Week 5: Multi-Criteria Decisions - Goal Progamming

Multi-criteria optimization (MCO) problems involve **optimizing two or more conflicting objectives** simultaneously. Unlike single-objective optimization, where a single "best" solution exists, MCO problems lack a single optimal solution.
It is common to encounter many real-world problems, especially in engineering, economics, and decision-making contexts, where there are multiple objectives that need to be optimized, and these objectives may conflict with each other.
For example, a company may like to design a highly reliable product at the lowest cost. However, maximizing quality of the product may involve use of better materials and thus may increase the cost which we would like to decrease. Techniques used for solving such problems include goal programming and analytic hierarchy process (AHP). In this course, we will discuss goal programming which falls within the current framework of optimization models.

Goal programming is used when the problem has more than one objectives, such as minimize cost and at the same time maximize quality. Each objective is treated as a constraint and the right-hand side (RHS) value is set to a target or 'goal' value that is desired to be met. If there exist solutions that meet the desired target values for the various objectives then an optimal solution can be found as usual. However, this might not always be possible to find a feasible solution that satisfies all the constraints and goals, i.e. the goal programming problem when solved results in 'no feasible solution found'. In such cases we would like a find solutions that meet the goals as closely as possible or violate them as little as possible. This is done by treating the 'goal' constraints as flexible ones by introducing permissible deviations from the desired goal values of the RHS. All other constraints are treated as hard constraints as usual that must be met. Such solutions are called 'satisficing' solutions.
The following steps are used in building a goal programming model (Charnes et al., 1955):

1. Identify the objectives of the problems and the corresponding target values. Consider each objective as a goal by allowing deviations.
2. Identify decision variables of the problem.
3. For each goal (objective) write the goal constraint using deviation variables. Goal constraints are soft constraints that can be violated. Deviation variables capture the deviation from the target set for each goal.
4. Write system constraints which are hard constraints that cannot be violated.
5. Write the objective function as a minimization of weighted sum of the deviation variables, where the weights are the penalties assigned for violation of goals.

Goal programming is widely used in various fields, including:

- **Business:** Resource allocation, production planning, financial investment decisions.
- **Engineering:** Design optimization considering multiple performance criteria and cost constraints.
- **Public Policy:** Planning and resource allocation, balancing economic, social, and environmental objectives.

Overall, goal programming offers a valuable approach to solving MCO problems by prioritizing goals, measuring deviations, and finding solutions that best satisfy all objectives within the given constraints.

Non - Preemptive and Preemptive Goal Programming

Goal programming problems are classified into preemptive and non-preemptive goal programming. In preemptive goal programming, the goals are defined as hierarchy of goals. That is, goals are defined at different priority levels. On the other hand, in a non-preemptive goal programming, all goals are treated as equally important.

Non-Preemptive Goal Programming:

- Equal Treatment: All goals are viewed as equally important and are not explicitly prioritized.
- Deviation Minimization: The primary objective is to minimize the sum or weighted sum of deviations from all the set goals.
- Focus: Non-preemptive programming emphasizes finding a solution that meets all goals reasonably well, without explicitly focusing on achieving any specific goal first.
- Applications: Suitable for situations where all objectives hold comparable importance and no clear hierarchy exists.

Preemptive Goal Programming:

- Prioritization: Goals are assigned priority levels, with higher-level goals taking precedence over lower-level ones.
- Sequential Optimization: The solution process follows a sequential approach. First, a solution is found to achieve the highest-priority goal as closely as possible.
- Subsequent Goals: Subsequent steps focus on achieving the next highest-priority goal. Additional constraints are imposed to ensure that the level at which the higher-priority goals are met is maintained.
- Focus: Preemptive programming prioritizes achieving higher-level goals first and then attempts to satisfy lower-level goals as much as possible within the remaining feasible space.
- Applications: Ideal for scenarios with a clear hierarchy of importance among the objectives, where achieving certain goals is more crucial than others.

Goal Programming Using Excel Solver

While Microsoft Excel's Solver tool is primarily designed for linear and nonlinear optimization problems with a single objective function, you can still use it for basic goal programming by formulating the problem appropriately.
Here's a simplified guide to using Solver for goal programming in Excel:

1. **Define Your Objectives and Constraints**: Identify the multiple objectives you want to achieve and the constraints that must be satisfied. Assign target levels or goals for each objective.
2. **Set Up Your Excel Worksheet**: Organize your data in an Excel worksheet. Define decision variables, objective functions, target levels, and constraints.

3. **Formulate the Goal Programming Model**: Based on your objectives, constraints, and target levels, formulate the goal programming model in Excel. Create a cell for each deviation variable introduced for goal constraints, representing the difference between achieved and target levels for each objective.
4. **Set Up the Objective Function**: Create an objective function that minimizes the sum of deviations (or another appropriate aggregation function such as a weighted sum) from the target levels for each objective.
5. **Enter Constraints**: Enter any constraints related to decision variables and other conditions to be met.
6. **Invoke Solver**: Once your model is set up, go to the Solver tool in Excel (usually found under the Data or Analysis tab). Set the objective function to minimize the sum of deviations subject to the constraints.
7. **Define Solver Parameters**: Configure Solver to adjust decision variables within defined ranges. Specify any additional hard constraints.
8. **Run Solver**: Run Solver to find the optimal solution that minimizes deviations from the target levels while satisfying the hard constraints.
9. **Review Results**: Once Solver finds a solution, review the results to ensure they make sense in the context of your problem. Excel will display the values of decision variables that minimize the total deviation and the deviation values.