PHY2048 Physics with Calculus I

Section 584761

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Exam 1 (Chapters 2 – 6)

February 14, 2018

Name: Solutions

Instructions:

This exam is composed of **10 multiple choice questions** and **5 free-response problems**. To receive a perfect score (100) on this exam, 4 of the 5 free-response problems must be completed. The fifth free-response problem **may <u>not</u> be answered for extra credit**. Each multiple choice question is worth 2 points, for a total of 20 points, and each free-response problem is worth 20 points, for a total of 80 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total	

MULTIPLE CHOICE QUESTIONS

1. The equations for kinematics can only be used under which condition(s)?
   (a) Constant velocity
   (b) Constant velocity and constant acceleration
   (c) Constant acceleration
   (d) They can be used under any condition

   Kinematics applies whenever the acceleration is constant. Option (a) isn't correct because it's too restrictive, as is option (b). Option (d) is incorrect because, as we know, there is a restriction to the use of kinematic equations. So $\boxed{\text{option (c)}}$ is correct.

2. A jogger is running along a circular track, which has a circumference of 400m (about one-quarter mile). After she completes one lap, which of the following statements about her motion is true?
   (a) Her distance traveled was 0m and her displacement was 400m
   (b) Her distance traveled was 0m and her displacement was 0m
   (c) Her distance traveled was 400m and her displacement was 400m
   (d) Her distance traveled was 400m and her displacement was 0m

   Since she started and ended at the same place, her displacement is zero. However, she did run an entire lap around the track, so her distance isn't zero; it's 400m, the length of the track. So $\boxed{\text{option (d)}}$ is correct.

3. An object moving in an orbit with a constant speed – for instance, the Moon around the Earth – is not accelerating.
   (a) True
   (b) False

   Even though speed is constant, the *velocity* is not constant because the object's direction changes as it orbits. Therefor it must be accelerating, and so $\boxed{\text{option (b)}}$ is correct.

4. A projectile is launched at 50 m/s at an angle of 30^o. At the peak of its trajectory, what is its speed?
   (a) 0m/s
   (b) 25m/s
   (c) 43m/s
   (d) 50m/s

   At the peak of the trajectory, it's the *vertical* velocity that's zero, not the total velocity. So the total velocity is just the x component: $v_{0x} = v_0 \cos\theta = (50)\cos(30) = 43\text{m/s}$. So $\boxed{\text{option (c)}}$ is correct.

5. A projectile moves through the air in the shape of a parabola. Which of the following statements is true about this motion?
   (a) The motion is always symmetric, due to being parabolic
   (b) The motion is symmetric, but only if the projectile starts and ends at the same height
   (c) The motion is symmetric, but only if the projectile is launched at 45^o
   (d) The motion is never symmetric

A parabola is only symmetric if it begins and ends at the same height, so $\boxed{\text{option (b)}}$ is correct.

6. Out of the following options, choose the **best** description of a force:
   (a) An influence on an object's motion due to a fundamental interaction between two objects
   (b) Something that causes an object to move
   (c) Something that causes an object to accelerate
   (d) Something that satisfies at least Newton's first and second laws
   What's important about this problem is we need to choose the **best** option. While a force is something that causes an object to move and something that causes an object to accelerate, recall that there are things called "pseudoforces" like the Coriolis force that cause acceleration but are not forces. This is because they don't satisfy Newton's third law. So options (b) and (c) aren't a very complete description of a force. A real force will satisfy all three of Newton's laws, so option (d) is wrong. So the most correct option is $\boxed{\text{option (a)}}$, because it is the only option that clearly illustrates Newton's third law, which none of the other options account for.

7. A man stands at rest on a flat surface. This is because:
   (a) His weight and the normal force acting on him cancel out, due to Newton's first law
   (b) His weight and the normal force acting on him cancel out, due to Newton's second law
   (c) His weight and the normal force acting on him cancel out, due to Newton's third law
   (d) His weight and the normal force acting on him cancel out, but not for any of the above reasons
   His weight and normal force *are* equal and opposite, but not because of Newton's third law, as most students might think at first. Newton's third law applies to a pair of forces acting on **different** objects, whereas both of these forces are acting on the same object. The normal force and weight are equal and opposite because the man isn't accelerating vertically, so the upward force must cancel the downward force. This is Newton's second law. So the correct answer is $\boxed{\text{option (b).}}$

8. Box B sits at rest on top of Box A. If you push Box A, causing it to accelerate, and Box B moves along on top of Box A undisturbed, what force is causing the motion of Box B?
   (a) No force; Newton's first law says that Box B can move under its own inertia
   (b) The pushing force you apply on the boxes
   (c) The upward interaction force that Box A puts on Box B
   (d) The friction between Box A and Box B
   Since Box B moves along with Box A, which is accelerating, there must be a force on Box B, so option (a) is wrong. Option (b) is wrong because your push is a contact force, so it only acts on Box A. While there is an upward force on Box B due to Box A, this force is perpendicular to the horizontal acceleration of Box B, so it can't be the force producing the motion on Box B, and therefore option (c) is wrong. It is in fact static friction that causes the motion of Box B, so $\boxed{\text{option (d)}}$ is correct.

