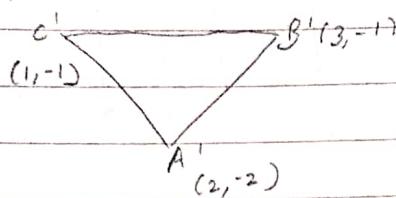
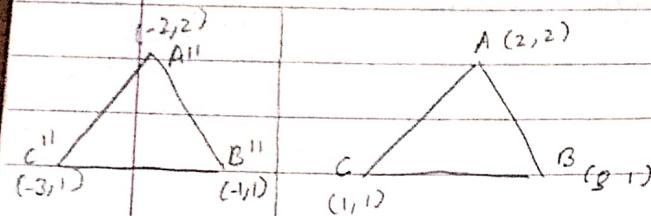


Reflection

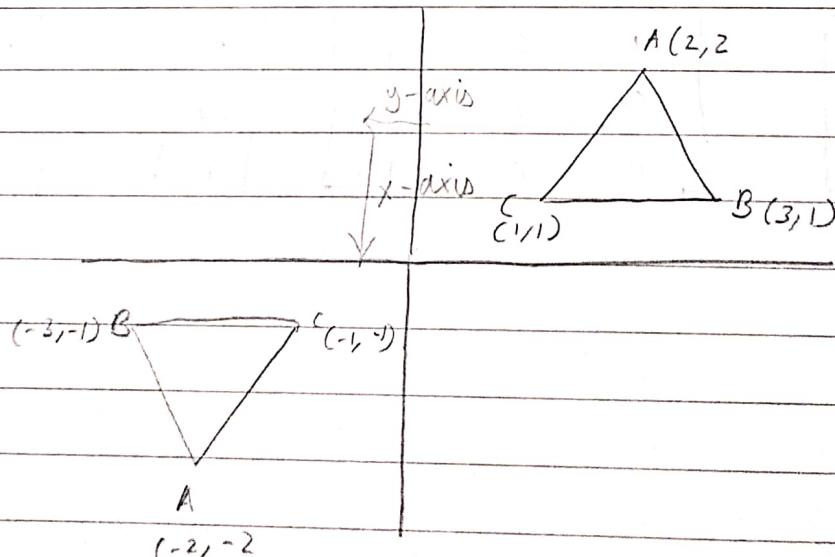
Reflection about x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

IV quadrant
-y

Reflection about y-axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

II quadrant
-x

Reflection about the origin

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

III quadrant
-x, -y

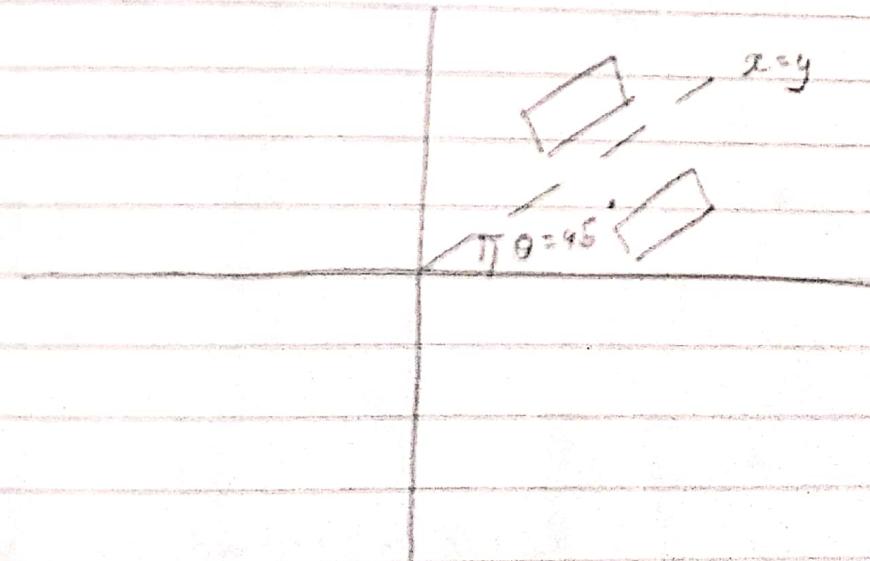
Ex. $A(1, 1)$ $B(2, 3)$ $C(4, 2)$

- Reflection about origin [P]

$$(1, 1) \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = (-1, -1)$$

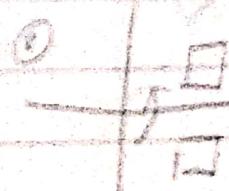
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} = (-2, -3)$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} = (-4, -2)$$



① $R(-\theta)$

$\theta = 45^\circ$



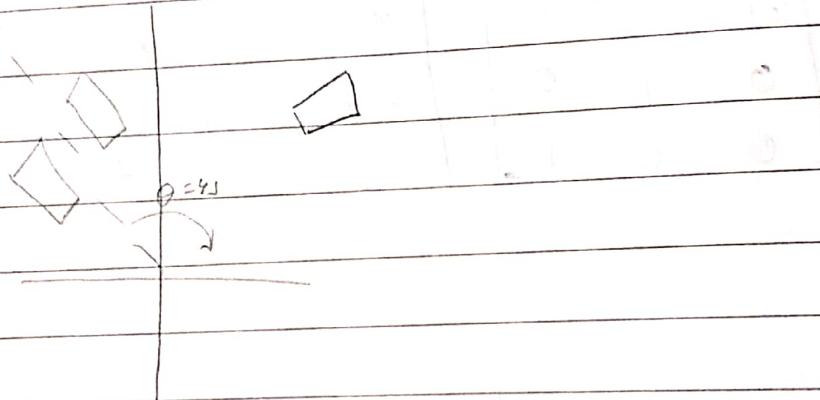
② Reflection about x-axis

③ $R(\theta)$

$$[R_0] \begin{bmatrix} \text{left side} \\ x=0 \end{bmatrix} [R(-45^\circ)] = \boxed{\quad}$$

