UNIT 5 **Reinforcement Learning**

We examine how an agent can learn from success and failure, from reward and punishment. We will study how agents can learn *what to do* in the absence of labeled examples of what to do.

Consider, for example, the problem of learning to play chess. A supervised learning agent needs to be told the correct move for each position it encounters, but such feedback is seldom available. In the absence of feedback from a teacher, an agent can learn a transition model for its own moves and can perhaps learn to predict the opponent's moves, but *without some feedback about what is good and what is bad, the agent will have no grounds for deciding which move to make.* The agent needs to know that something good has happened when it (accidentally) checkmates the opponent, and that something bad has happened when it is checkmated—or vice versa, if the game is suicide chess. This kind of feedback is called a **reward**, or **reinforcement**. In games like REINFORCEMENT chess, the reinforcement is received only at the end of the game. The task of **reinforcement learning** is to use observed rewards to learn an optimal (or nearly optimal) policy for the environment.

the agent has a complete model of the environment and knows the reward function, here we assume no prior knowledge of either.
In many complex domains, reinforcement learning is the only feasible way to train a program to perform at high levels. For example, in game playing, it is very hard for a human to provide accurate and consistent evaluations of large numbers of positions, which would be needed to train an evaluation function directly from examples. Instead, the program can be told when it has won or lost, and it can use this information to learn an evaluation function that gives reasonably accurate estimates of the probability of winning from any given position.

Reinforcement learning might be considered to encompass all of AI: an agent is placed in an environment and must learn to behave successfully therein. To keep the chapter manageable, we will concentrate on simple environments and simple agent designs. For the most part, we will assume a fully observable environment, so that the current state is supplied by each percept. On the other hand, we will assume that the agent does not know how the environment works or what its actions do, and we will allow for probabilistic action outcomes. Thus, the agent faces an unknown Markov decision process.

A **utility-based agent** learns a utility function on states and uses it to select actions that maximize the expected outcome utility.
A **Q-learning** agent learns an **action-utility function**, or **Q-function**, giving the expected utility of taking a given action in a given state.
A **reflex agent** learns a policy that maps directly from states to actions.

A utility-based agent must also have a model of the environment in order to make decisions, because it must know the states to which its actions will lead. For example, in order to make use of a backgammon evaluation function, a backgammon program must know what its legal moves are *and how they affect the board position*. Only in this way can it apply the utility function to the outcome states. A Q-learning agent, on the other hand, can compare the expected utilities for its available choices without needing to know their outcomes, so it does not need a model of the environment. On the other hand, because they do not know where their actions lead, Q-learning agents cannot look ahead; this can seriously restrict their ability to learn.

Passive learning, where the agent's policy is fixed and the task is to learn the utilities of states; this could also involve learning a model of the environment.

Active learning, where the agent must also learn what to do. The principal issue is **exploration**: an agent must experience as much as possible of its environment in order to learn how to behave in it.
An agent can use inductive learning to learn much faster from its experiences.

PASSIVE REINFORCEMENT LEARNING:
To keep things simple, we start with the case of a passive learning agent using a state-based representation in a fully observable environment. In passive learning, the agent's policy π is fixed: in state s, it always executes the action π(s). Its goal is simply to learn how good the policy is—that is, to learn the utility function $U^{\pi}(s)$.

The main difference is that the passive learning agent does not know the **transition model** P(s' |s,a), which specifies the probability of reaching state s' from state s after doing action a; nor does it know the **reward function** R(s), which specifies the reward for each state.

