PHY2053 General Physics I

Section 584771

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Exam 3 (Chapters 12 – 14, 17)

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Name: SOLUTIONS

MULTIPLE CHOICE QUESTIONS

1. Imagine some planet X existed, with double the mass of the Earth and triple the radius. What would the gravitational acceleration be at the surface of X?

   (a) 2.18 m/s^2
   (b) 4.36 m/s^2
   (c) 6.53 m/s^2
   (d) 9.8 m/s^2

2. What is the radius of the Earth's orbit around the Sun? Note that $M_{sun} = 1.99 \times 10^{30}$ kg and 1 yr $= 3.15 \times 10^7$ s.

   (a) 1.5×10^{11} m
   (b) 2.12×10^{11} m
   (c) 3×10^{11} m
   (d) 4.2×10^{11} m

3. A spring oscillates at some frequency f. If the spring constant doubled, and the acceleration of gravity halved (which would happen if it was moved to another planet), what would the resulting frequency of oscillations be?

   (a) $f/\sqrt{2}$
   (b) $\sqrt{2}f$
   (c) $2f$
   (d) $2\pi f$

4. A spring with a force constant of 120 N/m is anchored to the ceiling and hangs vertically. A 500g mass hangs at rest at the end of the spring. How far must the spring stretch to reach equilibrium?

   (a) 2cm
   (b) 4cm
   (c) 20cm
   (d) 40cm

5. A 200 N/m spring oscillates with an amplitude of 3.5cm. If a 0.75kg mass is attached to the end of the spring, what is the maximum speed of the mass?

   (a) 0.57 m/s
   (b) 1.75 m/s
   (c) 5.25 m/s
   (d) 7.87 m/s

6. Say there exists a wave with a frequency f and a wavelength λ, moving at a speed v. If the wavelength were doubled, what would the speed of the wave be?

   (a) $v/2$
   (b) v
   (c) $2v$
   (d) $4v$

7. A wave, with wavelength λ, passes into a different medium, where the wave speed doubles. In the new medium, the wavelength is:

(a) $\lambda/2$
(b) λ
(c) 2λ
(d) 4λ

8. A man in a boat drops 5m from peak to trough of the waves he is riding. What is the amplitude of the water waves?

(a) 2.5m
(b) 5m
(c) 7.5m
(d) 10m

9. What volume does 2.5×10^{24} particles of an ideal gas occupy at 300K and 3×10^6 Pa?

(a) 0.0035 m^3
(b) 0.0066 m^3
(c) 0.0079 m^3
(d) 0.012 m^3

$$PV = Nk_BT \Rightarrow V = \frac{Nk_BT}{P} = \frac{(2.5\times10^{24})(1.38\times10^{-23})(300)}{(3\times10^6)}$$
$$= 0.0035m^3$$

10. What is the rms speed of an atomic helium gas at 500K? Note that $m_{He} = 6.65 \times 10^{-27}$ kg.

(a) 1019 m/s
(b) 1248 m/s
(c) 1441 m/s
(d) 1764 m/s

$$V_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23})(500)}{(6.65\times10^{-27})}}$$
$$= 1764 \text{ m/s}$$

(c) • WRONG r, 3/7

(b) • $h = r - R_E$ Given 1/7

(a) • $v = \frac{2\pi r}{T}$, CORRECT T, WRONG r 3/6

FREE-RESPONSE PROBLEMS

1. The international space station passes overhead every 90 minutes in the sky.

6pt (a) ✗ What is the orbital speed of the space station?

7pt (b) ✗ What is the altitude of the space station's orbit?

7pt (c) ✗ What is the acceleration of the space station?

$T_{ORB} = 90\text{ min}$
$= 5400\text{ s}$

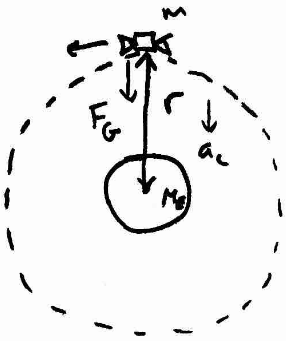
(a) $v_{ORB} = \frac{2\pi r}{T_{ORB}}$

$= \frac{2\pi(6.65\times10^6)}{(5400)}$

$= \boxed{7{,}738\ \text{m/s}}$

(ROUGHLY 7.7 km/s)

(b)



$F_G = ma_c$

$\Rightarrow G\frac{\cancel{m}M_E}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$

$= \left(\frac{2\pi r}{T}\right)^2$

$\Rightarrow \frac{GM_E}{r} = \frac{4\pi^2 r^2}{T^2} \Rightarrow r^3 = \frac{GM_E}{4\pi^2}T^2$

$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{4\pi^2}(5400)^2}$

$= 6.65\times10^6\ \text{m}$ ← (NOT FINAL ANSWER!)

