12. Divergent **13.** Divergent **14.** Convergent **15.** Convergent
16. Convergent **17.** Divergent **18.** Convergent.

Problems 9.4, page 376

1. Conv. for $x < 1$; Div. for $x \geq 1$ **2.** Conv. for $x < 1$; Div. for $x \geq 1$
3. Conv. for $x \leq 1$; Div. for $x > 1$ **4.** Conv. for ≥ 1 ; Div. for $x < 1$
5. Convergent for all values of p **6.** Convergent **7.** Convergent
8. Convergent **9.** Conv. for $x < 1$; Div. for $x \geq 1$ **10.** Convergent
11. Convergent **12.** Convergent **13.** Divergent
14. Convergent **15.** Conv. for $x < 1$, Div. for $x > 1$; Conv. for $p > 1$ and Div. for $p \leq 1$
16. Divergent **17.** Conv. if $\beta > \alpha > 0$; Div. if $\alpha \geq \beta > 0$.

Problems 9.5, page 379

1. Conv. for $x \leq 1$; Div. for $x > 1$ **2.** Conv. for $x \leq 1$; Div. for $x > 1$
3. Conv. for $x < 1$; Div. for $x \geq 1$ **4.** Conv. for $x < 2$; Div. for $x \geq 2$
5. Conv. for $x < e$; Div. for $x \geq e$ **6.** Conv. for $x \leq 1$; Div. for $x > 1$
7. Conv. for $x \leq 1$; Div. for $x > 1$ **8.** Conv. for $x^2 \leq 1$; Div. for $x^2 > 1$
9. Conv. for $x^2 < 4$; Div. for $x^2 \geq 4$ **10.** Convergent
11. Conv. for $x < 1/e$; Div. for $x \geq 1/e$ **12.** Conv. for $x < 1$; Div. for $x \geq 1$
13. Diverges
14. Conv. for $x < 1$; Div. for $x > 1$. When $x = 1$, Conv. for $b - a > 1$, Div. for $b - a \leq 1$.

Problems 9.6, page 381

1. Convergent **2.** Convergent **3.** Convergent
4. Convergent **5.** Conv. for $x < 1$; Div. for $x \geq 1$
6. Conv. for $x < \frac{1}{2}$; Div. for $x \geq \frac{1}{2}$ **7.** Convergent.

Problems 9.7, page 383

1. Oscillatory **2.** Convergent **3.** Convergent **4.** Convergent
5. Convergent **6.** Oscillatory **7.** Convergent **8.** Convergent
9. Convergent **10.** Oscillatory.

Problems 9.8, page 387

1. (*i*) and (*ii*) conditionally convergent
3. (*i*) Conditionally convgt. for $0 < p \leq 1$; (*ii*) Conditionally convgt
4. Absolutely convergent for (*i*) $0 < x < 1$; (*ii*) $-1 < x \leq 1$; (*iii*) $|x| \leq 1$.
5. Convergent for $x \leq 1$ and not convergent for $x > 1$
6. (*i*) $-1 < x \leq 1$; (*ii*) $-1 < x \leq 1$;
7. $-e < x \leq e$ **8.** (*i*) Absolutely convergent (*ii*) convergent
9. Absolutely convergent.

Problems 9.9, page 388

1. Conv. for $x < 1$; Div. for $x \geq 1$ **2.** Convergent **3.** Divergent
4. Convergent **5.** Divergent **6.** Conv. for $x < 1$; Div. for $x \geq 1$
7. Conv. for $x < 1$; Div. for $x \geq 1$ **8.** Conv. for $x < 1$; Div. for $x \geq 1$
9. Conv. for $x < 1/4$; Div. for $x \geq 1/4$ **10.** Conv. for $x < 2$; Div. for $x \geq 2$

11. Convergent for all x **12.** Conv. for $x < 1$; Div. for $x \geq 1$ **13.** Convergent
14. Absolutely convergent **15.** Convergent
16. Convergent for $p > 1$; divergent for $p \leq 1$.

Problems 9.10, page 391

1. Uniformly convergent for $0 \leq x \leq 1$. **2.** to
5. Uniformly convergent for all real values of x **6.** Uniformly convergent for $0 \leq x \leq 1/a$
10. (i) and (ii) Both converge uniformly for all real values of x.

Problems 9.11, page 392

1. (c) **2.** (d) **3.** (a) **4.** (b) **5.** (c) **6.** (d)
7. (a) **8.** (b) **9.** (b) **10.** (d) **11.** (c)
12. (a) $(-1, 1)$ (b) $(-1/2, 1/2)$ **13.** $-1 < x \leq 1$ **14.** $k > 1$ **15.** $a_n < k$ **16.** Oscillatory
17. All values of x **18.** $k < 1$ **19.** Convergent. **20.** Divergent. **21.** $q - p > 1$
22. Divergent **23.** Convergent. **24.** $0 < x < 4$ **25.** yes **26.** True **27.** Convergent
28. Divergent **29.** $x > 1$ **30.** $0 \leq x \leq 1$ **31.** (b) **32.** (c) **33.** (d)
34. (b) **35.** True.

Problems 10.1, page 400

1. $$\frac{2\sinh a\pi}{\pi}\left\{\left(\frac{1}{2a} - \frac{a\cos x}{1^2+a^2} + \frac{a\cos 2x}{2^2+a^2} - \ldots\right) + \left(\frac{\sin x}{1^2+a^2} - \frac{2\sin 2x}{2^2+a^2} + \frac{3\sin 3x}{3^2+a^2} - \ldots\right)\right\}$$

$$\frac{\pi}{\sinh \pi} = 2\left[\frac{1}{2^2+1} - \frac{1}{3^2+1} + \frac{1}{4^2+1} - \ldots\right].$$

Problems 10.2, page 401

1. No **2.** No **3.** Yes.

Problems 10.3, page 404

1. $\frac{1}{2}\pi - \frac{4}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\ldots\right)$ **2.** $\frac{I_0}{\pi} + \frac{1}{2}I_0 \sin x - \frac{2I_0}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}, \frac{1}{2}$

3. $\frac{\pi^2}{6} - 2\left(\cos x - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \ldots\ldots\right) - \frac{1}{\pi}\left\{\left(\frac{2}{1^3} - \frac{\pi^2}{1}\right)\sin x - \left(\frac{2}{2^3} - \frac{\pi^2}{2}\right)\sin 2x + \ldots\right\}$

4. $2\left(\pi - \frac{4}{\pi}\right)\sin x - \pi \sin 2x + \frac{2}{3}\left(\pi - \frac{4}{9\pi}\right)\sin 3x - \frac{\pi}{2}\sin 4x + \ldots\ldots$

5. $\frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\ldots \infty\right)$.

Problems 10.4, page 408

1. $-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin n\pi x}{n}$

2. (*i*) $\frac{a^2}{3}\ \frac{a^2}{11^2}\left\{\frac{1}{12}\cos\frac{2\pi x}{a}+\frac{1}{2^2}\cos 4\pi\frac{x}{a}+\ldots\ldots\right\}\ \frac{a^2}{\pi}\left\{\frac{1}{1}\sin\frac{2\pi x}{a}\ \frac{1}{2}\sin\frac{4\pi x}{a}+\ldots\ldots\right\}$

(*ii*) $f(t)=\frac{2}{3}+\frac{4}{\pi^2}\left(\cos\pi t-\frac{\cos 2\pi t}{2^2}+\frac{\cos 3\pi t}{3^2}-\ldots\ldots\right)$

4. $\frac{3}{2}-\frac{12}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\left\{\frac{\cos(2n-1)\pi x}{3}\right\}$

5. $\frac{E}{\pi}+\frac{E}{2}\sin\omega t-\frac{2E}{\pi}\left[\frac{1}{1.3}\cos 2\omega t+\frac{1}{3.5}\cos 4\omega t+\frac{1}{5.7}\cos 6\omega t+\ldots\right]$

6. $f(x)=\frac{\pi}{4}+2\sum_{n=1}^{\infty}(-1)^{n+1}\sin n\pi x$; put $x = 1/2$.

Problems 10.5, page 412

1. $\frac{a^2}{3}+\sum_{n=1}^{\infty}\frac{4a^2}{n^2\pi^2}(-1)^n\cos\frac{n\pi x}{a}$

3. $1-\frac{1}{2}\cos x-\frac{2}{1.3}\cos 2x+\frac{2}{2.4}\cos 3x-\frac{2}{3.5}\cos 4x-\ldots$

5. $\frac{1}{2}\pi-\frac{4}{\pi}\left(\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\ldots\ldots\right)$

6. (*i*) $\frac{2}{\pi}-\frac{4}{\pi}\left(\frac{\cos 2x}{3}+\frac{\cos 4x}{15}+\ldots\ldots+\frac{\cos 2nx}{4n^2-1}+\ldots\ldots\right)$ (*ii*) $\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4(-1)^{m+1}}{(4m^2-1)\pi}\cos\frac{2m\pi x}{l}$

7. $\frac{\pi}{2}+1-\frac{4}{\pi}\left(\cos x+\frac{1}{3^2}\cos 3x+\frac{1}{5^2}\cos 5x+\ldots\right);\frac{\pi^2}{8}$

8. $\frac{4k}{\pi}\left(\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\ldots\ldots\infty\right)$.

Problems 10.6, page 416

2. $\frac{\pi}{2}-\frac{4}{\pi}\left[\cos x+\frac{1}{3^2}.\cos 3x+\frac{1}{5^2}.\cos 5x+\ldots\ldots\right]$; $2\left[\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x-\ldots\ldots\right]$

3. $\frac{\pi^2}{3}-4\left[\cos x-\frac{1}{2^2}\cos 2x+\frac{1}{3^2}\cos 3x-\frac{1}{4^2}\cos 4x+\ldots\infty\right]$

4. $\sum_{n=2}^{\infty}\frac{1}{n}\sin 2nx$

5. $\frac{1}{3}+\frac{4}{\pi^2}\sum_{1}^{\infty}\frac{1}{n^2}\cos n\pi x$

6. $\frac{8}{\pi^3}\left(\frac{\sin\pi t}{1^3}+\frac{\sin 3\pi t}{3^3}+\frac{\sin 5\pi t}{5^3}+\ldots\ldots\right)$

7. $\frac{1}{n\pi}[1-(-1)^n]$

8. $-\frac{1}{2}\sin x+\sum_{n=2}^{\infty}\frac{(-1)^n.2n}{n^2-1}\sin nx$

9. $\sum_{n=1}^{\infty}\frac{2n\pi}{1+n^2\pi^2}(1-e\cos n\pi)\sin n\pi x$

10. $\sum_{n=1}^{\infty}\frac{4}{\pi n^3}[1-(-1)^n]\sin nx$

12. $\frac{l}{4}+\sum_{n=1}^{\infty}\frac{2l}{(n\pi)^2}\left\{2\cos\frac{n\pi r}{2}-1-(-1)^n\right\}\cos\frac{n\pi x}{l}$

13. $\frac{8}{\pi}\cos\frac{\pi}{4}\left[\frac{\sin 2x}{1.3}-\frac{\sin 6x}{5.7}+\frac{\sin 10x}{9.11}+\ldots\ldots\right]$

14. $\frac{2l^2h}{a(l-a)\pi^2}\left[\sin\frac{\pi a}{l}\sin\frac{\pi x}{l}+\frac{1}{2^2}\sin\frac{2\pi a}{l}\sin\frac{2\pi x}{l}+\frac{1}{3^2}\sin\frac{3\pi a}{l}\sin\frac{3\pi x}{l}+\ldots\ldots\right]$.

Problems 10.7, page 419

3. $\pi^4/96$.

Problems 10.8, page 420

1. $\displaystyle\sum_{-\infty}^{\infty} \frac{(\sinh al \cos n\pi l - i \cosh al \sin n\pi l)(a + in\pi)}{(a^2 + n^2\pi^2)} e^{in\pi x/l}$

2. $\displaystyle\frac{2}{\pi}\left\{1 - \frac{e^{2it} + e^{-2it}}{1.3} - \frac{e^{4it} + e^{-4it}}{3.5} - \frac{e^{6it} + e^{-6it}}{5.7} - \ldots\ldots\right\}$

3. $\displaystyle\frac{a}{\pi} \sin a\pi \sum_{-\infty}^{\infty} \frac{(-1)^n e^{inx}}{a^2 - n^2}$

4. $\displaystyle\sin h\, 9 \sum_{-\infty}^{\infty} \frac{(-1)^n (9 + n\pi i)}{81 + (n\pi)^2} e^{n\pi ix/3}$

5. $\displaystyle\frac{a}{2} - \frac{a}{\pi}\left[(e^u - e^{-u}) + \frac{1}{3}(e^{3u} - e^{-3u}) + \frac{1}{5}(e^{5u} - e^{-5u}) + \ldots\ldots\right]$ where $u = i\pi x/l$.

Problems 10.9, page 423

1. $11.733 - 7.733 \cos 2x - 2.833 \cos 4x + \ldots\ldots - 1.566 \sin 2x - 0.116 \sin 4x + \ldots\ldots$

2. $1.45 + (-0.37 \cos x + 0.17 \sin x) - (0.1 \cos 2x + 0.06 \sin 2x)$

3. $a_0 = 41.66$, $a_1 = -8.33$, $b_1 = -1.15$

4. -0.0731

5. $y = 2.102 + 0.558 \cos x + 1.531 \sin x + 0.354 \cos 2x + 0.145 \sin 2x$

6. $7.8 \sin \theta + 1.5 \sin 2\theta - 9.2 \sin 3\theta + 11.6 \sin 4\theta - \ldots\ldots$.

Problems 10.10, page 424

1. $2\pi/3$

2. $\frac{1}{2}\{f(c-0) + f(c+0)\}$

3. $(-1, 1)$ such that $f(x) = -f(-x)$

4. $f(x) = A$ when $0 < x < \pi$ and $f(x) = -A$ when $\pi < x < 2\pi$

5. Sine

6. § 10.11 (3)

7. Zero

8. not defined

9. odd

10. Cosine

11. even

12. $x = k/n$

13. Zero

14. Cosine

15. Zero

16. $\displaystyle\int_0^2 x^2 \cos \frac{n\pi x}{2}\, dx$

17. $\displaystyle\frac{1}{T}\int_\alpha^{\alpha+2T} f(x) \sin \frac{n\pi x}{T}\, dx$

18. § 10.3

19. Zero

20. $\displaystyle a_0 = \frac{2}{l}\int_0^l f(x)\, dx$, $\displaystyle a_n = \frac{2}{l}\int_0^l f(x) \cos \frac{n\pi x}{l}\, dx$

21. $\displaystyle\frac{4}{\pi}\left\{\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\ldots\right\}$

22. π

23. Zero

24. $2l$

25. $\displaystyle\sum_{n=-\infty}^{\infty} (-1)^n \frac{(1 - in\pi)}{1 + n^2\pi^2} \sinh 1 e^{in\pi x}$

26. False

27. $-\pi/2$

28. odd

29. Zero

30. 3.5355

31. zero

32. $-1/2$

33. $\frac{1}{2}a_n$

34. $\frac{\pi^2}{8}$

35. $\displaystyle\frac{\pi}{2} - \frac{4}{\pi}\left\{\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots\right\}$

36. $\displaystyle f(x) = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x$

37. $x^2 - x$

38. $x(l + x)$

39. True

40. False

41. False.

Problems 11.1, page 429

1. $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = 0$
2. $\dfrac{d^2y}{dx^2} + 4y = 0$
3. $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - xy = 0$
4. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$
5. $\dfrac{d^3y}{dx^3} - 7\dfrac{dy}{dx} + 6y = 0$
6. $2xy\dfrac{dy}{dx} + x^2 - y^2 = 0$
7. $(x^2 - 25)\left(\dfrac{dy}{dx}\right)^2 + x^2 = 0$
8. $y\dfrac{dy}{dx} = 2a.$
10. $y''' - 3y'' + 3y' - y = 0.$

Problems 11.2, page 431

1. $\sqrt{(1 - x^2)} + \sqrt{(1 - y^2)} = c$
2. $\log\dfrac{x}{y} - \dfrac{1}{x} - \dfrac{1}{y} = c$
3. $\tan x \tan y = c$
4. $(1 + x^2)^{3/2} - 3\sqrt{(1 + y^2)} = c$
5. $\tan y = c(1 - e^x)$
6. $2e^{-y} = e^{-x^2} + 1$
7. $x = 2\cos y$
8. $(x^2 + 1)(y^2 + 1) = c$
9. $3e^{2x} - 2e^{3y} + 8x^3 = c$
10. $(1 - ay)(a + x) = cy$
11. $(x + 1)(2 - e^y) = c$
12. $a\log\left(\dfrac{x - y - a}{x - y + a}\right) = 2y + c.$
13. $y = \tan^{-1}(x + y + 1) + c$
14. $\tan(x + y) = \sec(x + y) + x + c$
15. $x = \operatorname{cosec}(x + y + 1) - \cot(x + y + 1) + c$
16. $\log\sin(y - x) = \dfrac{1}{2}x^2 + cz$
17. $\cos xy + \dfrac{1}{2x^2} = c.$

Problems 11.3, page 432

1. $x(x^2 - 3y^2) = c$
2. $cy^3 = x^2e^{-x/y}$
3. $(x/y)^3 = 3\log cy$
4. $y + \sqrt{(x^2 + y^2)} = c$
5. $y^2 = 2x[y + x\log(cx)]$
6. $x(c + y) = ay^2$
7. $y = 2x\tan^{-1}(cx)$
8. $e^{x/y} = y + c$
9. $\log y - \dfrac{x^2}{4y^2}\left(z\log\dfrac{y}{x} + 1\right) = c$
10. $\log x = \dfrac{1}{2}\left[\dfrac{y}{x} - \dfrac{1}{2}\sin\left(\dfrac{2y}{x}\right)\right] + c$
11. $xy\cos(y/x) = c.$

Problems 11.4, page 434

1. $(X^2 + 2Y^2)^2 = c\left(\dfrac{\sqrt{2}Y - X}{\sqrt{2}Y + X}\right)$ where $X = x + 1$, $Y = y - 1$
2. $(y - x)^3 = c(y + x - 2)$
3. $(x + y)^7 = c(x - y - 2/3)^3$
4. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
5. $3(2y - x) + \log(3x + 3y + 4) = c$
6. $x - y + \dfrac{3}{4}\log(8x - 12y - 5) = c$
7. $\log(x + y + \frac{1}{3}) + \frac{3}{2}(y - x) = c.$

Problems 11.5, page 437

1. $y = ce^{-\tan x} + \tan x - 1$
2. $y = \log x + c/\log x$
3. $y\sec^2 x = \sec x - 2$
4. $y\cosh x = c + \dfrac{2}{3}\cosh^3 x$
5. $y\sqrt{(1 - x^2)} = \sin^{-1} x + c$
6. $y = c(1 - x)^2 + (1 - x^2)$
7. $y(1 + \sin x) = c - x^2/2$
8. $2r\sin^2\theta + \sin^4\theta = c$
9. $ye^{x^2} = 2x + c$

10. $x = y^3 + cy$ **11.** $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$ **12.** $xy^{-2} = c - e^{-y}$

13. $xe^{\tan^{-1} y} = \tan^{-1} y + c$ **14.** $xe^y = c + \tan y$.

Problems 11.6, page 439

1. $y^{-1} \sec x = \tan x + c$ **2.** $1/r = \sin\theta + c\cos\theta$ **3.** $x^2 + (4x^5 + c)y^4 = 0$

4. $1/y = x^2 - 2 + ce^{-x^2/2}$ **5.** $y^2 = x^2 + cx - 1$ **6.** $y/x = \log y + c$

7. $\sin y = (1 + x)(e^x + c)$ **8.** $e^{x+y} = \frac{1}{2}e^{2x} + c$ **9.** $\tan y = x^3 - 3x^2 + 6x - 6 + ce^{-x}$

10. $\cos y = \cos x(\sin x + c)$ **11.** $\sqrt{x} = \sqrt{y}(\log\sqrt{y} + c)$ **12.** $y^{-1} = \frac{1}{2}\log x + \frac{1}{4} + cx^2$.

Problems 11.7, page 442

1. $x^3 + y^3 - 3axy = c$ **2.** $x^4 + 2x^2y^2 - y^4 - 2a^2x^2 - 2b^2y^2 = c$

3. $x^3 - 6x^2y - 6xy^2 + y^3 = c$ **4.** $\frac{x^5}{5} - x^2y^2 + xy^4 + \cos y = c$ **5.** $e^{xy} + y^2 = c$

6. $x^5 + x^3y^2 - x^2y^3 - y^5 = c$ **7.** $x^3 + 3x^2y^2 + y^4 = c$ **8.** $x^2 - y^2 = cy^3$

9. $3y\cos 2x + 6y + 2y^3 = c$ **10.** $e^x = \sec x \tan y + c$ **11.** $x^2y + xy - x\tan y + \tan y = c$.

Problems 11.8, page 445

1. $ax + \tan^{-1} y/x = c$ **2.** $x^2 + y^2 - 2a^2\tan^{-1}(y/x) = c$ **3.** $y + cx + \log x + 1 = 0$

4. $3\log x - (y/x)^3 = c$ **5.** $\log(y/x) + \frac{1}{2}x^2y^2 = c$ **6.** $xy + \log(x/y) - (1/xy) = c$

7. $(y + 2/y^2)x + y^2 = c$ **8.** $4x^4y + 4x^3y^2 - x^4 = c$ **9.** $2\cos(xy) + x^{-2} = c$

10. $\log(x/y) = c + xy$ **11.** $(x/y) + e^{x^3} = c$ **12.** $4(xy)^{1/3} - \frac{2}{3}(x/y)^{3/2} = c$

13. $4y\log x = y^2 + c$.

Problems 11.9, page 446

1. $(x - y + c)(x^2 + y^2 + c) = 0$ **2.** $(2y - x^2 + c)(y + x + ce^{-x} - 1) = 0$

3. $x^2 + y^2 = cx$ **4.** $(y - cx)(y^2 - x^2 - c) = 0$

5. $(y - c)(y + x^2 - c)(xy + cy + 1) = 0$.

Problems 11.10, page 448

1. $x + c = \frac{a}{2}\left[\log\frac{p-1}{\sqrt{(1+p^2)}} - \tan^{-1} p\right]$, with the given relation

2. $xy = c^2x + c$ **3.** $y = 2\sqrt{(xc)} + c^2$ **4.** $2cy = c^2x^2 + 1$

5. $x = (\log p - p + c)(p - 1)^2$, with the given relation

6. $x = \sin p + c$, with the given relation.

Problems 11.11, page 449

1. $y = c(x - c)^2$ **2.** $y^2 = 2cx + c^3$

3. $(y + \alpha p)\sqrt{(p^2 - 1)} + a\cosh^{-1} p = c$, with the given relation

4. $y + (1 + p^2)^{-1} = c$, with the given relation.

Problems 11.12, page 450

1. (*i*) Gen. sol. : $y = cx + a/c^2$; Singular sol. : $2ax^2 = (2ac + x)^3$

(*ii*) Gen. sol. : $c = \log(cx - y)$; Singular sol. : $y = x(\log x - 1)$

(*iii*) Gen. sol. $y = cx + \sqrt{(a^2c^2 + b^2)}$; Singular sol. $y + \sqrt{1 - x^2} = 0$

(*iv*) Gen. sol. $y = cx - \sin^{-1} c$; Singular sol. $y = \sqrt{x^2 - 1} - \sin^{-1} \dfrac{\sqrt{x^2 - 1}}{x}$

2. $y = cx + (c - 2c^2)$ **3.** $(y - cx)(c - 1) = c$

4. $(y - cx)(c + 1) + ac^2 = 0$ **5.** $y^2 = cx^2 + c^2$ [**Hint :** Put $x^2 = u, y^2 = v$]

6. $xy = cy - c^2$ [**Hint :** Put $u = y, v = xy$] **7.** $y^2 = cx^2 - \dfrac{2c}{1 + c}$.

Problems 11.13, page 450

1. (*i*) **2.** (*ii*) **3.** (*iii*) **4.** (*i*) **5.** $\log y + c = x^2/2y^2$

6. $yx^2 = x^3 + c$ **7.** $e^x + x^2y + cy = 0$. **8.** (*iii*) **9.** $x^2 + y^2 + 2\tan^{-1} y/x = c$

10. $\log x + c = y^3/3x^3$ **11.** (*i*) **12.** $y^2 + 1/x + ce^{-y^2/2} = 2$

13. $y = cx + a/c^2$ **14.** $c = \log(cx - y)$ **15.** $xy = c$ or $x^2 - y^2 = c$

16. 2 **17.** $xy = c$ **18.** (*b*) **19.** (*b*)

20. $(1 + x^2)^{3/2} + (1 + y^2)^{3/2} = c$ **21.** $y = 5e^{-x}$ **22.** $x + y = u$

23. x^{-5} **24.** § 11.11 (3) **25.** $5x^4y^2 + 2(x^5 + y^5) = c$

26. $\sin(y/x) = cx$ **27.** (*a*) **28.** (*c*) **29.** $x + y\, dy/dx = 0$

30. $e^{-x^2} + 2\cos y = c$ **31.** (*c*) **32.** False **33.** False.

Problems 12.1, page 454

1. (*i*) $9y + 4x^2 = 0$; (*ii*) $3(x + 3y) = 2(1 - e^{3x})$ **2.** $y + 1 = 2e^{x^2/2}$

3. $x^2 + y^2 = cx$ **4.** $y = \sqrt{(a^2 - x^2)} + a \log\left(\dfrac{\alpha - \sqrt{a^2 - x^2}}{x}\right) + c$

5. $y^2 = 4x$ **6.** $y = ae^{cx}$ **7.** $y = ax + b$

8. $x = 3y^2$ **9.** (*i*) $r(\theta - \alpha) = c$; (*ii*) $r = a + b\cos\theta$

10. $r^2 = a^2 \sin 2\theta$ **11.** $c^2x^2 = 2cy + 1$ **12.** $r = ae^{\theta \cot \alpha}$.

Problems 12.2, page 457

1. $2x^2 + y^2 = c$ **2.** $x^2 + 2y^2 = c^2$ **3.** $3y^2 + 2x^2 = c^2$

4. $x^2 + y^2 + 2\mu y - c = 0$ **5.** This system is self-orthogonal **6.** $r = c(1 - \cos\theta)$

7. $r = b(\cos\theta - \sin\theta)$ **8.** $r = 2b/(1 - \cos\theta)$ **9.** $r^2 = c^2 \sin 2\theta$

10. $r^n\, n \sin\theta = b$ **13.** $x^2 + y^2 + cx + 1 = 0$ **14.** $y = cx$.

Problems 12.3, page 462

1. $V = \sqrt{\left(\dfrac{mg}{k}\right)} \tanh\left(\dfrac{9k}{m}t + c\right)$ **3.** $\dfrac{1}{k} \log_e 2$

5. $2\sqrt{v_o}/k$ **6.** $v^2 = 2gx - \dfrac{\lambda}{m} x^2$

10. $y = (\sqrt{150} - 0.001328t)^2$; $t_1 = 45$ min. 1 sec., $t_2 = 1$ hr. 16 min. 51 sec., $t_3 = 1$ hr. 38 min. 13 sec

11. 17 min. 4 sec.

