PHY2048 Physics with Calculus I

Section 584761

Prof. Douglas H. Laurence

Exam 3 (Chapters 9, 15 – 16, 19 – 21)

April 23, 2018

Name: SOLUTIONS

Instructions:

This exam is composed of **10 multiple choice questions** and **5 free-response problems.** To receive a perfect score (100) on this exam, 4 of the 5 free-response problems must be completed. The fifth free-response problem **may not be answered for extra credit.** Each multiple choice question is worth 2 points, for a total of 20 points, and each free-response problem is worth 20 points, for a total of 80 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total	

MULTIPLE CHOICE QUESTIONS

1. Imagine some planet X existed, with double the mass of the Earth and triple the radius. What would the gravitational acceleration be at the surface of X?
   - (a) 2.18 m/s^2
   - (b) 4.36 m/s^2
   - (c) 6.53 m/s^2
   - (d) 9.8 m/s^2

$a_G = G\frac{M}{r^2}$; $g = G\frac{M_E}{R_E^2} = 9.8\,m/s^2$

$g_X = G\frac{(2M_E)}{(3R_E)^2} = \frac{2}{9}G\frac{M_E}{R_E^2} = \frac{2}{9}(9.8) = \boxed{2.18\,m/s^2}$

2. What is the period of a geosynchronous orbit?
   - (a) 1 hr
   - (b) 1 day
   - (c) 1 month
   - (d) 1 yr

DEFINITION: GEO = EARTH

SYNCHRONOUS = IN SYNCH (OBVIOUSLY)

3. What is the radius of the Earth's orbit around the Sun? Note that $M_{sun} = 1.99 \times 10^{30}$ kg and 1 yr $= 3.15 \times 10^7$ s.
   - (a) 1.5×10^{11} m
   - (b) 2.12×10^{11} m
   - (c) 3×10^{11} m
   - (d) 4.2×10^{11} m

$F_G = ma_c \Rightarrow G\frac{\cancel{m}M}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$ ($M = M_{sun}$; ATTRACTING BODY)

$\Rightarrow v^2 = G\frac{M}{r}$; $v = \frac{2\pi r}{T} \Rightarrow \frac{4\pi^2 r^2}{T^2} = G\frac{M}{r}$ ($T = 1yr$)

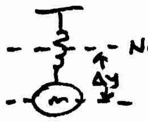
$\Rightarrow r^3 = \frac{GM}{4\pi^2}T^2 \Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(1.99\times10^{30})}{4\pi^2}(3.15\times10^7)^2} = \boxed{1.5\times10^{11}\,m}$

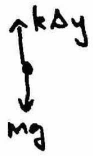
4. A spring oscillates at some frequency f. If the spring constant doubled, and the acceleration of gravity halved (which would happen if it was moved to another planet), what would the resulting frequency of oscillations be?
   - (a) $f/\sqrt{2}$
   - (b) $\sqrt{2}f$
   - (c) $2f$
   - (d) $2\pi f$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow f' = \frac{1}{2\pi}\sqrt{\frac{(2k)}{m}}$ (NO CHANGE w/ g)

$\Rightarrow f' = \sqrt{2}\cdot\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \boxed{\sqrt{2}f}$

5. A spring with a force constant of 120 N/m is anchored to the ceiling and hangs vertically. A 500g mass hangs at rest at the end of the spring. How far must the spring stretch to reach equilibrium?
   - (a) 2cm
   - (b) 4cm
   - (c) 20cm
   - (d) 40cm





$\Rightarrow k\Delta y = mg$

$\Rightarrow \Delta y = \frac{mg}{k} = \frac{(0.5)(9.8)}{(120)} = 0.04\,m$

$= \boxed{4\,cm}$

6. A 200 N/m spring oscillates with an amplitude of 3.5cm. If a 0.75kg mass is attached to the end of the spring, what is the maximum speed of the mass?
   - (a) 0.57 m/s
   - (b) 1.75 m/s
   - (c) 5.25 m/s
   - (d) 7.87 m/s

$U_{MAX} = K_{MAX} \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv_{MAX}^2$

$\Rightarrow v_{MAX} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{(200)}{(0.75)}}(0.035) = \boxed{0.57\,m/s}$

7. A beam of light with frequency 1×10^{14} Hz passes from air, where $n = 1$, to water, where $n = 1.33$. What is the frequency of the light in water? Note that the speed of light in any medium is $v = c/n$ and $c = 3 \times 10^8$ m/s,

FREQUENCY DOESN'T CHANGE BETWEEN 2 MEDIA!

