PHYS2350 General Physics I/Lab

Section EV1

Prof. Douglas H. Laurence

Exam 1 (Chapters 2 – 4)

September 28, 2017

Name: Solutions

ID: ______________________

Instructions: This exam is composed of **5 multiple choice questions** and 4 free-response problems, of which you **only answer 3. Do not answer all 4.** I will only grade 3 problems, and if you answer all 4, I will grade the first 3 problems.

Answer each free response problem on a separate sheet of paper. Work your solution out on scratch paper first, and then neatly write your solution for submission. **Do not include your scratch paper with your submission.** Since you won't be turning in your scratch paper, make sure all necessary information is included in your submission.

Each multiple choice question is worth 5 points, and each free-response problem is worth 25 points, so there will be 25 points from the multiple choice questions and 75 points from the free-response problems, totalling 100 points.

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

1) For the vectors $\vec{A} = -3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 5\hat{k}$, which of the following statements is true?

a) $\vec{A}$ and $\vec{B}$ are parallel

b) $\vec{A}$ and $\vec{B}$ are perpendicular

c) $\vec{A}$ and $\vec{B}$ are neither parallel nor perpendicular

d) There isn't enough information to be sure of any of the above statements

First off, recognize that option (d) is definitely incorrect: the vectors are completely defined, so we should definitely be able to figure out the angle between them. There are several ways to approach this problem, but the easiest is to simply test if one of the first two statements are true using the dot and cross products. The dot product will tell us if two vectors are perpendicular (because it would be zero), and the cross product will tell us if two vectors are parallel (because it would be zero). The dot product is easier to perform than the cross product, so we'll start with that:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = (-3)(1) + (4)(2) + (-1)(5) = -3 + 8 - 5 = 0$$

So the vectors are perpendicular, making option (b) the correct answer.

2) A ball is thrown straight up into the air with a speed of 7m/s. What is the velocity of the ball after 1s? Consider up as the positive direction.

a) 2.8m/s

b) 0m/s

c) -2.8m/s

d) -5.2m/s

We know the initial velocity of the ball, the acceleration of the ball (assumed to be the gravitational acceleration, g), and we know the time; what we're looking for is the final velocity, so we can use the following equation to solve the problem:

$$v = v_0 + at$$

The problem declares upward to be the positive direction, so $v_0 = +7\text{m/s}$, $a = -9.8\text{m/s}^2$, and $t = 1\text{s}$. Plugging these in, we find:

$$v = (7) + (-9.8)(1) = 7 - 9.8 = -2.8\text{m/s}$$

So, the answer is option (c). The negative sign simply means that the object is moving downwards after 1s.

3) You're driving your car at 15m/s when the driver in front of you slams on their brakes. If you have to stop your car in a distance of 20m, what is the smallest acceleration that would prevent you from hitting the car?

a) 5.63m/s^2

b) 0.38m/s^2

c) 11.25m/s^2

d) 7.23m/s^2

We're given the initial velocity and the distance, so we still need one more variable to solve a kinematic problem. Nothing is mentioned explicitly, but we know that we need the car to come to a stop within this distance, so the final velocity must be zero. Choosing the forward direction to be positive, this means that $v_0 = +15\text{m/s}$, $v = 0$, and $\Delta x = +20\text{m}$. It also means that the acceleration should be negative, in order to decrease the velocity. Using the following equation, we can solve for the acceleration:

$$\underbrace{v^2}_{=0} = v_0^2 + 2a\Delta x = 0 \qquad \Rightarrow \qquad a = \frac{-v_0^2}{2\Delta x} = \frac{-(15)^2}{2(20)} = -5.63\text{m/s}^2$$

Remember that a is a vector, so the negative sign simply indicates the direction. Unlike in the previous problem, where some answers *did* have the proper sign, all answer choices here are positive. Sometimes there will be an ambiguity like this, but you always have to choose the *most correct* answer choice. In this case, the problem is simply asking for the magnitude of the acceleration, which is just 5.63m/s, making option (a) the correct choice.

