L/m = constant = r \times v$

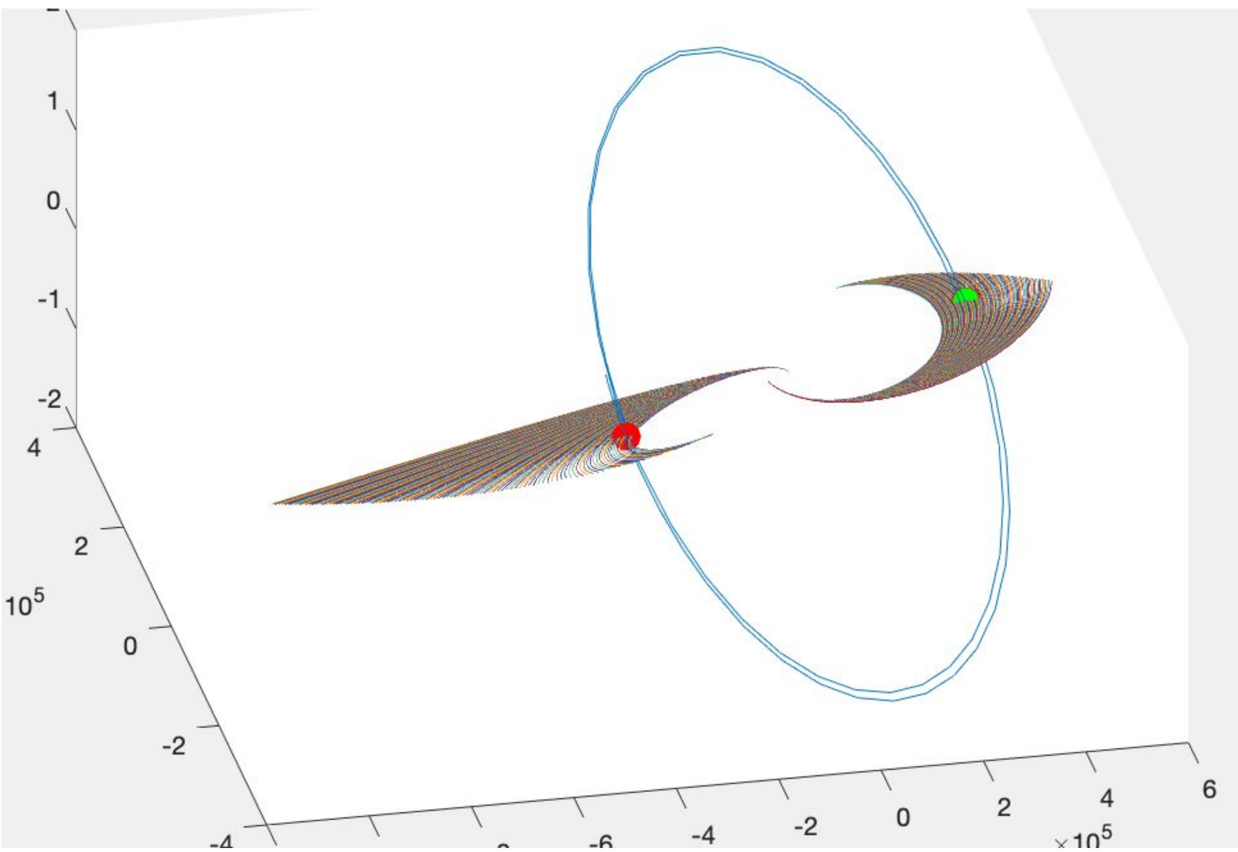
Since, both the apogee and perigee make 90 degrees angle with the velocity vector, we can write:

$r_{perigee}.\ v_{perigee}\ sin\ 90 = r_{apogee}.\ v_{apogee}\ sin\ 90 = \ constant$

$r_{perigee}.\ v_{perigee}$ = $r_{apogee}.\ v_{apogee}$, which is just the other representation of kepler's law which says the area swept is constant.

When the moon is in the plane of the satellite's orbit, its position vector satisfies **r.n =** 0 , where n is a unit vector normal to the plane of the satellite's orbit. This is how we determine possible impact points in the moon's orbit.

The choices and results obtained throughout the process is given below.



Results

1. Where are the two points where the lunar impact may take place?

- It is preferable to attempt to hit the moon at one of the two special points where the moon’s orbit crosses the satellite’s orbit plane. I passed in the initial condition of apogee and perigee position and velocity into the differential equation to obtain the values at different times. After calculations I obtained the following results for perigee and apogee impact by changing velocity which is explained later.