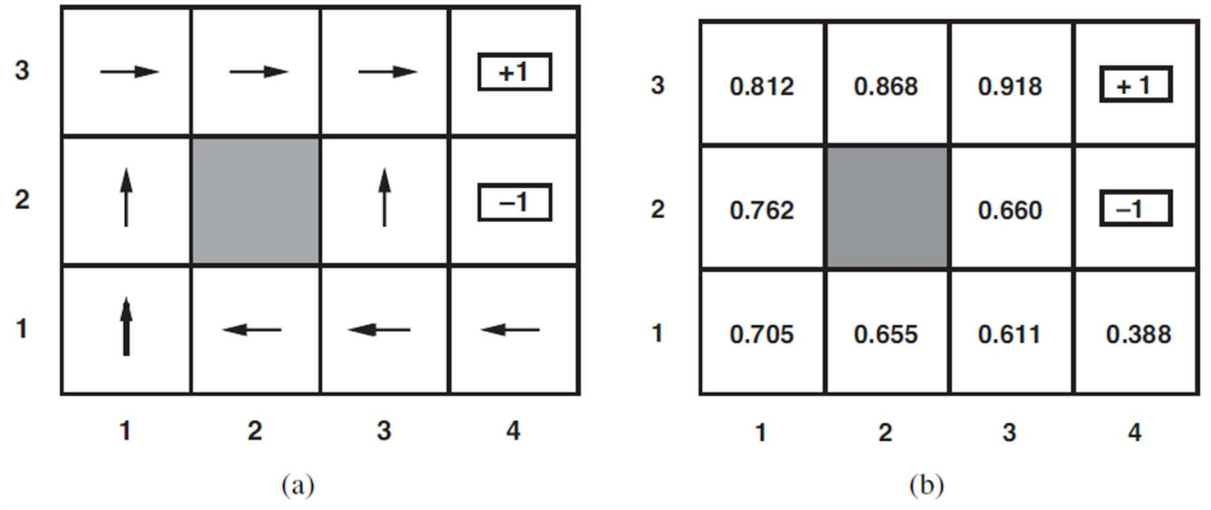


Figure 21.1 (a) A policy π for the 4×3 world; this policy happens to be optimal with rewards of $R(s) = -0.04$ in the nonterminal states and no discounting. (b) The utilities of the states in the 4×3 world, given policy π.

The agent executes a set of **trials** in TRIAL the environment using its policy π. In each trial, the agent starts in state (1,1) and experiences a sequence of state transitions until it reaches one of the terminal states, (4,2) or (4,3). Its percepts supply both the current state and the reward received in that state.

Typical trials might look like this:.
(1, 1)**-.04**->(1, 2)**-.04**->(1, 3)**-.04**->(1, 2)**-.04**->(1, 3)**-.04**->(2, 3)**-.04**->(3, 3)**-.04**->(4, 3)**+1**
(1, 1)**-.04**->(1, 2)**-.04**->(1, 3)**-.04**->(2, 3)**-.04**->(3, 3)**-.04**->(3, 2)**-.04**->(3, 3)**-.04**->(4, 3)**+1**
(1, 1)**-.04**->(2, 1)**-.04**->(3, 1)**-.04**->(3, 2)**-.04**->(4, 2)**-1** .

each state percept is subscripted with the reward received. The object is to use the information about rewards to learn the expected utility Uπ(s) associated with each nonterminal state s. The utility is defined to be the expected sum of (discounted) rewards obtained if policy π is followed.

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(S_t)\right]$$

where R(s) is the reward for a state, St (a random variable) is the state reached at time t when executing policy π, and S0 =s. We will include a **discount factor** γ.

Direct utility estimation

A simple method for **direct utility estimation** The idea is that the utility of a state is the expected total reward from that state onward (called the expected **reward-to-go**), and each trial provides a *sample* of this quantity for each state visited.
For example, the first trial in the set of three given earlier provides a sample total reward of 0.72 for state (1,1), two samples of 0.76 and 0.84 for (1,2), two samples of 0.80 and 0.88 for (1,3), and so on. Thus, at the end of each sequence, the algorithm calculates the observed reward-to-go for each state
and updates the estimated utility for that state accordingly, just by keeping a running average for each state in a table.

It is clear that direct utility estimation is just an instance of supervised learning where each example has the state as input and the observed reward-to-go as output. This means that we have reduced reinforcement learning to a standard inductive learning problem.
Learning techniques for those representations can be applied directly to the observed data.
Direct utility estimation succeeds in reducing the reinforcement learning problem to an inductive learning problem, about which much is known. Unfortunately, it misses a very important source of information, namely, the fact that the utilities of states are not independent! *The utility of each state equals its own reward plus the expected utility of its successor states.* That is, the utility values obey the Bellman equations for a fixed policy.