4) An elevator can carry a maximum of 650kg. If the maximum tension supported by the cables is 10,000N, what is the largest acceleration the elevator can have at maximum weight?

a) 5.58m/s^2

b) 8.42m/s^2

c) 14.58m/s^2

d) 25.18m/s^2

There are two forces acting on the elevator: the tension in the cables, pointing upwards, and the weight of the elevator (and its content), pointing downwards. There is also an acceleration of the elevator (our unknown). The problem doesn't specify in which direction the elevator accelerates, but let's analyze what Newton's second law claims about the tension for an acceleration in either direction. Remember: always choose the positive direction to be in the direction of the acceleration. If the acceleration is downward, the tension is negative and the weight is positive, and Newton's second law is:

$$W - T = ma \qquad \Rightarrow \qquad T = W - ma$$

This means that an acceleration can only *decrease* the tension in the cables, so this isn't the right direction for the acceleration; the acceleration must be up in order to hit the maximum tension in the cables. With up being positive, the tension is now positive and the weight is now negative, and Newton's second law states:

$$T - W = ma \qquad \Rightarrow \qquad a = \frac{T - W}{ma} = \frac{10{,}000 - (650)(9.8)}{650} = 5.58\text{m/s}^2$$

So, the correct answer is (a). Notice one thing, though: the problem didn't say anything about the mass of the elevator, so I assumed it was weightless. If nothing is said or implied, then I can't assume anything else.

5) Two boxes are placed on a table: Box A is placed directly onto the table and Box B is placed on top of Box A. If a pushing force is applied on Box A, and the two boxes move forward with a constant acceleration, which of the following statements is true? Assume all surfaces have friction.

a) Box A and Box B both experience static friction

b) Box A and Box B both experience kinetic friction

c) Box A experiences static friction and Box B experiences kinetic friction

d) Box A experiences kinetic friction and Box B experiences static friction

If both boxes are moving with a constant acceleration, we know two things automatically. First, since box A is definitely moving, it feels a kinetic friction due to sliding across the floor. Second, since box B is definitely accelerating, a force must be propelling it. According to Newton's first law, box B wants to remain at rest and not accelerate, but that would require box A to slide out from under it. Static friction is going to act to oppose this sliding, and carry box B along with box A. So, box A experiences kinetic friction and box B experiences static friction, making option (d) the correct one.

FREE-RESPONSE PROBLEMS

1) For the vectors $\vec{A} = 2\hat{i} - 3\hat{j}$ and $\vec{B} = -\hat{i} + 4\hat{k}$, and the scalars $c = 3$ and $d = -2$. Calculate the following quantities:

a) $\vec{F} = c\vec{A} - d\vec{B}$

b) $\vec{G} = c\left(\vec{A} \times \vec{B}\right)$

c) $\vec{H} = \left(\vec{A} \cdot \vec{B}\right)\vec{A}$

Each of these operations are calculable using algebra, and the vectors are already given in their algebraic expressions. (As opposed to being drawn as arrows, which would be their geometric representation.)

a) When multiplying vectors by scalars, all we have to do is multiply each of the vectors components by the scalar:

$$c\vec{A} = (cA_x)\hat{i} + (cA_y)\hat{j} + (cA_z)\hat{k} = (3)(2)\hat{i} + (3)(-3)\hat{j} + (2)(0)\hat{k} = 6\hat{i} - 9\hat{j}$$

$$d\vec{B} = (dB_x)\hat{i} + (dB_y)\hat{j} + (dB_z)\hat{k} = (-2)(-1)\hat{i} + (-2)(0)\hat{j} + (-2)(4)\hat{k} = 2\hat{i} - 8\hat{k}$$

Then, subtracting vectors simply means subtracting their components, so:

$$\vec{F} = c\vec{A} - d\vec{B} = (cA_x - dB_x)\hat{i} + (cA_y - dB_y)\hat{j} + (cA_z - dB_z)\hat{k} = (6-2)\hat{i} + (-9-0)\hat{j} + (0-(-8))\hat{k} = \boxed{4\hat{i} - 9\hat{j} + 8\hat{k}}$$

b) First, we'll do the cross product, then we'll multiply the result by the scalar c. To do the cross product, treat it like regular scalar multiplication, in the sense that the multiplication distrbutes:

$$\vec{A} \times \vec{B} = (2\hat{i} - 3\hat{j}) \times (-\hat{i} + 4\hat{k}) = (2)(-1)(\hat{i} \times \hat{i}) + (2)(4)(\hat{i} \times \hat{k}) + (-3)(-1)(\hat{j} \times \hat{i}) + (-3)(4)(\hat{j} \times \hat{k})$$