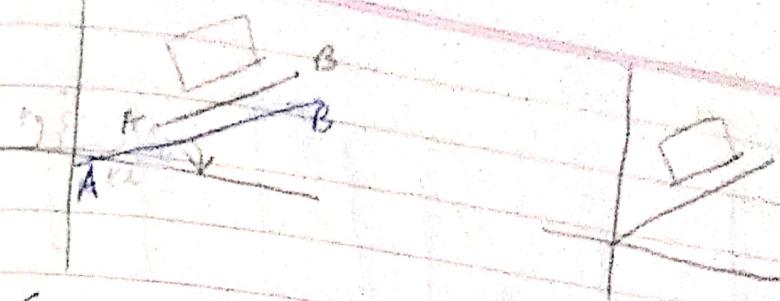
$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



- ① $R(-\theta)$
- ② Reflection about y-axis
- ③ $R(\theta)$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ - Reflection about } x = -y$$



A Translation (t_x, t_y)
Rotation (θ)

Reflection about x -axis
 $R(-\theta)$

$T(-t_x, -t_y)$

i. flip the quadrilateral about x -axis. given $(10, 8) (22, 8)$
 $(34, 17), (10, 27)$

Reflection about x -axis, y -axis

- Reflection about x -axis:

$$(10, 8) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \\ 1 \end{bmatrix} = (10, -8)$$

$$(22, 8) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 22 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 22 \\ -8 \\ 1 \end{bmatrix} = (22, -8)$$

$$(34, 17) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 34 \\ 17 \\ 1 \end{bmatrix} = \begin{bmatrix} 34 \\ -17 \\ 1 \end{bmatrix} = (34, -17)$$

$$(10, 27) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 27 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -27 \\ 1 \end{bmatrix} = (10, -27)$$

Reflection about y-axis :

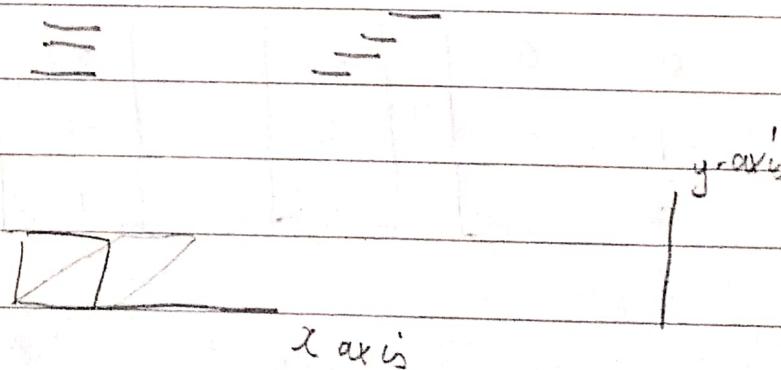
$$(10, 8) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 1 \end{bmatrix} = (-10, 8)$$

$$(22, 8) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -22 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -22 \\ 8 \\ 1 \end{bmatrix} = (-22, 8)$$

$$(34, 17) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -34 \\ 17 \\ 1 \end{bmatrix} = \begin{bmatrix} -34 \\ 17 \\ 1 \end{bmatrix} = (-34, 17)$$

$$(10, 27) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10 \\ 27 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 27 \\ 1 \end{bmatrix} = (-10, 27)$$

SHEAR

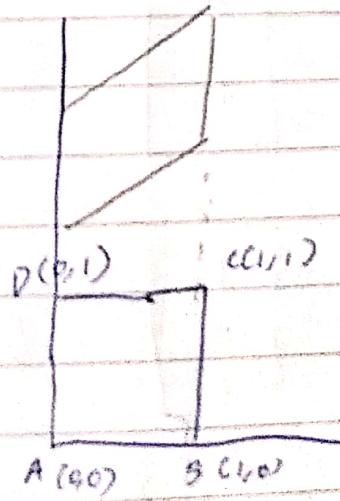
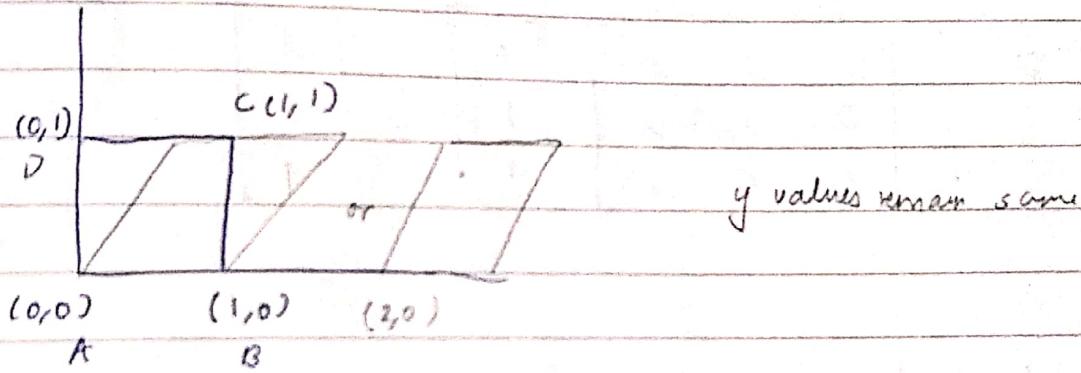


Date: YOUVA
 x -direction shear relative to x -axis

$$\begin{bmatrix} 1 & \text{shx} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

y direction shear relative to y -axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \text{shy} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Ex $\text{shy} = 0.5$ $\text{shx} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\text{sky} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

~~x - direction shear relative to x-axis~~

$$A \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (0, 0)$$

$$B \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = (1, 0)$$

$$C \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = (3, 1)$$

$$D \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = (2, 1)$$

~~y - direction shear relative to y axis~~

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (0, 0)$$

$$B \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = (1, 0)$$

$$C \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix} = (1, 0.5)$$

$$D \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix} = (0, 0.5)$$

120

x -direction shear relative to other reference line.

