Lecture Notes from October 19, 2017

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Abstract

Since I was running 30 minutes late due to traffic, I want to make sure that anyone who wasn't in class can see the material we covered, so I've typed up these notes summarizing the lecture. We finished chapter 6, work & energy, and we got about two-thirds of the way through the material for chapter 7, momentum.

1 Work & Energy

1.1 (Brief) Review of Previous Material

We left off by introducing the concept of energy conservation, but before continuing with that, I want to present a brief review of the material covered so far in this chapter. We began by defining the energy due to the motion of an object, known as the **kinetic energy**:

$$K = \frac{1}{2}mv^2 \tag{1}$$

How does an object gain kinetic energy if it begins from rest? Well, in order to do so, it has to gain a speed, which means it must be accelerated, which means a force must act on it. Along this line of thought, we define a quantity known as the **work** done by a force, which measures how much the energy of an object changes:

$$W = F\Delta x\cos\theta \tag{2}$$

There are a couple of things to note about this equation. First, the angle θ is the angle between the force $\vec{f}$ acting on the object and the displacement $\Delta\vec{x}$ the object undergoes. Second, the work doesn't change the "energy" of an object, it specifically changes the *kinetic* energy of an object. However, since the kinetic energy is changed due to an acceleration, and an acceleration is due to the *net* force acting on an object, the *total* work due to all of the forces combined acting on an object changes the kinetic energy of the object:

$$W_{tot} = \Delta K \tag{3}$$

This is known as the **work-energy theorem**.

To use the work-energy theorem, we need a way to calculate the total work done on an object. There are two ways of doing this. You can calculate the net force $\sum\vec{F}$ and find the work due to that,

$$W_{tot} = \sum F\Delta x\cos\theta \tag{4}$$

where θ is now the angle between the *net* force and the displacement. The other way to find the total work is to find the work due to each individual force and simply add those works:

$$W_{tot} = \underbrace{W_1 + W_2 + W_3 + ...}_{\text{all forces}} \tag{5}$$

So far, the kinetic energy and the work deal with an object when it's in motion, but there is an energy that isn't associated with an object in motion. This energy is a stored energy known as **potential energy**, as in the "potential to move" or the "potential to do work." Potential energy is stored when certain types of forces act on an object called **conservative forces**; there is no potential energy associated with non-conservative forces. Gravity, which is a conservative force, has a potential energy given by:

$$U = mgy \tag{6}$$

where y is a position along a vertical axis. You are free to choose where you want your origin to be (where $y = 0$), but the positive direction *must* be upward.

The last lecture ended with the introduction of **total mechanical energy**:

$$E = K + U \tag{7}$$

This is often referred to simple as the "energy" or the "total energy," however this doesn't represent the sum of all the possible types of energy; there are still other types of energy like heat that are ignored. The energy this cares about is the *mechanical* energy, which is the energy associated with movement, i.e. the kinetic and the potential energy.

1.2 Conservative and Non-conservative Forces

Now that we've reviewed the previously taught material, we can continue with our discussion of energy. I mentioned in the last section that only conservative forces produce a potential energy on an object, while non-conservative forces do not. These names are chosen for a specific reason: conservative forces *conserve* total mechanical energy while non-conservative forces do not. What this means is that if an object moves from point A to point B, and is only under the influence of conservatives forces, then whatever energy it had at point A, E_A, is equal to whatever energy it has at point B, E_B; the energy is conserved. This would not be true if there was a non-conservative force acting on the object as it moved from A to B.

What exactly is a conservative force? And what exactly is a non-conservative force? A conservative force is a force whose work **does not depend on the path taken by the object**; that is, the work is path-independent. Consider Figure 1.

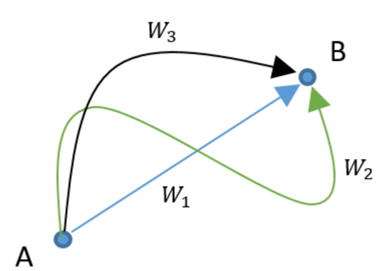

Figure 1: An object taking three possible paths between points A and B

In the figure, an object can move from point A to point B along one of three paths. The work done along path 1 is W_1, the work done along path 2 is W_2, and the work done along path 3 is W_3. What it means to be a conservative force, what it means for the work to be path independent, is that these works along these different paths are all the same:

$$W_1 = W_2 = W_3$$

The above claim would not be true for non-conservative forces, hence the name.

