PHY2049 Physics with Calculus II

Section 589357

Prof. Douglas H. Laurence

Exam 2 (Chapters 27 – 31)

July 16, 2018

Name: Solutions

Instructions:

This exam is composed of **10 multiple choice questions** and **4 free-response problems**. To receive a perfect score (100) on this exam, 3 of the 4 free-response problems must be completed. The fourth free-response problem **may** be answered for extra credit, giving a total possible score of 125/100 on the exam. Each multiple choice question is worth 2.5 points, for a total of 25 points, and each free-response problem is worth 25 points, for a total of 75 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

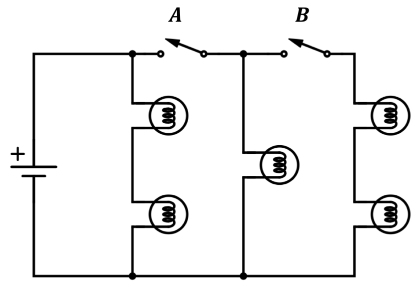
For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS



1. Above, A and B are switches: If a switch is closed, current can run through the wire, but if a switch is open, current cannot. In which configuration will the total power emitted by all lightbulbs be greatest?

   (a) Both A and B open
   (b) A open but B closed
   (c) Both A and B closed
   (d) All configurations will emit the same total power

   TOTAL POWER IS <u>ALWAYS</u> EQUAL TO POWER OUTPUT OF BATTERY.

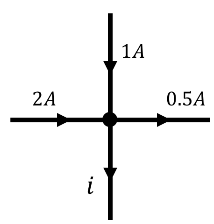
2. Consider a cylindrical resistor of radius r, a length L, and a resistance R. If you doubled the radius and doubled the length, what would the new resistance be?

   (a) $R/8$
   (b) $R/4$
   (c) $R/2$
   (d) $2R$

$$R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2}$$

$$R' = \rho \frac{(2L)}{\pi(2r)^2} = \rho \frac{\cancel{2}L}{\pi \cdot \cancel{4}_{2} r^2} = \frac{1}{2} \rho \frac{L}{\pi r^2} = \boxed{\frac{R}{2}}$$

R



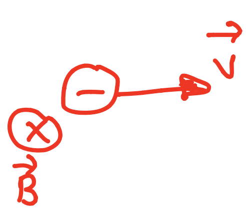
3. What is the unknown current i in the figure above?

   (a) 0.5A
   (b) 1.5A
   (c) 2.5A
   (d) 3.5A

$$\left.\begin{array}{l} \sum i_{in} = 2A + 1A = 3A \\ \sum i_{out} = 0.5A + i \end{array}\right\} 3A = 0.5A + i \Rightarrow \boxed{i = 2.5A}$$

4. An electron moves to the right in a magnetic field into the page. What direction is the magnetic force on this electron?

   (a) Up
   (b) Into the page
   (c) Down
   (d) Out of the page



RHR SAYS UP, BUT AN ELECTRON IS <u>NEGATIVE</u>, SO THE FORCE IS $\boxed{\text{DOWN.}}$

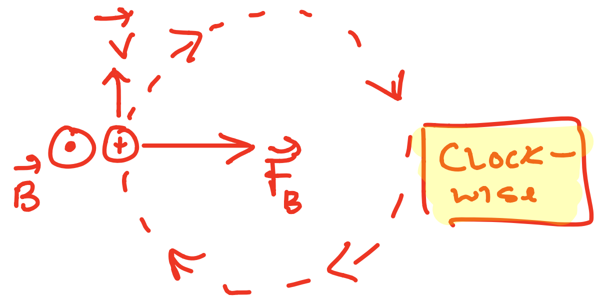
$$F_B = F_{centripetal} \Rightarrow qvB = ma_c = m\frac{v^2}{r}$$

$$\Rightarrow qB = \frac{mv}{r} \Rightarrow r = \frac{mv}{qB} = \frac{(1.67\times10^{-27})(1500)}{(1.6\times10^{-19})(0.01)} = 0.0016\text{m}$$