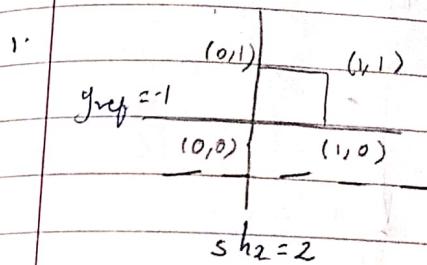
$$= \begin{bmatrix} 1 & sh_2 & -sh_2 x_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$y = y_{ref}$

y -direction shear relative to other reference line

$$= \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y x_{ref} \\ 0 & 0 & 1 \end{bmatrix}$$

$x = x_{ref}$



$$sh_y = 1/2, x_{ref} = -1$$

$$sh_2 = 2$$

- x direction shear relative to other ref line

$$= \begin{bmatrix} 1 & 2 & -2x-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = (2, 0)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} = (4, 1)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} = (5, 1)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = (3, 0)$$

y-direction shear relative to other reference line = $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1/2 & 1 & -1/2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \\ 1 \end{bmatrix} = (0, 0.5)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.5 \\ 1 \end{bmatrix} = (0, 1.5)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = (1, 2)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = (1, 1)$$

Composite Transformations

① Translation

$$\begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx_1 + tx_2 \\ 0 & 1 & ty_1 + ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Given 2 successive translation vectors tx_1, ty_1 & tx_2, ty_2 that are applied to a coordinate position P , then

$$P' = \{ T_{(tx_2, ty_2)} \circ T_{(tx_1, ty_1)} \} \cdot P$$

$$= T_{(tx_2 + tx_1, ty_2 + ty_1)} \cdot P$$

② Rotation

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 0 \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R' =$$

The 2 successive rotations apply to a point P produce the transform positions $P' = \{ R(\theta_2) \circ R(\theta_1) \} \cdot P$

$$= R(\theta_2 + \theta_1) \cdot P$$

③ Scaling

$$\begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx_2 \cdot Sx_1 & 0 & 0 \\ 0 & Sy_2 \cdot Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = \{ S_{(Sx_2, Sy_2)} \circ S_{(Sx_1, Sy_1)} \} \cdot P$$

$$= S(Sx_2 \cdot Sx_1, Sy_2 \cdot Sy_1) \cdot P$$

(4)

General - Pivot Rotation

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

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$\theta = 90^\circ$
 $t_x = -x_r$ $t_x = x_r$
 $t_y = -y_r$ $t_y = y_r$

(5)

General Fixed Point Scaling

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{x_1} & 0 & 0 \\ 0 & s_{y_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore R \rightarrow L$ to begin
 \therefore Top to down TL

$$= \begin{bmatrix} s_{x_1} & 0 & x_r \\ 0 & s_{y_1} & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_{x_1} & 0 & s_{x_1} x_r (1-sx_1) \\ 0 & s_{y_1} & s_{y_1} y_r (1-sy_1) \\ 0 & 0 & 1 \end{bmatrix}$$

(6)

General Scaling Direction

$$\begin{bmatrix} \cos(\theta) & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1 \cos^2\theta + s_2 \sin^2\theta & -(s_2 - s_1) \cos\theta \sin\theta & 0 \\ (s_2 - s_1) \cos\theta \sin\theta & s_2 \cos^2\theta + s_1 \sin^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Given points $A(0,0)$, $B(1,0)$, $C(1,1)$, $D(0,1)$
 Scale the object S_1 by 1, S_2 by 2 where S_1 is at
 angle of 45° with respect to x-axis.

A

$$= \begin{bmatrix} S_1 \cos^2 \theta + S_2 \sin^2 \theta & (S_2 - S_1) \cos \theta \sin \theta & 0 \\ (S_2 - S_1) \cos \theta \sin \theta & S_2 \cos^2 \theta + S_1 \sin^2 \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} & (2-1) \frac{1}{2} & 0 \\ (2-1) \cdot \frac{1}{2} & 2 \cdot \frac{1}{2} + 1 \cdot 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{3/2} = 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

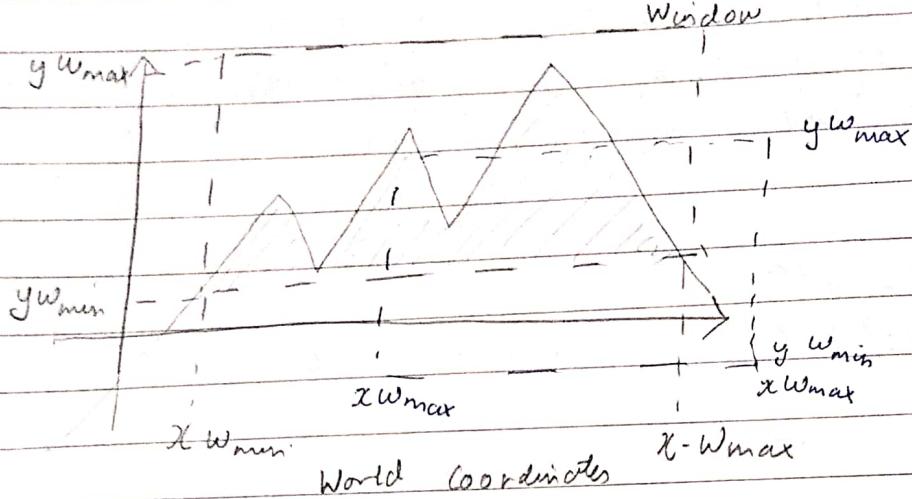
A $\begin{bmatrix} 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (0,0)$

