Chapter 17:
The Stars

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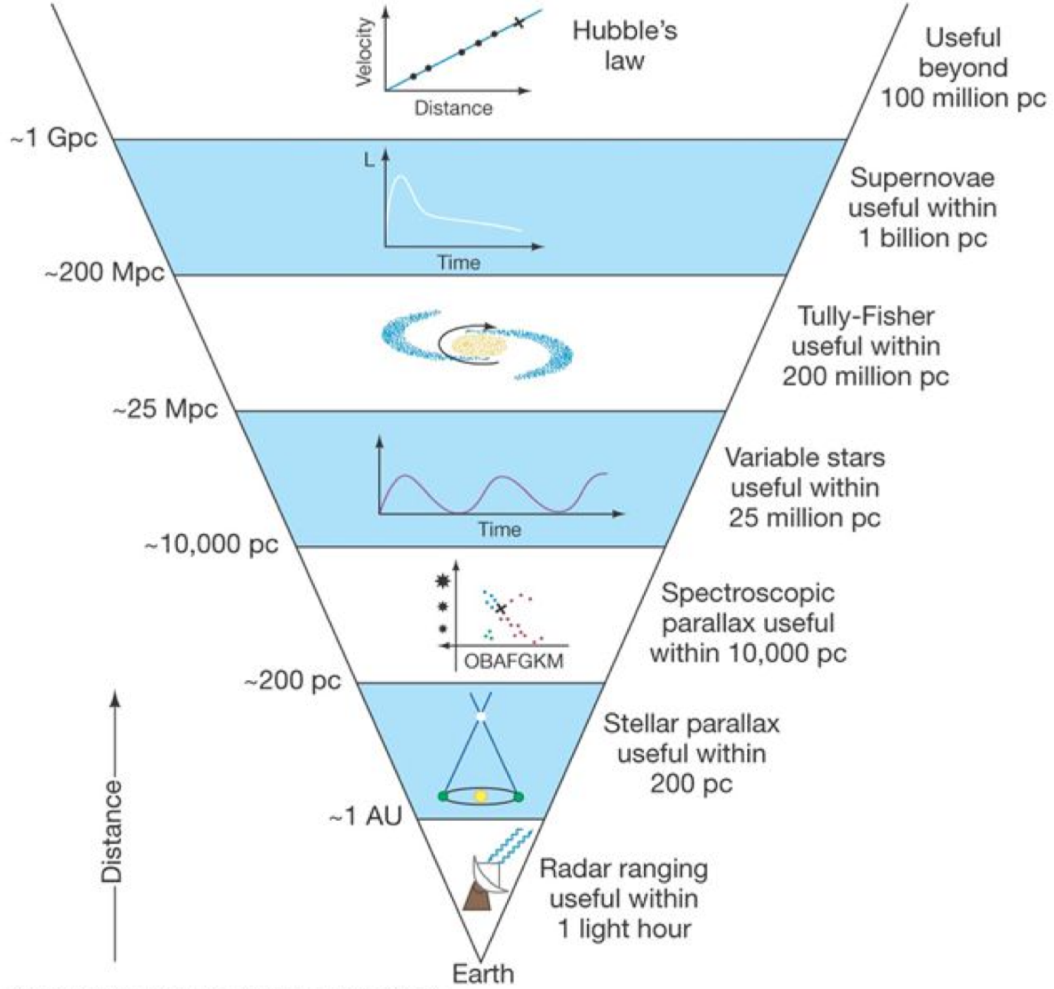
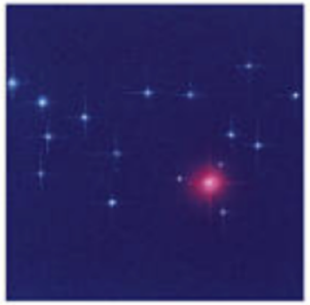
AST 1004

Summer 2018

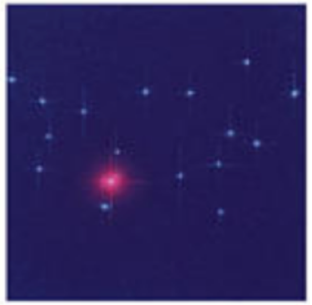
Cosmic Distance Ladder



Stellar Parallax

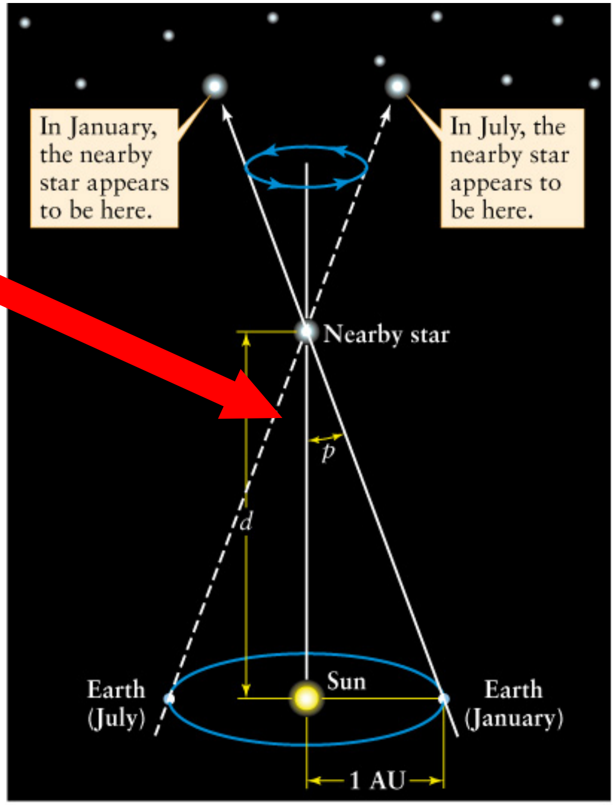


As seen in January

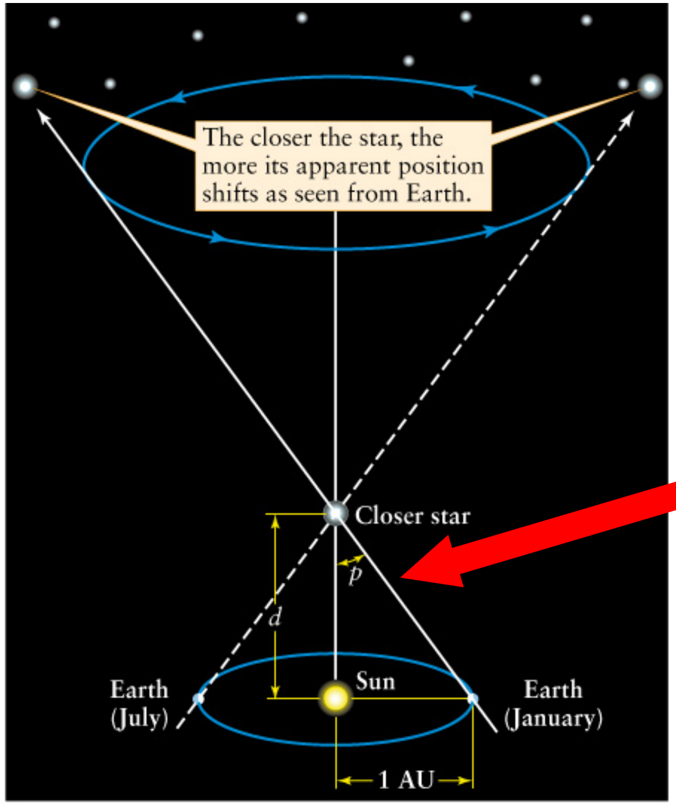


As seen in July

Smallerparallax
→ larger distance



(a) Parallax of a nearby star



(b) Parallax of an even closer star

Larger parallax
→ smaller distance

Stellar Parallax Calculations

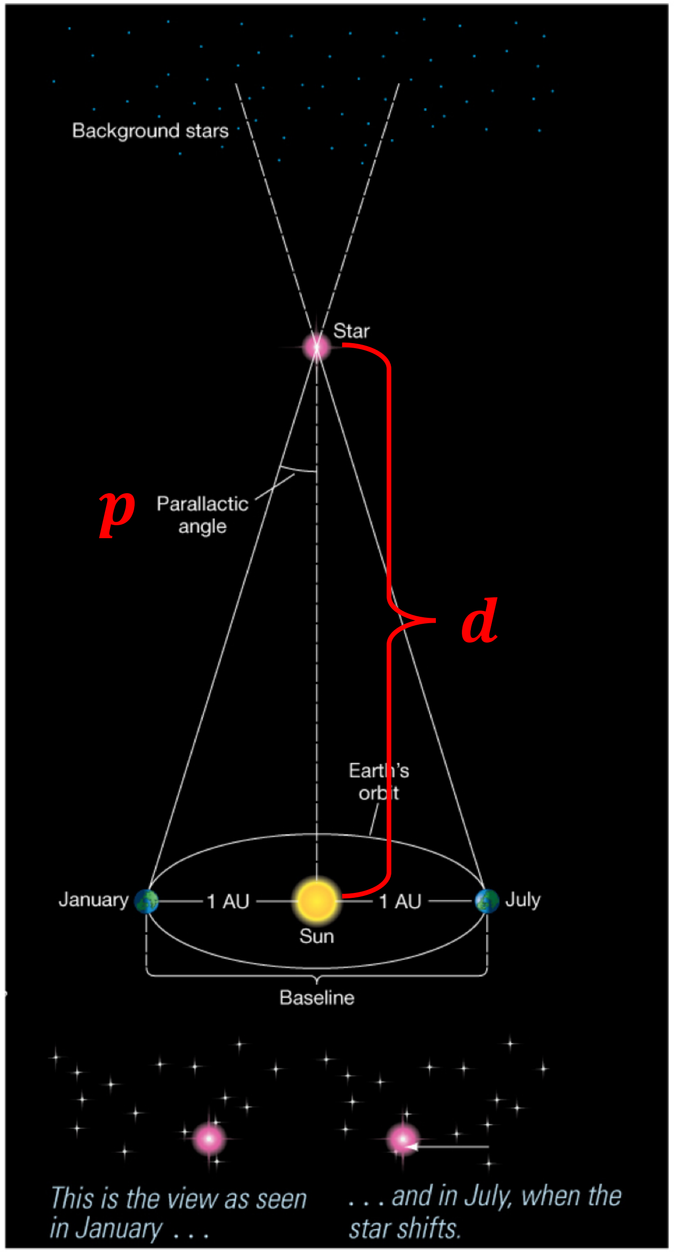
$$d = \frac{1}{p}$$

Distance in parsecs (pc)

Angle in arcseconds (“)

$$1 \text{ pc} = 3.26 \text{ ly}$$

Same order of magnitude!

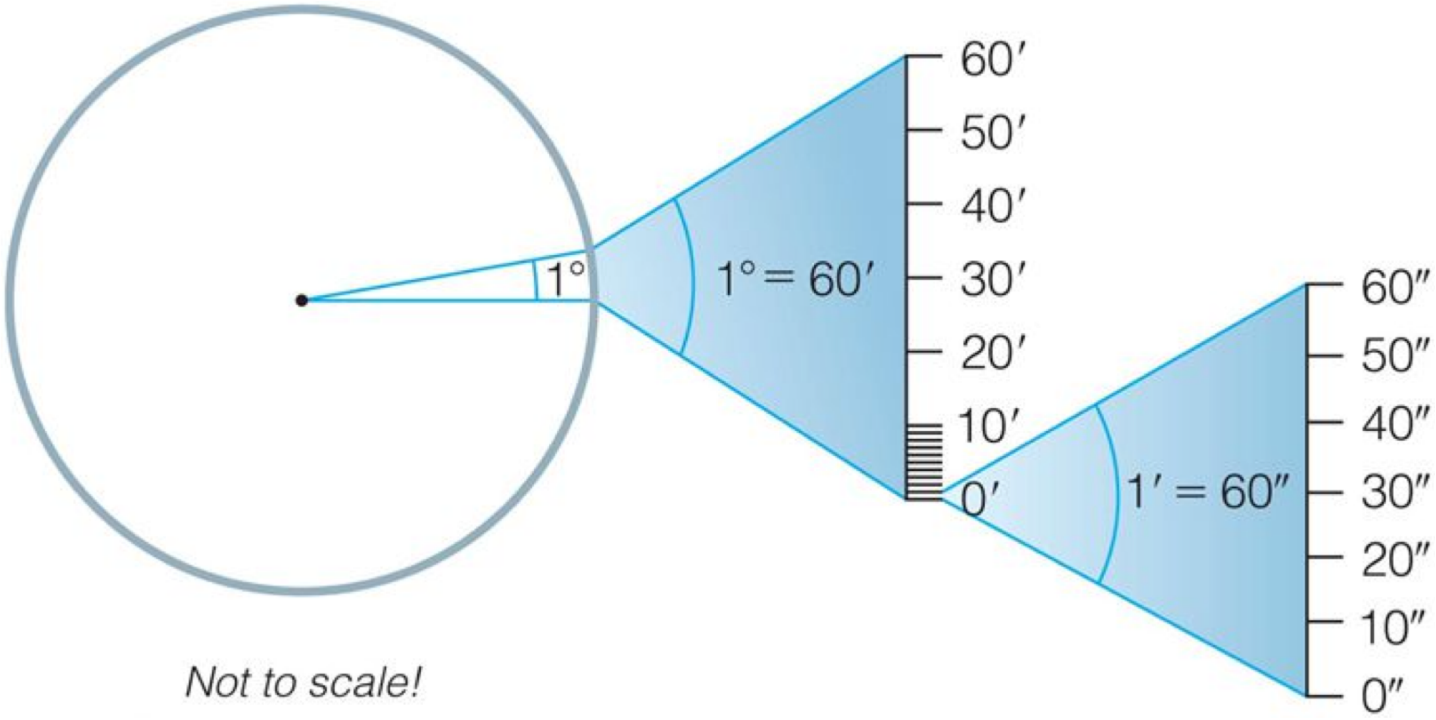


Arcminutes and Arcseconds

$$1 \text{ circle} = 360^o$$

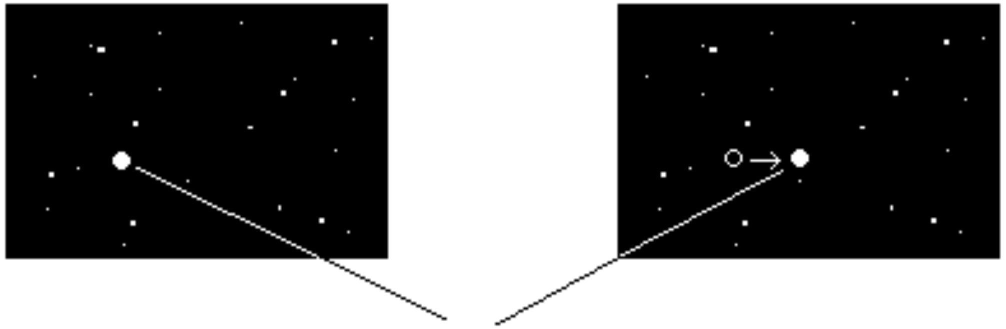
$$1^o = 60'$$

$$1' = 60''$$



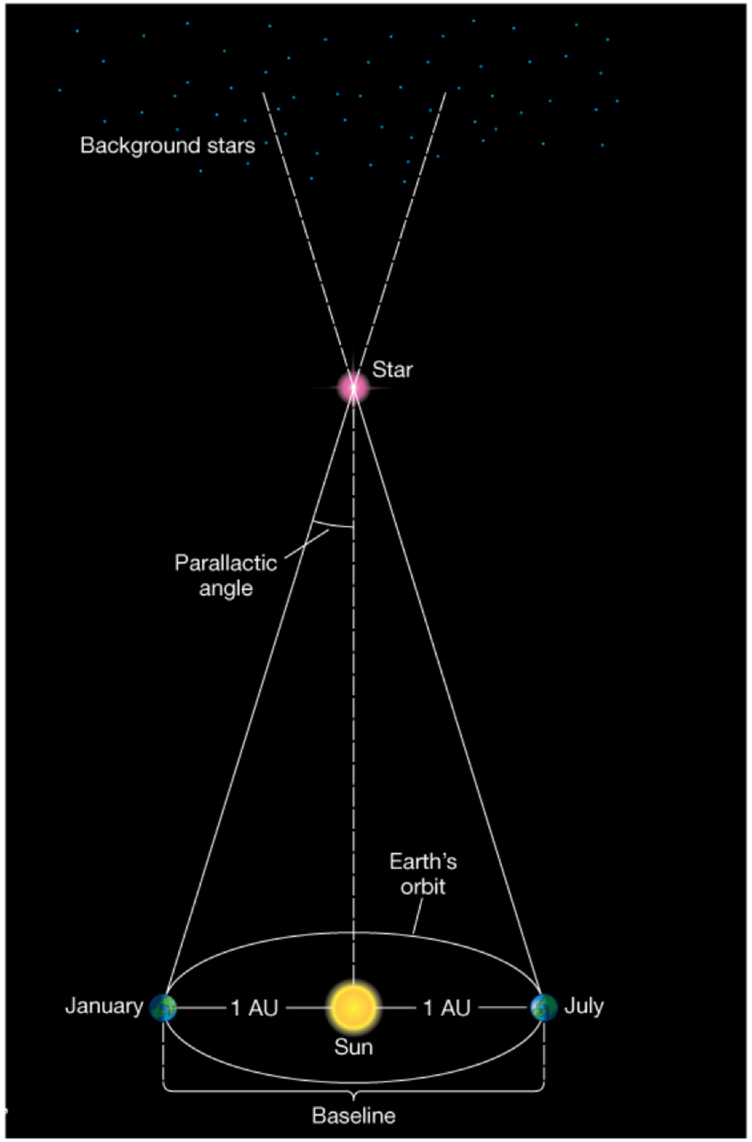
Measuring Parallax

Spring Fall



This star has moved 0.5 seconds of arc between photos.