Now all we need to do is evaluate the four cross products between the unit vectors using the cyclic permutations provided in the formula sheet. First, $\hat{i} \times \hat{i} = 0$ because $\hat{i}$ is parallel with $\hat{i}$, and the cross product between parallel vectors is always zero. Using the cyclic permutations, we see that $\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{i} = -\hat{k}$, and $\hat{j} \times \hat{k} = +\hat{i}$. So,

$$\vec{A} \times \vec{B} = (8)(-\hat{j}) + (3)(-\hat{k}) + (-12)(\hat{i}) = -12\hat{i} - 8\hat{j} - 3\hat{k}$$

Thus, multiplying by c,

$$\vec{G} = c\left(\vec{A} \times \vec{B}\right) = (3)(-12)\hat{i} + (3)(-8)\hat{j} + (3)(-3)\hat{k} = \boxed{-36\hat{i} - 24\hat{j} - 9\hat{k}}$$

c) First, we'll take the dot product:

$$\vec{A} \cdot \vec{B} = (2)(-1) + (-3)(0) + (0)(4) = -2$$

Notice that the result of the dot product is a scalar, so the operation of $(\vec{A} \cdot \vec{B})\vec{A}$ is the multiplication of a scalar and a vector, so:

$$\left(\vec{A} \cdot \vec{B}\right)\vec{A} = (-2)(2)\hat{i} + (-2)(-3)\hat{j} = \boxed{-4\hat{i} + 6\hat{j}}$$

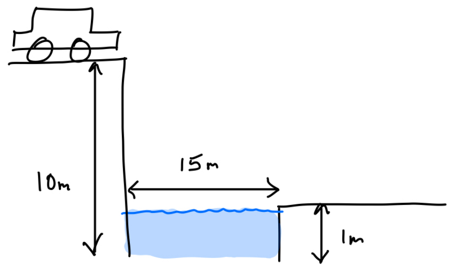


Figure for Problem 2

2) A stunt for a film requires a driver to jump a car across a river, as shown in the figure above.

a) What is the minimum speed the car must be going to safely clear the gap?

b) How fast would the car be moving when it lands on the other bank?

c) If the stunt driver were to perform the same stunt, but this time leave the upper bank at a 30^o incline, at the same speed found in part (a), would the driver still make it across the river?

a) To clear the gap, the car must be moving at a speed large enough to cover the width of the gap, 15m, in however long it takes the car to drop from the upper bank to the lower bank, which is a distance of 10m - 1m = 9m. So, we nee to solve for time using the vertical direction, and then solve for speed using the horizontal direction. In the vertical direction, we know that the initial speed is $v_{0y} = 0$, since the car is initially moving horizontally. We also know that $\Delta y = 0$ and $a_y = 9.8\text{m/s}^2$ (i.e. gravitational acceleration). Note that since the car is moving entirely downward, I've chosen that to be the positive direction. Solving for time:

$$\Delta y = \underbrace{v_{0y}t}_{=0} + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(9)}{9.8}} = 1.36\text{s}$$

So, the car has 1.36s to cross a 15m gap, which means that the speed in the horizontal direction, i.e. the car's initial speed, must be:

$$v_0 = v_{0x} = \frac{\Delta x}{t} = \frac{15}{1.36} = \boxed{11.0\text{m/s}}$$

b) When the driver reaches the other bank, he will still be going, in the horizontal direction, at 11.0m/s. However, the driver will have also gained speed in the vertical direction due to his drop. All the values (initial speed, acceleration, and time) are the same in the vertical direction as in part (a), so we can just go ahead and solve for the final vertical velocity:

$$v_y = \underbrace{v_{0y}}_{=0} + a_y t = a_y t = (9.8)(1.36) = 13.3\text{m/s}^2$$