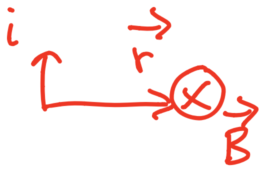
$$\Rightarrow \boxed{R = 1.6\text{mm}}$$

5. A proton, $q = +e$ and $m = 1.67\times10^{-27}$ kg, moves to the right with $v = 1500$ m/s in a magnetic field $B = 0.01$T out of the page. The protons moves in a:

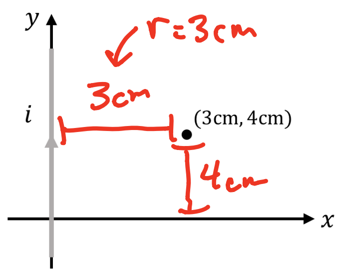
   (a) Clockwise circle, of radius $R = 1.6$mm
   (b) Counterclockwise circle, of radius $R = 1.6$mm
   (c) Clockwise circle, of radius $R = 638.7$m
   (d) Counterclockwise circle, or radius $R = 638.7$m



CLOCKWISE



INTO PAGE



6. A very long wire, as shown in the figure above, produces a magnetic field at the point indicated. If $i = 500$A, what is the magnitude and direction of this magnetic field?

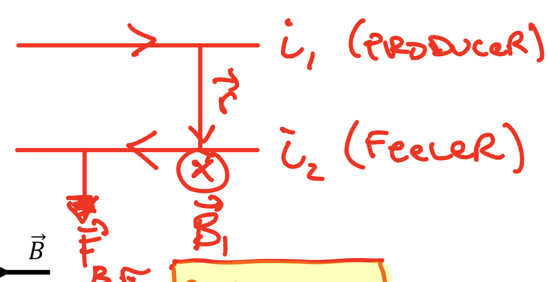
   (a) 0.0025T into the page
   (b) 0.0025T out of the page
   (c) 0.0033T into the page
   (d) 0.0033T out of the page

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7})(500)}{2\pi(0.03)}$$

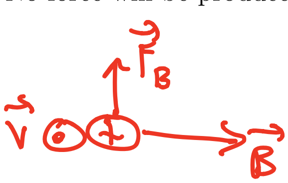
$$\Rightarrow \boxed{B = 0.0033\text{T}}$$

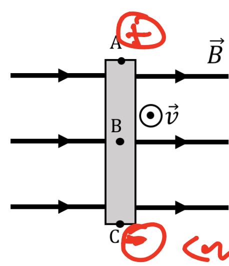
7. Two parallel wires carrying currents in opposite directions will produce a force on each other that is:

   (a) Attractive
   (b) Repulsive
   (c) Neither attractive nor repulsive
   (d) No force will be produced in this situation



