

Data Structure

Data Structures: Introduction

PreCAT Scope : Data Structures

- In Section B => **7 Questions** are there for this subject and mostly all questions are **theory based or concepts / pseudocode oriented**.
- In this course the main focus is on **implementation of basic data structures** and **introduction to an advanced data structures** to build a base which is required to learn and implement advanced data structures and algorithms in CDAC courses.



Data Structures: Introduction

+ Introduction

- Data structure
- Algorithm and analysis of an algorithm

+ Array

- Concept & definition
- **Searching Algorithms:**
 1. Linear Search (Algorithm & Implementation)
 2. Binary Search (Algorithm & Implementation)
- **Sorting Algorithms:**
 1. Selection Sort (Algorithm & Implementation)
 2. Bubble Sort (Algorithm & Implementation)
 3. Insertion Sort (Algorithm & Implementation)
 4. Quick Sort (Only Algorithm)
 5. Merge sort (Only Algorithm)



Data Structures: Introduction

+ **Linked List**

- Concept & definition
- Types of Linked List
- Operations on Linked List : addition, deletion & traversal.
- Difference between an array and linked list.

+ **Stack**

- Concept & definition
- Implementation of stack data structure (by using an array)
- Stack applications algorithms:
 1. Conversion of infix expression into its equivalent prefix
 2. Conversion of infix expression into its equivalent postfix
 3. Conversion of prefix expression into its equivalent postfix
 4. Postfix expression evaluation



Data Structures: Introduction

+ Queue

- Concept & definition
- Types of queue
- Implementation of linear queue & circular queue
- Applications of queue data structure

+ Introduction to an advanced data structure

- Tree : terminologies
- Graph : terminologies
- Hash Table



Data Structures: Introduction

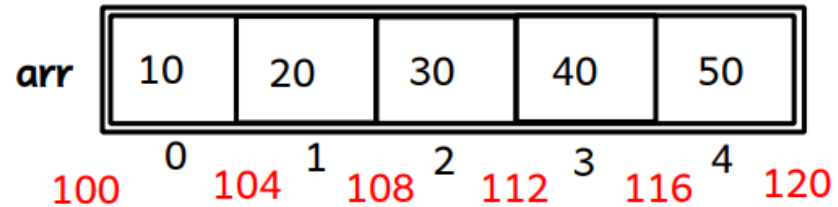
Q. What is Data Structure?

- Data Structure is **a way to store data elements into the memory (i.e. into the main memory) in an organized manner** so that operations like **addition, deletion, traversal, searching, sorting** etc... can be performed on it efficiently.



Data Structures: Introduction

Array: It is a **basic/linear data structure**, which is a collection/list of **logically related similar type of data elements**, gets stored into the memory at **contiguous locations**.



arr: int [] --> arr is an array variable which is a collection/list of 5 int elements

arr = 100 - array name itself is a base address of block of 20 bytes.

arr[0] : int	--> arr[0] = 10, & arr[0] = 100	arr[0] = *(arr+0) = *(100+0) = *(100) = 10
arr[1] : int	--> arr[1] = 20, & arr[1] = 104	arr[1] = *(arr+1) = *(100+1) = *(104) = 20
arr[2] : int	--> arr[2] = 30, & arr[2] = 108	arr[2] = *(arr+2) = *(100+2) = *(108) = 30
arr[3] : int	--> arr[3] = 40, & arr[3] = 112	arr[3] = *(arr+3) = *(100+3) = *(112) = 40
arr[4] : int	--> arr[4] = 50, & arr[4] = 116	arr[4] = *(arr+4) = *(100+4) = *(116) = 50



Data Structures: Introduction

Structure: It is a **basic/linear data structure**, which is a collection/list of **logically related similar and dissimilar type of data elements**, gets stored into the memory collectively as a **single entity** (as a single record).

```
struct employee {  
    int empid;//4 bytes  
    char name[32];//32 bytes  
    float salary;//4 bytes  
};
```

emp

111	"sachin pawar"	9999.99
empid	name	salary

100 104 136 140

```
struct employee emp;
```

sizeof structure = sum of size of all its members ==> sizeof(struct student) = 40 bytes

- We can access members of the structure by its variable through dot operator (.)

emp: struct employee

emp.empid: int

emp.name : char []

emp.salary: float

- We can access members of the structure by its pointer variable through an arrow operator (->)

```
struct employee *pe = &emp;
```

pe: struct student * -> sizeof(pe) = 4 bytes

pe->empid: int

pe->name : char []

pe->salary: float



Data Structures: Introduction

Two types of **Data Structures** are there:

1. Linear / Basic Data Structures: data elements gets stored into the memory in a linear manner (e.g. sequentially) and hence can be accessed **linearly / sequentially**.

- Array
- Structure & Union
- Linked List
- Stack
- Queue

2. Non-linear / Advanced Data Structures: data elements gets stored into the memory in a **non-linear manner (e.g. hierarchical)** and hence can be accessed **non-linearly**.

- Tree (Hierarchical)
- Graph
- Hash Table



Data Structures: Introduction

Q. What is a Program?

- A Program is a finite set of instructions written in any programming language (like C, C++, Java, Python, Assembly etc...) given to the machine to do specific task.

Q. What is an Algorithm?

- An algorithm is a finite set of instructions written in human understandable language (like english), if followed, accomplishes a given task.

Q. What is a Pseudocode?

- An algorithm is a finite set of instructions written in human understandable language (like english) **with some programming constraints**, if followed, accomplishes a given task, such an algorithm also called as **pseudocode**.
- **An algorithm is a template whereas a program is an implementation of an algorithm.**



Data Structures: Introduction

Example: An algorithm to do sum of all array elements

Algorithm ArraySum(A, n)//whereas A is an array of size n

```
{  
    sum=0;//initially sum is 0  
    for( index = 1 ; index <= size ; index++ ) {  
        sum += A[ index ];//add each array element into the sum  
    }  
    return sum;  
}
```

- In this algorithm, **traversal/scanning** operation is applied on an array. Initially sum is 0, each array element gets added into to the sum by traversing array sequentially from the first element till last element and final result is returned as an output.



Data Structures: Introduction

- An Algorithm is a solution of a given problem.
- Algorithm = Solution
- One problem may has many solutions:

e.g.

Searching => to search a given key element in a collection/list of elements.

Searching solutions/algorithms : **linear search & binary search**

Sorting => to arrange data elements in a collection/list of elements either in an ascending order or in a descending order (bydefault in an ascending order).

Sorting solutions/algorithms : **bubble sort, selection sort, insertion sort, quick sort, merge sort etc...**

- If one problem has many solutions we need to select an efficient solution or algorithm.



Data Structures: Introduction

- **Analysis of an algorithm** is a work of determining how much **time** i.e. computer time and **space** i.e. computer memory it needs to run to completion.

- There are two measures of an analysis of an algorithms:

- 1. Time Complexity** of an algorithm is the amount of time i.e. computer time required for it to run to completion.

- 2. Space Complexity** of an algorithm is the amount of space i.e. computer memory required for an algorithm to run to completion.

Asymptotic Analysis: It is a **mathematical** way to calculate time complexity and space complexity of an algorithm **without implementing it in any programming language**.

- In this type of analysis, analysis can be done on the basis of **basic operation** in that algorithm.

e.g. in searching & sorting algorithms comparison is the basic operation and hence analysis gets done on the basis of no. of comparisons, in addition of matrices algorithms addition is the basic operation and hence on the basis of addition operation.



Data Structures: Introduction

"Best case time complexity": if an algo takes min amount of time to run to completion then it is referred as best case time complexity.

"Worst case time complexity": if an algo takes max amount of time to run to completion then it is referred as worst case time complexity.

"Average case time complexity": if an algo takes neither min nor max amount of time to run to completion then it is referred as an average case time complexity.

"Asympotic Notations":

1. Big Omega (Ω): this notation is used to denote best case time complexity – also called as **asymptotic lower bound**

2. Big Oh (O): this notation is used to denote worst case time complexity – also called as **asymptotic upper bound**

3. Big Theta (Θ): this notation is used to denote an average case time complexity – also called as **asymptotic tight bound**



Data Structures: Searching Algorithms

1. Linear Search/Sequential Search:

Step-1: accept key from the user

Step-2: compare the value of key with each array element sequentially by traversing it from the first element till either key is found or maximum till the last element. If key is found then return true otherwise return false.

```
Algorithm LinearSearch( A, size, key){  
    for( index = 1 ; index <= size ; index++ ){  
        if( key == A[ index ] )  
            return true;  
    }  
    return false;  
}
```



Data Structures: Searching Algorithms

Best Case: If key is found at very first position in only 1 no. of comparison then it is considered as a best case and running time of an algorithm in this case is **$O(1)$** => and hence time complexity = **$\Omega(1)$**

Worst Case: If either key is found at last position or key does not exist, maximum **n** no. of comparisons takes place, it is considered as a worst case and running time of an algorithm in this case is **$O(n)$** => and hence time complexity = **$O(n)$**

Average Case: If key is found at any in between position it is considered as an average case and running time of an algorithm in this case is **$O(n/2)$** => and hence time complexity = **$\theta(n)$**



Data Structures: Searching Algorithms

2. Binary Search/Logarithmic Search:

- This algorithm follows **divide-and-conquer** approach.
- To apply binary search on an array prerequisite is that **array elements must be in a sorted manner**.