B $\begin{bmatrix} 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \\ 1 \end{bmatrix} = (1.5, 0.5)$

C $\begin{bmatrix} 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 & 2 \\ 0.5 & 2 \\ 1 & 1 \end{bmatrix} = (2, 2)$

D $\begin{bmatrix} 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.5 \\ 1 \end{bmatrix} = (0.5, 1.5)$

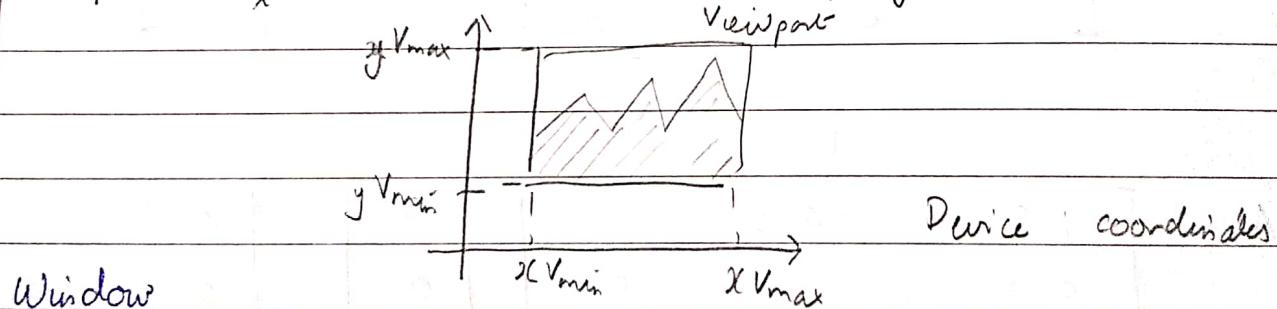
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2-D VIEWING

(Transferring an image from world co-ordinate system to device co-ordinate system \rightarrow 2-D Viewing Transformation.)

Window - Tells the area to be captured

View port - Where it has to be displayed on display device.



A world coordinate area selected for display is called a window

viewport

An area on a display device to which a window is mapped

Mapping of point of world coordinate scene to device coordinates is referred to as device transformation.