Problems 12.4, page 465

1. 0.0006931 sec
2. $\frac{10}{L^2 + R^2}(R \sin t - L \cos t + Le^{-Rt/L})$
3. $i = \frac{1}{5}(1 - e^{-100})$
4. $i = ke^{-t/RC} + \frac{\omega CE_m}{\sqrt{(1 + R^2C^2\omega^2)}} \sin(\omega t + \theta)$ where $\theta = \cot^{-1}(RC\omega)$.

Problems 12.5, page 467

1. 52.5 mts
2. 48°C
3. B drinks hotter coffee
4. 490,000 cal
5. 2.16 cm.

Problems 12.6, page 469

1. 604.9
2. 2 log 3/log 2
3. $(1 - 1/p)^{21}$ times the original amount
4. 64.5 days
5. 21.5 gm
6. $t = 300 - 5 \log 2 + 5 \log \frac{0.7 - x}{0.5 - x}$
7. 3 hr. 50 min. 16 sec
8. $100(2 - e^{-t/20})$; 13.9 min.

Problems 12.7, page 469

1. $6(1 - e^{-3})$
2. 54 m
3. 90.25%
4. $r(\theta - \alpha) = c$
5. $y = ae^{cx}$
6. rectangular hyperbola
7. $x^2 - y^2 = c$
8. The system is self-orthogonal
9. $2\sqrt{v_0}/k$
10. 2 log 3/log 2
11. Sunil
12. (*d*)
13. (*c*)
14. (*d*)
15. 2.21
16. (*c*)
17. (*c*)
18. (*a*)
19. False
20. True.

Problems 13.1, page 474

1. $\frac{2}{3}e^{2t} \sin 3t$
2. $y = e^x(4 \cos 3x - \sin 3x)$
3. $y = c_1 + (c_2 + c_3x)e^{-x/2}$
4. $y = c_1e^{-x} + e^{x/2}\left(c_2 \cos \frac{\sqrt{3x}}{2} + c_3 \sin \frac{\sqrt{3x}}{2}\right)$
5. $y = (c_1 + c_2x + c_3x^2)e^x$
6. $y = (c_1 + c_2x) \cos 2x + (c_3 + c_4x) \sin 2x$
7. $y = c_1e^{-x} + c_2e^{2x} + e^{x/2}(c_3 \cos x/\sqrt{2} + c_4 \sin x/\sqrt{2})$
8. $y = (c_1 + c_2x) \cos x + (c_3 + c_4x) \sin x + c_5e^x$.

Problems 13.2, page 486

1. $y = (c_1 + c_2x)e^{3x} + 3x^2e^{3x} + \frac{7}{25}e^{-2x} - \frac{1}{9} \log 2$
2. $y = \frac{3}{5}e^{-2x}(\cos x + 3 \sin x) - \frac{e^x}{10} - \frac{e^{-x}}{2}$
3. $x = c_1 \cos nt + c_2 \sin nt + \frac{kx}{2n} \sin(nt + \alpha)$
4. $x = e^{-t}(c_1 \cos \sqrt{2}t + c_2 \sin \sqrt{2}t) + \frac{1}{4}(\sin t - \cos t)$
5. $y = c_1e^{-x} + c_2e^{-2x} + 1 + \frac{1}{10}(3 \sin 2x - \cos 2x)$

6. $y = c_1e^x + c_2e^{3x} + \frac{1}{884}(10\cos 5x - 11\sin 5x) + \frac{1}{20}(\sin x + 2\cos x)$

7. $y = c_1 + (c_2 + c_3x)\,e^{-x} - \frac{x^2}{2}e^{-x} + \frac{3}{50}\cos 2x - \frac{2}{25}\sin 2x$

8. $y = (c_1 + c_2x)\,e^{-x} + \frac{1}{2} + \frac{1}{5}(2\sin 2x + \cos 2x)$

9. $y = (c_1 + c_2x)e^x + c_3\,e^{3x} + \frac{1}{8}(xe^{3x} - x^2e^x)$ **10.** $y = c_1e^x + c_2e^{-x} + \frac{e^x}{12}(2x^3 - 3x^2 + 9x)$

11. $y = c_1 + c_2e^x + c_3e^{-x} + xe^x - (x^2 + x) - 2\sin x$

12. $y = e^{3x}(c_1\cos 4x + c_2\sin 4x) + \frac{1}{17}e^{2x} + \frac{1}{565}(23\sin x + 6\cos x) + \frac{x}{25} + \frac{6}{625}$

13. $y = (c_1 + c_2x)\cos x + (c_3 + c_4x)\sin x + x^4 - 24x^2 + 72 + \frac{1}{225}\sin 4x - \frac{1}{9}\sin 2x$

14. $y = c_1\,e^{-2x} + c_2e^{-3x} - \frac{e^{-2x}}{10}(\cos 2x + 2\sin 2x)$

15. $y = e^{-x/2}\left\{(c_1 + x/4)\cos(x\sqrt{3/2}) + (c_2 + x/4\sqrt{3})\sin(x\sqrt{3/2}\right\} + e^{x/2}\left\{c_3\cos\sqrt{3x/2} + c_4\sin\sqrt{3x/2}\right\}$

16. $y = e^{-x}(c_1\cos\sqrt{2x} + c_2\sin\sqrt{2x}) + \frac{e^x}{41}(4\sin x + 5\cos x).$

17. $y = c_1\,e^{-x} + c_2e^{-3x} - \frac{e^{-x}}{5}(\sin x + 2\cos x) + \frac{e^{3x}}{22}\left(x - \frac{5}{11}\right)$

18. $y = c_1\cos\sqrt{2x} + c_2\sin\sqrt{2x} + \frac{e^{3x}}{11}\left(x^2 - \frac{12}{11}x + \frac{50}{121}\right) + \frac{e^x}{17}(4\sin 2x - \cos 2x)$

19. $y = c_1e^x + c_2e^{-x} + c_3\cos x + c_4\sin x - (1/5)\cos x\cosh x$

20. $y = c_1 + (c_2 + c_3\,x)\,e^{-x} + \frac{e^{2x}}{18}\left(x^2 - \frac{7x}{8} + \frac{11}{6}\right) + \frac{1}{100}(3\sin 2x + 4\cos 2x)$

21. $y = c_1\cos 4x + c_2\sin 4x + \frac{1}{7}\left(x\sin 3x - \frac{6}{7}\cos 3x\right)$

22. $y = (c_1 + c_2x)\,e^{-x} + \frac{1}{2}\cos x + \frac{1}{2}(x - 1)\sin x$

23. $y = c_1e^x + c_2e^{-x} - \frac{1}{2}(x\sin x + \cos x) + (xe^x/12)(2x^2 - 3x + 9)$

24. $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x} + e^{-2x}\,.\,e^{e^x}$

25. $y = c_1\cos ax + c_2\sin ax - \frac{1}{a^2}\cos ax\log(\sec ax + \tan ax).$

Problems 13.3, page 490

1. $y = (c_1 - x/a)\cos ax + [c_2 + (1/a^2)\log\sin ax]\sin ax$

2. $y = c_1\cos x + c_2\sin x + \cos x\log(\cos x) + x\sin x$

3. $y = c_1\cos x + c_2\sin x - \cos x\log(\sec x + \tan x)$

4. $y = c_1\cos x + c_2\sin x + \frac{x}{2}\sin x - \frac{x^2}{4}\cos x$ **5.** $y = (c_1 + c_2x)\,e^x + xe^x\log x$

6. $y = (e^x + e^{2x})\log(1 + e^x) + (c_1 - 1 - x)\,e^x + (c_2 - x)\,e^{2x}$

7. $y = e^x(c_1\cos x + c_2\sin x) - e^x\cos x\log(\sec x + \tan x)$

8. $y = c_1 + c_2e^{2x} - \frac{1}{2}e^x\sin x$ **9.** $y = c_1\cos x + c_2\sin x + \sin x\log(1 + \sin x) - x\cos x - 1$

10. $y = c_1\,e^x + c_2\,e^{2x} + \frac{1}{2}(x^2 + 3x + 3.5 - 2xe^x)$ **11.** $y = c_1\cos x + c_2\sin x - x\sin x$

12. $y = c_1e^{2x} + c_2e^{3x} + xe^{3x} + \frac{1}{10}(\sin x + \cos x)$ **13.** $y = c_1e^{x} + c_2e^{-2x} - \frac{1}{4}(2x + 1) - \frac{1}{10}(\cos x + 3\sin x)$

14. $y = e^{x}(c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x) + \frac{1}{27}(9x^3 + 18x^2 + 6x - 8) + \frac{1}{4}(\cos x - \sin x)$

15. $y = c_1 + c_2e^{2x} - \frac{1}{2}e^{x}\sin x.$

Problems 13.4, page 495

1. $y = c_1x^2 + c_2x^3 - x^2 \log x$ **2.** $y = c_1x^4 + c_2x^{-1} + \frac{x^4}{5}\log x$

3. $y = (c_1 + c_2 \log x)x^2 + \frac{1}{4} + 2x + \frac{1}{2}x^2(\log x)^2$

4. $y = c_1x^2 + c_2x^{-1} + \frac{1}{3}(x^2 - 1/x)\log x$ **5.** $u = \frac{kr}{8}(a^2 - r^2)$

6. $c_1x^{-1} + c_2x^{-2} + \frac{1}{2}\log x - \frac{3}{4}$

7. $y = c_1x^{-1} + \sqrt{x}\{c_2 \cos[(\sqrt{3}/2)\log x] + c_3 \sin[(\sqrt{3}/2)\log x]\} + \frac{1}{2}x + \log x$

8. $y = c_1x^{-2} + x[c_2 \cos(\sqrt{3}\log x) + c_3 \sin(\sqrt{3}\log x)] + 8\cos(\log x) - \sin(\log x)$

9. $y = c_1x^{-1} + \{c_2 \cos(\log x) + c_3 \sin(\log x)\}x + 5x + 10 \log x/x$

10. $y = \frac{1}{x}(c_1 + c_2 \log x) + \frac{1}{x}\log\frac{x}{1-x}$ **11.** $y = x^{-2}(c_1 + c_2 \log x) + \frac{x}{9}\left(\log x - \frac{2}{3}\right)$

12. $y = c_1x^3 + c_2x^{-4} + \frac{x^3}{98}\log x(7 \log x - 2)$

13. $y = c_1(2x + 3)^a + c_2(2x + 3)^b - \frac{3}{14}(2x + 3) + \frac{3}{4}$ where $a, b = \frac{3 \pm \sqrt{57}}{4}$

14. $y = c_1(x - 1) + c_2(x - 1)^2 + c_3(x - 1)^{-2} + \log(x + 1) + 1$

15. $y = c_1 \cos \log(1 + x) + c_2 \sin \log(1 + x) - \frac{1}{3}\sin[2 \log(1 + x)]$

16. $y = c_1(3x + 2)^{1/3} + c_2(3x + 2)^{-1} + \frac{1}{27}\left[\frac{1}{15}(3x + 2)^2 + \frac{1}{4}(3x + 2) - 7\right]$

Problems 13.6, page 499

1. $x = (c_1 + c_2x)e^{3x}$; $y = [(1 - 2x)(c_2 - 2c_1)]e^{3x}$

2. $x = e^t + e^{-t}$, $y = e^{-t} - e^t + \sin t$ **3.** $x = c_1e^t + c_2e^{-5t} + \frac{6}{7}e^{2t}$; $y = c_2e^{-5t} - c_1e^t + \frac{8}{7}e^{2t}$

4. $x = e^{6t}(c_1 \cos t + c_2 \sin t)$, $y = e^{6t}[(c_1 - c_2)\cos t + (c_1 + c_2)\sin t]$

5. $x = \frac{1}{5}e^t + \frac{2}{5}e^{-t} - c_1 \sin 2t + c_2 \cos 2t$, $y = \frac{2}{5}e^t + \frac{1}{5}e^{-t} + c_1 \cos 2t + c_2 \sin 2t$

6. $x = c_1e^t + c_2e^{-5t} + \frac{3}{7}e^{2t} - \frac{2}{5}t - \frac{13}{25}$, $y = c_1e^t - c_2e^{-5t} - \frac{4}{7}e^{2t} - \frac{3t}{5} - \frac{12}{25}$

7. $x = -t - \frac{2}{3}$, $y = \frac{1}{2}t^2 + \frac{4}{3}t + c$ **8.** $y = c_1e^x + c_2e^{-2x} + 2e^{-x}$, $z = 3c_1e^x + 2c_2e^{-2x} + 3e^{-x}$

9. $x = c_1e^{-t} + c_2e^{3t} - \frac{1}{5}(\cos t - 2 \sin t)$; $y = 2c_1e^{-t} - 2c_2e^{3t} + \frac{1}{5}(\sin t + 2 \cos t)$

10. $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$, $y = \frac{1}{2}\left(-t + \frac{1}{t}\right)$

11. $x = c_1e^{-t} + c_2e^{3t} - \frac{1}{5}(\cos t - 2 \sin t)$, $y = 2c_1e^{-t} - 2c_2e^{3t} + \frac{2}{5}\cos t + \frac{1}{5}\sin t$

12. $x = (c_1 + c_2t)e^{-t} + (c_3 + c_4t)e^t$, $y = -\frac{1}{2}[c_1 + c_2(1 + t)]e^{-t} + \frac{1}{2}[c_4(1 - t) - c_3]e^t$

13. $x = c_1e^t + c_2e^{-t} + c_3 \cos t + c_4 \sin t - \frac{t}{4} \cos t + \frac{t}{4} \sin t$

$y = -c_1e^t - c_2e^{-t} + c_3 \cos t + c_4 \sin t + \frac{1}{4}(2 + t)(\sin t - \cos t)$

14. $x = \frac{8}{9}\left(1 - \cos \frac{3}{2}t\right), y = \frac{4}{3}t - \frac{8}{9} \sin \frac{3}{2}t.$

Problems 13.7, page 500

1. $y = c_1e^{ax} + c_2e^{-ax} + c_3 \cos ax + c_4 \sin ax$

2. $-\frac{1}{25}(3 \sin 2x + 4 \cos 2x)$ **3.** 1/6 **4.** $e^x(x - 1)$

5. (*b*) **6.** $y = c_1 + (c_2 + c_3 x + c_4 x^2) e^{2x}$ **7.** (*a*)

8. $y = e^x (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x$ **9.** $y = \cos x + 2 \sin x$

10. (*ii*) **11.** $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x$

12. $\frac{1}{10} \cosh 3x$ **13.** $y = a \log x + 6$

14. $y = (c_1 + c_2 x)\, e^{\sqrt{2}x} + (c_3 + c_4x) e^{-\sqrt{2}x}$ **15.** $\frac{1}{6}x^3e^{-x}$

16. $\frac{1}{2}e^{2x}$ **17.** $\sin 2x$ **18.** $\frac{1}{2}x^2 e^{-x}$

19. $y = (c_1 + c_2 x) e^{-x/2} + c_3$ **20.** (*c*) **21.** (*a*)

22. $x e^{-t}$ **23.** $\frac{d^2y}{dt^2} + 7y = 2e^t.$ **24.** (*c*)

25. (*a*) **26.** (*b*)

27. $y = (c_1 + c_2 \log x) x$ **28.** $x^2y_2 + xy_1 - y = 0$ **29.** $\frac{1}{9} \log 2$

30. e^t **31.** (*d*)

32. $y = c_1e^{-x} + c_2 e^{2(1+\sqrt{2})x} + c_3 e^{2(1-\sqrt{2})x}$ **33.** $\frac{d^3y}{dx^3} + 2$

34. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ **35.** False **36.** False.

Problems 14.1, page 506

1. 38 sec **4.** $x = \frac{ue^{-\lambda nt}}{n\sqrt{1 - \lambda^2}} \sin [nt\sqrt{(1 - \lambda^2)}]$

6. It must be shortened by 1/8640 of its length

7. It must be increased by 0.0074 ft./sec^2 **8.** 4321/4319

9. $k^2 > 4\mu$, $\theta = c_1e^{\frac{-k+\lambda}{2}t} + c_2e^{\frac{-k-\lambda}{2}t}$ where $\lambda = \sqrt{(k^2 - 4\mu)}$

$k^2 = 4\mu$, $\theta = (c_1 + c_2t)\, e^{-kt/2}$

$k^2 < 4\mu$, $\theta = c_1\, e^{-kt/2} \cos \left(\frac{\sqrt{4\mu - k^2}}{2}t + c_2\right)$

10. $x = \frac{F_0}{2n^2}(\sin nt - n \cos nt).$

Problems 14.2, page 513

2. 1 ft. ; $\pi/2\sqrt{2}$ sec ; $4\sqrt{2}$ ft./sec **4.** $\pi/\sqrt{7}$

5. $x = e^{-5t}\left\{\cos\sqrt{220t} + (5/\sqrt{220})\sin\sqrt{220}\,t\right\}$. **6.** 0.45 sec. ; 1.15 sec

8. $x = \frac{10}{21}e^{-t} - \frac{17}{27}e^{-8t} - \frac{\sqrt{2}}{3}\sin\left(9t + \frac{\pi}{4}\right)\frac{\sqrt{2}}{3}, \frac{2\pi}{9}$ sec., $9/2\pi$ cycles/sec

9. $0.8\,(2\sin 4t - \cos 4t)$

10. (*i*) $x = Ae^{-kt}\cos\left\{t\sqrt{(b^2 - k^2)} + B\right\} + \{e^{-kt}/(b^2 + k^2 - n^2)\}\sin nt$

(*ii*) $x = Ae^{-kt}\cos\left\{t\sqrt{(b^2 - k^2)} + B\right\} - (te^{-kt}/2n)\cos nt$

Problems 14.3, page 517

2. $i = I\sin(T/\sqrt{LC})$ **3.** $i = 2Eke^{-RT/2L}\sin(kt/2L)$, where $k = \sqrt{\left(\frac{4L - CR^2}{C}\right)}$

4. $R^2 > 4L/C$ for over damping ; $R^2 = 4L/C$ for critical damping ; $R^2 < 4L/C$ for under damping ; critical resistance $= 2\sqrt{L/C}$

5. $q = e^{-500t}(0.002\cos 1323t + 0.0008\sin 1323t)$

8. (*i*) $i = Ae^{-\alpha t}\cosh(\beta t + \gamma)$; (*ii*) $i = Ae^{-\alpha t}\cos(\beta t + \gamma) + \frac{E}{R}\cos\phi\sin(pt + \phi)$

where $\alpha = -\frac{R}{2L}$, $\beta = \pm\left\{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}\right\}$ and $\phi = \tan^{-1}\{(1 - CLp^2)/CRp\}$.

Problems 14.4, page 525

4. $y = \frac{wl}{2Pn}\operatorname{cosec}\frac{nl}{2}\cos\left(nx - \frac{nl}{2}\right) - \frac{wl}{2nP}\cot\frac{nl}{2} + \frac{w}{2P}(x^2 - lx)$

6. $y = \frac{F}{P}[n\sin nx - l\cos nx + l - x]$ **7.** $\pi^2EI/4l^2$

8. $\frac{W}{2a^2}\left(\operatorname{sech}\frac{al}{2} - \sec al\right)$

Problems 14.5, page 528

1. $\frac{2u\sin\alpha}{g}; \frac{u^2\sin 2\alpha}{g}$ **2.** (*i*) $\frac{2u^2\sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta}$; (*iii*) $\frac{u^2}{g(1 + \sin\beta)}$

4. $4x^2 + k^2y^2 = 4$ **5.** $i_1 = \frac{a}{p + \omega}\sin pt, i_2 = \frac{a}{p + \omega}\cos pt.$

6. $i_1 = \frac{E}{R}\left(\frac{2}{3} - \frac{1}{2}e^{-Rt/L} - \frac{1}{6}e^{-3Rt/L}\right)$; $i_2 = \frac{E}{R}\left(\frac{1}{3} - \frac{1}{2}e^{-Rt/L} + \frac{1}{6}e^{-3Rt/L}\right)$

7. $x = a(nt - \sin nt), y = a(1 - \cos nt)$

8. $x = \frac{E}{H\omega}(1 - \cos\omega t), y = \frac{E}{H\omega}(\omega t - \sin\omega t)$, where $\omega = eH/m$.

Problems 14.6, page 529

1. (*b*) **2.** (*b*) **3.** (*c*) **4.** (*b*)
5. (*b*) **6.** (*b*) **7.** (*b*) **8.** 60 sec

9. $30/\pi\sqrt{LC}$ **10.** 0.0074 sec **11.** resonance **12.** $EI\dfrac{d^2y}{dx^2} - Py = \dfrac{w}{2}(x^2 - lx)$

13. $y = 0$ and $\dfrac{dy}{dx} = 0$

Problems 15.1, page 531

1. $y = -x^2 \sin x - 4x \cos x + c_1x + c_2$

2. $y = \dfrac{x^4}{24} + \dfrac{x^3}{6}\log x - \dfrac{11}{35}x^3 + c_1x^2 + c_2x + c_3$

Problems 15.2, page 533

1. $2(y^{1/4} - 1) = x$

2. $\sqrt{(y^2 - 8y)} + 4\cosh^{-1}(\frac{1}{4}y - 1) = 3x$

3. $r = \dfrac{\sqrt{(v^2 - a^2\omega^2)}}{\omega}\sinh\left[\omega t + \sinh^{-1}\dfrac{a\omega}{\sqrt{(v^2 - a^2\omega^2)}}\right]$

4. $t = \dfrac{h^{3/2}}{\sqrt{(2g)a}}\left[\cos^{-1}\sqrt{\dfrac{x}{h}} - \sqrt{\left(\dfrac{hx - x^2}{h}\right)}\right]$

Problems 15.3 page 534

1. $y = c_1 - x^2 - c_2/x$

2. $y = c_1x + (c_1^2 + 1)\log(x - c_1) + c_2$

3. $15c_1^2y = 4(c_1x + a^2)^{5/2} + c_2x + c_3$

4. $x^2 + y^2 = a^2$

5. $\theta = \dfrac{m}{\mu a}\log\left(1 + \dfrac{\mu a\omega t}{m}\right)$

6. $v = \dfrac{1}{r_1 - r_2}\left[v_1r_1 - v_2r_2 - \dfrac{(v_2 - v_1)r_1r_2}{r}\right].$

Problems 15.4, page 536

1. $y = 2x - 2\log(1 - c_1e^{2x}) + c_2$

2. $y^2 = x^2 + c_1x + c_2$

3. $\log y = c_1e^x + c_2e^{-x}$

4. $(\log y - 1)(c_1x + c_2) = 1$

5. $(x - a)^2 + y^2 = c^2$, circles whose centres are on the x-axis.

Problems 15.5, page 537

1. $y = e^{x^2}(c_1x + c_2)$

2. $y = (x^2 - x + c_1x)e^x + c_2x$

3. $y = e^x(c_1\log x + x + c_2)$

4. $cy = 1 + (k - x)\cot x$

5. $y = \left[c_1 - \dfrac{1}{2}\cos x - \dfrac{1}{5}c_2e^{-2x}(\cos x + 2\sin x)\right]e^x.$

Problems 15.6, Page 539

1. $y = c_1\cos(\sin x) + c_2\sin(\sin x)$

2. $y = c_1\cos(1/x) + c_2\sin(1/x)$

3. $y = c_1e^t + c_2e^{-t} - t$ where $t = \cos x$

4. $y = c_1\cos(2\tan^{-1}x) + c_2\sin(2\tan^{-1}x)$

5. $y = c_1e^{\sqrt{2}\sin x} + c_2e^{-\sqrt{2}\sin x} + \sin^2 x.$

Problems 15.7, page 540

1. $nx - lz = c(mz - ny)$ **2.** $x^2 + y^2 + z^2 = cx$ **3.** $xy^2 = cz^3$

4. $x^2 + y^2 - xz = cz$ **5.** $y(x + z) = c(x + y + z)$ **6.** $x + y + z + \log(xyz) = c.$

Problems 15.8, page 541

1. $x^3 - y^3 = c_1, x^2 - z^2 = c_2$ **2.** $lx + my + nz = c_1, x^2 + y^2 + z^2 = c_2$ **3.** $\dfrac{x-y}{y-z} = c_1, \dfrac{z-x}{y-z} = c_2$

4. $x^2 - y - 2xy = c_1,\ x^2 - y^2 - z^2 = c_2$ **5.** $xyz = c_1, x^2 + y^2 + z^2 = c_2$ **6.** $y = c_1 z, x^2 + y^2 + z^2 = c_2 z.$

Problems 16.1, page 544

1. $y = a_1\left(x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots\right)$

2. $y = a_0\left(1 - \dfrac{x^4}{4\cdot 3} + \dfrac{x^8}{8\cdot 7\cdot 4\cdot 3} - \dfrac{x^{12}}{12\cdot 11\cdot 8\cdot 7\cdot 4\cdot 3} + \ldots\ldots\right)$

$+\ a_1\left(x - \dfrac{x^5}{5\cdot 4} + \dfrac{x^9}{9\cdot 8\cdot 5\cdot 4} - \dfrac{x^{13}}{13\cdot 12\cdot 9\cdot 8\cdot 5\cdot 4} + \ldots\ldots\right)$

3. $y = a_0\left(1 - \dfrac{x^2}{2} + \dfrac{x^4}{2\cdot 4} - \dfrac{x^6}{2\cdot 4\cdot 6} + \ldots\ldots\right) + a_1\left(x - \dfrac{x^3}{3} + \dfrac{x^5}{3\cdot 5} - \dfrac{x^7}{3\cdot 5\cdot 7} + \ldots\ldots\right)$

4. $y = 4 + 5x - 4x^2 - \dfrac{5}{3}x^3 - \dfrac{x^5}{3} - \dfrac{x^7}{7} - \ldots\ldots$ **5.** $y = a_0\left(1 + \dfrac{x^2}{2} - \dfrac{x^4}{8} + \dfrac{x^6}{16} - \dfrac{5x^8}{128} + \ldots\right) + a_1 x$

6. $y = a_0(1 - x^2 + \frac{1}{3}x^4 - \frac{1}{5}x^6 + \ldots\ldots) + a_1 x.$

Problems 16.2, page 550

1. $y = c_1 \cos\sqrt{x} + c_2 \sin\sqrt{x}$ **2.** $y = a_0\left(1 - x^2 + \dfrac{x^4}{4}\ldots\ldots\right) + a_1\left(x - \dfrac{x^3}{2} + \dfrac{3x^5}{10}\ldots\ldots\right)$

3. $y = (c_1 + c_2 \log x)\left[1 + x + \dfrac{1}{(2!)^2}x^2 + \dfrac{1}{(3!)^3}x^3 + \ldots\ldots\right] - 2c_2\left[x + \dfrac{1}{(2!)^2}\left(1 + \frac{1}{2}\right)x^2 + \dfrac{1}{(3!)^2}\left(1 + \frac{1}{2} + \frac{1}{3}\right)x^3 + \ldots\ldots\right]$

4. $y = c_1(1 + x + x^2/4 + x^3/4\,.\,7 + \ldots) + c_2 x^{2/3}(1 + \frac{1}{3}x + x^2/3\,.\,6 + x^3/3\,.\,6\,.\,9 + \ldots)$

5. $y = a_0\left(1 - 2x + \dfrac{3}{2!}x^2 - \dfrac{4}{3!}x^3 + \ldots\right) + a_1\left[y_1 \log x + a_0\left(3x - \dfrac{13}{4}x^2 + \ldots\right)\right]$

6. $y = a_0 x\left(1 + \dfrac{x}{5} + \dfrac{x^2}{70} + \ldots\right) + a_1 x^{-1/2}\left(1 - x - \dfrac{x^2}{2} + \ldots\right)$

7. $y = c_1 x^{-\frac{1}{2}}\left(1 + \dfrac{x}{2} + \dfrac{x^2}{40} + \ldots\ldots\right) + c_2 x^{1/4}\left(1 + \dfrac{x}{14} + \dfrac{x^2}{616} + \ldots\ldots\right)$

$+\ c_2\sqrt{x}(x + x^2/2\,.\,3 + x^4/2\,.\,4\,.\,3\,.\,7 + x^6/2\,.\,4\,.\,6\,.\,3\,.\,7\,.\,11 + \ldots)$

8. $y = a_0\sqrt{x}(1 - x) + a_1\left(1 - 3x + \dfrac{3x^2}{1\cdot 3} + \dfrac{3x^3}{3\cdot 5} + \dfrac{3x^4}{5\cdot 7} + \ldots\right)$

9. $y = a_0(1 - \frac{2}{3}x + \frac{1}{3}x^2 + \ldots) + a_1x^4(1 - 2x + 3x^2 - 4x^3 + \ldots)$

10. $y = c_1\left(1 + 3x^2 + \frac{3}{5}x^4 \ldots\ldots\right) + c_2x^{3/2}\left(1 + \frac{3}{8}x^2 - \frac{1.3}{8.16}x^4 + \frac{1.3.5}{8.16.24}x^6 + \ldots\ldots\right).$

Problems 16.3, page 557

1. 0.224, 0.44.