Step-1 : accept key from the user

Step-2 : calculate **mid position** of an array by the formula, **$\text{mid} = (\text{left} + \text{right}) / 2$** (by means of calculating mid position big size array gets divided logically into two subarrays, from **left to mid-1 = left subarray** & **mid+1 to right = right subarray**).

Step-3 : compare the value of key with element which is at mid position. if key matches with element at mid position means key is found and return true.

Step-4 : if key do not matches then check, is the value of key is less than element which is at mid position, if yes then goto **search key only into the left subarray by skipping whole right subarray otherwise (means of the value of key is greater than element which is at mid position) goto search key only into the right subarray by skipping whole left subarray**.

Step-5 : repeat Step-2, Step-3 & Step-4 till either key is not found or max till the subarray is valid, if subarray becomes invalid means key is not found and hence return false in this case.



Data Structures: Searching Algorithms

```
Algorithm BinarySearch(A, n, key) //A is an array of size "n", and key to be search
{
    left = 1;
    right = n;

    while( left <= right )
    {
        //calculate mid position
        mid = (left+right)/2;
        //compare key with an ele which is at mid position
        if( key == A[ mid ] )//if found return true
            return true;

        //if key is less than mid position element
        if( key < A[ mid ] )
        {
            right = mid-1; //search key only in a left subarray
        }
        else //if key is greater than mid position element
        {
            left = mid+1; //search key only in a right subarray
        }
    } //repeat the above steps either key is not found or max any subarray is valid
    return false;
}
```



Data Structures: Searching Algorithms

Best Case: if the key is found in an array in very first iteration at mid position only **1 no. of comparison** it is considered as a best case and running time of an algorithm in this case is

$O(1) \Rightarrow \Omega(1)$.

i.e. If key is found at **root position** it is considered as a best case.

Worst Case: if either key is not found or key is found at **leaf position** it is considered as a worst case and running time of an algorithm in this case is $O(\log n) \Rightarrow O(\log n)$.

Average Case: if key is found at **non-leaf position** it is considered as an average case and running time of an algorithm in this case is $O(\log n) \Rightarrow \theta(\log n)$.



Data Structures: Sorting Algorithms

1. Selection Sort:

- In this algorithm, in first iteration, first position gets selected and element which is at selected position gets compared with all its next position elements, **if selected position element found greater than any other position element then swapping takes place and in first iteration smallest element gets settled at first position.**
- In the second iteration, second position gets selected and element which is at selected position gets compared with all its next position elements, again if selected position element found greater than any other position element then swapping takes place and **in second iteration second smallest element gets settled at second position**, and so on **in maximum (n-1) no. of iterations** all array elements gets arranged in a sorted manner.



Data Structures: Sorting Algorithms

Iteration-1	Iteration-2	Iteration-3	Iteration-4	Iteration-5
<div><div>302060501040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>103060502040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102060503040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030605040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030406050</div><div>012345</div><div>sel_pospos</div></div>
<div><div>203060501040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>103060502040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102050603040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030506040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030405060</div><div>012345</div><div></div></div>
<div><div>203060501040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>103060502040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030605040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030406050</div><div>012345</div><div></div></div>	
<div><div>203060501040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102060503040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102030605040</div><div>012345</div><div></div></div>		
<div><div>103060502040</div><div>012345</div><div>sel_pospos</div></div>	<div><div>102060503040</div><div>012345</div><div></div></div>			
<div><div>103060502040</div><div>012345</div><div></div></div>				
<div><div>103060502040</div><div>012345</div><div></div></div>				



Data Structures: Sorting Algorithms

Best Case : $\Omega(n^2)$

Worst Case : $O(n^2)$

Average Case : $\theta(n^2)$

2. Bubble Sort:

- In this algorithm, **in every iteration elements which are at two consecutive positions gets compared**, if they are already in order then no need of swapping between them, but if they are not in order i.e. if previous position element is greater than its next position element then swapping takes place, and by this logic **in first iteration largest element gets fixed at last position, in second iteration second largest element gets fixed at second last position** and so on, **in max (n-1) no. of iterations** all elements gets arranged in a sorted manner.



Data Structures: Sorting Algorithms

Iteration-1	Iteration-2	Iteration-3	Iteration-4	Iteration-5
<div><div>302060501040</div><div>012345</div><div>pospos+1</div></div>	<div><div>203050104060</div><div>012345</div><div>pospos+1</div></div>	<div><div>203010405060</div><div>012345</div><div>pospos+1</div></div>	<div><div>201030405060</div><div>012345</div><div>pospos+1</div></div>	<div><div>102030405060</div><div>012345</div><div>pospos+1</div></div>
<div><div>203060501040</div><div>012345</div><div>pospos+1</div></div>	<div><div>203050104060</div><div>012345</div><div>pospos+1</div></div>	<div><div>203010405060</div><div>012345</div><div>pospos+1</div></div>	<div><div>102030405060</div><div>012345</div><div>pospos+1</div></div>	<div><div>102030405060</div><div>012345</div><div></div></div>
<div><div>203060501040</div><div>012345</div><div>pospos+1</div></div>	<div><div>203050104060</div><div>012345</div><div>pospos+1</div></div>	<div><div>201030405060</div><div>012345</div><div>pospos+1</div></div>	<div><div>102030405060</div><div>012345</div><div></div></div>	
<div><div>203050601040</div><div>012345</div><div>pospos+1</div></div>	<div><div>203010504060</div><div>012345</div><div>pospos+1</div></div>	<div><div>201030405060</div><div>012345</div><div></div></div>		
<div><div>203050106040</div><div>012345</div><div>pospos+1</div></div>	<div><div>203010405060</div><div>012345</div><div></div></div>			
<div><div>203050104060</div><div>012345</div><div></div></div>				



Data Structures: Sorting Algorithms

Best Case : $\Omega(n)$ - if array elements are already arranged in a sorted manner.

Worst Case : $O(n^2)$

Average Case : $\theta(n^2)$

3. Insertion Sort:

- In this algorithm, one array is considered logically as two sets in which **initially first set contains one element** and **second set contains remaining all elements i.e. $(n-1)$ no. of elements.**

- in every iteration one element will be picked sequentially from second set referred as a **key element**, which will be inserted into the first set at its appropriate position i.e. while inserting key element into the first set it makes sure that all the elements in a first set are arranged in a sorted manner, and to do this key element gets compared with elements which are there into the first set from **left to right** and wherever its appropriate position found it will be **inserted** into the first set, by repeating this logic in **max $(n-1)$ iterations** all the elements from second set will be inserted into the first set and all array elements gets arranged into the first set in a sorted manner.



Data Structures: Sorting Algorithms

Best Case : $\Omega(n)$ - if array elements are already arranged in a sorted manner.

Worst Case : $O(n^2)$

Average Case: $\theta(n^2)$

- Insertion sort algorithm works efficiently for already sorted input array by design.
- Insertion sort algorithm is an efficient algorithm for smaller input size array.

4. Merge Sort:

- This algorithm follows **divide-and-conquer** approach.
- In this algorithm:

Step-1: big size array gets divided logically into smallest size subarrays (i.e. having size 1)

Step-2: keep on merge already sorted subarrays into a single array in a sorted manner step by step and finally whole array gets sorted.

- This algorithm works fine for **even** as well **odd** input size array.
- This algorithm takes extra space to sort array elements, and hence its space complexity is more.

