PHY2049 Physics with Calculus II

Section 589357

Prof. Douglas H. Laurence

Exam 3 (Modern Physics & Electromagnetic Waves)

July 30–31, 2018

Take-home

Name: Solutions

Instructions:

This exam is a take-home exam, due on **Tuesday July 31, 2018, by 11:59pm**. You must work on the exam by yourself, though you are, of course, allowed to access all lecture notes, your textbook, the internet, etc. You are not allowed to seek help from any tutors, professors, etc.; as I stated, you must work on the exam by yourself.

This exam is composed of **10 multiple choice questions** and **4 free-response problems**. To receive a perfect score (100) on this exam, 3 of the 4 free-response problems must be completed. The fourth free-response problem **may <u>not</u> be answered for extra credit**. Each multiple choice question is worth 2.5 points, for a total of 25 points, and each free-response problem is worth 25 points, for a total of 75 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

1. What is the cyclotron frequency of an electron, $m = 9.11 \times 10^{-31}$ kg, in a magnetic field of $B = 0.01$T?

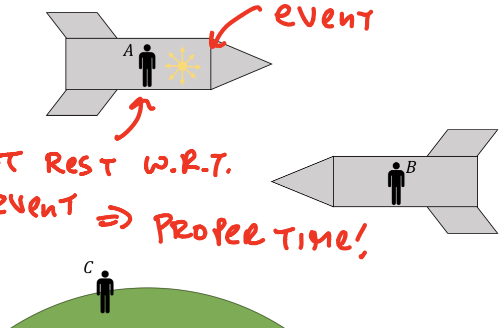
   (a) 1.46×10^{-51} s^{-1}
   (b) 5.69×10^{-10} s^{-1}
   (c) 5.69×10^{-14} s^{-1}
   (d) 1.76×10^{9} s^{-1}

$$\omega = \frac{qB}{m} = \frac{(1.6\times10^{-19})(0.01)}{(9.11\times10^{-31})} = 1.76\times10^{9}\,s^{-1}$$

2. A blue laser has a wavelength of 450nm. What would the frequency of this laser be?

   (a) 1.5×10^{-15} Hz
   (b) 0.007 Hz
   (c) 135 Hz
   (d) 6.66×10^{14} Hz

$$c = \lambda f \Rightarrow f = \frac{c}{\lambda} = \frac{(3\times10^{8})}{(450\times10^{-9})} = 6.66\times10^{14}\,Hz$$



event

AT REST W.R.T. event $\Rightarrow$ PROPER TIME!

3. In the above figure, there is a strobe light flashing every 0.1s on the ship with observer A. Which observer measures the proper time of the flashes of light?

   ↳ PROPER TIME

   (a) A
   (b) B
   (c) C
   (d) D

4. Refer to the figure for the previous problem. If the ship carrying observer A is moving at $0.8c$ relative to the Earth, and the ship carrying observer B is moving at $0.9c$ relative to the Earth, how frequently does observer C see the strobe light flash?

   (a) Once every 0.44s
   (b) Once every 1s
   (c) Once every 1.7s
   (d) Once every 2.29s

$$\Delta t' = \gamma \Delta t_p = \frac{1}{\sqrt{1-(0.8)^2}}(1s) = 1.7s$$

5. Refer to the previous problem (A moves at $0.8c$ and B moves at $0.9c$). How long does the ship carrying observer B appear to be according to observer A if the ship was manufactured to be 100m long?

