Tribhuvan University
Institute of Sciences and Technology
**SCHOOL OF MATHEMATICAL SCIENCES**
**First Assessment 2079**

*Subject* **Multivariable Calculus for Data Science** — *Full Marks* 45
*Course No* **MSMT 554** — *Pass Marks*: 22.5
*Level* **MDS. /I Year /II Semester** — *Time* 2 hrs

*Candidates are required to give their answer in their own words as far as practicable.*

## Group A [5 ×3=15]

1. Define scalar triple product of three vectors and give its geometrical meaning. Using scalar triple product, verify that the vectors $\vec{u} = \vec{i} + 5\vec{j} - 2\vec{k}$, $\vec{v} = 3\vec{j} - \vec{j}$, and $\vec{w} = 5\vec{i} + 9\vec{j} - 4\vec{k}$ are coplanar. [1+2]

2. Find the parametric equations and symmetric equation for the lines through (2,1,0) and perpendicular to both the vectors $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$. [3]

3. Define curvature of a vector function $\vec{r} = \vec{r}(t)$. Find the curvature of the vector function $\vec{r}(t) = p\cos t\,\vec{i} + p\sin t\,\vec{j}$, where $p$ is constant. [1+2]

4. Find the limit, if it exists, or show that the limit does not exist:
   a) $f(x, y) = \dfrac{5y^4\cos 2x}{x^4 + y^4}$ b) $f(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}}$ [1.5+1.5]

5. Explain why the function $f(x, y) = \sqrt{x + e^{4y}}$ is differentiable at the given point (3, 0). Find the linearization $L(x, y)$ of the function $f(x, y)$ at that point. [1+2]

## Group B [5 ×6=30]

6. Prove the Parallelogram Law $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2|\vec{a}|^2 + 2|\vec{b}|^2$ for any two vectors $\vec{a}$ and $\vec{b}$. Give its geometric interpretation. Also if $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal, show that the vectors $\vec{a}$ and $\vec{b}$ must have the same length. [2+1+3]

**OR**

Derive a vector equation of a straight line
   a) through the given point $\vec{a}$ and parallel to the vector $\vec{b}$
   b) through two points $\vec{a}$ and $\vec{b}$. Also, find a vector equation for the line through the point (1, 0, 6) and perpendicular to the plane $x + 3y + z = 5$. [2+2+2]

7. Derive the expression for the derivative of scalar triple product of three vectors. Find the derivative of the scalar triple product of the vectors
   $t\vec{i} + t^2\vec{j} + t\vec{k}$, $(t+1)\vec{i} + (t+2)\vec{j} - 3t\vec{k}$ and $t^2\vec{i} + 2t\vec{j} + t\vec{k}$ at $t = 2$. [2+4]

8. Find the domain and range of the function $f(x, y) = \sqrt{16 - 4x^2 - y^2}$. Describe the graph of $f$. Sketch a contour map of this surface using level curves corresponding to $c$ = 1, 2, 3,4, 5. [1.5+1.5+1.5+1.5]

9. a) Let $f(x, y)$ be defined on an open disk $D$ that contains the point $(a, b)$. Prove that if the functions $f_{xy}$ and $f_{yx}$ are continuous on $D$, then $f_{xy}(a, b) = f_{yx}(a, b)$. [4]
   b) Show that $z = e^x \sin y$ satisfies the equation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$. [2]

**10. a)** Prove that if $x = x(t)$ and $y = y(t)$ are differentiable functions of $t$ and $z = f(x, y)$ is a differentiable function of $x$ and $y$, then $z = f(x(t), y(t))$ is a differentiable function of $t$ and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt},$$

where the ordinary derivatives are evaluated at $t$ and the partial derivatives are evaluated at $(x, y)$. [3]

**b)** Calculate $\partial z/\partial u$ and $\partial z/\partial v$ using the following functions:

$z = f(x, y) = 3x2 - 2xy + y2$, $x = x(u, v) = 3u + 2v$, $y = y(u, v) = 4u - v$. [3]