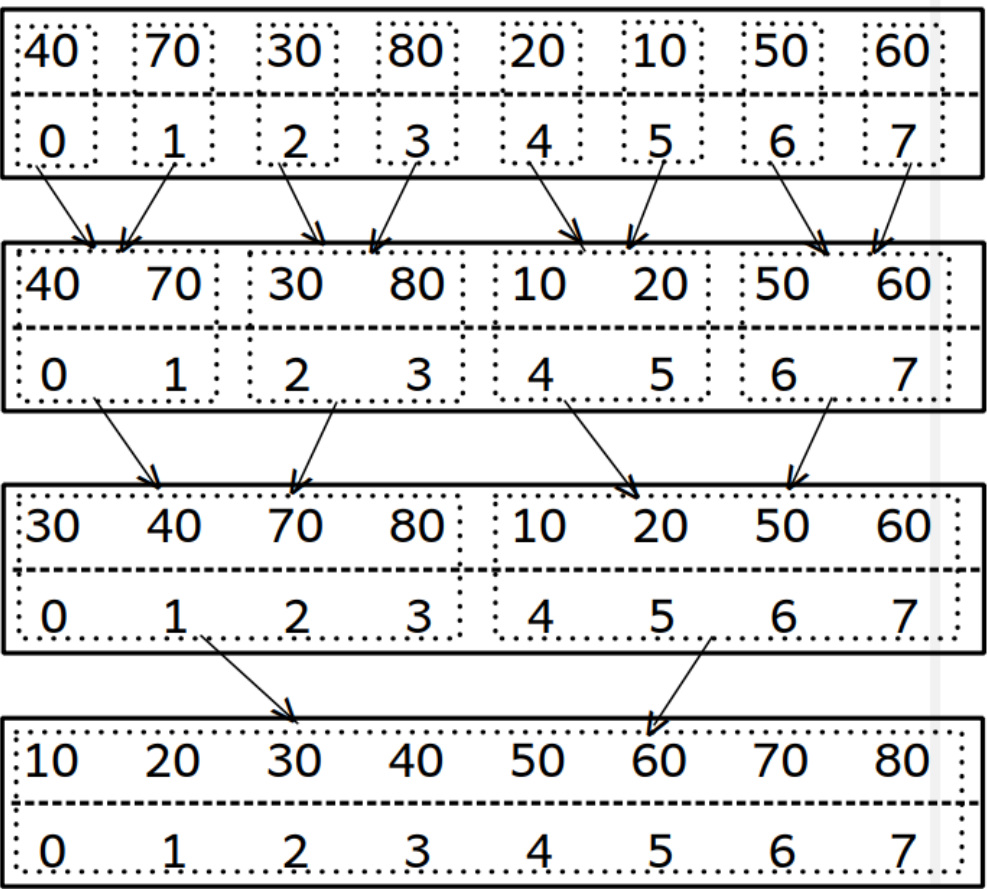
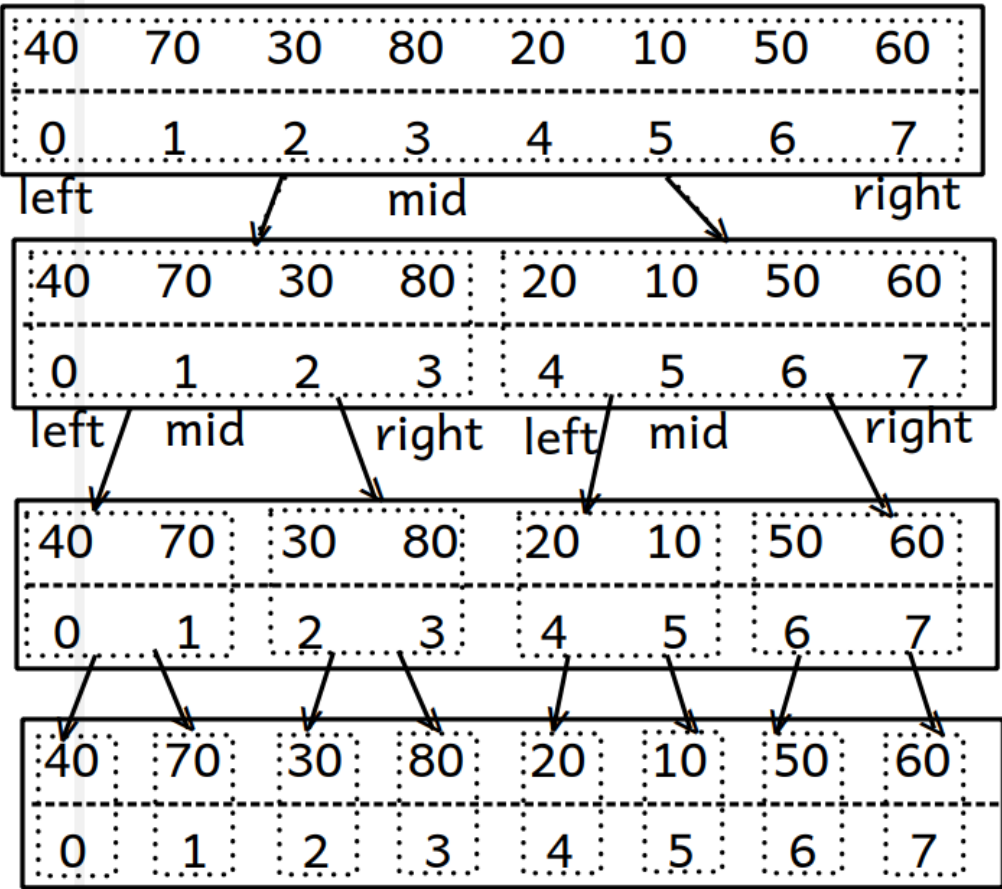


Data Structures: Sorting Algorithms

Merge Sort

Dividing big size array into smallest size subarrays

Merge already sorted arrays



Data Structures: Sorting Algorithms

Best Case : $\Omega(n \log n)$

Worst Case : $O(n \log n)$

Average Case : $\theta(n \log n)$

5. Quick Sort:

- This algorithm follows **divide-and-conquer** approach.
- In this algorithm the basic logic is a **partitioning**.

- Partitioning:

Step-1: select pivot element (select leftmost element as a pivot element).

Step-2: **shift elements towards left side which are smaller than pivot** and **shift elements towards right side which are greater than pivot**, due to this, **pivot element gets settled at its appropriate position**, and array gets divided logically into two partitions in such a way that elements which are at left of pivot is referred as **left partition** and elements which are at its right referred as a **right partition**.



Data Structures: Sorting Algorithms

Best Case : $\Omega(n \log n)$

Worst Case : $O(n^2)$ - worst case rarely occurs

Average Case : $\theta(n \log n)$

- Quick sort algorithm is an efficient sorting algorithm for larger input size array.



Data Structures: Linked List

- Limitations of an array data structure:

- 1. Array is static**, i.e. size of an array is fixed, its size cannot be either grow or shrink during runtime.
- 2. Addition and deletion operations on an array are not efficient as it takes $O(n)$ time**, and hence to overcome these two limitations of an Array, **Linked List** data structure has been designed.

Linked List : It is a **basic / linear data structure**, which is a **collection / list of logically related similar type of data elements** in which, **an address of first element in that collection / list is stored into a pointer variable referred as a head pointer** and **each element contains data as well as link of its next element** (as well as previous element) i.e. each element contains an address of its next (as well as an address of its its previous element).

- Elements in a linked list are **explicitly linked** with each other
- An element in a Linked List is also called as a **Node**.



Data Structures: Linked List

- **Basically there are two types of linked list:**

- 1. Singly Linked List :** It is type of linked list in which each node contains an address of its next node only i.e. each node has only one/single link.

- further there are two subtypes of singly linked list

- i. singly linear linked list**

- ii. singly circular linked list**

- 2. Doubly Linked List :** It is type of linked list in which each node contains an address of its next node as well as an addr of its prev node i.e. each node has two links.

- further there are two subtypes of doubly linked list

- i. doubly linear linked list**

- ii. doubly circular linked list**



Data Structures: Linked List

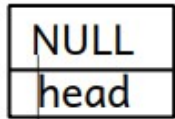
- 1. Singly Linear Linked List:** It is a type of linked list in which
- **head** always contains an address of first element / node, if list is not empty.
 - each node has two parts:
 - i. data part:** contains data of any primitive / non-primitive type.
 - ii. pointer part (next):** contains an address of its next element / node.
 - last node points to NULL, i.e. next part of last node contains NULL.



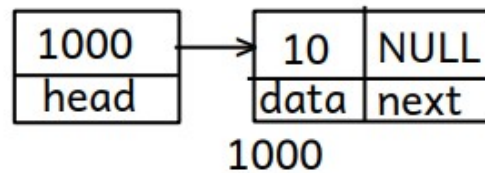
Data Structures: Linked List

SINGLY LINEAR LINKED LIST

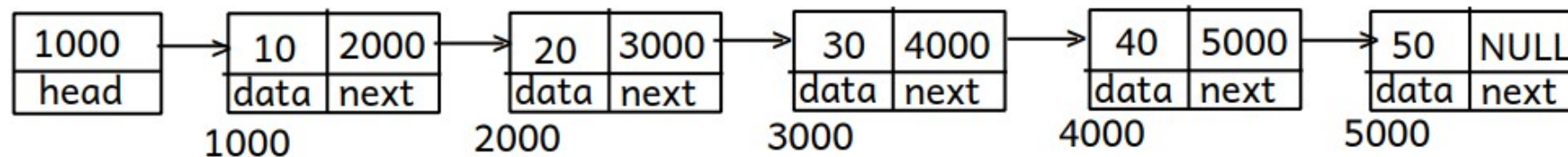
1) singly linear linked list --> list is empty



2) singly linear linked list --> list contains only one node



3) singly linear linked list --> list contains more than one nodes



Data Structures: Linked List

Limitations of Singly Linear Linked List:

- Add node at last position & delete node at last position operations are not efficient as it takes $O(n)$ time.
- We can start traversal only from first node and can traverse the SLL only in a forward direction.
- Previous node of any node cannot be accessed from it.
- **Any node cannot be revisited** – to overcome this limitation **Singly Circular Linked List** has been designed.