$$d = \frac{1}{p} = \frac{1}{0.25''} = 4\text{pc}$$



Limits of Parallax

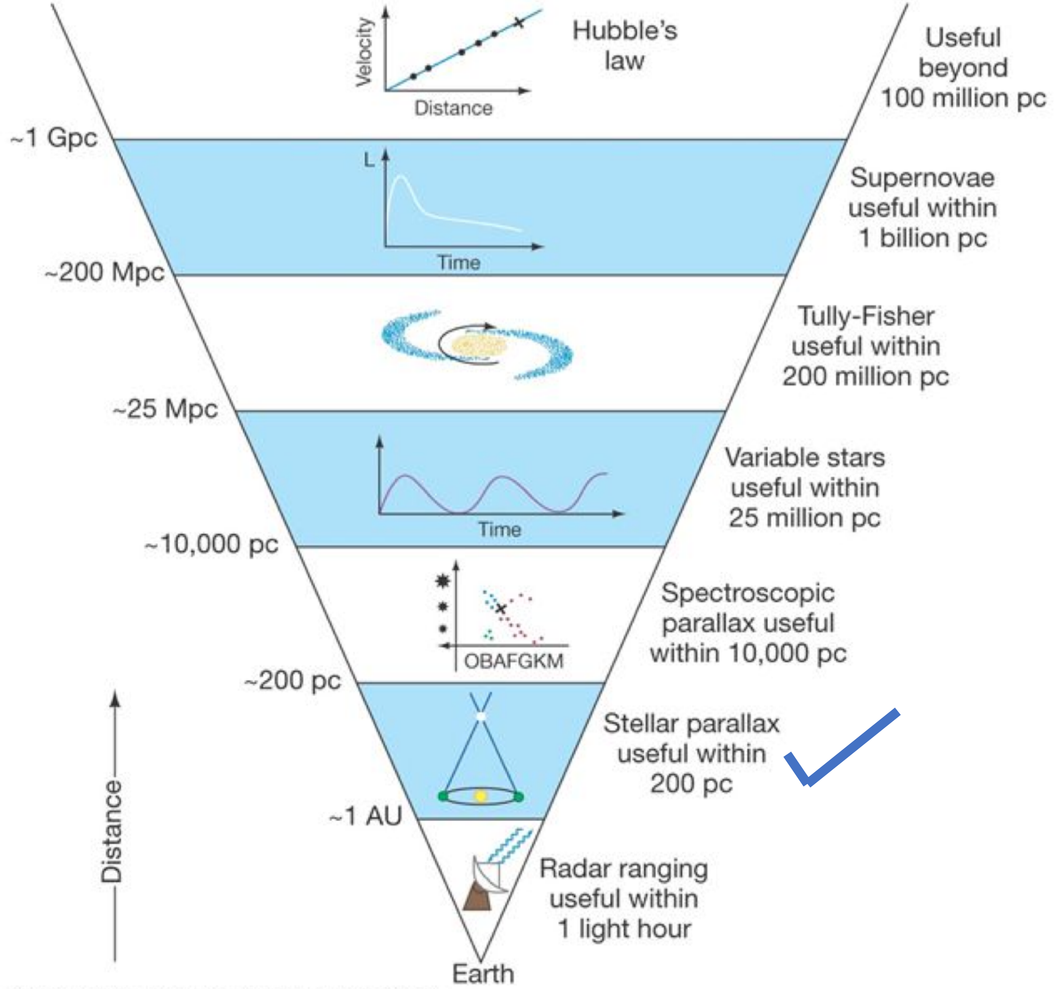
Angles smaller than 0.01'' are hard to measure from Earth

$$d_{\max} = \frac{1}{0.01''} = 100\text{pc}$$

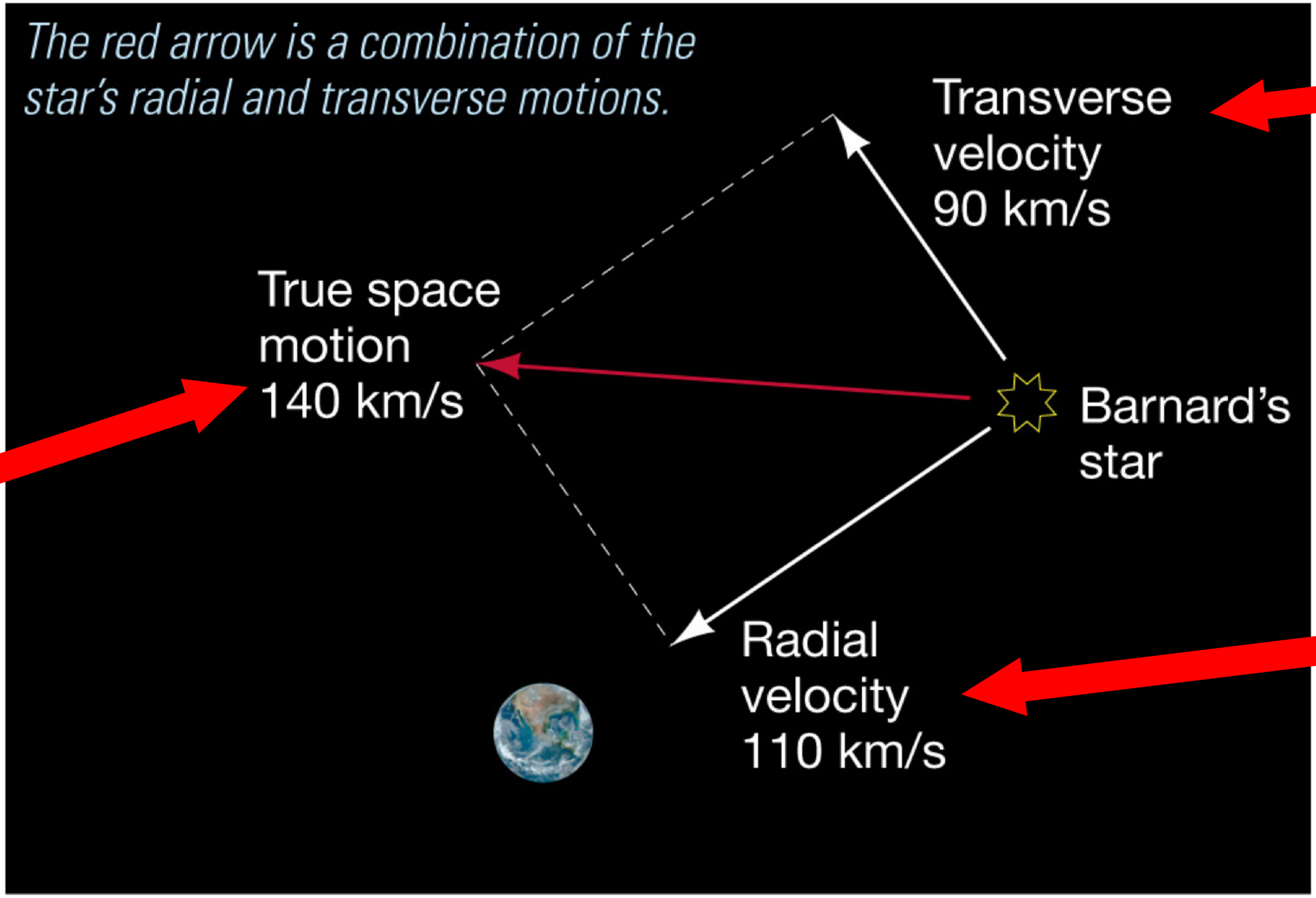
Angles smaller than 0.001'' are hard to measure from Space

$$d_{\max} = \frac{1}{0.001''} = 1000\text{pc}$$

Cosmic Distance Ladder



Motion of Stars



Found using geometry

Found using Pythagorean theorem:

$$v = \sqrt{v_t^2 + v_r^2}$$

$$= \sqrt{(90)^2 + (110)^2}$$

$$= 142 \text{ km/s}$$

Found using Doppler Effect

Translational Motion

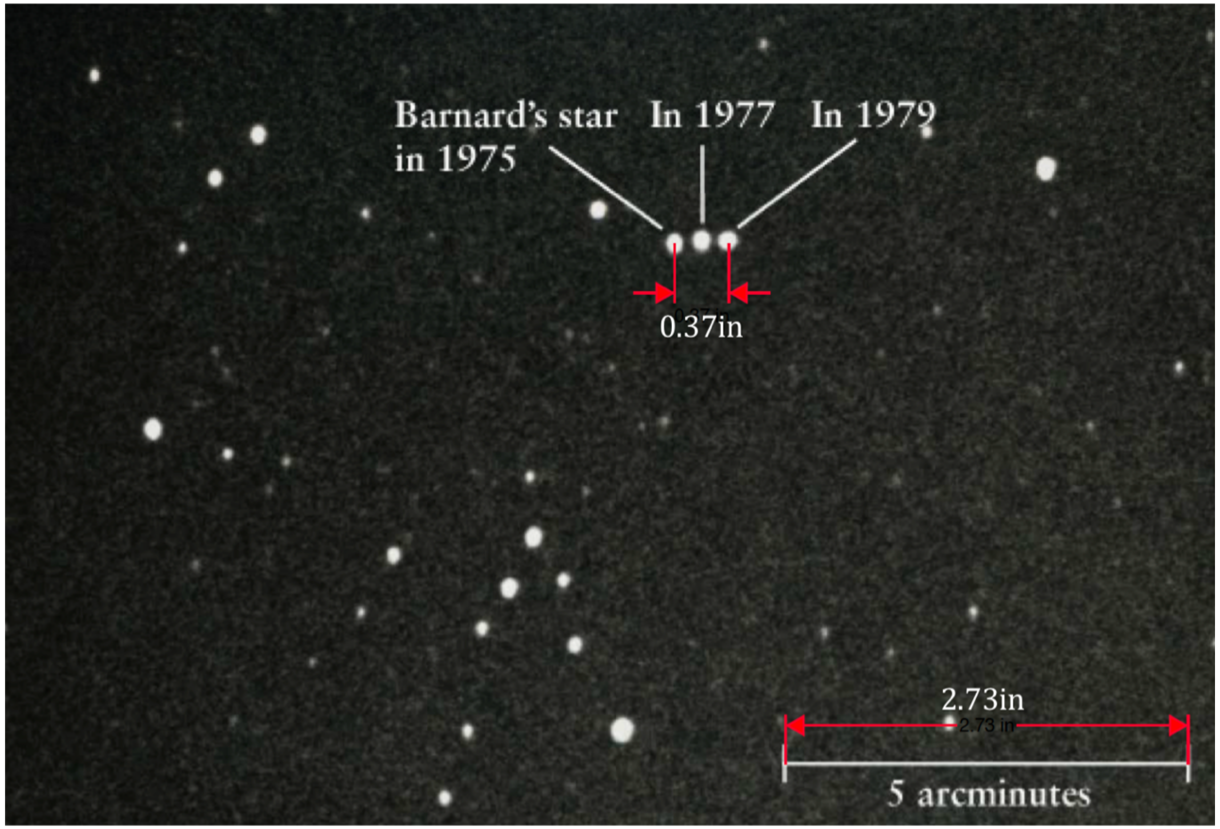
$$0.37\text{in}\times\frac{5'}{2.37\text{in}} = 0.68'\times\frac{60''}{1'} = 40.8''$$

Distance traveled (arcsec/year)

$$v_t = 4.74\mu d$$

Transverse velocity (km/s)

Distance to star (pc)



$$v_t = 4.7\times\left(\frac{40.8''}{4\text{ yr}}\right)\times(1.8\text{ pc}) = 86.9\text{ km/s}$$

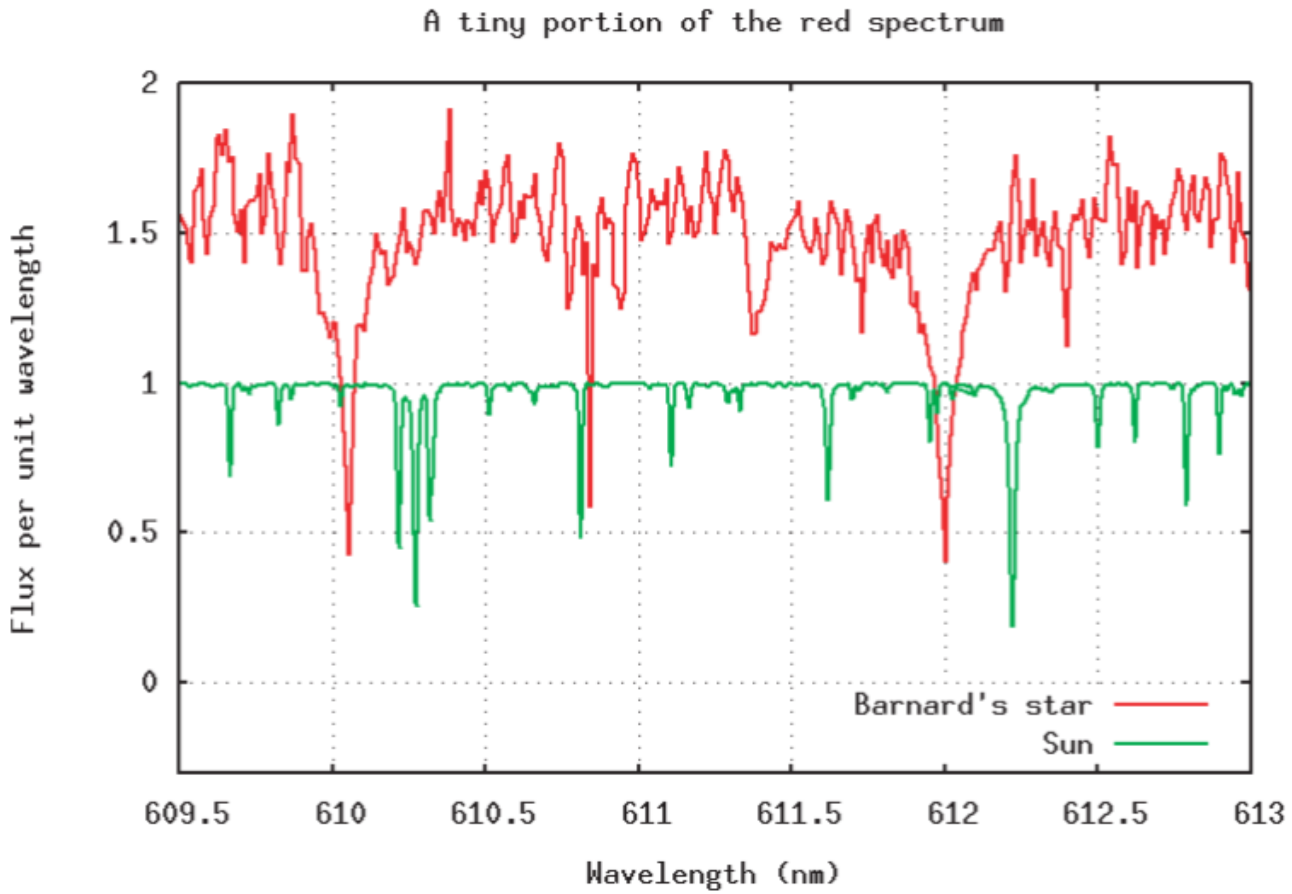
Radial Motion

Shifted wavelength

$$\frac{\lambda}{\lambda_0} - 1 = \frac{v_r}{c}$$

Proper wavelength

Figure 1: Barnard Star and Sun's Spectrum



$$\frac{(612\ \text{nm})}{(612.25\ \text{nm})} - 1 = -0.000408\ \ (\text{negative} \rightarrow \text{towards})$$

$$v_r = 0.000408 \times \left(3\times10^5\,\frac{\text{km}}{\text{s}}\right) = 122.4\ \text{km/s}$$

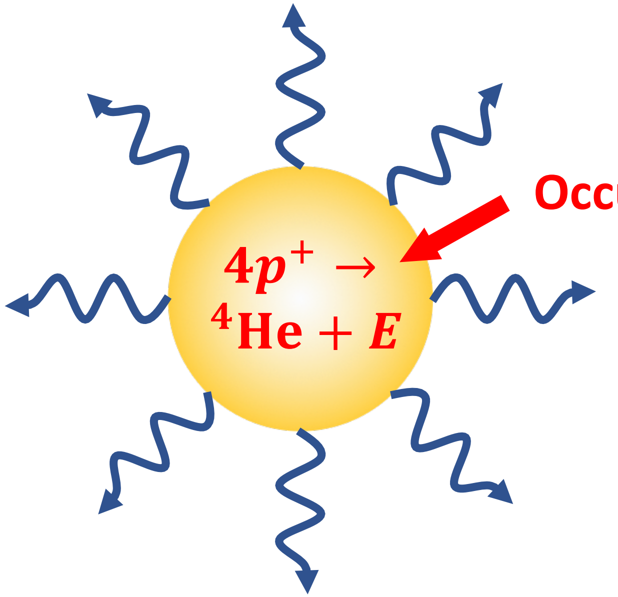
Luminosity

Brightness is what we call **Luminosity**, L

→ It's the amount of energy released per second (Watts, W)