Another way to state this same requirement for conservative forces is to consider a circular path, like those shown in Figure 2:

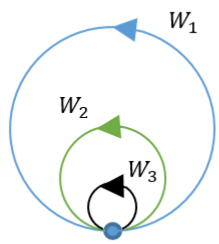

Figure 2: Three possible circular paths, starting and ending at the same point

As before, each of the works associated with each path, W_1, W_2, and W_3, must all be equal to each other. But there's another, special consequence to this: since the starting and ending point are the same, and the work done around a loop is independent of the radius of the loop, then the work along any loop must also be equal to the work done around a loop of *zero* radius. In order for the object to travel around a loop of zero radius, it must stay in place (obviously), and if it stays in place, no work is done. So, not only do the three works equal each other, but they all equal zero. It's true that **the work around a loop due to a conservative force is always zero.**

Let's take a look at gravity, and show that it's a conservative force. Consider an object moving in a square loop, as in Figure 3.

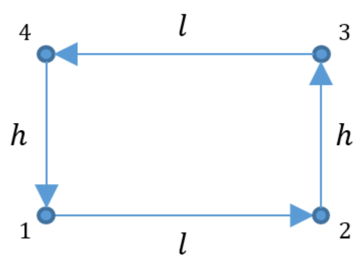

Figure 3: An object moving in a square loop under the influence of gravity

Consider this loop to be oriented such that the path $2 \to 3$ points upward. If I move a mass m from point 1 to point 2, how much work is done? Well, the force is gravity, mg, and the displacement is l, but the angle is 90^o since gravity points down and the displacement is to the right. Using equation (2), we see that $\cos(90^o) = 0$, so $W_{1\to 2} = 0$. What about the next path? Well, going from 2 to 3, the object is under the same force, mg, undergoes a displacement of h, but this time the angle isn't 90^o; since the displacement is up and gravity points down, the angle is 180^o, and $\cos(180^o) = -1$. So the work is $W_{2\to 3} = -mgh$. As with path $1 \to 2$, the force of gravity is perpendicular to the displacement along path $3 \to 4$, so $W_{3\to 4} = 0$. Finally, moving down the path $4 \to 1$, everything is the same as along the path $2 \to 3$ except that now *both* the force of gravity and the displacement point down, so the angle between them is 0^o, and $\cos(0^o) + 1$, so $W_{4\to 1} = +mgh$. Thus, the total work along the square loop is:

$$W_{tot} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + W_{4\to 1} = 0 - mgh + 0 + mgh = 0$$

Just as expected, the work done around a closed loop is zero since gravity is a conservative force.

But what about the force of friction? To test friction, we'll consider the same path as in Figure 3, but instead of the path involving moving up and down, imagine the path was on the surface of a table, so we only considered friction and could ignore gravity. Along the path $1 \to 2$, the force is the kinetic friction f_k, the displacement is l, and the angle is 180^o because friction always points *opposite* the direction of motion. So, $W_{1\to 2} = -f_k l$. Along path $2 \to 3$, the force is still f_k, the displacement is h, and the angle is still 180^o. This is the key difference between friction and gravity: friction *always* points opposite to the direction of motion, so the angle between it and the displacement will *always* be 180^o. So, $W_{2\to 3} = -f_k h$. Likewise, $W_{3\to 4} = -f_k l$ and $W_{4\to 1} = -f_k h$. So, the total work done around the square loop is:

$$W_{tot} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + W_{4\to 1} = -f_k l - f_k h - f_k l - f_k h = -2f_k l - 2f_k$$

This work will never be zero unless there is no force (i.e. $f_k = 0$) or the object doesn't move (that is, $l = h = 0$). But in either case we wouldn't expect any force to produce any work. So, we have to conclude that friction is a non-conservative force.

In practice, you won't have to test a force in the manner I did above. Any force that has a potential energy associated with it, like gravity, is a conservative force. All other forces are non-conservative. The only conservative forces you will see in this class are gravity and the spring force, which we won't get to until a later chapter.

1.3 Energy Conservation

So, now that we know what conservative and non-conservative forces are, we can start discussing energy conservation. If an object moves from some point A to some point B, and no non-conservative forces are acting on it, then its initial energy equals its final energy:

$$E_i = E_f \tag{8}$$

This is known as the **conservation of total mechanical energy**, or energy conservation for short. This equation is more useful if we substitute in the definition of energy, as given by equation (7):

$$K_i + U_i = K_f + U_f \tag{9}$$

Using energy conservation is extremely straightforward: find your kinetic and potential energies at your initial point, find your kinetic and potential energies at your final point, and then plug them into the above equation and solve. Obviously, one of the energies is going to be unknown, otherwise you wouldn't be solving for anything, so for that energy you'll just plug in the equation or keep it as its variable.