2. Singly Circular Linked List: It is a linked list in which

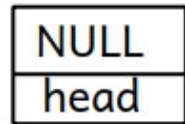
- head always contains an address of first element / node, if list is not empty.
- each node has two parts:
 - i. data part** : contains data of any primitive / non-primitive type.
 - ii. pointer part (next)** : contains an address of its next element / node.
- last node points to first node, i.e. next part of last node contains an address of first node.



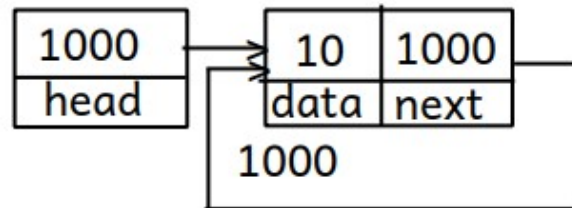
Data Structures: Linked List

SINGLY CIRCULAR LINKED LIST

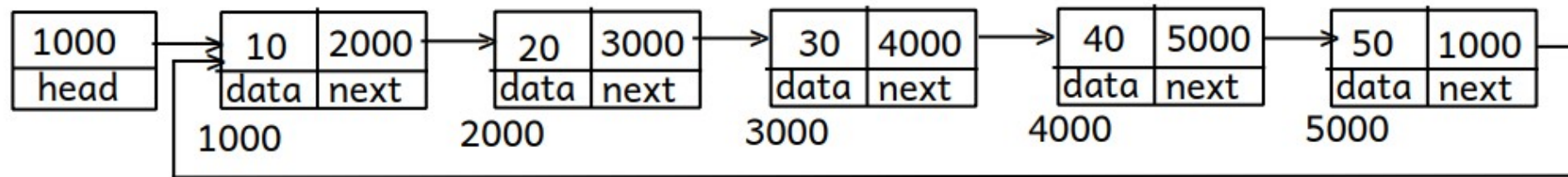
1) singly circular linked list --> list is empty



2) singly circular linked list --> list contains only one node



3) singly circular linked list --> list contains more than one nodes



Data Structures: Linked List

Limitations of Singly Circular Linked List:

- Add last, delete last & add first, delete first operations are not efficient as it takes $O(n)$ time.
- We can start traversal only from first node and can traverse the SCLL only in a forward direction.
- Previous node of any node cannot be accessed from it

to overcome these limitations of **singly linked list** **doubly linked list** has been designed.

3. Doubly Linear Linked List: It is a type of linked list in which

- head always contains an address of first element / node, if list is not empty.
- each node has three parts:

i. data part : contains data of any primitive / non-primitive type.

ii. pointer part (next) : contains an address of its next element / node.

iii. pointer part (prev) : contains an address of its previous element / node.

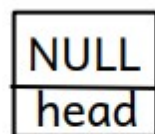
- next part of last node & prev part of first node point to NULL.



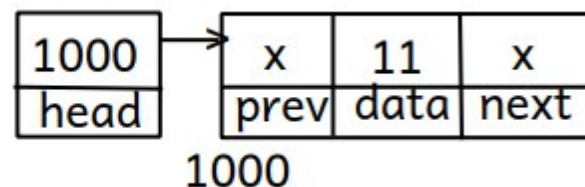
Data Structures: Linked List

DOUBLY LINEAR LINKED LIST

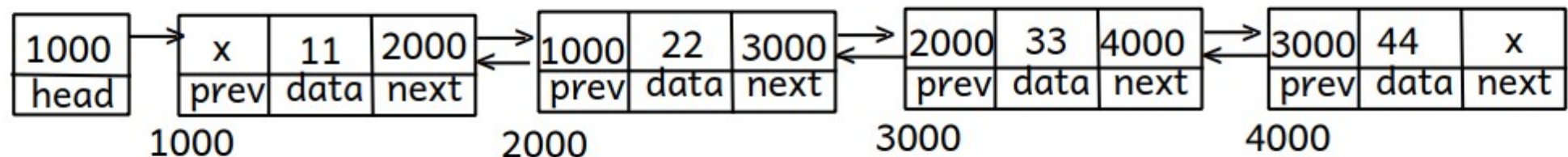
1. doubly linear linked list --> list is empty



2. doubly linear linked list --> list is contains only one node



3. doubly linear linked list --> list is contains more than one nodes



Data Structures: Linked List

Limitations of Doubly Linear Linked List:

- **Add last** and **delete last** operations are not efficient as it takes $O(n)$ time.
- **We can start traversal only from first node**, and hence to overcome these limitations Doubly Circular Linked List has been designed.

4. Doubly Circular Linked List: It is a linked list in which

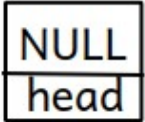
- head always contains an address of first node, if list is not empty.
- each node has three parts:
 - i. data part:** contains data of any primitive / non-primitive type.
 - ii. pointer part(next):** contains an address of its next element/node.
 - iii. pointer part(prev):** contains an address of its previous element/node.
- next part of last node contains an address of first node & prev part of first node contains an address of last node.



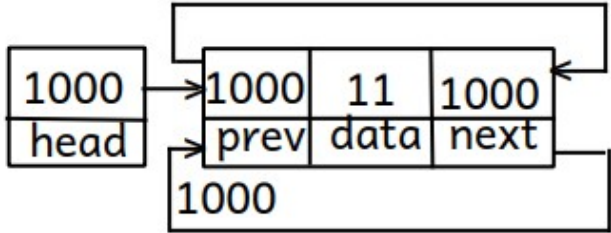
Data Structures: Linked List

DOUBLY CIRCULAR LINKED LIST

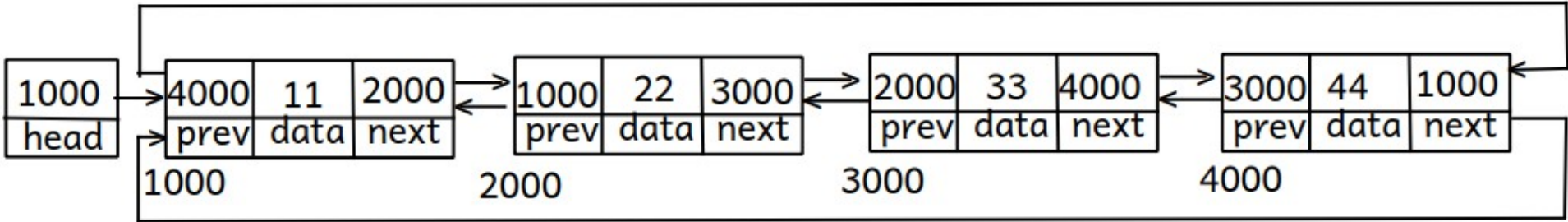
1. doubly circular linked list --> list is empty



2. doubly circular linked list -> list is contains only one node



3. doubly circular linked list --> list is contains more than one nodes



Data Structures: Linked List

Advantages of Doubly Circular Linked List:

- DCLL can be traversed in forward as well as in a backward direction.
- **add last, add first, delete last & delete first** operations are efficient as it takes **O(1)** time and are convenient as well.
- Traversal can be start either from first node or from last node.
- Any node can be revisited.
- Previous node of any node can be accessed from it

Array v/s Linked List:

- Array is **static** data structure whereas linked list is dynamic data structure.
- Array elements can be accessed by using **random access** method which is efficient than linked list elements which can be accessed by **sequential access** method.
- Addition & Deletion operations are efficient on linked list than on an array.
- Array elements gets stored into the **stack section**, whereas linked list elements gets stored into **heap section**.
- In a linked list extra space is required to maintain link between elements, whereas in an array to maintain link between elements is the job of **compiler**.



Data Structures: Stack

Stack: It is a **basic/linear data structure**, which is a **collection/list of logically related similar type of data elements** into which data elements can be added as well as deleted from only one end referred **top** end.

- In this collection/list, element which was inserted last only can be deleted first, so this list works in **last in first out/first in last out** manner, and hence it is also called as **LIFO list**/FILO list.

- We can perform basic three operations on stack in **O(1)** time: **Push, Pop & Peek**.

1. Push : to insert/add an element onto the stack at top position

step1: check stack is not full (if stack is not full then only an ele can be push onto the stack).

step2: increment the value of top by 1

step3: insert an element onto the stack at top position.

2. Pop : to delete/remove an element from the stack which is at top position

step1: check stack is not empty (if stack is not empty then only an element can be popped from the stack).

step2: decrement the value of top by 1.



