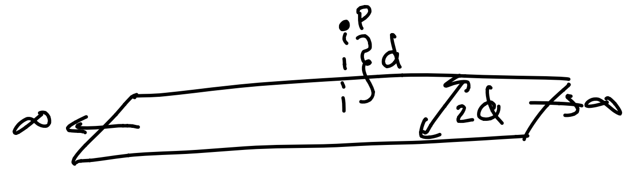
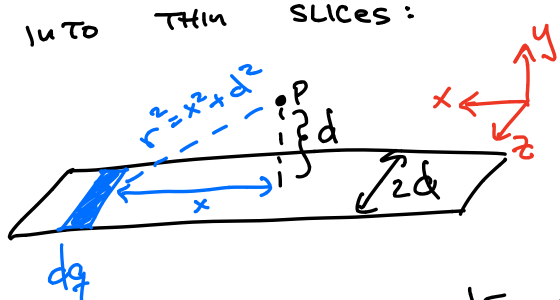
# COULOMB'S LAW APPROACH FOR PROB 24.17

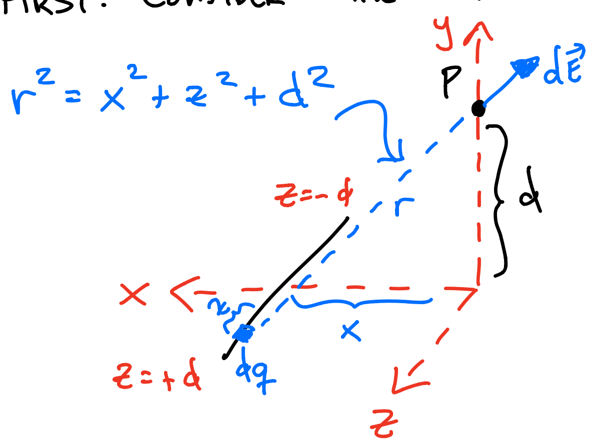
WHAT IS $\vec{E}$ AT POINT P?




BREAK BAND UP INTO THIN SLICES:



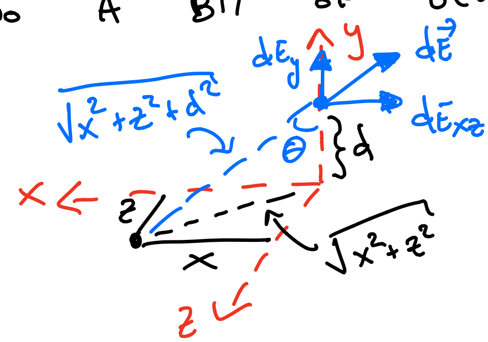

WE DON'T KNOW THE ELECTRIC FIELD $dE$ AT P UNTIL WE FIND THE ELECTRIC FIELD DUE TO A THIN STRIP FIRST. CONSIDER THE THIN STRIP TO BE A LINE OF CHARGE:




$$dE = k\frac{dq}{r^2}$$

$$= k\frac{dq}{(x^2+z^2+d^2)}$$

TO ISOLATE THE y-COMPONENT OF $d\vec{E}$, WE NEED TO DO A BIT OF GEOMETRY:




$$dE_y = dE\cos\theta$$

$$= dE \cdot \frac{d}{\sqrt{x^2+z^2+d^2}}$$

$$\Rightarrow dE_y = \frac{kd}{(x^2+z^2+d^2)^{3/2}}\,dq$$

CONVERT THE INTEGRAL OVER $dq$ INTO ONE OVER $dz$:

$$dq = \lambda\, dz$$

$$\Rightarrow dE_y = \frac{kd\lambda}{(x^2+z^2+d^2)^{3/2}}\,dz$$

$$\Rightarrow E_y = kd\lambda \int_{-d}^{d} \frac{dz}{(x^2+z^2+d^2)^{3/2}}$$

NOTE: $$\int \frac{dx}{(x^2+a^2+b^2)^{3/2}} = \frac{x}{(a^2+b^2)\sqrt{x^2+a^2+b^2}}$$

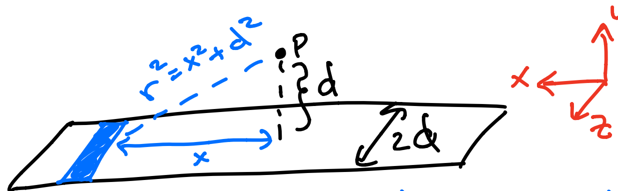
$$\Rightarrow E_y = kd\lambda \cdot \left. \frac{z}{(x^2+d^2)\sqrt{z^2+x^2+d^2}} \right|_{-d}^{d}$$

$$= 2kd\lambda \frac{d}{(x^2+d^2)\sqrt{x^2+2d^2}}$$

REPLACING $\lambda = q/2d$,

$$E_y = \frac{kqd}{(x^2+d^2)\sqrt{x^2+2d^2}}$$

NOW, INTEGRATING THIS SLIVER ALONG THE BAND:




$dq \leftarrow dq = \sigma \, dA = \sigma \cdot 2d \cdot dx$

$$dE_y = \frac{kd}{(x^2+d^2)\sqrt{x^2+2d^2}} \, dq = \frac{2k\sigma d^2}{(x^2+d^2)\sqrt{x^2+2d^2}} \, dx$$

$$\Rightarrow E_y = 2k\sigma d^2 \int_{-\infty}^{\infty} \frac{dx}{(x^2+d^2)\sqrt{x^2+2d^2}}$$

NOTE: $$\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)\sqrt{x^2+2a^2}} = \frac{\pi}{2a^2}$$

$$\Rightarrow E_y = \cancel{2}k\sigma \cancel{d^2} \cdot \frac{\pi}{\cancel{2}\cancel{d^2}} = \pi k \sigma$$

OR, SUBSTITUTING $k = \frac{1}{4\pi\varepsilon_0}$,

$$\boxed{E_y = \frac{\sigma}{4\varepsilon_0}}$$