  The distance between apogee and perigee was obtained to be 7.73*10^5 kms, which is very close to the real distance between the moon's perigee and apogee.

  In the image Red point represents perigee and the green point is apogee. As we can see the plane of the moon and the satellite's path intersect at those points. The array of paths represents the possible paths by which we hit the moon. The report will show how rocket’s velocity can be used to do this efficiently in the next sections.

- The results I obtained from calculating the travel time from apogee and perigee are:

```
time_to_hit_from_apogee =

   4.7390e+05
```

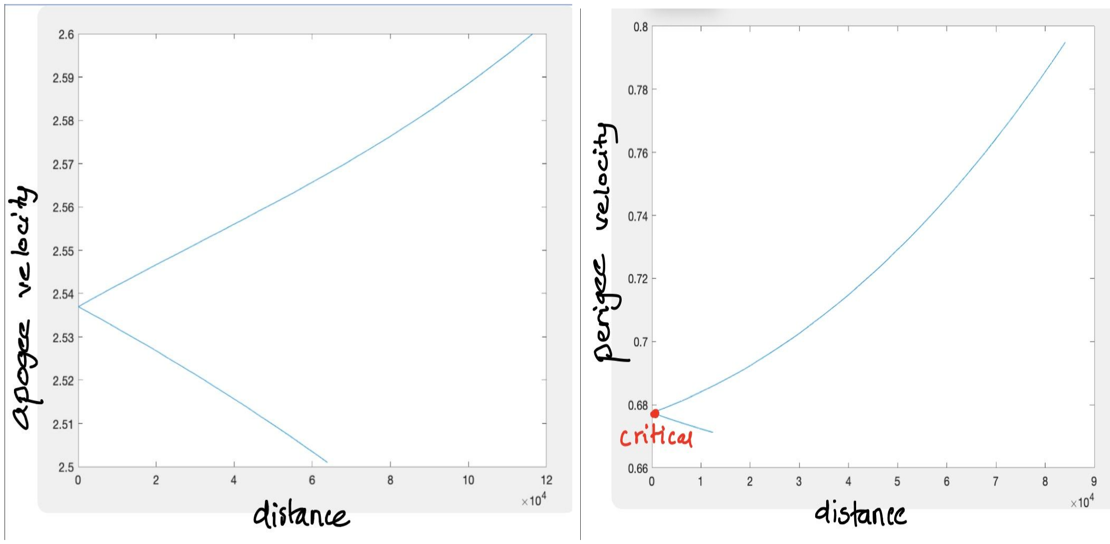
```
time_to_hit_from_perigee =

   2.9643e+05
```

  Time to hit from the apogee was obtained to be around 5.5 days.
  Time to hit from the perigee was obtained to be around 3.43 days
  I want to hit the moon in the first possible orbital point efficiently. So I chose the perigee of the moon to be April 4th 2:08 PM. That would mean the launch dates are:

Apogee	**Perigee**
March 31st at around 2AM	Midnight after April 2nd

Since I want to finish the mission to take less time, I would choose Midnight after April 2nd as my launch date.

From the range of speed the critical additional velocity required for the satellite was found. The plot is included below:



As we can observe the apogee's trajectory required 2.5370 km/s and for the perigee is 0.6775 km/s which is less than apogee's. It is the velocity in which the satellite gets closest to the moon. It was obtained by making an event function that stops when the distance is zero. Impact Velocity was found to be:

```
vel_impact =

    3.5560
```

Since I want to spend as little fuel as possible. I want to maintain the trajectory at low velocity and faster time. Thus GTO should be fired at perigee at midnight after April 2nd 2020. The satellite takes 3.43 days to hit the moon on its trajectory.The impact velocity was found by taking the magnitude of impact of **norm**(V_moon - V_satellite) which is around 3.5560 km/s.

Since, the impact velocity is good enough for the impact I choose perigee. All other necessary calculation of energy conservation and making sure the orbit were correct is given in the Appendix.