   ↳ PROPER LENGTH

   (a) 43.6m
   (b) 60m
   (c) 167m
   (d) 229m

$$L' = \frac{L_p}{\gamma} = \sqrt{1-(0.9)^2}\,(100m)$$
$$= 43.6m$$

6. What is the total relativistic energy of an electron, $m = 0.51$ MeV$/c^2$, moving at a speed of $0.99c$?

   (a) 0.07 MeV
   (b) 0.08 MeV
   (c) 3.11 MeV
   (d) 3.62 MeV

$$E = \gamma m c^2 = \frac{1}{\sqrt{1-(0.99)^2}}(0.51\ \mathrm{MeV}/c^2)c^2$$
$$= 3.62\ \mathrm{MeV}$$

7. If an electron, with a mass 9.11×10^{-31} kg, moves at a speed of 300 km/s, what is its de Broglie wavelength?

↳ non-relativistic!

   (a) 0.39 nm
   (b) 2.42 nm
   (c) 390 nm
   (d) 2424 nm

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{(6.626\times10^{-34})}{(9.11\times10^{-31})(300\times10^{3})} = 2.42\times10^{-9}\ m$$
$$= 2.42\ nm$$

8. What is the ionization energy of Li^{2+} in the ground state?

↳ 3 protons, missing $2e^-$ ⇒ $1e^-$ ⇒ hydrogen-like!

   (a) 13.6 eV
   (b) 30.6 eV
   (c) 54.5 eV
   (d) 122.4 eV

$$\Delta E = (-13.6\ eV)\underbrace{Z^2}_{3}\Big(\underbrace{\cancel{\frac{1}{n_f^2}}}_{\infty \to 0} - \underbrace{\frac{1}{n_i^2}}_{1}\Big) = (13.6)\cdot 9 = 122.4\ eV$$

9. What frequency of light must an electron in hydrogen absorb to jump from the $n = 2$ state to the $n = 5$ state?

   (a) 2.86 Hz
   (b) 4.08 Hz
   (c) 6.91×10^{14} Hz
   (d) 9.86×10^{14} Hz

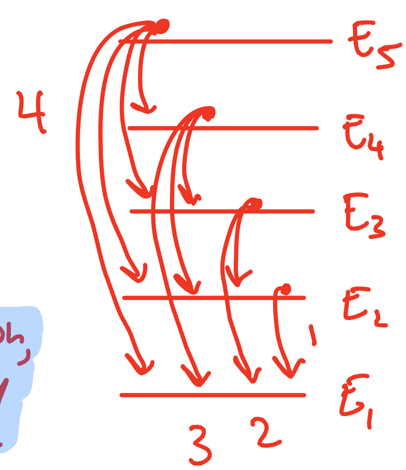
$$\Delta E = (-13.6\ eV)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (-13.6\ eV)\left(\frac{1}{5^2} - \frac{1}{2^2}\right) = 2.86\ eV$$

Photon energy is 2.86 eV & $E = hf$

$$\Rightarrow f = \frac{E}{h} = \frac{(2.86)}{(4.136\times10^{-15})} = 6.91\times10^{14}\ Hz$$

10. An electron absorbs a single photon to jump from the $n = 1$ state to the $n = 5$ state. How many different photons can be emitted?

   (a) 4
   (b) 5
   (c) 12
   (d) 24



$4+3+2+1 = 10$

Not an option, automatically correct for all.

(I accidentally did $4\times3\times2\times1 = 24$)

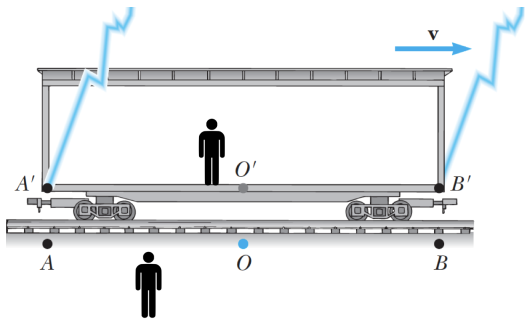
FREE-RESPONSE PROBLEMS

1. This problem will be a conceptual problem on special relativity.

   (a) What are the two postulates of special relativity? Briefly explain each one (you can give an example to help explain).

   (b) Conceptually explain why the second postulate of special relativity leads to time being relative.