(a) 7.5×10^{13} Hz
(b) 1×10^{14} Hz
(c) 1.33×10^{14} Hz
(d) 1.5×10^{14} Hz

8. Say there exists a wave with a frequency f and a wavelength λ, moving at a speed v. If the wavelength were doubled, what would the speed of the wave be?

(a) $v/2$
(b) v
(c) $2v$
(d) $4v$

SPEED DOESN'T DEPEND UPON λ OR f!

9. A wave, with wavelength λ, passes into a different medium, where the wave speed doubles. In the new medium, the wavelength is:

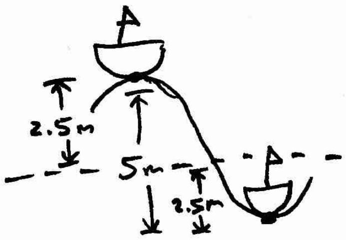
f REMAINS THE SAME, SO

$v = \lambda f \Rightarrow \lambda = \frac{v}{f}$; $\lambda' = \frac{(2v)}{f} = 2\frac{v}{f} = \boxed{2\lambda}$

(a) $\lambda/2$
(b) λ
(c) 2λ
(d) 4λ

10. A man in a boat drops 5m from peak to trough of the waves he is riding. What is the amplitude of the water waves?

(a) 2.5m
(b) 5m
(c) 7.5m
(d) 10m



PEAK-TO-TROUGH IS DOUBLE THE AMPLITUDE!

(c) • WRONG r, 3/7

(b) • $h = r - R_E$ GIVEN 1/7

(a) • $V = \frac{2\pi r}{T}$, CORRECT T, WRONG r 3/6

FREE-RESPONSE PROBLEMS

1. The international space station passes overhead every 90 minutes in the sky.

6pt (a) ✗ What is the orbital speed of the space station?

7pt (b) ✗ What is the altitude of the space station's orbit?

7pt (c) ✗ What is the acceleration of the space station?

$T_{ORB} = 90\,\text{MIN}$
$= 5400\,\text{s}$

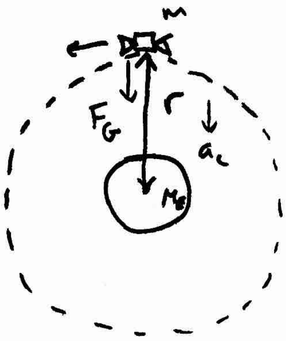
(a) $V_{ORB} = \frac{2\pi r}{T_{ORB}}$

$= \frac{2\pi(6.65\times10^{6})}{(5400)}$

$= \boxed{7{,}738\ \text{m/s}}$

(ROUGHLY 7.7 km/s)

(b)



$F_G = ma_c$

$\Rightarrow G\frac{\cancel{m}M_E}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$

$= \left(\frac{2\pi r}{T}\right)^2$

$\Rightarrow \frac{GM_E}{r} = \frac{4\pi^2 r^2}{T^2} \Rightarrow r^3 = \frac{GM_E}{4\pi^2}T^2$

$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{4\pi^2}(5400)^2}$

$= 6.65\times10^{6}\,\text{m}$ ← (NOT FINAL ANSWER!)

ALTITUDE: $h = r - R_E$

$= 6.65\times10^{6} - 6.4\times10^{6}$

$= \boxed{2.5\times10^{5}\ \text{m}}$

c) $a = a_c = \frac{v^2}{r} = \frac{(7738)^2}{(6.65\times10^{6})} = \boxed{9\ \text{m/s}^2}$

(r, NOT h)

2. Two masses, $m_1 = 2.5$kg and $m_2 = 1.7$kg, are placed along the x-axis, with m_1 at (0, 0) and m_2 at ~~(0, 3cm)~~ (3 cm, 0). In all questions, **ignore the Earth's gravity**.