References:

Moon Distances- dateandtime.com
GTO- https://www.agi.com/products/satellite-design-and-operations
Kepler's and Gravitation- introduction to analytical mechanics

Appendix:

Conserved quantities:

1. Total Energy
2. Angular momentum of the planet.

$$E = -\frac{GMm}{r} + \frac{1}{2} m v^2$$

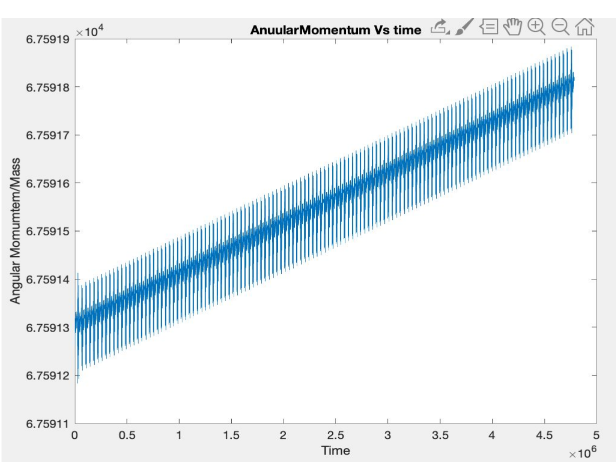
$$\Rightarrow \frac{E}{m} = -\frac{GM}{r} + \frac{1}{2} \cdot (v_x^2 + v_y^2 + v_z^2)$$

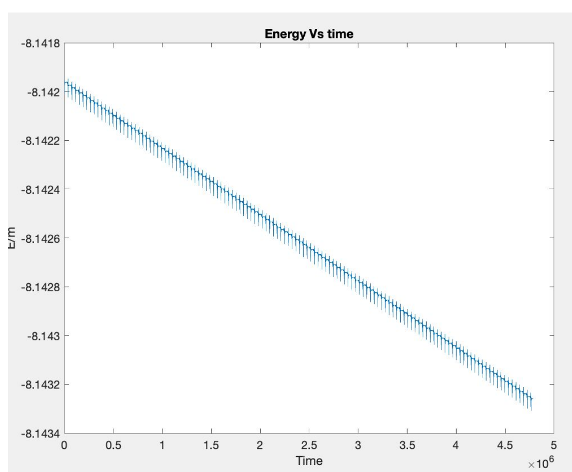
$$\vec{H} = \vec{r} \times m\vec{v} = (x\hat{i} + y\hat{j} + z\hat{k}) \times m(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$\Rightarrow \frac{|\vec{H}|}{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} = (yv_z - zv)\hat{i} - \hat{j}(xv_z - z \cdot v_x) + \hat{k}(x \cdot v_y - y \cdot v_x)$$

$$v_a = \sqrt{\left(\frac{2GM}{(r_a + r_p)} \frac{r_p}{r_a}\right)}$$

The angular momentum is divided by the mass because we do not know the mass of the satellite. As we can see it only fluctuates a bit around 6.75*10^4 and is approximately constant. The graphs are constant as expected:





To calculate the equations of motion for matlab:

We know,

Magnitude of the force is $F_g = \frac{GMm}{r^2}$, where constants are...

$r = \sqrt{x^2+y^2+z^2}$

The direction of the force is always acting towards the origin:

Explain: $-\frac{\vec{r}}{r}$

$$F = -\frac{GMm}{r^2}\hat{r} = -\frac{GMm}{r^3}\vec{r} = -\frac{GMm}{r^3}(x\hat{i}+y\hat{i}+z\hat{i})$$

$$\vec{F} = m\vec{a} \Rightarrow -\frac{GMm}{r^3}(x\hat{i}+y\hat{i}+z\hat{i}) = m(\ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k})$$

This 3D vector represents two differential eqns of motion

$\ddot{x} = Kx$, $\ddot{y} = Ky$, $\ddot{z} = Kz$, $K = -\frac{GM}{r^3}$

(7) Assume at $t=0$, the satellite is at perigee of the orbit, $x = R$, velocity $\frac{dx}{dt}$ = correct init. vel.

$$\vec{r}_p = C\cdot[(\cos\omega\cdot\cos\Omega - \sin\omega\sin\Omega\cos\theta)\,\hat{i} + (\cos\omega\cdot\sin\Omega + \sin\omega\cdot\cos\Omega\cdot\cos\theta)\,\hat{j} + (\sin\omega\cdot\sin\theta)\,\hat{k}]$$

$r_p - R_e = 250$ km

$r_p = 250 + 6378.145$ km

$= 6628.145$ km

$\vec{r}_p = x\hat{i} + y\hat{i} + k\hat{i}$

$\theta = 6^\circ$

$w = 178^\circ \Rightarrow \omega =$

$W = 200^\circ \Rightarrow \Omega =$

This is how we get initial conditions for the satellite and the moon.