REPULSIVE

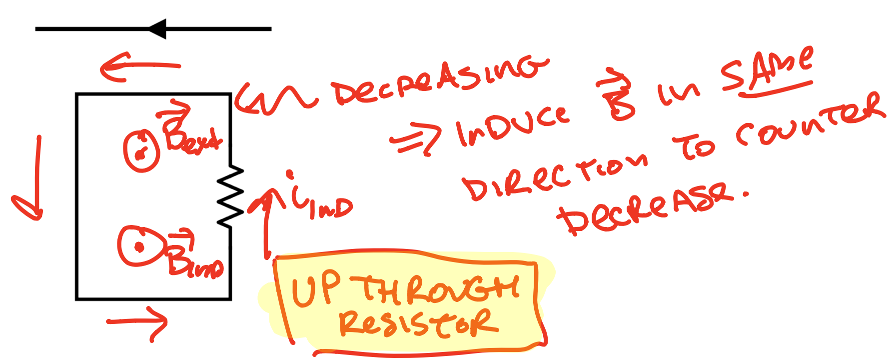




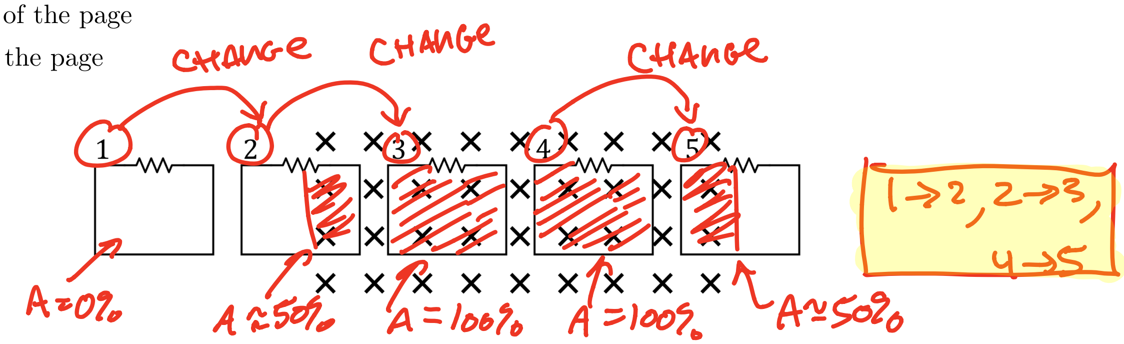
cm Low at ⊖ end

8. A conductor moves out of the page in a horizontal magnetic field, as shown above. Which point on the conductor is at the lowest potential?

   (a) A
   (b) B
   (c) C
   (d) All are at equal potential

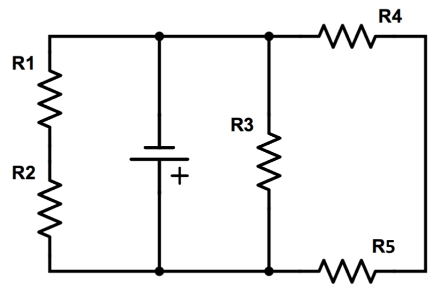


9. In the above figure, the wire carries a current that is decreasing. In what direction through the resistor will the induced current move?
   (a) Up
   (b) Down
   (c) Out of the page
   (d) Into the page



10. A loop moves to the right, passing through a region of uniform magnetic field into the page, as shown above. During which motions will there be an induced current in the loop?
    (a) 1-to-2 and 2-to-3
    (b) 1-to-2, 2-to-3, and 3-to-4
    (c) 1-to-2 and 4-to-5
    (d) 1-to-2, 2-to-3, and 4-to-5

FREE-RESPONSE PROBLEMS



1. In the circuit above, $R_1 = 1\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_4 = 1\Omega$, $R_5 = 2\Omega$, and the battery is 5V.

   (a) What is the equivalent resistance of this circuit?

   (b) What is the current through R_3?

   (c) How much power is produced by R_4?

   (d) What is the voltage across R_1?

(a)

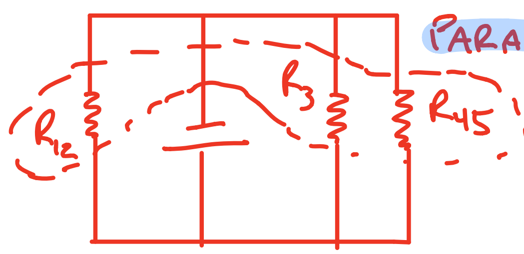
$R_{46} = R_4 + R_5 = 1 + 2 = 3\Omega$

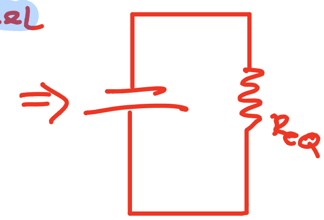
$R_{12} = R_1 + R_2 = 1 + 3 = 4\Omega$

$$\frac{1}{R_{eq}} = \frac{1}{R_{12}} + \frac{1}{R_3} + \frac{1}{R_{45}} = \frac{1}{4} + \frac{1}{2} + \frac{1}{3}$$

