

PreCAT Scope : Data Structures

- In Section B => 7 Questions are there for this subject and mostly all questions are theory based or concepts / pseudocode oriented.
- In this course the main focus is on **implementation of basic** data structures and introduction to an advanced data structures to build a base which is required to learn and implement advanced data structures and algorithms in CDAC courses.



+ Introduction

- Data structure
- Algorithm and analysis of an algorithm

+ Array

- Concept & definition
- Searching Algorithms:
- 1. Linear Search (Algorithm & Implementation)
- 2. Binary Search (Algorithm & Implementation)
- Sorting Algorithms:
- 1. Selection Sort (Algorithm & Implementation)
- 2. Bubble Sort (Algorithm & Implementation)
- 3. Insertion Sort (Algorithm & Implementation)
- 4. Quick Sort (Only Algorithm)
- 5. Merge sort (Only Algorithm)



+ Linked List

- Concept & definition
- Types of Linked List
- Operations on Linked List: addition, deletion & traversal.
- Difference between an array and linked list.

+ Stack

- Concept & definition
- Implementation of stack data structure (by using an array)
- Stack applications algorithms:
- 1. Conversion of infix expression into its equivalent prefix
- 2. Conversion of infix expression into its equivalent postfix
- 3. Conversion of prefix expression into its equivalent postfix
- 4. Postfix expression evalution



+ Queue

- Concept & definition
- Types of queue
- Implementation of linear queue & circular queue
- Applications of queue data structure

+ Introduction to an advanced data structure

- Tree : terminologies
- Graph: terminologies
- Hash Table

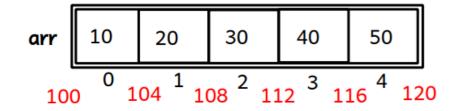


Q. What is Data Structure?

- Data Structure is a way to store data elements into the memory (i.e. into the main memory) in an organized manner so that operations like addition, deletion, traversal, searching, sorting etc... can be performed on it efficiently.



Array: It is a basic/linear data structure, which is a collection/list of logically related similar type of data elements, gets stored into the memory at contiguos locations.



arr: int [] --> arr is an array variable which is a collection/list of 5 int elements

arr = 100 - array name itself is a base address of block of 20 bytes.



```
# Structure: It is a basic/linear data structure, which is a collection/list of logically related similar and disimmilar type of data
elements, gets stored into the memory collectively as a single entity (as a single record).
struct employee {
       int empid;//4 bytes
                                                        "sachin pawar"
                                                                                      9999.99
                                               111
       char name[32];//32 bytes
                                      emp
                                                                                       salary
                                              empid
       float salary;//4 bytes
                                                         name
};
                                          100
                                                   104
                                                                                  136
                                                                                             140
struct employee emp;
sizeof structure = sum of size of all its members ==> sizeof(struct student) = 40 bytes
- We can access members of the structure by its variable through dot operator (.)
emp: struct employee
emp.empid: int
emp.name : char []
emp.salary: float
- We can access members of the structure by its pointer variable through an arrow operator (->)
struct employee *pe = &emp;
pe: struct student * -> sizeof(pe) = 4 bytes
pe->empid: int
pe->name: char[]
pe->salary: float
```



Two types of **Data Structures** are there:

- 1. Linear / Basic Data Structures: data elements gets stored into the memory in a linear manner (e.g. sequentially) and hence can be accessed linearly / sequentially.
 - Array
 - Structure & Union
 - Linked List
 - Stack
 - Queue
- 2. Non-linear / Advanced Data Structures: data elements gets stored into the memory in a non-linear manner (e.g. hierarchical) and hence can be accessed non-linearly.
 - Tree (Hierarchical)
 - Graph
 - Hash Table



Q. What is a Program?

- A Program is a finite set of instructions written in any programming language (like C, C++, Java, Python, Assembly etc...) given to the machine to do specific task.

Q. What is an Algorithm?

- An algorithm is a finite set of instructions written in human understandable language (like english), if followed, acomplishesh a given task.