Problems 16.4, page 562

1. $y = c_1J_{1/2}(x) + c_2J_{-1/2}(x)$

2. $y = c_1J_{2/5}(x) + c_2J_{-2/5}(x)$

3. $y = x^n[c_1J_n(kx) + c_2y_n(kx)]$ where $n = \frac{1}{2}(1 - \alpha)$

4. $y = x[c_1J_1(2x) + c_2Y_1(2x)]$

5. $y = c_1\sqrt{x}J_1(2\sqrt{x}) + c_2\sqrt{x}\,Y_1(2\sqrt{x})$

7. $y = c_1\sqrt{x}J_n(x) + c_2\sqrt{x}\,J_{-n}(x)$

11. $x^2 = \sum_{n=1}^{\infty} \frac{2}{\alpha_n^2} \cdot \frac{1}{J_2^2(3\alpha_n)}(3\alpha_nJ_1(3\alpha_n) - 2J_2(2\alpha_n))$

Problems 16.5, page 570

3. (*i*) $2P_3 + 4P_1$;

(*ii*) $\frac{2}{5}P_3 + \frac{4}{3}P_2 - \frac{2}{5}P_1 - \frac{7}{3}P_0$;

(*iii*) $\frac{8}{5}P_3 - 4P_2 + \frac{47}{5}P_1 + 4$

(*iv*) $\frac{8}{35}P_4 + \frac{6}{5}P_3 - \frac{2}{21}P_2 + \frac{34}{5}P_1 - \frac{224}{105}P_0$

9. (*i*) $f(x) = -\frac{7}{3}P_0(x) - \frac{2}{5}P_1(x) + \frac{4}{3}P_2(x) + \frac{2}{5}P_3(x)$;

(*ii*) $f(x) = -\frac{32}{15}P_0(x) - \frac{4}{5}P_1(x) - \frac{40}{21}P_2(x) + \frac{2}{5}P_3(x) + \frac{8}{35}P_4(x)$.

Problems 16.6, page 572

1. $x^3 = \frac{1}{4}(3T_1 + T_3)$.

Problems 16.7, page 575

1. $y_n(x) = \sin nx$, $n = 1, 2, \ldots$

2. $y_n(x) = \sin[(2n + 1)\pi x/2l]$, $n = 0, 1, 2, \ldots$

3. $y_n(x) = \cos nx$, $n = 0, 1, 2, \ldots$

4. $1, \sin x, \cos x, \sin 2x, \cos 2x, \ldots$

5. $y_n(x) = \sin\left[(2n + 1)\frac{\pi}{2}\log|x|\right]$, $n = 0, 1, 2, \ldots$

6. $[xe^{-x}y']' + ne^{-x^2}y = 0$, $p(x) = e^{-x}$

7. $[e^{-x^2}y']' + 2ne^{-x^2}y = 0$, $p(x) = e^{-x^2}$.

Problems 16.8, page 575

1. $\frac{1}{3}(10 - 9P_1 + 8P_2)$

2. $\sqrt{(2/\pi x)}\cos x$

3. $\frac{2}{(2n + 1)}$

4. zero

5. zero

6. $J_n(x) = \frac{x}{2n}(J_{n+1}(x) + J_{n-1}(x))$

7. $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$

8. $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

9. $\sqrt{(2/\pi x)}\sin x$

10. $x^n J_{n-1}(x)$ **11.** $\frac{1}{2}(3x^2-1)$ **12.** zero

13. True **14.** $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$ ■ **15.** $\alpha \neq \beta$

16. $2P_3 + 4P_1$ **17.** $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$

18. $-J_1(x)$ **19.** False **20.** True **21.** True **22.** True **23.** True
24. False **25.** (*b*) **26.** (*c*) **27.** (*iv*) **28.** (*iii*) **29.** (*iii*)
30. (*iv*) **31.** (*iii*) **32.** (*iii*) **33.** (*iii*) **34.** (*iii*) **35.** (*iv*)
36. 0, 1.

Problems 17.1, page 579

1. $z = px + qy + p^2 + q^2$ **2.** $z^2(p^2+q^2+1) = c^2$ **3.** $p^2 + q^2 = \tan^2\alpha$
4. $p + q = px + qy$ **5.** $z^2(p^2+q^2+1) = 9$ **6.** $py - qx = 0$
7. $py + qx = 0$ **8.** $qx - py = x + y$ **9.** $xys = px + py - z$

10. $xyr = 2\,(px + qy - 2z)$ **11.** $\frac{\partial^2 z}{\partial y^2} = \frac{\partial z}{\partial y}$ **12.** $x(y-z)p + y(z-x)q = z(x-y)$
y

13. $z\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}$ **14.** $p + q = mz$ **15.** $px^2 + qy = 2y^2$

16. $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ **17.** $\frac{\partial^2 v}{\partial t^2} = \frac{a^2}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right)$ **18.** $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial t} + \frac{\partial^2 z}{\partial t^2} = 0$

19. $p(x-2z) + q(2z-y) = y - x$ **20.** $(y-z)p + (z-x)q = x - y.$

Problems 17.2, page 581

1. $z = \frac{x^2}{2}\log x + axy + \phi(x) + \psi(y)$ **2.** $z = \frac{1}{6}x^3y + xf(y) + \phi(y)$

3. $u = -e^{-t}\sin x + \phi(x) + \psi(t)$ **4.** $z = f(x) + x\phi(y) + \psi(y) - \frac{1}{12}\sin(2x+3y)$

5. $z = e^x\cosh y + e^{-x}\sinh x$ **6.** $z = \sin x + e^y\cos x.$

Problems 17.3, page 584

1. $x = z^3 f(x/y)$ **2.** $\sqrt{x} - \sqrt{y} = f(\sqrt{x} - \sqrt{z})$ **3.** $x^2 + y^2 + z^2 = f(x+y+z)$

4. $[\cos(x+y) + \sin(x+y)]e^{y-x} = \phi\left[z^{\sqrt{2}}\tan\left(\frac{x+y}{2} + \frac{\pi}{8}\right)\right]$ **5.** $x^2 - y^2 = f(y^2 - z^2)$

6. $\phi\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$ **7.** $x\log(x+y) - z = f(x+y)$ **8.** $x^2 + y^2 + 2z = [\log(xy)]$

9. $x^2 + y^2 - z^2 = f(x+y+z)$ **10.** $x + y + z = f(xyz)$ **11.** $\phi(x^2+y^2+z^2, xyz) = 0$
12. $x^2 + y^2 = f(y^2 - yz)$ **13.** $\phi(y/z, x^2+y^2+z^2) = 0$ **14.** $x^2 + y^2 + z^2 = f(y^2 - 2yz - z^2)$

15. $f\left(\frac{y}{z}, \frac{z}{x} - \frac{y}{x} + x^2\right) = 0.$

Problems 17.4, page 587

1. $z = ax - ay/(1 + a) + b$

2. $z = ax + \sqrt{(1 - a^2)}y + c$

3. $4z(1 + a^2) = (x + ay + b)^2$

4. $(1 - a + az) = (x + ay + b)^2$

5. $2z = ay^2 - [a/(a + 1)]x^2 + b$

6. $z = a(x - y) - (\cos x + \cos y) + b$

7. $\frac{8}{9}z = (x + a)^{3/2} + (y + a)^{3/2} + b$

8. $3z = (x + a)^3 + (y - a)^3 + b$

9. $z = \dfrac{a^2}{2}\sinh^{-1}\dfrac{x}{a} + \dfrac{x_1\sqrt{(x^2 + a^2)}}{2} + \dfrac{y\sqrt{(y^2 - a^2)}}{2} - \dfrac{a^2}{2}\cosh^{-1}\dfrac{y}{a} + b$

10. $z = ax + by + \sin(a + b)$

11. $z = \dfrac{1}{6}(zx + a)^3 + a^2y + b$

12. $z = ax + by - 2\sqrt{(ab)}$

13. $z = axy + a^2(x + y) + b.$

Problems 17.5, page 590

1. $z = \{\sqrt{(ax)} + \sqrt{(b + y)}\}^2/(1 + a)$

2. $z = ax^by^{1/b}$

3. $\dfrac{z^2}{2} \pm \left\{\dfrac{z}{2}\sqrt{z^2 - 4a^2} - 2a^2\log\left(z + \sqrt{z^2 - 4a^2}\right)\right\} = 2ax + 2y + b$

4. $\log(z - ax) = y - a\log(a + y) + b$

5. $2\sqrt{(z - a - b)} = \sqrt{ax} + \dfrac{1}{\sqrt{a}}y + c$

6. $z = axe^{-y} - \frac{1}{2}a^2e^{-2y} + b.$

Problems 17.6, page 595

1. $z = f_1(y) + f_2(y + 2x) + xf_3(y + 2x)$

2. $z = f_1(y - x) + f_2(y + 2x) + xf_3(y + 2x) + \dfrac{e^{x+2y}}{27}$

3. $z = f_1(x + y) + xf_2(x + y) + \dfrac{x^2}{2} \times e^{x+y}$

4. $z = f_1(y + x) + zf_2(y + x) + f_3(y + 2x) - e^{2x+y}$

5. $z = f_1(y + x) + xf_2(y + x) - \sin x$

6. $y = f_1(x - at) + f_2(x + at) - \dfrac{E}{p^2}\sin pt$

7. $z = f_1(y) + f_2(y + 2x) + xf_3(y + 2x) + 3x\cos(3x + 2y)$

8. $f_1(y\,x) + f_2(y - 2x) + f_3(y + 3x) + \dfrac{1}{75}\sin(x + 2y) + \dfrac{2}{3}x^3.$

9. $z = f_1(y + x) + f_2(y + 2x) + \dfrac{1}{12}e^{2x-y} - xe^{x+y} - \dfrac{1}{3}\cos(x + 2y)$

10. $z = f_1(y) + f_2(y + x) + \frac{1}{3}(\sin x\cos 2y + 2\cos x\sin 2y)$

11. $z = f_1(y) + f_2(y + x) + \dfrac{1}{2}[\sin(x + 2y) + \cos(x + 2y)] - \dfrac{1}{6}[\sin(x - 2y) + \cos(x - 2y)]$

12. $z = f_1(y + x) + f_2(y - x) + \dfrac{3}{28}e^{x-y}[\sin(x + 2y) - 2\cos(x + 2y)]$

13. $z = f_1(y - x) + f_2(y - 2x) + 4x^3y - 3x^4$

14. $z = f_1(y - x) + xf_2(y - x) + \dfrac{1}{4}(x^4 - 2x^3y + 2x^2y^2)$

15. $z = f_1(y - x) + f_2(y + 2x) + ye^x$

16. $z = f_1(y - x) + xf_2(y - x) + f_3(y + x) + \dfrac{e^x}{25}(\cos 2y + 2\sin 2y)$

17. $z = f_1(y - x) + xf_2(y - x) + x\sin y.$

Problems 17.7, page 597

1. $z = e^{-x}\phi_1(y) + e^x\phi_2(y-x) - \dfrac{xe^{-x}}{2}$
2. $z = e^x\phi_1(y+x) + e^{2x}\phi_2(y+x) + \frac{1}{2}e^{2x-y}$
2. $z = e^x\phi_1(y-x) + e^{3x}\phi_2(y-2x) + x + 2y + 6$
4. $z = f_1(y) + e^{-x}f_2(y+x) + \frac{1}{3}x^3 - x^2 + xy^2 + 6x$
5. $z = e^x\phi_1(y) + e^{-x}\phi_2(x+y) + \frac{1}{2}\cos(x+2y)$
6. $z = f_1(x) + e^{3y}f_2(2y-x) + \dfrac{3}{50}[4\cos(3x-2y) + 3\sin(3x-2y)]$

Problems 17.8, page 598

1. $z = \phi_1(x) + \phi_2(x+y+z)$
2. $z = \phi_1(y+\sin x) + \phi_2(y-\sin x)$
3. $z = \phi_1(xy^2) + \phi_2(x^2y)$
4. $z = \phi_1(y/x) + \phi_2(x^2+y^2) + xy$
5. $y = \phi_1(z) + e^x\phi_2(z)$
6. $y = \phi_1(x+y+z) + x\phi_2(x+y+z)$.

Problems 17.9, page 598

1. order two & degree two
2. $z = f_1(y+2x) + xf_2(y+2x)$
3. $z = -x^2\sin xy + yf(x) + \phi(x)$
4. $x^2+y^2+z^2 = f(x+y+z)$
5. $-\dfrac{1}{2}\sin(x+y)$
6. $xp + yq = z$
7. $z = ax + (1-\sqrt{a})^2y + c$
8. $\sqrt{x} - \sqrt{y} = f(6/x - \sqrt{z})$
9. $x\log(x+y) = z + f(x+y)$
10. First
11. $z = 2x + y\log x + f(xy)$
12. $\partial z/\partial x = \partial z/\partial y$
13. $z = f_1(y) + f_2(y+x) + f_3(y+2x)$
14. $4y^2p = q^2$
15. $u = \int f(y)\,dy + \phi(x)$
16. $c = 1$
17. $u = \dfrac{1}{6}x^3y + xf(y) + \phi(y)$
18. $f_1(y+x) + f_2(y+6x)$
19. (*iv*)
20. (*iii*)
21. (*iii*)
22. (*iii*)
23. (*ii*)
24. (*iv*)
25. (*iv*)
26. (*i*)
27. False
28. False
29. True
30. True
31. False.

Problems 18.1, page 601

1. $z = ce^{4ax^3}.e^{-3ay^4}$
2. $u = ce^{k(1/y - 1/x)}$
3. $u = 8e^{-12x-3y}$
4. $u = 3e^{x-y} - e^{2x-5y}$
5. $u = 3e^{-5x-3y} + 2e^{-3x-2y}$
6. $u = \dfrac{1}{\sqrt{2}}\sinh\sqrt{2x} + e^{-3y}\sin x.$

Problems 18.2, page 610

1. $y(x,t) = \displaystyle\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}\cos\frac{n\pi ct}{l}$, when $b_n = \dfrac{2}{l}\displaystyle\int_0^b f(x).\sin\frac{n\pi x}{l}dx$
2. $y(x,t) = \displaystyle\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi ct}{l} + b_n\sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi x}{l}$ where

   $a_n = \dfrac{2}{l}\displaystyle\int_0^l f(x)\sin\frac{n\pi x}{l}dx,\ b_n = \frac{2}{n\pi c}\int_0^l g(x)\sin\frac{n\pi x}{l}dx$
3. $y = \dfrac{8k}{\pi^2}\left(\sin\dfrac{\pi x}{l}\cos\dfrac{\pi ct}{l} - \dfrac{1}{3^2}\sin\dfrac{3\pi x}{l}\cos\dfrac{3\pi ct}{l} + \ldots\right)$
4. $y = \dfrac{8h}{\pi^2}\left(\sin\dfrac{\pi x}{l}\cos\dfrac{\pi ct}{l} - \dfrac{1}{3^2}\sin\dfrac{3\pi x}{l}\cos\dfrac{3\pi ct}{l} + \ldots\right)$

6. $y(x, t) = \dfrac{4l^2c}{a\pi^3}\left\{\sin\dfrac{\pi x}{l}\sin\dfrac{\pi at}{l} - \dfrac{1}{33}\sin\dfrac{3\pi x}{l}\sin\dfrac{3\pi at}{l}\ldots\right\}$

7. (*i*) $y(x, t) = a(x - x^2 - c^2t^2)$; (*ii*) $y(x, t) = \dfrac{a}{2}(1 - \cos 2\pi x \cos 2\pi ct)$.

Problems 18.3, page 617

1. $u(x, t) = \dfrac{400}{\pi^2}\sum_{n=0}^{\infty}\dfrac{(-1)^n}{(2n+1)^2}e^{-[(2n+1)c\pi/100]^2 t}\sin\dfrac{(2n+1)\pi x}{100}$

2. $\sum_{n=odd}\dfrac{8a}{n^3\pi^3}\sin\dfrac{n\pi x}{l}e^{(n\pi c/l)^2 t}$

3. $u(x, t) = \dfrac{40}{\pi}\sum_{n=1}^{\infty}\dfrac{[1-4(-1)^n]}{n}\sin\left(\dfrac{n\pi x}{30}\right)\bar{e}\,\dfrac{(an\pi)^2 t}{900}$

4. $u(x, t) = -3x + 90 - \dfrac{80}{\pi}\sum_{1}^{\infty}\dfrac{1}{n}\sin\dfrac{n\pi x}{5}e^{-c^2n^2\pi^2 t/25}$

5. $u(x, t) = \dfrac{5}{2} - \dfrac{4l}{\pi^2}\sum_{1}^{\infty}\dfrac{1}{(2n-1)^2}\cos\dfrac{(2n-1)\pi x}{l}\,.\,e^{-(2n-1)^2c^2\pi^2 t/25}$

6. $u(x, t) = 50 - \dfrac{400}{\pi^2}\sum_{1}^{\infty}\dfrac{2}{(2n-1)^2}\cos\dfrac{(2n-1)\pi x}{50}\,.\,e^{-c^2\pi^2(2n-1)^2 t/2500}$

7. $\theta = \dfrac{4\theta_0}{\pi}\left[e^{-(\pi/2l)^2 kt}\cos\dfrac{\pi x}{2l} - \dfrac{1}{3}e^{-(3\pi/2l)^2 kt}\cos\dfrac{3\pi}{2l}x + \dfrac{1}{5}e^{-(5\pi/2l)^2 kt}\cos\dfrac{5\pi}{2l}x - \ldots\ldots\right]$

8. $V = V_0e^{-\sqrt{(n/2k)}x}\sin[nt - \sqrt{(n/2k)}\,x]$.

Problems 18.4, page 623

1. $u = -\dfrac{8}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\dfrac{\sin nx \sinh n(\pi - y)}{n(n^2-4)\sinh nx}$

5. $u(x, y) = \dfrac{3200}{\pi^3}\sum_{1}^{\infty}\dfrac{\sin\dfrac{(2n-1)\pi x}{20}\sinh\dfrac{(n-1)\pi y}{20}}{(2n-1)^2\sinh(2n-1)\pi}$

8. $u(x, y) = u_0\cosh\dfrac{\pi x}{a}\cosh\dfrac{\pi}{a}(b-y)\operatorname{sech}\dfrac{\pi b}{a}$.

Problems 18.5, page 626

1. $u(r, \theta) = \dfrac{8k}{\pi}\sum_{1}^{\infty}\left(\dfrac{r}{a}\right)^{2n-1}\dfrac{\sin(2n-1)\theta}{(2n-1)^3}$

2. $u(r, \theta) = \dfrac{3200}{\pi^2}\sum_{1}^{\infty}\left(\dfrac{r}{10}\right)^{2n-1}\dfrac{\sin(2n-1)\theta}{(2n-1)^3}$

3. $u(r, \pi) = \dfrac{2}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\left(\dfrac{a}{r}\right)^{2n}\dfrac{r^{4n}-b^{4n}}{a^{4n}-b^{4n}}\cdot\dfrac{\sin 2n\theta}{n^3}$

4. $u(r, \theta) = \sum\dfrac{2k}{n\pi}\left(\dfrac{r}{a}\right)^{4n}(1-\cos n\pi)\sin 4n\theta$

5. $u(r, \theta) = 50 - \dfrac{200}{\pi}\sum_{1}^{\infty}\dfrac{1}{2n-1}\left(\dfrac{r}{a}\right)^{2n-1}\sin(2n-1)\theta$

6. $u(r, \theta) = \cos\theta\left(\dfrac{200}{r} - \dfrac{r}{2}\right) + \sin\theta\left(2r - \dfrac{200}{r}\right)$. **7.** $u(r, \theta) = 4\cos\theta\,(r - 1/r) + 4\sin\theta\,(r + 1/r)$.

Problems 18.6, page 630

1. $u = \sum_{m=1}^{\infty}\dfrac{J_2(\alpha_m)}{\alpha_m^2 J_1^2(\alpha_m)}\cos\alpha_m t\, J(\alpha_m r)$.

Problems 18.7, page 634

1. $e = e_0 \sin \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{(LC)}}$; $i = i_0 - e_0 \sqrt{\left(\frac{C}{L}\right)} \cos \frac{\pi x}{l} \sin \frac{\pi t}{l\sqrt{CL}}$

3. $v = \frac{20(l-x)}{l} + \frac{24}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{l} \, exp\,(-n^2\pi^2 t / RCl^2)$

$i = \frac{20}{lR} + \frac{24}{lR} \sum_{n=1}^{\infty} (-1)^n \frac{n\pi x}{l} exp\,(-n^2\pi^2 t / RCl^2)$

4. $v = V_0 \cos \{pt - px\sqrt{(LC)}\}$.

Problems 18.9, page 638

1. $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}, \frac{\partial^2 i}{\partial t^2} = LC\frac{\partial^2 i}{\partial t^2}$

2. $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$

3. If $u(x, t)$ is the temperature, then temperature gradient at a point is $\partial u/\partial x$ for all t.

4. elliptic

5. $\partial^2 y/\partial t^2 = c^2 \partial^2 y/\partial x^2$

6. $u = \frac{10}{l}x + 30$

7. parabolic partial differential equation

8. $r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

9. $y = \frac{2h}{l}x, 0\ x < \frac{l}{2}; y = \frac{l}{2h}(2h - y), \frac{l}{2} < x < l$

10. $y(0, t) = 0, y(l, t) = 0, \left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

11. zero

12. $u(0, y) = 0, v(a, y) = 0, 0 < y < a; u_x(x, 0) = 0$ for all t and $u\ (x, a) = u$ for $0 < x < a$

13. $\frac{\partial u\ (0, t)}{\partial x} = 0, \frac{\partial u(l, t)}{\partial x} = 0$ for all t

14. $y(x, t) = f(x + ct) + f(x - ct)$

15. $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ where c^2 is the diffusivity

16. $u(x) = x^2 + 20$

17. § 18.8-(6), (7), (8)

18. $u = 8e^{-12x-3y}$

19. $z = 4e^{3x+t}$

20. α^2 (= $k/s\rho$) is called the diffusivity of the substance (cm²/sec)

21. $\frac{\partial^2 v}{\partial x^2} = RC\frac{\partial v}{\partial t}, \frac{\partial^2 i}{\partial x^2} = RC\frac{\partial c}{\partial t}$

22. $u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} e^{-c^2 n^2 \pi^2 t/l^2}$

23. § 18.7 – (3), (4), (5)

25. False.

Problems 19.1, page 646

1. (*i*) $\sqrt{2}\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$. (*ii*) $-8i/25$

2. $\frac{-y}{x^2 + y^2 - 2x + 1}$

3. $x = \pm 1.5, y = \pm 2$

5. A circle: centre (–1, 1) radius $\sqrt{2}$

8. $-1 + i\sqrt{3}, -1 - i\sqrt{3}, 1 - i\sqrt{3}; 4\sqrt{3}$

9. $-2 + 0.\,i, 1 - i\sqrt{3}$

10. $-1 - i$, $\sqrt{2}(\mp \sin 15° \pm i \cos 15°), \sqrt{2}(\mp \cos 15° \pm i \sin 15°)$

11. (*i*) Annular region between the circles of radii 2 and 4 with centre $(-3, 0)$ including boundary of inner circle; (*ii*) Region of complex plane above the line $y = 2$; (*iii*) Infinite region bounded by the lines $\theta = \pi/3$ and $\theta = \pi/2$; (*iv*) Real axis and region above it between $x = \pm 2$

13. (*i*) Ellipse with foci at $z = \pm 1$ and major for axis = 3

(*ii*) (*a*) Right bisector of the line joining $z = 3$ and $z = -1$;

(*b*) Circle through the points $z = 3$ and $z = -1$;

14. (*i*) Right bisector of the points 0 and 2; (*ii*) Circle through the points $\frac{1}{3}$ and 3

15. (*i*) A straight line; (*ii*) Circle with centre (1, 1/2) and radius $\sqrt{5/2}$.

Problems 19.2, page 650

5. $4m\pi/n(n+1)$

7. $2^{n+1}\sin^n\dfrac{\alpha-\beta}{2}\cos n\left(\dfrac{\pi+\alpha+\beta}{2}\right)$.

Problems 19.3, page 653

1. (*i*) $(2)^{1/8}\,[0.98 \pm i(0.195)]6$; $(2)^{1/2}\,[-0.195 \pm i\,(0.98)]$

(*ii*) $(2)^{1/5}\cos\dfrac{4n+3}{10}\pi$, where $n = 0, 1, 2, 3, 4$; (*iii*) $\pm 2\sqrt{2}$

(*iv*) $2^3\sqrt{2}\ \cos r\pi/9$, where $r = 1, 7$ or 13

3. $\pm i,\ \frac{1}{2}\left(\sqrt{3} \pm i\right), \frac{1}{2}\left(-\sqrt{3} \pm i\right)$

5. (*i*) $1, -1, \cos(\pm\pi/5), \cos(\pm 3\pi/5)$

(*ii*) $-1,\ \frac{1}{2}\left(1 \pm i\sqrt{3}\right), \pm(1+i)/\sqrt{2}, \pm(-1+i)/\sqrt{2}$

(*iii*) $\dfrac{1 \pm i}{\sqrt{2}}, \dfrac{-1 \pm i}{\sqrt{2}}; 1, \cos\left(\pm\dfrac{2\pi}{5}\right), \cos\left(\pm\dfrac{4\pi}{5}\right)$.