Knowing the final speed in both the horizontal and vertical directions means we can find the (total) final speed by using the Pythagorean theorem:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(11.0)^2 + (13.3)^2} = \boxed{17.3\text{m/s}}$$

c) If the driver left the upper bank at a 30^o incline, his initial velocity would no longer be in the horizontal direction, but would be at an angle. If his initial speed was that of part (a), then we can break this new initial velocity up into its x- and y-components:

$$v_{0x} = v_0 \cos\theta = (11.0)\cos(30) = 9.53\text{m/s} \quad \text{and} \quad v_{0y} = v_0 \sin\theta = (11.0)\sin(30) = 5.5\text{m/s}$$

To find out if the driver makes it, we have to figure out how long he's in the air for, and then using that time, find out how far he travels horizontally. If this range is greater than 15m, the distance of the gap, he successfully crosses. If it's less, he doesn't. To begin, we'll need to find the time up to the peak of the trajectory. The peak of a trajectory is always described by $v_y = 0$, so

$$\underbrace{v_y}_{=0} = v_{0y} - gt = 0 \qquad \Rightarrow \qquad t_{up} = \frac{v_{0y}}{g} = \frac{5.5}{9.8} = 0.56\text{s}$$

Now that we know how long he takes to travel up, we need to find how long it takes for him to fall. This will *not* be the same as the amount of time it takes to peak; this is only true when the trajectory is symmetric, meaning the initial and final heights are the same, which is not true in this case. What we need to do is figure out how far he has to fall and use the fact that his initial speed (from the peak going down) is zero to calculate the time of the drop. Of course, this means that we need to know how high up the peak is. Luckily, we can solve for this too:

$$\underbrace{v_y^2}_{=0} = v_{0y}^2 - 2g\Delta y = 0 \qquad \Rightarrow \qquad \Delta y = \frac{v_0^2}{2g} = \frac{(5.5)^2}{2(9.8)} = 1.54\text{m}$$

Note that I still used $v_y = 0$ and $v_{0y} = 5.5\text{m/s}$, because I was considering the motion from the start to the peak. Now that we know the peak is 1.54m above the upper bank, and the upper bank is 9m above the lower bank, the peak must be 1.54m + 9m = 10.54m above the lower bank. This is how far the car is going to drop. So, the time to drop is:

$$\Delta y = \underbrace{v_{0y}t}_{=0} + \frac{1}{2}a_y t^2 \qquad \Rightarrow \qquad t_{down} = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(10.54)}{9.8}} = 1.47\text{s}$$

Note that in this problem I've used down as the positive direction, so I had to switch the sign on the acceleration. You could keep the upward direction positive, but then you'd have to make $\Delta y = -10.54\text{m}$. Now that we know the time up and the time down, the total time is $t = 0.56\text{s} + 1.47\text{s} = 2.03\text{s}$. How far does the car move horizontally during that time?

$$\Delta x = v_{0x}t = (9.53)(2.03) = \boxed{19.3\text{m} > 15\text{m}}$$

So, yes, the driver does make it across. Notice that I used $v_{0x} = 9.53\text{m/s}$, which is what we found the initial horizontal velocity to be when we considered the driver launching at an angle, *not* what we solved for in part (a).

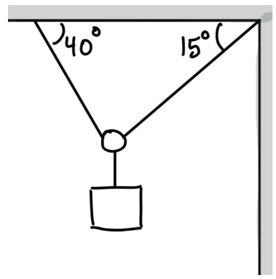


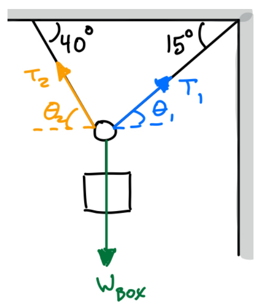
Figure for Problem 3

3) A 10 kg box hangs from a ring as shown in the above figure. Two ropes are anchored to the ceiling which support the ring.