Q. What is a Pseudocode?

- An algorithm is a finite set of instructions written in human understandable language (like english) with some programming constraints, if followed, acomplishesh a given task, such an algorithm also called as pseudocode.
- An algorithm is a template whereas a program is an implementation of an algorithm.



```
Example: An algorithm to do sum of all array elements
Algorithm ArraySum(A, n)//whereas A is an array of size n
  sum=0;//initially sum is 0
  for( index = 1 ; index <= size ; index++ ) {</pre>
  sum += A[ index ];//add each array element into the sum
  return sum;
```

- In this algorithm, **traversal/scanning** operation is applied on an array. Initially sum is 0, each array element gets added into to the sum by traversing array sequentially from the first element till last element and final result is returned as an output.



- An Algorithm is a solution of a given problem.
- Algorithm = Solution
- One problem may has many solutions:

e.g.

Searching => to search a given key element in a collection/list of elements.

Searching solutions/algorithms: linear search & binary search

Sorting => to arrange data elements in a collection/list of elements either in an ascending order or in a descending order (bydefault in an ascending order).

Sorting solutions/algorithms: bubble sort, selection sort, insertion sort, quick sort, merge sort etc...

- If one problem has many solutions we need to select an efficient solution or algorithm.



- **Analysis of an algorithm** is a work of determining how much **time** i.e. computer time and **space** i.e. computer memory it needs to run to completion.
- There are two measures of an analysis of an algorithms:
- **1. Time Complexity** of an algorithm is the amount of time i.e. computer time required for it to run to completion.
- **2. Space Complexity** of an algorithm is the amount of space i.e. computer memory required for an algorithm to run to completion.
- **Asymptotic Analysis:** It is a **mathematical** way to calculate time complexity and space complexity of an algorithm **without implementing it in any programming language.**
- In this type of analysis, analysis can be done on the basis of **basic operation** in that algorithm.
- e.g. in searching & sorting algorithms comparison is the basic operation and hence analysis gets done on the basis of no. of comparisons, in addition of matrices algorithms addition is the basic operation and hence on the basis of addition operation.



- "Best case time complexity": if an algo takes min amount of time to run to completion then it is referred as best case time complexity.
- "Worst case time complexity": if an algo takes max amount of time to run to completion then it is referred as worst case time complexity.
- "Average case time complexity": if an algo takes neither min nor max amount of time to to run to completion then it is referred as an average case time complexity.

"Asympotic Notations":

- 1. Big Omega (Ω): this notation is used to denote best case time complexity also called as asymptotic lower bound
- 2. Big Oh (O): this notation is used to denote worst case time complexity also called as asymptotic upper bound
- 3. Big Theta (θ) : this notation is used to denote an average case time complexity also called as **asymptotic tight bound**



1. Linear Search/Sequential Search:

```
Step-1: accept key from the user
```

Step-2: compare the value of key with each array element sequentially by traversing it from the first element till either key is found or maximum till the last element. If key is found then return true otherwise return false.

```
Algorithm LinearSearch( A, size, key){
  for( index = 1 ; index <= size ; index++ ){
   if( key == A[ index ] )
    return true;
  }
  return false;
}</pre>
```



Best Case: If key is found at very first position in only 1 no. of comparison then it is considered as a best case and running time of an algorithm in this case is O(1) => and hence time complexity $= \Omega(1)$

Worst Case: If either key is found at last position or key does not exists, maximum \mathbf{n} no. of comparisons takes place, it is considered as a worst case and running time of an algorithm in this case is O(n) => and hence time complexity = O(n)

Average Case: If key is found at any in between position it is considered as an average case and running time of an algorithm in this case is O(n/2) => and hence time complexity $= \Theta(n)$



2. Binary Search/Logarithmic Search:

- This algorithm follows divide-and-conquer approach.
- To apply binary search on an array prerequisite is that array elements must be in a sorted manner.