Data Structures: Stack

3. Peek : to get the value of an element which is at top position without push & pop.

step1: check stack is not empty

step2: return the value of an element which is at top position

Stack Empty : top == -1

Stack Full : top == SIZE-1

Applications of Stack:

- Stack is used by an OS to **control of flow of an execution of program.**
- In recursion internally an OS uses a stack.
- undo & redo functionalities of an OS are implemented by using stack.
- Stack is used to implement advanced data structures algorithms like **DFS: Depth First Search** traversal in tree & graph.
- Stack is used in **expression conversions algorithms and expression evaluation algorithms.**



Data Structures: Stack

- Algorithm to convert given infix expression into its equivalent postfix expression:

Initially we have, an Infix expression, an empty Postfix expression & empty Stack.

```
# algorithm to convert given infix expression into its equivalent postfix expression
step1: start scanning infix expression from left to right
step2:
    if( cur ele is an operand )
        append it into the postfix expression
    else//if( cur ele is an operator )
    {
        while( !is_stack_empty(&s) && priority(topmost ele) >= priority(cur ele) )
        {
            pop an ele from the stack and append it into the postfix expression
        }

        push cur ele onto the stack
    }
step3: repeat step1 & step2 till the end of infix expression
step4: pop all remaining ele's one by one from the stack and append them into the
postfix expression.
```



Data Structures: Stack

- Algorithm to convert given infix expression into its equivalent prefix expression:

Initially we have, an Infix expression, an empty Prefix expression & empty Stack.

```
# algorithm to convert given infix expression into its equivalent prefix:
step1: start scanning infix expression from right to left
step2:
    if( cur ele is an operand )
        append it into the prefix expression
    else//if( cur ele is an operator )
    {
        while( !is_stack_empty(&s) && priority(topmost ele) > priority(cur ele) )
        {
            pop an ele from the stack and append it into the prefix expression
        }

        push cur ele onto the stack
    }
step3: repeat step1 & step2 till the end of infix expression
step4: pop all remaining ele's one by one from the stack and append them into the
prefix expression.
step5: reverse prefix expression - equivalent prefix expression.
```



Data Structures: Queue

Queue: It is a basic/linear data structure which is collection/list of logically related similar type of data elements into which elements can be added from one end referred as **rear** end, whereas elements can be deleted from it from another end referred as **front** end.

- In this list, **element which was inserted first can be deleted first**, so this list works in **first in first out** manner, hence this list is also called as **FIFO list/LILO list**.

- Two basic operations can be performed on queue in $O(1)$ time.

1. Enqueue: to insert/push/add an element into the queue from rear end.

2. Dequeue: to delete/remove/pop an element from the queue which is at front end.

- There are different types of queue:

1. Linear Queue (works in a fifo manner)

2. Circular Queue (works in a fifo manner)

3. Priority Queue: it is a type of queue in which elements can be inserted from rear end randomly (i.e. without checking priority), whereas an element which is having highest priority can only be deleted first.

- Priority queue can be implemented by using linked list, whereas it can be implemented efficiently by using **binary heap**.

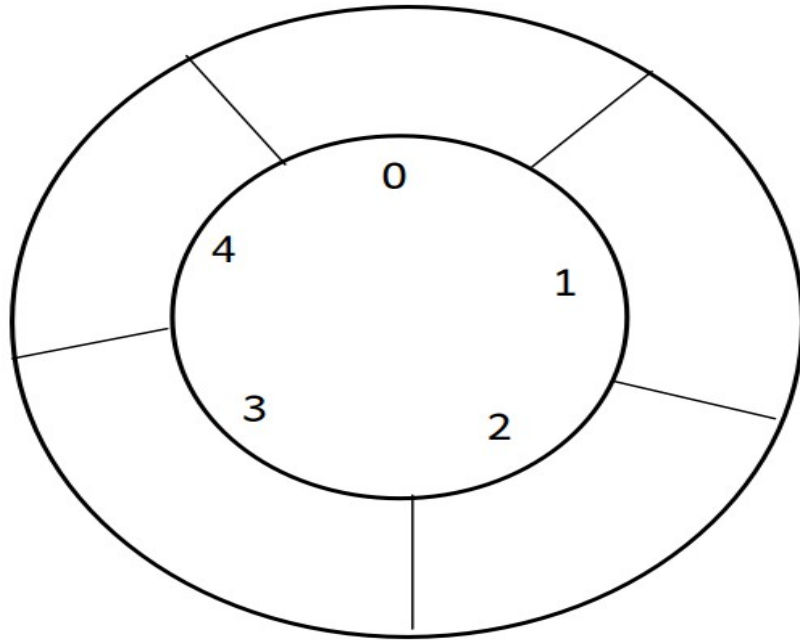
4. Double Ended Queue (deque) : it is a type of queue in which elements can added as well as deleted from both the ends.



Data Structures: Queue

front=-1

rear=-1



Circular Queue

is_queue_full : front == (rear+1)%SIZE

is_queue_empty : rear == -1 && front == rear

1. "enqueue": to insert/add/push an element into the queue from rear end:

step1: check queue is not full

step2: increment the value of rear by 1 [rear = (rear+1)%SIZE]

step3: push/add/insert an ele into the queue at rear position

step4: if(front == -1)

front = 0

2. "dequeue": to remove/delete/pop an element from the queue which is at front position.

step1: check queue is not empty

step2:

if(front == rear)//if we are deleting last ele

front = rear = -1;

else

increment the value of front by 1 [i.e. we are deleting an ele from the queue]. [front = (front+1)%SIZE]



Data Structures: Queue

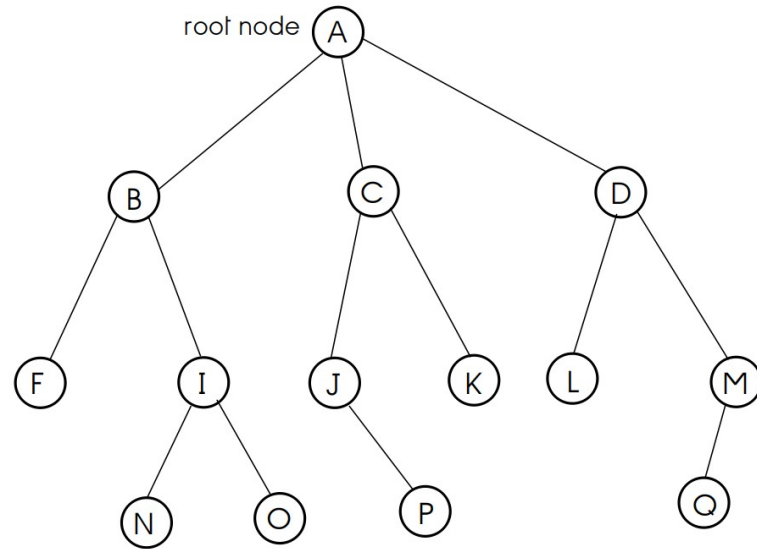
Applications of Queue:

- Queue is used to implement OS data structures like **job queue, ready queue, message queue, waiting queue** etc...
- Queue is used to implement OS algorithms like **FCFS CPU Scheduling, Priority CPU Scheduling, FIFO Page Replacement** etc...
- Queue is used to implement an advanced data structure algorithms like **BFS: Breadth First Search** Traversal in tree and graph.



Data Structures: Tree

Tree: It is a **non-linear, advanced** data structure which is a collection of finite no. of logically related similar type of elements in which, there is a first specially designated element referred as a **root element**, and remaining all elements are connected to it in a **hierarchical manner**, follows **parent-child relationship**.



Tree: Data Structure



Data Structures: Tree

- **siblings/brothers:** child nodes of same parent are called as siblings.
- **ancestors:** all the nodes which are in the path from root node to that node.
- **descedents:** all the nodes which can be accessible from that node.
- **degree of a node** = no. of child nodes having that node
- **degree of a tree** = max degree of any node in a given tree
- **leaf node/external node/terminal node:** node which is not having any child node OR node having degree 0.
- **non-leaf node/internal node/non-terminal node:** node which is having any no. of child node/s OR node having non-zero degree.
- **level of a node** = level of its parent node + 1
- **level of a tree** = max level of any node in a given tree (by assuming level of root node is at level 0).
- **depth of a tree** = max level of any node in a given tree.
- as tree data structure can grow upto any level and any node can have any number of child nodes, operations on it becomes unefficient, so restrictions can be applied on it to achieve efficiency and hence there are different types of tree.