→ It's an **inherent quantity** of a star

→ Sometimes called **Absolute brightness**



Occurs 9.58×10^{37} times every second

$$L_\odot = 3.828 \times 10^{26}\ \mathrm{W}$$

Flux

We don't measure the brightness of a star, but it's **apparent brightness**

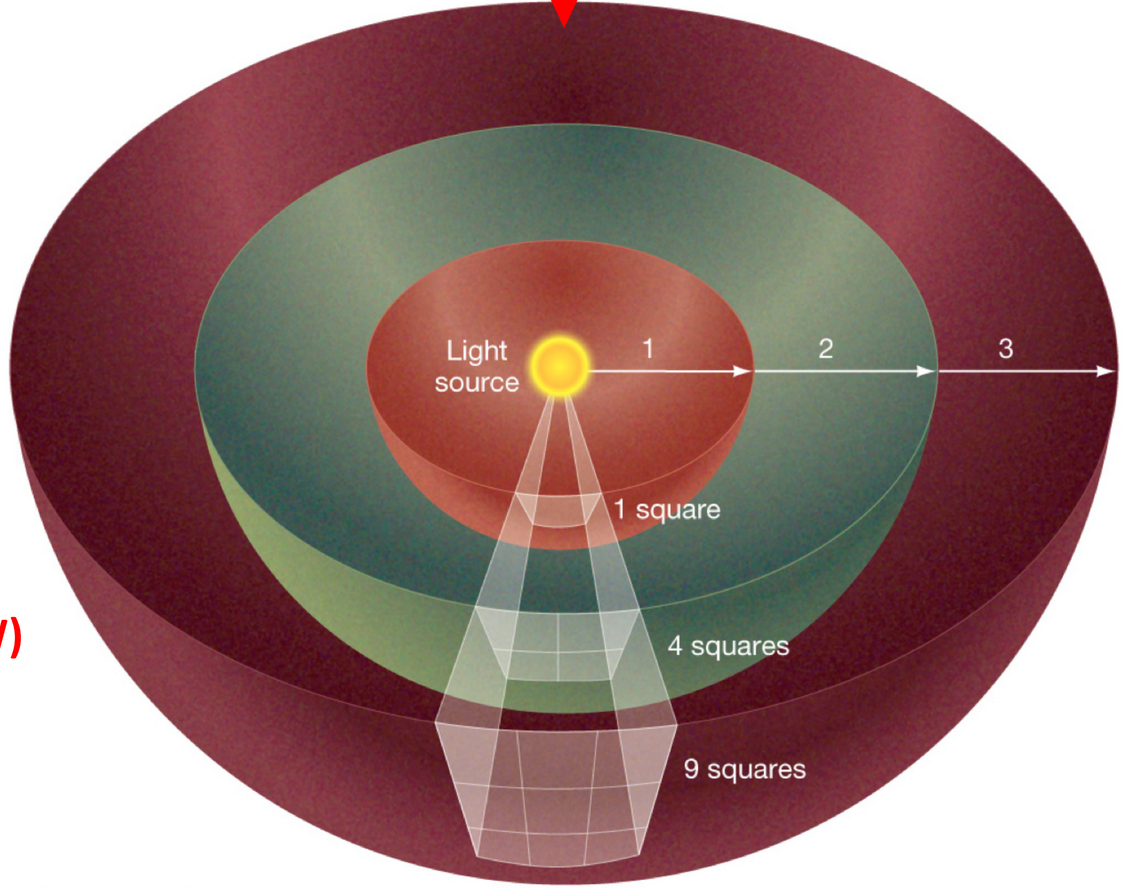
- This is known as **Flux**, F (W/m^2)
- Flux measures how the luminosity spreads out over a sphere for isotropic emission

$$F = \frac{L}{\underbrace{4\pi r^2}_{\text{Surface area of sphere (m}^2\text{)}}}$$

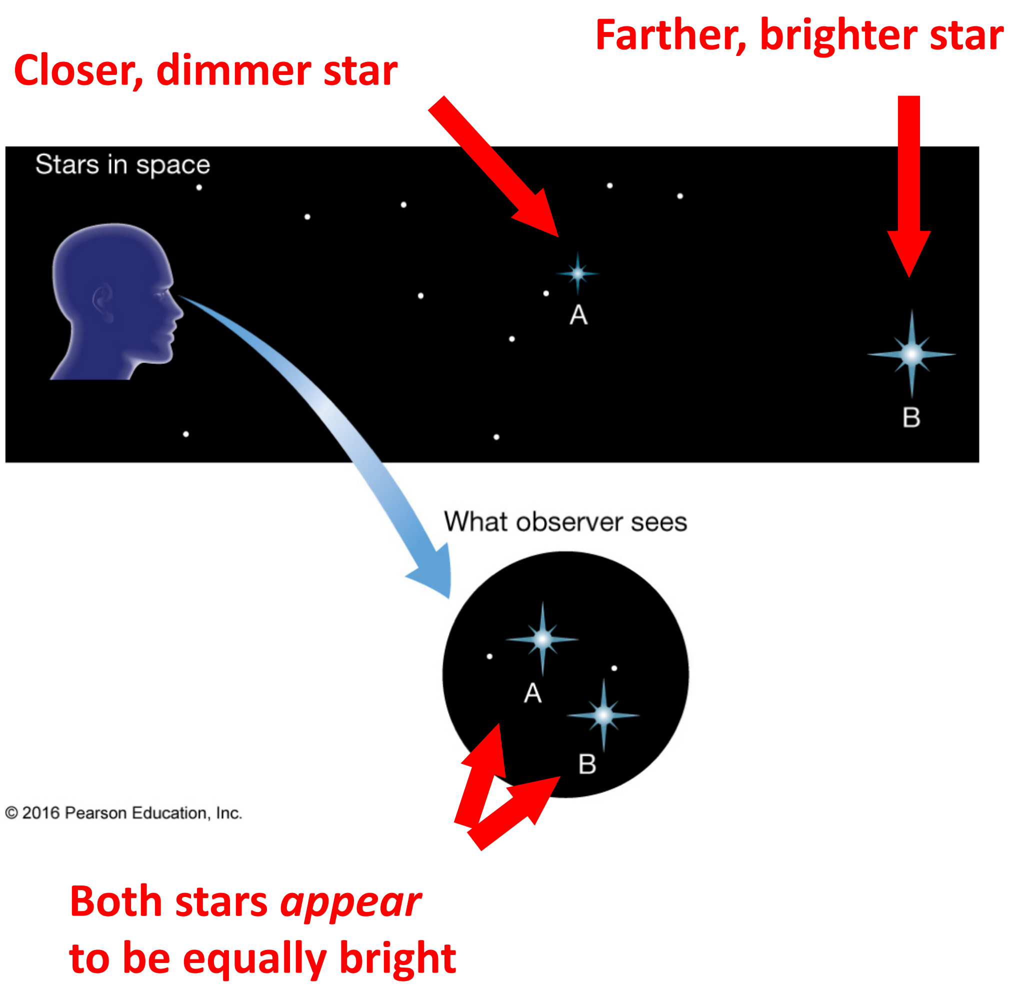
Luminosity (W)

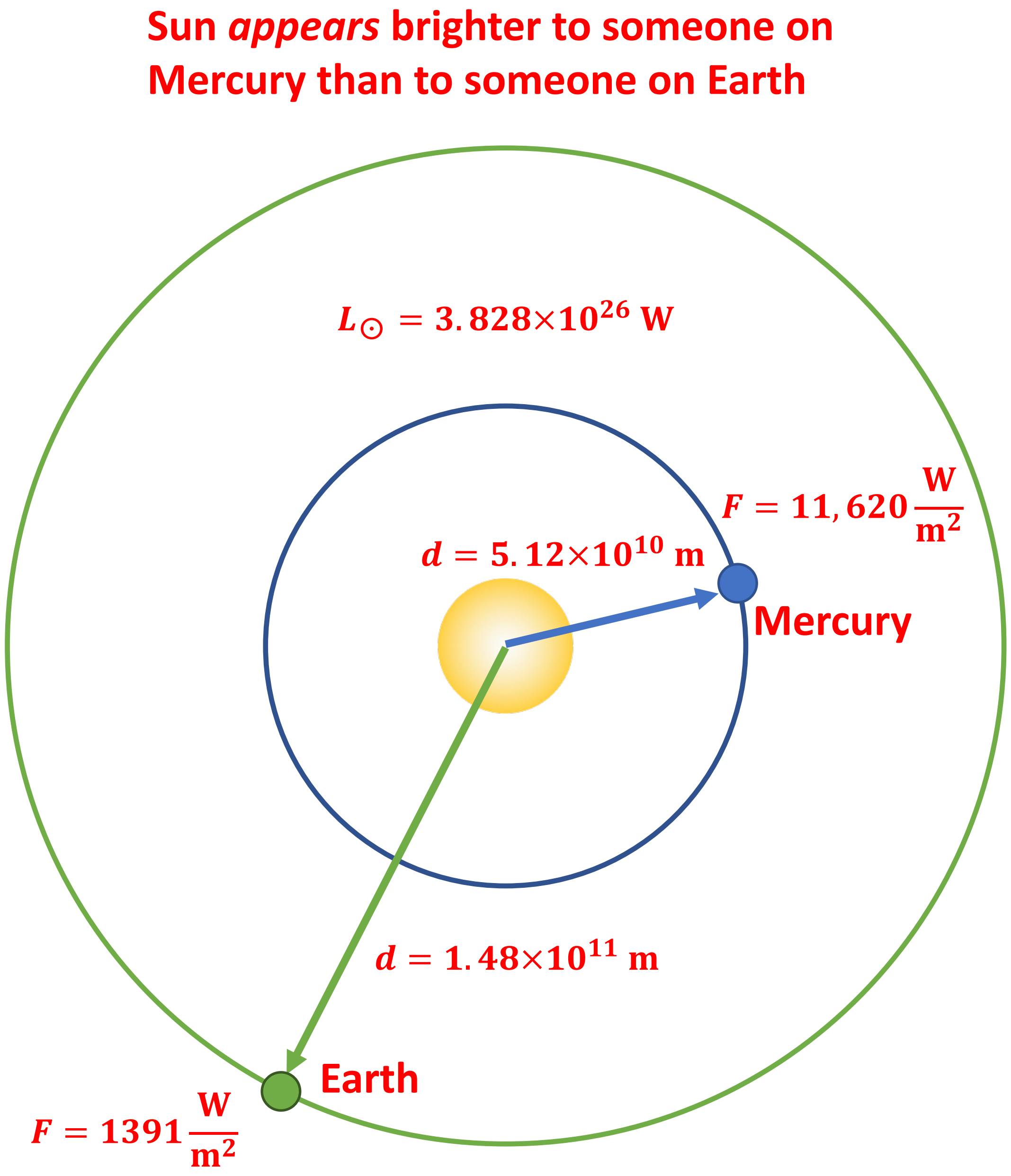
Flux (W/m^2)

Isotropic Emission!



Flux and Distance





Magnitude (of Brightness)

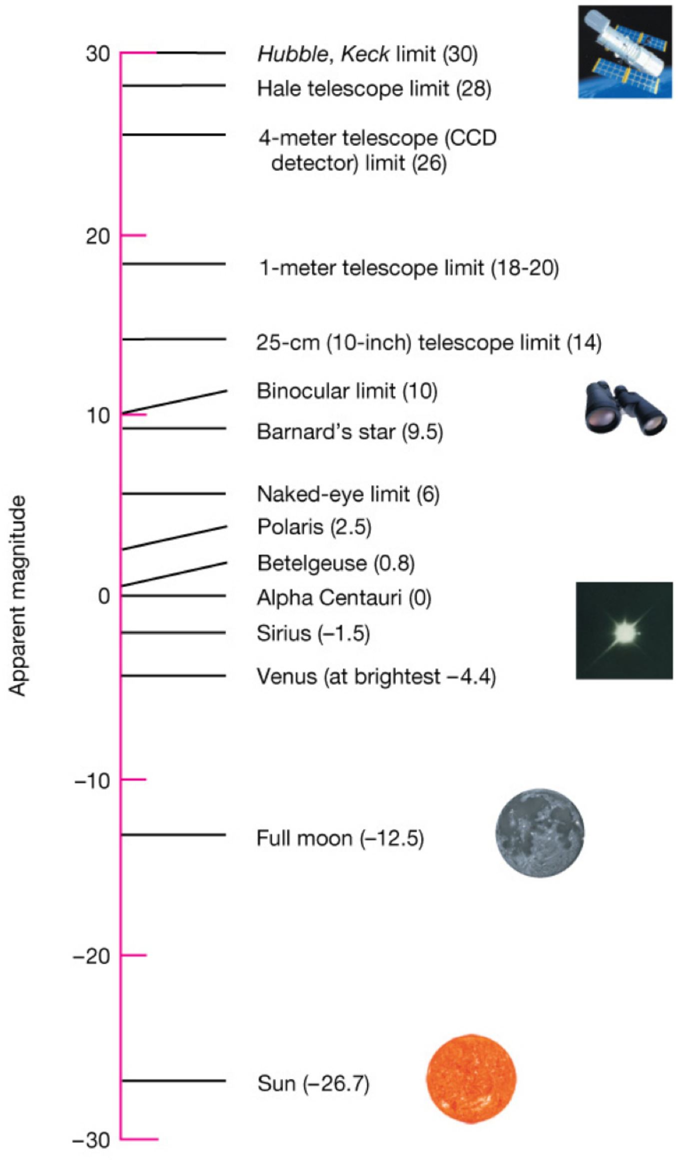
A common way to measure brightness is not in terms of luminosity or flux, but in terms of **magnitude** (of brightness)

- The **absolute magnitude (M)** replaces luminosity
- The **apparent magnitude (m)** replaces flux

Absolute magnitude is defined as the apparent magnitude measured if the star was at a distance of 10pc.

Magnitude runs "in reverse" – the larger the magnitude, the dimmer the object is.

- 0 is the magnitude of Vega
- 6 is the dimmest object the human eye can see
- Negative magnitude are objects brighter than Vega
- The Sun is the brightest object in the sky at -26.7



Brighter objects have smaller apparent magnitudes.

Converting Brightnesses

http://www.1728.org/magntudj.htm

Apparent magnitude

$$M = m - 5\log d + 5$$

Absolute magnitude

Distance to star (in pc)

Absolute magnitude

$$L = 2.51^{4.83 - M}$$

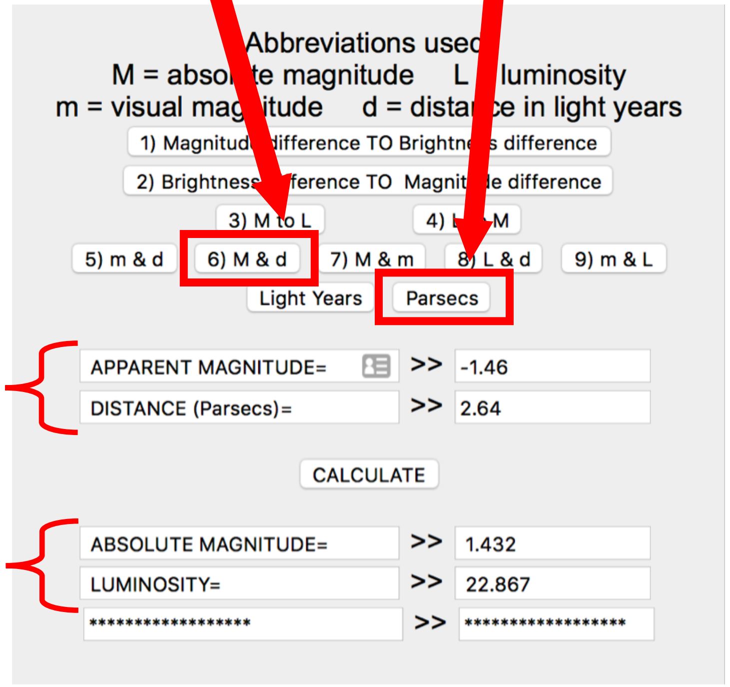
Luminosity (in $L_\odot$)

Use units of parsecs for distance

Use apparent magnitude (m) and distance (d)

Input your apparent magnitude (m) and distance (d)

Output absolute magnitude (M) and luminosity ($L_\odot$)



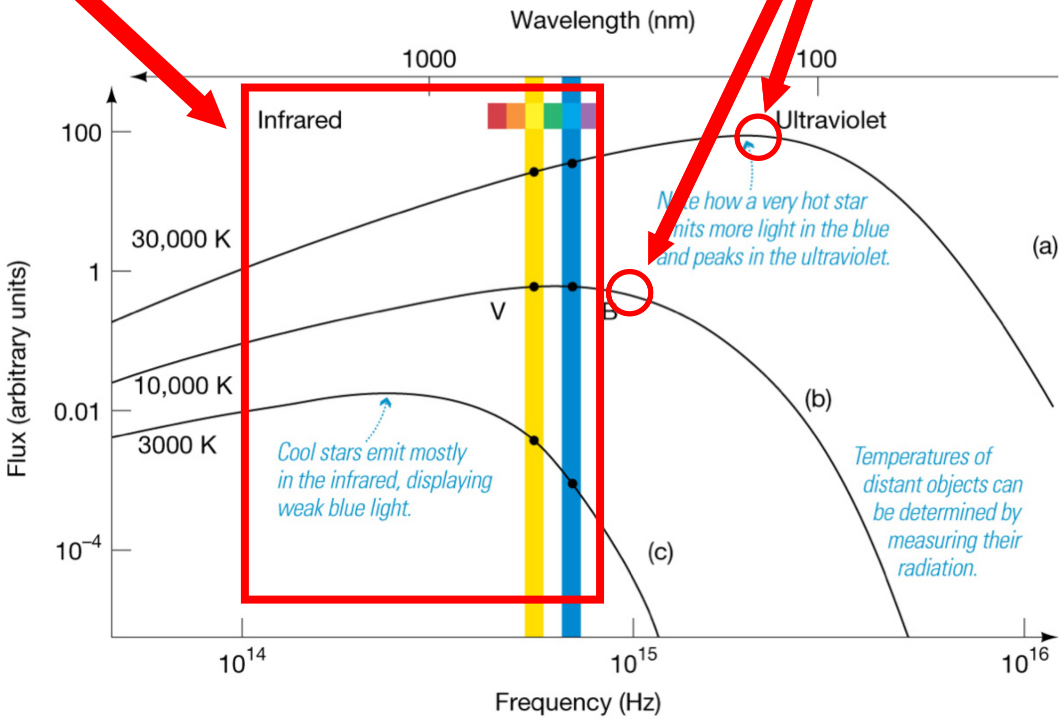
Stellar Temperature

$b = 2.9\times10^{-3}\ \mathrm{m\cdot K}$

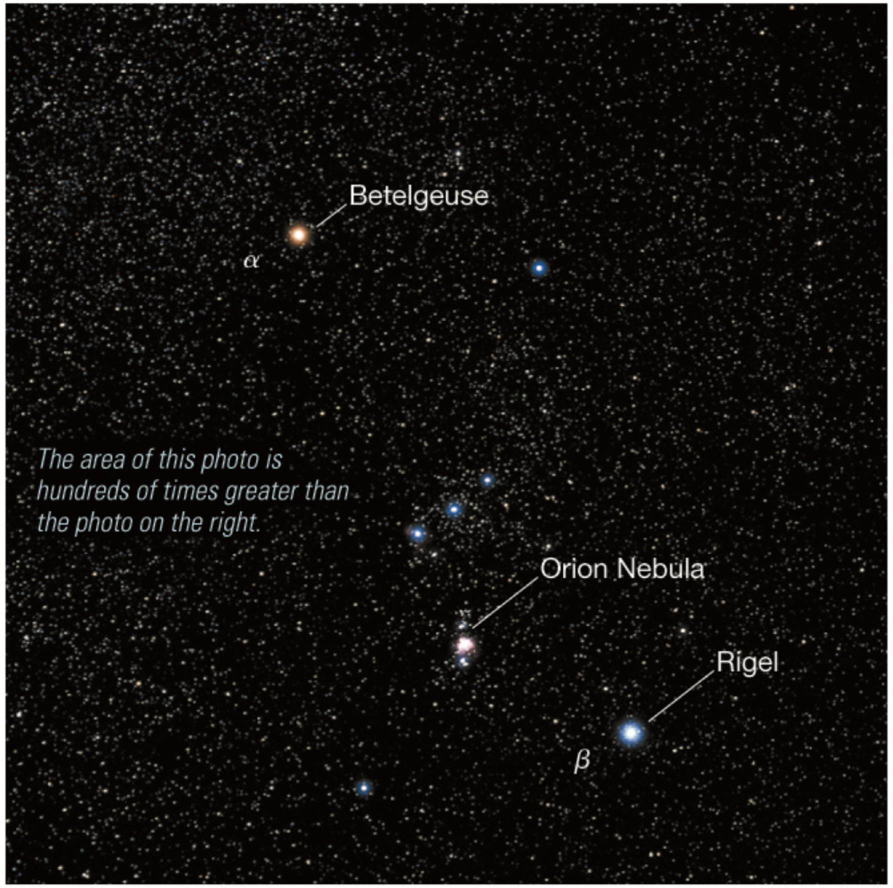
Wein's Law: $T = \dfrac{b}{\lambda_{max}}$

Range of frequencies available to Earth-based telescope.