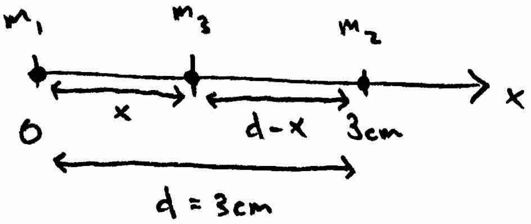
a) What is the gravitational force between the masses?

b) Where could a third mass, $m_3 = 0.75$kg, be placed such that the total gravitational force on it would be zero?

c) What would the gravitational force on m_3 be due to either m_1 or m_2 individually?

a) $F_{12} = G\frac{m_1 m_2}{r_{12}^2} = (6.67\times10^{-11})\frac{(2.5)(1.7)}{(0.03)^2} = \boxed{3.15\times10^{-7}\ N}$

b) $F_{13} = F_{23}$



$\Rightarrow \cancel{G}\frac{m_1 \cancel{m_3}}{x^2} = \cancel{G}\frac{m_2 \cancel{m_3}}{(d-x)^2}$

$\Rightarrow m_1(d-x)^2 = m_2 x^2$

$\Rightarrow \sqrt{m_1}(d-x) = \sqrt{m_2}\,x$

$\Rightarrow \sqrt{m_1}\,d = (\sqrt{m_1}+\sqrt{m_2})\,x$

$\Rightarrow x = \frac{\sqrt{m_1}}{\sqrt{m_1}+\sqrt{m_2}}\,d = \frac{\sqrt{2.5}}{\sqrt{2.5}+\sqrt{1.7}}(3\text{cm}) = \boxed{1.64\text{cm}}$

c) $F_{13} = F_{23}$ ← (so we only need to solve for one force.)

$F_{13} = G\frac{m_1 m_3}{x^2} = (6.67\times10^{-11})\frac{(2.5)(0.75)}{(0.0164)^2} = \boxed{4.65\times10^{-7}\ N}$

(d) • $V_{MAX} = 12.5$, NOT 12.5+0.56. 5/6 (a) $\omega = 11.18 s^{-1}$, BUT INCORRECT T, 4/5

3. A 150N/m spring oscillates with a 1.2kg mass attached to it.

5pt a) ~~a)~~ What is the period of the mass' oscillation?

5pt b) ~~b)~~ If the mass were originally released from rest when the spring was stretched by 5cm, what is the maximum speed of the mass?

4.5pt c) ~~c)~~ What is the amplitude of the oscillations?

6pt d) ~~d)~~ If, after some time, the mass was struck while at equilibrium such that it gained a momentum of 15 Ns, what would the new amplitude of the oscillations be?

a) $\omega = \sqrt{\frac{k}{m}}$ $(\omega = 2\pi f)$ $\Rightarrow$ $f_{sp} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $(T = \frac{1}{f})$

$\Rightarrow T_{sp} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(1.2)}{(150)}} = \boxed{0.56s}$

b) $E = \cancel{K_i}^{0} + U_i = \frac{1}{2}kx_i^2 = \frac{1}{2}(150)(0.05)^2 = 0.188J$

(SINCE ENERGY IS CONSERVED)

$E = K_{MAX}$ ($U = 0$ WHEN K IS MAXIMUM)

$= \frac{1}{2}mV_{MAX}^2$

$\Rightarrow V_{MAX} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.188)}{(1.2)}} = \boxed{0.56\,m/s}$

c) AMPLITUDE OCCURS WHEN $K=0$. THE MASS STARTED AT $x = 5cm$ WITH $K=0$, SO THE AMPLITUDE IS JUST 5cm.

$\boxed{A = 5cm}$

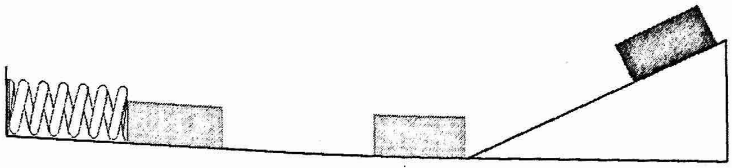
d) $\Delta p = m\Delta V \Rightarrow \Delta V = \frac{\Delta p}{M} = \frac{15}{1.2} = 12.5\,m/s$ (THE MASS GAINS 12.5 m/s AT EQUILIBRIUM)