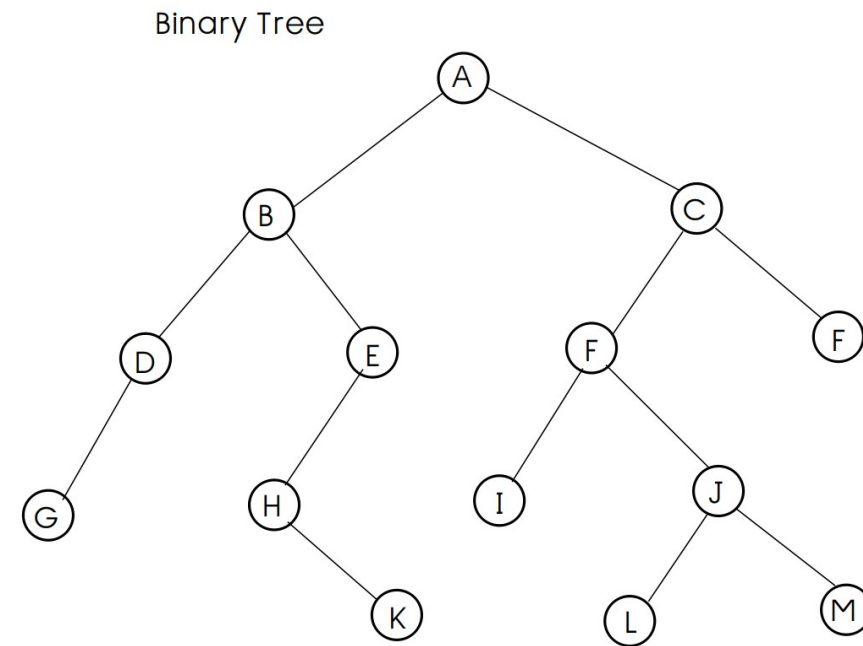
Data Structures: Tree

- **Binary tree:** it is a tree in which each node can have max 2 number of child nodes, i.e. each node can have either 0 OR 1 OR 2 number of child nodes.

OR

Binary tree: it is a set of finite number of elements having three subsets:

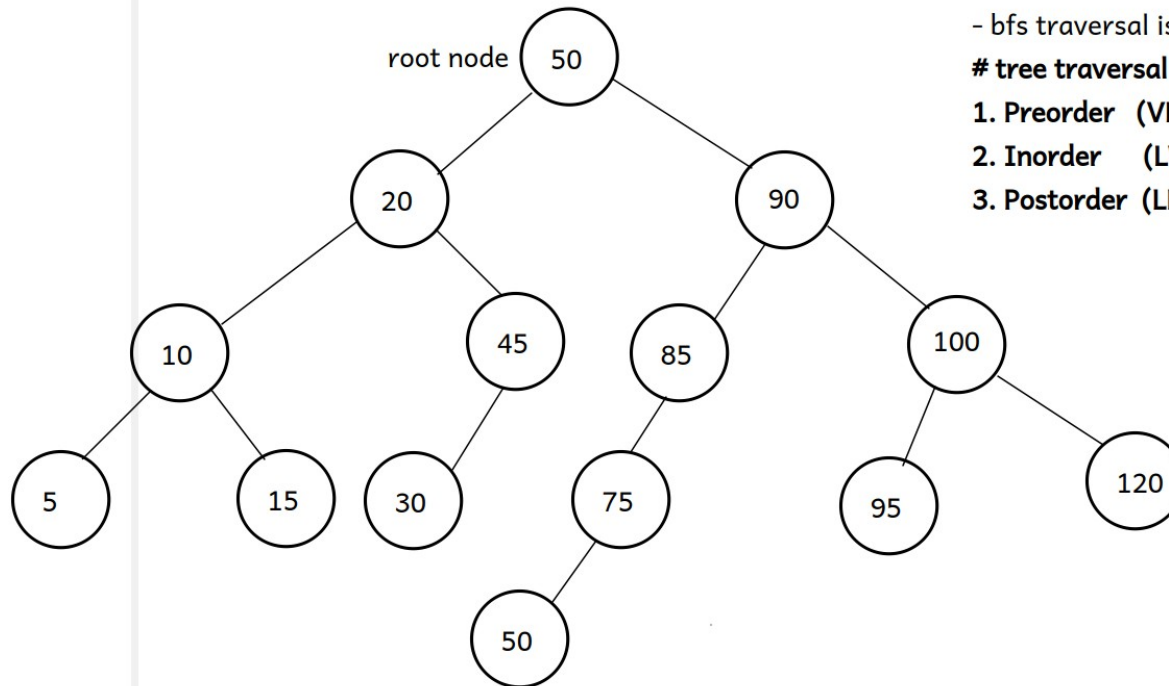
1. root element
2. left subtree (may be empty)
3. right subtree (may be empty)



Data Structures: Tree

- **Binary Search Tree(BST)**: it is a **binary tree** in which left child is always smaller than its parent and right child is always greater than or equal to its parent.

input order of an ele's for BST: 50 20 90 85 10 45 30 100 15 75 95 120 5 50



1. **dfs traversal**: 50 20 10 5 15 45 30 90 85 75 50 100 95 120

2. **bfs traversal**: 50 20 90 10 45 85 100 5 15 30 75 95 120 50

- bfs traversal is also called as "levelwise traversal".

tree traversal methods on BST:

1. **Preorder (VLR)** : 50 20 10 5 15 45 30 90 85 75 50 100 95 120

2. **Inorder (LVR)**: 5 10 15 20 30 45 50 50 75 85 90 95 100 120

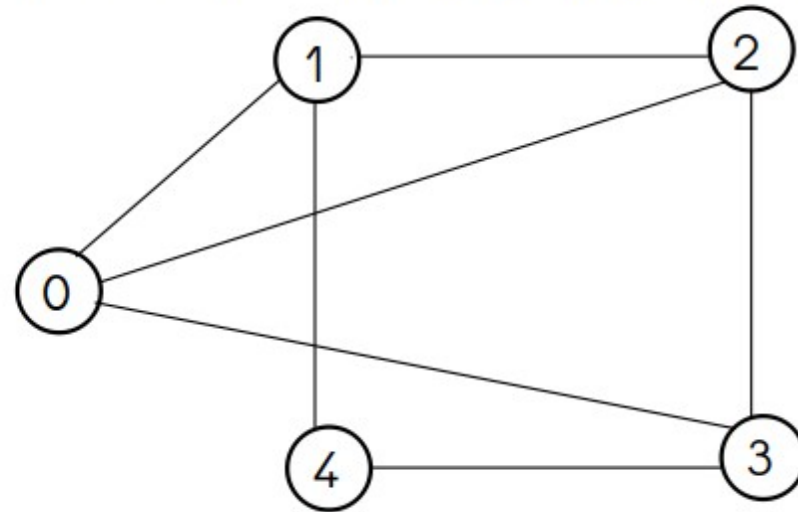
3. **Postorder (LRV)**: 5 15 10 30 45 20 50 75 85 95 120 100 90 50



Data Structures: Graph

Graph: It is **non-linear/advanced** data structure, which is a collection of logically related similar and dissimilar type of elements which contains set of finite no. of elements referred as a **vertices**, also called as **nodes**, and set of finite no. of ordered/unordered pairs of vertices referred as an **edges**, also called as an **arcs**, whereas it may carry **weight/cost/value** (cost/weight/value may be -ve).

$G(V,E): V=\{0,1,2,3,4\}; E=\{(0,1),(0,2),(0,3),(1,2),(1,4),(2,3),(3,4)\}$



Data Structures: Graph

Applications of Graph:

- In Computer science graphs are used to represent the flow of computation.
- **Google maps** uses graphs for building transportation systems, where intersection of two(or more) roads are considered to be a vertex and the road connecting two vertices is considered to be an edge, thus their navigation system is based on the algorithm to calculate the shortest path between two vertices.
- In **Facebook**, users are considered to be the vertices and if they are friends then there is an edge running between them. Facebook's Friend suggestion algorithm uses graph theory. Facebook is an example of undirected graph.
- In **World Wide Web**, web pages are considered to be the vertices. There is an edge from a page u to other page v if there is a link of page v on page u . This is an example of Directed graph. It was the basic idea behind Google Page Ranking Algorithm.
- In **Operating System**, we come across the Resource Allocation Graph where each process and resources are considered to be vertices. Edges are drawn from resources to the allocated process, or from requesting process to the requested resource. If this leads to any formation of a cycle then a deadlock will occur.

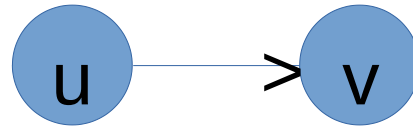


Data Structures: Graph

- If there exists a direct edge between two vertices then those vertices are referred as an **adjacent vertices** otherwise **non-adjacent**.



$$(u, v) == (v, u)$$



$$(u, v) != (v, u)$$

- if we can represent any edge either (u,v) OR (v,u) then it is referred as **unordered pair of vertices i.e. undirected edge**.

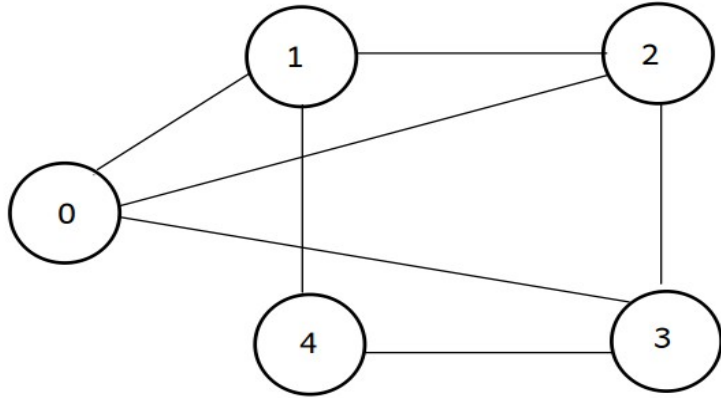
e.g. $(u,v) == (v,u) \Rightarrow$ unordered pair of vertices \Rightarrow undirected edge \Rightarrow graph which contains undirected edges referred as undirected graph.