   (c) Conceptually explain why the second postulate of special relativity leads to distance being relative.



   (d) Refer to the figure above. According to an observer on the side of the train tracks, in frame O, the lightning strikes points A and B at the same time. According to an observer on the train, in frame O', would the lightning strike points A' and B' at the same time? Given this conclusion, would we say that simultaneity (the act of being simultaneous) is relative or not?

2. A muon produced in a laboratory at rest will decay in 2.2 μs, according to an observer in the lab. If the muon were produced in a laboratory so that it were moving at 0.99c, it would decay in a different amount of time, according to an observer in the lab.

(a) Which time is the proper time, the time observed when the muon is at rest or the time observed when the muon is moving at 0.99c?

(b) How long would an observer in the lab measure the muon's lifetime to be when it moves at 0.99c?

(c) Muons produced in the upper atmosphere typically travel at 0.99c. According to an observer at rest on the Earth's surface, if a muon were produced at an altitude of ~~1500m~~ 15,000m, at what altitude would the muon decay?

(d) If an observer could travel alongside the muon, so that the muon appeared to be at rest, how far would this observer measure the muon to travel before decaying?

(a) WHEN THE MUON IS AT REST.

(b) $\Delta t' = \gamma \Delta t_p = \frac{1}{\sqrt{1-.99^2}}(2.2\mu s) = 15.6\,\mu s$

(c) $d = v\Delta t'$ ACCORDING TO AN OBSERVER AT REST ON EARTH, SO,

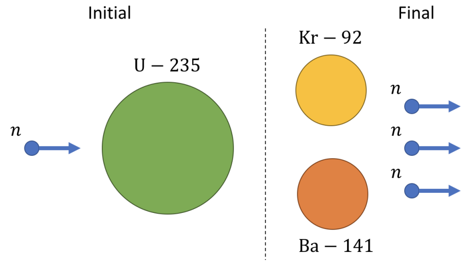
$$d = (0.99 \times 3\times10^8)(15.6\times10^{-6}) = 4633\text{m}$$

IF THE ALTITUDE PRODUCED IS 15,000m THEN THE ALTITUDE DETECTED IS

$$H = 15,000 - 4633 = 10,367\text{m}$$

(d) RIDING ALONG W/ THE MUON, THE DISTANCE TRAVELED IS:

$$d' = v\Delta t_p = (0.99 \times 3\times10^8)(2.2\times10^{-6})$$
$$= 653\text{m}$$



3. You should treat the fission of uranium-235 (U-235) as shown in the figure above: a neutron, moving with a kinetic energy of 0.025 eV, hits a stationary U-235 nucleus. This nucleus splits into two "daughter" nuclei kryton-92 (Kr-92) and barium-141 (Ba-141), both of which are at rest after the fission event, and emits 3 neutrons. Note the following:

$$1 \text{ Ns} = 1.87 \times 10^{27} \text{ eV}/c$$

$$m_n = 940 \text{ MeV}/c^2$$

$$m_{U-235} = 218.942 \text{ GeV}/c^2$$

$$m_{Kr-92} = 85.629 \text{ GeV}/c^2$$

$$m_{Ba-141} = 131.261 \text{ GeV}/c^2$$

(a) If the kinetic energy of the incoming neutron is 0.025 eV, what is the momentum of the incoming neutron?

