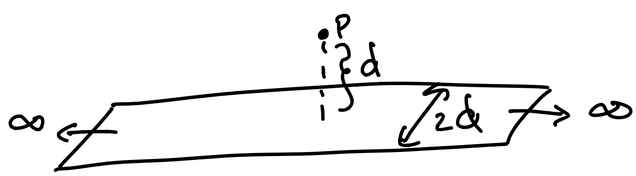
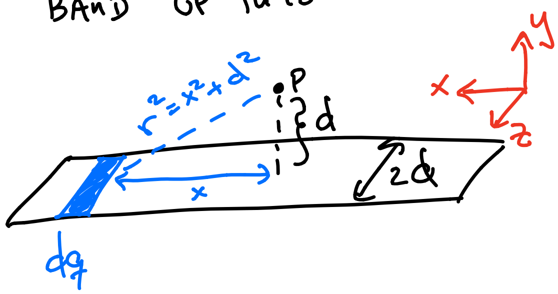
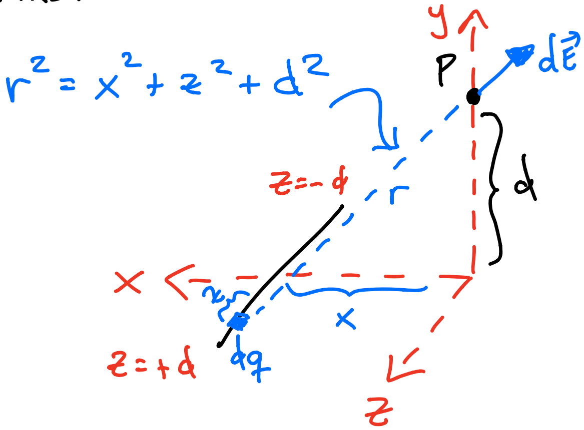


What is $\vec{E}$ at point P?

Break band up into thin slices:

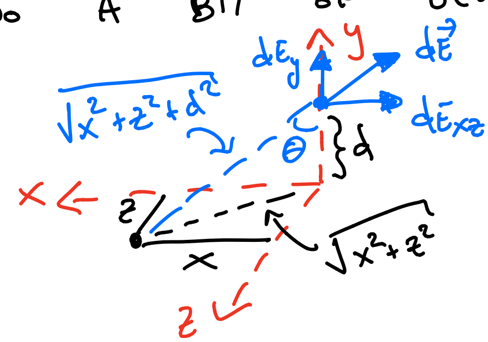


We don't know the electric field dE at P until we find the electric field due to a thin strip first. Consider the thin strip to be a line of charge:



$$dE = k\frac{dq}{r^2} = k\frac{dq}{(x^2+z^2+d^2)}$$

To isolate the y-component of $d\vec{E}$, we need to do a bit of geometry:



$$dE_y = dE\cos\theta = dE\cdot\frac{d}{\sqrt{x^2+z^2+d^2}}$$

$$\Rightarrow dE_y = \frac{kd}{(x^2+z^2+d^2)^{3/2}}\, dq$$

Convert the integral over dq into one over dz:

$$dq = \lambda\, dz$$

$$\Rightarrow dE_y = \frac{kd\lambda}{(x^2+z^2+d^2)^{3/2}}\, dz$$

$$\Rightarrow E_y = kd\lambda \int_{-d}^{d} \frac{dz}{(x^2+z^2+d^2)^{3/2}}$$

Note: $$\int \frac{dx}{(x^2+a^2+b^2)^{3/2}} = \frac{x}{(a^2+b^2)\sqrt{x^2+a^2+b^2}}$$

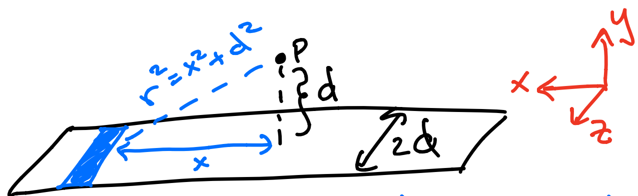
$$\Rightarrow E_y = kd\lambda \cdot \left. \frac{z}{(x^2+d^2)\sqrt{z^2+x^2+d^2}} \right|_{-d}^{d}$$

$$= 2kd\lambda \frac{d}{(x^2+d^2)\sqrt{x^2+2d^2}}$$

Replacing $\lambda = q/2d$,

$$E_y = \frac{kqd}{(x^2+d^2)\sqrt{x^2+2d^2}}$$

NOW, INTEGRATING THIS SLIVER ALONG THE BAND:



$dq \rightsquigarrow dq = \sigma\, dA = \sigma \cdot 2d \cdot dx$

$$dE_y = \frac{kd}{(x^2+d^2)\sqrt{x^2+2d^2}}\, dq = \frac{2k\sigma d^2}{(x^2+d^2)\sqrt{x^2+2d^2}}\, dx$$

$$\Rightarrow E_y = 2k\sigma d^2 \int_{-\infty}^{\infty} \frac{dx}{(x^2+d^2)\sqrt{x^2+2d^2}}$$

<u>NOTE:</u> $$\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)\sqrt{x^2+2a^2}} = \frac{\pi}{2a^2}$$

$$\Rightarrow E_y = \cancel{2}k\sigma \cancel{d^2} \cdot \frac{\pi}{\cancel{2}\cancel{d^2}} = \pi k \sigma$$

OR, SUBSTITUTING $k = \frac{1}{4\pi\varepsilon_0}$,

$$\boxed{E_y = \frac{\sigma}{4\varepsilon_0}}$$