$\Rightarrow V_{MAX} = 0.56\,m/s + 12.5\,m/s = 13.06\,m/s$ (NEW MAX SPEED)

$K_{MAX} = U_{MAX} \Rightarrow \frac{1}{2}mV_{MAX}^2 = \frac{1}{2}kA^2$

$\Rightarrow A = \sqrt{\frac{m}{k}}V_{MAX} = \sqrt{\frac{1.2}{150}}(13.06) = \boxed{1.17m}$ (NEW AMPLITUDE AT NEW V_{MAX})

(b). FIND $U_{spr} = \frac{1}{2}(100)(0.075)^2$ 2/7



4. A spring with constant 100 N/m is compressed by 7.5cm. At the end of the spring is a 55g mass (unattached to the spring) which is fired horizontally by the spring, as shown in the figure above. Assume that the ramp is always frictionless.

6 a) ~~x~~ At what point does the mass lose contact with the spring?

7 b) ~~x~~ At what speed does the mass lose contact with the spring?

7 c) ~~x~~ If the horizontal surface is frictionless, to what height does the mass reach?

a) AT THE EQUILIBRIUM POSITION OF THE SPRING, WHICH IS AT ITS NATURAL LENGTH WHEN HORIZONTAL. THIS IS THE POINT WHEN THE SPEED OF THE BLOCK IS A MAXIMUM, & THE SPRING WILL SLOW DOWN PAST THIS POINT.

b) $U_{MAX} = K_{MAX}$

$\Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv_{MAX}^2$ (POSITION IS AN AMPLITUDE INITIALLY, AND MASS LOSES CONTACT WITH SPRING WHEN V IS MAXIMUM.)

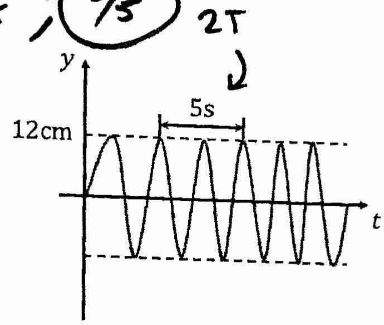
$\Rightarrow v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{(100)}{(0.055)}}(0.075) = \boxed{3.20\ m/s}$

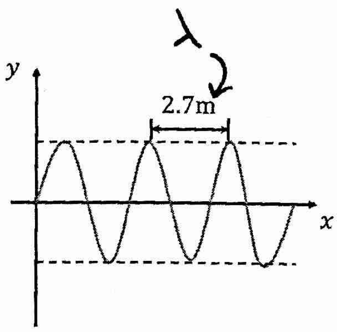
c) ENERGY IS CONSERVED ALONG THE HORIZONTAL SURFACE AND UP THE RAMP, SO WHATEVER KINETIC ENERGY THE MASS HAS ALONG THE HORIZONTAL SURFACE IS TURNED INTO GRAVITATIONAL POTENTIAL ENERGY UP THE RAMP:

$K_i = U_{g,f} \Rightarrow \frac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh \Rightarrow h = \frac{v_i^2}{2g} = \frac{(3.20)^2}{2(9.8)} = \boxed{0.52m}$

(b) • $f = 5$, $\frac{1}{5}$ (c) • $A = 24\text{cm}$, $\frac{3}{5}$

• $f = \frac{1}{5}$; $\frac{3}{5}$





5. Based on the graphs above, find the following characteristics of the wave:

5 a) The wavelength

5 b) The frequency

5 c) The amplitude

5 d) The wavespeed

a) WAVELENGTH IS DISTANCE BETWEEN PEAKS, WHICH CAN BE FOUND ON THE y VS. x GRAPH:

$$\boxed{\lambda = 2.7\text{m}}$$

b) THE FREQUENCY IS $\frac{1}{T}$, AND THE PERIOD (T) CAN BE FOUND ON THE y VS. t GRAPH AS THE TIME BETWEEN PEAKS. NOTICE THAT THE INDICATED TIME, 5s, IS THE TIME BETWEEN 3 PEAKS, NOT 2, WHICH REPRESENTS TWO CYCLES, SO 5s IS TWICE THE PERIOD, OR

$$2T = 5\text{s} \Rightarrow T = 2.5\text{s}$$

$$\Rightarrow f = \frac{1}{T} = \frac{1}{2.5} = \boxed{0.4 \text{ Hz}}$$

c) THE AMPLITUDE IS THE MAXIMUM y-DISPLACEMENT, WHICH IS $\boxed{12\text{cm.}}$

d) $v = \lambda f = (2.7)(0.4) = \boxed{1.08 \text{ m/s}}$

FORMULA SHEET

- Constants:

$$g = 9.8\text{m/s}^2$$

- Vectors:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$
$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