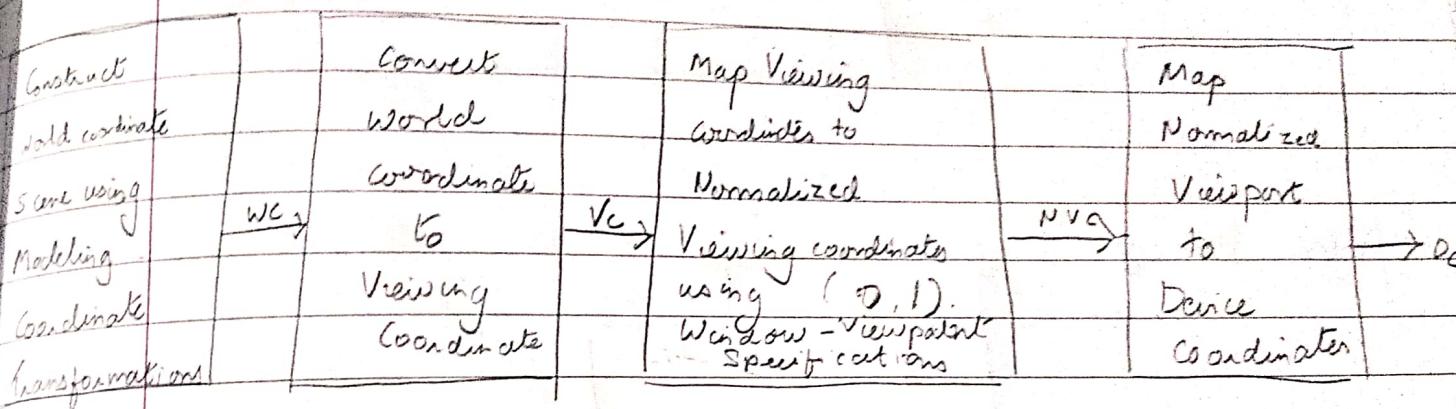


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X

2D Viewing transformation Pipeline



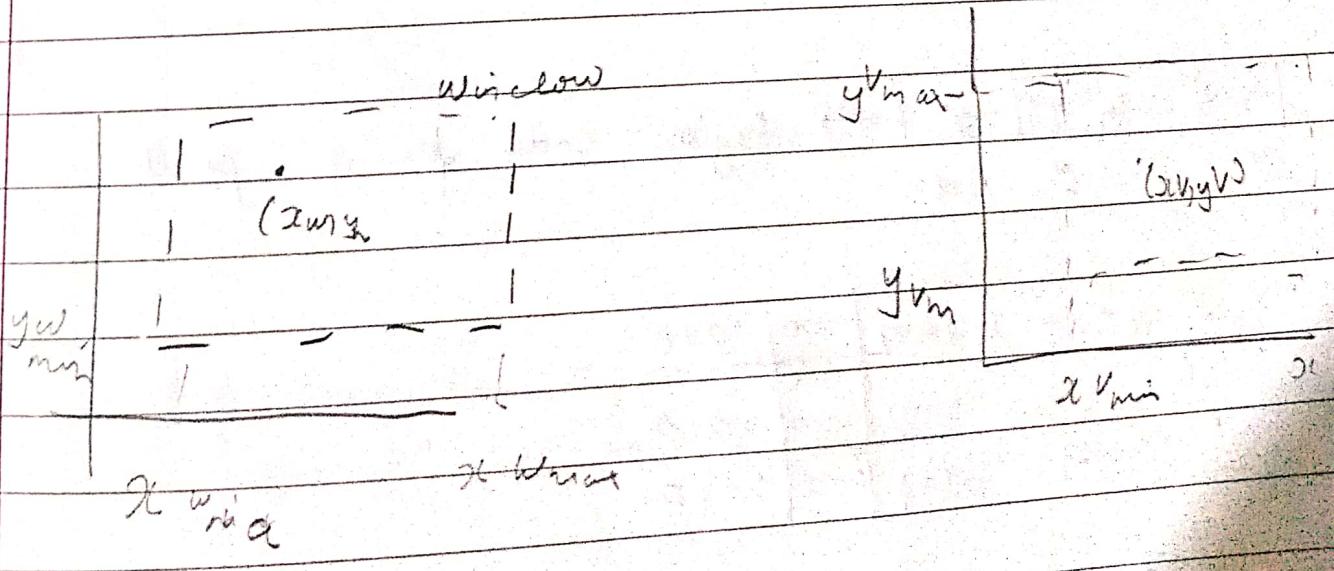
MC - Modelling coordinates

WC - World coordinates

VC - Viewing coordinates

NVC - Normalized Viewing Coordinates

DC - Device coordinates



$$x_V - x_{V\min}$$

$$x_{V\max} - x_{V\min}$$

$$x_W - x_{W\min}$$

$$x_{W\max} - x_{W\min}$$

$$y_V - y_{V\max}$$

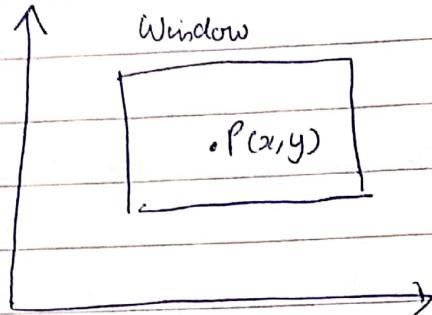
$$y_{V\max} - y_{V\min}$$

$$y_W - y_{W\max}$$

$$y_{W\max} - y_{W\min}$$

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Point Clipping



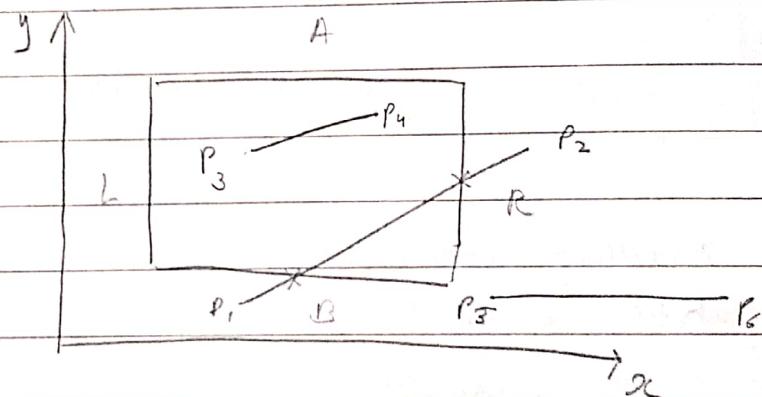
$$x < x_{W\max}$$

$$x > x_{W\min}$$

$$y < y_{W\max}$$

$$y > y_{W\min}$$

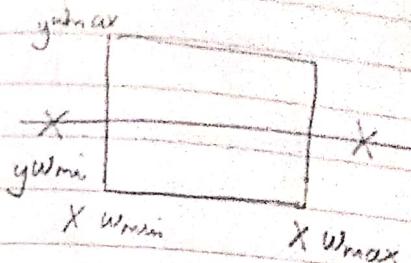
Line Clipping



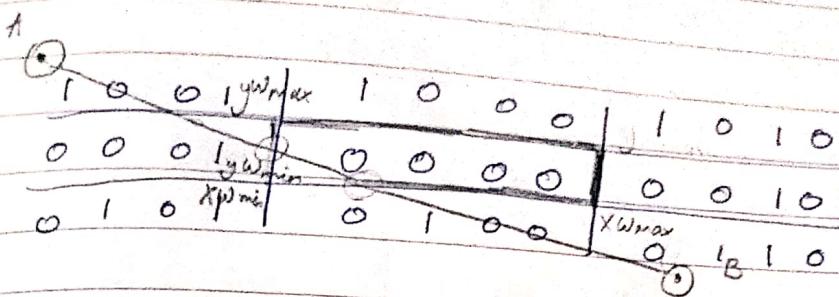
Cohen-Sutherland Line Clipping

Above	Below	Right	Left	→ Region code for a point.
4	3	2	1	\oplus

$P_2 = 1010$	$P_1 = 1001$	$R = 1000$	$B = 0001$	$L = 0101$
$R_1 = 0100$	$R_2 = 0100$	$B_1 = 0000$	$B_2 = 0010$	$L_1 = 0110$



Clipping - discarding unwanted part
(outside the window)



$1001 \rightarrow$ Region Code for A
(AND) $0110 \rightarrow$ Region Code for B
0000

$$x = x_{w\min}$$

$$y = ?$$

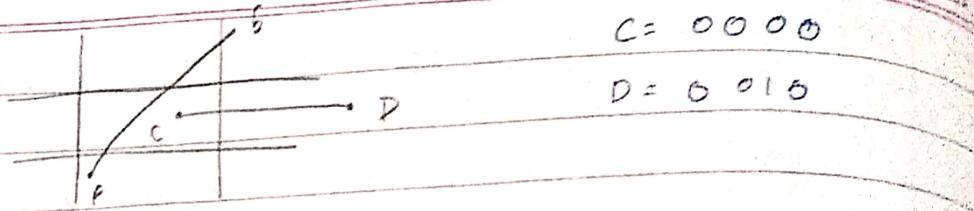
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Replace $(x_2, y_2) \rightarrow (x, y)$

$$m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

* (If any point has region code = 000, then no perform AND operation \rightarrow directly clipping)