Step-1: accept key from the user

Step-2: calculate **mid position** of an array by the formula, **mid = (left+right)/2** (by means of calculating mid position big size array gets divided logically into two subarrays, from **left to mid-1 = left subarray** & **mid+1 to right = right subarray**).

Step-3: compare the value of key with element which is at mid position. if key matches with element at mid position means key is found and return true.

Step-4: if key do not matches then check, is the value of key is less than element which is at mid position, if yes then goto search key only into the left subarray by skipping whole right subarray otherwise (means of the value of key is greater than element which is at mid position) goto search key only into the right subarray by skipping whole left subarray.

Step-5: repeat Step-2, Step-3 & Step-4 till either key is not found or max till the subarray is valid, if subarray becomes invalid means key is not found and hence return false in this case.



```
Algorithm BinarySearch(A, n, key) //A is an array of size "n", and key to be search
  left = 1;
  right = n;
  while( left <= right )</pre>
    //calculate mid position
    mid = (left+right)/2;
    //compare key with an ele which is at mid position
    if( key == A[ mid ] )//if found return true
      return true;
    //if key is less than mid position element
    if( key < A[ mid ] )</pre>
      right = mid-1; //search key only in a left subarray
    else//if key is greater than mid position element
      left = mid+1;//search key only in a right subarray
  }//repeat the above steps either key is not found or max any subarray is valid
  return false;
```



Best Case: if the key is found in an array in very first iteration at mid position only **1 no. of comparison** it is considered as a best case and running time of an algorithm in this case is

$$O(1) => \Omega(1).$$

i.e. If key is found at **root position** it is considered as a best case.

Worst Case: if either key is not found or key is found at **leaf position** it is considered as a worst case and running time of an algorithm in this case is $O(\log n) => O(\log n)$.

Average Case: if key is found at non-leaf position it is considered as an average case and running time of an algorithm in this case is $O(\log n) = > \Theta(\log n)$.



1. Selection Sort:

- In this algorithm, in first iteration, first position gets selected and element which is at selected position gets compared with all its next position elements, if selected position element found greater than any other position element then swapping takes place and in first iteration smallest element gets setteled at first position.
- In the second iteration, second position gets selected and element which is at selected position gets compared with all its next position elements, again if selected position element found greater than any other position element then swapping takes place and **in second iteration second smallest element gets setteled at second position**, and so on **in maximum (n-1) no. of iterations** all array elements gets arranged in a sorted manner.



| iteration-1 | iteration-2 | iteration-3 | iteration-4 | iteration-5 |
|---|---|---|---|---|
| 30 20 60 50 10 40 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos |
| 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 10 20 30 40 50 60 0 1 2 3 4 5 |
| 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 10 20 30 40 60 50 0 1 2 3 4 5 | |
| 0 1 2 3 4 5 sel_pos pos | 0 1 2 3 4 5 sel_pos pos | 10 20 30 60 50 40 0 1 2 3 4 5 | | |
| 0 1 2 3 4 5 sel_pos pos | 10 20 60 50 30 40 0 1 2 3 4 5 | | | |
| 10 30 60 50 20 40 0 1 2 3 4 5 | | | | |



Best Case : $\Omega(n^2)$

Worst Case : $O(n^2)$

Average Case : $\theta(n^2)$

2. Bubble Sort:

- In this algorithm, in every iteration elements which are at two consecutive positions gets compared, if they are already in order then no need of swapping between them, but if they are not in order i.e. if previous position element is greater than its next position element then swapping takes place, and by this logic in first iteration largest element gets fixed at last position, in second iteration second largest element gets fixed at second last position and so on, in max (n-1) no. of iterations all elements gets arranged in a sorted manner.