What to do about peaks outside of that range?



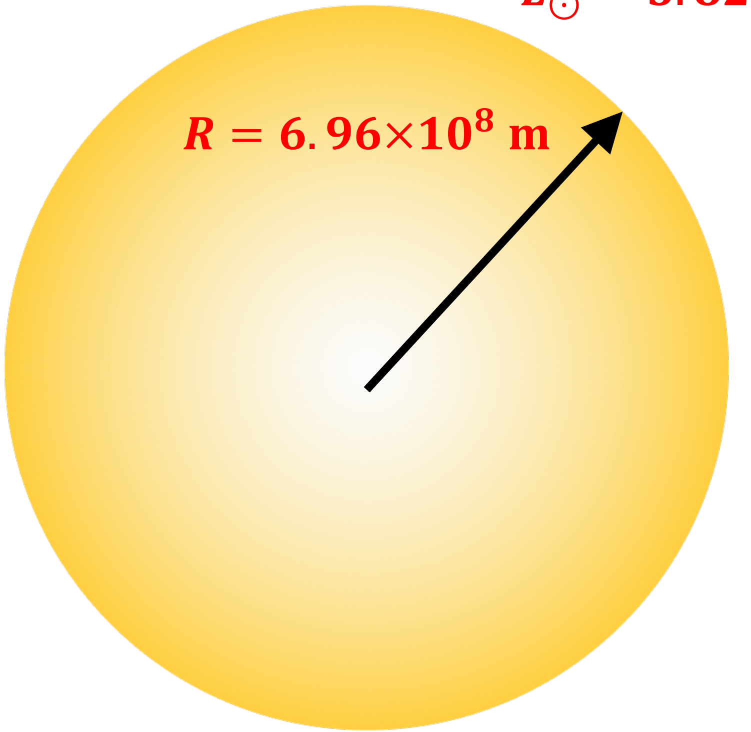
Need a method that doesn't depend on entire spectrum!



Stefan-Boltzmann Law

$\boldsymbol{T = 5800\ \mathrm{K}}$

$\boldsymbol{L_{\odot} = 3.828 \times 10^{26}\ \mathrm{W}}$



A blackbody with a surface temperature T and has an **irradiance I**, the flux at the surface of the blackbody, given by:

$$I = \sigma T^4$$

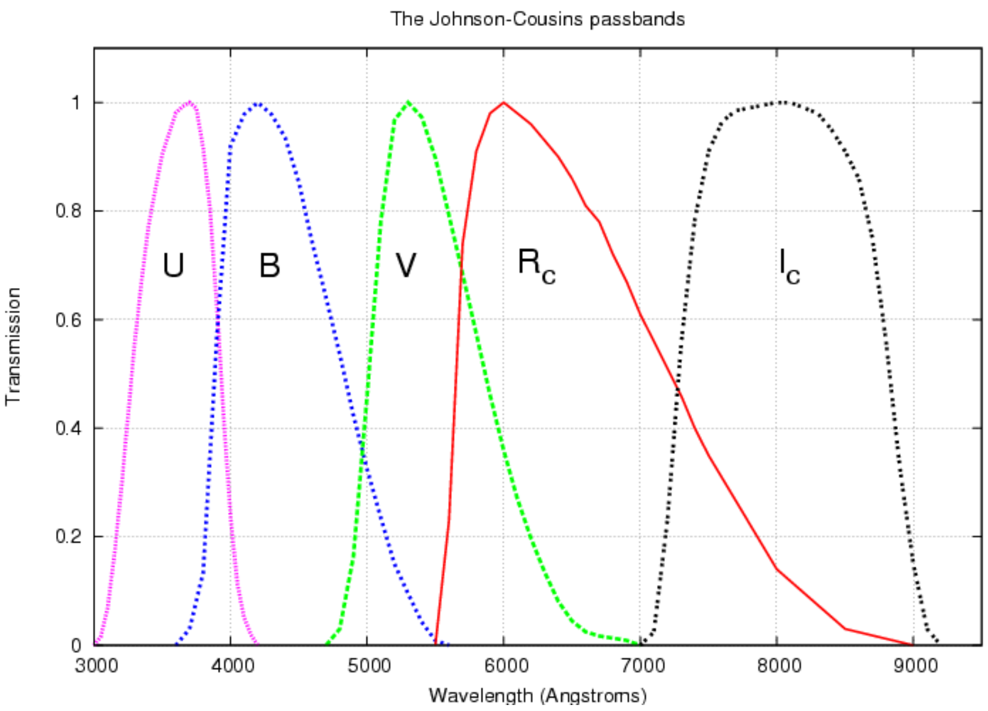
$\boldsymbol{\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}}$

Solar irradiance:

$$\boldsymbol{I = \left(5.67\times10^{-8}\right)(5800)^4 = 6.42\times10^{7}\ \frac{\mathrm{W}}{\mathrm{m}^2}}$$

$$\boldsymbol{L = 4\pi R^2 \cdot I = 4\pi\left(6.96\times10^{8}\right)^2\left(6.42\times10^{7}\right) = 3.91\times10^{26}\ \mathrm{W}}$$

Temperature-Color Relationship



Apparent magnitude in B

Apparent magnitude in V

Color Index: $B - V = m_B - m_V$

→ By definition, B – V = 0 for Vega. So B – V < 0 is a star bluer than Vega, and B – V > 0 is a star redder than Vega. T ~ 10,000K for Vega, so if B – V < 0, T > 10,000K, and if B – V > 0, T < 10,000K.

$$T \approx \frac{9000\mathrm{K}}{(B - V) + 0.93}$$

→ Works for stars with $4000\mathrm{K} \leq T \leq 11000\mathrm{K}$, or $-0.1 \leq B - V \leq 1.4$

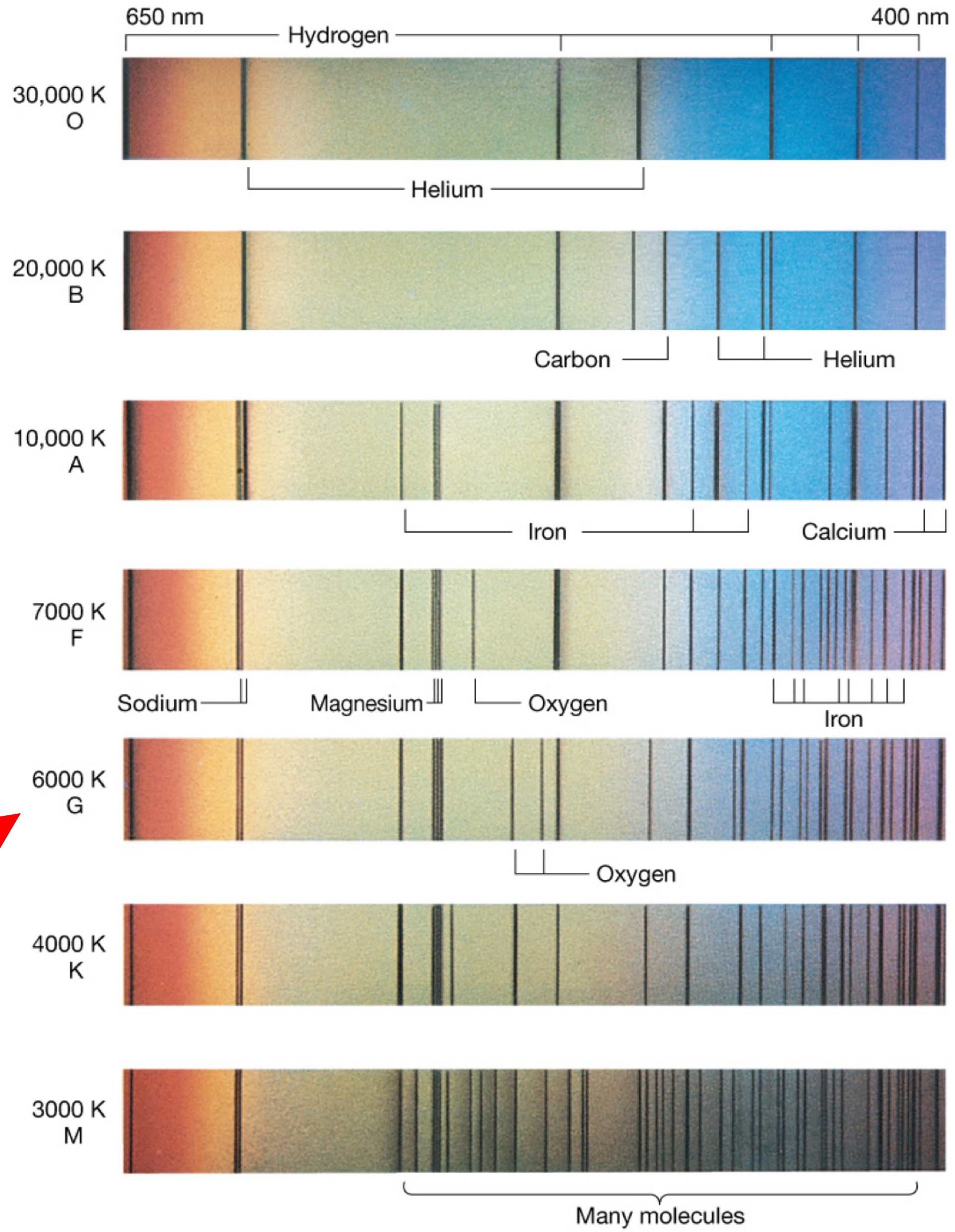
Stellar Classification

Stars are classified by their **emission spectra**, which varies with temperature. The common classification system is the **Harvard system**.

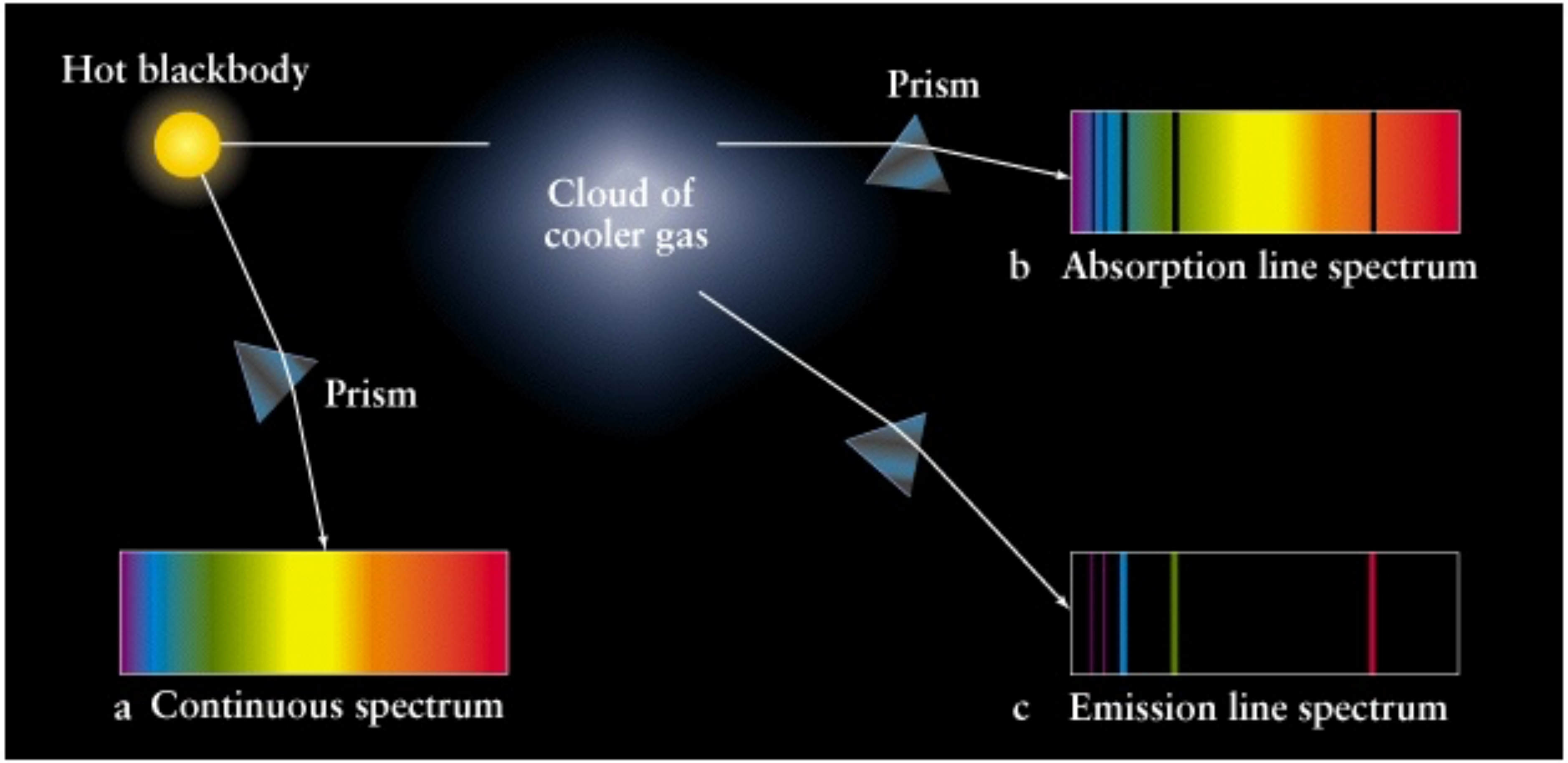
From **hottest-to-coldest**, the classes are:

- **O** (Oh)
- **B** (Be)
- **A** (A)
- **F** (Fine)
- **G** (Guy/Girl)
- **K** (and Kiss)
- **M** (Me)