C = 0000

D = 0010

$$x = x_{W\max}$$

$$y = y_1 + m(x - x_1)$$

E = 1010

F \Rightarrow 0100

$$\begin{array}{r} 0000 \\ - 0000 \end{array} \quad ; \quad 0000$$

be possibly
previous window

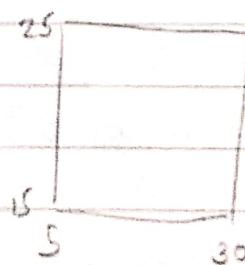
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Replace (x_2, y_2)
 (x, y)

$$y = w_y y_{W\max} / y_{W\min}$$

$$x = x_1 + \frac{y - y_1}{m}$$

1. Clip the line with the coordinates $(x_1, y_1) = (2, 20)$ & $(x_2, y_2) = (10, 15)$ $z_1 = 2$ $w_x = 5$ $w_y = 30$ $w_z = 15$ $w_r = 25$



Region Code for $(x_1, y_1) = (2, 20)$

$$\text{Bit 1: } x - x_{W\min} = x - w_b = 2 - 5 = -3 \Rightarrow \text{neg} \Rightarrow 1$$

$$\text{Bit 2: } x_{W\max} - x = w_r - x = 30 - 2 = 28 \Rightarrow \text{pos} \Rightarrow 0$$

$$\text{Bit 3: } y - y_{W\min} = y - w_b = 20 - 15 = 5 \Rightarrow \text{pos} \Rightarrow 0$$

$$\text{Bit 4: } y_{W\max} - y = w_r - y = 25 - 20 = 5 \Rightarrow \text{pos} \Rightarrow 0$$

Region code is 10001

Region Code for $(x_2, y_2) = (10, 15)$

$$\text{Bit 1: } x - x_{W\min} = x - w_L = 10 - 5 = 5 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 2: } x_{W\max} - x = w_R - x = 20 - 10 = 10 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 3: } y - y_{W\min} = y - w_B = 15 - 10 = 5 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 4: } y_{W\max} - y = w_T - y = 25 - 15 = 10 \Rightarrow +ve \Rightarrow 0$$

Region code is 00001

∴ Since Region Code for 1st point is non-zero value
(i.e. 0001, ∴ Perform left clipping).

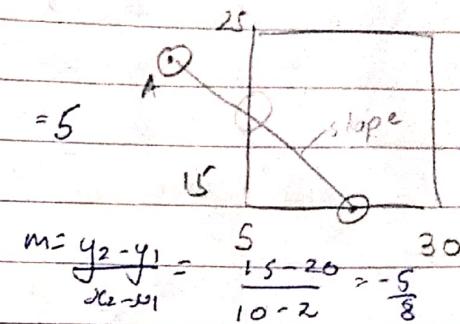
To find Intersection Point

$$x = \frac{x_{W\max} / \text{for right} - x_{W\min} / \text{for left}}{5} = 5$$

$$y = y_1 + m(x - x_1)$$

$$= 15 - \frac{5}{8}(5-10)$$

$$= 18.125$$



(5, 18.125)

Region Code for $(x'_1, y'_1) = (5, 18.125)$

$$\text{Bit 1: } x - x_{W\min} = x - w_L = 5 - 5 = 0 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 2: } x_{W\max} - x = w_R - x = 20 - 5 = 15 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 3: } y - y_{W\min} = y - w_B = 18.125 - 10 = 8.125 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 4: } y_{W\max} - y = w_T - y = 25 - 18.125 = 6.875 \Rightarrow +ve \Rightarrow 0$$

Region code for point 1 is 0000 & Region Code for second pt
0000.

Hence we have clipped the line segment

2. $(x_1, y_1) = (2, 16)$ $(x_2, y_2) = (10, 15)$

Region code for $(x_1, y_1) = (2, 16)$

$$\begin{aligned} \text{Bit 1} &= x - x_{W_{\min}} = x - w_L = 40 - 2 - 5 = +3 \Rightarrow +ve \Rightarrow 1 \\ \text{Bit 2} &= x_{W_{\max}} - x = w_R - x = 30 - 2 = +28 \Rightarrow +ve \Rightarrow 0 \\ \text{Bit 3} &= y - y_{W_{\min}} = y - w_B = 16 - 15 = +1 \Rightarrow +ve \Rightarrow 0 \\ \text{Bit 4} &= y_{W_{\max}} - y = w_T - y = 25 - 16 = 9 \Rightarrow +ve \Rightarrow 0 \end{aligned}$$

\therefore Region code is 1010

Region code for $(x_2, y_2) = (10, 15)$

$$\begin{aligned} \text{Bit 1} &= x - x_{W_{\min}} = x - w_L = 10 - 5 = 5 \\ \text{Bit 2} &= x_{W_{\max}} - x = w_R - x = 30 - 5 = 25 \\ \text{Bit 3} &= y - y_{W_{\min}} = y - w_B = 15 - 15 = 0 \\ \text{Bit 4} &= y_{W_{\max}} - y = w_T - y = 25 - 15 = 10 \end{aligned}$$

\therefore Region code is 0101

$$\begin{array}{r} 1000 \\ 0101 \\ \hline \cancel{0} \end{array} \quad \begin{array}{r} 0001 \\ 00101 \\ \hline 0001 \end{array}$$

\therefore The result of the AND operation is a non-zero value,
the line is completely outside the window region,
Hence, the line is discarded.

