$$E[X] = \sum_{k=-\infty}^{+\infty} k\, P(X=k) \longleftarrow \text{Population mean of any Discrete RV}$$

(Discrete) Binomial RV

$$E[\log X] = \sum_{k=-\infty}^{+\infty} \log_e k \cdot P(X=k)$$

$$E[X^2] = \sum_{k=-\infty}^{+\infty} k^2 \cdot P(X=k)$$

$$\text{Shoe Size} = \frac{[5+4+3+3+5+6+7+8]}{8} = \bar{X}$$

$$\frac{(5-\bar{x})^2 + (4-\bar{x})^2 + (3-\bar{x})^2 + (3-\bar{x})^2 + (5-\bar{x})^2 + (6-\bar{x})^2 + (7-\bar{x})^2 + (8-\bar{x})^2}{8}$$

$$\frac{2\times(3-\bar{x})^2 + 1\times(4-\bar{x})^2 + 2\times(5-\bar{x})^2 + 1\times(6-\bar{x})^2 + 1\times(7-\bar{x})^2 + 1\times(8-\bar{x})^2}{8}$$

$$\left(\frac{2}{8}\right)\cdot(3-\bar{x})^2 + \left(\frac{1}{8}\right)(4-\bar{x})^2 + \left(\frac{2}{8}\right)(5-\bar{x})^2 + \left(\frac{1}{8}\right)\cdot(6-\bar{x})^2 + \left(\frac{1}{8}\right)(7-\bar{x})^2$$

$$+ \frac{1}{8}(8-\bar{x})^2$$

$$(3-\mu)^2 \cdot P(X=3) + (4-\mu)^2 \cdot P(X=4) + (5-\mu)^2 \cdot P(X=5) + \dots$$

$$\sum_{k=3}^{8} (k-\mu)^2 \cdot P(X=k)$$

$$E[\mu]$$

$$\underbrace{E\left[(X-\mu)^2\right]} = \underbrace{\sum_{k=-\infty}^{+\infty} (k-\mu)^2 \cdot P(X=k)}$$

$$\to E[\mu] = \sum_{k=-\infty}^{+\infty} \mu \cdot P(X=k).$$

$$= \mu P(X=-\infty) + \dots + \mu \cdot P(X=0) + \dots + \mu P(X=+\infty)$$

$$= \mu \left[\sum_{k=-\infty}^{+\infty} P(X=k)\right] = 1 \Rightarrow \mu$$

$$\boxed{E[\text{constant}] = \text{constant}}$$

$$\sum_{k=-\infty}^{+\infty} 2\mu k \cdot P(X=k)$$

$$E\left[(X-\mu)^2\right] = E\left[X^2 + \mu^2 - 2\mu X\right]$$

$$= E\left[X^2\right] + E\left[\mu^2\right] - E\left[2\mu X\right]$$

$$= \boxed{2\mu \sum_{k=-\infty}^{+\infty} k \cdot P(X=k)}$$

$$E[X] = \mu$$

$$= E[X^2] + \mu^2 - 2\mu^2$$

Less the Complex

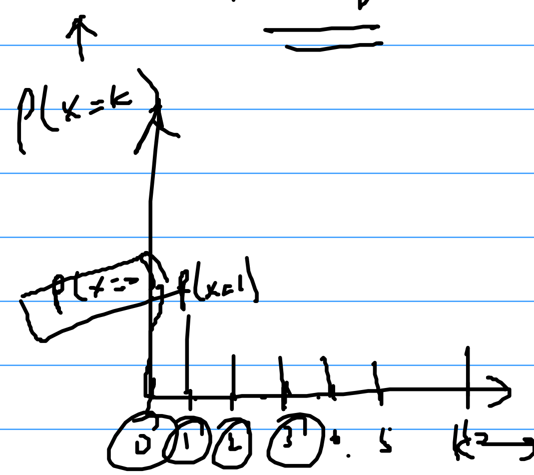
$$= E[X^2] - \mu^2 = \underbrace{E[X^2] - (E[X])^2}$$

$E[(X-\mu)^2] \rightarrow$ More Time Complex

$N=3$, p, q $\quad N=10$

$$E[X] = \sum_{k=-\infty}^{\infty} k \cdot P(X=k)$$

$$= \sum_{k=0}^{3} k \cdot P(X=k)$$



$$P(X=k) = {}^{N}C_k \, (p)^k (q)^{(N-k)}$$

$$= \cancel{0 \cdot P(X=0)} + 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3 \cdot P(X=3)$$

$$= 1 \cdot {}^{3}C_1 (p)^1 (q)^2 + 2 \cdot {}^{3}C_2 (p)^2 \cdot q^1 + 3 \cdot {}^{3}C_3 (p)^3 \cdot q^0$$

$$= 3pq^2 + 6p^2q + 3p^3$$

$$= 3p(q^2 + 2pq + p^2) \Rightarrow 3p \cdot (p+q)^2 = 3p$$

$(1-p)$

$E[X]$

$$P(X=k) = \frac{e^{-\lambda}\,\lambda^{k}}{k!} \qquad \lambda = np = c$$

$$P(K=k) = {}^{N}C_{k}\,(p)^{k}\cdot(q)^{(n-k)}$$

RNN