Sun in a G-type star, with $T = 5800$ K

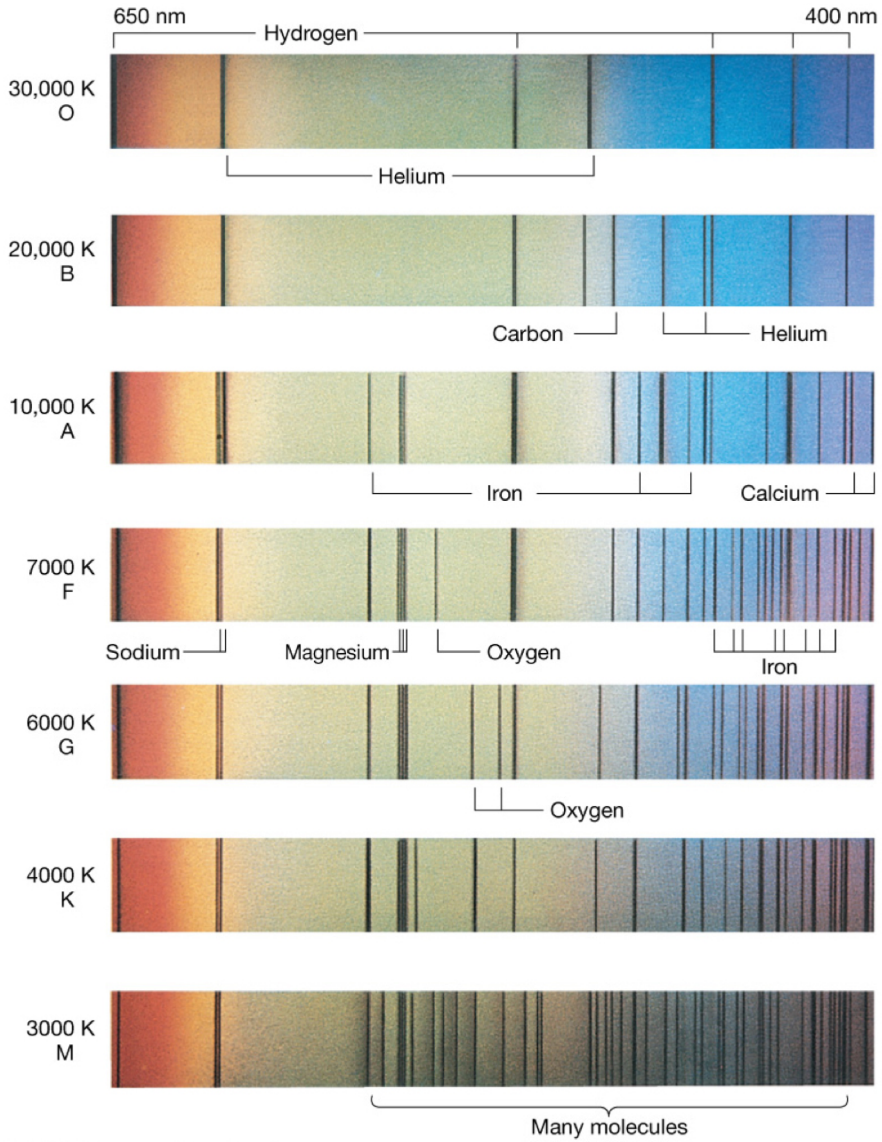


Reminder: Emission vs. Absorption Spectra

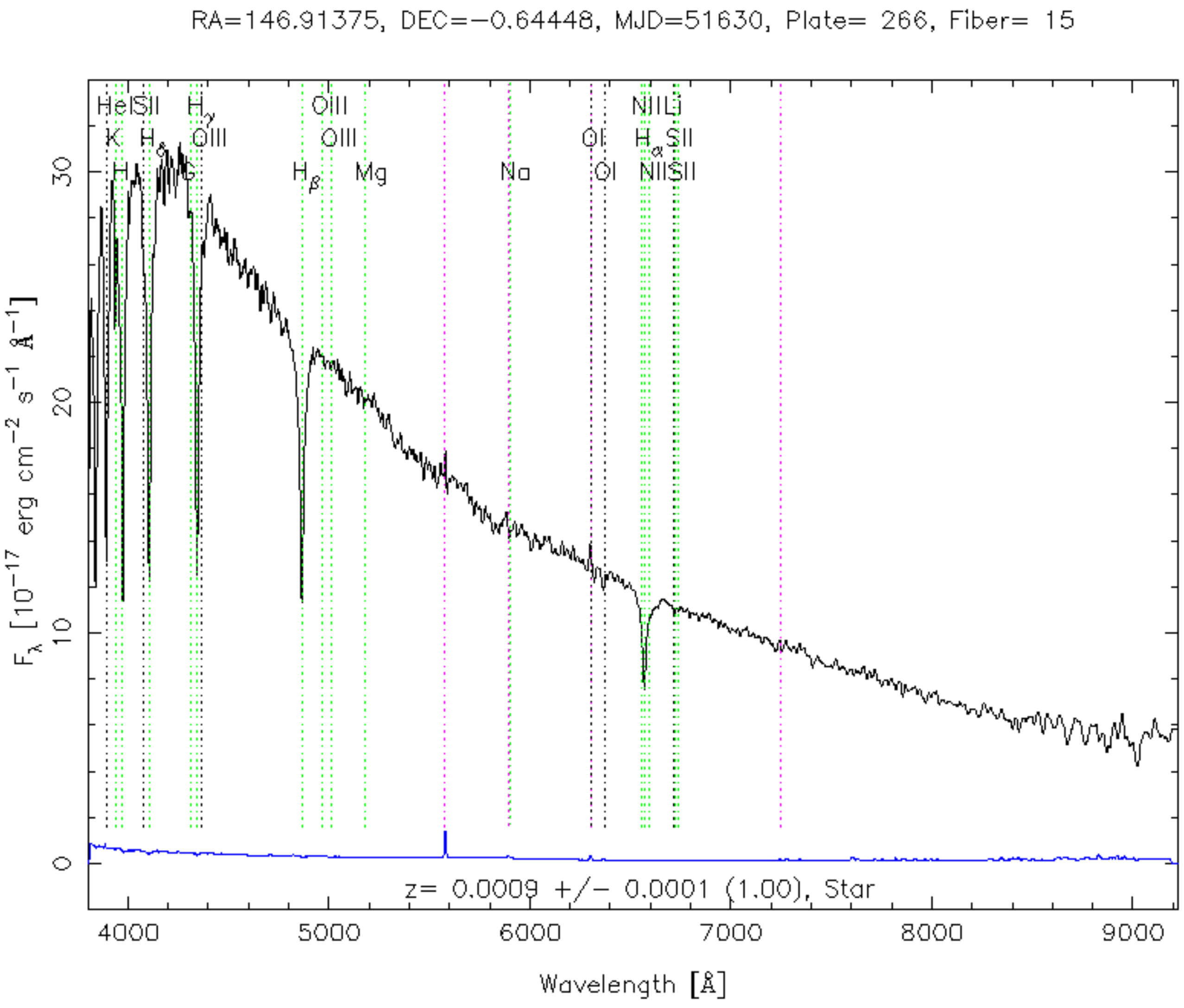


Spectral Class Characteristics

Spectral Type	Temperature (Kelvin)	Spectral Lines
O	28,000 - 50,000	Ionized helium
B	10,000 - 28,000	Helium, some hydrogen
A	7500 - 10,000	Strong hydrogen, some ionized metals
F	6000 - 7500	Hydrogen, ionized calcium (labeled H and K on spectra) and iron
G	5000 - 6000	Neutral and ionized metals, especially calcium; strong G band
K	3500 - 5000	Neutral metals, sodium
M	2500 - 3500	Strong titanium oxide, very strong sodium



Example 1: Spectrum of Unknown Star



Look for hydrogen lines first!

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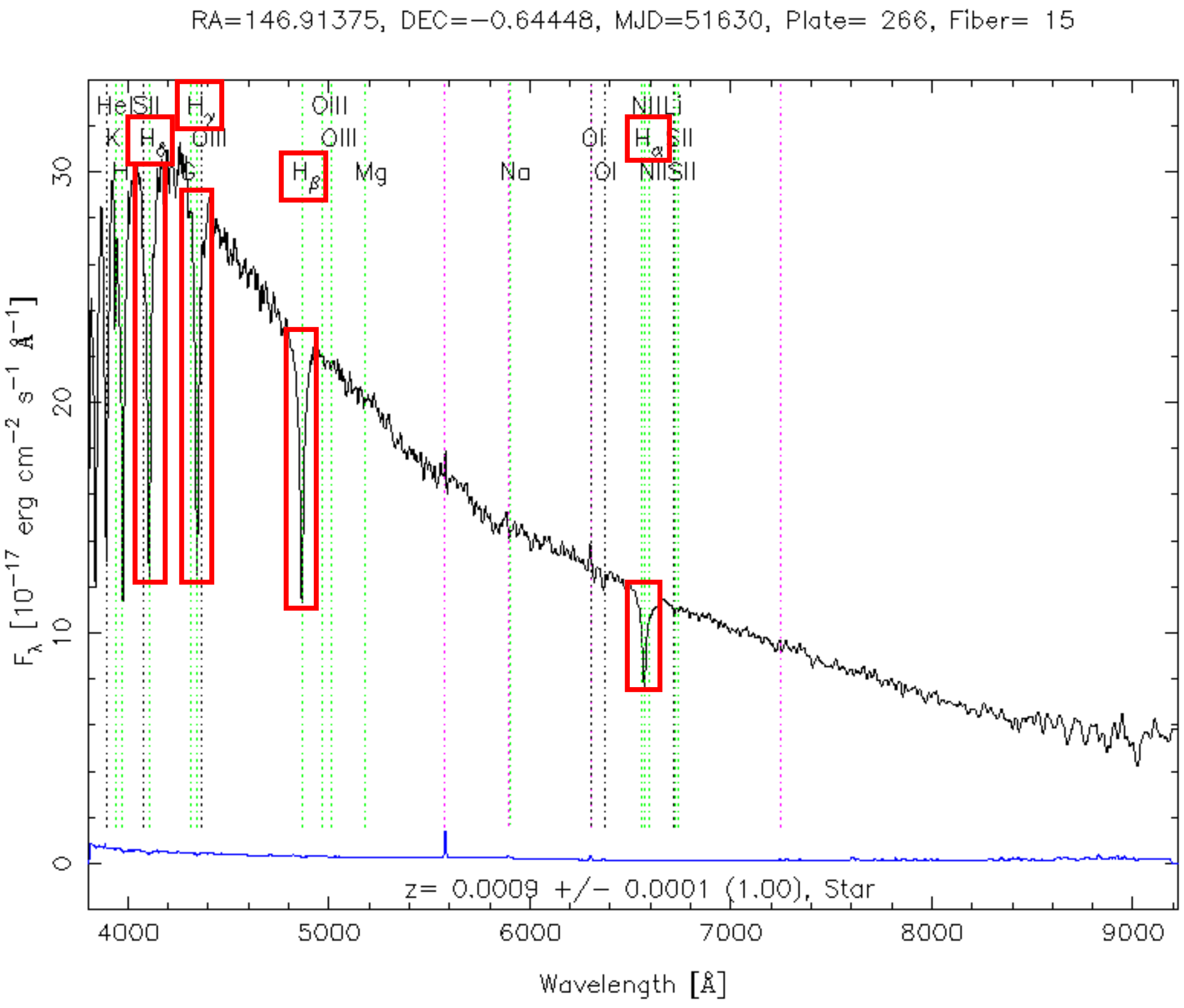

Hydrogen lines

→ B, A, or F type star

Spectral Type	*Temperature (Kelvin)*	*Spectral Lines*
[illegible]	28,000 - 50,000	Ionized helium
B	10,000 - 28,000	Helium, some hydrogen
A	7500 - 10,000	Strong hydrogen, some ionized metals
F	6000 - 7500	Hydrogen, ionized calcium (labeled H and K on spectra) and iron
[illegible]	5000 - 6000	Neutral and ionized metals, especially calcium; strong G band
[illegible]	3500 - 5000	Neutral metals, sodium
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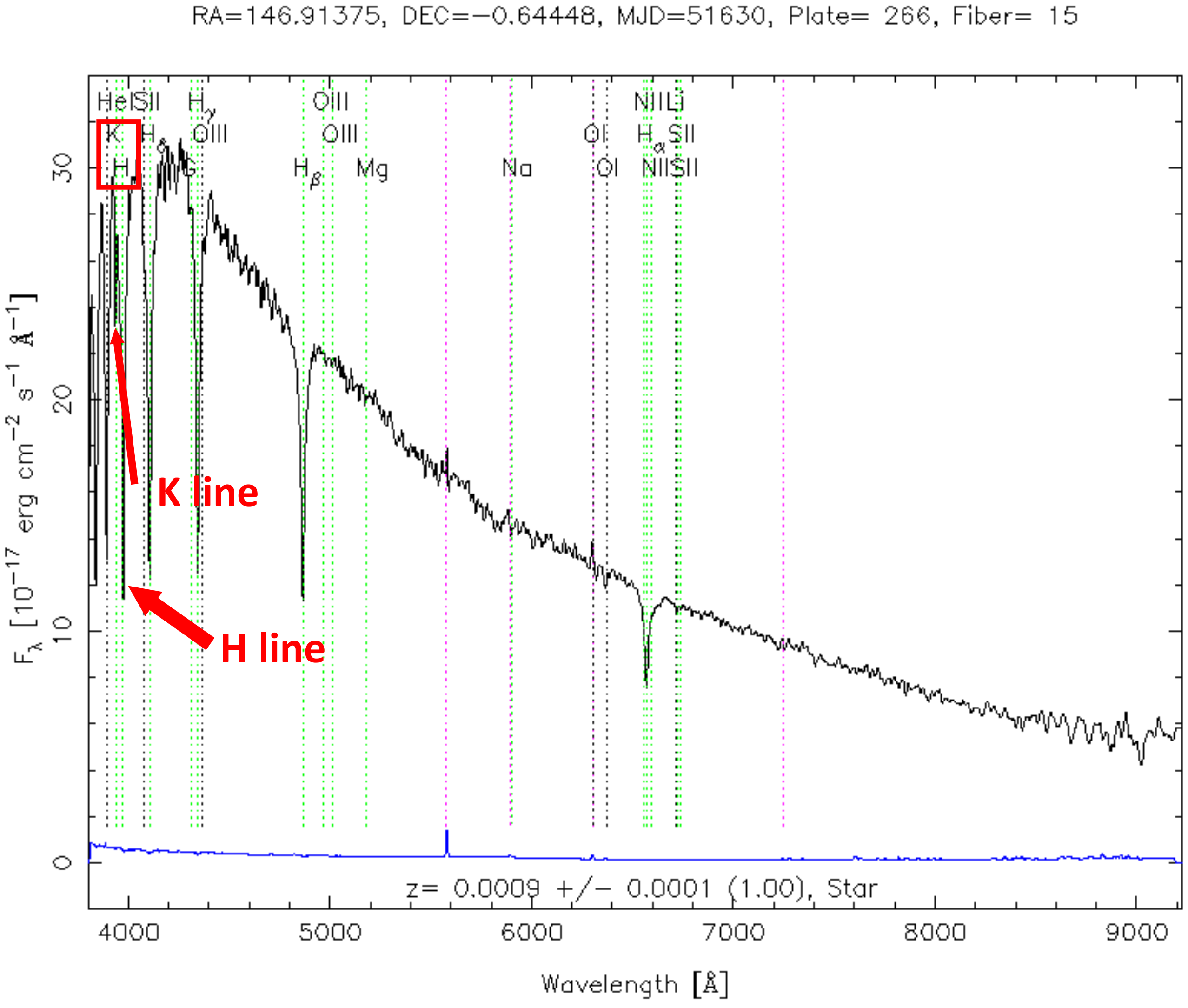

H and K lines present
→ F type star!

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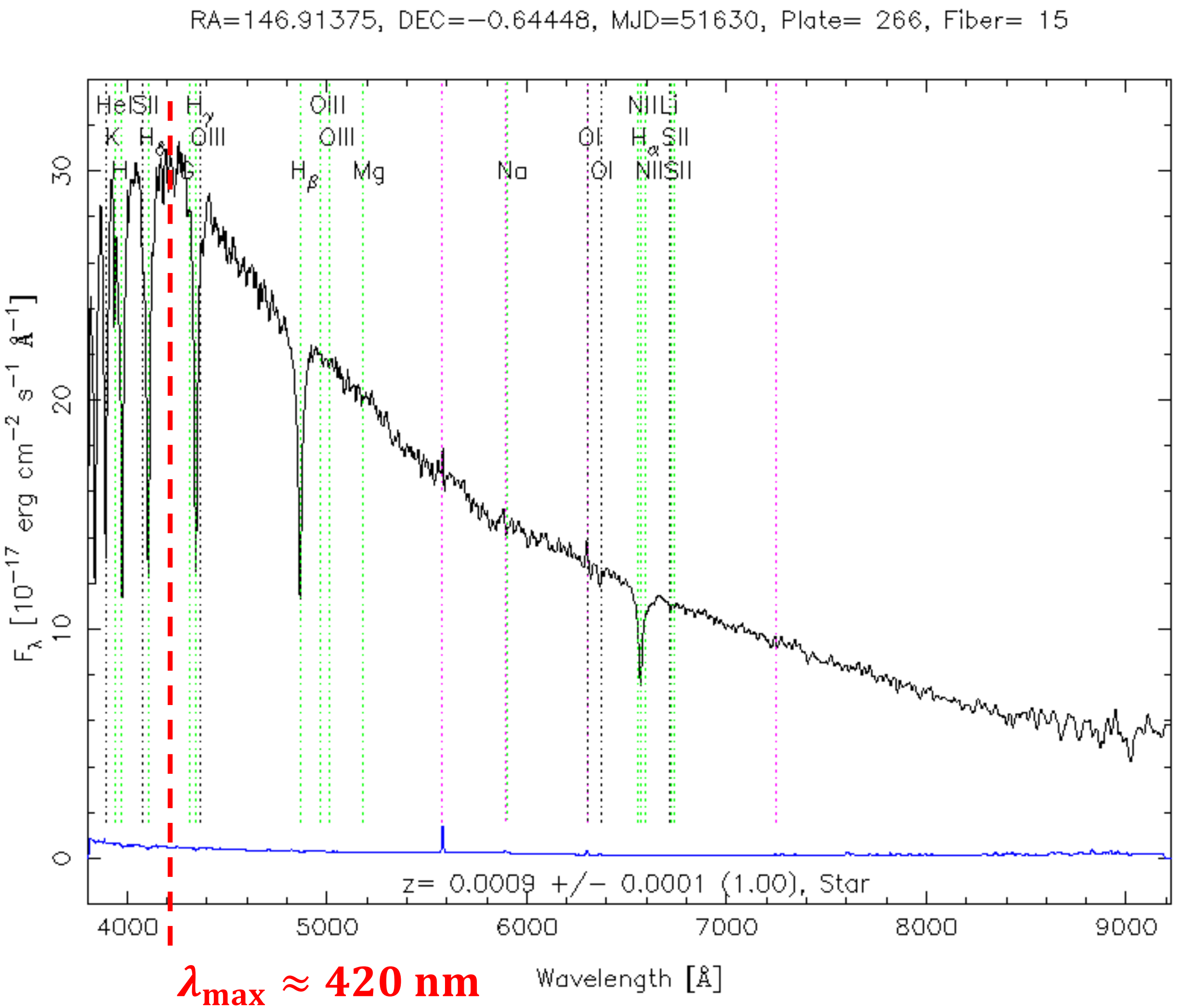

Confirmation with Wein's Law:

$$T = \frac{b}{\lambda_{\max}} = \frac{2.9\times10^{6}\ \text{nm}\cdot\text{K}}{420\ \text{nm}} = \boxed{6905\ \text{K}}$$