3. $(10, 12)$, $(34, 27)$

$$0100$$

Region code for $(x_1, y_1) = (10, 12)$

$$\begin{aligned} \text{Bit 1} &= x - x_{W_{\min}} = x - w_L = 10 - 5 = 5 \Rightarrow +ve \Rightarrow 0 \\ \text{Bit 2} &= x_{W_{\max}} - x = w_R - x = 30 - 10 = 20 \Rightarrow +ve \Rightarrow 0 \\ \text{Bit 3} &= y - y_{W_{\min}} = y - w_B = 12 - 15 = -3 \Rightarrow -ve \Rightarrow 1 \\ \text{Bit 4} &= y_{W_{\max}} - y = w_T - y = 25 - 12 = 13 \Rightarrow +ve \Rightarrow 0 \end{aligned}$$

The Region code is 0 1 0 0

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$$\text{Region code for } (x_2, y_2) = (34, 27)$$

$$\text{Bit 1} = x - x_{w_{\min}} = x - w_1 = 34 - 5 = 29 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 2} = x_{w_{\max}} - x = w_1 - x = 30 - 34 = -4 \Rightarrow -ve \Rightarrow 1$$

$$\text{Bit 3} = y - y_{w_{\min}} = y - w_0 = 27 - 15 = 12 \Rightarrow +ve \Rightarrow 0$$

$$\text{Bit 4} = y_{w_{\max}} - y = w_0 - y = 25 - 27 = -2 \Rightarrow -ve \Rightarrow 1$$

\therefore The Region code is 1010

$$\begin{array}{r} 0100 \\ 1010 \\ \hline 0000 \end{array} \quad | \quad \begin{array}{r} 1010 \\ 0100 \\ \hline \end{array}$$

\therefore first point is non zero (i.e. 0100), perform bottom clipping.

To find intersection point

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - 12}{34 - 10} = \frac{15}{24} = \frac{5}{8}$$

$$y = y_1 - \frac{5}{8}(x_1 - x_1)$$

To find intersection point

$$y = y_{w_{\min}} = 15$$

$$P_1. \quad IS = 12 + \frac{5}{8}(x - 10) \quad IS = 12 + \frac{5}{8}(x - 10)$$

$$IS = 12 + \frac{5}{8}x - \frac{50}{8}$$

$$x = 14.8$$

$$x = 14.8$$

$$\therefore (x'_1, y'_1) = (14.8, 15) \quad (14.8, 15)$$

$$y = y_{W\max} = 25$$

$$25 = 12 + \frac{5}{8}(x - 10)$$

$$= 30.8$$

$$y = y_{W\min} = 15$$

$$25 = 12 + \frac{5}{8}(x - 10)$$

$$x = 30.8$$

$$(25, 25)$$

$$(30.8, 25)$$

$$x = x_{W\max} = 30$$

$$y = y_{W\min} = 15$$

$$y = 27 + \frac{5}{8}(30 - 34)$$

$$= 24.5$$

$$\therefore (x_2', y_2') = (30, 24.5)$$

Region code for $(x_1', y_1') = (14.8, 15)$

$$\text{Bit 1} = x - x_{W\min} = 14.8 - 5 = 9.8 \Rightarrow \text{true} \Rightarrow 0$$

$$\text{Bit 2} = x_{W\max} - x = 30 - 14.8 = 15.2 \Rightarrow \text{true} \Rightarrow 0$$

$$\text{Bit 3} = y - y_{W\min} = 15 - 15 = 0 \Rightarrow \text{true} \Rightarrow 0$$

$$\text{Bit 4} = y_{W\max} - y = 25 - 15 = 10 \Rightarrow \text{true} \Rightarrow 0$$

\therefore The region code is 0000

Region code for $(x_2', y_2') = (30, 24.5)$

$$\text{Bit 1} = x - x_{W\min} = 30 - 5 = 25 \Rightarrow \text{true} \Rightarrow 0$$

$$\text{Bit 2} = x_{W\max} - x = 30 - 30 = 0 \Rightarrow \text{true} \Rightarrow 0$$

$$\text{Bit 3} = y - y_{W\min} = 24.5 - 15 = 9.5 \Rightarrow \text{true} \Rightarrow 0$$

$$\text{Bit 4} = y_{W\max} - y = 25 - 24.5 = 0.5 \Rightarrow \text{true} \Rightarrow 0$$

\therefore The region code is 0000

\therefore The region code of both points is 0000

\therefore we have clipped the line.

$$y = w_T / w_B$$

$$x = w_L / w_B$$

$$z = w_R / w_B$$

Region Code for $(x_1, y_1) = (10, 12)$

Bit 1 = $x - xw_{\max} = 10 - 12 = -2 \Rightarrow \text{neg} \Rightarrow 0$

Bit 2 = $xw_{\max} - x = 12 - 10 = 2 \Rightarrow \text{pos} \Rightarrow 0$

Bit 3 = $y - yw_{\min} = 12 - 10 = 2 \Rightarrow \text{pos} \Rightarrow 0$

Bit 4 = $yw_{\max} - y = 12 - 15 = -3 \Rightarrow \text{neg} \Rightarrow 1$

\therefore Region Code for $(10, 12)$ is 0100

Region Code for $(x_2, y_2) = (34, 27)$

Bit 1 = $x - xw_{\min} = 34 - 5 = 29 \Rightarrow \text{pos} \Rightarrow 0$

Bit 2 = $xw_{\max} - x = 30 - 34 = -4 \Rightarrow \text{neg} \Rightarrow 1$

Bit 3 = $y - yw_{\min} = 27 - 15 = 12 \Rightarrow \text{pos} \Rightarrow 0$

Bit 4 = $yw_{\max} - y = 25 - 27 = -2 \Rightarrow \text{neg} \Rightarrow 1$

\therefore Region code for $(34, 27)$ is 1010

Since both region codes are non-zero values, we perform AND operation.