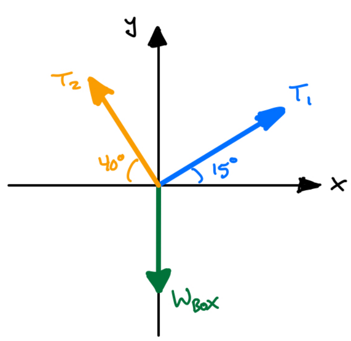
a) Draw a free body diagram for the ring. *Make sure to include any appropriate angles.*

b) What is the tension in each rope supporting the ring?

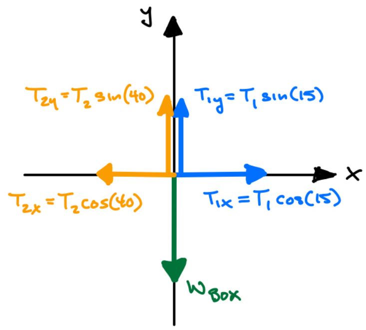
a) Before drawing the free body diagram, I'll draw the forces on top of the figure.



The box is going to put a force on the ring equal to its weight, the rope to the right is going to put a tension force on the ring which I called T_1, and the rope to the left is going to put a tension force on the ring which I called T_2. There are no other forces acting on the ring because 1) we are not told that there are any forces acting on this ring specifically, 2) we've included the weight and the tension forces, which we know should be there, and 3) Newton's second law can be satisfied by these three forces (since the ring isn't moving and these forces *could* cancel out). These are the three categories of forces on a free body diagram that I talked about in class. The last thing to do before drawing the free body diagram is to figure out the angles θ_1 and θ_2 that I indicated. Notice that the orange and blue dashed lines are both horizontal, meaning they are parallel to the ceiling that the ropes are anchored to. Since "rope 1" intersects both the ceiling and the blue line, θ_1 and the 15^o angle are alternate interior angles, so $\theta_1 = 15^o$. The same argument is made for $theta_2 = 40^o$. With this information, the free body diagram can be drawn:



b) To solve for the tension in each rope, we need to break the forces T_1 and T_2 down into their components. Both the angles 15^o and 40^o are made to the x-axis, so both the T_1 and T_2 x-components "get the cosine. Thus, our free body diagram looks like the following figure with its forces given by components:



Now that all the forces lie on axes, we can write down Newton's second law in the x- and y-directions. Since the ring isn't moving, we know that all the forces must balance, so the forces to the left equal the forces to the right, and the forces up equal the forces down:

$$T_2 \cos(40) = T_1 \cos(15) \quad \text{and} \quad T_2 \sin(40) + T_1 \sin(15) = W_{box}$$

We have two equations and two unknowns, T_1 and T_2, so we can solve the problem without any more information. Using the equation in the x-direction, we see that:

$$T_2 \cos(40) = T_1 \cos(15) \quad \Rightarrow \quad T_2 = T_1 \frac{\cos(15)}{\cos(40)}$$

Plugging this in for T_2 in the equation in the y-direction:

$$W_{box} = T_2 \sin(40) + T_1 \sin(15) = \left(T_1 \frac{\cos(15)}{\cos(40)} \right) \sin(40) + T_1 \sin(15)$$

Notice that there's a $\sin(40)$ in the numerator associated and a $\cos(40)$ in the denominator of the first term on the right-hand-side of the above equation, which we can say is equal to $\tan(40)$. So, our equation becomes:

$$W_{box} = T_1 \cos(15) \tan(40) + T_1 \sin(15) = T_1 (\cos(15) \tan(40) + \sin(15))$$

$$\Rightarrow \quad T_1 = \frac{W_{box}}{\cos(15) \tan(40) + \sin(15)} = \frac{(10)(9.8)}{\cos(15) \tan(40) + \sin(15)} = \boxed{91.6\text{N}}$$

Now that we know T_1, we can plug it into our equation for T_2 (the one we solved using our equation in the x-direction):

$$T_2 = T_1 \frac{\cos(15)}{\cos(40)} = (91.6) \frac{\cos(15)}{\cos(40)} = \boxed{115.5\text{N}}$$

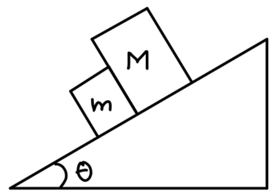


Figure for Problem 4

4) Two boxes, of mass m and M, are placed on a plane with an inclination of θ, as shown in the above photo, and released from rest. The surface of the incline has a coefficient of kinetic friction of μ_k.

a) Draw a free body diagram for each box.