$$= \frac{3}{12} + \frac{6}{12} + \frac{4}{12} = \frac{13}{12}$$

$$\Rightarrow R_{eq} = \frac{12}{13} \Rightarrow \boxed{R_{eq} = 0.9\,\Omega}$$



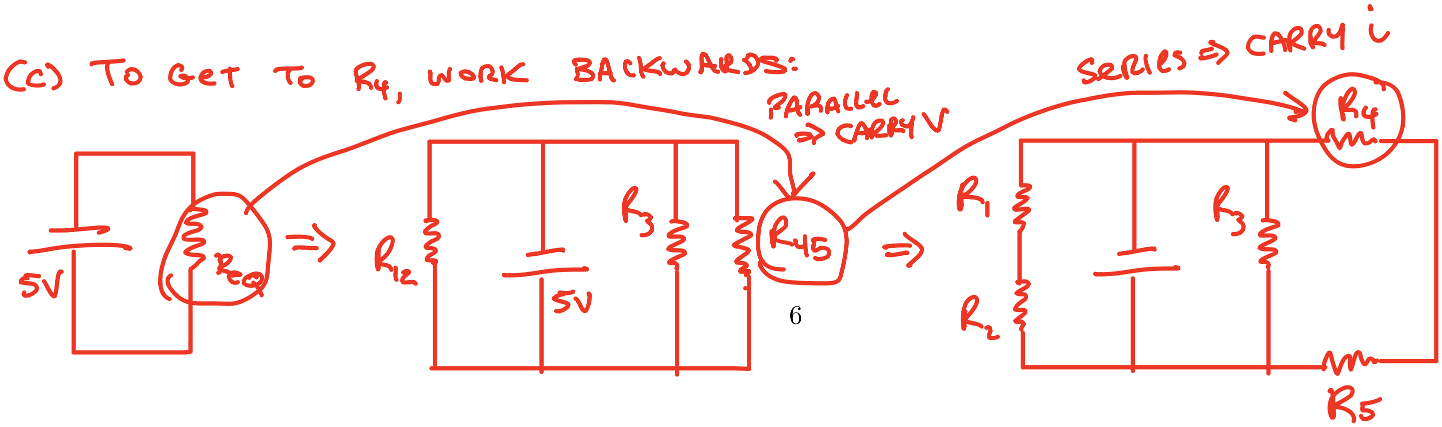


(b) R_3 ALREADY in PARALLEL w/ BATTERY (SEE FIGURE ABOVE), SO THE VOLTAGE V_3 EQUALS THE VOLTAGE OF THE BATTERY, 5V.

SO,

$$i_3 = \frac{V_3}{R_3} = \frac{5}{2} \Rightarrow \boxed{i_3 = 2.5A}$$

(c) TO GET TO R_4, WORK BACKWARDS:



VOLTAGE ACROSS R_{45} SAME AS ACROSS R_{EQ}, SO $V_{45} = 5V$.
TO SOLVE FOR ANYTHING TO DO WITH R_4, WE NEED TO KNOW THE CURRENT THROUGH R_{45}:

$$i_{45} = \frac{V_{45}}{R_{45}} = \frac{5}{3} = 1.67A$$

THIS IS THE SAME AS THE CURRENT THROUGH R_4 (AND R_5), $i_4 = 0.71A$.
KNOWING i_4, WE CAN FIND POWER OUTPUT BY R_4:

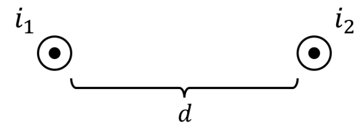
$$P_4 = i_4^2 R_4 = (1.67)^2(1) \Rightarrow \boxed{P_4 = 2.8W}$$