(*iv*) $\cos(2n+1)\pi/6$, where $n = 0, 1, 2, 3, 4, 5$; $2(2m+1)\pi/3$, where $m = 0, 1, 2$

6. $(-1+i)/\sqrt{2}, (1-i)/\sqrt{2}$

7. $\pm 1, \pm i, \pm\left(\cos\dfrac{\pi}{6} \pm i\sin\dfrac{\pi}{6}\right), \pm\left(\cos\dfrac{\pi}{3} \pm i\sin\dfrac{\pi}{3}\right)$; Last four values

9. $x^3 + x^2 - 2x - 1 = 0$.

Problems 19.4, page 655

1. $32\cos^5\theta - 24\cos^3\theta + 6\cos\theta$

13. $-(2)^{-11}(\sin 12\theta - 2\sin 10\theta - 4\sin 8\theta + 10\sin 6\theta + 5\sin 4\theta - 20\sin 2\theta)$

14. $\sin^5\theta = A\sin\theta - B\sin 3\theta + C\sin 5\theta$.

Problems 19.5, page 661

1. (*i*) $e^{x^2-y^2}\cos 2xy,\ e^{x^2-y^2}\sin 2xy$ (*ii*) ie^5. (*iii*) $e^{16}\cos 30, e^{16}\sin 30$

5. $[(pq'-p'q)(qr'-q'r)]^2 = [(pq'-p'q)^2 + (qr'-q'r)^2](pr'-p'r)^2$

10. $\dfrac{1}{64}(\cosh 7\theta + 7\cosh 5\theta + 21\cosh 3\theta + 35\cosh\theta)$

17. $-\log^3$

18. $-13/12$.

Problems 19.6, page 664

10. $\pm \frac{\pi}{4} + \frac{i}{4} \log \frac{1+\sin\theta}{1-\sin\theta}$ according as $\cos\theta$ is + ve or – ve

11. $\sin^{-1}\left(\sqrt{\sin\theta}\right) + i \log\left[\sqrt{(1+\sin\theta)} - \sqrt{\sin\theta}\right]$.

Problems 19.7, page 667

1. (*i*) $\log_e 10 + i\,[\tan^{-1}(4/3) \pm 2n\pi]$; (*ii*) $\log_e 1 + i\,(\pi + 2n\pi)$

4. (*i*) $\sqrt{2}e^{-(2n-\frac{1}{4})\pi}, (2n - \frac{1}{4})\pi + \log\sqrt{2}$; (*ii*) $e^{-\pi^2/8}, (\pi/4)\log_e 2$

9. $\sqrt{[\frac{1}{2}(\cos 2x + \cosh 2y)]} - i\tan^{-1}(\tan x \tanh y)$

10. (*i*) $2n\pi \pm i \log(2 + \sqrt{3})$; (*ii*) $-\frac{1}{2}\log 3 + (n + \frac{1}{2})i\pi$.

Problems 19.8, page 669

1. $e^{\sin\theta\cos\theta}\cos(\theta + \sin^2\theta)$

2. $\sin\alpha\cos(\cos\beta)\cosh(\sin\beta) - \cos\alpha\sin(\cos\beta)\sinh(\sin\beta)$

3. $\tan^{-1}\dfrac{x\sin\alpha}{1 + x\cos\alpha}$, except when $x = 1$ and $\alpha = (2n+1)\pi$

4. $\log(2\cos\theta/2)$

5. $-\frac{1}{2}\tan^{-1}(\cos\beta\,\mathrm{cosech}\,\alpha)$

6. $\frac{1}{2}\tan^{-1}\dfrac{2c\sin\alpha}{1-c^2}$

7. $(2\cos\theta)^{-1/2}\cos\theta/2$

8. $\sin\dfrac{n(\pi-\alpha)}{2}/(2\sin\alpha/2)^n$

9. $\sin\left(\alpha + \dfrac{n-1}{2}\beta\right)\sin\dfrac{n\beta}{2}\,\mathrm{cosec}\,\dfrac{\beta}{2}$

10. $\dfrac{\cos\{\alpha + \frac{1}{2}(n-1)\beta\}\sin n\frac{\beta}{2}}{\sin\frac{1}{2}\beta}$

11. $\dfrac{\sin\alpha(\cos\alpha - \sin\alpha)}{1 - \sin 2\alpha + \sin^2\alpha}$

12. $\dfrac{1 - x\cos\theta - x^n\cos n\theta + x^{n+1}\cos\overline{n-1}\,\theta}{1 - 2x\cos\theta + x^2}$.

Problems 19.9, page 670

2. 0.053 radians **3.** 1°59′ **4.** 39.7.

Problems 19.10, page 670

1. (*b*) **2.** (*c*) **3.** (*b*) **4.** (*c*)
5. (*b*) **6.** (*d*) **7.** odd **8.** $32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$
9. $6(1-2i)$ **10.** $2i\sin n\theta$ **11.** $\frac{1}{25}(-6+17i)$ **12.** $\cosh x\cos y$
13. $\frac{1}{19}$ radians **14.** 1 **15.** $-\cos x\sinh y$ **16.** $e^{-\pi/4\sqrt{2}}$ **17.** $\alpha+\beta+\gamma$
18. $2i\,n\pi$ **19.** real **20.** $\sinh\phi$ **21.** $\sinh 2\phi/(\cosh 2\theta + \cosh 2\phi)$
22. $16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$ **23.** an equilateral **24.** $\pi/2$
25. $x = \pm 1, y = -4$. **26.** a circle **27.** True **28.** True
29. True **30.** True **31.** False **32.** True
33. False.

Problems 20.1, page 682

4. $a = 1, b = -6, c = 1, d = 2, e = 4$

6. (*i*) and (*ii*) Not analytic. (*iii*) Analytic

7. $p = -1$

11 & 12. $f(z)$ is not analytic at origin although C-R equations are satisfied there

14. (*i*) $z^3 + 3z^2 + 1 + ic$; (*ii*) $\cos z$; (*iii*) $\log z$; (*iv*) i/z ; (*v*) $e^z + i(c - z)$; (*vi*) $ze^{2z} + ic$; (*vii*) $z \sin z$; (*viii*) $x^2e^z + ic$

15. (*i*) $(1 + i)/z + c$; (*ii*) $\cos z + c$; (*iii*) e^z ; (*iv*) $\bar{z}e^{-\bar{z}} + c$; (*v*) $1 + ize^{-z}$; (*vi*) $(2 \cos x \cosh y)/(\cos 2x + \cosh 2y)$

16. (*i*) $f(z) = c - iz^3$; (*ii*) $f(z) = \left(\frac{1}{2} - \cot \frac{z}{2}\right)$ (*iii*) $\frac{\cot z}{1+c} + c\left(\frac{1+3i}{5}\right)e^z + c$

17. $\psi = 3xy^2 - x^3 + c$

18. $2 \tan^{-1}(y/x)$; $2 \log z + c$

20. $v = C - e^{-2xy} \cos(x^2 - y^2)$; $f(z) = C - i\, e^{iz^2}$

22. (*i*) $x^4 - 6x^2y^2 + y^4 = c$ (*ii*) $x^2 - y^2 + 2e^x \sin y = c$ (*iii*) $r^2 \sin 2\theta = c'$

23. (*i*) $x/(x^2 + y^2)$; (*ii*) $\frac{1}{2} \log(x^2 + y^2)$

24. (*i*) $a(1 + \cos\theta + i \sin\theta \log r)$; (*ii*) $(r + 1/r)\cos\theta + (r - 1/r)\sin\theta + c$

27. $-2 \tan^{-1}[(y - 2)/(x - 1)]$, $2i \log(z - 1 - 2i)$.

Problems 20.2, page 687

1. $z = \pm i$

2. $w = -1/z$

3. $w = (2i - 6z)/(iz - 3)$; fixed points $z = i, 2i$; no critical points

4. $w = \dfrac{(20 + 18i) - (32 + 12i)z}{(29 + 17i) - (11 - 3i)z}$

5. (*i*) $w = i(1 - z)/(1 + z)$; (*ii*) $w = \dfrac{(4i - 2) + (5 - 3i)z}{2i + (1 + i)z}$ (*iii*) $w = (1 - z)/(1 + z)$.

Problems 20.3, page 692

1. (*i*) $I(w) > 0$; (*ii*) Region bounded by the parabolas $v^2 = 4(1 \pm u)$; and $u^2 = 1 - 2v$; (*iii*) Region bounded by parabolas $v^2 = \frac{1}{4} \pm u$, $v^2 = 4(1 \pm u)$; (*iv*) Region boundary $\rho = 2\sqrt{\rho} \cos\frac{\phi}{2} + 3$.

2. $w = z^6$

3. Lines parallel to x and y axes map into two families of rectangular hyperbolas in the w-plane which cut each other at right angles. Lines parallel to u and v axes map into two families of parabolas in the z plane which cut each other orthogonally. It is conformal at $z = 0$

4. (*a*) Line $4v + 1 = 0$

5. (*b*) Every circle through the origin ($z = 0$) transforms into a st. line not passing through the origin ($w = 0$). If a line passes through $z = 0$, its image is a line through $w = 0$. (*b*) Circle with centre (1/2, 1/2) and radius $(1/\sqrt{2})$ (*c*) Lemniscate $p^2 = \cos 2\phi$

10. $z = \pm a$

11. See § 20.10 (3)

14. See § 20.10(4).

Problems 20.4, page 694

1. $z = \cosh w$

3. $w = \sin z$

4. $w = \log z$.

Problems 20.5, page 696

1. (*a*) $(5 - i)/6$ (*b*) $(5 + i)/6$

2. (*i*) $4 + (25/3)i$; (*ii*) $4 + 8i$

3. $\frac{1}{6}(64i - 103)$

6. (*i*) i ; (*ii*) $2i$; (*iii*) 0

7. (*a*) $\frac{1}{3}(i - 1)$, (*b*) $\frac{1}{6}(5i - 3)$

9. (*i*) $\frac{2}{3}$; (*ii*) $-\frac{2}{3}$.

Problems 20.6, page 702

1. (*i*) 0 ; (*ii*) $2\pi i$ **2.** 0 **3.** (*a*) zero ; (*b*) zero
4. (*i*) $5\pi i$; (*ii*) $\pi i/2$ (*iii*) $4\pi i$ **5.** (*a*) $-10\,\pi i$ (*b*) $2\pi ie$ **6.** (*i*) $4\pi i$; (*ii*) $2\pi ie^{-1}$; (*iii*) $-\pi i$
7. (*i*) $8\pi i$; (*ii*) 0 **8.** (*i*) 0 ; (*ii*) $2\pi(6+13i)$; (*iii*) $12\pi i$ **9.** zero.

Problems 20.7, page 709

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\,n(z-1)^n$; Convgt. in $|z-1|<1$

2. $f(z)=\frac{1}{3}+\frac{u}{9}(z+i)-\frac{7}{27}(z+1)^2+\ldots\ldots$ Region of convergence is $|z+i|<1$

3. (*i*) $\frac{1}{2}(z-1)-\frac{1}{2^2}(z-1)^2+\frac{1}{2^3}(z-1)^3-\ldots\ldots$ (*ii*) $-(z-\pi/2)+(z-\pi/2)^3/3!-(z-\pi/2)^5/5!+\ldots$

(*iii*) (*a*) $f(z)=-\frac{1}{5}\sum_{n=0}^{\infty}(-1)^n\,(z+1)^n-\frac{1}{20}\sum_{n=0}^{\infty}\left(\frac{z+1}{4}\right)^n$ *in the region* $|z+1|<1$. Also, $(z+1)<4$

(*b*) $f(z)=-\frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{z-1}{5}\right)^n\frac{1}{10}\sum_{n=0}^{\infty}\left(\frac{z-1}{2}\right)^n$ in the region $|z-1|<2$. Also $|z-1|<3$

4. (*i*) $\frac{1}{z+1}+\frac{1}{(z+1)^2}+\frac{1}{(z+1)^3}+\ldots+\frac{1}{2}\left[1+\frac{z+1}{2}+\frac{(z+1)^2}{2^2}+\frac{(z+1)^3}{2^3}\right]$

(*ii*) $\frac{1}{4z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\ldots\right)-\frac{1}{12}\left(1-\frac{z}{3}+\frac{z^2}{9}-\ldots\right)$. (*iii*) $-\frac{1}{2(z-1)}-3\sum_{n=1}^{\infty}\frac{(z-1)^{n-1}}{2^{n+1}}$

5. (*i*) $e\left[(z-1)^{-2}+(z-1)^{-1}+\frac{1}{2!}+\frac{1}{3!}(z-1)+\frac{1}{4!}(z-1)^2+\ldots\right.$

(*ii*) $e^2(z-1)^{-3}+2e^2(z-1)^{-2}+2e^2(z-1)^{-1}+\frac{4e^2}{3}+\frac{2e^2}{3}(z-1)+\ldots$

6. (*i*) $\sum_{n=1}^{\infty}(-1)^{n-1}n\,.\,(z-1)^{-n}$ for $|z-1|>1$. (*ii*) $-\sum_{n=2}^{\infty}\frac{z^{2n-5}}{2(n-1)!}$

7. (*i*) $1+\frac{3}{z}\left(1-\frac{2}{z}+\frac{2^2}{z^2}-\frac{2^3}{z^3}+\ldots\ldots\right)-\frac{8}{3}\left(1-\frac{z}{3}+\frac{z^2}{3^2}-\frac{z^3}{3^3}+\ldots\ldots\right)$

(*ii*) $\frac{2}{z+2}+\frac{3}{(z+2)^2}+\frac{3^2}{(z+2)^3}+\ldots+\frac{1}{5}\left(1+\frac{z+2}{5}+\frac{(z+2)^2}{5^2}+\frac{(z+2)^3}{5^3}+\ldots\right)$

(*iii*) (*a*) $\frac{7}{z}-\frac{9}{z^2}-\frac{45}{z^3}-\frac{81}{z^4}+\ldots\ldots$ (*b*) $\frac{5}{2(z-3)}+\frac{7}{12}-\frac{z-3}{24}-\frac{5(z-3)^2}{432}+\frac{7(z-3)^2}{864}+\ldots\ldots$

8. (*a*) $\frac{z}{4}-\frac{5z^2}{16}+\frac{21}{64}z^5-\ldots\ldots$; (*b*) $\frac{1}{3}\left(\frac{1}{z^5}-\frac{1}{z^3}-\frac{z}{4}+\frac{z^3}{16}-\frac{z^5}{64}+\ldots\ldots\right)$; (*c*) $\frac{1}{z^3}-\frac{3}{z^5}+\ldots\ldots$

9. $z=0, z=2$ are the isolated singularities **10.** $z=0$ is an isolated essential singularity
11. $z=0$, is a non-isolated essential singularity
12. $z=1$ is a pole of order 2 **13.** $z=1$ is a pole of order 4
14. $z=a$ is a double pole and $z=0, \pm 1, \pm 2, \ldots\ldots$ are simple poles.

Problems 20.8, page 715

1. $-\frac{1}{t}-2i+3t+4i\,t^2+\ldots$ where $t=z-i$; -1

2. (*i*) $3e/2$; (*ii*) Res $(z = -ai/2)$

3. (*i*) Res $f(0) = -1/2$, Res $f(2) = 2\frac{1}{2}$;

(*ii*) Res $f(-1) = 0$, Res $f(i) = \dfrac{1+2i}{2(1-i)}$, Res $f(-i) = \dfrac{2i-1}{2(i-1)}$

(*iii*) Res $f(-1) = 1$, Res $f(i) = \dfrac{2+i}{-1+i}$, Res $f(-i) = \dfrac{-2+i}{1+i}$

4. (*i*) Res $f(0) = -4/3$; (*ii*) Res $f(i) = \frac{1}{2}e^{-1}$; Res $f(-i) = \frac{1}{2}e$; (*iii*) $-i\,\pi/4$, Res $f(n\pi) = 1$, n an integer.

5. (*i*) 0 ; (*ii*) $16\pi i/(2 + 3i)$

6. (*i*) $\dfrac{\pi i e^{-4}}{5}$ (*ii*) 0 ; (*iii*) $i\pi/4$

7. (*i*) πi ; (*ii*) $\pi/2\,(3 + 2i)$, (*iii*) zero.

8. (*i*) $-2\pi i$; (*ii*) πi ; (*iii*) $\pi/16$

9. (*i*) $-2\pi i$; (*ii*) $8\pi i/3e^2$; (*iii*) $\dfrac{\pi e^{2i}}{2}$; (*iv*) $\dfrac{8\pi}{3} i e^{-2}$

10. (*i*) $\dfrac{21\pi i}{16}$; (*ii*) $2\pi i \sec 1(1 + \tan 1)$; (*iii*) $-2\pi i$

11. $2\pi i$

12. $\dfrac{1}{z^3} - \dfrac{1}{6z} + \dfrac{7z}{360} - \dfrac{31z^6}{15120} + \ldots\ldots; -\dfrac{1}{3}\pi i$.

Problems 20.10, page 723

1. (*i*) **2.** (*iii*) **3.** (*a*) **4.** (*iii*) **5.** (*ii*) **6.** (*ii*)
7. (*ii*) **8.** (*ii*) **9.** (*iii*) **10.** (*ii*) **11.** (*ii*) **12.** (*i*)
13. (*ii*) **14.** (*iii*) **15.** $v(x, y) = x^2 - y^2 + 2y + c$
16. $3x^2 y - y^3 + c$ **17.** $u + iv$ is an analytic function **18.** $2i/3$
19. 1 **20.** $z = \dfrac{1}{2}(a + b)$ **21.** $2u + 1 = 0$
22. $z = 1, \dfrac{1}{2}(1 \pm \sqrt{3})$ **23.** $z = 0$ **24.** -1
25. $z - \dfrac{z^2}{2} + \dfrac{z^3}{3} - \dfrac{z^4}{4} + \ldots$ **26.** zero **27.** (*iii*)
28. $\dfrac{1}{2}\left[\dfrac{d^2}{dz^2}\{(z-a)^3 f(z)\}\right] z = a.$ **29.** zero **30.** $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}, \dfrac{\partial u}{\partial y} = -\dfrac{\partial u}{\partial x}$
31. zero **32.** $e^x \sin y$ **33.** zero
34. $z = 2$ **35.** no point in the z-plane **36.** $\dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}, \dfrac{\partial v}{\partial r} = -\dfrac{1}{r}\dfrac{\partial u}{\partial \theta}$
37. zero
38. A simply connected region is one in which any closed curve, lying entirely within it, can be shrunk to a point without going out of the region
39. which is analytic or regular **40.** $z = 1, 2$
41. $(z_1 - z_2)(z_3 - z_4)/(z_1 - z_4)(z_3 - z_2)$ **42.** i
43. $\dfrac{1}{2} + \dfrac{3}{4}z + \dfrac{7}{8}z^2 + \dfrac{15}{16}z^3 + \ldots$ **44.** $\pm i$
45. 1 **46.** Magnification and rotation
47. The coefficient of $(z - a)^{-1}$ in the expansion of $f(z)$ around an isolated singularity $(z = a)$ is called the residue of $f(z)$ at that point.
48. $2\pi i$ **49.** zero **50.** $z = 1, 2$
51. $-\dfrac{1}{2}\left\{1 + \dfrac{z}{2} + \dfrac{z^2}{4} + \dfrac{z^3}{8} + \ldots\right\}$ **52.** $i/2$ **53.** $z = 0, 2$
54. § 20.14 **55.** $\pm i$ **56.** zero
57. Zeroes are at $z = \pm 1$, singularity is at $z = 0$ **58.** -1

59. essential singularity **60.** zero

61. $\sin z = \frac{1}{\sqrt{2}}\left\{1 + (z - \pi/4) - \frac{(z-\pi/4)^2}{2!} - \frac{(z-\pi/4)^3}{3!} + \frac{(z-\pi/4)^4}{4!} + \ldots\right\}$

62. a circle with centre (3, 2) and radius 2 in w-plane. **63.** $n\pi$, n an integer

64. $\phi(a)/\psi'(a)$ **65.** circles **66.** True **67.** False **68.** True **69.** True
70. False **71.** True **72.** True **73.** True **74.** True **75.** True
76. True **77.** False **78.** False **79.** False **80.** True **81.** True
82. True.

Problems 21.1, page 732

1. $\frac{1}{s-2} + \frac{24}{s^4} + \frac{3(s-2)}{s^4+9}$. **2.** $\frac{1}{s} + \frac{\sqrt{\pi}}{s^{3/2}} + \sqrt[3]{\left(\frac{\pi}{s}\right)}$ **3.** $\frac{3s-20}{s^2-25}$

4. $\frac{s\cos b - a\sin b}{s^2+a^2}$ **5.** $\frac{s^2-2s+4}{s(s^2+4)}$ **6.** $\frac{2(s^2-5)}{(s^2+1)(s^2+25)}$

7. $\frac{\sqrt{\pi} - e^{1(1/4s)}}{2s^3/2}$ **8.** $\frac{5}{4}\left\{\frac{1}{s^2+1} - \frac{3/2}{s^2+9} + \frac{1/2}{s^2+25}\right\}$ **9.** $\frac{s(s^2+28)}{(s^2+4)(s^2+36)}$

10. $\frac{b}{(s+a)^2-b^2}$ **11.** $\frac{60}{s-2} - \frac{s-2}{s^2-4s+20}$ **12.** $\frac{30(s+3)}{(s^2+6s+13)(s^2+6s+73)}$

13. $\frac{2}{(s+1)(s^2+2s+5)}$ **14.** $\frac{1}{8}\left\{\frac{3}{s-2} - \frac{4(s-2)}{s^2-4s+8} + \frac{s-4}{s^2-8s+32}\right\}$

15. $\frac{a(s^2+2a^2)}{s^4+4a^4}$ **16.** $\frac{3}{2}\left[\frac{1}{s^2-9} + \frac{s^2-13}{s^4-10s^2+169}\right]$ **17.** $\frac{2}{(s+2)^3}$

18. $\frac{1}{n} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^4}$ **19.** $4\frac{(4s^2+4s-1)}{(4s^2+1)^2}$

20. $\frac{4}{s} - \frac{e^{-s}}{s}$ **21.** $\frac{1+e^{-\pi s}}{s^2+1}$ **22.** $\frac{e^{-\pi s/3}}{s^2+1}$

23. $e^{-2\pi s/3}\frac{s}{s^2+1}$ **24.** $\frac{2}{s^3} - \frac{e^{-2s}}{s^3}(2+3s+3s^2) + \frac{e^{-3s}}{s^2}(5s-1)$

25. $\frac{4}{(s-1)(s^2-2s+5)}$.

Problems 21.2, page 734

1. $(1/s^2T) - e^{-sT}/s(1-e^{sT})$ **2.** $[Ew/(s^2+w^2)]\coth(\pi s/2w)$ **3.** $(a/s)\tanh(as/2)$

4. $(1/s^2)\tanh\frac{1}{2}as$ **5.** $\frac{1}{\sqrt{(s^2+a^2)}}$ **6.** $(s^2-2as+a^2+b^2)^{-1/2}$

7. $\frac{2}{(s-2)\sqrt{(s+1)}}$.

Problems 21.3, page 740

1. $\frac{s+1}{s(s^2+2s+2)}$ **4.** $\frac{2(3s^2+4)}{s^2(s^2+4)^2}$ **5.** $\frac{16}{(s^2+4)^2}$

6. $\dfrac{2s^3 - 6a^2 s}{(s^2 + a^2)^3}$

7. $\dfrac{2as}{(s^2 - a^2)^2}$

8. $\dfrac{6(s-2)}{(s^2 - 4s + 13)^2}$

9. $\dfrac{8(s+2)}{s^2 + 4s + 20}$

10. $\dfrac{2(s^3 + 6s^2 + 9s + 2)}{(s^2 + 4s + 5)^3}$

11. $\log\{(s+b)/(s+a)\}$

12. $\cot^{-1}(s)$

13. $\frac{1}{2}\log\{(s^2 + 36)/(s^2 + 16)\}$

14. $\dfrac{1}{2}\log\left(\dfrac{s^2 + b^2}{(s-a)^2}\right)$

15. $\cot^{-1}(s+1)$

16. $\dfrac{1}{2}\log\left(\dfrac{s^2 + 9}{s^2}\right)$

17. $\cot^{-1} s - \frac{1}{2} s \log(1 + s^{-2})$

18. $\dfrac{1}{s - \log 2} + \dfrac{2s}{(s^2+1)^2} + \dfrac{1}{2}\log\left(\dfrac{s^2+9}{s^2+4}\right)$

19. (*i*) $\log 2/3$; (*ii*) $\pi/8$; (*iii*) $12/169$; (*iv*) $\dfrac{8(s+1)}{s(s^2 + 2s + 17)}$

21. (*i*) $\dfrac{1}{s}\cot^{-1}(s)$; (*ii*) $\dfrac{1}{s}\cdot\dfrac{s+1}{s^2 + 2s + 2}$; (*iii*) $\dfrac{\cot^{-1}(s-1)}{s}$.

Problems 21.4, page 743

1. $\dfrac{1}{2}\left(\cos\dfrac{5t}{2} - \sin\dfrac{5t}{2}\right) - 4\cosh 3t + 6\sinh 3t$

2. $e^{3t} - e^{2t}$

3. $3e^{t/2} + 2e^{t/3}$

4. $e^{2t} + 2e^{-4t}$

5. $\dfrac{1}{3}(8e^{2t} - e^{-t})$

6. $\cosh t$

7. $e^t + e^{-2t} - 2e^{3t}$

8. $2e^{3t} - \frac{3}{5}e^{2t} - \frac{2}{5}e^{7t}$

9. $\dfrac{1}{2}e^t - e^{2t} + \dfrac{5}{2}e^{3t}$

10. $\frac{1}{2}t\sinh t$

11. $\frac{1}{3}t(e^t - e^{-2t})$

12. $\frac{1}{13}(3e^{3t} - 3\cos 2t + 2\sin 2t)$

13. $\frac{1}{2}(\sin t - te^{-t})$

14. $\frac{1}{2}[\cos at + \cosh at]$

15. $(\frac{1}{3}a^2)[e^{at} - e^{-at/2}\{\cos(\sqrt{3}at/2) + \sqrt{3}\sin(\sqrt{3}at/2)\}]$

16. $\frac{1}{3}(5\sin t - \sin 2t)$

17. $\frac{1}{3}e^{-2t}(6\cos 3t - 7\sin 3t)$

18. $\dfrac{1}{5}(1 + e^{-t})\sin t + \dfrac{3}{5}(1 - e^{-t})\cos t$

19. $\frac{1}{3}e^{-t}(\sin t + \sin 2t)$

20. $(2/\sqrt{3})\sinh(\frac{1}{2}t)\sin(\frac{1}{2}\sqrt{3}t)$

21. $\cos at \sinh at$.