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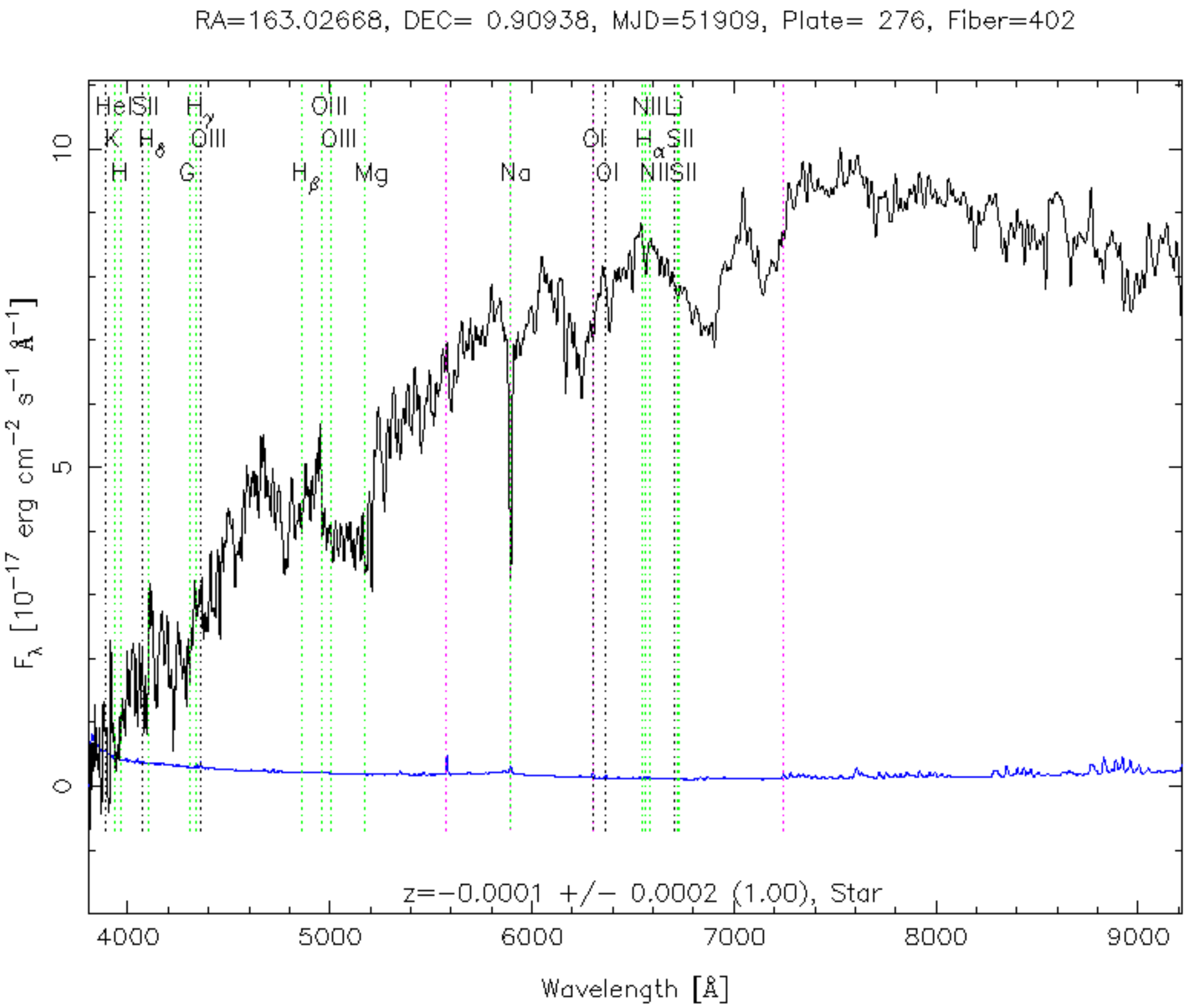
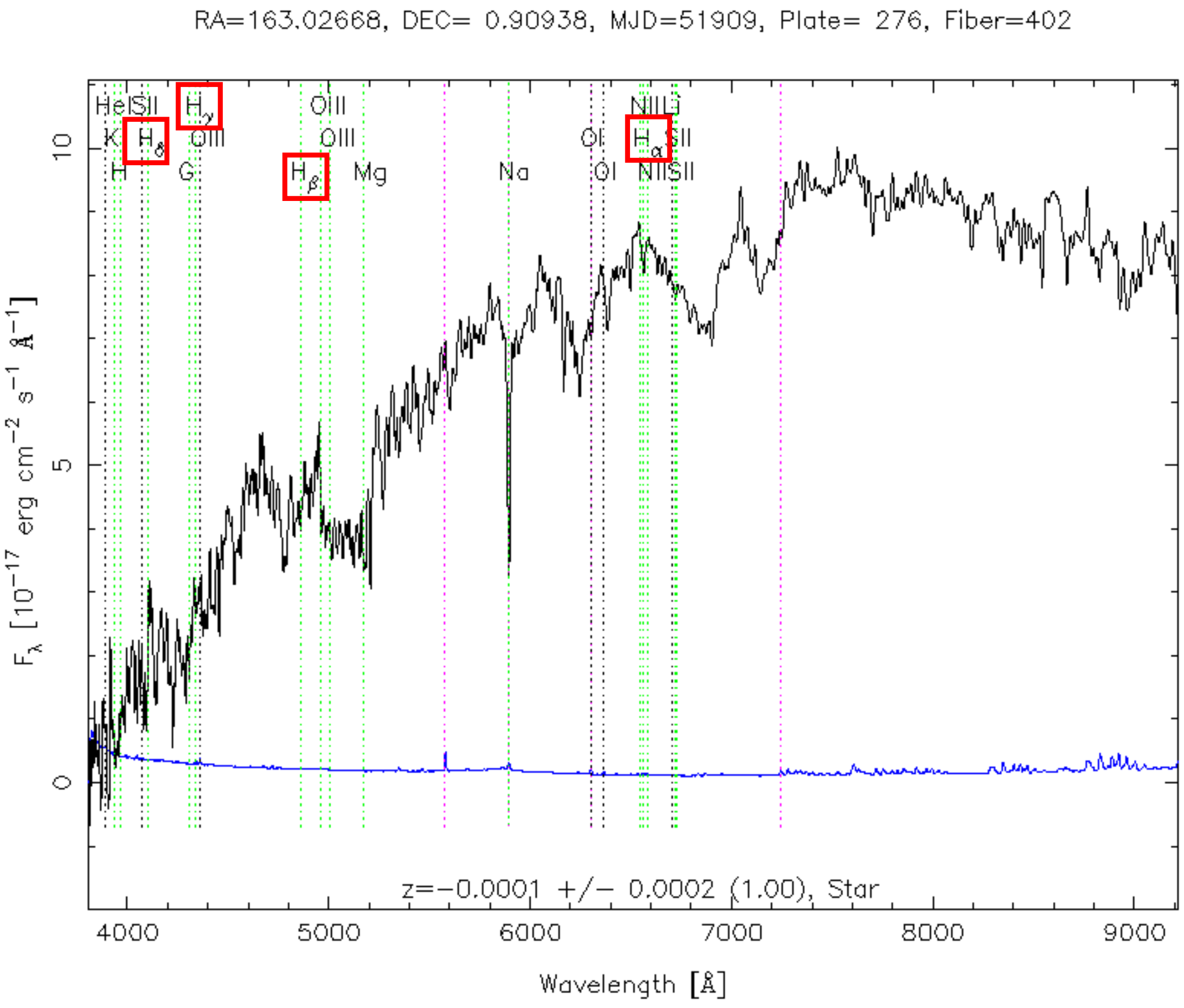


Look for hydrogen lines first!

Example 2: Spectrum of Unknown Star



No hydrogen lines

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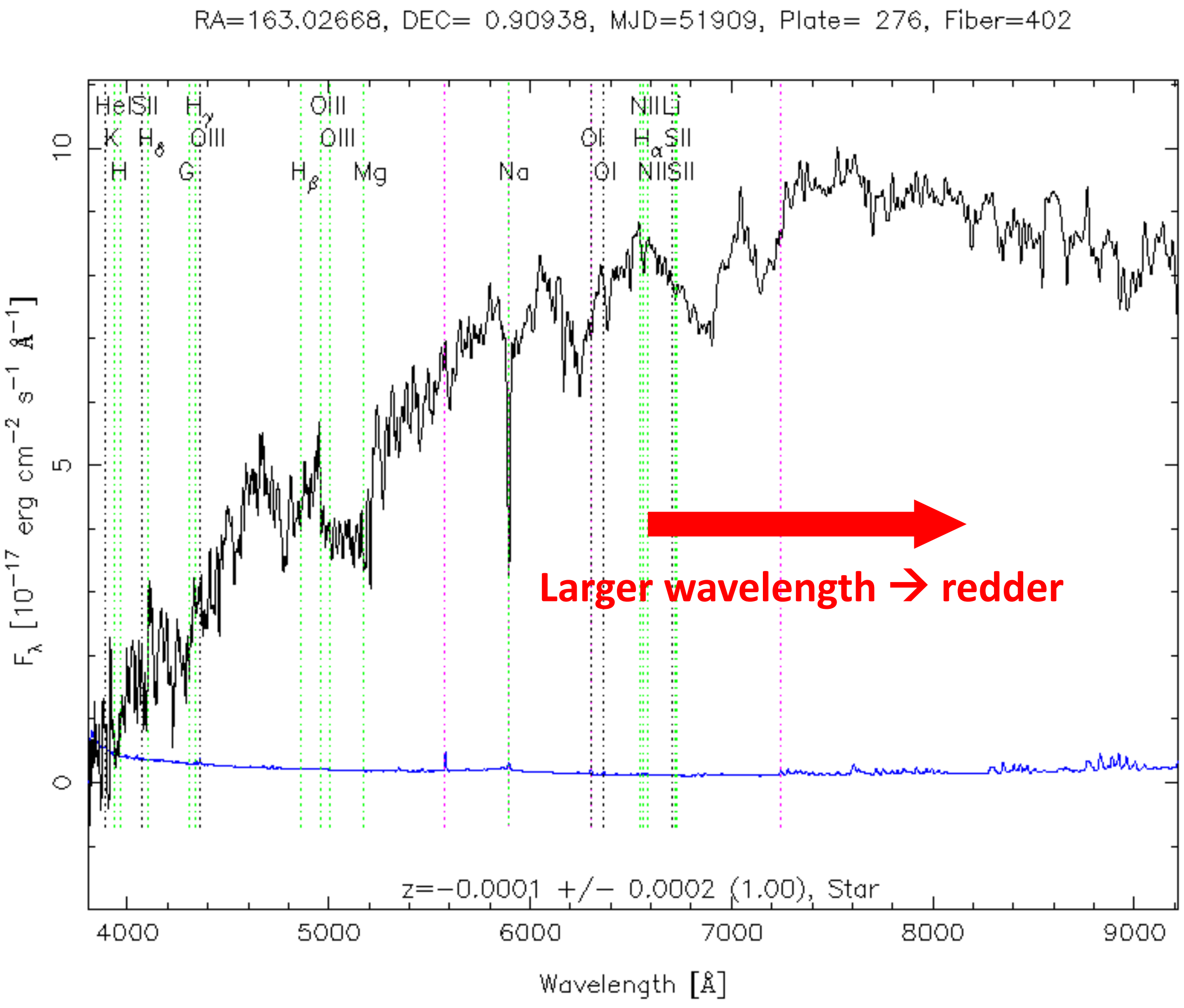

Redder than previous F-type star → colder than F-type!

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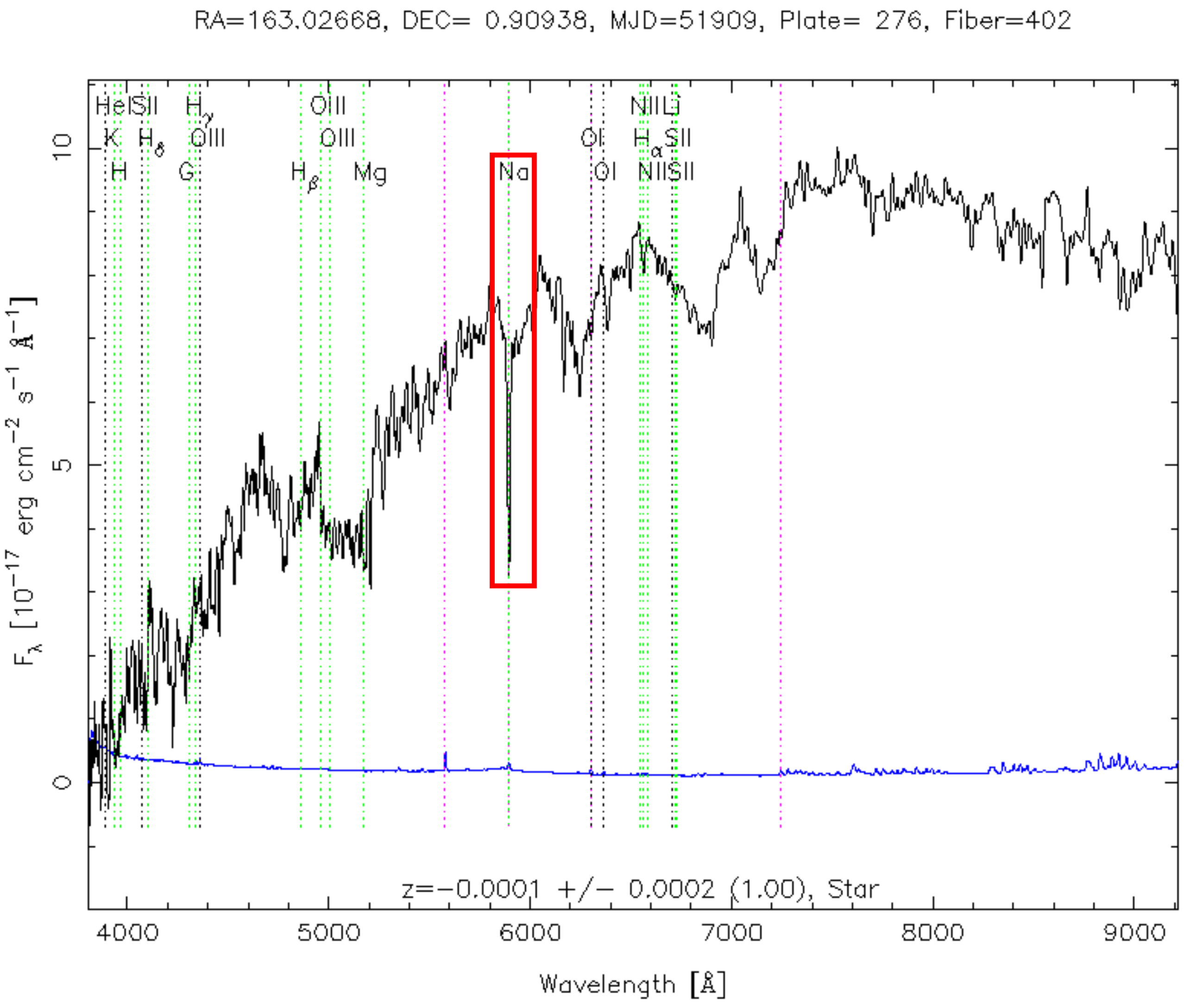

Prominent sodium (Na) line → K or M type star

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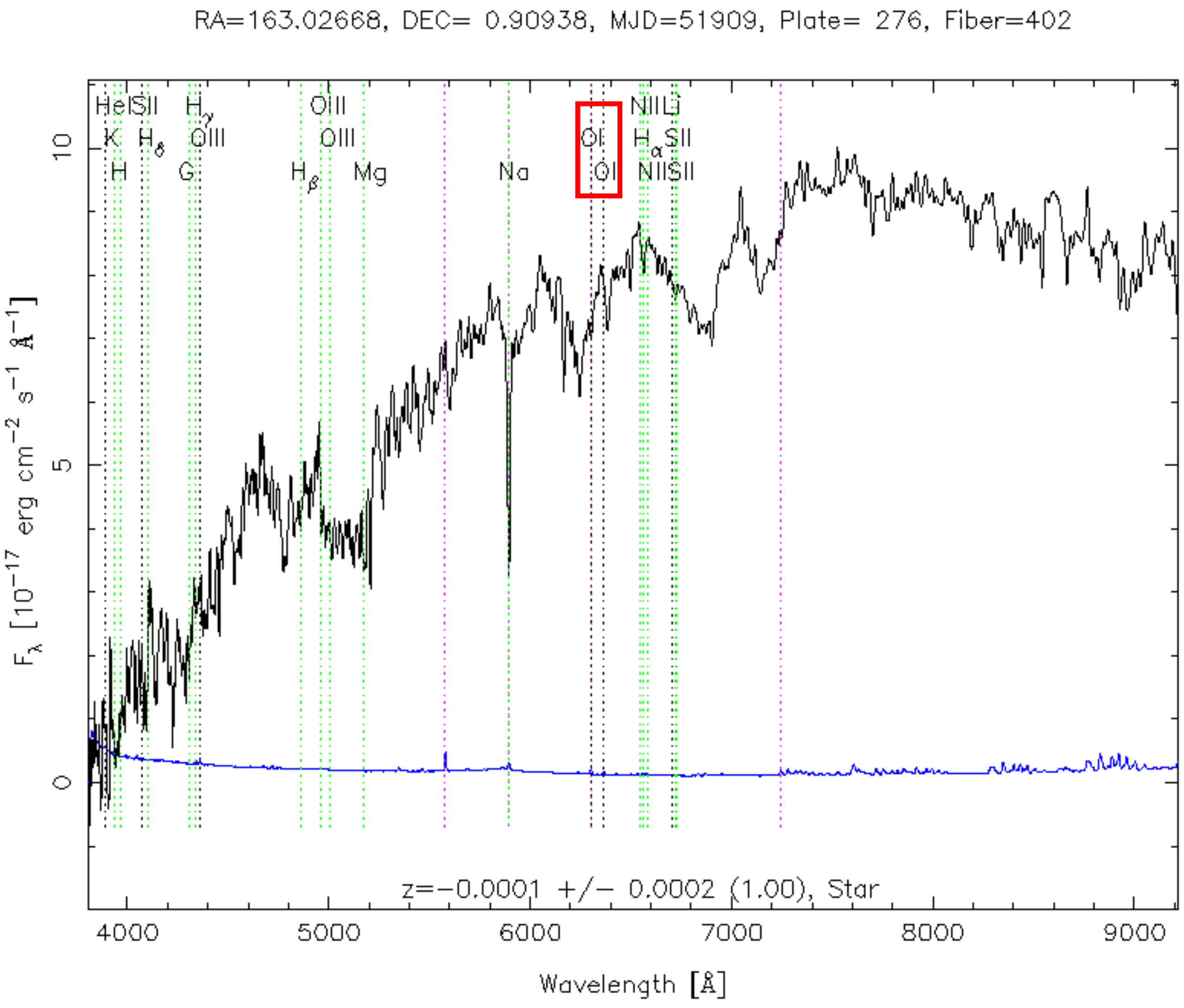

No titanium-oxide (TiO) lines → K-type star!