ALTITUDE: $h = r - R_E$

$= 6.65\times10^6 - 6.4\times10^6$

$= \boxed{2.5\times10^5\ \text{m}}$

c) $a = a_c = \frac{v^2}{r} = \frac{(7738)^2}{(6.65\times10^6)} = \boxed{9\ \text{m/s}^2}$

(r, NOT h)

(d) $\cdot$ $V_{MAX} = 12.5$, NOT $12.5 + 0.56$. 5/6

(c) $\omega = 11.18 s^{-1}$, BUT ~~...~~ INCORRECT T, 4/5

3. A 150N/m spring oscillates with a 1.2kg mass attached to it.

5pt a) ~~a.~~ What is the period of the mass' oscillation?

5pt b) ~~b.~~ If the mass were originally released from rest when the spring was stretched by 5cm, what is the maximum speed of the mass?

4 ~~5~~pt c) ~~c.~~ What is the amplitude of the oscillations?

6pt d) ~~d.~~ If, after some time, the mass was struck while at equilibrium such that it gained a momentum of 15 Ns, what would the new amplitude of the oscillations be?

a) $\omega = \sqrt{\frac{k}{m}}$ $(\omega = 2\pi f)$ $\Rightarrow$ $f_{sp} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $(T = \frac{1}{f})$

$\Rightarrow T_{sp} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(1.2)}{(150)}} = \boxed{0.56s}$

b) $E_i = \cancel{K_i}^{0} + U_i = \frac{1}{2}kx_i^2 = \frac{1}{2}(150)(0.05)^2 = 0.188 J$

(SINCE ENERGY IS CONSERVED)

$E = K_{MAX}$ ($U = 0$ WHEN K IS MAXIMUM)

$= \frac{1}{2}mV_{MAX}^2$

$\Rightarrow V_{MAX} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.188)}{(1.2)}} = \boxed{0.56 m/s}$

c) AMPLITUDE OCCURS WHEN $K = 0$. THE MASS STARTED AT $x = 5cm$ WITH $K = 0$, SO THE AMPLITUDE IS JUST 5cm.

$\boxed{A = 5cm}$

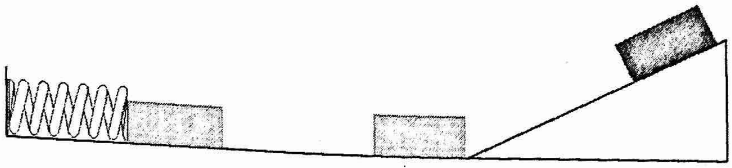
d) $\Delta p = m\Delta V \Rightarrow \Delta V = \frac{\Delta p}{M} = \frac{15}{1.2} = 12.5 m/s$ (THE MASS GAINS 12.5 m/s AT EQUILIBRIUM)

$\Rightarrow V_{MAX} = 0.56 m/s + 12.5 m/s = 13.06 m/s$ (NEW MAX SPEED)

$K_{MAX} = U_{MAX} \Rightarrow \frac{1}{2}mV_{MAX}^2 = \frac{1}{2}kA^2$

$\Rightarrow A = \sqrt{\frac{m}{k}} V_{MAX} = \sqrt{\frac{1.2}{150}}(13.06)$ 7 $= \boxed{1.17m}$ (NEW AMPLITUDE AT NEW V_{MAX})

(b). FIND $\mathcal{U}_{spr} = \frac{1}{2}(100)(0.075)^2$ 2/7



4. A spring with constant 100 N/m is compressed by 7.5cm. At the end of the spring is a 55g mass (unattached to the spring) which is fired horizontally by the spring, as shown in the figure above. Assume that the ramp is always frictionless.

6 a) ~~X~~ At what point does the mass lose contact with the spring?

7 b) ~~X~~ At what speed does the mass lose contact with the spring?

7 c) ~~X~~ If the horizontal surface is frictionless, to what height does the mass reach?

a) AT THE EQUILIBRIUM POSITION OF THE SPRING, WHICH IS AT ITS NATURAL LENGTH WHEN HORIZONTAL. THIS IS THE POINT WHEN THE SPEED OF THE BLOCK IS A MAXIMUM, & THE SPRING WILL SLOW DOWN PAST THIS POINT.

b) $U_{MAX} = K_{MAX}$

$\Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv_{MAX}^2$ (POSITION IS AN AMPLITUDE INITIALLY, AND MASS LOSES CONTACT WITH SPRING WHEN V IS MAXIMUM.)

$$\Rightarrow v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{(100)}{(0.055)}}(0.075) = \boxed{3.20\ m/s}$$

c) ENERGY IS CONSERVED ALONG THE HORIZONTAL SURFACE AND UP THE RAMP, SO WHATEVER KINETIC ENERGY THE MASS HAS ALONG THE HORIZONTAL SURFACE IS TURNED INTO GRAVITATIONAL POTENTIAL ENERGY UP THE RAMP:

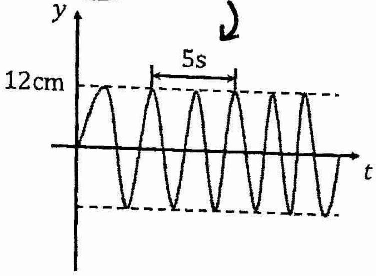
$$K_i = U_{g,f} \Rightarrow \frac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh \Rightarrow h = \frac{v_i^2}{2g} = \frac{(3.20)^2}{2(9.8)} = \boxed{0.52m}$$

(b) • f = 5 ; 1/5

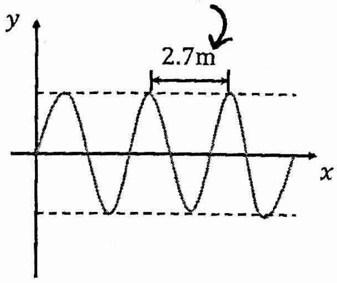
• f = 1/5 ; 3/5

(c) • A = 24 cm, 3/5

2T



λ



5. Based on the graphs above, find the following characteristics of the wave:

5 a) ~~1.~~ The wavelength

5 b) ~~2.~~ The frequency

5 c) ~~3.~~ The amplitude

5 d) ~~4.~~ The wavespeed

a) WAVELENGTH IS DISTANCE BETWEEN PEAKS, WHICH CAN BE FOUND ON THE y vs. x GRAPH:

$$\boxed{\lambda = 2.7\text{m}}$$

b) THE FREQUENCY IS $\frac{1}{T}$, AND THE PERIOD (T) CAN BE FOUND ON THE y vs. t GRAPH AS THE TIME BETWEEN PEAKS. NOTICE THAT THE INDICATED TIME, 5s, IS THE TIME BETWEEN 3 PEAKS, NOT 2, WHICH REPRESENTS TWO CYCLES, SO 5s IS TWICE THE PERIOD, OR

$$2T = 5\text{s} \Rightarrow T = 2.5\text{s}$$

$$\Rightarrow f = \frac{1}{T} = \frac{1}{2.5} = \boxed{0.4\ \text{Hz}}$$

c) THE AMPLITUDE IS THE MAXIMUM y-DISPLACEMENT, WHICH IS $\boxed{12\text{cm.}}$

d) $V = \lambda f = (2.7)(0.4) = \boxed{1.08\ \text{m/s}}$

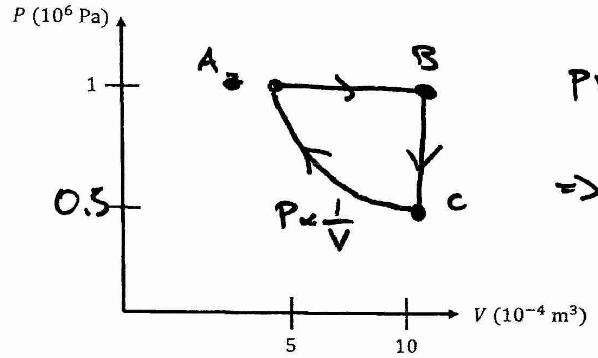
5. 3.4 mol of an ideal gas begins with an initial pressure of 1×10^6 Pa and an initial volume of 5×10^{-4} m^3.

5 a) If the gas is expanded isobarically to 1×10^{-3} m^3, use the ideal gas law to derive an equation to find the new temperature.

5 b) What is the new temperature after the compression?

5 c) If the pressure of the gas is decreased isochorically to the original temperature, what is the final pressure of the gas?

5 d) If the gas is then compressed isothermally to its initial state (its initial value of P and V), fill out the following pressure vs. volume diagram with the complete process of the gas. *Hint: the isothermal line is not a straight line; check the ideal gas law to find its shape.*



$PV = \boxed{Nk_BT}$ CONSTANT FOR ISOTHERMAL

$\Rightarrow P = \frac{\text{CONSTANT}}{V}$

a) ISOBARIC $\Rightarrow$ CONSTANT P

$PV = Nk_BT \Rightarrow \frac{P}{Nk_B} = \frac{T}{V} \Rightarrow \boxed{\frac{T_1}{V_1} = \frac{T_2}{V_2}}$

b) $T_2 = \frac{V_2}{V_1} T_1 = \frac{(1\times10^{-3})}{(5\times10^{-4})}(17.7) = \boxed{35.4\text{K}}$

$P_1V_1 = \underset{nR}{Nk_B}T_1 \Rightarrow T_1 = \frac{P_1V_1}{\underset{nR}{Nk_B}} = \frac{(1\times10^6)(5\times10^{-4})}{(3.4)(8.314)} = 17.7\text{ K}$

c) ISOCHORIC $\Rightarrow$ CONST V

$PV = Nk_BT \Rightarrow \frac{V}{Nk_B} = \frac{T}{P} \Rightarrow \frac{T_1}{P_1} = \frac{T_2}{P_2}$

$\Rightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{(17.7)}{(35.4)}(1\times10^6) = 0.5\times10^6$ Pa

d) See ABOVE.