Problems 21.5, page 750

1. $\dfrac{1}{25}(e^{-5t} + 5t - 1)$

2. $\dfrac{1}{8} - \dfrac{1}{4}\left(t^2 + t + \dfrac{1}{2}\right)e^{-2t}$

3. $\dfrac{1}{a^2}\cos\left(\dfrac{bt}{a}\right)$

4. $\dfrac{1}{a^2}\left(t - \dfrac{1}{a}\sin at\right)$

5. $\dfrac{1}{2}t^2 + \cos t + 1$

6. $\dfrac{1}{2}te^{-2t}\sin 2t$

7. $t\sin at$

8. $\dfrac{1}{2}(a^2t^2 - 4at + 2)e^{-at}$

9. $\dfrac{1 - e^t}{t}$

10. $\dfrac{1}{t}(e^{-bt} - e^{-at})$

11. $e^{-t} - e^{-2t} - e^{-3t}$

12. $\dfrac{1}{t}(\cos at - \cos bt)$

13. $\dfrac{2}{t}(1 - \cosh at)$

14. $\dfrac{2}{t}(e^t - \cos t)$

15. $\dfrac{\sin 2t}{t}$

16. $\dfrac{\sin t}{t}$

17. $\dfrac{2(\sinh t - t\cosh t)}{t^2}$

18. $\dfrac{e^{-bt} - e^{-at}}{a - b}$

19. $\frac{1}{2a^3}(\sin at - at\cos at)$ **20.** $\frac{1}{a^3}(at - \sin at)$ **21.** $t(e^{-t}+1) + 2(e^{-t}-1)$

22. $\frac{1}{16}(e^{2t} - e^{-2t} - 4te^{-2t})$ **23.** $\frac{1}{13}(3\sin 3t + 2\cos 2t - 2e^{-2t})$ **24.** $\frac{t^2}{2} + \cos t - 1$

25. $\frac{e^{-2t}}{54}(\sin 3t - 3t\cos 3t)$.

Problems 21.6, page 754

1. $y = \frac{7}{4}e^{-t} - \frac{3}{4}e^{-3t} - \frac{1}{2}te^{-t}$ **2.** $x = \frac{at}{2}\sinh t$ **3.** $y = t - 3\sin t + \cos t$

4. $y = 2t + 3 + \frac{1}{2}(e^{3t} - e^{t}) - 2e^{2t}$ **5.** $y = 4e^{2t}(1+t) - 7e^{t}$ **6.** $y = 2\cos 5t + t\sin 5t$

7. $y = \frac{1}{2\omega}\sin\omega t$ **8.** $y = \frac{1}{8}e^{t} - \frac{1}{40}e^{-3t} - \frac{1}{10}(2\sin t + \cos t)$

9. $y = \frac{1}{2}(\cos kt + \cosh kt)$ **10.** $y = \frac{1}{8}[(3-t^2)\sin t - 3t\cos t]$ **11.** $y = \frac{11}{3}e^{-t}(\sin t + \sin 2t)$

12. $y = \frac{-12}{5} + \frac{12}{5}e^{-t}\cos 2t + \frac{7}{10}e^{-t}\sin 2t$ **13.** $y = e^{2t}(x^2 - 6x + 12) - e^{t}(15x^2 + 7x + 11)$

14. $x = \frac{4}{9}\sin 2t - \frac{5}{9}\sin t - \frac{1}{3}t\cos 2t$ **15.** $y = \frac{1}{2}\left(\frac{3\sin t}{t} - \cos t\right)$

16. $y = e^{2t}$ **17.** $y = t$ **18.** $y = 3J_0(2t)$

21. $(n\sin at - a\sin nt)F_0/mn(n^2 - a^2)$, where $n^2 = k/m$.

Problems 21.7, page 756

1. $x = \frac{1}{2}(e^{t} + \cos t + 2\sin t - t\cos t)$, $y = \frac{1}{2}(t\sin t - e^{t} + \cos t - \sin t)$

2. $x = e^{t} + e^{-t}$, $y = e^{-t} - e^{t} + \sin t$ **3.** $x = 2 + t^2/2$, $y = -1 - t^2/2$

4. $x = \frac{1}{10}(5 - 2e^{-t} - 3e^{-6t/11})$, $y = \frac{1}{5}(e^{-t} - e^{-6t/11})$ **5.** $x = e^{6}(1+2t) + 2e^{3t}$, $y = \sinh t + \cosh t - e^{-3t} - te^{t}$

6. $i_1 = \frac{a}{p+\omega}(\sin\omega t + \sin pt)$; $i_2 = \frac{a}{p-\omega}(\cos\omega t - \cos pt)$.

Problems 21.8, page 762

1. $\frac{2}{s^2+4}(e^{-2\pi s} - e^{-4\pi s})$

2. (*i*) $(1-2t)u(t-\pi) + 2tu(t)$, $\frac{2}{s^2} + \left(\frac{1-2\pi}{s} - \frac{2}{s^2}\right)e^{-as}$

(*ii*) $t^2[u(t) - u(t-2)]$, $\frac{2(1-e^{-2s})}{s^3} - \frac{4e^{-2s}(1+s)}{s^2}$

(*iii*) $\{u(t) - u(t-T)\}\cos(\omega t + \phi)$; $[(s\cos\phi - \omega\sin\phi) - e^{-sT} \times \{s\cos(\phi + \omega T) - \omega\sin(\phi + \omega T)\}]/(s^2 + \omega^2)$

3. (*i*) $\frac{s}{s^2+1} + \left(\frac{1}{s} + \frac{s}{s^2+1}\right)e^{-\pi s} - \left(\frac{1}{s} - \frac{1}{s^2+1}\right)e^{-2\pi s}$

(*ii*) $\frac{1}{s^2+1} + e^{-\pi s}\left(\frac{s}{s^2+4} - \frac{s}{s^2+1}\right) + e^{-2\pi s}\left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right)$

(*iii*) $\frac{2}{s^3}\{1 + e^{-2s}(2s^2 - 1) - 2e^{-4s}(1+4s)\}$

4. (*i*) $e^{-s}/(s-1)$; (*ii*) $2e^{-s}/s^3$; (*iii*) $e^{-2s}\left\{\frac{24}{s^4}+\frac{42}{s^3}+\frac{28}{s^2}+\frac{25}{s}\right\}$; (*iv*) $e^{-s}(2+2s+s^2+s^3)/s^3$

5. $20e^{-2}$

6. (*i*) $-\sin t \,.\, u(t-\pi)$; (*ii*) $\frac{1}{3}\, e^{-4(t-2)} \sin 3(t-2) \,.\, u(t-2)$;

(*iii*) $\frac{1}{2}\, e^{-(t-1)}(t-1)^2\, u(t-1)$ (*iv*) $3-4(t-1)\,u(t-1)+4(t-3)\,u(t-3)$

7. $y=\frac{1}{2}\sin 2t+\frac{1}{4}(1-\cos 2t)-\left\{\frac{1}{4}1-\cos(t-1)\,u(t-1)\right\}$

8. $x=3-2\cos t+2\{t-4-\sin(t-4)\}\,u(t-4)$.

9. $y(x)=\begin{cases}\dfrac{2Wx^2(3l-5x)}{81EI}, 0<x<l/3\\ \dfrac{2Wx^2(3l-5x)}{81EI}+\dfrac{W}{6EI}\left(x-\dfrac{l}{3}\right)^3, \dfrac{l}{3}<x<l\end{cases}$

10. $y(x)=\frac{wl^2}{16EI}x^2-\frac{wl}{12EI}x^3+\frac{w}{24EI}x^4-\frac{w}{24EI}(x-l/2)^4\,u(x-l/2)$

11. $x=\frac{I}{mn}e^{-\mu t/2m}\sin nt$; $\frac{dx}{dt}=\frac{I}{m}e^{-\mu t/2m}\left(\cos nt-\frac{\mu}{2mn}\sin nt\right)$, where $n^2=\frac{k}{m}-\frac{\mu^2}{4m^2}$.

Problems 21.9, page 764

1. $\frac{2s}{(s^2+1)^2}$ **2.** (*d*) **3.** (*b*)

4. $\frac{1}{s^2-4s+5}$ **5.** te^{-2t} **6.** $\frac{1}{3}e^{-2t}\sin 3t$

7. $s\bar{f}(s)-f(0)$ **8.** $\frac{1}{8}(2-3t)\,e^{-3t/2}$ **9.** $\frac{1}{2}\left[\frac{1}{s}+\frac{s}{s^2-16}\right]$

10. $\frac{1}{s-\log 2}$ **11.** $\frac{k!}{(s+1)^{k+1}}$ **12.** $\frac{2}{13}$

13. $e^{-at/s}$ **14.** $\Gamma(3/2)/s^{3/2}$ **15.** $s^2f(s)-sf(0)-f'(0)$

16. $\frac{1}{4}\left[\frac{3s}{s^2+16}+\frac{s}{s^2+144}\right]$ **17.** $\cot^{-1}(s/a)$ **18.** $\frac{e^{-3t}\,t^4}{24}$

19. $\frac{s\cos 3-2\sin 3}{s^2+4}$ **20.** (*c*) **21.** $f(t-a)\,u(t-a)$

22. e^{-as} **23.** $1-3t+2t^2$ **24.** $\int_0^t f(t)\,dt$

25. $\int_0^T \frac{e^{-st}\,f(t)\,dt}{(1-e^{-st})}$ **26.** $\frac{1}{(s+1)^2(s+2)}$ **27.** $\frac{2(s-3)}{s^2-6s+34}+\frac{12}{s^2-6s+25}$

28. $1/(s-\log 4)$ **29.** $\frac{e^{-3t}}{\sqrt{(\pi t)}}$. **30.** (*d*)

31. $\frac{t}{2}\sin\frac{t}{2}$ **32.** (*c*) **33.** (*c*) **34.** (*ii*)

35. (*iii*) **36.** (*ii*) **37.** (*iv*) **38.** (*i*)

39. $\int_a^b f(x)dx$ **40.** (ii) **41.** (iii) **42.** (d)

43. True **44.** False **45.** False.

Problems 22.1, page 776

1. $\frac{2}{\pi}\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\,d\lambda\,;\ \frac{\pi}{2}$ for $|x|<1$, $\frac{\pi}{4}$ for $|x|=1$, 0 for $|x|>1$

2. $(i)\ \frac{4}{\pi}\int_0^\infty \frac{\sin\omega-\omega\cos\omega}{\omega^3}\cos\omega x\,d\omega$ $(ii)\ \frac{2}{\pi}\int_0^\infty \cos\omega x\,\frac{a}{a+\omega^2}\,d\omega$

4. $(i)\ \frac{2\sin as}{s}$, π ; $(ii)\ 2\,\{(a^2s^2-2)\sin as+2as\cos as\}/s^3$

5. $\frac{4}{s^3}(\sin sa - sa\cos sa)$ **6.** $(i)\ \sqrt{(3\pi)}\,e^{-3s^2/4}$ $(ii)\ \frac{\sqrt{\pi}}{2}\,e^{(3is-s^2/4)}$

7. $\frac{1-\cos 2s}{s}$, $\frac{\sin 2s}{s}$ **8.** $\sqrt{(\pi/2)}\,e^{-as}$ **9.** $\frac{a}{a^2+s^2}$; $\frac{\pi}{2a}\,e^{-a\lambda}$

10. 1 **13.** $(i)\ \frac{\pi}{2a^2}(1-e^{-as})$ $(ii)\ \tan^{-1}(s/a)$

11. $\frac{1}{a\sqrt{2}}\,e^{-s^2/4a^2}$; $\frac{1}{2a^3\sqrt{2}}\,e^{-s^2/4a^2}$

14. $(i)\ \frac{1}{2}\left\{\frac{\sin[a(1-s)]}{1-s}-\frac{\sin[a(1+s)]}{1+s}\right\}$ $(ii)\ (2\cos s-\cos 4s-1)/s^2-(2\sin s)/s$

15. $2/(\pi s^2)$ **16.** $F_s(p)=-32(-1)^p/p\pi$; $F_c(p)=32\frac{(-1)^p-1}{p^2\pi^2}$

17. $(\pi/s)\cos s/c$ **19.** $f(x)=(2+2\cos x-4\cos 2x)/\pi x$ **20.** $2/\pi(1+x^2)$.

Problems 22.2, page 780

2. $\frac{1}{4}\int_{-\infty}^{\infty} e^{-||x-t|+|t||}\,dt$ **5.** $2\left(\frac{1-\cos s}{s^2}\right)$.

Problems 22.3, page 783

1. $\frac{1}{9}[e^{-t}+e^{2t}(3t-1)]$ **2.** $\frac{1}{5}(e^{-2t}-2\sin t-\cos t)$ **3.** $(\sinh at-at)/a^2$

4. $\frac{1}{2}[e^x(x-1)+\cos x]$ **5.** $\frac{1}{2}(\sin t-t\cos t)$.

Problems 22.4, page 791

1. $y=30\,e^{-75t}\cos 5x$ **2.** $\sum_{n=1}^{\infty}\frac{V_o}{n\pi}(1-\cos n\pi)\,e^{-n^2t}\sin nx_0$

3. $u(x,t)=\frac{2}{\pi}\int_0^\infty e^{-s^2t}\left\{\frac{\sin(1+x)s+\sin(1-x)s}{s}\right\}ds$

6. $\theta(x,t)=\theta_0\,erf\left(\frac{x}{2\sqrt{(kt)}}\right)+\theta_0\sum_{n=1}^{\infty}(-1)^n\left\{erf\,\frac{nl-x}{2\sqrt{(kt)}}-erf\,\frac{nl+x}{2\sqrt{(kt)}}\right\}$.

Problems 22.5, page 792

1. $F_c(s) = \int_0^\infty f(t) \cos st \, dt$ 2. $s^2/2$

3. The Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transforms.

4. $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-isx} \, ds$ 5. $\int_{-\infty}^{\infty} t^n f(t) e^{ist} \, dt$ 6. $e^{isa} F(s)$

7. $\frac{2}{\pi} \int_0^\infty \sin \lambda x \int_0^\infty f(t) \sin \lambda t \, dt \, d\lambda$ 8. $\frac{1}{a}$ 9. $-s^2 [F(u)]$

10. $\frac{2}{\pi} \int_0^\infty \sin(\lambda x) \, d\lambda \int_0^\pi \sin(\lambda t) \, dt$ 11. $-\frac{n\pi}{l} \int_0^1 f(x) \cdot \cos \frac{n\pi x}{l} \, dx$ 12. $\frac{1}{a} F\left(\frac{\lambda}{a}\right)$

13. $f(x) = \frac{2}{\pi^3} \sum_{p=1}^{\infty} \left(\frac{1 - \cos p\pi}{p^2}\right) \sin px$ 14. $\frac{1}{2} F(s/2)$

15. $1/(s^2 + 1)$ 16. True 17. False 18. True

19. True 20. False 21. True.

Problems 23.1, page 800

1. (i) $e^{a/z}$; (ii) $z(e^{-z} - 1)$; (iii) $\frac{z}{z - e^{i\theta}}$

2. (i) $\frac{2z}{(z-1)^2} + \frac{z/\sqrt{2}}{z^2 - \sqrt{2z} + 1} + \frac{z}{z-1}$; (ii) $\frac{z^3 - 3z^2 + 4z}{(z-1)^3}$; (iii) $\frac{z(z+1)}{(z-1)^3} + \frac{3z}{(z-1)^2} + \frac{2z}{z-1}$

6. (i) $\frac{z^2 \sin \theta}{z^2 - 2z \cos \theta + 1}$ (ii) $\cos \alpha \, \frac{z(z - \cos \pi/8)}{z^2 \; 2z \cos \pi/8 + 1} - \sin \alpha \, \frac{z \sin \pi/8}{z^2 - 22 \cos \pi/8 + 1}$

7. $\frac{z(z - \cos \theta)}{z^2 - 2z \cos \theta + 1}, |z| > 1; \frac{z(z^2 \cos \theta - 2z + \cos \theta)}{(z^2 - 2z \cos \theta + 1)^2}$ 8. $\frac{z^2}{z^2 + 1}, |z| > 1; \frac{z^2}{z^2 + a^2}$

9. (i) $\frac{z}{z - e^{-a}}$; (ii) $\frac{z e^{-a}}{(z - e^{-a})^2}$; (iii) $\frac{(z + e^{-a}) e^{-a}}{(z - e^{-a})^3}$

12. $\frac{z^2(1 + 3z^2)}{(1 - z)(1 + z^2)}$ 13. $u_2 = 2, u_3 = 11.$

Problems 23.2 page 804

1. $\frac{z}{z-4}$; $|z| > 4$ 2. $\frac{z}{2-z}$; $|z| < 2$ 3. $\frac{3z}{(4-z)(z-3)}$; $3 < |z| < 4.$

4. $\frac{5z}{(4-5)^2}$; $|z| > 5$ 5. $-\log(1 - 3/z)$; $|z| > 3$ 6. $e^{3/z}$, ROC is z-plane

7. $(1 - e^a/z)^{-1}$; $|z| > |e^a|$.

Problems 23.3, page 807

1. $\frac{1}{2}(3^{n+1} - 1)$ 2. $(n + 1) a^n$ 3. $\frac{1}{2} n (n - 1)$

5. $4a^n$ 6. $(1/3)^n - 2^n$ 7. n

8. $\frac{3}{4}\left\{\frac{1}{(2)^{n-1}} + \frac{1}{(-4)^n}\right\}$ 9. $(n^2 + 7n + 4)(4)^{n-1}$

13. $u_n = (2)^{n-1} + (3)^{n-1} + (4)^{n-1}\ (n > 0)$

11. $2 + (2)^n + 3(n-1)\,2^n, (n \geq 1)$

12. $(-1)^{n+1} - 2n + \cos n\pi/2$

13. $1 - e^{-at}$

14. (*i*) $\left(-\frac{1}{3} - \frac{z}{3^2} - \frac{z^2}{3^3} - \frac{z^3}{3^4} \cdots\right) + \left(\frac{1}{2} + \frac{z}{2^2} + \frac{z^2}{2^3} + \frac{z^3}{2^4} + \cdots\right)$; (*ii*) $(-2^{n-1})\,z^{-n}, n > 0$

(*iii*) $(3^{n-1} - 2^{n-1})\,z^{-n}, n \geq 1, 0, n \leq 0$

15. $\frac{1}{2}\,(n-1)\,(n-2)5^{n-3}, n \geq 3$ and $= 0, n < 3$

16. $2(-i)^{n-1} - (-2)^{n-1}$

17. $\frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)^n$

18. $\frac{1}{3} - \frac{1}{3}\left(-\frac{1}{2}\right)^{n-1}$

19. $u_n = 1 + \frac{1}{2}\,[(i)^{n-2} + (-i)^{n-2}]$

20. $2n \sin(n\pi/2), n = 0, 1, 2, \ldots$

Problems 23.4, page 811

1. $y_k = \frac{8}{5}\left(\frac{1}{2}\right)^k - \frac{3}{5}\left(\frac{-1}{3}\right)^k$

2. $y(n) = (n-1)\,(-1)^{n-2}y(n-2) - 2^n$

3. $y_n = 2^{n-1} + (-2)^{n-1}$

4. $f(n) = 2 + (-4)^n$

5. $y_n = \frac{4}{3}\left[2\,(-1)^n + (2)^n\right]$

6. $36\left[\frac{1}{2} - (2)^n + \left(\frac{1}{2}\right)^n\right]$

7. $y_n = (c_1 + c_2 n)\,3^n + \frac{1}{2}\,n(n-1)\,3^{n-2}$

8. $y_n = c_1 + c_2 \,.\, 3^n + 5^n/8$

9. $y_n = 2\left[\left(\frac{1}{4}\right)^n - \left(-\frac{1}{4}\right)^n\right]$

10. 5.2^n

11. $y_n = \frac{1}{3}(-1)^n - \frac{2}{5}(-3)^n + \frac{1}{15}(2)^n$

12. $y_k = 1 - 2k + 2^k$

13. $y_n = c_1\,4^n + \left(c_2 - \frac{n}{4}\right)2^n + 2n - \frac{8}{3}$

14. $y_k = \frac{1}{2}(k+2)\frac{1}{5^k}\cos\frac{k\pi}{2}$

15. $y_n = \left[\frac{1}{4} - \frac{9}{4}(-3)^n\right]\mu(n)$

16. $y_n = (-2)^{n-1}, (n \geq 1)$.

Problems 23.5, page 811

1. $z/(z-1)$

2. $\sum_{n=0}^{\infty} u_n\, z^{-n}$

3. $z/(z-1)^2$

4. $az/(z-a)^2$

5. $\frac{z \sin\theta}{z^2 - 2z\cos\theta + 1}$

6. $e^{1/z}$

7. $(z^2 + z)/(z-1)^3$

8. $Z(au_n + bv_n + cw_n) = aZ(u_n) + bZ(v_n) + cZ(w_n)$

9. 2^{n-1}

10. $(-1)^{n-1}\,n$

11. $u_0 = \underset{z \to \infty}{Lt}\ \{Z(u_n)\}$

12. False

13. False

14. True

15. True

16. False

17. False

18. False.

Problems 24.1, page 815

1. $a = 2.28, b = 6.1879, p = 30.46$

2. $a = 1120, b = 55.1$

3. $a = 0.2, b = 0.0044$

4. $a = 0.5012, n = 0.5$

5. $a = 0.115, b = 11.8$

6. $a = 4.1, b = 0.43$

7. $a = 0.0498, b = -0.02$.

Problems 24.2, page 819

1. $y = 13.6x$ **2.** 15.2 thousand tons **3.** $Y = 0.004P + 0.048$
4. $R = 70.052 + 0.292t$ **5.** $a = 0.545, b = 0.636$
6. (*a*) $y = 4.193 + 1.117x$ (*b*) $y = 8 - 0.5\,x$
7. $y = 1.243 - 0.004x + 0.22x^2$ **8.** $y = 0.34 - 0.78x + 0.99x^2$ **9.** $y = 18.866 + 66.158x - 4.333$
10. $R = 3.48 - 0.002V + 0.0029V^2$ **11.** $V = 2.593 - 0.326T + 0.023\,T^2$.

Problems 24.3, page 823

1. $6.32, b = 0.0095$ **2.** $a = 1.52, b = 0.49$ **3.** $a = 3, b = 2$
4. $y = 7.187 - 5.16\,\frac{1}{x}$; 4.894 **5.** $a = 0.988, b = 3.275$ **6.** $y = 2.978\,x^{0.5143}$; 5.8769
7. $a = 0.1839, b = 0.0221$ **8.** $f(t) = 0.678\,e^{-3t} + 0.312\,e^{-2t}$ **9.** $a = 146.3, k = -0.412$
10. $a = 99.86, b = 1.2$.

Problems 24.4, page 826

1. $a = 11.1, b = 0.71$ **2.** $y = 46.05 + 6.1\,x$ **3.** $a = 0.0028, b = 0.01, c = 4.18$
4. $a = 15.8, b = 2.1, c = -0.5$ **5.** $a = 1.459, b = 0.062$.

Problems 24.5, page 828

1. $y = 0.12 + 0.47x$ **2.** $y = 1.184 + .523x$ **3.** $y = 1.53 + 0.063x + 0.074x^2$
4. $y = 0.485 + 0.397x + 0.124x^2$.

Problems 24.6, page 829

1. $Y = aX + b$ where $X = x, Y = y/x$
2. $Y = A + BX$, where $X = \log_{10} p, Y = \log_{10} v, A = \frac{1}{r}\log b, B = -1/r$
3. § 24.4 **4.** $y = aX + c$, where $X = x^b$ **5.** (*ii*)
6. $\Sigma y = nA + B\Sigma x, \Sigma xy = A\Sigma x + B\Sigma x^2$ where $y = \log_{10} y, A = \log_{10} a, B = \log_{10} b$
7. § 24.12 **8.** Zero
9. $y = aX + b$ where $X = x^2/\log_{10} z, Y = y/\log_{10} x$ **10.** $a = 0.0167, b = 1.05$.
11. The moments of the observed values of y are respectively equal to the moments of the calculated values of y
12. $a = 1.7, b = 1.26$ **13.** $y = a + bx$ where $x = 1/x, y = 1/y$
14. (*r*) **15.** (*b*)

Problems 25.1, page 837

1. 336.79
2. 64% get more than 50 marks ; median 54.7, $Q_1 = 46$, $Q_3 = 61.5$
3. Mean = 27.9 ; Median = 25.66 ; Mode = 21.85
4. Mean = 32.58 ; Median = 32.6 ; Mode = 35.1
5. 3.1% **6.** 1.3% **7.** 192 km/hr
8. 60 km/hr **9.** 38.6 ; 36.2
10. Median = 12.2 days ; Mode = 11.4 days

Problems 25.2, page 842

1. 4.45, 0.39 **3.** 4, 7 **4.** 10.04, 10.13, 11.69, 5.54, 2.35

5. 32, 32.6, 12.4 **6.** Q.D. = 10.9, S.D. = 15.26 **7.** 1.845 ; 1.8175
8. 0.55, 1.24 ; first, yes **9.** Height **10.** A
11. B is a better player and more consistent
12. A is more efficient, B is more consistent **13.** 161.3, 5.68.

Problems 25.3, page 845

1. $\mu_1 = \mu_3 = 0, \mu_2 = 2, \mu_4 = 11$; $\beta_1 = 0, \beta_2 = 2.75$ **2.** 8.85 ; 5.25 ; 0.32 ; 1.09
3. – 0.2064 **4.** 0.22 ; 1.157 **6.** 0 ; 2.9
7. $\beta_1 = 0.493$; $\beta_2 = 0.655$; platykurtic.

Problems 25.4, page 854

1. $r = 0.81$; $x = 0.5y + 0.5, y = 1.3x + 1.1$ **2.** $r = 0.96$
3. $r = 0.92$ **4.** $r = -0.055$ **5.** 0.7291
6. $r = 0.4517$ **7.** $r = 0.632$; $y = 0.467 + 0.8x, x = 0.167 + 0.5y$
9. $m = (\beta - b)/(a - \alpha)$ **10.** $\bar{x} = 4, \bar{y} = 7, r = -0.5$ **11.** $\bar{x} = 9.06, \bar{y} = 5.52, r = 0.46$
12. $r = 0.7395$; $\bar{x} = -0.1034$; $\bar{y} = 0.5172$ **13.** 134.5
14. 1.28 inch **15.** 0.8545 **16.** 0.932.