**OR**

**a)** Prove that if $f$ is a differentiable function of $x$ and $y$, then $f$ has a directional derivative in the direction of any unit vector $u = (a, b)$ and $D_u f(x, y) = f_x(x, y)a + f_y(x, y)b$. [3]

**b)** Find the direction for which the directional derivative of $f(x, y) = 3x^2 - 4xy + 2y^2$ at $(-2, 3)$ is a maximum. What is the maximum value? Find the maximum rate of change of $f(x, y) = \sqrt{x^2 + y^4}$ at $(-2, 3)$ and the direction in which this maximum rate of change occurs. [3]

***

Tribhuvan University
Institute of Sciences and Technology
**SCHOOL OF MATHEMATICAL SCIENCES**
**Second Assessment 2079**

*Subject:* **Multivariable Calculus for Data Science** *Full Marks:* 45
*Course No:* **MDS 554** *Pass Marks:* 22.5
*Level:* **MDS/I Year /II Semester** *Time:* 2 hrs

*Candidates are required to give their answer in their own words as far as practicable.*

**Attempt All Questions.**

**Group A [5 × 3 = 15]**

1. Find the normal vector and binormal vector of the space curve $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ where $x = t^2, y = t^2, z = t^3$ at point (1, 1, 1).

2. Evaluate $\iint_D (10x^2y^3 - 6)\, dA$, where $D$ is the region bounded by $x = -2y^2$ and $x = y^3$.

3. Evaluate $\iiint_E (12y - 8x)\, dV$ where $E$ is the region behind $y = 10 - 2z$ and in front of the region in the $xz$-plane bounded by $z = 2x$, $z = 5$ and $x = 0$.

$z = 5$ to $2x$
$y = 10$ to

4. If $\vec{F} = (2x + y)\,\vec{i} + (3y - x)\,\vec{j}$, evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $C$ is the curve in the $xy$-plane consisting of the straight lines from (0, 0) to (2, 0) and then to (3, 2).

5. If a closed surface $S$ enclosed a volume $V$ and $\vec{F} = x\,\vec{i} + 2y\,\vec{j} + 3z\,\vec{k}$, Using Gauss' theorem show that $\iint_S \vec{F} \cdot \vec{n}\, ds = 6V$.

$0 \le y \le 10 - 2z$
$\le z \le 2x$
$0 \le x \le 5/2$

**Group B [5 × 6 = 30]**

6. Find the maximum and minimum values of $f(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.

**OR**

The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

7. a) Use a double integral to determine the volume of the solid that is inside the cylinder $x^2 + y^2 = 16$, below $z = 2x^2 + 2y^2$ and above the $xy$-plane.

   b) Use a triple integral to determine the volume of the region below $z = 6 - x$, above $z = -\sqrt{4x^2 + 4y^2}$ inside the cylinder $x^2 + y^2 = 3$ with $x \le 0$.

   $-\sqrt{3 - x^2}$

8. a) Evaluate $\iint_R (x + 2y)\, dA$ where $R$ is the triangle with vertices (0, 3), (4, 1) and (2, 6) using the transformation $x = \frac{1}{2}(u - v), y = \frac{1}{4}(3u + v + 12)$ to $R$.

   b) Determine the surface area of the portion of $y = 2x^2 + 2z^2 - 7$ that is inside the cylinder $x^2 + z^2 = 4$.

9. Define line integral. Is the the vector field $\vec{F} = (x^2 - yz)\,\vec{i} + (y^2 - zx)\,\vec{j} + (z^2 - xy)\,\vec{k}$ a irrigational? Justify. Also find a scalar function ø such that $\vec{F} = \nabla ø$.

**OR**

State Green's theorem in the plane. Prove that the area enclosed by a simple closed curve $C$ is given by $= \frac{1}{2}\int_C (x\,dy - y\,dx)$. Verify Green's theorem in the plane for $\int_C (2xy - x^2)\,dx + (x + y^2)\,dy$ where C is the closed curve given by $y = x^2$, $x^2 = y$.