(b) Using momentum conservation, what is the momentum of each outgoing neutron? *Hint: for simplicity, assuming that each neutron has the same outgoing velocity.*

(c) Using relativistic energy conservation, what is the amount of energy released in this fission process? *Hint: compare the momentum to the mass of each neutron; do you need to include the momentum in this calculation?*

(d) If a nuclear power plant operates based on this reaction, and is outputting 3000 MW of power, how many fission events are occurring each second? *Hint: don't forget that a W is a J/s, so you need to convert from eV to J.*

(a) IF $K = 0.025\text{eV}$ AND $E_0 = mc^2 = 0.51$ MeV, THIS NEUTRON IS NON-RELATIVISTIC. SO,

$$K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

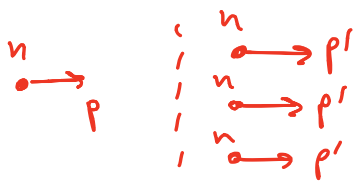
NOTE THAT: $K = 0.025\text{eV} = 4 \times 10^{-21}$ J

$$\Rightarrow p = \sqrt{2(1.67 \times 10^{-27})(4 \times 10^{-21})} = 3.66 \times 10^{-24} \text{ Ns}$$

$$= 6844 \text{ eV}/c$$

m_n in kg

(b) Initially, momentum is just momentum of incoming neutron. Finally, momentum is just the momentum of the 3 outgoing neutrons, which we consider to all be equal. So,



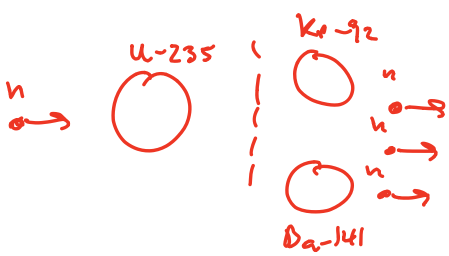
$$\Rightarrow p = 3p'$$

$$\Rightarrow p' = \frac{1}{3}p = \frac{1}{3}(6844)$$

$$\approx 2281 \text{ eV}/c$$

$$= 1.22 \times 10^{-24} \text{ Ns}$$

(c) $pc \ll m_n c^2$, so we can say the total energy $\approx mc^2$.



$$E_i = m_n c^2 + m_{U235} c^2$$

$$E_f = m_{Kr92} c^2 + m_{Ba141} c^2 + 3m_n c^2 + E_{REL}$$

$$E_i = E_f \Rightarrow \cancel{m_n c^2} + m_{U235} c^2 = m_{Kr92} c^2 + m_{Ba141} c^2 + \overset{2}{\cancel{3}} m_n c^2 + E_{REL}$$

$$\Rightarrow E_{REL} = m_{U235} c^2 - m_{Kr92} c^2 - m_{Ba141} c^2 - 2m_n c^2$$

$$= 218.942 - 85.629 - 131.261 - 2(0.94)$$

$$= 0.172 \text{ GeV} = 172 \text{ MeV}$$

$$= 2.75 \times 10^{-11} \text{ J}$$

940 MeV = 0.94 GeV

(d) If $P = 3000$ MW, then <u>in 1s</u>, $E_{OUT} = 3000$ MJ.

$$E_{OUT} = n E_{REL} \Rightarrow n = \frac{E_{OUT}}{E_{REL}} = \frac{3000 \times 10^6}{2.75 \times 10^{-11}} = 1.09 \times 10^{20}$$

\# of events

energy released per event

So, there are 1.09×10^{20} events per second.

4. Consider the first 8 orbitals of hydrogen, i.e. $n = 1$, $n = 2$, ..., $n = 8$.

(a) How many different photons could be emitted from a transition from $n = 8$ to a lower orbital?

(b) What is the wavelength of a photon emitted for the transition $n = 6$ to $n = 3$?

(c) What transition would produce the highest frequency photon? What is this frequency?