Problems 25.5, page 855

1. (*d*) **2.** (*d*) **3.** (*a*) **4.** (*a*)
5. (*b*) **6.** (*a*) **7.** (*b*) **8.** Coefficient of correlation
9. No **10.** 13.83 **11.** $\bar{x} = 2, \bar{y} = 3, r = \sqrt{3}$
12. Zero **13.** $\frac{1}{2}(Q_3 - Q_1)$ **14.** –1
15. § 25.9 **16.** $\dfrac{\Sigma XY}{n\,\sigma_x\,\sigma_y}$ **17.** $(\bar{x}, \bar{y})$
18. Reliability or consistency **19.** $\sqrt{\beta_1}$ **20.** degree of peakedness
21. $y - \bar{y} = r\,\dfrac{\sigma_y}{\sigma_x}(x - \bar{x})$ **22.** $100\,\sigma/\bar{x}$ **23.** $\tan^{-1}\left\{\dfrac{1-r^2}{r}\cdot\dfrac{\sigma_x\,\sigma_y}{\sigma_x^2+\sigma_y^2}\right\}$
24. 3 **25.** Two regression coefficients **26.** perpendicular
27. greater **28.** ± 1 **29.** $1 - \dfrac{6\,\Sigma\,d_i^2}{n^3 - n}$
30. Coefficient of standard deviation **31.** zero
32. – 1 and 1 **33.** – 0.6 **34.** False
35. True **36.** False

Problems 26.1, page 858

1. 4096 **2.** 360 ; 120 **3.** 120 ; 115 **4.** (*a*) 676000 (*b*) 468000

Problems 26.2, page 868

1. (*i*) 7/12 ; (*ii*) $P(A/B) = 3/4, P(A \cup B) = 7/12, P(A'\,B') = 3/8$
2. (*a*) 1/36 ; (*b*) 1/6, Yes **3.** 36 : 30 : 25
4. 1/7 **5.** (*i*) 1/4, (*ii*) 7/8, (*iii*) 11/16 **6.** 15/1024

7. 3/28 **8.** 20/81 **9.** (*i*) 2816/4165 ; (*ii*) 2197/2025,
10. 0.11 **11.** (*a*) 6.739 ; (*b*) 0.024 **12.** (*a*) 1/114 ; (*b*) 685/1140
13. 2/801 **14.** 10/21 **15.** $1-(1-p_1)(1-p_2)\ldots(1-p_n)$; 0.518
16. 15/17 **17.** 1/2 **18.** 5/12 **19.** (*i*) 83/110 (*ii*) 25/83
20. $1-2/(n-1)$ **21.** 7/20 **22.** (*a*) 1/6 ; (*b*) 3/4 **23.** 61/90
24. 0.72 **25.** 0.2223 **26.** 0.88

Problems 26.3, page 871

1. 3/11 **2.** 25/69, 28/69, 16/69 **3.** 0.3175, 0.254 **4.** 15/59.

Problems 26.4, page 878

1. $k = 1$; $\mu = .8$, $\sigma^2 = 2.232$ **2.** $2\sqrt{5}$
3. $F(x) = 0, -\infty < x < 0$
$= 1/8, 0 \le x \le 1$
$= 1/2, 1 \le x \le 2$
$= 7/8, 2 \le x \le 3$
$= 1, 3 \le x \le \infty$ **5.** ₹ 32
6. 2 **7.** $f(x)$ is a p.d.f. $\bar{x} = \frac{1}{2}, \sigma^2 = \frac{1}{20}$ **8.** (*i*) 9/16, 7/16 ; (*ii*) $k = 0.45$
9. (*i*) 0.37, (*ii*) 0.63 **10.** 4/9
12. $y_0 = 3/4$; Mean = 1 ; Variance = 1/5. **13.** 0.2
14. 1/3, 2/9 **15.** $F(x) = 0, x < x_1$; $(x - x_1)/(x_2 - x_1), x_1 \le x < x_2$; $1, x \ge x_2$.

Problems 26.5, page 881

1. $n = 4, p = q = \frac{1}{2}; \frac{15}{16}$ **2.** ${}^4C_r(1/6)^{4-r}(5/6)^r$; $r = 0, 1, 2, 3, 4$
3. (*a*) 0.02579 ; (*b*) 0.04571 ; (*c*) 1.024×10^{-7} **4.** 0.3456
5. 45927/50000 **6.** (*i*) 0.246 ; (*ii*) 0.345
7. (*i*) ${}^{20}C_1(1/20)(19/20)^{19}$; (*ii*) $\sum_{r=0}^{5} {}^{20}C_r (1/20)^r (19/20)^{20-r}$ (*iii*) 19
8. (*a*) 0.08 ; (*b*) 0.26 ; (*c*) 0.92 **9.** (*a*) 250 ; (*b*) 25 ; (*c*) 500
10. (*i*) 0.59049 ; (*ii*) 0.32805 ; (*iii*) 0.08146 **11.** 11
12. 99.83 **13.** 0.3585, 0.3773, 0.1887 ; 0.0596
14. 600 **15.** $100\,(.432 + .568)^5$
16. $200\,(0.554 + 0.446)^6$.

Problems 26.6, page 884

1. (*i*) 2 ; (*ii*) $\frac{2}{3}e^{-2}$ **2.** $P(0) = 0.2636, P(3) = 0.1041, P(> 3) = 0.1506$
4. $(10)^{15}e^{-10}/15! = 0.035$ **5.** 0.08
6. (*i*) 0.2231 (*ii*) 0.1913 **7.** 0.0008
8. $m = 0.51 = \sigma^2$; Poisson frequencies of 0, 1, 2, 3, 4 accidents are 180.1, 91.9, 23.4, 4, 0.6
9. 0.6 **11.** Theoretical frequencies are 44, 43, 21, 7, 1
12. Theoretical frequencies are 109, 142, 92, 40, 13, 3, 1, 0, 0, 0, 0.

Problems 26.7, page 890

2. (*i*) 0.1644 ; (*ii*) 0.7686 **3.** (*i*) 0.095 ; (*ii*) – 0.995 **4.** 36.4
5. (*i*) 16, (*ii*) 2 **6.** 294 **7.** 543
8. (*i*) 79 ; (*ii*) 35% ; (*iii*) 11 **10.** 52 **11.** 67
12. ₹ 866 **13.** $y = \dfrac{100}{\sqrt{(3.4\pi)}} e^{-\frac{(x-2)^2}{3.4}}$ **14.** $\mu = 13.64$. $\sigma = 3.98$.

Problems 26.8, page 892

1. (*a*) 0.0287, (*b*) 0.9672, (*c*) 0.5111 **2.** (*a*) 0.7854, (*b*) 0.1815, (*c*) 0.1815
3. (*a*) 0.97815, (*b*) 0.00595, (*c*) 0.01209.

Problems 26.9, page 893

3. $\frac{1}{2}(n+1), \frac{1}{12}(n^2-1)$ **5.** Mean = $a + b$, variance = b^2
6. $[(1-e^t)/t]^2$ **8.** $(1-t)^{3/4}$.

Problems 26.10, page 894

1. (*a*) **2.** (*b*) **3.** (*d*) **4.** (*b*) **5.** 1/7 **6.** 1/2
7. (*b*) **8.** (*b*) **9.** (*a*) **10.** (*c*) **11.** 0.1288 **12.** 2
13. 0.21 **14.** 0.24
15.

X:	0	1	2
$p(x)$:	1/4	2/4	1/4

16. § 25.5 **17.** 0.7837
18. zero **19.** equal **20.** $P(A) + P(B)$
21. $\beta_1 = 0, \beta_2 = 3$ **22.** 120 **23.** 0.2646
24. 1/9 **25.** 0.2222 **26.** 1/6
27. e^{-3} **28.** 5/36 **29.** 2
30. symmetrical **31.** $1 - e^{-m}$ **32.** six
33. 0.6915 **34.** $(q + pe^t)^n$ **35.** 4 : 5
36. 1/6 **37.** 3.5 **38.** $\sqrt{2}$
39. unity **40.** $n \to \infty, p \to 0$ such that np is fixed
41. $P(A) + P(B)$ **42.** ${}^yC_x / {}^{y+z}C_x$ **43.** np
44. $P(A \cup B) = 0.72, P(A \cap B') = 0.1653$ **45.** $\left(\frac{1}{3} + \frac{2}{3}\right)^{18}$
46. 1 **47.** $\mu_r' = \left[\dfrac{d^r}{dt^r}(\Sigma\, p_i\, e^{tx_i})\right]_{t=0}$ **48.** 1/6
49. 3/4 **50.** 1/2 **51.** 0.2 **52.** 2/7
53. $(q + p)^n$ **54.** 2/3 **55.** Mean and S.D. **56.** 1/13
57. 2 **58.** § 26.6 **59.** $e^{-4/3}$ **60.** 1/3
61. True **62.** False **63.** True **64.** False
65. False **66.** True **67.** False **68.** False
69. $k = 2$ **70.** 8 **71.** $f(x) = \lambda e^{-\lambda x}$ for $x > 0$, λ is a parameter
72. 25/12 **73.** 2 **74.** l **75.** 4
76. 8 **77.** n, m degrees of freedom **78.** $\int_{-\infty}^{\infty} e^{t(x-a)} f(x)\, dx$

79. $-5/7$ **80.** $\dfrac{e^{-x}x^{l-1}}{\Gamma(l)}, 0 < x < \infty$ **81.** $\dfrac{1}{2}(b+a)$

82. 50% **83.** 3/8 **84.** 1 **85.** 3/4

86. (*iii*) **87.** $F(x) = \int_{-\infty}^{x} f(x)\,dx$ **88.** $\dfrac{\lambda^{2r}e^{-\lambda}}{(2r)!}$ **89.** $-\infty < t < \infty$

90. 6 **91.** 1/9 **92.** False **93.** 2

94. $4(1-x)^3$ **95.** 0.264

Problems 27.1, page 901

1. Die is biased **2.** No **3.** Yes

4. 8.91% and 15.07% **5.** Consistent **6.** $p = 65/500$, S.E. = 0.015

7. 37.5% ; 30.3 and 44.7 respectively **8.** No

9. Difference is not significant

10. $Z \sim 6.56$ so that the difference is significant **11.** No.

Problems 27.2, page 904

1. No **2.** Mean weight lies between 64.6 and 69.4 lbs.

3. 0.0774 **5.** 62 **6.** 2.696 **7.** No

8. No **9.** (*i*) Yes, (*ii*) No **10.** Yes.

Problems 27.3, page 910

1. 0.25 **2.** $t = 0.62$, Yes **3.** 11.887 and 12.113 cm

4. Refute the claim **5.** Process is not under control **6.** No

7. Sample mean = 575.2 kg., S.E. = 2.75 kg **8.** Accept null hypothesis

9. Yes with 75% confidence. **10.** No **11.** No

12. No

Problems 27.4, page 914

1. 0.41 **2.** Hypothesis is correct

3. Significant at 5% level **4.** Yes

5. f_e: 33.82 161.78 315.98 308.48 150.54 29.4

$\chi^2 = 7.97$. Binomial distribution gives a good fit at 5% level

6. f_o: 305 365 210 80 18 12

f_e: 301.2 361.4 216.8 86.7 26 7.9 ; $\chi^2 = 3.097$

Poisson distribution gives a good fit at 5% level

7. f_o: 314 355 204 85 29 12

f_e: 301 362.2 217.3 86.9 26.1 6.5

Poisson distribution can be fitted to the data

8. $\chi^2 = 1.2$. The fit is quite good at 5% level.

Problems 27.5, page 917

1. First variance cannot be regarded as significantly greater than the second

2. Not significant as $F = 2.1$ and $F_{0.05} = 4.15$ **4.** Not significant as $F = 2.4$ and $F_{0.01} = 3.2$

5. Product of firm B cannot be said to be of better quality than those of firm A.

6. Not significant at 1% level and just significant at 5% level as $F = 2$, $F_{0.01} = 2.62$ and $F_{0.05} = 1.98$

7. $F = 1.49$, Not-significant **8.** $F = 1.025$; Yes

Problems 27.6, page 917

1. § 27.3 (3) **2.** § 27.15 **3.** § 27.3 (2)

4. We are testing the hypothesis that one process is better than another

5. § 27.11 **6.** 1 **7.** 50 **8.** $(0, \infty)$

9. II **10.** 8 ; 16 **11.** $r = n - 1$ **12.** False

13. Less than 30 **14.** § 27.17 **15.** $-\infty < t < \infty$ **16.** (*ii*)

17. True

Problems 28.1, page 926

1. (*i*) 2.687, (*ii*) 1.46, (*iii*) 2.375, (*iv*) 2.875 **2.** (*i*) 0.519, (*ii*) 2.875, (*iii*) 1.146, (*iv*) 0.367

3. (*i*) – 0.686, (*ii*) 2.7065, (*iii*) 0.686, (*iv*) 1.4036

4. (*i*) 0.853, (*ii*) 0.607, (*iii*) 2.798, (*iv*) 3.789, (*v*) – 0.134

5. 1.861 **6.** (*i*) 1.532, (*ii*) 2.095, (*iii*) 1.834, (*iv*) 1.226

7. (*i*) 1.855 (*ii*) 2.198 (*iii*) 1.662 **8.** – 16.56

9. (*i*) 0.853, (*ii*) – 1.9338, (*iii*) 2.7985, (*iv*) 4.545

10. (*i*) 0.518, (*ii*) 0.052, (*iii*) 0.695, (*iv*) 2.911 **11.** $x_{n+1} = \frac{1}{2}(x_n + N/x_n)$; (*i*) 3.605 (*ii*) 3.162

12. 3.4482 **13.** 2.3784

14. (*i*) 0.055 (*ii*) 0.258 (*iii*) 0.4347

Problems 28.2, page 929

1. (*i*) 1.532, (*ii*) 0.684, (*iii*) 3.18, (*iv*) 1.168 **2.** 1.674

3. 2.231 **4.** – 1.328 **5.** 2.924

Problems 28.3, page 936

1. $x = 7, y = -9, z = 5$ **2.** $x = -51/4, y = 115/8, z = 35/4$ **3.** $x = 1, y = 2, z = 3$

4. $x_1 = 2, x_2 = -1, x_3 = 3$ **5.** $x_1 = 1, x_2 = 2, x_3 = -1, x_4 = -2$ **6.** $x = 1, y = 3, z = 5$

7. $x = 8.7, y = 5.7, z = -1.3$ **8.** $x = 1, y = 2, z = 3$ **9.** $x = 7, y = -9, z = 5$

10. $x_1 = 2, x_2 = 1/5, x_3 = 0, x_4 = 4/5$ **11.** $x = y = z = 1$ **12.** $x = 1, y = 2, z = 3$

13. $x = 35/18, y = 29/18, z = 5/18$ **14.** $x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$ **15.** $\begin{bmatrix} 1.2 & -0.4 & 0.2 \\ -0.2 & -0.1 & 0.3 \\ -0.4 & 0.3 & 0.1 \end{bmatrix}$.

Problems 28.4, page 942

1. $x = 2.556, y = 1.722, z = -1.055$

2. (*a*) $x = 2.426, y = 3.573, z = 1.926$ (*b*) $x = 2.426, y = 3.573, z = 1.926$

3. $x = 1, y = 1, z = 1$ **4.** $x = 0.998, y = 1.723, z = 2.024$

6. $x = 1.052, y = 1.369, z = 1.962$ **8.** $x = -13.223, y = 16.766, z = -2.306$

9. $x = 1, y = 2, z = 3, u = 4$ **10.** $x = 1.36, y = 2.103, z = 2.845$

11. $x = y = z = 1$ **12.** $x = 52.5, y = 44.5, z = 59.7$

13. $x = 1.93, y = 3.57, z = 2.43$

Problems 28.5, page 943

1. $x = 2, y = 1$
2. $x = -1.853, y = -1.927$
3. $x = 0.7974, y = 0.4006$
4. $x = 3.162, y = 6.45$
5. $x = -3.131, y = 2.362$

Problems 28.6, page 945

1. (*a*) 5.38, $\begin{bmatrix} 0.46 \\ 1 \end{bmatrix}$; (*b*) 4.418, $\begin{bmatrix} 1 \\ 0.618 \end{bmatrix}$
2. 3.41 ; $[0.74, -1, 0.67]'$
3. (*a*) 6, $[1, 1, -1]'$ (*b*) 8, $[1, -0.5, 0.5]'$
4. (*a*) 7 ; $[2.099/7, 0.467/7, 1]$ (*b*) 25.182, $[1, 0.045, 0.068]'$ (*d*) 11.66 $[0.025, 0.422, 1.000]$.

Problems 28.7, page 945

1. Newton-Raphson method
2. $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})}$
3. Chord AB
4. $x_{n+1} = \dfrac{1}{2}(x_n + N/x_n)$
5. initial approximation x_0 is chosen sufficiently close to the root
6. diagonal
7. (*c*)
8. (*a*)
9. $x_{n+1} = \dfrac{1}{3}(2x_n + N/x^2_n)$
10. $x_{n+1} = \dfrac{1}{2}(x_n + x_{n-1})$
11. $x_2 = x_0 - \dfrac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$
12. (*a*)
13. $x_{n+1} = x_n(2 - Nx_n)$
14. (*b*)
15. Newton-Raphson method
16. § 28.6
17. Upper triangular matrix
18. False
19. True
20. $x = 1, y = 1$.

Problems 29.1, page 952

1. 0.4
2. -7459
5. 239
6. 4.68, 2.68, 55.8, 99.88
8. (*i*) $1 - 2\sin(x + 1/2)\sin 1/2$; (*ii*) $\tan^{-1}(1/2n^2)$;
(*iii*) $192\,[x(x+4)(x+8)(x+12)(x+16)]$ (*iv*) $-2/[(x+2)(x+3)(x+4)]$
9. (*i*) $e^{3x}[e^3 \log(1 + 1/x) + (e^3 - 1)\log 2x]$ (*ii*) $2^x(1-x)/(1+x)$
(*iii*) $(a-1)^n a^x$; (*iv*) $(-1)^n n!/[x(x+1)(x+2)\ldots(x+n)]$.
12. (*i*) -36 ; (*ii*) $24 \times 2^{10} \times 10!$
14. $u = [x]^4 - 6[x]^3 + 13[x]^2 + x + 9$
15. $4x^3 - 12x^2 + 8x + 1$; $12x(x-1)$
16. $\dfrac{1}{2}[x]^4 + 3[x]^3 + 4[x] + c$
17. $y(4) = 74, y(6) = 261$
19. 15.

Problems 29.2, page 957

1. $\left(\dfrac{\Delta^2}{E}\right)u_x = u_{x+h} - 2u_x + u_{x-h}$; $\dfrac{\Delta^2 u_x}{Eu_x} = \dfrac{u_{x+2h} - 2u_{x+h} + u_x}{u_{x+h}}$
2. (*i*) $2(\cos h - 1)\sin x$; (*ii*) $6x$; (*iii*) $2(\cos h - 1)[\sin(x+h) + 1]$; (*iv*) 8
8. *Error* = 10
9. 31
10. $f(1.5) = 0.222, f(5) = 22.022$
11. $y(4) = 74, y(6) = 261$
12. -99
13. $y_4 = 1$ approx
15. (*i*) $n(3n^2 + 6n + 1)$; (*ii*) $\dfrac{n(n+1)(n+2)(n+3)}{4}$
16. $2/(1-x)^3$.

Problems 29.3, page 961

1. 5.54 **2.** 6.36 **3.** 1.1312 **4.** 0.788
5. ₹ 110.52 **6.** 8666 **7.** 352 ; 219 **8.** 0.9623, 0.2903
9. 24 approx **10.** $f(x) = 9x - 4x^2$ **11.** 1.625 **12.** 0.1955
13. 4.219 **14.** 2530 **15.** $y_1 = 0.1, y_{10} = 100$ **16.** $u_2 = 42, u_4 = 49$
17. 10, 22 **18.** 755.

Problems 29.4, page 971

1. 19.4 **2.** 12.826 **3.** 54000 **4.** 3.2219
5. 3.0375 **6.** 395 **7.** 3.347 **8.** 9
9. 3250.875 **11.** 2.5283 by all formulae.

Problems 29.5, page 974

1. 14.63 **2.** 2.8168 **3.** 0.89 **4.** 100
5. $648 + 30x - x^2$ **6.** $x^3 - 3x^2 + 5x - 6$ **7.** $x^5 - 9x^4 + 18x^3 - x^2 + 19x - 18$
8. 3 **9.** $\dfrac{0.5}{x-1} - \dfrac{0.5}{x+1} + \dfrac{1}{x-2}$

Problems 29.6, page 977

1. 1 **2.** 3.09 **3.** 448, 3150 **4.** 133.19
5. $f(x) = \dfrac{1}{24}x^3 - 25x + 24 - \dfrac{7}{6}x^2 + \dfrac{557}{60}x - 25$ **6.** $f(x) = \dfrac{1}{20}x^3$ **7.** $f(x) = x^4 - 3x^3 + 5x^2 - 6$
8. 31.

Problems 29.7, page 978

1. 11.5 **2.** 6.5928 **3.** 37.23

Problems 29.8, page 978

1. \$ 7.3 **2.** (*b*)

3.

x	$f(x)$	I.D.D.	II.D.D.
5	7		
		2.9	
15	36		0.87
22	160	17.7	

4. Intermediate value of the variables. **5.** \$ 7.8

6. $\dfrac{[x_1, x_2, x_3, x_4] - [x_0, x_1, x_2, x_3]}{x_4 - x_0}$ **7.** $\dfrac{-1}{4}$ and $\dfrac{1}{4}$ **8.** \$ 7.14

9. $f(x) = \dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + \dfrac{(x-x_0)(x-x_1)}{(x_1-x_0)(x_1-x_2)} + \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

10. $\dfrac{13}{5}$ **11.** (*c*) **12.** 1.857

13. *Extrapolation* is the process of estimating the value of a function outside the given range of values.

14. $1/(abc)$ **15.** (*a*) **16.** $x^3 - 7x^2 - 18x - 12$
17. (*b*) **18.** $6h^2(x + h)$.

Problems 30.1, page 987

1. – 27.9, 117.67 **2.** 4.75, 9 **3.** 0.63, 6.6
4. (*a*) 0.493, – 1.165 (*b*) 0.4473, – 0.1583 ; (*c*) 0.4662, – 0.2043 **5.** 2.8326
6. – 0.06 ; 0.5 **7.** 0.175 **8.** 13.13 m/sec
9. (*i*) – 52.4, (*ii*) – 0.0191 **10.** 44.92 **11.** 0.085
12. 3.82 rad/sec, 6.75 rad/sec^2 **13.** 0.2561 **15.** 0.0186
16. 135 **17.** $y_{\max}(1) = 0.25, y_{\min}(0) = 0$ **18.** *Max* $f(10.04) = 1340.03$

Problems 30.2, page 995

1. (*i*) 0.695 (*ii*) 0.693 (*iii*) 0.693 **2.** (*i*) 0.7854 (*ii*) 0.7854, (*iii*) 0.78535, (*iv*) 0.7854
3. 1.61 **4.** – 6.436 **5.** (*i*) 70.16 (*ii*) 0.635
6. 0.6305 **7.** (*i*) 2.0009 ; (*ii*) 1.1873 **8.** (*i*) 1.1249 (*ii*) 0.911
9. (*a*) 1.8276551, .0001924 ; (*b*) 1.8278472, .0000003 ; (*c*) 1.8278470, 0000005 ; (*d*) 1.8278474, .00000001
10. 1.3028 **11.** 403.67 **12.** 7.78
13. 710 sq.ft **14.** 3.032 **15.** 408.8 cub. cm.
16. 1.063 sec ; 1.064 sec **17.** 552 m ; 3 m/sec^2. **18.** 30.87 m/sec.
19. 29 min nearly.

Problems 30.3, page 996

1. (*c*) **2.** $\frac{1}{h}\left[\Delta f(a) - \frac{1}{2}\Delta^2 f(a) + \frac{1}{3}\Delta^3 f(a) - \cdots\right]$
3. *h* should be small **4.** 0.775 **5.** $2\frac{2}{3}$
6. 30.8 **7.** (*b*)
8. larger number of sub-intervals **9.** 0.7854 **10.** (*d*)
11. $y'_{x_n}\frac{1}{h} = \left[\nabla_{y_n} + \frac{1}{2}\nabla^2{}_{y_n} + \frac{1}{3}\nabla^3{}_{y_n} + \cdots\right]$ **12.** a multiple of 6
13. (*c*) **14.** 0.783 **15.** 0.69
16. 1.36125 **17.** 1.36
18. if the entire curve is itself a parabola
19. even and multiples of 3 **20.** False

Problems 31.1, page 999

1. $y_{x+3} - 2y_{x+2} + 2y_{x+1} = 0$ **2.** $\Delta y_n = (-1)^{n+1}/(n+1)$ **3.** $u_{n+1} - 2u_n = 0$
4. (*i*) $(x+2)y_{x+2} - 2(2x+1)y_{x+1} + xy_x = 0$; (*ii*) $(x^2+x)y_{x+2} - (2x^2+4x)y_{x+1} + (x^2+3x+2)y_x = 0$
5. (*ii*) $y_{n+2} - 8y_{n+1} + 15y_n = 0$; (*ii*) $y_{n+2} - 6y_{n+1} + 4y_n = 0$
6. (*i*) $(x-1)y_{x+2} - (3x-2)y_{x+1} + 2xy_x = 0$; (*ii*) $y_{x+2} - 4y_x = 0$;
(*iii*) $y_{x+3} - 6y_{x+2} + 11y_{x+1} - 6y_x = 0$.

Problems 31.2, page 1002

1. $u_p = (c_1 + c_2 p)\,3p$ **2.** $y_n = c_1 \cos\frac{2n\pi}{3} + c_2 \sin\frac{2n\pi}{3}$ **3.** $u_n = c_1 \cos n\pi/2 + c_2 \sin n\pi/2$
4. $y_n = c_1 \,.\, 2^n + c_2 \,.\, 3^n$. **5.** $y_n = (2)^{n-1} + (-2)^{n-1}$ **6.** $u_k = c_1(-1)^k + (c_2 + c_3 k)\,2^k$.
7. $f(x) = (c_1 + c_2 x)(-1)^x + c_3 \,.\, 2^x$ **8.** $u_n = 2n + (-2)^n$. **9.** $y_n = 6 + (n-3)\,2^n$
10. $u_n = 2^{n/2}\,[c_1 \cos n\pi/4 + c_2 \sin n\pi/4]$
11. $y_m = 2^m\left\{c_1 \cos\frac{m\pi}{4} + c_2 + \sin\frac{m\pi}{4} + c_3 \cos\frac{3m\pi}{4} + c_4 \sin\frac{3m\pi}{4}\right\}$ **14.** $y_n = c_1(-1)^n + c_2(10)^n$

Problems 31.3, page 1005

1. $y_n = c_1(-1)^n + c_2(6)^n - 2^n/12$

2. $y_n = \left(\frac{n}{15} - \frac{1}{25}\right)(-3)^n + \frac{2^n}{25}$

3. $y_p = c_1 + c_2 p + c_3 p^2 + \frac{1}{6}\, p(p-1)(p-2)$

4. $y_n = 2^n\left(\frac{2}{\sqrt{3}}\sin\frac{n\pi}{3} - 2\cos\frac{n\pi}{3}\right) + 2$

5. $y = c_1 + c_2 \,.\, 3^x + \frac{1}{2}\, x \,.\, 3^{x-1}$

6. $y_x = (c_1 + c_2 x)\, 2^x + 3x(x-1)\, 2^{x-3} + 5 \,.\, 4^{x-1}$

7. $u_n = c_1 + c_2(-1)^n + \frac{1}{2}\,\frac{\cos\left(\frac{n}{2} - 1\right) - \cos\frac{n}{2}}{1 - \cos 1}$

8. $y_p = c_1\cos\frac{p}{2} + c_2\sin\frac{p}{2} + \frac{p\cos\left(p - \frac{1}{2}\right)}{2\sin\frac{1}{2}}$

9. $y_x = c_1 + 2^x + c_2(-2)^x - \frac{1}{27}\,(9x^2 + 12x + 11)$

10. $y_n = c_1(-1)^n + c_2\cos\frac{n\pi}{3} + c_3 + \sin\frac{n\pi}{3} + \frac{1}{2}\, n(n-3)$

11. $y_n = (c_1 + c_2 n)(3)^n + c_3(-1)^n + \frac{1}{3}(2)^n - \frac{3n}{4}$

12. $y_n = (c_1 + c_2 n)\, 2^{-n} + \frac{2^n}{9} + n(n-1)\left(\frac{1}{2}\right)^{n-1}$

13. $y_n = c_1(-2)^n + c_2(-3)^n + \frac{n}{12} - \frac{7}{144}$

14. $u_x + (c_1 + c_2 x)(-3)^x + \frac{2^x}{25}(5x - 2) + \frac{2}{4}\, x^{x-2} + \frac{7}{16}$

15. $y_n = c_1(-2)^n + 2^n(c_2\cos n\pi/3 + c_3\sin n\pi/3) + \frac{3}{16}\,(2)^n + 2^{n-4}(2n+3)$

16. $u_n = \left\{c_1 + c_2 n + \frac{1}{48}\, n(n-1)^2(n-2)\right\}2^n.$

17. $y_k = c_1 \,.\, 2^k + c_2 \,.\, 3^k + \frac{4^k}{2}(k^2 - 13k + 61)$

18. $y_n = 2^n\left\{(c_1 + n)\cos\frac{n\pi}{3} + c_2 + \sin\frac{n\pi}{3}\right\}.$

Problems 31.4, page 1006

1. $y_x = a + b(-1)^x + x,\ z_x = a + b(-1)^{x+1} - (x+1)$

2. $y_x = (a + bx)(-1)^x - \frac{1}{9} \,.\, 2^{x+2},\ z_x = \frac{2^x}{9} - (-1)^x\left[a + b\left(x - \frac{1}{2}\right)\right]$

3. $u_n = 2.4^n - 2\frac{1}{2}\, n(n-1),\ v_n = 4^n + 2 + \frac{1}{n} + n(n+1)$

4. $u_x = -2a + b(-2)^x - c + \frac{1}{2x}\,(3 - x),\ v_x = a + c + b(-2)^x + \frac{1}{2}x(x-1).$

Problems 31.5, page 1007

1. $y_{i+1} - 2y_i + y_{i-1} = -\frac{l_m}{P}\, y_i$. Solve it for y_i.