$$U^{\pi}(s) = R(s) + \gamma \sum_{s'} P(s' \mid s, \pi(s)) U^{\pi}(s') \ .$$

direct utility estimation misses opportunities for learning. For example, the second of the three trials given earlier reaches the state (3,2), which has not previously been visited. The next transition reaches (3,3), which is known from the first trial to have a high utility. The Bellman equation suggests immediately that (3,2) is also likely to have a high utility, because it leads to (3,3), but direct utility estimation learns nothing until the end of the trial. More broadly, we can view direct utility estimation
as searching for U in a hypothesis space that is much larger than it needs to be, in that it includes many functions that violate the Bellman equations. For this reason, the algorithm often converges very slowly.

function PASSIVE-ADP-AGENT(*percept*) **returns** an action
  inputs: *percept*, a percept indicating the current state s' and reward signal r'
  persistent: π, a fixed policy
    mdp, an MDP with model P, rewards R, discount γ
    U, a table of utilities, initially empty
    N_{sa}, a table of frequencies for state–action pairs, initially zero
    $N_{s'|sa}$, a table of outcome frequencies given state–action pairs, initially zero
    s, a, the previous state and action, initially null

  if s' is new **then** $U[s'] \leftarrow r'$; $R[s'] \leftarrow r'$
  if s is not null **then**
    increment $N_{sa}[s, a]$ and $N_{s'|sa}[s', s, a]$
    for each t such that $N_{s'|sa}[t, s, a]$ is nonzero **do**
      $P(t \mid s, a) \leftarrow N_{s'|sa}[t, s, a] \;/\; N_{sa}[s, a]$
  $U \leftarrow$ POLICY-EVALUATION(π, U, mdp)
  if s'.TERMINAL? **then** $s, a \leftarrow$ null **else** $s, a \leftarrow s', \pi[s']$
  return a

Adaptive dynamic programming:

An **adaptive dynamic programming** (or ADP) agent takes advantage of the constraints among the utilities of states by learning the transition model that connects them and solving the corresponding Markov decision process using a dynamic programming method.

For a passive learning agent, this means plugging the learned transition model P(s'|s,π(s)) and the observed rewards R(s) into the Bellman equations to calculate the utilities of the states. we can adopt the approach of **modified policy iteration**, using a simplified value iteration process to update the utility estimates after each change to the learned model. Because the model usually changes only slightly with each observation, the value iteration process can use the previous utility estimates as initial values and should converge quite quickly.

The process of learning the model itself is easy, because the environment is fully observable. This means that we have a supervised learning task where the input is a state–action pair and the output is the resulting state. In the simplest case, we can represent the transition model as a table of probabilities. We keep track of how often each action outcome occurs and estimate the transition probability P(s'|s,a) from the frequency with which s' is reached when executing a in s. For example, in the three trials, Right is executed three times in (1,3) and two out of three times the resulting state is (2,3), so P((2, 3) | (1, 3), Right) is estimated to be 2/3.

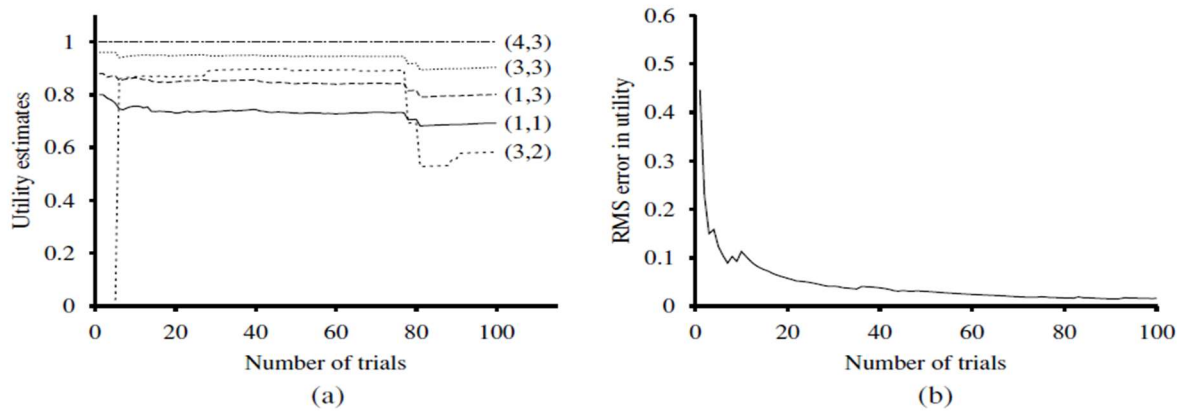


Figure 21.3 The passive ADP learning curves for the 4×3 world, given the optimal policy shown in Figure 21.1. (a) The utility estimates for a selected subset of states, as a function of the number of trials. Notice the large changes occurring around the 78th trial—this is the first time that the agent falls into the -1 terminal state at (4,2). (b) The root-mean-square error (see Appendix A) in the estimate for $U(1, 1)$, averaged over 20 runs of 100 trials each.