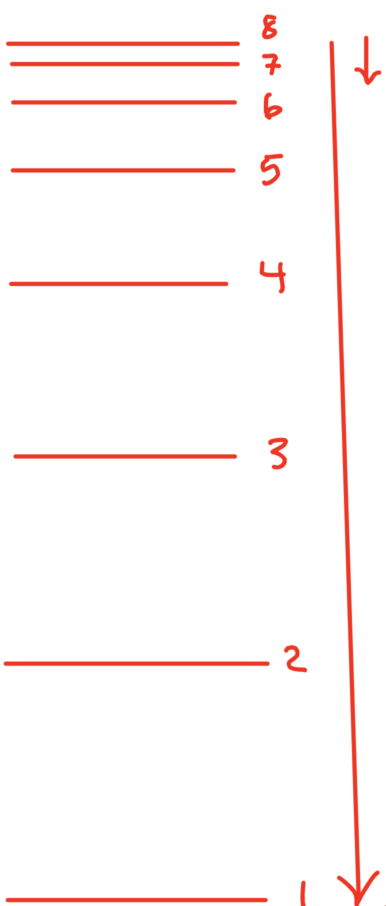
(d) What transition would produce the lowest frequency photon? What is this frequency?

(a) # OF PHOTONS $= 7+6+5+4+3+2+1 = 28$

(b) $\Delta E_{6\to 3} = (-13.6\,eV)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (-13.6)\left(\frac{1}{3^2} - \frac{1}{6^2}\right)$

$= -1.13\,eV \Rightarrow$ PHOTON HAS $E = +1.13\,eV$

$E_{ph} = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(1240\,\cancel{eV}\cdot nm)}{(1.13\,\cancel{eV})} = 1097\,nm$



SMALLEST ΔE $\Rightarrow$ SMALLEST f

(c) $\Delta E_{8\to 1} = (-13.6)\left(\frac{1}{1^2} - \frac{1}{8^2}\right) = -13.4\,eV$

$E_{ph} = \frac{h}{f} \Rightarrow f = \frac{E}{h} = \frac{13.4\,eV}{4.136 \times 10^{-15}\,eV\cdot s}$

$= 3.24 \times 10^{15}\,Hz$

(d) $\Delta E_{8\to 7} = (-13.6)\left(\frac{1}{7^2} - \frac{1}{8^2}\right) = -0.065\,eV$

$\Rightarrow f = \frac{E}{h} = \frac{0.065}{4.136 \times 10^{-15}} = 1.57 \times 10^{13}\,Hz$

LARGEST ΔE $\Rightarrow$ LARGEST f

FORMULA SHEET

- Vectors:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

- Physics I Formulae:

$$\sum \vec{F} = m\vec{a}$$

$$W = \vec{F} \cdot \Delta\vec{x} \quad \text{or} \quad W = \int \vec{F} \cdot d\vec{x}$$

$$W_{tot} = \Delta K$$

$$W_{cons} = -\Delta U$$

$$K = \frac{1}{2}mv^2$$

$$K_i + U_i = K_f + U_f$$

$$\vec{F} = -\vec{\nabla}U \quad \text{where} \quad \vec{\nabla}f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$a_c = \frac{v^2}{r}$$

- Electric Forces and Fields:

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\left.\begin{aligned} k &= 8.99 \times 10^9 \ \frac{\text{Nm}^2}{\text{C}^2} \\ \epsilon_0 &= 8.85 \times 10^{-12} \ \frac{\text{F}}{\text{m}} \end{aligned}\right\} \ k = \frac{1}{4\pi\epsilon_0}$$

$$Q = Ne$$

$$F = k\frac{q_1 q_2}{r^2}$$

$$\vec{F} = q\vec{E}$$

$$E = k\frac{q}{r^2}$$

$$\text{or} \quad E = \int k\frac{dq}{r^2} \quad \text{with} \quad dq = \text{density} * \text{space element}$$

$$\text{where} \quad \lambda = \frac{Q}{L} \quad \text{or} \quad \sigma = \frac{Q}{A} \quad \text{or} \quad \rho = \frac{Q}{V} \quad \text{(densities)}$$

$$\Phi_E = \vec{E} \cdot \vec{A} \quad \text{or} \quad \Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi_{tot} = \frac{q_{enc}}{\epsilon_0}$$