For example, say an object is at rest on an inclined plane, a height h above the ground, and I want to know its speed at the bottom of the incline. I know that the kinetic energy initially (at the top) is zero, since it starts at rest, and the potential energy is $U_i = mgh$ (if I set $y = 0$ to be the bottom of the incline). My final potential energy is zero (once again, because $y = 0$ is at the bottom of the incline), and I don't know the final kinetic energy, but I know the equation for it. Plugging all of this into our conservation of energy equation, we get:

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + mgh = \frac{1}{2}mv^2 + 0 \quad \Rightarrow \quad v = \sqrt{2gh}$$

This assumes that the incline is frictionless, which I would want to assume since the problem didn't mention friction. But what if there was friction? Friction is a non-conservative force, so I could no longer use equation (9). In order to understand this better, we should look at the relationship between work and the three forms of energy we have: kinetic, potential, and total. We can imagine that there are three types of work: total work, W_{tot}, the work due to conservative forces, W_{cons}, and the work due to non-conservative forces, W_{nc}. We know that the total work is related to the kinetic energy. Since conservative forces produce potential energies, the work due to conservative forces should be related to the potential energy. Finally, since non-conservative forces don't allow total energy to be conserved, the work due to non-conservative forces should be related to total energy. Specifically, these relationships are:

$$W_{tot} = \Delta K \quad \text{(the work-energy theorem)} \tag{10a}$$

$$W_{cons} = -\Delta U \quad \text{(it's \textbf{very} important to remember the negative sign)} \tag{10b}$$

$$W_{nc} = \Delta E \tag{10c}$$

If we want to know what happens when there are non-conservative forces, let's consider the case where $\Delta E \neq 0$, which would happen because non-conservative forces don't conserve energy. Using the definition of total energy, given by equation (7), we see that

$$\Delta E = \Delta(K + U) = \underbrace{(K_f + U_f) - (K_i - U_i)}_{\Delta\, =\, \text{final}\, -\, \text{initial}}$$

Moving the $(K_i + U_i$ over to the left-hand-side with ΔE, we get:

$$K_i + U_i + \Delta E = K_f + U_f$$

But don't forget equation (10c). Plugging that in, the above equation becomes:

$$K_i + U_i + W_{nc} = K_f + U_f \tag{11}$$

which I will call the equation of **energy non-conservation**. As long as you can calculate the amount of work done by any non-conservative force, you can still use the same approach to solve an energy problem as if energy were conserved.

Let's think back to the previous **example**, with the object at rest on top of the incline. If there were friction down the slope, say of magnitude f_k, then we couldn't use energy conservation and we'd have to use equation (11). In order to do so, we'd have to calculate the work due to friction. The force is f_k, as I just stated, let's say the distance down the slope was Δx, and the angle between the displacement and friction is 180^o, as always. So, the work due to friction is $W_{nc} = -f_k \Delta x$. The only thing about this problem is that we weren't given the distance down the slope, Δx: we were given the height of the object on the slope. However, if you knew the inclination angle of the slope, it's a simple matter of trigonometry to find the distance down the slope. After you know W_{nc}, solving the problem is basically the same as it was before: find K_i, U_i, K_f and U_f, plug them in along with W_{nc} into the equation, and solve for whatever you're asked for.

Energy physics is mainly used to solve problems that deal with a change in an object's speed due to a change in an object's position. This is because the equation for energy conservation only contains kinetic energy, which depends upon speed, and potential energy, which depends upon position. So any questions like "an object starts at some height and drops to some height, what's the speed after the drop?" are most easily solved using energy physics. **The big advantage of energy physics** over kinematics is that kinematics only works if the acceleration is constant, while energy physics works under any circumstance if energy is conserved, or any circumstance where you can calculate W_{nc} if energy isn't conserved.

1.4 Power

The last thing to talk about in this chapter is the concept of **power**, which is the rate of change of energy, or the rate at which work is being done:

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{t} \tag{12}$$

Either of the above equations will get you to the right answer. Note that the units of power are

$$[P] = \text{W (Watts)} = \frac{\text{J}}{\text{s}}$$

Solving problems involving power are extremely straightforward. **For example**, imagine a machine was pulling some box up an incline at a constant velocity. In this scenario, since the velocity is constant the kinetic energy isn't changing, but the potential energy is increasing as the object moves up the slope. If the gain in potential energy were, say, 10J, and it took the machine 2.5s to pull the object up, then the power would be

$$P = \frac{\Delta E}{\Delta t} = \frac{10\text{J}}{2.5\text{s}} = 4\text{W}$$

There is a special case for power: the case of an object being propelled at a constant velocity. In this case, the equation for power becomes:

$$P = Fv \tag{13}$$

where v is the (constant) speed of the object and F is the force propelling the object.