9. You're trying to move a heavy object, with a mass of 200kg, along a floor with friction. The coefficients of friction between the object and the floor are $\mu_s = 0.5$ and $\mu_k = 0.4$. If you push on the object with a force of 450 N, what is the magnitude of the friction acting on the object?
   (a) 0N
   (b) 450N
   (c) 784N
   (d) 980N

If you're trying to move an object, you first need to break static friction. The maximum static friction on this object is:

$$f_{s,max} = \mu_s N = (0.5)(200 * 9.8) = 980\text{N}$$

This is **not** the correct answer, however, because you aren't pushing on the object with 980N. You push on the object with 450N, which isn't enough force to overcome static friction. Since static friction is an adaptive force, it will simply cancel out the force you apply so that the object remains at rest. So the correct answer is $f_s = 450\text{N}$, which is $\boxed{\text{option (b).}}$

10. A table is said to "have a breaking point of 980N." Using your knowledge to interpret this statement, if the table already has a 20kg block resting on it, how many 10kg blocks can be added to the table before it breaks? *Hint: Recall that we talked about why a surface wouldn't normally break in class.*

    (a) 2

    (b) 5

    (c) 8

    (d) 10

    The phrase "have a breaking point of 980N" should be interpreted as "the maximum normal force is 980N." So, how much weight can you place on the table without exceeding the maximum normal force, and thus breaking the table? 980N, of course. This corresponds to:

    $$m = \frac{W}{g} = \frac{980}{9.8} = 100\text{kg}$$

    There is *already* 20kg on top of the table, so you can only add 100 - 20 = 80kg to the table. If each block has a mass of 10kg, then you can only add 80/10 = 8 blocks without breaking the table. Thus, $\boxed{\text{option (c)}}$ is correct.

FREE-RESPONSE PROBLEMS

1. Consider two vectors: $\vec{A}$ has a magnitude of 10 and makes an angle of 25^o clockwise from the $+x$ axis, and $\vec{B}$ has a magnitude of 5 and makes an angle of 40^o counter-clockwise from the $+x$ axis.

a) Give both $\vec{A}$ and $\vec{B}$ in vector notation, e.g. "$\vec{C} = 3\hat{\imath} + 4\hat{\jmath}$".

b) Find the vector $\vec{A} + 2\vec{B}$, and give it in vector notation.

c) What is $\vec{A} \cdot \vec{B}$?

d) What is $|\vec{A} \times \vec{B}|$?

a) In order to write $\vec{A}$ and $\vec{B}$ in vector notation, we need to find their components:

$$A_x = A\cos\theta = 10\cos(25) = 9.1$$
$$A_y = A\sin\theta = 10\sin(25) = 4.2$$
$$B_x = B\cos\theta = 5\cos(40) = 3.8$$
$$B_y = B\sin\theta = 5\sin(40) = 3.2$$

Before putting this into vector notation, it's important to recognize that even though the number I found for A_y above is positive, $\vec{A}$ actually points below the x axis, so it's y component is actually negative. So, the vectors, in vector notation, are:

$$\boxed{\begin{aligned}\vec{A} &= 9.1\hat{\imath} - 4.2\hat{\jmath} \\ \vec{B} &= 3.8\hat{\imath} + 3.2\hat{\jmath}\end{aligned}}$$

b) Adding vectors is straightforward in vector notation:

$$\vec{A} + 2\vec{B} = (9.1\hat{\imath} - 4.2\hat{\jmath}) + 2(3.8\hat{\imath} + 3.2\hat{\jmath}) = (9.1\hat{\imath} - 4.2\hat{\jmath}) + (7.6\hat{\imath} + 6.4\hat{\jmath})$$

$$\Rightarrow \quad \boxed{\vec{A} + 2\vec{B} = 16.7\hat{\imath} + 2.2\hat{\jmath}}$$

c) We can find the dot product in two ways: either using the components of $\vec{A}$ and $\vec{B}$ or their magnitudes and directions:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (9.1)(3.8) + (-4.2)(3.2) \quad \Rightarrow \quad \boxed{\vec{A} \cdot \vec{B} = 21.1}$$

or:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = (10)(5)\cos(25 + 40) = 21.1$$

The reason to use $\theta = 25 + 40$ is because $\vec{B}$ is 40^o above the x axis while $\vec{A}$ is 25^o below the x axis, so you have to add them to find the angle between both vectors. Notice that we get the same answer either way we compute the dot product.

d) To find the magnitude of the cross product, we'll just us the formula in the formula sheet:

$$|\vec{A} \times \vec{B}| = AB\sin\theta = (10)(5)\sin(25 + 40) \quad \Rightarrow \quad \boxed{|\vec{A} \times \vec{B}| = 45.3}$$

The reason to use the angle $25 + 40$ is the same as in the previous part.