- Electric Potential Energy and Electric Potential:

$$U = k\frac{q_1 q_2}{r}$$

$$\phi = k\frac{q} {r} \quad \text{or} \quad \phi = \int k\frac{dq}{r}$$

$$dq = \text{density} * \text{space element}$$

$$U = q\phi \quad \text{and} \quad \Delta U = q\Delta\phi$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\vec{F} = -\vec{\nabla} U$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$U = -\int \vec{F}\cdot d\vec{x} \quad \text{or} \quad \Delta U = -\int_S \vec{F}\cdot d\vec{x}$$

$$\phi = -\int \vec{E}\cdot d\vec{x} \quad \text{or} \quad \Delta\phi = -\int_S \vec{E}\cdot d\vec{x}$$

$$V = \Delta\phi$$

- Capacitance and Dielectrics:

$$Q = CV$$

$$\left.\begin{aligned} C &= \epsilon_0\frac{A}{d} \\ E &= \frac{\sigma}{\epsilon_0} \end{aligned}\right\} \text{Parallel plate capacitors}$$

- Direct Current Circuits

$$R = \rho\frac{L}{A}$$

$$V_R = iR$$

$$P = Vi = i^2 R = \frac{V^2}{R}$$

$$\sum\nolimits_{\text{loop}} V = 0 \;\; \text{(Kirchhoff's Loop Rule)}$$

$$\sum i_{\text{in}} = \sum i_{\text{out}} \;\; \text{(Kirchhoff's Junction Rule)}$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + ... \;\; \text{(series)}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \;\; \text{(parallel)}$$

- Magnetism

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{\text{Tm}}{\text{A}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \vec{r}}{r^3}\ \text{(Biot-Savart law, point charge)}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{id\vec{l} \times \vec{r}}{r^3}\ \text{(Biot-Savart law, current)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{enc}}\ \text{(Ampere's law)}$$

$$B = \frac{\mu_0 i}{2\pi r}\ \text{(long wire)}$$

$$B = \frac{\mu_0 i}{2R}\ \text{(center of a loop)}$$

$$B = \mu_0 \frac{N}{L} i = \mu_0 n i\ \text{(center of a solenoid)}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}\ \text{(point charge)}$$

$$\vec{F}_B = i\vec{l} \times \vec{B}\ \text{(wire)}$$

- Electromagnetic Induction

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}$$

- Electromagnetic Waves

$$c = 3 \times 10^8\ \text{m/s}$$

$$h = 6.626 \times 10^{-34}\ \text{Js} = 4.136 \times 10^{-15}\ \text{eVs}$$

$$c = \lambda f$$

$$E = hf = \frac{hc}{\lambda}$$

$$\omega = \frac{qB}{m}\ \text{(cyclotron frequency)}$$

- Special Relativity

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t' = \gamma \Delta t_p$$

$$L' = \frac{L_p}{\gamma}$$

$$p = \gamma m v$$

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2 = (\gamma - 1)mc^2$$

$$E = \sqrt{p^2c^2 + m^2c^4} = \gamma mc^2$$

$$E = K + mc^2$$

- Quantum Mechanics

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ Js} = 6.582 \times 10^{-16} \text{ eVs}$$

$$hc = 1.986 \times 10^{-25} \text{ Jm} = 1240 \text{ eV} \cdot \text{nm}$$

$$a_0 = 5.29 \times 10^{-11} \text{ m} \ \text{(Bohr radius)}$$

$$\lambda = \frac{h}{p} \ \text{(de Broglie wavelength)}$$

For the following, $n = 1, 2, 3, ...$

$$L = rp = n\hbar \ \text{(angular momentum quantization)}$$

$$r = n^2 a_0 \ \text{(orbit quantization)}$$

$$v = \frac{ke^2}{n\hbar} \ \text{(orbital velocity quantization)}$$

$$E = (-13.6 \text{ eV})\frac{Z^2}{n^2}$$

$$\Delta E = (-13.6 \text{ eV})Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$