Let's go back to our previous **example** about the box being pulled up the slope. If the box was being pulled upwards at 5m/s, then the force that the machine is exerting on the box is:

$$P = Fv \quad \Rightarrow \quad F = \frac{P}{v} = \frac{4\text{W}}{5\text{m/s}} = 0.8\text{N}$$

That's pretty much all there is to power.

2 Momentum

2.1 Definition

First, let's define the **momentum** of an object. It's similar to the kinetic energy, in that it depends on the mass of the object and the motion of the object. But whereas the kinetic energy depends on the speed of the object, the momentum depends on the *velocity* of the object, making it a vector quantity:

$$\vec{p} = m\vec{v} \tag{14}$$

Originally, Newton didn't publish his second law as $\sum \vec{F} = m\vec{a}$; this is actually only true in the special case where mass is a constant. The true form of Newton's second law is:

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \tag{15}$$

So, instead of a force being thought of as something that causes an acceleration, we should properly think of a force as something that produces a change in an object's momentum.

To get our previous version of Newton's second law back, the above equation becomes (because of math; don't worry about it):

$$\sum \vec{F} = \vec{v}\frac{\Delta m}{\Delta t} + m \underbrace{\frac{\Delta \vec{v}}{\Delta t}}_{=\vec{a}} = \vec{v}\frac{\Delta m}{\Delta t} + m\vec{a}$$

As we can see, in the case where the mass of an object doesn't change, $\Delta m = 0$ and the first term disappears, leaving only $m\vec{a}$, exactly as we had before.

Problems involving the first term aren't super common, but a common **example** is a child carrying a wagon behind them. If it were raining, the wagon would be filling with water, so the mass of the wagon would be changing, and that would put a force on the child. If the wagon were filling at 10mL per second, then 10g of water would be added to the wagon each second (1mL of water has a mass of 1mL), so:

$$\frac{\Delta m}{\Delta t} = 10\frac{\text{g}}{\text{s}}$$

If the child pulled the wagon at a constant velocity of 2m/s, then the acceleration term in Newton's second law would be zero (since the velocity isn't changing), and the force the wagon puts on the child would be:

$$F = v\frac{\Delta m}{\Delta t} = (2)(10) = 20\text{N}$$

2.2 Systems

Something that I've hinted at before, and is integral to the study of momentum, is the concept of a **system**. A system is a bit of a vague concept, but it's clear in the context of Newton's second law. When using Newton's second law, we need to find $\sum \vec{F}$; that is, add up all the forces. Well, all the forces *acting on what*? We add up all the forces acting on our chosen system, which can be any collection of objects we choose. So long as we consider all the forces acting on every object in our system, Newton's second law doesn't care, and we'll be able to solve our problem.

Imagine that we had 4 objects that were each interacting with each other. In the following figure, Figure 4, I've chosen my system to include objects 1, 2, and 3, leaving object 4 outside of the system.

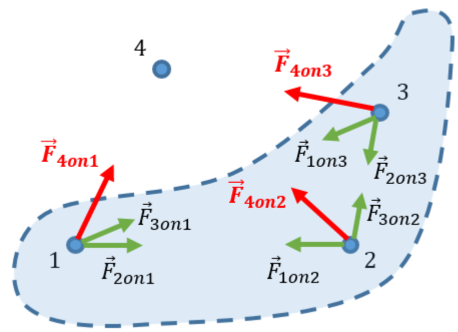

Figure 4: A system of objects 1, 2, and 3.