There are two mathematical approaches:
The first approach, **Bayesian reinforcement learning**, assumes a prior probability P(h) for each hypothesis h about what the true model is; the posterior probability P(h | **e**) is obtained in the usual way by Bayes' rule given the observations to date. Then, if the agent has decided to stop learning, the optimal policy is the one that gives the highest expected utility.Let u^{π}_{h} be the expected utility, averaged over all possible start states, obtained by executing policy π in model h. Then we have

$$\pi^* = \operatorname*{argmax}_{\pi} \sum_{h} P(h \mid \mathbf{e}) u_h^{\pi}$$

The second approach, derived from **robust control theory**, allows for a *set* of possible models H and defines an optimal robust policy as one that gives the best outcome in the *worst case* over H

$$\pi^* = \operatorname*{argmax}_{\pi} \min_{h} u_h^{\pi}$$

the set H will be the set of models that exceed some likelihood threshold on P(h | **e**), so the robust and Bayesian approaches are related.

Temporal-difference learning
Solving the underlying MDP as in the preceding section is not the only way to bring the Bellman equations to bear on the learning problem. Another way is to use the observed transitions to adjust the utilities of the observed states so that they agree with the constraint equations. Consider, for example, the transition from (1,3) to (2,3) in the second trial. Suppose that, as a result of the first trial, the utility estimates are Uπ(1, 3)=0.84 and Uπ(2, 3)=0.92. Now, if this transition occurred all the time, we would expect the utilities to obey the equation

$$U^{\pi}(1,3) = -0.04 + U^{\pi}(2,3)$$

so Uπ(1, 3) would be 0.88. Thus, its current estimate of 0.84 might be a little low and should be increased. More generally, when a transition occurs from state s to state s', we apply the following update to $U^{\pi}(s)$.

$$U^{\pi}(s) \leftarrow U^{\pi}(s) + \alpha(R(s) + \gamma U^{\pi}(s') - U^{\pi}(s))$$

α is the **learning rate** parameter. Because this update rule uses the difference in utilities TEMPORAL- between successive states, it is often called the **temporal-difference.**
All temporal-difference methods work by adjusting the utility estimates towards the ideal equilibrium that holds locally when the utility estimates are correct.

First, notice that the update involves only the observed successor s', whereas the actual equilibrium conditions involve all possible next states. One might think that this causes an improperly large change in $U^{\pi}(s)$ when a very rare transition occurs; but, in fact, because rare transitions occur only rarely, the *average value* of $U^{\pi}(s)$ will converge to the correct value.

Furthermore, if we change α from a fixed parameter to a function that decreases as the number of times a state has been visited increases, then $U^{\pi}(s)$ itself will converge to the correct value. The performance of the passive TD agent on the 4×3 world. It does not learn quite as fast as the ADP agent and shows much higher variability, but it is much simpler and requires much less computation per observation. Notice that *TD does not need a transition model to perform its updates.* The environment supplies the connection between neighboring states in the form of observed transitions.