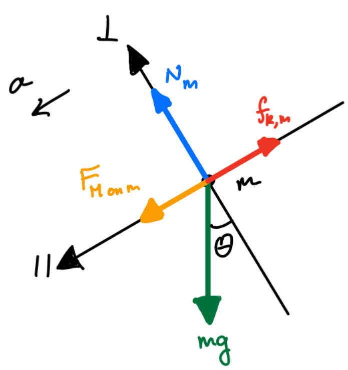
b) Show that the acceleration of the boxes down the slope is

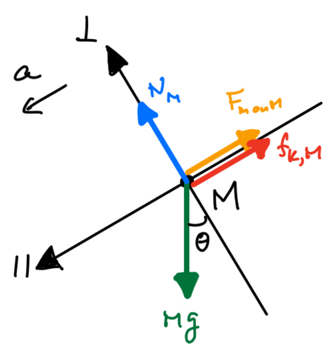
$$a = (\sin\theta - \mu_k \cos\theta)\, g$$

c) What force does the mass M put on the mass m as they move down the slope?

d) Why does the answer to part (c) make sense, other than the fact that the equation says so? *Hint: use the acceleration from part (b) to justify your answer.*

a) As the problem says, we need a free body diagram *for each box*. That means we have to consider not only the three types of forces (told, know, and adaptive) that we'd normally look to put on a free body diagram, but we also have to include any interactive forces between the two objects, as determined by Newton's third law. The free body diagrams will be labelled by m and M to signify which refers to which, though anything that does the job will work (1 and 2, A and B, etc). Both boxes have a weight, which points straight down; both boxes have a normal force, which points perpendicular to the surface (remember that "normal" means perpendicular); and both boxes have a frictional force, which points up the slope (opposing the downward sliding of the boxes. This frictional force will be kinetic friction because the boxes will be sliding. Lastly, m is going to apply a contact force on M up the slope (away from m), and M is going to apply a contact force on m down the slope (away from M). Using a rotated coordinate system, I've drawn the two free body diagrams below.





Note where I've placed the incline angle of the slope; this angle always goes here, since all incline angles are going to be measured from the ground.

b) Since the angle is made to the perpendicular axis, this axis "gets the cosine." This means that both weights break into their components like:

$$W_{\|} = W\sin\theta \qquad \text{and} \qquad W_{\perp} = W\cos\theta$$

So, we can now write out Newton's second law for both boxes. Each box is going to have two equations, one in the parallel and one in the perpendicular directions, for a total of four equations, but only the parallel equations will be our dynamical (i.e. relating to motion) equations, since the acceleration is along the parallel direction. The perpendicular equations will be static equations, so we can just say that the forces in one direction along the perpendicular axis equal the forces along the opposite direction. In both cases, the only forces on the perpendicular axis are the normal force and the perpendicular component of the weight, so Newton's second law in the perpendicular direction says:

$$N_m = mg\cos\theta \quad \text{and} \quad N_M = Mg\cos\theta$$

We talked in class about these equations. Because this problem is about the acceleration, which is in the parallel direction, these equations are "uninteresting." However, if we want to know what the friction forces are on each box, we do need to know the normal force, so we would need to arrive at these equations. Using the equation for kinetic friction, $f_k = \mu_k N$, we have:

$$f_{k,m} = \mu_k N_m = \mu_k mg\cos\theta \quad \text{and} \quad f_{k,M} = \mu_k N_M = \mu_k Mg\cos\theta$$