We can broadly divide forces into two categories (this has nothing to do with the division of forces into conservative and non-conservative, as we did in the last chapter): internal forces and external forces. **Internal forces** are any forces that originate from within the system, such as the force of object 1 on 2 or the force of object 3 on 1. **External forces** are any forces that originate from outside the system, such as the forces that object 4 puts on each object within the system, or the force of gravity due to the Earth, which is also external to the system. With this division, we can split the net force on the system:

$$\sum \vec{F}_{sys} = \sum \vec{F}_{int,sys} + \sum \vec{F}_{ext,sys} \tag{16}$$

Something interesting happens with the net internal force, though. Let's write it out:

$$\sum \vec{F}_{int,sys} = \vec{F}_{2on1} + \vec{F}_{3on1} + \vec{F}_{1on2} + \vec{F}_{3on2} + \vec{F}_{1on3} + \vec{F}_{2on3}$$

Because of Newton's third law, each force that's produce by one object on another has an equal an opposite force put back on it. Looking at the above equation, we see:

$$\sum \vec{F}_{int,sys} = \vec{F}_{2on1} + \vec{F}_{3on1} + \underbrace{\vec{F}_{1on2}}_{=-\vec{F}_{2on1}} + \vec{F}_{3on2} + \underbrace{\vec{F}_{3on1}}_{=-\vec{F}_{1on3}} + \underbrace{\vec{F}_{2on3}}_{=-\vec{F}_{3on2}} = 0$$

Because of Newton's third law, all the internal forces cancel out. This is one of the most important consequences of Newton's third law: **the net internal force on a system is always zero**. So, the net force on a system becomes, according to equations (15) (16):

$$\sum \vec{F}_{sys} = \sum \vec{F}_{ext,sys} = \frac{\Delta \vec{p}_{sys}}{\Delta t} \tag{17}$$

According to the above equation, the *only* type of force that can change a system's momentum is an external force; no internal force can change the momentum of a system. For instance, you cannot pull yourself up to the ceiling by lifting on your belt. The force you put on your belt is internal to the system that is your person, and since the net internal force is always zero, it can have no affect on your momentum.

2.3 Momentum Conservation

This leads to the concept of **momentum conservation**: if the net external force on a system is zero, then the *total* momentum of that system is conserved. This doesn't mean that the momenta of individual objects within your system have to remain the same, only that the quantity

$$\vec{p}_{sys} = \underbrace{\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...}_{\text{all objects in system}} \tag{18}$$

must not change.

Many students get hung up on the word "conservation." A common mistake is to think that energy conservation or momentum conservation *always* applies. Energy is not always conserved; momentum is not always conserved. **For example**, if I dropped a marker, is its momentum conserved on its way down? What about its energy? Its energy is definitely conserved on the way down, since the only force (ignoring air resistance) is gravity, which is conservative. But the momentum of the marker is *definitely not* conserved. We can see this instantly because the marker's speed is increasing, so its momentum is definitely changing. The reason this is so is because there is an external force acting on the marker: gravity. If there is ever an external force acting on a system, that system's momentum is not conserved.

Consider another **example**: a cannon is aligned as shown in Figure 5.

Figure 5: A cannonball fired horizontally

Initially, before the cannonball is fired, the total momentum of the cannon-cannonball system is zero, since each object is at rest. After the cannonball is fired, it has gained a momentum $\vec{p}_{cb}$ to the right. Assuming that there are no external forces acting on the cannon (gravity is balanced out by the normal force, so we don't have to worry about it), the momentum of the system is conserved, and the cannon recoils with the same momentum but in the opposite direction, $-\vec{p}_{cb}$.

Let's change this up a bit. Imagine now the cannon is oriented as shown in Figure 6.

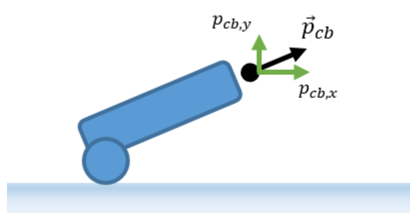

Figure 6: A cannonball fired at an angle

Is the momentum of the system still conserved? Well, since the cannonball is fired at an angle, it has a horizontal component of momentum, $p_{cb,x}$, and a vertical component of momentum, $p_{cb,y}$, but the cannon can *only* recoil in the horizontal direction. This means that momentum cannot possibly be conserved, since the cannon doesn't gain any momentum downward to balance out the gained upward momentum of the cannonball. In the horizontal direction, there's no reason to assume there are any external forces, so the cannon will recoil with the same momentum as the cannonball in the x-direction, $p_{cb,x}$, but there *must* have been an external force in the vertical direction since there was a change in the momentum of the system.

This vertical force was the normal force. The first hint is that, according to equation (17), the external force must point in the same direction as the change in momentum. Since the system gained an upward momentum, the force must point upward. Conceptually, you can think about it like this: momentum wants to be conserved, so the cannon wants to gain a momentum downward, but the floor is there and that prevents the cannon from gaining that downward momentum.

So, the moral of the story is this: momentum is not always conserved; be careful when you apply momentum conservation and make sure you're applying it in the right case.