- if we cannot represent any edge either (u,v) OR (v,u) then it is referred as an **ordered pair of vertices i.e. directed edge**.

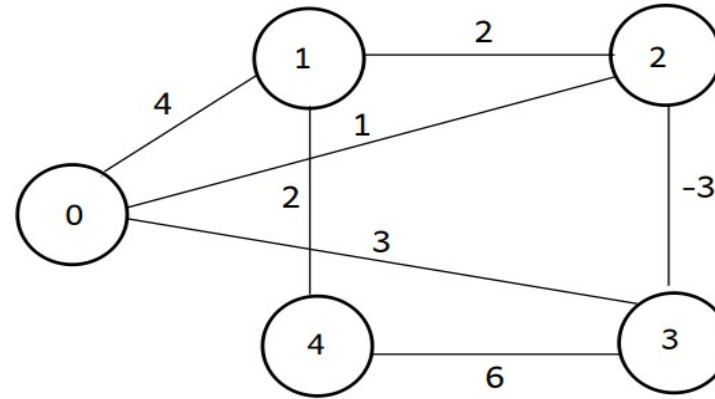
$\langle u, v \rangle != \langle v, u \rangle \Rightarrow$ ordered pair of vertices \Rightarrow directed edge \rightarrow graph which contains set of directed edges referred as directed graph (di-graph).



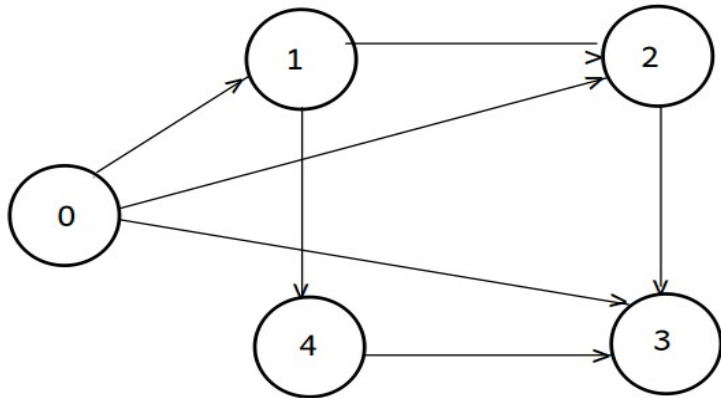
Data Structures: Graph



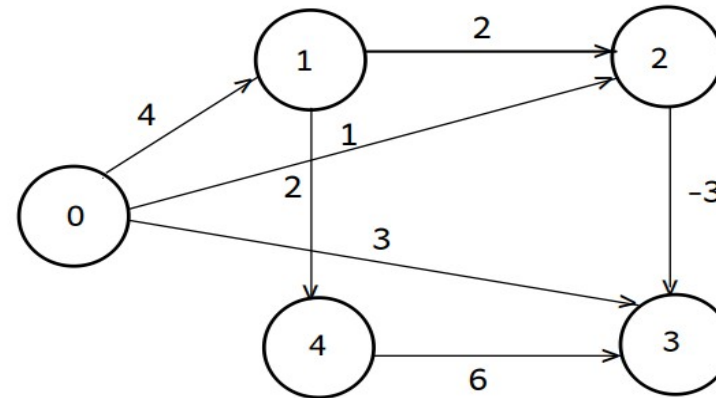
undirected unweighted graph



undirected weighted graph



directed unweighted graph



directed weighted graph



Data Structures: Graph

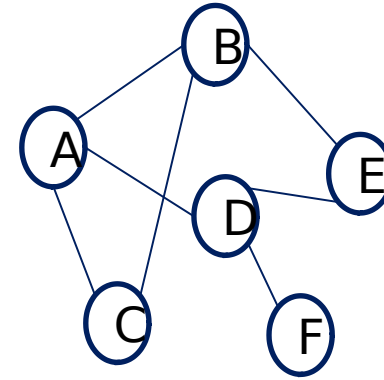
- **Path:** Path is set of edges connecting two vertices.
- **Cycle:** in a given graph, if in any path starting vertex and end vertex are same, such a path is called as a cycle.
- **Loop:** if there is an edge from any vertex to that vertex itself, such edge is called as a loop. Loop is the smallest cycle.
- **Connected Vertices:** if there exists a direct/indirect path between two vertices then those two vertices are referred as a connected vertices otherwise not-connected.
- adjacent vertices are always connected but vice-versa is not true.
- **Connected Graph:** if any vertex in a graph is connected to remaining all vertices then such a graph is called as connected graph, otherwise it is referred as disconnected/not connected.



Graph

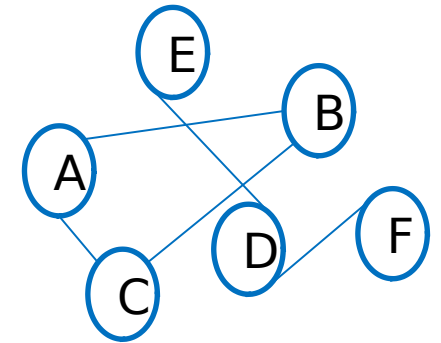
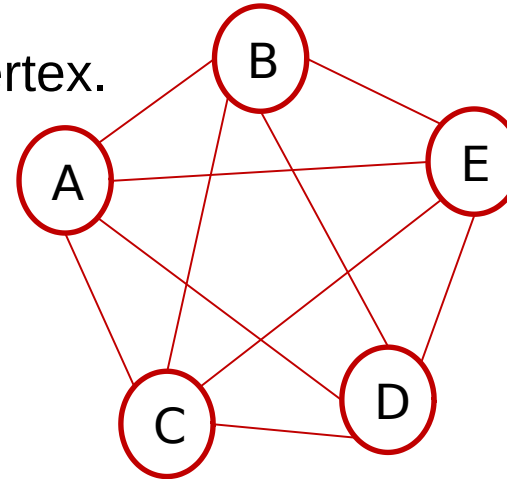
- **Connected graph**

- From each vertex some path exists for every other vertex.
- Can traverse the entire graph starting from any vertex.



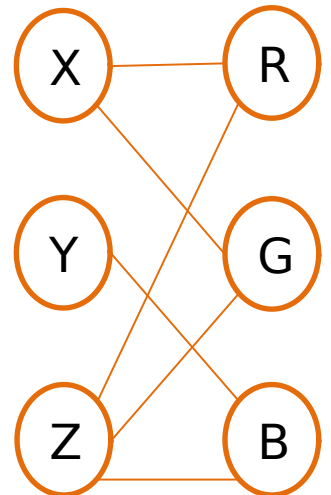
- **Complete graph**

- Each vertex of a graph is adjacent to every other vertex.
- Un-directed graph: Number of edges = $n(n-1) / 2$
- Directed graph: Number of edges = $n(n-1)$