- Kinematics:

$$\Delta x = v_0t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum\vec{F} = m\vec{a}$$
$$W = mg$$
$$f_{s,max} = \mu_s N$$
$$f_k = \mu_k N$$

- Work & Energy:

$$W = \vec{F}\cdot\Delta\vec{x} \quad \text{or} \quad W = \int\vec{F}\cdot d\vec{x}$$
$$W_{tot} = \Delta K$$
$$W_{cons} = -\Delta U$$
$$K = \frac{1}{2}mv^2$$
$$U_g = mgy$$
$$K_i + U_i + W_{nc} = K_f + U_f$$
$$\vec{F} = -\vec{\nabla}U$$
$$\vec{\nabla}f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

- Momentum & Collisions:

$$\vec{p} = m\vec{v}$$
$$\sum\vec{F} = \frac{d\vec{p}}{dt}$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$\vec{v}_{1i} - \vec{v}_{2i} = \vec{v}_{2f} - \vec{v}_{1f}$$

- Rotational Mechanics

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\tau = rF\sin\theta$$

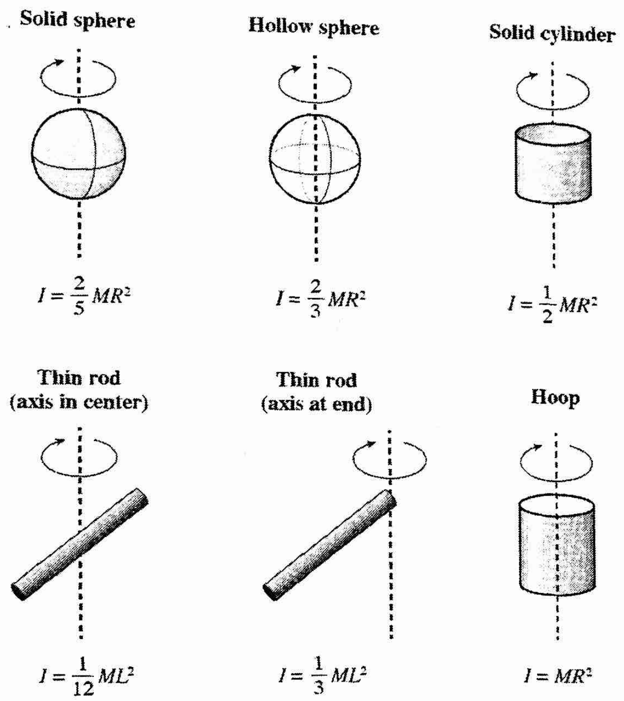
$$\sum\tau = I\alpha \quad \text{or} \quad \sum\tau = \frac{dL}{dt}$$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$L = I\omega \quad \text{or} \quad L = rp$$

$$I = \int r^2 dm$$

$$I_{new} = I_{cm} + md^2$$



- Gravity:

$$G = 6.67 \times 10^{-11}\ \frac{\mathrm{N}m^2}{\mathrm{kg}^2}$$

$$F_G = G\frac{mM}{r^2}$$

$$a_G = G\frac{M}{r^2}$$

$$U_G = -G\frac{mM}{r}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$v_{orb} = r\omega_{orb} = \frac{2\pi r}{T_{orb}}$$

- Oscillations:

$$F_{sp} = kx$$

$$U_{sp} = \frac{1}{2}kx^2$$

$$\omega_{sp} = \sqrt{\frac{k}{m}}$$

$$\omega_{pend} = \sqrt{\frac{g}{l}}$$

$$f = 1/T$$

$$\omega = 2\pi f$$

- Waves:

$$v = \lambda f$$

$$f_{beat} = |f_1 - f_2|$$