2.4 Collisions

A case where momentum *is* always conserved is during a collision. During a collision, the momenta of the individual colliding objects change, but they change due to the impact force the objects put on one another. If we consider the colliding objects to be one system, then those are internal forces. There shouldn't be any case where there is an external force (since they'd be colliding along some surface, gravity would be balanced out by the normal force), so the momentum of the system should be conserved. **This holds for any collision**.

While momentum is always conserved in a collision, energy is not. Unless otherwise told, **do not assume energy is conserved in a collision**. Imagine two cars moving towards each other and crashing, the crash causing their wrecked cars to come to a stop. Initially, each car had some kinetic energy, and the system had a total energy equal to the sum of their kinetic energies. After the collision, though, both cars are at rest, and therefore the total energy is zero; that initial energy was lost, and therefore energy isn't conserved. There is a specific type of collision, which I will get to in a second, during which energy is conserved.

There are three types of collisions that you need to know:

- **Inelastic**: this is your standard collision, during which only momentum is conserved and energy is lost.

- **Elastic**: this is the opposite of an inelastic collision, in the sense that energy is also conserved in addition to momentum. Note that I used the word "energy" specifically, while an elastic collision is typically defined as conserving kinetic energy. During the collision, which lasts an instant, the heights of the colliding objects aren't going to change, so there will be no change in potential energy. So saying kinetic energy is conserved is the same as saying energy is conserved in this case.

- **Perfectly inelastic collision**: this type of collision is "perfect" in the sense that, if an inelastic collision loses energy, a perfectly inelastic collision loses the maximum amount of energy possible. However, the thing to note about perfectly inelastic collisions is that the colliding objects *always* stick together after the collision. Momentum is conserved in this collision, as always.

The last thing we covered in the lecture were the equations associated with each type of collision. Imagine that a mass m_A was moving with an initial velocity $\vec{v}_{Ai}$ towards a mass m_B which is moving with an initial velocity of $\vec{v}_{Bi}$. After the objects collide, we'll say m_A has a final velocity of $\vec{v}_{Af}$ and m_B has a final velocity of $\vec{v}_{Bf}$. Note that momentum conservation means that the initial total momentum equals the final total momentum, and that the total momentum is just the sum of the momenta of mass A and mass B, so the equation for momentum conservation is:

$$m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = m_A\vec{v}_{Af} + m_B\vec{v}_{Bf} \tag{19}$$

Keeping this in mind, the equations for each type of collision are as follows:

- **Inelastic**: only momentum conservation applies, so the only applicable equation is equation (19):

  $$m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = m_A\vec{v}_{Af} + m_B\vec{v}_{Bf}$$

- **Elastic**: momentum is conserved in this type of collision, so equation (19) applies:

  $$m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = m_A\vec{v}_{Af} + m_B\vec{v}_{Bf}$$

  but kinetic energy is also conserved in an elastic collision, so we have the equation:

  $$\frac{1}{2}m_A v_{Ai}^2 + \frac{1}{2}m_B v_{Bi}^2 = \frac{1}{2}m_A v_{Af}^2 + \frac{1}{2}m_B v_{Bf}^2 \tag{20}$$

  Do not use this equation to solve a collision problem! Combining the equation for momentum conservation and the equation for kinetic energy conservation is a bad idea. We all learned how to solve systems of equations in algebra, but they were typically systems of *linear* equations. The conservation of kinetic energy is quadratic (it has v^2 terms), which makes it a total pain in the ass to solve. Luckily, someone thought of a very clever way to combine the equations for momentum and kinetic energy conservation to create a third, linear equation:

  $$\vec{v}_{Ai} - \vec{v}_{Bi} = \vec{B}f - \vec{A}f \tag{21}$$

  This equation, as well as the equation for momentum conservation, should be combined to solve elastic collision problems, as this will be a system of linear equations and can easily be solved using substitution or elimination.

- **Perfectly Inelastic**: only momentum is conserved in these collisions, as previously said, but we can make a simplification to the momentum conservation equation since the colliding objects stick together post-collision:

  $$m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = (m_A + m_B)\vec{v}_f \tag{22}$$

  All the above equation says is that, after the collision, you have a single object with the combined mass of the two colliding objects moving at some final velocity. Using this equation makes solving perfectly inelastic collision problems simpler.

This is the end of the lecture. We'll be solving collision problems and wrapping up chapter seven at the beginning of the lab session on Tuesday, October 24, 2017, followed by the review for the upcomming exam.