| iteration-1 | iteration-2 | iteration-3 | iteration-4 | iteration-5 |
|---|---|---|---|----------------------------------|
| 30 20 60 50 10 40 | 20 30 50 10 40 60 | 20 30 10 40 50 60 | 20 10 30 40 50 60 | 10 20 30 40 50 60 |
| 0 1 2 3 4 5 pos pos+1 | 0 1 2 3 4 5 pos pos+1 | 0 1 2 3 4 5 pos pos+1 | 0 1 2 3 4 5 pos pos+1 | 0 1 2 3 4 5 pos pos+1 |
| 20 30 60 50 10 40 0 1 2 3 4 5 | 20 30 50 10 40 60 0 1 2 3 4 5 | 20 30 10 40 50 60 0 1 2 3 4 5 | 10 20 30 40 50 60 0 1 2 3 4 5 | 10 20 30 40 50 60 0 1 2 3 4 5 |
| 20 30 60 50 10 40 0 1 2 3 4 5 pos pos+1 | 20 30 50 10 40 60 0 1 2 3 4 5 | 20 10 30 40 50 60 0 1 2 3 4 5 | 10 20 <u>30 40 50 60</u> 0 1 2 3 4 5 | |
| 20 30 50 60 10 40 0 1 2 3 4 5 pos pos+1 | 20 30 10 50 40 60 0 1 2 3 4 5 pos pos+1 | 20 10 30 40 50 60 0 1 2 3 4 5 | | |
| 20 30 50 10 60 40 0 1 2 3 4 5 pos pos+1 | 20 30 10 40 50 60 0 1 2 3 4 5 | | | |
| 20 30 50 10 40 60 0 1 2 3 4 5 | | | | |



Best Case: $\Omega(n)$ - if array elements are already arranged in a sorted manner.

Worst Case : O(n²)

Average Case : $\theta(n^2)$

3. Insertion Sort:

- In this algorithm, one array is considered logically as two sets in which initially first set contains one element and second set contains remaining all elements i.e. (n-1) no. of elements.
- in every iteration one element will be picked sequentially from second set referred as a **key element**, which will be inserted into the first set at its appropriate position i.e. while inserting key element into the first set it makes sure that all the elements in a first set are arranged in a sorted manner, and to do this key element gets compared with elements which are there into the first set from **left to right** and wherever its appropriate position found it will be **inserted** into the first set, by repeating this logic in **max (n-1) iterations** all the elements from second set will be inserted into the first set and all array elements gets arranged into the first set in a sorted manner.



Best Case : $\Omega(n)$ - if array elements are already arranged in a sorted manner.

Worst Case : O(n²)

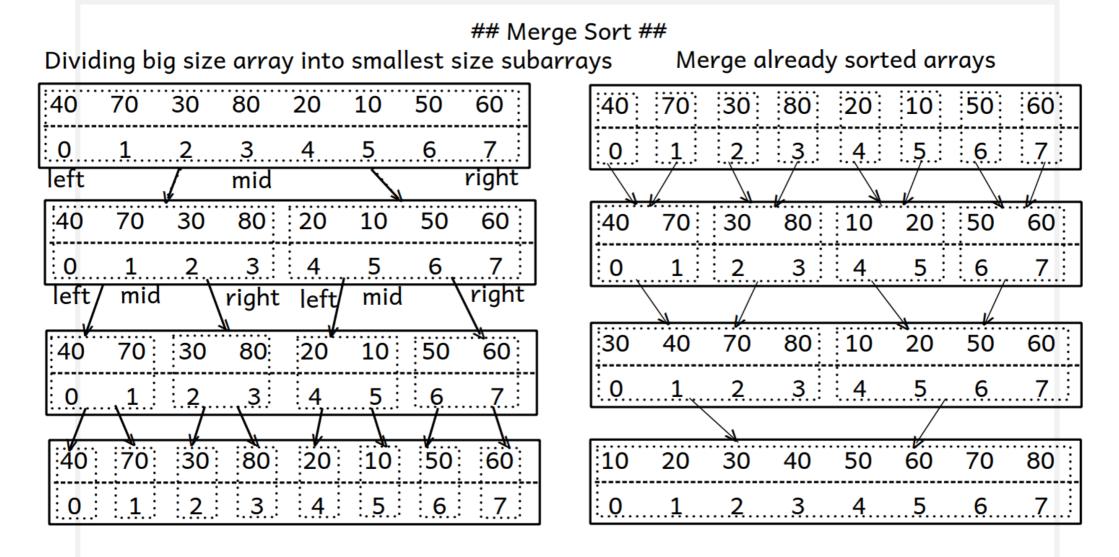
Average Case: $\theta(n^2)$

- Insertion sort algorithm works efficiently for already sorted input array by design.
- Insertion sort algortihm is an efficient algorithm for smaller input size array.