Problems 31.6, page 1007

1. $y_{n+2} - 5y_{n+1} + 6y_n = 0$

2. $u_n = c_1 + c_2 n + c_3 n^2$

3. $u_n = c_1 + c_2(-2)^n + c_3(3)^n$

4. $y_n = c + 2^n.$

5. $y_n = c(2)^n - (n+1)$

6. $(x^2 + x)y_{x+2} - (2x^2 + 4x)y_{x+1} - (x^2 + 3x + 2)y_x = 0$

7. $y_n = c(2)^k + 1$

8. $y_n = (2)^{n+2} + (-2)^{n-1}$

9. Third

10. $(x+2)y_{n+2} - 2(n+1)y_{n+1} + ny_n = 0$ **11.** Second

12. $y_n = (C_1 + C_2 n)2^n$ **13.** $\frac{1}{2}x(x-1)(3)^{x-2}$ **14.** $y_{n+2} - 6y_{n+1} + 9y_n = 0$

15. True.

Problems 32.1, page 1012

1. $y = 1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}$ **2.** 0.0214 **3.** $y = \frac{1}{3}x^3 - \frac{1}{81}x^9 + \ldots$

4. (*a*) 0.9138, (*b*) 0.1938

5. $y(1.1) = 0.1103$, $y(1.2) = 0.2428$. Exact $y(1.1) = 0.1103$ and $y(1.2) = 0.2428$

6. 1.1053425 **7.** 1.1272 **8.** 1.005.

Problems 32.2, page 1017

1. 1.1831808 **2.** 1.1448 **3.** 4.5559

4. $y(0.1) = 0.095$, $y(0.2) = 0.181$, $y(0.3) = 0.259$ **5.** $y(0.2) = 1.2046$, $y(0.4) = 1.4644$

6. 2.2352 **7.** 1.0928 **8.** 5.051.

Problems 32.3, page 1021

1. 1.7278 **2.** 1.1749 **3.** 1.0207, 1.0438

4. 2.5005 **5.** $y(0.1) = 0.9052$, $y(0.2) = 0.8213$

6. $y(0.1) = 2.9919$, $y(0.2) = 2.9627$ **7.** 0.3487 **8.** 0.8489

9. 1.1678 **10.** 1.0911, 1.1677, 1.2352, 1.2902, 1.338.

Problems 32.4, page 1026

1. 3.795 **2.** 1.2797 **3.** $y(1.4) = 3.0794$

4. $y(4.5) = 1.023$ **5.** $y(0.4) = 2.162$ **6.** $y(0.4) = 0.441$.

Problems 32.5, page 1030

1. 0.2416 **2.** 1.0408 **3.** 0.6897

4. $y(4.4) = 1.019$ **5.** 2.5751 **6.** $y(1.4) = 0.949$.

Problems 32.6, page 1034

1. $y_3 = 1 + \frac{x}{2} + \frac{3}{40}x^5 + \frac{1}{40}x^6 + \frac{1}{192}x^9$. $z_3 = \frac{1}{2} + \frac{3}{8}x^4 + \frac{1}{10}x^5 + \frac{3}{34}x^8 + \frac{7}{340}x^9 + \frac{1}{256}x^{12}$

2. $y(0.1) = 0.105$, $z(0.1) = 0.999$; $y(0.2) = 0.22$, $Z(0.2) = 0.997$

3. $y(0.1) = 2.0845$, $z(0.1) = 0.5867$ **4.** $y_2 = 1 + \frac{1}{2}x + \frac{3}{40}x^5$

5. $y(0.1) = 0.5075$ **6.** $y(0.2) = 0.9802$, $y(0.2) = -0.196$

7. -0.5159 **8.** $\theta(0.2) = 0.8367$, $(d\theta/dt)_{0.2} = 3.6545$.

Problems 32.7, page 1038

1. 0.14031 **2.** $y(.25) = y(.75) = 2.4$, $y(.5) = 3.2$

3. $y(1.25) = 1.3513$, $y(1.5) = 1.635$, $y(1.75) = 1.8505$

4. $y(.25) = -0.3473$, $y(.5) = -0.9508$, $y(.75) = -1.7257$

5. $n = 2$, $y(0.5) = 0.1389$, true value $= 0.1505$; $n = 4$, $y(0.5) = 0.147$

6. $y(0.25) = 0.062, y(0.5) = 0.25, y(0.75) = 0.562$

7. $y(1) = 1.0171, y(2) = 1.094$

8. $y(1/3) = 1.1539, y(2/3) = 3.9231\ ; y(1) = 7.4615.$

Problems 32.8, page 1038

1. (*b*)

2. $y_{i+1} - 2y_i + y_{i-1} + h^2 y_i = 5h^2$

3. $1 + x + x^2 + x^3/6$

4. § 31.4

5. § 31.3

6. (*b*)

7. $y_4 = y_2 + \dfrac{h}{3}(f_2 + 4f_3 + f_4)$

8. Modified Euler's method

9. $y_1 = y_0 + \dfrac{h}{24}(55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3})$

10. Milne's method and Adam-Bashforth method

11. four

12. $y = 1 + \dfrac{x^2}{2} + \dfrac{x^4}{8}$

13. (*b*)

14. $y_4 = y_0 + \dfrac{4h}{3}(2f_1 - f_2 + 2f_3)$

15. $y_1 = y_0 + \dfrac{h}{24}(9f_1 + 19f_0 - 5f_{-1} + f_{-2})$

16. 1.1818

17. $\dfrac{dy}{dx} = z, \dfrac{dz}{dx} + y(1 + yz) = 0$

18. § 31.7

19. starting values

20. Picards and Runge-Kutta methods

21. It agrees with Taylor's series solution upto the term in h^4

22. (*d*)

23. $y_{i+1} + 2(h^2 - 1)\, y_i + y_{i-1} = 0$

24. (*a*)

25. True

26. False

27. False.

Problems 33.1, page 1041

1. Parabolic

2. Hyperbolic

3. (*i*) Parabolic (*ii*) Elliptic (*iii*) Elliptic

4. Outside the ellipse $(x/0.5)^2 + (y/0.25)^2 = 1$.

Problems 33.2, page 1050

1. $u_1 = 1.999, u_2 = 2.999, u_3 = 3.999$

2. 2.37, 5.6, 9.87, 2.89, 6.14, 9.89, 3.02, 6.17, 9.51

3. $u_1 = 10.188, u_2 = 0.5, u_3 = 1.188, u_4 = 0.25, u_5 = 0.625, u_6 = 1.25$

4. $u_1 = 26.66, u_2 = 33.33, u_3 = 43.33, u_4 = 46.66$

5. $u_1 = 0.99, u_2 = 1.49, u_3 = 0.49$

6. $u_1 = 1.57, u_2 = 3.71, u_3 = 6.57, u_4 = 2.06, u_5 = 4.69, u_6 = 8.06, u_7 = 2, u_8 = 4.92, u_9 = 9$

7. $u_1 = -3, u_2 = -2, u_3 = -2.$

Problems 33.3, page 1054

1.

j \ i	0	1	2	3	4
0	0	3	4	3	0
1	0	2	3	2	0
2	0	1.5	2	1.5	0
3	0	1	1.5	1	0
4	0	0.75	1	0.75	0
5	0	0.5	0.75	0.5	0

2.

$j \backslash i$	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.08	0.15	0.20	0.23	0.24	0.23	0.20	0.15	0.08	0
2	0	0.075	0.14	0.19	0.22	0.23	0.22	0.19	0.14	0.075	0
3	0	0.07	0.133	0.18	0.21	0.22	0.21	0.18	0.133	0.07	0

3.

$j \backslash i$	0	1	2	3	4	5
0	0	24	84	144	144	0
1	0	42	84	114	72	0
2	0	42	78	78	57	0
3	0	39	60	67.5	39	0
4	0	30	53.25	49.5	33.75	0
5	0	26.6	39.75	43.5	24.75	0
6	0	19.88	35.06	32.25	21.75	0

4.

$j \backslash i$	0	1	2	3	4
0	0	0.5	1	0.5	0
1	0	0.5	0.5	0.5	0
2	0	0.25	0.5	0.25	0
3	0	0.25	0.25	0.25	0

Problems 33.4, page 1060

1.

$t = 0.3\ ;\ x =$	0.1	0.2	0.3	0.4	0.5
Numerical sol. $u =$	0.02	0.04	0.06	0.075	0.08
Exact sol. $u =$	0.02	0.04	0.06	0.075	0.08

2.

$j \backslash i$	0	1	2	3	4	5
0	0	20	15	10	5	0
1	0	7.5	15	10	5	0
2	0	−5	2.5	10	5	0
3	0	−5	−10	−2.5	5	0
4	0	−5	−10	−15	−7.5	0
5	0	−5	−10	−15	−20	0

3.

$j \backslash i$	0	1	2	3	4	5	6	7	8	9	10
0	0	10.19	10.16	10.21	10.24	10.25	10.24	10.21	10.16	10.09	0
1	0	5.08	10.15	10.20	10.23	10.24	10.23	10.20	10.15	5.08	0
2	0	0.06	5.12	10.17	10.20	10.21	10.20	10.17	10.12	0.06	0
3	0	0.04	0.08	5.12	10.15	10.16	10.15	10.12	10.08	0.04	0
4	0	0.02	0.04	0.06	5.08	10.09	10.08	10.06	10.04	0.02	0
5	0	0	0	0	0	0	0	0	0	−0.02	0

4.

$j \backslash i$	0	0.1	0.2	0.3	0.4	0.5
0.1	0	0.037	0.07	0.096	0.113	0.119
0.2	0	0.031	0.059	0.082	0.096	0.101
0.3	0	0.023	0.043	0.059	0.07	0.074
0.4	0	0.012	0.023	0.031	0.037	0.039
0.5	0	0	0	0	0	0

Problems 33.5, page 1060

1. (*b*) **2.** False **3.** a hyperbolic equation

4. Poisson's equation **5.** $u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1}$. **6.** $(u_{i-1,j} - 2u_{i,j} + u_{i+1,j})/h^2$

7. § 32.8 (2) **8.** $\frac{1}{4}(u_{i-1,j+1} + u_{i+1,j-1} + u_{i+1,j+1} + u_{i-1,j-1})$

9. hyperbolic **10.** Bendre-Schmidt

11. $u_{i,j+1} = 2(1-4\alpha^2)\,u_{i,j} + 4\alpha^2(u_{i-1,j} + u_{i+1,j} - u_{i,j-1})$ where $\alpha = k/h$.

12. $u_{i,j+1} = \frac{1}{2}(u_{i-1,j} + u_{i+1,j})$ **13.** \$33.5(2) **14.** $\lambda < \frac{1}{2}$

15. Elliptic

16. $u(0, t) = 0 = u(1, t), (t > 0)$; $u(x, 0) = f(x), 0 < x < l$; $\delta x/\delta t\,(x, 0) = 0, 0 < x < l$

17. $u_{i,j+1} = u_{i+1,j} - u_{i-1,j} - u_{i,j-1}$ **18.** $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = f(x, y)$

19. 100 **20.** $\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = 0$

21. $y < 0$ **22.** $k = 1/4$.

Problems 34.1, page 1063

1. Max. $Z = 1.2x_1 + 1.4x_2$; subject to $40x_1 + 25x_2 \le 1000$,
$35x_1 + 28x_2 \le 980$, $25x_1 + 35x_2 \le 875$ and $x_1, x_2 \ge 0$

2. Max. $Z = 3x_1 + 2x_2 + 4x_3$; subject to $4x_1 + 3x_2 + 5x_3 \le 2000$,
$2x_1 + 2x_2 + 4x_3 \le 2500$, $100 \le x_1 \le 150$, $200 \le x_2$ and $50 \le x_3$

3. Max. $Z = 3x_1 + 2x_2 + x_3$; subject to $3x_1 + 4x_2 + 3x_3 \le 42$,
$5x_1 + 3x_3 \le 45$, $3x_1 + 6x_2 + 2x_3 \le 41$ and $x_1, x_2, x_3 \ge 0$.

4. Max. $Z = 400x + 300y$; subject to $x + y \le 200$, $x \ge 20$, $y \ge 4x$, $x \ge 0$, $y \ge 0$

5. Min. $Z = x_1 + x_2$; subject to $2x_1 + x_2 \ge 12$, $5x_1 + 8x_2 \ge 74$,
$x_1 + 6x_2 \ge 24$ and $x_1, x_2, x_3 \ge 0$.

6. Max. $Z = 0.15x_1 + 0.25x_2$; subject to
$2x_1 + 5x_2 \le 480{,}000$, $5x_1 + 4x_2 \le 720{,}000$,
$8x_1 + 16x_2 \le 300{,}000$, $0 \le x_1 \le 25{,}000$ and $0 \le x_2 \le 7{,}000$

7. Min. $Z = 41x_1 + 35x_2 + 96x_3$; subject to
$2x_1 + 3x_2 + 7x_3 \ge 1250$, $x_1 + x_2 \ge 250$, $5x_1 + 3x_2 \ge 900$,
$6x_1 + 25x_2 + x_3 \ge 232.5$ and $x_1, x_2, x_3 \ge 0$

8. Min. $Z = 100x_1 + 250x_2 + 160x_3$; subject to
$0.94x_1 + x_2 + 1.04x_3 \le 0.98$, $10x_1 + 15x_2 + 17x_3 \ge 14$,
$470x_1 + 500x_2 + 520x_3 \ge 495$, $x_1 + x_2 + x_3 = 1$ and $x_1, x_2, x_3 \ge 0$.

Problems 34.2, page 1069

1. $x_1 = 100, x_2 = 50$; max. $Z = 550$

2. $x_1 = 8/15, x_2 = 12/5$, max. $Z = 24.8$

3. $x_1 = 15, x_2 = 0$; max. $Z = 300$

4. $x_1 = 1000, x_2 = 500$; max. $Z = 5500$

5. 450 units of product B only ; max. profit = ₹ 1800

6. $X = 2, Y = 4.5$; max. profit = ₹ 37

7. $A = 1.18$ units, $B = 0.53$ units ; max. profit = ₹ 14.50 approx.

8. 2000/11 units of product A and 1000/11 units of B ; max. profit = ₹ 10,000

9. $x_1 = 4, x_2 = 0$; max. $z = 8$

10. Unbounded solution

11. $x_1 = 2, x_2 = 4$; min. $Z = 64$

12. Production cost will be min. if G and J run for 12 and 4 days respectively.

Problems 34.3, page 1074

1. Max. $Z = 3x_1 + 5x_2 + 8x_3$; subject to $2x_1 - 5x_2 + s_1 = 6$,
$3x_1 + 2x_2 + x_3 - s_2 = 5$, $3x_1 + 4x_3 + s_3 = 3$;
$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

2. Min. $Z = 3x_1 + 2x_2 + 5x_3$, subject to $-5x_1 + 2x_2 + s_1 = 5$,
$2x_1 + 3x_2 + 4x_3 - s_2 = 7$, $2x_1 + 5x_3 + s_3 \ge 3$
$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$.

3. Max. $Z = 3x_1 - 2x_2 + 4x_4 - 4x_5$; subject to
$x_1 + 2x_2 + x_4 - x_5 + s_1 = 8$, $2x_1 - x_2 + x_4 - x_5 - s_2 = 2$,
$-4x_1 + 2x_2 + 3x_4 - 3x_5 = 6$; $x_1, x_2, x_4, x_5, s_1, s_2 \ge 0$

4. (*i*) $x_1 = 2, x_3 = 1$ (Basic) ; $x_3 = 0$ (Non-basic). (*ii*) $x_1 = 5$, $x_3 = -1$ (Basic) ; $x_2 = 0$ (Non-basic) ; (*iii*) $x_2 = 5/3, x_3 = 2/3$ (Basic) ; $x_1 = 0$ (Non-basic). All the three basic solutions are non-degenerate

6. Basic solutions are (*i*) $x_1 = 2, x_2 = 1$ (Basic) and $x_3 = 0$;
(*ii*) $x_1 = x_3 = 1$ (Basic) and $x_2 = 0$; (*iii*) $x_2 = -1, x_3 = 2$ (Basic) and $x_1 = 0$
(*a*) First two solutions are non-degenerate basic feasible solutions
(*b*) First solution is optimal and $Z_{max} = 5$.

Problems 34.4, page 1081

1. $x_1 = 2, x_2 = 4$, max. $Z = 14$

2. $x_1 = 0, x_2 = 20$; max. $Z = 200$

3. $x_1 = 7/3, x_2 = 4/3$; max. $Z = 16$

4. $x_1 = 5, x_2 = x_3 = 0$; max $Z = 50$

5. $x_1 = 0, x_2 = 100, x_3 = 230$; max. $Z = 1350$

6. $x_1 = 89/41, x_2 = 50/41, x_3 = 62/41$; max. $Z =$ ₹ 765/41

7. $x_1 = 4, x_2 = 5, x_3 = 0$; min $Z = -11$

8. $x_1 = 280/13, x_2 = 0, x_3 = 20/13, x_4 = 180/13$; max. $Z = 2280/13$

9. $x_1 = 0, x_2 = 400$ units ; max. profit = ₹ 1200

10. $x_1 = 125, x_2 = 250$ units ; max profit = ₹ 2250

11. $x_1 = 400$ gms, $x_2 = 0$; min. cost = ₹ 2

12. $x_1 = 0, x_2 = x_3 = 50$; max. profit = ₹ 700

13. $x_1 = 0.5, x_2 = x_3 = 0.04$ units ; min. cost = ₹ 5.80

14. Averages for corn, wheat, soyabeans are 250, 625, zero respectively to achieve a max. profit of ₹ 32,000.

Problems 34.5, page 1088

1. $x_1 = 0, x_2 = 2, x_3 = 0$; max. $Z = 4$

2. $x_1 = 3, x_2 = 2, x_3 = 0$; max. $Z = 8$

3. $x_1 = x_2 = -6/15$; max. $Z = -48/5$

4. $x_1 = 23/3, x_2 = 5, x_3 = 0$; max $Z = 85/3$

5. $x_1 = x_2 = x_3 = 5/2, x_4 = 0$; max. $Z = 15$

6. $x_1 = 21/13, x_2 = 10/13$; max. $Z = 31/13$

7. Infeasible

8. $x_1 = 23/3, x_2 = 5, x_3 = 0$; max. $Z = 85/3$

9. $x_1 = 55/7, x_2 = 30/7, x_3 = 0$; max. $Z = 155/7$

10. $x_1 = 2, x_2 = 0$; max. $Z = 18$

11. Degenerate solution : $x_1 = 0$ (non-basic) ; $x_2 = 1, x_3 = 0$ (basic) ; max. $Z = 3$.

Problems 34.6, page 1091

1. Min. $W = 26y_1 + 7y_2$; subject to $6y_1 + 4y_2 \geq 10$
$5y_1 + 2y_2 \geq 13, 3y_1 + 5y_2 \geq 19$; $y_1, y_2, y_3 \geq 0$

2. Max. $W = 11y_1 + 7y_2 + y_3 + 5y_4$; subject to $3y_1 + 2y_2 - y_3 + 3y_4 \leq 2$,
$4y_1 + 3y_2 + 2y_3 + 2y_4 \leq 4, y_1 - 2y_2 + 3y_3 + 2y_4 \leq 3$;
$y_1, y_2, y_3, y_4 \geq 0$

3. Min. $W = -3y_1 + y_2 + 4y_3$; subject to $y_1 + 3y_2 - 2y_3 \leq -3$,
$y_1 + y_3 \geq 16, y_1 - 2y_2 + y_3 \leq -7$;
$y_1, y_2 \geq 0, y_3$, unrestricted in sign

4. Max. $W = -5y_1 + 9y_2 + 8y_3$; subject to $-2y_1 + 4y_2 - 8y_3 \leq 3$,
$3y_1 - 2y_2 + 4y_3 \leq -2, -y_1 + 3y_3 = 1$;
$y_1, y_2 \geq 0, y_3$ unrestricted

5. Min. $y = 3y_1 + 4y_2 + y_3 + 6y_4$; subject to $5y_1 - 2y_2 + y_3 - 3y_4 \geq 2$,
$6y_1 + y_2 - 5y_3 - 3y_4 \geq 5, -y_1 + 4y_2 + 3y_3 + 7y_4 \geq 6, y_1, y_2, y_3, y_4 \geq 0$.

Problems 34.7, page 1094

1. $x_1 = x_2 = 0, x_3 = 5/2$; min. $Z = 2.5$

2. $x_1 = 4, x_2 = 2$; max. $Z = 10$.

3. $x_1 = 7, x_2 = 0$, max. $Z = 21$

4. $x_1 = 0, x_2 = 100, x_3 = 230$; max. $Z = 1350$.

Problems 34.8, page 1097

1. $x_1 = 0, x_2 = 1$; max. $Z = -1$

2. $x_1 = 3/5, x_2 = 6/5$; min. $Z = 12/5$

3. $x_1 = 6, x_2 = 2, x_3 = 0$; min. $Z = 10$

4. $x_1 = 65/23, x_2 = 0, x_3 = 20/23, x_4 = 0$; min. $Z = 215/23$.

Problems 34.9, page 1104

1. $x_{11} = 200, x_{12} = 50, x_{22} = 175, x_{24} = 125, x_{33} = 275, x_{34} = 125$; min. cost = ₹ 12075

2. $x_{13} = 14, x_{21} = 6, x_{22} = 5, x_{23} = 1, x_{32} = 5$; min. cost = 143

3. $x_{11} = 50, x_{12} = 100, x_{21} = 150, x_{33} = 150, x_{42} = 100, x_{43} = 50$; min. tonnage = 3300

4. $x_{11} = 140, x_{13} = 60, x_{21} = 40, x_{22} = 120, x_{33} = 90$; min. cost = ₹ 5920

5. $x_{11} = 5, x_{14} = 2, x_{22} = 3, x_{23} = 7, x_{32} = 5, x_{34} = 13$;
min. cost = ₹ 799 and maximum saving = ₹ 201

6. $x_{11} = 150, x_{13} = 20, x_{22} = 160, x_{24} = 40, x_{33} = 90, x_{34} = 90$; max. profit = 4920

7. $x_{13} = 2, x_{22} = 1, x_{23} = 2, x_{31} = 4, x_{33} = 1$; min. cost = 33

8. $x_{13} = 0, x_{15} = 800, x_{21} = 400, x_{24} = 100, x_{32} = 400, x_{33} = 200,$
$x_{34} = 300, x_{43} = 300$; min. cost = 9200.

Problems 34.10, page 1109

1. $x_{11} = x_{22} = x_{33} = 1$; min. cost = ₹ 18
2. $A \to 2, B \to 3, C \to 4, D \to 1$; min. $Z = 38$
3. $I \to B, II \to A, III \to D, IV \to C$; min. cost = ₹ 49
4. $A \to$ Dyn. Prog., $B \to$ Queuing Th., $C \to$ Reg. Analysis, $D \to$ L.P. ; min. time = 28 hrs
5. (*i*) $A \to I, B \to II, C \to III, D \to IV$; (*ii*) $A \to I, B \to III, C \to II, D \to IV$
6. $1 \to IV, 2 \to II, 3 \to VI, 4 \to I, 5 \to III, 6 \to V$; max. profit = ₹ 270

Problems 34.11, page 1110

1. § 34.5 Def. 2
2. it provides an optimality test
3. § 34.11
4. § 34.16 (1)
5. § 34.13
6. § 34.6 (1)
7. Min. $W = 7y_1 + 5y_2$, subject to $2y_1 + 3y_2 \le 4$,
   $3y_1 - 2y_2 \le 9$, $2y_1 + 4y_2 \le 2$, $y_1 \ge 0$, y_2 is unrestricted in sign
8. § 34.12 (2)
9. § 34.14
10. Minimize $Z = (2x_{11} + 3x_{12} + 11x_{13} + 4x_{14}) + (5x_{21} + 6x_{22} + 8x_{23} + 7x_{24})$,
    subject to $x_{11} + x_{12} + x_{13} + x_{14} = a_1 (= 15)$, $x_{21} + x_{22} + x_{23} + x_{24} = a_2 (= 20)$,
    $x_{11} + x_{21} = b_1 (= 10)$, $x_{12} + x_{22} = b_2 (= 5)$; $x_{13} + x_{23} = b_3 (= 12)$; $x_{14} + x_{24} = b_4 (= 8)$ and $x_{ij} \ge 0$.
    [$\because$ $\Sigma a_i = \Sigma b_j = 35$]
11. (*i*) $x_1 = 3, x_2 = 5, x_3 = 0$; (*ii*) $x_1 = 0.5, x_2 = 0, x_3 = 2.5$
12. § 34.5 (Def. 4)
13. § 34.15
14. balanced
15. § 34.9
16. § 34.7 (3)
17. optimal
18. Minimize $y = 5y_4 - 3y_3$, subject to $y_4 + y_3 = 5$, $2y_4 - 5y_3 \ge 6$, $y_3 \ge 0$ and y_4 unrestricted
19. 5
20. Max. $Z = 5/19$
21. § 34.7
22. § 34.16
23. § 34.7 [2 (*ii*)]
24. *Min.* $W = 2y_1 + 4y_2 + 3y_3$, subject to $-y_1 + y_2 + y_3 \ge 2$, $2y_1 + y_2 \ge 1$, $y_1, y_2 \ge 0$
25. North west corner rule and Vogeli approximation method
26. Slash or surplus variables.