$$P_1 \rightarrow 0100$$

$$\begin{array}{r} P_2 \rightarrow 1010 \\ \hline 0000 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - 12}{34 - 10} = \frac{15}{24} = \frac{5}{8}$$

Consider Point $(10, 12)$ having region code 0100

We clip against bottom boundary

$$x = \frac{y - y_1 + x_1}{m} = \frac{15 - 12 + 10}{\frac{5}{8}} = 14.8$$

$$y = 15 = w_B$$

Hence, the new point is $(14.8, 15)$

Region Code for $(x'_1, y'_1) = (14.8, 15)$

$$\text{Bit 1} = x - xw_{\min} = 14.8 - 5 = 9.8 \Rightarrow \text{pos} \Rightarrow 0$$

$$\text{Bit 2} = xw_{\max} - x = 30 - 14.8 = 15.2 \Rightarrow \text{pos} \Rightarrow 0$$

$$\text{Bit 3} = y - yw_{\min} = 15 - 15 = 0 \Rightarrow \text{pos} \Rightarrow 0$$

$$\text{Bit 4} = yw_{\max} - y = 25 - 15 = 10 \Rightarrow \text{pos} \Rightarrow 0$$

\therefore The region code for $(34, 27)$ is 0000

T.B.C
1010

Consider point $(34, 27)$ having region code 1010
 We clip against Top boundary
 $x = \frac{y-y_1}{m} + x_1 = 30.8$

$y = w_r = 25$
 Hence, the new point obtained is $(30.8, 25)$

Region code for $(x_2', y_2') = (30.8, 25)$

Bit 1

0 1

Bit 2

1

Bit 3

0

Bit 4

0

- Region code for $(30.8, 25)$ is 0010

We clip against Right Boundary.

$$x = w_{R2} = 30$$

$$y = y_1 + m(x - x_1) = 24.5$$

Region code for $(x_2', y_2') = (30, 24.5)$

Bit 1

1

Bit 2

0

Bit 3

0

Bit 4

0

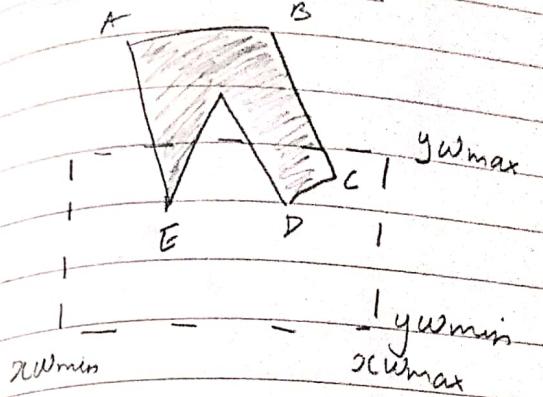
Region code for $(30, 24.5)$ is 0000

Since region code for both the points is 000, 0000.

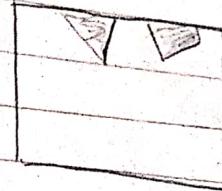
Hence, we have clipped the line

And the new points of line obtained is
 to $(14.8, 15)$ and $(30, 24.5)$

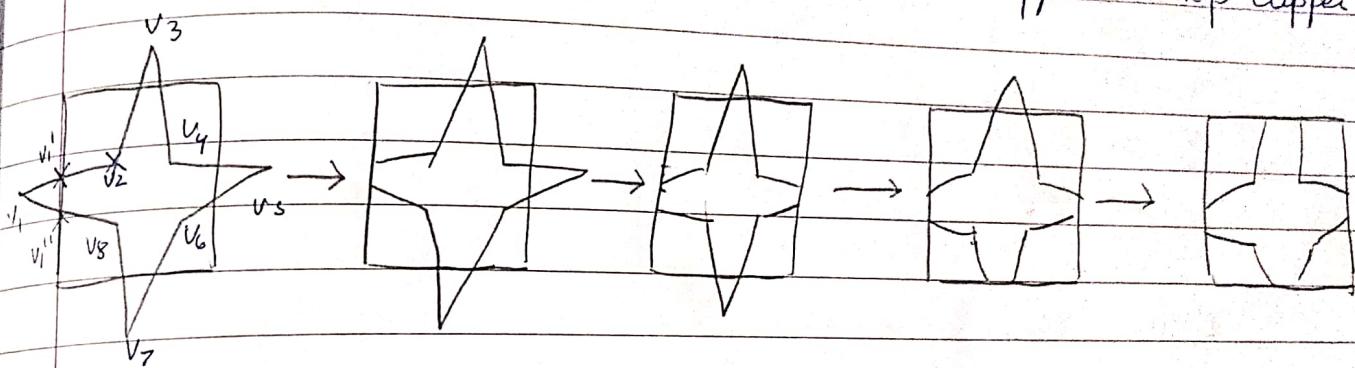
POLYGON CLIPPING



Clip:
L R B T



Sutherland Hodgeman Polygon Clipping
Left clipper \rightarrow Right clipper \rightarrow Bottom clipper \rightarrow Top clipper.



Output Vertex List

for edge v_i, v_j , save $\{v_i', v_j\}$ ($\text{out} \rightarrow \text{in}$)

$v_2 v_3$, Save $\{v_3\}$ ($\text{in} \rightarrow \text{in}$)

$v_8 v_1$ save $\{v_1', v_8\}$ ($\text{in} \rightarrow \text{out}$)

Conditions :-

② Start vertex \rightarrow End vertex

Save $\{\text{out}', \text{in}\}$

① $\text{out} \rightarrow \text{in}$

Save $\{\text{in}\}$

② $\text{in} \rightarrow \text{in}$

Save $\{\text{out}', \text{in}\}$ $\{\text{in}'\}$

③ $\text{in} \rightarrow \text{out}$

Save $\{\text{none}\}$

④ $\text{out} \rightarrow \text{out}$