FORMULA SHEET

- Constants:

$$g = 9.8\text{m/s}^2$$

- Vectors:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$
$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

- Kinematics:

$$\Delta x = v_0t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum\vec{F} = m\vec{a}$$
$$W = mg$$
$$f_{s,max} = \mu_s N$$
$$f_k = \mu_k N$$

- Work & Energy:

$$W = \vec{F}\cdot\Delta\vec{x} \quad \text{or} \quad W = \int\vec{F}\cdot d\vec{x}$$
$$W_{tot} = \Delta K$$
$$W_{cons} = -\Delta U$$
$$K = \frac{1}{2}mv^2$$
$$U_g = mgy$$
$$K_i + U_i + W_{nc} = K_f + U_f$$
$$\vec{F} = -\vec{\nabla}U$$
$$\vec{\nabla}f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

- Momentum & Collisions:

$$\vec{p} = m\vec{v}$$
$$\sum\vec{F} = \frac{d\vec{p}}{dt}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$\vec{v}_{1i} - \vec{v}_{2i} = \vec{v}_{2f} - \vec{v}_{1f}$$

- Rotational Mechanics

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\tau = rF\sin\theta$$

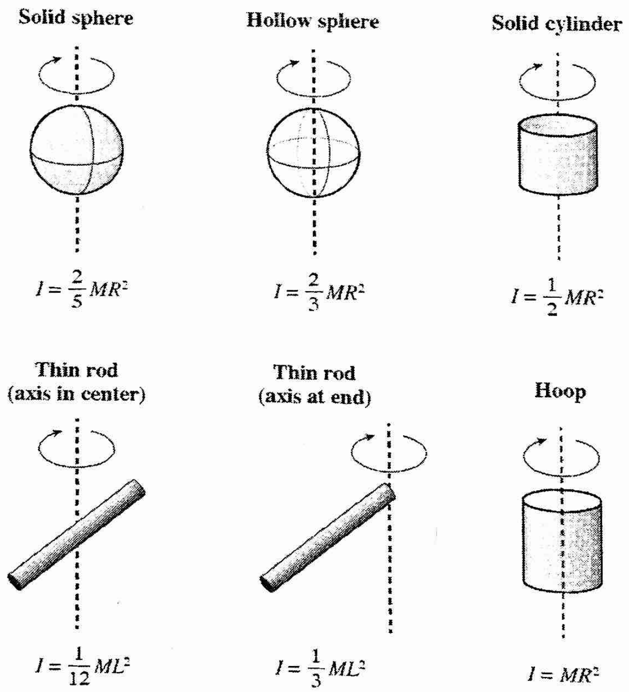
$$\sum \tau = I\alpha \quad \text{or} \quad \sum \tau = \frac{dL}{dt}$$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$L = I\omega \quad \text{or} \quad L = rp$$

$$I = \int r^2 dm$$

$$I_{new} = I_{cm} + md^2$$



- Gravity:

$$M_{Earth} = 5.97 \times 10^{24} \text{ kg}$$
$$M_{Sun} = 1.99 \times 10^{30} \text{ kg}$$
$$R_{Earth} = 6400 \text{ km}$$
$$G = 6.67 \times 10^{-11} \ \frac{\text{N}m^2}{\text{kg}^2}$$
$$F_G = G\frac{mM}{r^2}$$
$$a_G = G\frac{M}{r^2}$$
$$U_G = -G\frac{mM}{r}$$
$$v_{esc} = \sqrt{\frac{2GM}{R}}$$
$$v_{orb} = r\omega_{orb} = \frac{2\pi r}{T_{orb}}$$

- Oscillations:

$$F_{sp} = kx$$
$$U_{sp} = \frac{1}{2}kx^2$$
$$\omega_{sp} = \sqrt{\frac{k}{m}}$$
$$\omega_{pend} = \sqrt{\frac{g}{l}}$$
$$f = 1/T$$
$$\omega = 2\pi f$$

- Waves:

$$v = \lambda f$$
$$f_{beat} = |f_1 - f_2|$$

- Ideal Gases:

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$
$$R = 8.314 \text{ J/mol}\cdot\text{K}$$
$$N_A = 6.022 \times 10^{23} \text{ particles/mol}$$
$$K_{av} = \frac{3}{2}k_B T$$
$$PV = Nk_B T$$
$$v_{rms} = \sqrt{(v^2)_{av}}$$
$$T_K = T_{^\circ C} + 273$$