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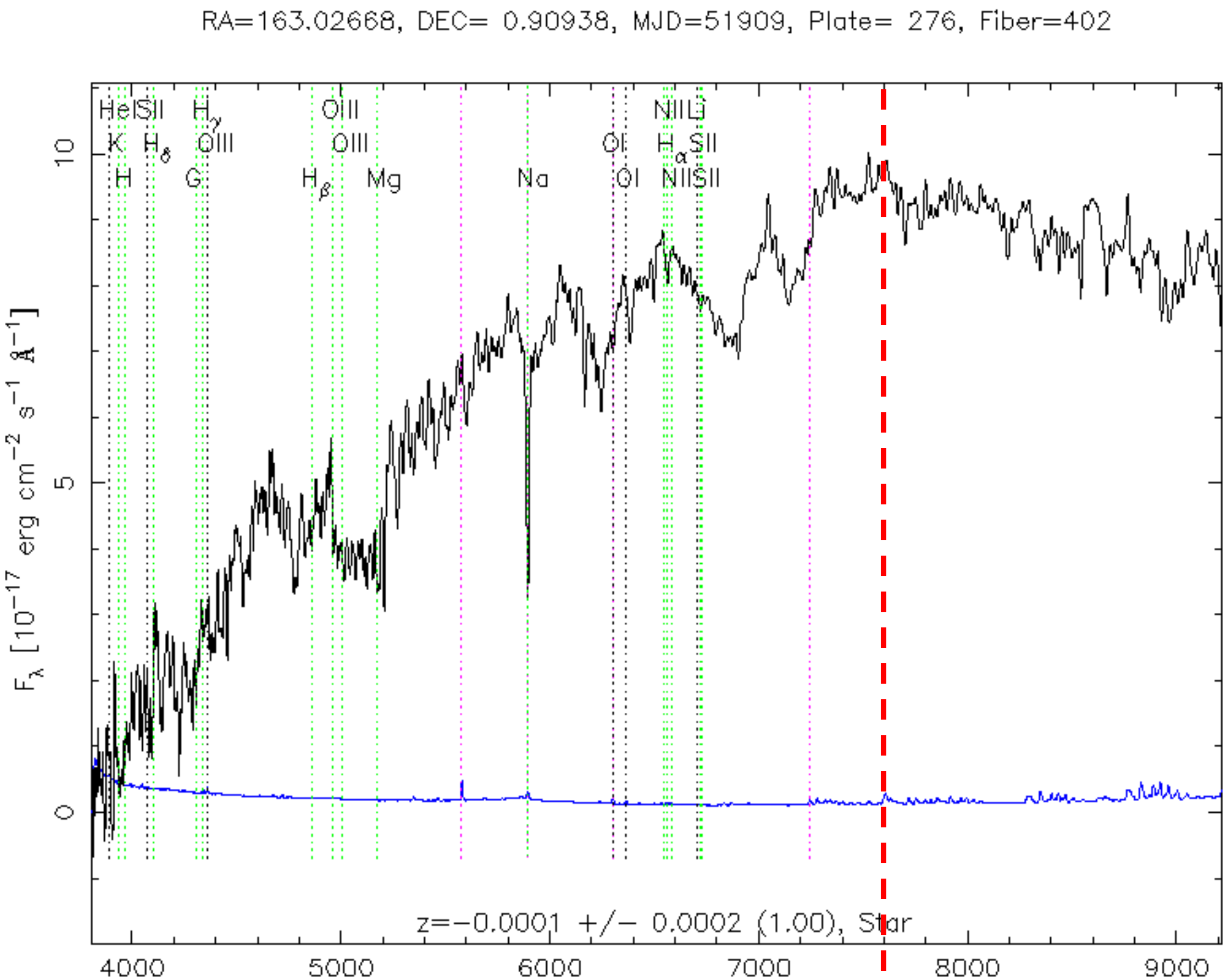

$\lambda_{\max} \approx 760\ \mathrm{nm}$

Confirmation with Wein's Law:

$$T = \frac{b}{\lambda_{\max}} = \frac{2.9\times10^{6}\ \mathrm{nm}\cdot\mathrm{K}}{760\ \mathrm{nm}} = \boxed{3816\ \mathrm{K}}$$

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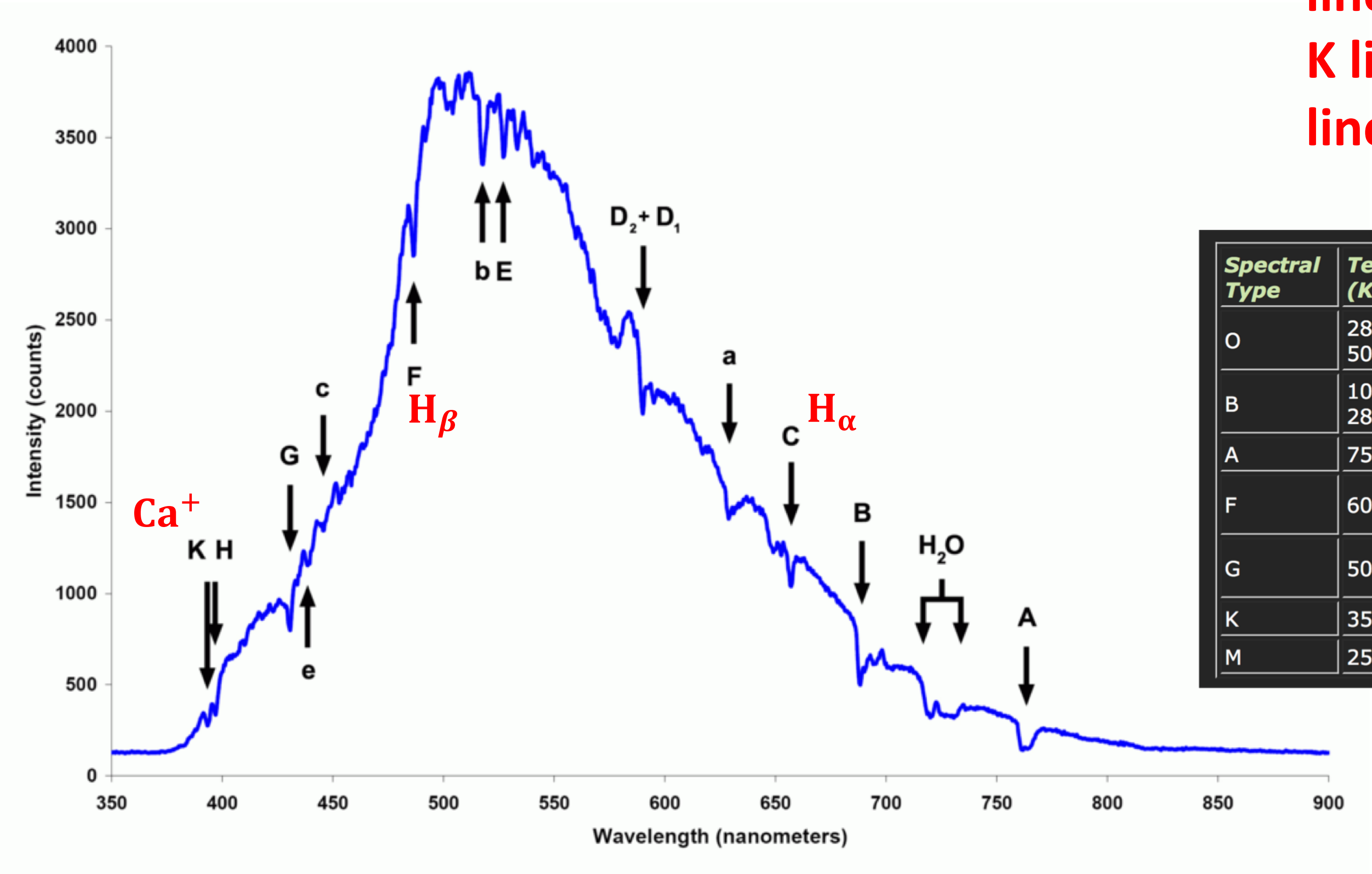
Example 3: The Sun



Weak/missing hydrogen lines, ionized calcium(H & K lines) present, strong G line → G-type star!

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Stellar Radii

If we can find the **luminosity** and the **irradiance** of a star, then we can figure out its **radius**:

Definition of flux

All in SI units

$$I = \boxed{\sigma T^4} = \boxed{\frac{L}{4\pi R^2}} \Rightarrow R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Stefan-Boltzmann law

Units of $R_\odot$

Units of $L_\odot$

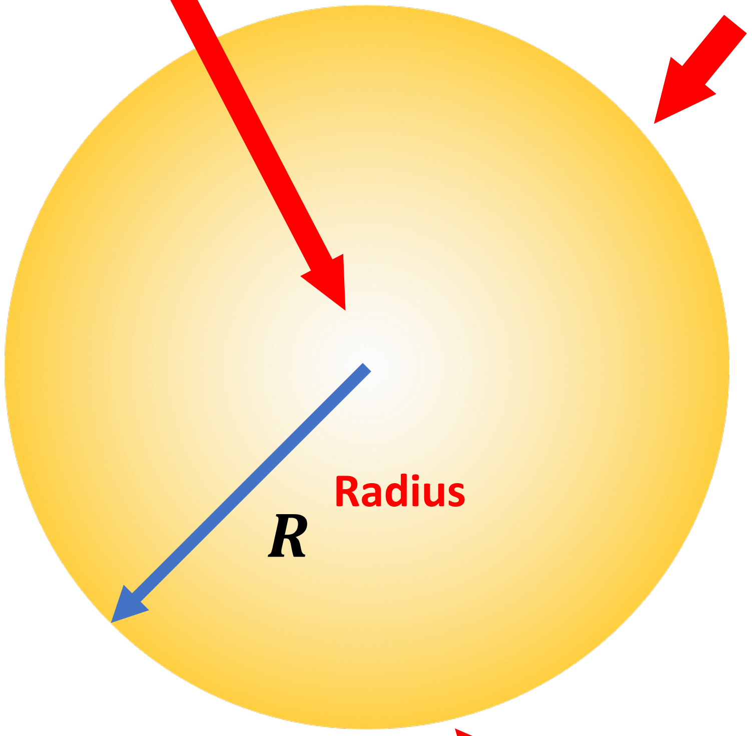
Simple radius-luminosity-temperature (RLT) relationship.

$$R = \frac{\sqrt{L}}{T^2}$$

Units of $T_\odot$

We treat the luminosity as coming from the center

Which spreads out over the surface, creating a flux

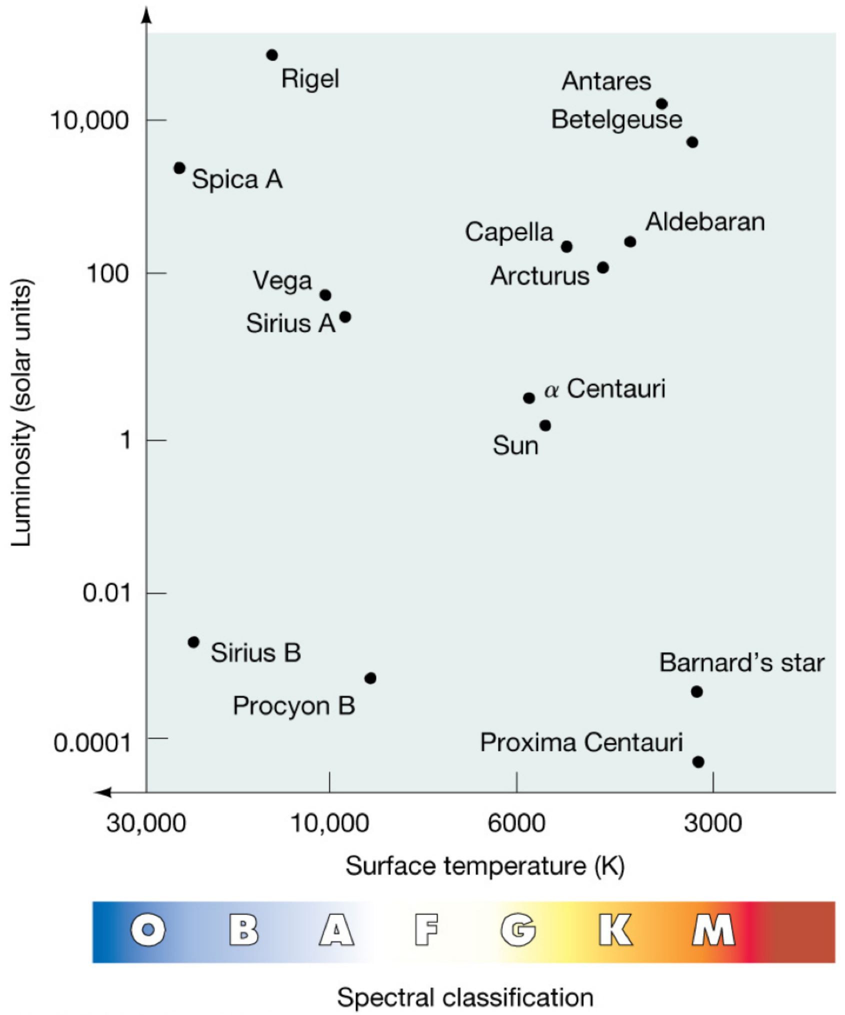


We call this flux the Irradiance I

Trends in Stellar Characteristics

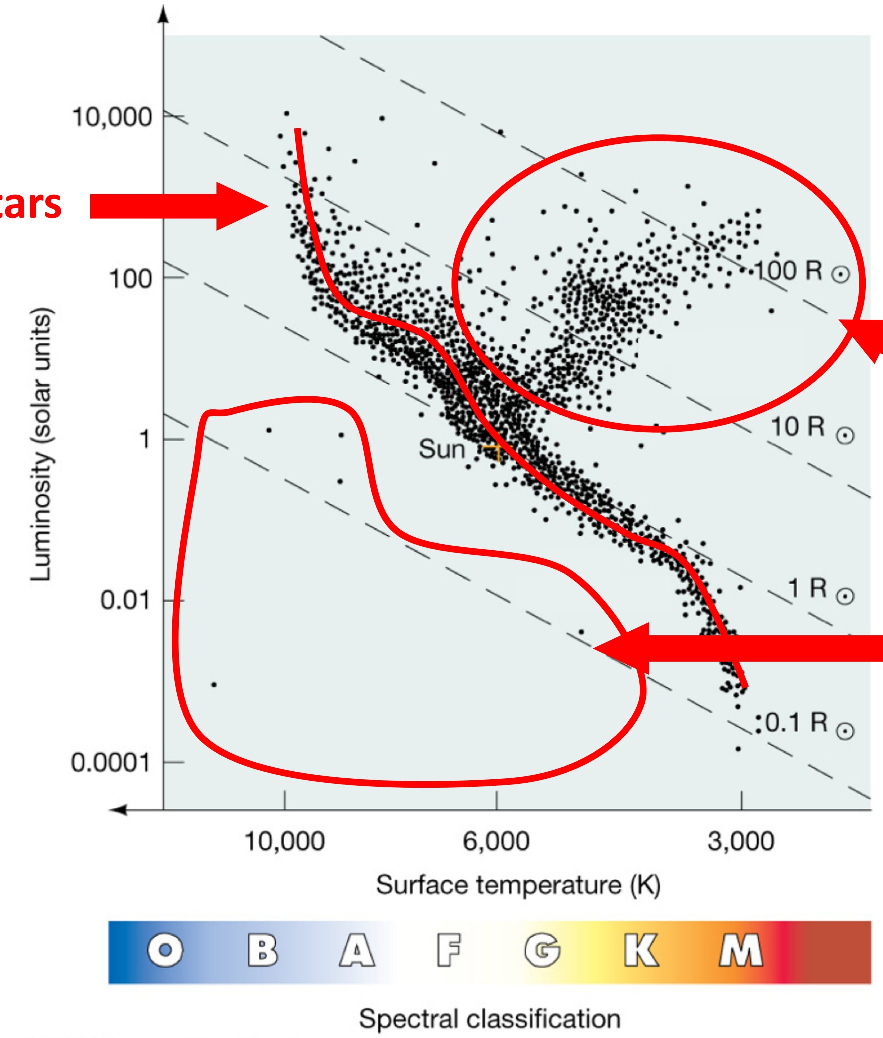
Plotting **luminosity vs. temperature** for our nearest neighbors doesn't look like much, as you can see in the figure. But if we expand this to farther stars, and stars at different points in their evolution, we find very clear trends in these characteristics.

- Note that while we're plotting luminosity vs. temperature, we're actually plotting against spectral class, so temperature increases to the **left** of the diagram, not to the right.
- Stars are going to be, essentially, of two types: **main sequence stars**, which are healthy stars burning hydrogen, and **off sequence stars**, which are at other points in their evolution.