4. Merge Sort:

- This algorithm follows divide-and-conquer approach.
- In this algorithm:
- **Step-1:** big size array gets divided logically into smallest size subarrays (i.e. having size 1)
- **Step-2:** keep on merge already sorted subarrays into a single array in a sorted manner step by step and finally whole array gets sorted.
- This algorithm works fine for **even** as well **odd** input size array.
- This algorithm takes extra space to sort array elements, and hence its space complexity is more.







Best Case : $\Omega(n \log n)$

Worst Case : O(n log n)

Average Case : $\theta(n \log n)$

5. Quick Sort:

- This algorithm follows **divide-and-conquer** approach.
- In this algorithm the basic logic is a partitioning.
- Partitioning:

Step-1: select pivot element (select leftmost element as a pivot element).

Step-2: shift elements towards left side which are smaller than pivot and shift elements towards right side which are greater than pivot, due to this, pivot element gets settled at its appropriate position, and array gets divided logically into two partitions in such a way that elements which are at left of pivot is referred as left partition and elements which are at its right referred as a right partition.



Best Case : $\Omega(n \log n)$

Worst Case: O(n²) - worst case rarely occures

Average Case : $\theta(n \log n)$

- Quick sort algortihm is an efficient sorting algorithm for larger input size array.



- Limitations of an array data structure:
- **1. Array is static**, i.e. size of an array is fixed, its size cannot be either grow or shrink during runtime.
- 2. Addition and deletion operations on an array are not efficient as it takes O(n) time, and hence to overcome these two limitations of an Array, Linked List data structure has been designed.
- # Linked List: It is a basic / linear data structure, which is a collection / list of logically related similar type of data elements in which, an address of first element in that collection / list is stored into a pointer variable referred as a head pointer and each element contains data as well as link of its next element (as well as previous element) i.e. each element contains an address of its next (as well as an address of its previous element).
- Elements in a linked list are **explicitly linked** with each other
- An element in a Linked List is also called as a Node.



- Basically there are two types of linked list:
- 1. Singly Linked List: It is type of linked list in which each node contains an address of its next node only i.e. each node has only one/single link.
- further there are two subtypes of singly linked list
- i. singly linear linked list
- ii. singly circular linked list
- 2. Doubly Linked List: It is type of linked list in which each node contains an address of its next node as well as an addr of its prev node i.e. each node has two links.
- further there are two subtypes of doubly linked list
- i. doubly linear linked list
- ii. doubly circular linked list



- 1. Singly Linear Linked List: It is a type of linked list in which
- **head** always contains an address of first element / node, if list is not empty.
- each node has two parts:
- i. data part: contains data of any primitive / non-primitive type.
- ii. pointer part (next): contains an address of its next element / node.
- last node points to NULL, i.e. next part of last node contains NULL.

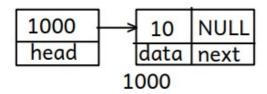


SINGLY LINEAR LINKED LIST

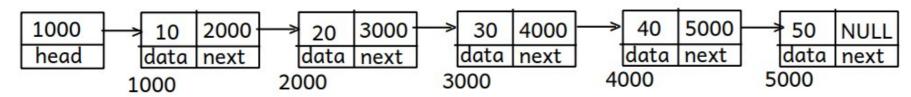
1) singly linear linked list --> list is empty



2) singly linear linked list --> list contains only one node



3) singly linear linked list --> list contains more than one nodes





Limitations of Singly Linear Linked List:

- Add node at last position & delete node at last position operations are not efficient as it takes O(n) time.
- We can starts traversal only from first node and can traverse the SLLL only in a forward direction.
- Previous node of any node cannot be accesed from it.
- Any node cannot be revisited to overcome this limitation Singly Circular Linked List has been designed.
- 2. Singly Circular Linked List: It is a linked list in which
- head always contains an address of first element / node, if list is not empty.
- each node has two parts:
- i. data part: contains data of any primitive / non-primitive type.
- ii. pointer part (next): contains an address of its next element / node.
- last node point to first node, i.e. next part of last node contains an address of first node.

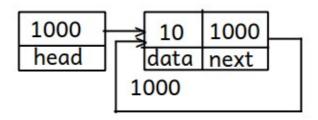


SINGLY CIRCULAR LINKED LIST

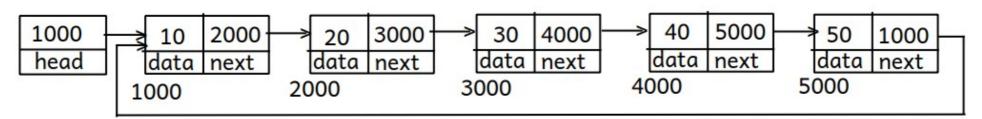
1) singly circular linked list --> list is empty



2) singly circular linked list --> list contains only one node



3) singly circular linked list --> list contains more than one nodes





Limitations of Singly Circular Linked List:

- Add last, delete last & add first, delete first operations are not efficient as it takes O(n) time.
- We can starts traversal only from first node and can traverse the SCLL only in a forward direction.
- Previous node of any node cannot be accessed from it to overcome this limitations of **singly linked list doubly linked list** has been designed.
- 3. Doubly Linear Linked List: It is a type of linked list in which
- head always contains an address of first element / node, if list is not empty.
- each node has three parts:
- i. data part: contains data of any primitive / non-primitive type.
- ii. pointer part (next): contains an address of its next element / node.
- iii .pointer part (prev): contains an address of its previous element / node.
- next part of last node & prev part of first node point to NULL.

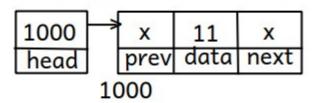


DOUBLY LINEAR LINKED LIST

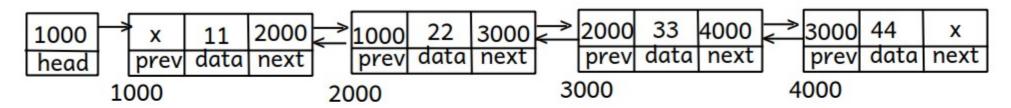
1. doubly linear linked list --> list is empty



2. doubly linear linked list --> list is contains only one node



3. doubly linear linked list --> list is contains more than one nodes



Limitations of Doubly Linear Linked List:

- Add last and delete last operations are not efficient as it takes O(n) time.
- We can starts traversal only from first node, and hence to overcome these limitations Doubly Circular Linked List has been designed.
- 4. Doubly Circular Linked List: It is a linked list in which
- head always contains an address of first node, if list is not empty.
- each node has three parts:
- i. data part: contains data of any primitive / non-primitive type.
- ii. pointer part(next): contains an address of its next element/node.
- iii .pointer part(prev): contains an address of its previous element/node.
- next part of last node contains an address of first node & prev part of first node contains an address of last node.