(d) WORKING BACK AS IN PART (c), WE KNOW $V_{12} = V_{EQ}$. TO BREAK R_{12} INTO R_1 & R_2, WE NEED TO CARRY THE CURRENT. SO:

$$i_{12} = \frac{V_{12}}{R_{12}} = \frac{5}{4} = 1.25A$$

SO, $i_1 = i_{12} = 1.25A$, AND THE VOLTAGE ACROSS R_1 IS:

$$V_1 = i_1 R_1 = (1.25)(1) \Rightarrow \boxed{V_1 = 1.25V}$$



2. As shown above, a very long wire carrying a current $i_1 = 2000$A out of the page is placed a distance $d = 7$cm away from a a second very long wire, carrying a current $i_2 = 2500$A out of the page.

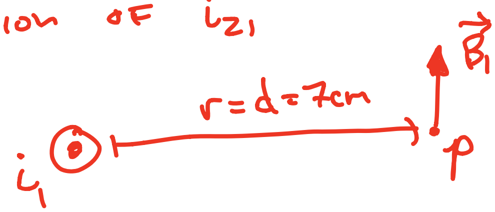
   (a) What is the magnitude of the force per unit length between each wire? Is it attractive or repulsive?

   (b) What is the magnetic field produced at the center of these two wires? Give both magnitude and direction.

   (c) What would the force on a 1m wire, carrying a current $i_3 = 1000$A into the page, be if it were placed halfway between the two wires in the above figure? Give both magnitude and direction.

(a) CONSIDER i_1 TO BE PRODUCER & i_2 TO BE FEELER.

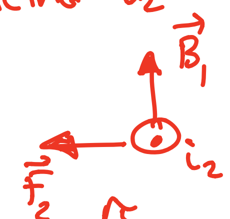
AT LOCATION OF i_2,



$$B_1 = \frac{\mu_0 i_1}{2\pi r} = \frac{(4\pi \times 10^{-7})(2000)}{2\pi\,(0.07)}$$

$$= 0.0057\text{ T}$$

PLACING i_2 IN PRESENCE OF $\vec{B}_1$, IT FEELS A FORCE:

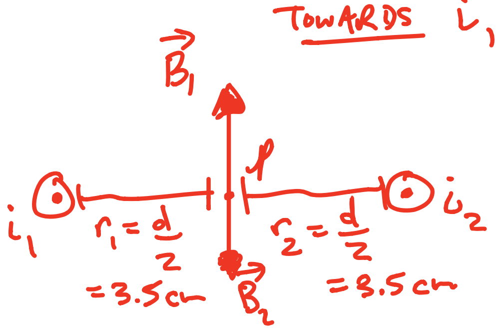


$$F_2 = i_2 \ell_2 B_1 \Rightarrow \frac{F_2}{\ell_2} = i_2 B_1 = (2500)(0.0057)$$

$$\Rightarrow \boxed{F_2/\ell_2 = 14.57\ \text{N/m}}$$

F_2 PULLS i_2 <u>TOWARDS</u> i_1 $\Rightarrow$ $\boxed{\text{ATTRACTIVE}}$

(b)



$$B_1 = \frac{\mu_0 i_1}{2\pi r_1} = \frac{(4\pi \times 10^{-7})(2000)}{2\pi\,(0.035)} = 0.011\text{ T}$$

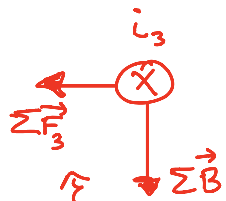
$$B_2 = \frac{\mu_0 i_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7})(2500)}{2\pi\,(0.035)} = 0.014\text{ T}$$