The Hertzsprung-Russell (H-R) Diagram

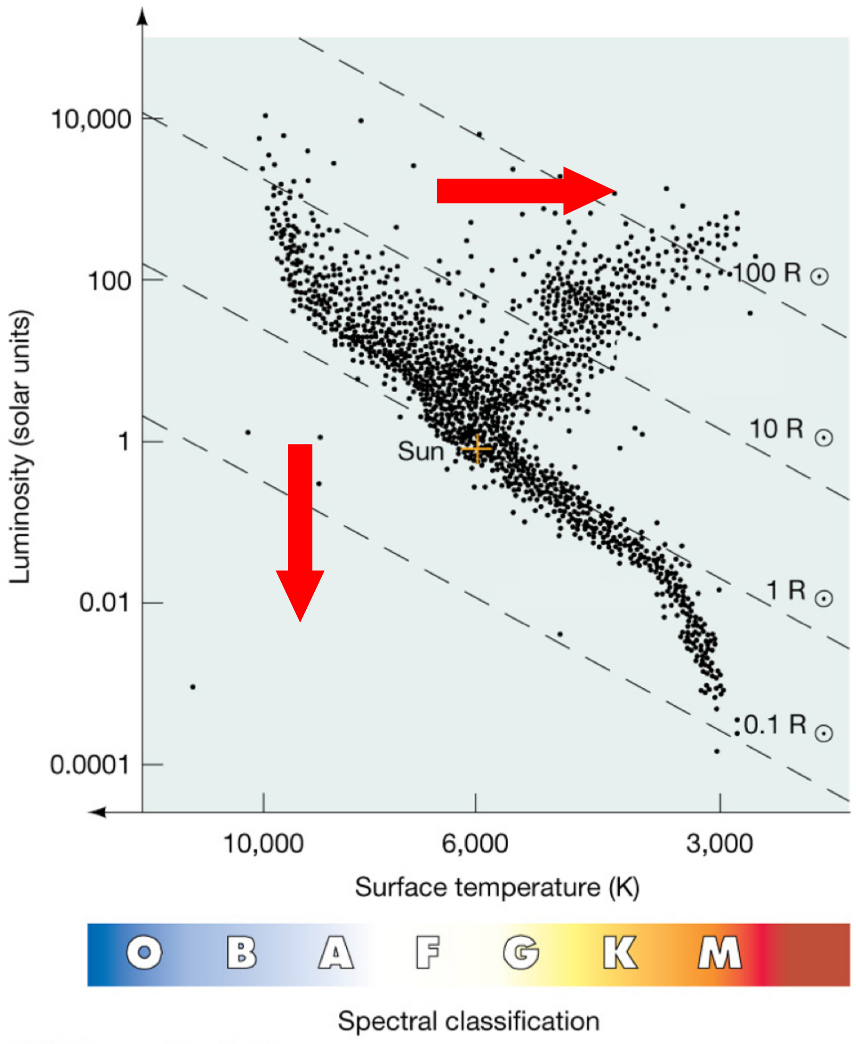
Healthy, main sequence stars



Two regions of off-sequence stars, which are at different points in their evolution after they run out of hydrogen to burn

Off-Sequence Regions

RLT Relationship: $R = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

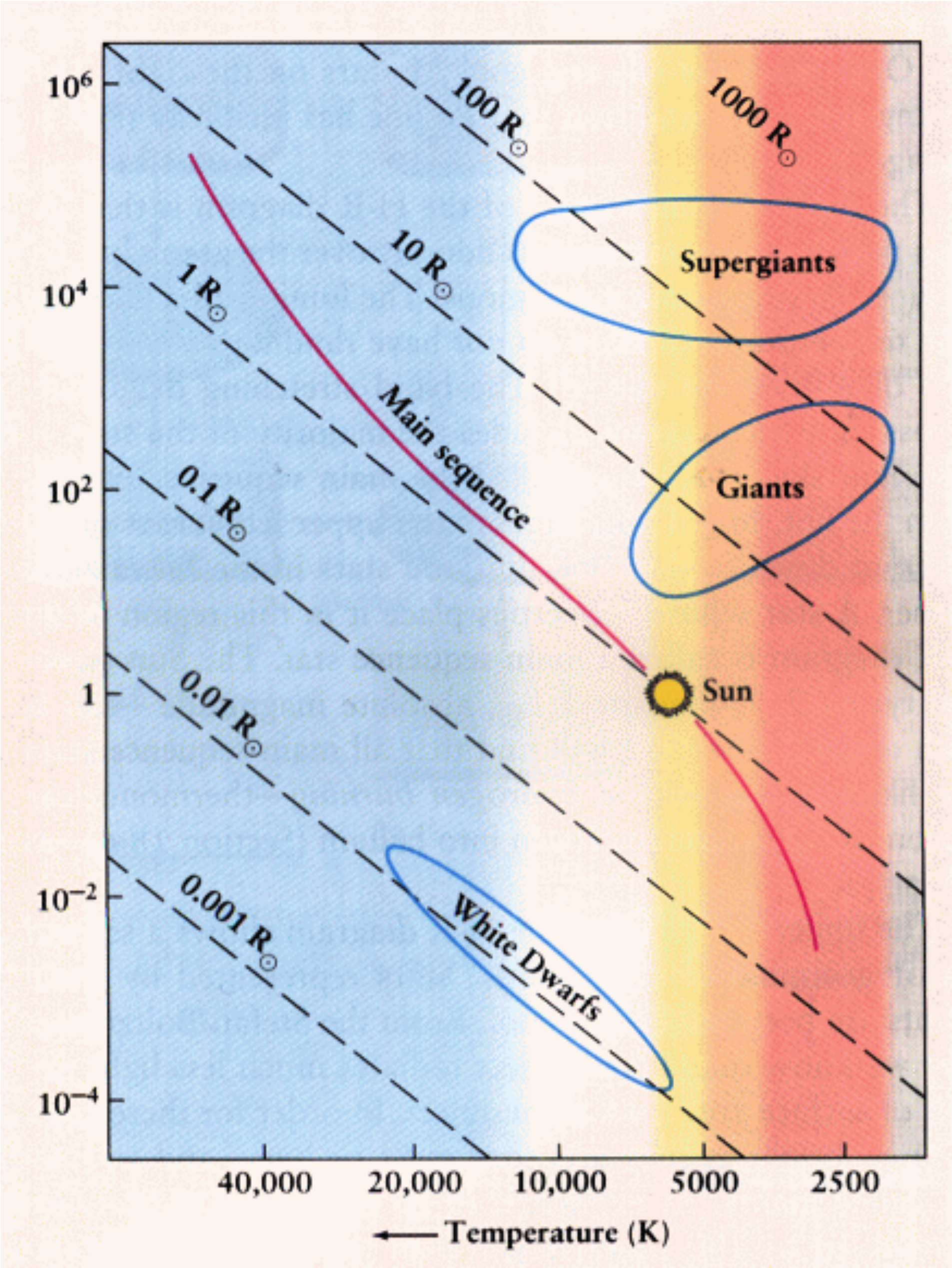


Moving to the right means decreasing T while keeping L constant. This means that R is going to increase. So, this region is known as the giant region, and stars within it are either giants or supergiants.

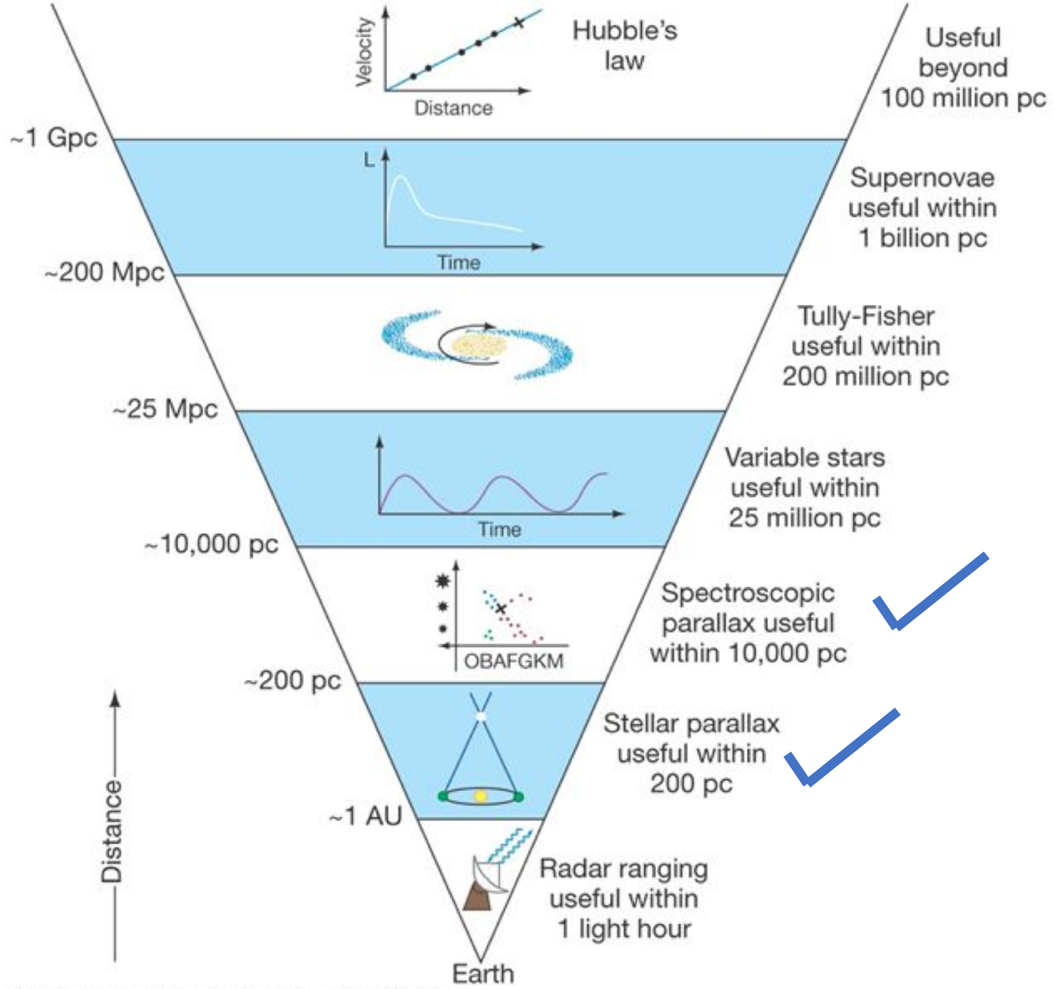
Moving down means keeping T constants while decreasing L. This means that R is going to decrease. Thus, this region is known as the dwarf region.

H-R Diagram in Detail

- 91% of stars are main sequence stars
- Along the main sequence, the highest percentage of stars are red, and the percentage drops as you move to the blue stars, which have the lowest percentage
- Giants and supergiants tend to be red and compose less than 1% of all stars
- The remaining 8% of stars are dwarfs, which can be blue, white, or red
- Because of the RLT relationship, knowing luminosity and temperature allows us to find the radius. The diagonal lines represent lines of constant radius on the diagram.



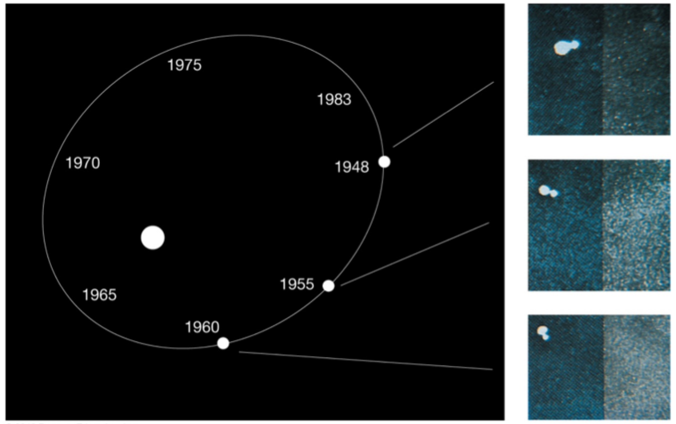
Cosmic Distance Ladder

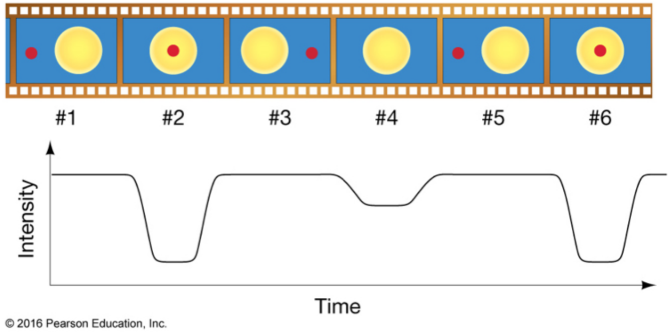


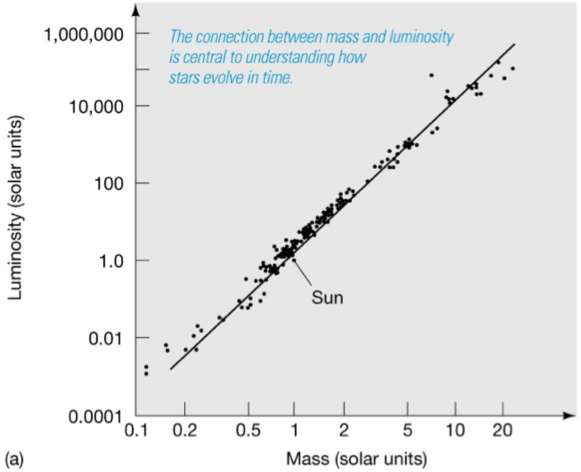
Stellar Mass

There are 3 main ways we'll try to find a star's mass:

1. **Stellar binaries**. Two stars that are orbiting a common center (a **barycenter**). These are typically called **visual binaries**.
2. **Stars with satellite**. Many stars have satellites like our Sun does; they could be planets like our Sun, or other stars. These are usually called **eclipsing binaries**.
3. **The Mass-Luminosity Relationship**. This is an empirical relationship that is different for different masses of stars, so before you can apply it, you have to have a good guess as to what the mass probably is.







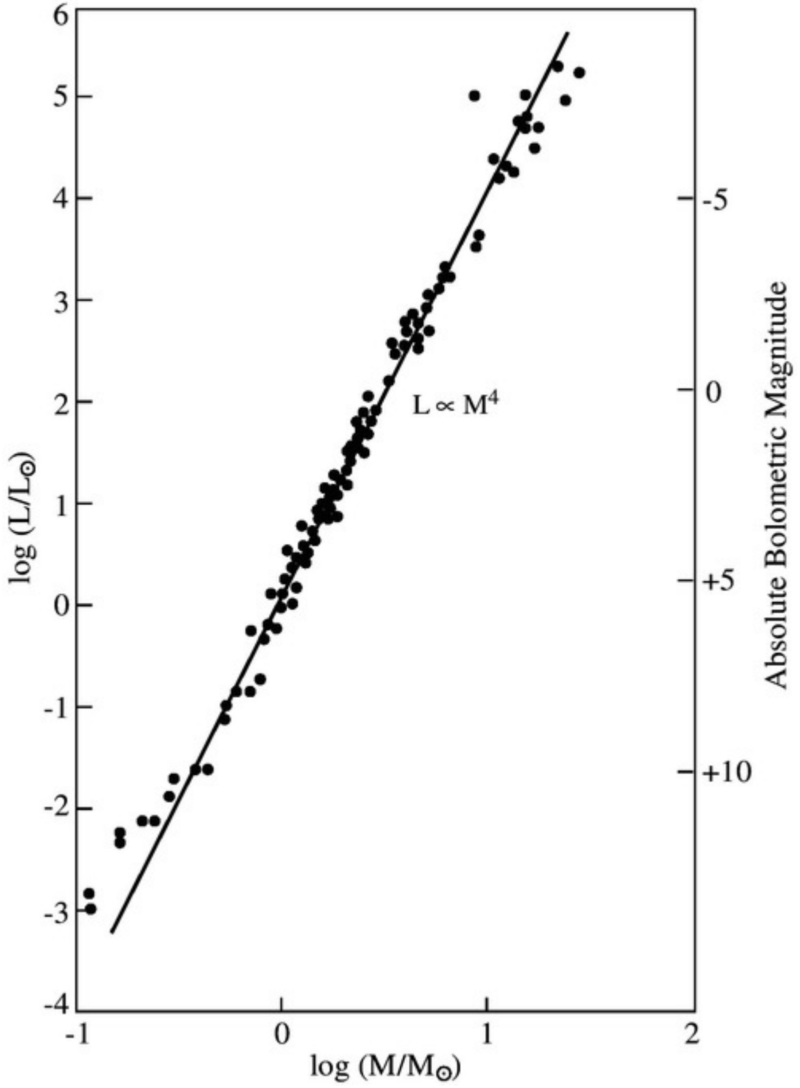
(a)

Mass-Luminosity Relationship

$$L = M^4$$

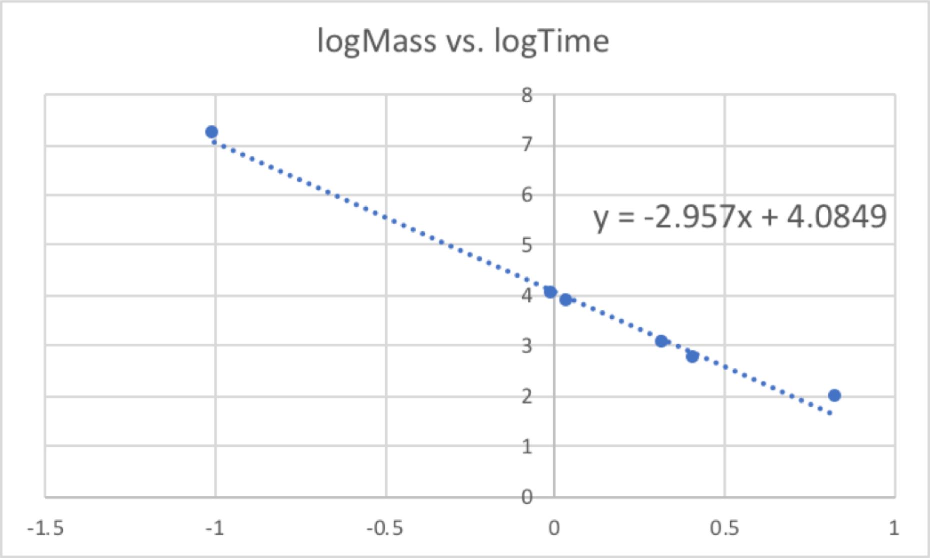
Luminosity in units of $L_\odot$

Mass in units of $M_\odot$



Stellar Lifetimes

Star	Estimated Lifetime (*M*/*L*) (10^6 years)
Spica B*	90
Vega	500
Sirius	1000
Alpha Centauri	7000
Sun	10,000
Proxima Centauri	16,000,000



$$\log M = -3 \log t + \text{const}$$

Mass ($M_\odot$)

$$M = \frac{1}{t^3}$$

Lifetime ($t_\odot$)

Stellar Masses Across the Universe