Problems 35.1, page 1118

1. (*i*) $y = \frac{1}{4}x^2 + c_1x + c_2$; (*ii*) $y = c_1x^{-1} + c_2$
3. $y = c_1 e^x + c_2 e^{-x} + \frac{xe^x}{2}$
4. $y = -x \cos x/2$
5. $y = (c + x) \sin x$
6. $y = \sinh (c_1x + c_2)$
7. $y = x^2 - 1$
8. $(x + c)^2 + y^2 = k^2$
9. $y = 2 \sin x$
12. The spirals of the family $r = a \sec (\phi \sin \alpha + b)$.

Problems 35.2, page 1120

1. $y = \pm 2 \sin m\pi x$, where m is an integer
2. $y = \lambda x^2 + ax + b$, where λ, a, b are determined from the isoperimetric and boundary conditions.
4. $y(x) = \frac{1}{2}(1 - \cos x) + \frac{1}{4}(2 - \pi) \sin x$.

Problems 35.3, page 1124

2. $y = (c_1 + c_2x) \cos x + (c_3 + c_4x) \sin x$,
   $z = (c_1 + c_2x) \cos x + (c_3 + c_4x) \sin x - 2c_2 \sin x + 2c_4 \cos x$
3. $y = a_n \sin nx, n = 1, 2, 3 \ldots$
4. $y = \cos x$
5. $y = -\frac{\lambda}{24\mu}(x^2 - a^2)^2$

6. (*i*) $y = c_1e^{2x} + c_2e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$;
(*ii*) $y = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5x^4 + c_6x^5 + x^7/7!$.

Problems 35.4, page 1126

1. (*i*) and (*ii*) $\bar{y} = \frac{5}{18}x(1-x)$ **2.** $\bar{y} = \frac{x}{4}(5x-1)$ **4.** $\bar{y} = 0.58 + 0.27x$

5. (*i*) and (*ii*) $c_1 = 0.93, c_2 = -0.05$ **6.** 0.05 **7.** $c_1 = 3.27, c_2 = -2.69$

8. $y = \frac{1}{2}(5x^2 - 3x)$.

Problems 36.1, page 1134

1. $y(x) = 6x - 5 + \int_0^x (5 - 6x + 6t)\, y(t)\, dt$.

2. $y(x) = x - \sin x + e^x(x-1) + \int_0^x [\sin x - e^x(x-t)]\, y(t)\, dt$

3. $y(x) = \int_0^x t(t-x)\, y(t)\, dt + \frac{1}{2}x^2$

4. $y(x) + \int_0^x [1 + x - 2t + (x-t)e^{-t}]\, y(t)\, dt = \frac{x^5}{20} - \frac{5x^3}{6} + x - 3$

5. $y(x) = \cos x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{2}\int_0^x (x-t)^2\, y(t)\, dt$

6. $y(x) = \sin x - \frac{1}{2} + \int_0^x \left\{\frac{1}{2}x(x-t)^2 - 1\right\} y(t)\, dt$

7. $y(x) - \int_0^x \{4 - 6(x-t) + 2(x-t)^2 - \frac{1}{6}(x-t)^3\}\, y(t)\, dt = \frac{1}{4}\cos 2x - \frac{19}{12} + \frac{32}{3}x - \frac{85}{6}x^2 + \frac{20}{3}x^3$

8. $y''(x) - 2xy'(x) - 3y(x) = 0$; $y(0) = 1, y'(0) = 0$

9. $y''(x) - y(x) + 3\sin x = 0$; $y(0) = 3, y'(0) = 0$

10. $y'''(x) + 6y(x) = 0$; $y(0) = 4, y'(0) = -3, y''(0) = 2$

11. $y'''(x) - 3y''(x) + 4y'(x) + 2y(x) + e^{-x} = 0$; $y(0) = 1, y'(0) = 2, y''(0) = 3$.

Problems 36.2, page 1137

1. $y(x) = \int_0^1 G(x,t)\, ty(t)\, dt + \frac{1}{2}x(x-1)$, where $G(x,t) = x(1-t), (x < t)$ and $= t(1-x), x > t$

2. $y(x) = \int_0^1 G(x,t)\, y(t)\, dt + \frac{1}{6}(x^3 - 3x + 6)$, where $G(x,t) = x, x < t$ and $= t, x > t$

3. $u(x) = \int_0^1 G(x,t)\, e^t\, u(t)\, dt + \frac{1}{6}(x^3 + x)$, where $G(x,t) = x(1-2t), x < t$ and $= t(1-2x), x > t$

4. $G(x,t) = \frac{\sinh x \sinh(t-1)}{\sinh 1}, x < t$ and $= \frac{\sinh t \sinh(x-1)}{\sinh 1}, x > t$

5. $u(x) = \lambda \int_0^1 G(x,t)\, t\, .\, u(t)\, dt$, where $G(x,t) = \frac{1}{2}x\left(\frac{1}{t-t}\right), x < t$ and $= \frac{1}{2}\cdot t\left(\frac{1}{x-x}\right), x > t$

Problems 36.3, page 1141

5. $y(x) = \frac{1}{2}(\sin x + \sinh x)$ **6.** $y(x) = x^2 + \frac{1}{12}x^4$ **7.** $y(x) = 1$

8. $y(x) = \pm 6J_0(4x)$ **9.** $y(x) = J_1(2x) \quad (x > 0)$

10. $y(x) = \frac{1}{2} e^{-2x} (\cos x + 3 \sin x) - \frac{1}{2} e^{-x}$

11. $y(x) = (1 - x) e^{-x} + 1/2 \sin x$

12. $y(x) = 1 + x^2 + x^4/24.$

13. $y(x) = 1 + x^2/2$

14. $y(x) = 1/2$

15. $y(x) = \frac{3\sqrt{3}}{4\pi} x^{1/3} (3x + 2).$

Problems 36.4, page 1145

1. Has no eigen values and eigen functions

2. Eigen value $\lambda = 1/4$; eigen function is $y(x) = x^2 + 3x/2$

3. Eigen values $\lambda = 8/(\pi - 2)$; eigen function is $y(x) = \sin^2 (x)$

4. Eigen value $\lambda = 1/\pi$, $y = \sin x$

5. Has no eigen values or eigen functions

6. Eigen values are $\lambda = \pm 1/\pi$; eigen functions are $y(x) = \cos x + \sin x$, $y(x) = \cos x - \sin x$

7. $y(x) = x + \frac{\lambda [(6 - \lambda) x - 4]}{12 + \lambda^2}$

8. $y(x) = x + \frac{\lambda}{12 (1 - 2\lambda) - \lambda^2} [10 + (6 + \lambda) x]$

9. $y(x) = x + \frac{\lambda}{(1 - \lambda\pi)\left(1 - \frac{1}{2}\lambda\pi\right) + 4\lambda^2} \left\{2\lambda\pi + \frac{1}{2}\pi^2\left(1 - \frac{1}{2}\lambda\pi\right) + \pi(1 - 2\lambda\pi) \sin x\right\}$

10. $y(x) = 2x - \pi + \frac{\pi^2 \sin^2 x}{\pi - 1}$

11. $y(x) = \frac{2}{2 - \lambda} \sin x, \lambda \neq 2$

12. $y(x) = x + \frac{2\lambda\pi}{1 + 2\lambda^2\pi^2} (\lambda\pi x - 4\lambda\pi \sin x + \cos x)$

13. There is no solution to the integral equation when $\lambda = 3$

14. $\lambda_1 = 2, \lambda_2 = -2$; $y_1(x) = 1 - x$, $y_2(x) = 1 - 3x$

15. (*i*) When $F(x) = x$, solution is $y(x) = x + \lambda \left\{\frac{2\lambda\pi^2}{\lambda^2\pi^2 - 1} \sin x + \frac{2\pi}{\lambda^2\pi^2 - 1} \cos x\right\}$

(*ii*) When $F(x) = 1$, solution is $y(x) = 1$.

Problems 36.5, page 1148

1. $y(x) = 1 - \frac{3\lambda x}{2(3 + \lambda)} (\lambda \neq -3)$

2. $y(x) = \frac{\sin x}{1 + \lambda\pi}$ only if $|\lambda| < \frac{1}{\pi}$

3. $y(x) = \frac{4 + 2\lambda (2 - 3x)}{4 - \lambda^2} (\lambda \neq 2)$

4. $y = e^x$

5. $y(x) = 1$

6. $y = \sin x$

7. $y(x) = 2.$

Problems 37.1, page 1154

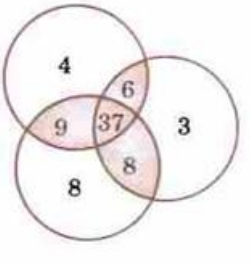
2. (*i*) *True,* since $\{a\}$ is a subset of the set $\{a, b, c\}$; (*ii*) and (*iii*) *False,* since the element a cannot be a subset of the set $\{a, b, c\}$; (*iv*) *True,* since the set $\{a, b\}$ is a subset of the set $\{a, b, c\}$; (*v*) *False,* since the set $\{a, b\}$ is not an element of the set $\{a, b, c\}$; (*vi*) *True,* since the null set ϕ is a subset of every set.

17. 20

18. 105

19. 136

20. Number of students not taking any of these courses is 71.



Problems 37.2, page 1160

1. (*a*) It is not true that Sam is a teacher and John is an honest boy ; (*b*) Sam is a teacher and John is not an honest boy ; (*c*) Sam is not a teacher iff John is an honest boy ; (*d*) If Sam is a teacher then John is not an honest boy.

2. (*a*) $(p \vee q) \Rightarrow r$ where $p \equiv I$ have no car, $q = I$ do not wear good dress, $r = I$ am not, a millionaire.

3.

p	q	$\sim q$	$p \Rightarrow q$	$p \Rightarrow q \wedge \sim q$
1	1	0	1	0
1	0	1	0	0
0	1	0	1	0
0	0	1	1	1

(*b*)

p	q	r	$p \Leftrightarrow q$	$r \vee q$	$(p \Leftrightarrow q) \wedge (r \vee q)$
1	1	1	1	1	1
1	1	0	1	1	0
1	0	1	0	1	0
1	0	0	0	0	0
0	1	1	0	1	0
0	1	0	0	1	0
0	0	1	1	1	1
0	0	0	1	0	0

7. (*i*) $T_p = [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31]$
$T_q = [1, 3, 9, 27, \ldots]$, $T_r = [1, 3, 9, 7]$
(*ii*) $T_r \leq T_q$

10. (*i*)

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

15. (*i*) Dual of $(p \wedge q) \vee r$ is $(p \wedge q) \vee r$ (*ii*) Dual of $(p \wedge q) \vee t$ is $(p \vee q) \wedge t$

Problems 37.3, page 1166

1. (*a*) $(\forall x \in A)(x + 2 < 10)$ (*b*) $(1\, x \in A)(x + 2 = 10)$

2. (*a*) $\forall x, (x^3 \neq x)$ (*b*) $\rightarrow x, (x + 5 \leq x)$
(*c*) None of the students are 26 or older (*d*) Some students do not live in the hostels.

3. $\forall x\, P(x)$ is false **4.** $\forall (x_1, x_2)\, Q(x_1 + x_2)Q$

5. $\forall (a, b)\, R(a + x = b)$

6. (*a*) $\forall x[Q(x) \rightarrow R(x)]$ (*b*) $\sim \forall x[Q(x) - R(x)]$, (*c*) $\exists x[Q(x) \wedge R(x)]$, (*d*) $\exists x\, [Q(x) \wedge \sim R(x)]$

8. $(\sim A \vee \sim A) \wedge (B \vee \sim A) \wedge (\sim A \vee C) \wedge (B \vee C)$

10. 1. $p \vee q$ (Premise), 2. $\sim p \rightarrow q$ (conditional equivalence)
3. $q \rightarrow s$ (Premise) 4. $\sim p \rightarrow s$ (2, 3 chain rule)
5. $p \rightarrow r$ (Premise) 6. $\sim s \rightarrow p$ (4, conditional equivalence)
7. $\sim s \rightarrow r$(5, 6 chain rule) 8. $s \vee r$ (7, conditional equivalence)

12. $(b\, x)\, R\, (x = \sqrt{Z})$ **13.** (*a*) Conclusion is not valid (*b*) Conclusion is not valid

Problems 37.4, page 1170

1.

x	y	z	$x \wedge y$	z'	$y \wedge z'$	$(x \wedge y) \vee (y \wedge z')$
0	0	0	0	1	0	0
0	0	1	0	0	0	0
0	1	0	0	1	1	1
1	0	0	0	1	0	0
1	1	0	1	1	1	1
1	0	1	0	0	0	0
0	1	1	0	0	0	0
1	1	1	1	0	0	1

2. $x \vee z' \wedge y = x \wedge y$

3. (*i*) $x' \vee y' \vee z'$ (*ii*) 0

14.

x_1	x_2	x_3	$x_1 \vee x_3$	x_3'	$x_2 \vee x_3'$	$x_1 \wedge (x_2 \vee x_3')$	P
0	0	1	1	0	0	0	0
0	1	0	0	1	1	0	0
0	1	0	0	1	1	0	0
1	0	0	1	1	1	1	1
1	1	0	1	1	1	1	1
0	1	1	1	0	1	0	0
1	0	1	1	0	0	0	0
1	1	1	1	0	1	1	1

Problems 37.5, page 1172

1. (*i*) 0 (*ii*) 0

2. (*i*) $(x \vee y \vee z) \wedge (x \vee y \vee z')$ (*ii*) $x \vee y \wedge (x \vee y') \wedge (x' \vee y)$

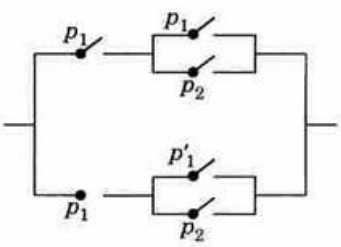
4. $(x \vee y \vee z) \wedge (x \vee y \vee z') \wedge (x \vee y' \vee z) \wedge (x \vee y' \vee z') \wedge (x \vee y' \vee z') \wedge (x' \vee y' \vee z) \wedge (x' \vee y' \vee z')$

5. $(x \wedge y \wedge z) \vee (x \wedge y \wedge z') \vee (x \wedge y' \wedge z) \vee (x \wedge y' \wedge z')$

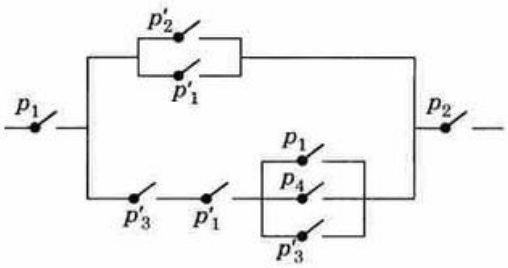
6. $F' = (x \wedge y \wedge z) \vee (x \wedge y' \wedge z) \vee (x' \wedge y \wedge z) \vee (x' \wedge y \wedge z') \vee (x' \wedge y' \wedge z)$

Problems 37.6, page 1174

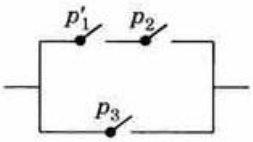
1. (*i*)



(*ii*)



2. (*i*) $p_1 \vee [p_2' \wedge (p_1 \vee p_2')]$ (*ii*) $[(p_1 \vee p_2) \vee (p_1 \vee p_3)] \wedge (p_1 \wedge p_2')$

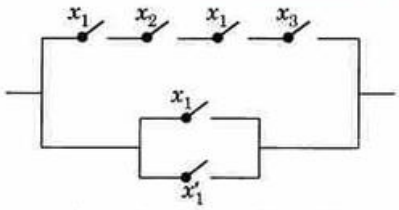


3. $x \wedge y$

4. $p_1' \wedge p_2 \vee p_3$;

6. (*a*) $x_1 \wedge x_2'$

(*b*)



(*c*) $(x_1 \vee x_2' \vee x_3) \wedge (x_1 \vee x_2' \vee x_3') \wedge (x_3 \vee x_2' \vee x_1) \wedge (x_3 \vee x_2' \vee x_1')$

Problems 37.7, page 1179

1. $F_2 = F_3$

2. F^c = [0.2 Ram, 0.7 Sham, 0.4 John, 0.3 Charu]

3. $F \cup G = [0.4\, x_1, 0.7\, x_2, 0.5\, x_3, 0.9\, x_4]$
$F \cap G = [0.3\, x_1, 0.6\, x_2, 0.1\, x_3, 0.8\, x_4]$

4. (*i*) Truth value of '*F* is not rich' is 0.2
(*ii*) Truth value of '*G* is not fat' is 0.4
(*iii*) Truth value of 'Mary is not beautiful' is 0.3

5. (*i*) $F \neq G$ (*ii*) *F* is not a subset of *G* ; *G* is not a subset of *F*.
(*iii*) $F^c = [1, 1, 1, 1, 0.9, 0.7, 0.5, 0.1, 0, 0]$
$F \cap G = [0, 0.1, 0.3, 0.5]$
$F \cup G = [0.1, 0.5, 0.9, 1, 0.9, 0.9, 1, 1]$

6. (*i*) Truth value of the cojunction of 'Latif and John are good players' is 0.6.
(*ii*) Truth value of the disjunction of 'Latif and John are good players' is 0.7.

8. Members and its degree of membership.

Problems 38.1, page 1186

1. (*i*) $\dfrac{d\phi}{dt} = \dfrac{d\phi}{\partial x^i} \cdot \dfrac{dx^i}{dt}$; (*ii*) $x^i x^i$

2. (*i*) $a_{11}(x^1)^2 + a_{22}(x^2)^2 + a_{33}(x^3)^2 + (a_{12} + a_{21})\, x^1x^2 + (a_{13} + a_{31})x^1x^3 + (a_{23} + a_{32})x^2x^3$
(*ii*) $g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 + 2g_{12}dx^1dx^2 + 2g_{23}dx^2dx^3 + 2g_{31}dx^3\,dx^1$.
(*iii*) $g_{11} = g_{12}\, g^{2p} + \ldots + g_{ln}\, g^{np}$

3. (*i*) δ^i_k ; (*ii*) δ^p_s.

6. (*i*) $\bar{A}_p{}^{qr} = \dfrac{\partial \bar{x}^q}{\partial x^j} \dfrac{\partial \bar{x}^r}{\partial x^k} \dfrac{\partial x^i}{\partial \bar{x}^p} A_i^{jk}$; (*ii*) $\bar{C}_{pq} = \dfrac{\partial x^m}{\partial \bar{x}^p} \dfrac{\partial x^n}{\partial \bar{x}^q} C_{mn}$

7. Yes, $A^{ij}{}_{kl}{}^m$, contravariant order 3, covariant order 2, Rank 5

9. (*a*) $2\rho \cos^2\phi - z \cos\phi + \rho^3 \sin^2\phi \cos^2\phi, -\rho^2 \sin 2\phi + \rho z \sin\phi + \rho^4 \sin\phi \cos^3\phi, \rho z \sin\phi$;
(*b*) $2r \sin^2\theta \cos^2\phi - r \sin\theta \cos\theta \cos\phi + r^3 \sin^4\theta \sin^2\phi \cos^2\phi + r^2 \sin\theta \cos^2\theta \sin\phi$;
$2r^2 \sin\theta \cos\theta \cos^2\phi - r^2 \cos^2\theta \cos\phi + r^4 \sin^3\theta \cos\theta \sin^2\phi \cos^2\phi - r^3 \sin^2\theta \cos\theta \sin\phi$;
$-2r^2 \sin^2\theta \sin\phi \cos\phi + r^2 \sin\theta \cos\theta \sin\phi + r^4 \sin^4\theta \sin\phi \cos^3\phi$

11. $(a \cos\phi + b \sin\phi) \sin\theta + c \cos\theta$; $\{(a \cos\phi + b \sin\phi) \cos\theta - c \sin\theta\}/r$; $(b \cos\phi - a \sin\phi)/r \sin\theta$.

Problems 38.2, page 1189

6. Rank = 1

Problems 36.3, page 1193

1. $g = 4, g^{11} = 2, g^{22} = 5, g^{33} = 1.5, g^{12} = 3, g^{23} = -2.5, g^{13} = -1.5,$

2. $g_{11} = 1, g_{22} = \rho^2, g_{33} = 1, g_{ij} = 0\ (i \neq j)$; $g^{11} = 1, g^{22} = \rho^{-2}, g^{33} = 1, g^{ij} = 0\ (i \neq j)$

3. $g = r^4 \sin^2\theta/(1 - r^2/R^2)$; $g^{11} = 1 - r^2/R^2, g^{22} = 1/r^2, g^{33} = (r \sin\theta)^{-2}, g^{ij} = 0\ (i \neq j)$.

Problems 38.4, page 1199

1. (*a*) $[ii, i] = \frac{1}{2}\partial g^{ij}/\partial x^i$, $[ii, k] = -\frac{1}{2}\partial g_{ii}/\partial x^k$,

$[ik, k] = [ki, k] = \frac{1}{2}\partial g_{kk}/\partial x^i$, $[ij, k] = 0$, when i, j, k are all different

(*b*) $\begin{Bmatrix} i \\ ii \end{Bmatrix} = \frac{1}{2} g^{ij} \frac{\partial g_{ii}}{\partial x^i}$, $\begin{Bmatrix} k \\ ii \end{Bmatrix} = -\frac{1}{2} g^{kk} \frac{\partial g_{ii}}{\partial x^k}$

$\begin{Bmatrix} k \\ ik \end{Bmatrix} = \begin{Bmatrix} k \\ ki \end{Bmatrix} = \frac{1}{2} g^{kk} \frac{\partial g_{kk}}{\partial x^i}$ (no summation over i or k)

$\begin{Bmatrix} k \\ ij \end{Bmatrix} = 0$, when i, j, k are all different

2. (*a*) All are zero

(*b*) $[21, 2] = \rho = [12, 2]$; $[22, 1] = \rho$, all others are zero

(*c*) $[21, 2] = r = [12, 2]$; $[31, 3] = r \sin^2\theta = [13, 3]$; $[32, 3]$ $= r^2 \sin\theta \cos\phi = [23, 3]$; $[22, 1] = -r$; $[33, 1] = -r \sin^2\theta$; $[33, 2] = -r^2 \sin\theta \cos\phi$; all others are zero

3. (*a*) All are zero

(*b*) $\begin{Bmatrix} 1 \\ 22 \end{Bmatrix} = -\rho$, $\begin{Bmatrix} 2 \\ 21 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 12 \end{Bmatrix} = \frac{1}{\rho}$, all others are zero

(*c*) $\begin{Bmatrix} 1 \\ 22 \end{Bmatrix} = -r$, $\begin{Bmatrix} 1 \\ 33 \end{Bmatrix} = -r \sin^2\theta$, $\begin{Bmatrix} 2 \\ 21 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 12 \end{Bmatrix} = \frac{1}{r}$,

$\begin{Bmatrix} 2 \\ 33 \end{Bmatrix} = -\sin\theta \cos\theta$, $\begin{Bmatrix} 3 \\ 31 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 13 \end{Bmatrix} = \frac{1}{r}$,

$\begin{Bmatrix} 3 \\ 32 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 23 \end{Bmatrix} = \cot\theta$, all others are zero

5. (*a*) $r^2 \sin\theta \cos\theta$; $r^2 \sin\theta \cos\theta$ (*b*) $-\sin\theta \cos\theta$; $\cot\theta$

6. (*a*) $-r \sin^2\theta$; $r^2 \sin\theta \cos\theta$ (*b*) $-r \sin^2\theta$; $\cot\theta$

8. (*a*) $u^{ij}{}_{,k} = \frac{\partial u^{ij}}{\partial x^k} + \begin{Bmatrix} i \\ ks \end{Bmatrix} u^{sj} + \begin{Bmatrix} i \\ ks \end{Bmatrix} u^{is}$; (*b*) $A^h{}_{ij,k} = \frac{\partial A^h{}_{ij}}{\partial x^k} - \begin{Bmatrix} s \\ ik \end{Bmatrix} A^h{}_{sj} + \begin{Bmatrix} h \\ ks \end{Bmatrix} A^s{}_{ij}$

10. $A^j{}_{k,q} B_n{}^{lm} + A^j{}_k B^{lm}{}_{n,q}$

11. (*a*) $\frac{1}{\rho}\left[\frac{\partial}{\partial p}(\rho A_\rho) + \frac{\partial}{\partial \phi}(A_\phi) + \frac{\partial}{\partial z}(\rho A_z)\right]$; (*b*) $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial r}(\sin\theta\, A_\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi}$

12. (*a*) $\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 v}{\partial \phi^2} + \frac{\partial^2 v}{\partial z^2} + \frac{1}{\rho}\frac{\partial v}{\partial \rho} = 0$ (*b*) $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v}{\partial \phi^2} + \frac{2}{r}\frac{\partial v}{\partial r} + \frac{\cot\theta}{r^2}\frac{\partial v}{\partial \theta} = 0.$

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Higher Engineering Mathematics

Dr. B.S. Grewal

The book provides a clear exposition of essential tools of applied mathematics from a modern point of view and meets complete requirements of engineering and computer science students. Every effort has been made to keep the presentation at once simple and lucid. It is written with the firm conviction that a good book is one that can be read with minimum guidance from the instructor. To achieve this, more than the usual number of solved examples, followed by properly graded problems have been given. Many of the examples and problems have been selected from recent papers of various university and other engineering examinations. Basic Concepts and Useful Information has been given in an Appendix. However, the subject matter has been set in eight main units:

- ***Algebra & Geometry*** : Solution of equations, Linear algebra: Deter-minants, Matrices, Vector algebra and Solid geometry.
- ***Calculus*** : Differential calculus, Partial differentiation, Integral calculus, Multiple integrals, Vector calculus.
- ***Series*** : Infinite series and Fourier series.
- ***Differential Equations*** : Differential equations of first order and their applications, Linear differential equations and their applications, Differential equations of different types, Series solution of differential equations and special functions, Partial differential equations and their applications.
- ***Complex Analysis*** : Complex numbers and functions, Calculus of complex functions.
- ***Transforms*** : Laplace transforms, Fourier transforms and Z-transforms.
- ***Numerical Techniques*** : Empirical Laws and Curve fitting, Statistical methods, Probability and Distributions, Sampling and Inference, Numerical methods, Finite differences and Interpolation, Difference equations, Numerical solution of Ordinary and Partial differential equations, Linear programming.
- ***Special Topics*** : Calculus of variations, Integral equations, Discrete mathematics, Tensors.

An exhaustive list of *'Objective Type of Questions'* has been given at the end of each chapter. *Standard Tables, Answers to Problems,* and a fairly comprehensive *Index* is given at the end.

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