CALL DOWN +, AND UP −, THEN:

$$\Sigma B = \underbrace{-B_1}_{\text{UP}} + \underbrace{B_2}_{\text{DOWN}} = -0.011 + 0.014 = \underbrace{+\,0.03\text{ T}}_{\text{DOWN}}$$

$$\Rightarrow \boxed{\Sigma B = 0.003\text{T DOWN}}$$

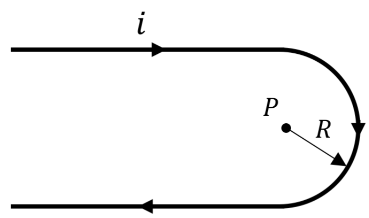
(c) IF WE PLACE A WIRE, CALL IT i_3, IN BETWEEN i_1 & i_2, IT WILL FEEL $\Sigma\vec{B}$ THAT WE CALCULATED IN PART (b).



$$\Sigma F_3 = i_3 \ell_3 \Sigma B = (1000)(0.003)$$

$$\Rightarrow \boxed{\Sigma F_3 = 3N}$$

$\Sigma\vec{F}_3$ IS TO THE $\boxed{\text{LEFT}}$



3. In the above figure, we want to break the wire up into three segments: 1) the long wire on top, carrying the current to the right; 2) the half-loop, carrying the current clockwise; and 3) the long wire on bottom, carrying the current to the right. In this problem, the current is $i = 200$A and the radius of the loop is $R = 17$cm.

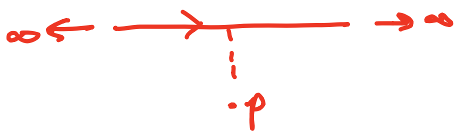
(a) What will the magnitude of the magnetic field be due to segments 1 and 3 at point P? *Hint: Don't forget that the point P is at the end of the wire, not somewhere in the middle.*

(b) What will the magnitude of the magnetic field be due to segment 2 at point P?

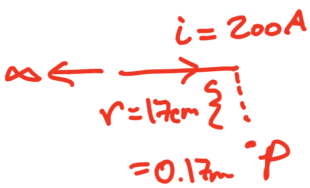
(c) What is the direction of the magnetic field produced by each segment?

(d) Using the answers to parts (a), (b), and (c), what is the net magnetic field at point P? This is the magnetic field due to the entire, U-shaped wire.

(a)



WE KNOW $B = \frac{\mu_0 i}{2\pi r}$ FOR THIS WIRE.



THIS WIRE IS EXACTLY <u>HALF</u> THE ABOVE, SO

$$B = \frac{\mu_0 i}{4\pi r} = \frac{(4\pi \times 10^{-7})(200)}{4\pi(0.17)}$$

$\Rightarrow$ EACH WIRE PRODUCES $\boxed{B = 0.00012\,T}$

(b)

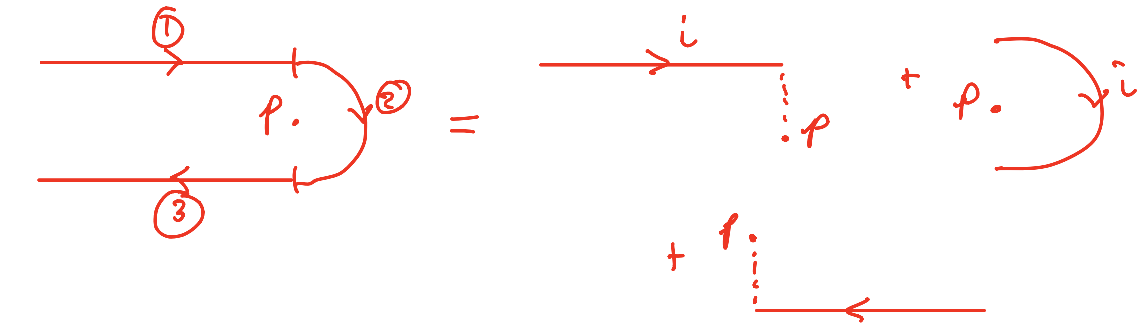
WE KNOW THIS FULL LOOP WILL PRODUCE $B = \frac{\mu_0 i}{2R}$ AT ITS CENTER.