**10.** **State Stokes' theorem in a surface S. Show that in a plane, Green's theorem is a particular case of Stokes' theorem. Verify Stokes' theorem for the vector function $\vec{F} = (x^2 + y^2)\,\vec{i} - 2xy\,\vec{j}$ taken round the rectangle in the xy-plane bounded by $x = 0, x = a, y = 0, y = b$.**

***

MDS554-2079

Tribhuvan University
Institute of Sciences and Technology
2079

*Subject*: **Multivariable Calculus for Data Science** | *Full Marks*: 45
*Course No*: **MDS 554** | *Pass Marks*: 22.5
*Level*: **MDS / I Year / II Semester** | *Time*: 2 hrs

*Candidates are required to give their answer in their own words as far as practicable.*

Attempt **ALL** questions.

## Group A [5×3 =15]

1. Find the normal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ ,where $x = t^2$, $y = t^2, z = t^3$ at point (1, 1, 1).

2. Find the equation of the tangent plane to $z = x^2\cos(\pi y) - \frac{6}{xy^2}$ at point $(2,-1)$.

3. Find and classify all the critical points of the function: $f(x, y) = (y-2)x^2 - y^2$.

4. Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the $xy$-plane defined by $0 \le x \le 2$, $0 \le y \le 1$.

5. With the help of Gauss's divergence theorem, show that
$$\iint_S \vec{F}\cdot\hat{n}\,ds = \frac{4}{3}\pi(a+b+c)$$
where $\vec{F} = ax\vec{i} + by\vec{j} + cz\vec{k}$ and $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

## Group B [5 ×6 =30]

6. Establish the vector equation of a straight line through two points $\vec{a}$ and $\vec{b}$. Find the vector equation of the line through the point (2, 1,0) and perpendicular to both the vectors $\vec{k}$ -2$\vec{j}$ and $\vec{j}$+2$\vec{k}$. Also find the scalar and vector projections of $\vec{q} = \vec{i} - \vec{j} + \vec{k}$ onto $\vec{p} = \vec{i} + \vec{j} + \vec{k}$. [2+2+2]

7. Derive the expression for the derivative of vector triple product of three vectors. Find the derivative of the scalar triple product of the vectors $\vec{p} = (a\cos t, b\sin t, 0)$, $\vec{q} = (-a\sin t, b\cos t, t)$ and $\vec{r} = (1, 2, 3)$ at $t = 0$. [3+3]

8. Prove that if $x = x(t)$ and $y = y(t)$ are differentiable functions of $t$ and $z = f(x, y)$ is a differentiable function of $x$ and $y$, then $z = f(x(t), y(t))$ is a differentiable function of $t$ and $\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$,
where the ordinary derivatives are evaluated at $t$ and the partial derivatives are evaluated at $(x, y)$. Also, find the maximum rate of change of $f(s, t) = te^{st}$ at point (0, 2) in the direction at which this maximum rate of change occurs.

 [3+3]

OR

Calculate $\partial z/\partial s$ and $\partial z/\partial t$ using the following functions: $z = e^{x+2y}$, $x = s/t$, $y = t/s$.
Also Show that $u = e^{-x}\cos y - e^{-y}\cos x$ satisfies the Laplace equation $u_{xx} + u_{yy} = 0$.

[3+3]

9. Evaluate $\iint_R xy^3\, dA$ where $R$ is the region bounded by $xy = 1, xy = 3, y = 2, y = 6$ $x = \frac{v}{6u}$

$y = 2t$

using the transformation $x = \frac{y}{6u}$, $y = 2v$. Using polar coordinates, find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$. [3+3]

10. State Green's theorem in the plane and use it to find the area of the circle of radius 4 unit. Verify Green's theorem in the plane for $\int_C (2xy - x^2)\, dx + (x + y^2)\, dy$ where $C$ is the closed curve given by the line $y = x$ and parabola $x = y^2$.

[1+2+3]

**OR**

Find the equation of the tangent plane to the surface with parametric equation $x = u^2, y = v^2$, and $z = u + 2v$. Verify Stokes' theorem for the vector function

at point (1,1,0,1)

$\vec{F} = x\vec{i} + y\vec{j}$ around the square boundary $x = 0, y = 0, x = a, y = a$. [3+ 3]

***

$\sqrt{a^2+1} = \frac{a^2}{1}$