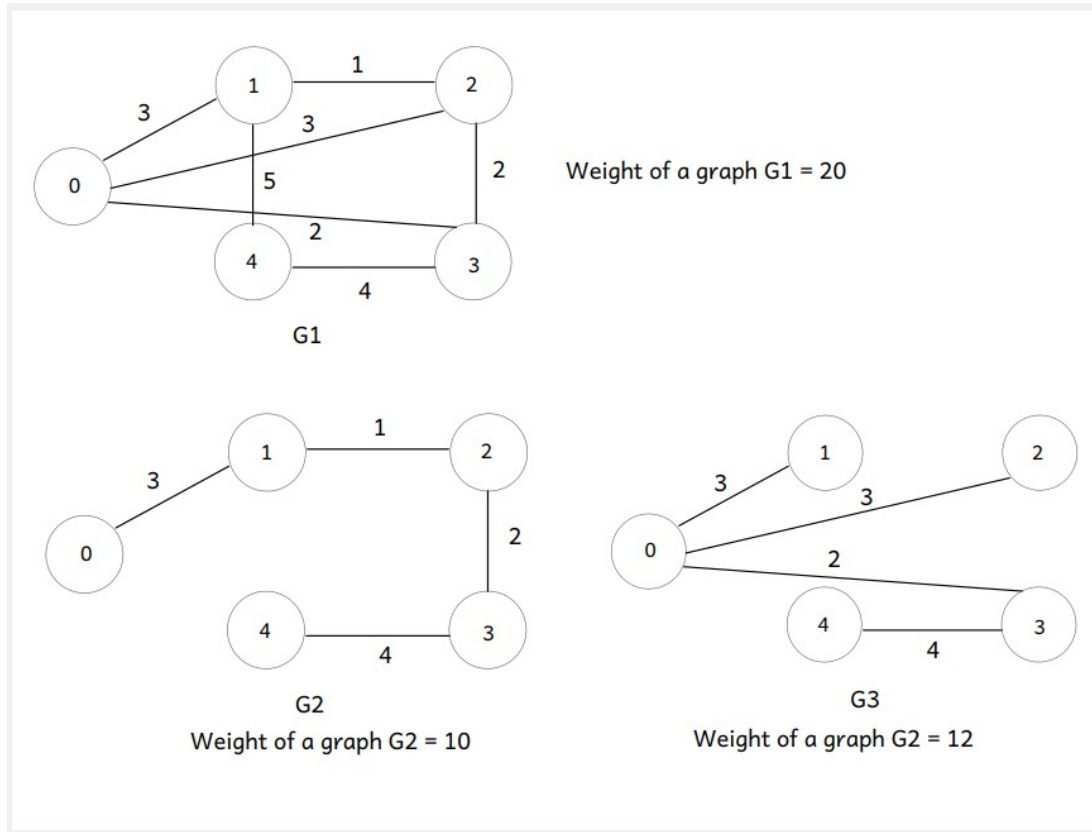
- **Bi-partite graph**

- Vertices can be divided in two disjoint sets.
- Vertices in first set are connected to vertices in second set.
- Vertices in a set are not directly connected to each other.



Data Structures: Graph

Spanning Tree:



- **Weight of a graph** = sum of weights of all its edge.
- **Spanning Tree:**
 - } Connected subgraph of a graph.
 - } Includes all V vertices and $V-1$ edges.
 - } Do not contain cycle.
 - } A graph may have multiple spanning trees.
- **Minimum Spanning Tree:** Spanning tree of a given graph having minimum weight.
 - } Used to minimize resources/cost.
 - } Algorithms to find MST of a given Graph:
 - **Prim's Algorithm** $\Rightarrow O(E \log V)$
 - **Kruskal's Algorithm** $\Rightarrow O(E \log V)$



Data Structures: Graph

Graph Traversal Algorithms:

Used to traverse all vertices in the graph.

DFS Traversal (using Stack) and BFS Traversal (by using Queue)

Shortest Path Algorithm:

Single source SPT algorithm used to find minimum distance from the given vertex to all other vertices.

Dijkstra's Algorithm (Doesn't work for -ve weight edges) $\Rightarrow O(V \log V)$.

Bellman Ford Algorithm $\Rightarrow O(VE)$.

All pair Shortest Path Algorithm:

To find minimum distance from each vertex to all other vertices.

Floyd Warshall Algorithm $\Rightarrow O(V^3)$

Johnson's Algorithm $\Rightarrow O(V^2 \log V + VE)$



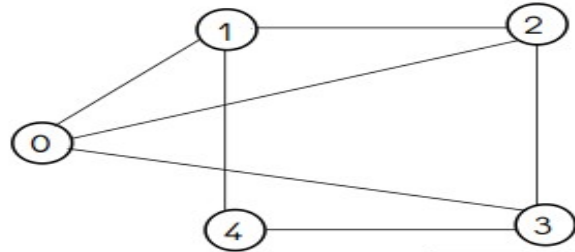
Data Structures: Graph

- There are two graph representation methods:

1. Adjacency Matrix Representation (2-D Array)

2. Adjacency List Representation (Array of Linked Lists)

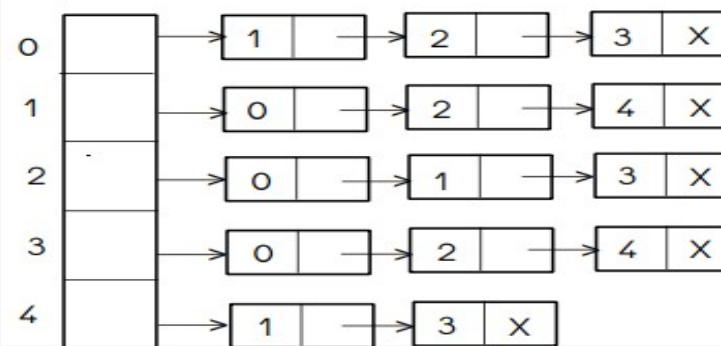
$G(V,E): V=\{0,1,2,3,4\}; E=\{ (0,1),(0,2),(0,3),(1,2),(1,4),(2,3),(3,4) \}$



Adjacency Matrix Representation

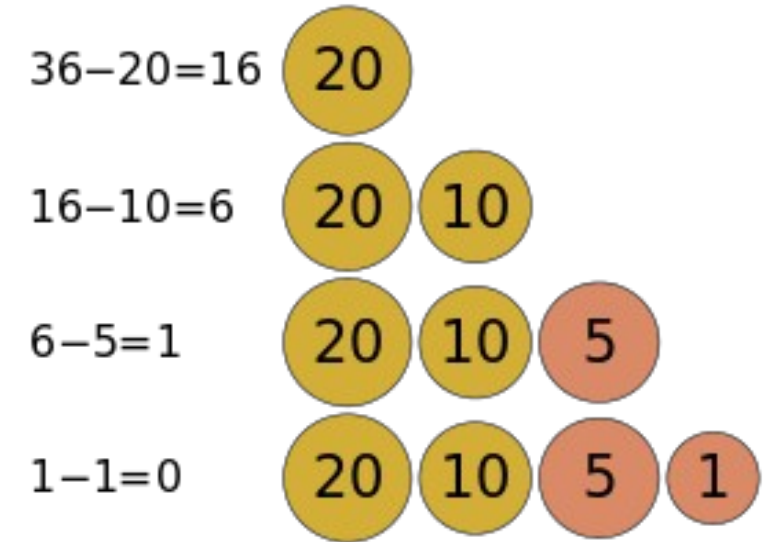
	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	1	0
3	1	0	1	0	1
4	0	1	0	1	0

Adjacency List Representation



Problem solving technique: Greedy approach

- - A greedy algorithm is any algorithm that follows the problem-solving heuristic of making the locally optimal choice at each stage with the intent of finding a global optimum.
- - We can make choice that seems best at the moment and then solve the sub-problems that arise later.
- - The choice made by a greedy algorithm may depend on choices made so far, but not on future choices or all the solutions to the sub-problem.
- - It iteratively makes one greedy choice after another, reducing each given problem into a smaller one.
- - A greedy algorithm never reconsiders its choices.
- - A greedy strategy may not always produce an optimal solution.



- Greedy algorithm decides minimum number of coins to give while making change.



Data Structures: Hash Table

Hash Table: Hash table is an **associative** data structure in which data is stored in **key-value pairs** so that for the given key value can be searched in minimal possible time. Ideal time complexity is **$O(1)$** . Internally it is an array (table) where values can be accessed by the index (slot) calculated from the key.

Hash Function: It is a mathematical function of the key that yields the slot of the hash table where the key-value is stored. Simplest example is: $f(k) = k \% \text{size}$.

Collision: There is a possibility that two keys result in the same slot. This is called a collision and must be handled using some **collision handling technique**. It can be handled by one of the following techniques.

1. Open Addressing
2. Chaining/Separate Chaining



Data Structures: Hash Table

Collision:

[illegible]

Data Structures: Hash Table

1. Open Addressing:

- In open addressing, all elements are stored in the hash table itself. When searching for an element, we one by one examine table slots until the desired element is found or it is clear that the element is not in the table.

- Open Addressing is done following ways:

A. Linear Probing: In linear probing, if collision occurs next free slot will be searched/probed linearly.

B. Quadratic Probing: In quadratic probing, if collision occurs next free slot will be searched/probed quadratically.

C. Double Hashing: In double hashing, if collision occurs next free slot will be searched/probed by using another hash function, so two hash functions can be use to find next/probe next free slot.



Data Structures: Hash Table

[illegible]

Data Structures: Hash Table

Load Factor:

m = Number of slots in the hash table

n = Number of keys to be inserted in the hash table

Load factor (α) = n / m (< 1)

Expected time to search/insert/delete $< 1 / (1 - \alpha)$

So Search, Insert and Delete take **$O(1/(1 - \alpha))$** time

Limitations of Open Addressing:

- Open addressing requires more computation.
- In open addressing table may becomes full, it can be used only if frequency and number of keys known in advanced.

2. Chaining:

- The idea is to make each cell of hash table point to a linked list of records that have same hash function value. Chaining is simple, but requires additional memory outside the table.



Data Structures: Hash Table

A	B	C	D	E	F	G	H	I	J	K	L
Hashing Input				Chaining							
insert values => 50, 700, 76, 85, 92, 73, 101											
Hash Function	Key % 7			50%7=1	700%7=0	76%7=6	85%7=1	92%7=1	73%7=3	101%7=3	
Hash Table with Capacity = 7											
slot											
0				700							
1				50	→	85	→	92			
2											
3				73	→	101					
4											
5											
6				76							