THIS IS EXACTLY HALF THE LOOP ABOVE, SO WILL PRODUCE:

$$B = \frac{\mu_0 i}{4R} = \frac{(4\pi \times 10^{-7})(200)}{4(0.17)} \Rightarrow \boxed{B = 0.00037\,T}$$

(c) ALL 3 SEGMENTS PRODUCE MAGNETIC FIELDS $\boxed{\text{INTO THE PAGE.}}$

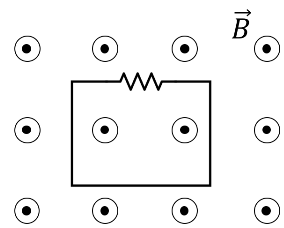
(d)



ALL 3 SEGMENTS INTO PAGE, SO:

$$\Sigma B = B_1 + B_2 + B_3 = 0.00012 + 0.00037 + 0.00012$$

$$\Rightarrow \boxed{\Sigma B = 0.00061\,T \text{ INTO THE PAGE}}$$



4. As shown above, a loop is placed in a magnetic field pointing out of the page. The loop has dimensions 10cm×15cm.

(a) When the magnetic field is $B = 1.5$T, what is the magnetic flux through the loop?

(b) If this magnetic field decreased at 0.2 T/s, what would the induced EMF on the loop be?

(c) If the resistor shown in the loop was 8Ω, what would the induced current in the loop be?

(d) In what direction through the resistor would the induced current flow?

(a) $\Phi_B = BA = (1.5)(\underbrace{0.1 \times 0.15}_{10\text{cm} \times 15\text{cm}}) \Rightarrow \boxed{\Phi_B = 0.0025 \text{ Wb}}$ OR Tm^2

(b) $\mathcal{E}_{IND} = \dfrac{\Delta \Phi_B}{\Delta t} = \dfrac{\Delta(BA)}{\Delta t} = A\dfrac{\Delta B}{\Delta t} = (0.1 \times 0.15)(0.2)$

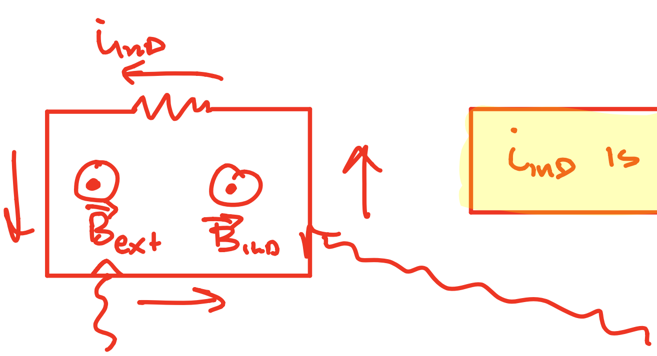
CONSTANT

B DECREASES AT 0.2 T/s

$\Rightarrow \boxed{\mathcal{E}_{IND} = 0.003 \text{ V}}$

(c) $i_{IND} = \dfrac{\mathcal{E}_{IND}}{R} = \dfrac{0.003}{8} \Rightarrow \boxed{i_{IND} = 0.00038 \text{ A}}$

(d)



$\boxed{i_{IND} \text{ IS TO THE LEFT THROUGH R}}$

IF B_{ext} IS DECREASING, THEN $\vec{B}_{IND}$ MUST POINT IN SAME DIRECTION TO COMBAT THE DECREASE IN $\vec{B}_{ext}$.