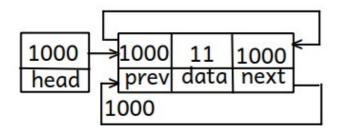


DOUBLY CIRCULAR LINKED LIST

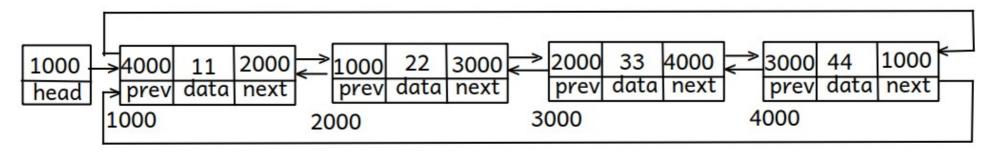
1. doubly circular linked list --> list is empty



2. doubly circular linked list -> list is contains only one node



3. doubly circular linked list --> list is contains more than one nodes





Advantages of Doubly Circular Linked List:

- DCLL can be traversed in forward as well as in a backward direction.
- add last, add first, delete last & delete first operations are efficient as it takes O(1) time and are convenient as well.
- Traversal can be start either from first node or from last node.
- Any node can be revisited.
- Previous node of any node can be accessed from it

Array v/s Linked List:

- Array is **static** data structure whereas linked list is dynamic data structure.
- Array elements can be accessed by using **random access** method which is efficient than linked list elements which can be accessed by **sequential access** method.
- Addition & Deletion operations are efficient on linked list than on an array.
- Array elements gets stored into the **stack section**, whereas linked list elements gets stored into **heap section**.
- In a linked list extra space is required to maintain link between elements, whereas in an array to maintain link between elements is the job of **compiler**.



Stack: It is a **basic/linear data structure**, which is **a collection/list of logically related similar type of data elements** into which data elements can be added as well as deleted from only one end referred **top** end.

- In this collection/list, element which was inserted last only can be deleted first, so this list works in **last in first out/first in last out** manner, and hence it is also called as **LIFO list**/FILO list.
- We can perform basic three operations on stack in O(1) time: Push, Pop & Peek.
- 1. Push: to insert/add an element onto the stack at top position
 - step1: check stack is not full (if stack is not full then only an ele can be push onto the stack).
 - step2: increment the value of top by 1
 - step3: insert an element onto the stack at top position.
- 2. Pop : to delete/remove an element from the stack which is at top position
- step1: check stack is not empty (if stack is not empty then only an element can be popped from the stack).
 - step2: decrement the value of top by 1.



3. Peek: to get the value of an element which is at top position without push & pop.

step1: check stack is not empty

step2: return the value of an element which is at top position

Stack Empty : top == -1

Stack Full : top == SIZE-1

Applications of Stack:

- Stack is used by an OS to control of flow of an execution of program.
- In recursion internally an OS uses a stack.
- undo & redo functionalities of an OS are implemented by using stack.
- Stack is used to implement advanced data structures algorithms like **DFS: Depth First Search** traversal in tree & graph.
- Stack is used in expression conversions algorithms and expression evaluation algorithms.



- Algorithm to convert given infix expression into its equivalent postfix expression:

Initially we have, an Infix expression, an empty Postfix expression & empty Stack.

```
# algorithm to convert given infix expression into its equivalent postfix expression
step1: start scanning infix expression from left to right
step2:
   if( cur ele is an operand )
        append it into the postfix expression
    else//if( cur ele is an operator )
        while( !is stack empty(&s) && priority(topmost ele) >= priority(cur ele) )
           pop an ele from the stack and append it into the postfix expression
        push cur ele onto the stack
step3: repeat step1 & step2 till the end of infix expression
step4: pop all remaining ele's one by one from the stack and append them into the
postfix expression.
```



- Algorithm to convert given infix expression into its equivalent prefix expression:

Initially we have, an Infix expression, an empty Prefix expression & empty Stack.

```
# algorithm to convert given infix expression into its equivalent prefix:
step1: start scanning infix expression from right to left
step2:
   if( cur ele is an operand )
        append it into the prefix expression
    else//if( cur ele is an operator )
        while( !is_stack_empty(&s) && priority(topmost ele) > priority(cur ele) )
           pop an ele from the stack and append it into the prefix expression
        push cur ele onto the stack
step3: repeat step1 & step2 till the end of infix expression
step4: pop all remaining ele's one by one from the stack and append them into the
prefix expression.
step5: reverse prefix expression - equivalent prefix expression.
```