Notice that I used the same coefficient of kinetic friction for each box; this is because the problem doesn't make a distinction in what μ_k should be for each box, so we have to assume it's the same. Now that we know what f_k is for each box, we've gone as far as we can without addressing the parallel direction. Defining down the slope as the positive direction for both boxes, we see that Newton's second law in the parallel direction says:

$$F_{M\,on\,m} + mg\sin\theta - f_{k,m} = ma \quad \text{and} \quad Mg\sin\theta - F_{m\,on\,M} - f_{k,M} = Ma$$

To solve for a, we have two equations (the two above), but we have 3 unknowns: $F_{M\,on\,m}$, $F_{m\,on\,M}$, and a. Always remember that Newton's third law relates the two interaction forces, so we can eliminate one of our unknowns: $F_{M\,on\,m} = F_{m\,on\,M} = F$. So, now there are only two unknowns and two equations, so the problem is solvable. We should rearrange each equation so that they are written like $F = ...$

$$F = ma - mg\sin\theta + f_{k,m} \quad \text{and} \quad F = Mg\sin\theta - f_{k,M} - Ma$$

This means we can set each equation equal to one another, $F = F$, and solve for a. Plugging in our equations for the friction forces, we see that:

$$ma - mg\sin\theta + \mu_k mg\cos\theta = Mg\sin\theta - \mu_k Mg\cos\theta - Ma$$

The Ma needs to be moved to the left-hand-side, and everything but ma from the left-hand-side needs to be moved to the right-hand-side:

$$ma + Ma = Mg\sin\theta - \mu_k Mg\cos\theta + mg\sin\theta - \mu_k mg\cos\theta$$

Here's where the algebra gets a little dicey. On the left-hand-side, it's clear that we can factor our an a, giving us $(M+m)a$. On the right-hand-side, we see that each term has a common factor of g, so we can factor that out. Then, we need to group the terms by mass. So, our equation becomes:

$$(M+m)a = (M\sin\theta - \mu_k M\cos\theta + m\sin\theta - \mu_k m\cos\theta)\,g = [M(\sin\theta - \mu_k\cos\theta) + m(\sin\theta - \mu_k\cos\theta)]\,g$$

Notice what we end up with on the right-hand-side: inside of the parentheses, the coefficient of M and m is the same, just $\sin\theta - \mu_k\cos\theta$, so that can be factored giving us:

$$(M+m)a = [(M+m)(\sin\theta - \mu_k\cos\theta)]\,g$$

Cancelling the factor of $M+m$ from both sides, we get our answer:

$$\boxed{a = (\sin\theta - \mu_k\cos\theta)g}$$

c) To find the force that M puts on m, we need to solve either of the equations for F that we have above, since the force M puts on m is the same as the force m puts on M:

$$F = ma - mg\sin\theta + \mu_k mg\sin\theta = m(\sin\theta - \mu_k\cos\theta)g - mg\sin\theta + \mu_k mg\sin\theta = \boxed{0}$$

Notice that I plugged in $f_{k,m} = \mu_k mg\sin\theta$.

d) Why does the above answer make sense? Why are the boxes not putting any force on each other? It all has to do with the acceleration. Notice something very important: it's mass-independent. That means box m is going down the slope at the same acceleration as box M *whether or not they are in contact.* Remember that, no matter what, they boxes have to be moving with the same acceleration, since they're moving as one, but if they were truly putting a force on one another, the acceleration would depend somehow on their masses (maybe the sum of their masses, for instance). Since the acceleration doesn't depend on mass, you could put m on the slope by itself and the motion it would undergo would be the exact same as if you put M right behind it. Since they move with the same acceleration on their own, they don't touch each other on the way down, and so the force between them is zero.

FORMULA SHEET

- Constants:

$$g = 9.8 \mathrm{m/s^2}$$

- Vectors:

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

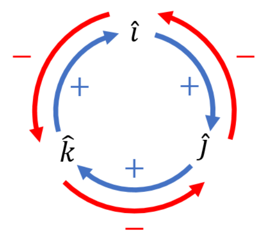


Figure 1: Cyclic permutations for cross product

- Kinematics:

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum \vec{F} = m\vec{a}$$

$$W = mg$$

$$f_{s,max} = \mu_s N$$

$$f_k = \mu_k N$$