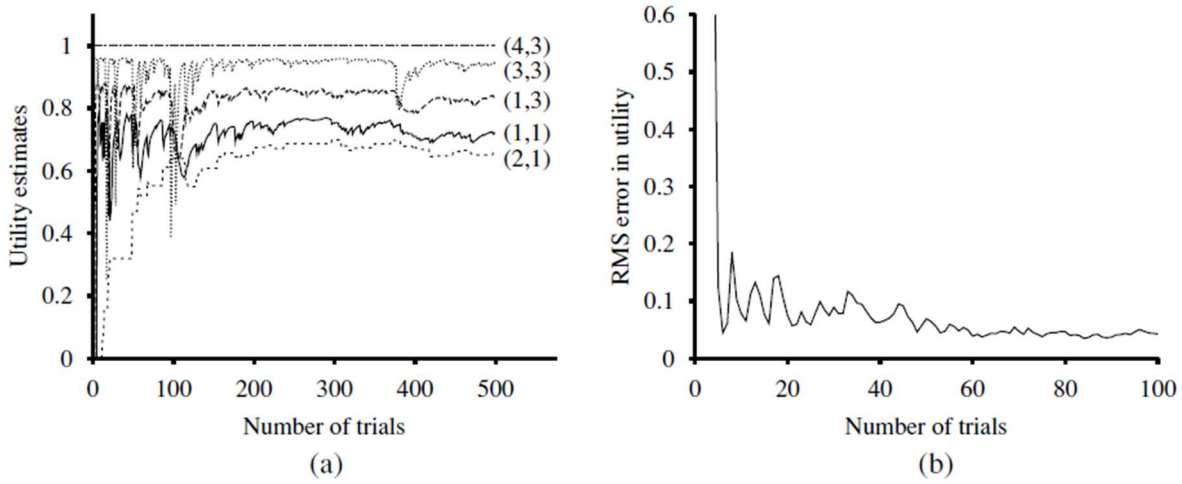
function PASSIVE-TD-AGENT(*percept*) **returns** an action
  inputs: *percept*, a percept indicating the current state s' and reward signal r'
  persistent: π, a fixed policy
    U, a table of utilities, initially empty
    N_s, a table of frequencies for states, initially zero
    s, a, r, the previous state, action, and reward, initially null

  if s' is new **then** $U[s'] \leftarrow r'$
  if s is not null **then**
    increment $N_s[s]$
    $U[s] \leftarrow U[s] + \alpha(N_s[s])(r + \gamma\, U[s'] - U[s])$
  if s'.TERMINAL? **then** $s, a, r \leftarrow$ null **else** $s, a, r \leftarrow s', \pi[s'], r'$
  return a

The ADP approach and the TD approach are actually closely related. Both try to make local adjustments to the utility estimates in order to make each state "agree" with its successors. One difference is that TD adjusts a state to agree with its *observed* successor, whereas ADP adjusts the state to agree with *all* of the successors that might occur, weighted by their probabilities. This difference disappears when the effects of TD adjustments are averaged over a large number of transitions, because the frequency of each successor in the set of transitions is approximately proportional to its probability. A more important difference is that whereas TD makes a single adjustment per observed transition, ADP makes as many as it needs to restore consistency between the utility estimates U and the environment model P. Although the observed transition makes only a

local change in P, its effects might need to be propagated throughout U. Thus, TD can be viewed as a crude but efficient first approximation to ADP.

Each adjustment made by ADP could be seen, from the TD point of view, as a result of a "pseudoexperience" generated by simulating the current environment model. It is possible to extend the TD approach to use an environment model to generate several pseudoexperiences—transitions that the TD agent can imagine *might* happen, given its current model. For each observed transition, the TD agent can generate a large number of imaginary transitions. In this way, the resulting utility estimates will approximate more and more closely those of ADP—of course, at the expense of increased computation time.



ACTIVE REINFORCEMENT LEARNING:
A passive learning agent has a fixed policy that determines its behavior. An active agent must decide what actions to take. Let us begin with the adaptive dynamic programming agent and consider how it must be modified to handle this new freedom.

First, the agent will need to learn a complete model with outcome probabilities for all actions, rather than just the model for the fixed policy. The simple learning mechanism used by PASSIVE-ADP-AGENT will do just fine for this. Next, we need to take into account the fact that the agent has a choice of actions. The utilities it needs to learn are those defined by the *optimal* policy; they obey the Bellman equations

$$U(s) = R(s) + \gamma \max_a \sum_{s'} P(s' \mid s, a) U(s')$$

These equations can be solved to obtain the utility function U using the value iteration or policy iteration algorithms. The final issue is what to do at each step. Having obtained a utility function U that is optimal for the learned model, the agent can extract an optimal action by one-step look-ahead to maximize the expected utility; alternatively, if it uses policy iteration, the optimal policy is already available, so it should simply execute the action the optimal policy recommends.

