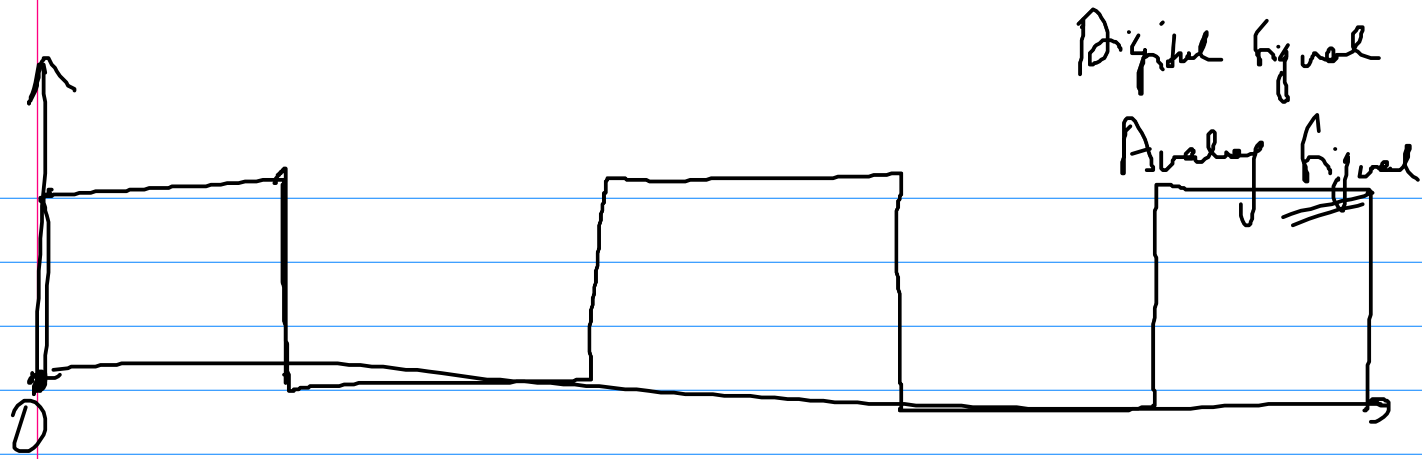
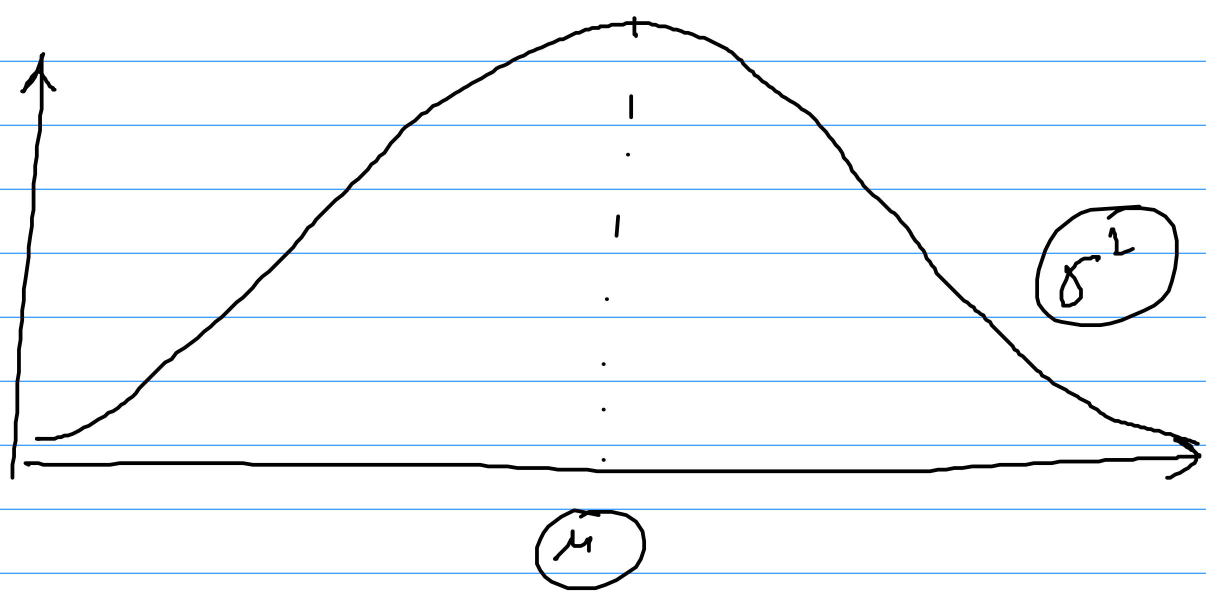


MAXIMUM LIKELIHOOD ESTIMATION :-



Step 1 :- Withdraw sample of size 'N'. Lets say that you get following values in a sample →

$x_1, x_2, x_3, \ldots, x_N$

(49.49 kmph, 22.78 kmph, ...68.7 kmph) lets assume

Let these values are IID. (Independent and Identically Distributed)

Step 2:- Construct Likelihood function from these values. Likelihood function determines the following probability—

$$P(X = x_1 \cap X = x_2 \cap \ldots\ldots X = x_N) =$$

$$\underbrace{P(X = x_1)}\cdot \underbrace{P(X = x_2)} \ldots\ldots\ldots \underbrace{P(X = x_N)}$$

$$\left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right] \,(\;)\,(\;)\,(\;) \ldots \left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x_N-\mu)^2}{2\sigma^2}}\right]$$

$$\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^N e^{\underbrace{-\sum_{i=1}^{N}(x_i-\mu)^2/2\sigma^2}} = L(\mu, \sigma)$$

$$-\frac{(x_1-\mu)^2}{2\sigma^2} - \frac{(x_2-\mu)^2}{2\sigma^2} - \ldots - \frac{(x_N-\mu)^2}{2\sigma^2}$$

$$\underbrace{f(x,y)} \Rightarrow \frac{\partial}{\partial x} f(x,y) = 0$$

$$\frac{\partial}{\partial \mu} L(\mu,\sigma) = 0 \Rightarrow \boxed{\mu = \frac{\sum_{i=1}^{N} x_i}{N}}$$

$$\frac{\partial}{\partial \sigma} L(\mu,\sigma) = 0 \Rightarrow \boxed{\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i-\mu)^2}{N}}}$$

$$\boxed{\sigma^2 = \frac{\sum_{i=1}^{N} (x_i-\mu)^2}{N}}$$

Step 1 :- $P(X=x_1 \wedge X=x_2 \wedge X=x_3 \wedge \ldots X=x_N)$

$$= \underbrace{P(X=x_1)} \cdot \underbrace{P(X=x_2)} \cdot \ldots \ldots \underbrace{P(X=x_N)}$$

$$P(X=k) = \frac{k}{m^2} e^{-k^2/2m^2}$$

$$= \left[\frac{x_1}{m^2} e^{-x_1^2/2m^2}\right] \cdots \left[\frac{x_N}{m^2} e^{-x_N^2/2m^2}\right]$$

$$= \frac{x_1 \cdot x_2 \cdot x_3 \cdots x_N}{(m^2)^N} e^{-\sum_{i=1}^{N} x_i^2 / 2m^2} \Rightarrow L(m)$$

$$\frac{\partial}{\partial m} L(m) \doteq 0 \Rightarrow \boxed{m = \sqrt{\frac{1}{2N} \sum_{i=1}^{N} x_i^2}}$$