FORMULA SHEET

- Vectors:

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

- Physics I Formulae:

$$\sum \vec{F} = m\vec{a}$$

$$W = \vec{F}\cdot\Delta\vec{x} \quad \text{or} \quad W = \int \vec{F}\cdot d\vec{x}$$

$$W_{tot} = \Delta K$$

$$W_{cons} = -\Delta U$$

$$K = \frac{1}{2}mv^2$$

$$K_i + U_i = K_f + U_f$$

$$\vec{F} = -\vec{\nabla}U \quad \text{where} \quad \vec{\nabla}f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$a_c = \frac{v^2}{r}$$

- Electric Forces and Fields:

$$e = 1.6\times10^{-19}\ \mathrm{C}$$

$$\left.\begin{aligned} k &= 8.99\times10^{9}\ \frac{\mathrm{Nm}^2}{\mathrm{C}^2} \\ \epsilon_0 &= 8.85\times10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}} \end{aligned}\right\} \ k = \frac{1}{4\pi\epsilon_0}$$

$$Q = Ne$$

$$F = k\frac{q_1q_2}{r^2}$$

$$\vec{F} = q\vec{E}$$

$$E = k\frac{q}{r^2}$$

$$\text{or} \quad E = \int k\frac{dq}{r^2} \quad \text{with} \quad dq = \text{density} * \text{space element}$$

$$\text{where} \quad \lambda = \frac{Q}{L} \quad \text{or} \quad \sigma = \frac{Q}{A} \quad \text{or} \quad \rho = \frac{Q}{V} \quad \text{(densities)}$$

$$\Phi_E = \vec{E}\cdot\vec{A} \quad \text{or} \quad \Phi_E = \int \vec{E}\cdot d\vec{A}$$

$$\Phi_{tot} = \frac{q_{enc}}{\epsilon_0}$$

- Electric Potential Energy and Electric Potential:

$$U = k\frac{q_1 q_2}{r}$$

$$\phi = k\frac{q}{r} \quad \text{or} \quad \phi = \int k\frac{dq}{r}$$

$$dq = \text{density} * \text{space element}$$

$$U = q\phi \quad \text{and} \quad \Delta U = q\Delta\phi$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\vec{F} = -\vec{\nabla} U$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$U = -\int \vec{F} \cdot d\vec{x} \quad \text{or} \quad \Delta U = -\int_S \vec{F} \cdot d\vec{x}$$

$$\phi = -\int \vec{E} \cdot d\vec{x} \quad \text{or} \quad \Delta\phi = -\int_S \vec{E} \cdot d\vec{x}$$

$$V = \Delta\phi$$

- Capacitance and Dielectrics:

$$Q = CV$$

$$\left.\begin{aligned} C &= \epsilon_0 \frac{A}{d} \\ E &= \frac{\sigma}{\epsilon_0} \end{aligned}\right\} \text{Parallel plate capacitors}$$

- Direct Current Circuits

$$R = \rho\frac{L}{A}$$

$$V_R = iR$$

$$P = Vi = i^2 R = \frac{V^2}{R}$$

$$\sum\nolimits_{\text{loop}} V = 0 \ \ \text{(Kirchhoff's Loop Rule)}$$

$$\sum i_{\text{in}} = \sum i_{\text{out}} \ \ \text{(Kirchhoff's Junction Rule)}$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + ... \ \ \text{(series)}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \ \ \text{(parallel)}$$

- Magnetism

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{\mathrm{Tm}}{\mathrm{A}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}\ \text{(Biot-Savart law, point charge)}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{id\vec{l}\times\vec{r}}{r^3}\ \text{(Biot-Savart law, current)}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{\text{enc}}\ \text{(Ampere's law)}$$

$$B = \frac{\mu_0 i}{2\pi r}\ \text{(long wire)}$$

$$B = \frac{\mu_0 i}{2R}\ \text{(center of a loop)}$$

$$B = \mu_0\frac{N}{L}i = \mu_0 n i\ \text{(center of a solenoid)}$$

$$\vec{F}_B = q\vec{v}\times\vec{B}\ \text{(point charge)}$$

$$\vec{F}_B = i\vec{l}\times\vec{B}\ \text{(wire)}$$

- Electromagnetic Induction

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}$$