Exploration
The results of one sequence of trials for an ADP agent that follows the recommendation of the optimal policy for the learned model at each step. The agent *does not* learn the true utilities or the true optimal policy! What happens instead is that, in the 39th trial, it finds a policy that reaches the +1 reward along the lower route via (2,1), (3,1), (3,2), and (3,3).

After experimenting with minor variations, from the 276th trial onward it sticks to that policy, never learning the utilities of the other states and never finding the optimal route via (1,2), (1,3), and (2,3). We call this agent the **greedy agent**.

Repeated experiments show that the greedy agent *very seldom* converges to the optimal policy for this environment and sometimes converges to really horrendous policies.

What the greedy agent has overlooked is that actions do more than provide rewards according to the current learned model; they also contribute to learning the true model by affecting the percepts that are received. By improving the model, the agent will receive greater rewards in

the future. An agent therefore must make a tradeoff between **exploitation** to maximize its reward—as reflected in its current utility estimates—and **exploration** to maximize its long-term well-being. Pure exploitation risks getting stuck in a rut. Pure exploration to improve one's knowledge is of no use if one never puts that knowledge into practice. In the real world, one constantly has to decide between continuing in a comfortable existence and striking out into the unknown in the hopes of discovering a new and better life. With greater understanding, less exploration is necessary.

Q-learning, which learns an action-utility representation instead of learning utilities. We will use the notation Q(s, a) to denote the value of doing action a in state s. Q-values are directly related to utility values as follows:

$$U(s) = \max_a Q(s, a)$$

Q-functions may seem like just another way of storing utility information, but they have a very important property: *a TD agent that learns a Q-function does not need a model of the form* P(s'|s,a)*, either for learning or for action selection.* For this reason, Q-learning is called a **model-free** method. As with utilities, we can write a constraint equation that must hold at equilibrium when the Q-values are correct:

$$Q(s, a) = R(s) + \gamma \sum_{s'} P(s' \mid s, a) \max_{a'} Q(s', a')$$

As in the ADP learning agent, we can use this equation directly as an update equation for an iteration process that calculates exact Q-values, given an estimated model, require that a model also be learned, because the equation uses P(s' | s, a). The temporal-difference approach, on the other hand, requires no model of state transitions—all it needs are the Q values. The update equation for TD Q-learning is

$$Q(s, a) \leftarrow Q(s, a) + \alpha(R(s) + \gamma \max_{a'} Q(s', a') - Q(s, a))$$

Notice that it uses exactly the same exploration function f as that used by the exploratory ADP agent—hence the need to keep statistics on actions taken (the table N). If a simpler exploration policy is used—say, acting randomly on some fraction of steps, where the fraction decreases over time—then we can dispense with the statistics.

Q-learning has a close relative called **SARSA** (for State-Action-Reward-State-Action). The update rule for SARSA is very similar to Equation

$$Q(s, a) \leftarrow Q(s, a) + \alpha(R(s) + \gamma\, Q(s', a') - Q(s, a))$$

where a' is the action *actually taken* in state s'. The rule is applied at the end of each s, a, r, s', a' quintuplet

whereas Q-learning backs up the *best* Q-value from the state reached in the observed transition, SARSA waits until an action is actually taken and backs up the Q-value for that action. Now, for a greedy agent that always takes the action with best Q-value, the two algorithms are identical. When exploration is happening, however, they differ significantly. Because Q-learning uses the best Q-value, it pays no attention to the actual policy being followed—it is an **off-policy** learning algorithm, whereas SARSA is an **on-policy** algorithm. Q-learning is more

flexible than SARSA, in the sense that a Q-learning agent can learn how to behave well even when guided by a random or adversarial exploration policy.

Both Q-learning and SARSA learn the optimal policy for the 4×3 world, but do so at a much slower rate than the ADP agent. This is because the local updates do not enforce consistency among all the Q-values via the model. The comparison raises a general question: is it better to learn a model and a utility function or to learn an action-utility function with no model? In other words, what is the best way to represent the agent function? This is an issue at the foundations of artificial intelligence. As we stated in Chapter 1, one of the key historical characteristics of much of AI research is its adherence to the **knowledge-based** approach. This amounts to an assumption that the best way to represent the agent function is to build a representation of some aspects of the environment in which the agent is situated.