2. An object is dropped from the roof of a building of unknown height. Assume that $t = 0$ when the object is dropped.

a) How far does the object drop between $t = 0$ to $t = 1s$?

b) How **much farther** does the object drop between $t = 1s$ and $t = 2s$ than between $t = 0$ and $t = 1s$?

c) If it takes the object 5s to hit the ground, how tall was the building?

d) If the building were twice as tall, how much longer would it take the object to hit the ground?

a) We know that $v_0 = 0$, $a = 9.8\text{m/s}^2$, and $t = 1\text{s}$ (during this time interval). So, the distance dropped is:

$$\Delta y = \cancel{v_0}t + \frac{1}{2}at^2 = \frac{1}{2}(9.8)(1)^2 \quad \Rightarrow \quad \boxed{\Delta y_{0\to 1s} = 4.9\text{m}}$$

b) There are a few different ways to figure this one out, and they should all give the same answer. The way I'm going to solve this is by finding the final velocity after the fall from $t = 0$ to $t = 1\text{s}$, and use that as the initial velocity in the fall from $t = 1\text{s}$ to $t = 2\text{s}$:

$$v = \cancel{v_0} + at = (9.8)(1) = 9.8\text{m/s}$$

So, during the drop form $t = 1\text{s}$ to $t = 2\text{s}$, $v_0 = 9.8\text{m/s}$, $a = 9.8\text{m/s}^2$, and $t = 1\text{s}$. So the distance traveled is:

$$\Delta y_{1s\to 2s} = v_0 t + \frac{1}{2}at^2 = (9.8)(1) + \frac{1}{2}(9.8)(1)^2 = 14.7\text{m}$$

However, this is **not** the answer! The question didn't ask how far the object traveled from 1s to 2s, but how *much farther* did it travel than during the first second it was falling. We know from part a that $\Delta y_{0\to 1s} = 4.9\text{m}$, so:

$$D = \Delta y_{1s\to 2s} - \Delta y_{0\to 1s} = 14.7 - 4.9 \quad \Rightarrow \quad \boxed{D = 9.8\text{m}}$$

c) We know that $v_0 = 0$, $a = 9.8\text{m/s}^2$, and now we are told that, to hit the ground, it takes $t = 5\text{s}$. So the distance that the object drops in 5s will be equal to the height of the building:

$$\Delta y = \cancel{v_0}t + \frac{1}{2}at^2 = \frac{1}{2}(9.8)(5)^2 \quad \Rightarrow \quad \boxed{H = 122.5\text{m}}$$

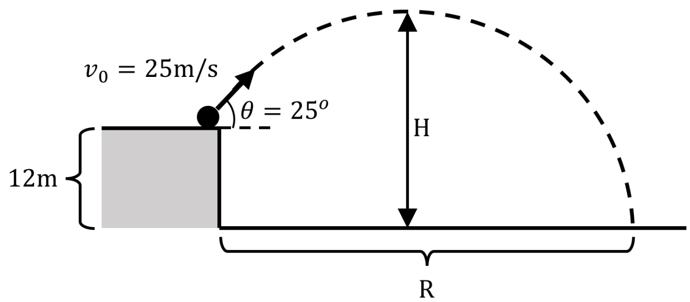
d) If the building were twice as tall, then $\Delta y = 245\text{m}$ to fall to the ground. v_0 would still be 0 and a would still be 9.8m/s^2, so the time taken would be:

$$\Delta y = \cancel{v_0}t + \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(245)}{(9.8)}} = 7.1\text{s}$$

Note the equation for t that I used right before I answered the question: if Δy doubled, t would increase by $\sqrt{2}$, so it would be $\sqrt{2} * 5 = 7.1\text{s}$, as we got above.

Now, the question is a bit ambiguous, but any of the following answers would be acceptable:

$$\boxed{\begin{array}{l} \text{Time to hit the ground } t = 7.1\text{s} \\ t = 7.1 - 5 = 2.1\text{s longer to hit the ground} \\ 7.1/5 = 1.4 \text{ times longer to hit the ground} \end{array}}$$



3. A projectile is launched with an initial speed of 25m/s at a launch angle of 25^o from a height of 12m above the ground, as shown in the figure above.

a) What is the maximum height of the projectile, H? *Hint: find the maximum height above the launch point, and use that to find the maximum height above the ground.*

b) How long does it take the projectile to reach the maximum height?

c) How long does it take the projectile to drop from the maximum height to the ground?

d) What is the range R of the projectile?

First, we should break the initial velocity up into components; this is always how we want to start 2-dimensional problems.

$$v_{0x} = v_0 \cos\theta = 25\cos(25) = 22.7\text{m/s}$$
$$v_{0y} = v_0 \sin\theta = 25\sin(25) = 10.6\text{m/s}$$

Now we can start answering the problem.

a) We know that $v_{0y} = 10.6$m/s, $a_y = -9.8\text{m/s}^2$, and $v_y = 0$ at the maximum height (this is the defining characteristic of the maximum height). So, the distance from the launch point to the peak is:

$$\cancel{v_y}^2 = v_{0y}^2 + 2a_y\Delta y \quad \Rightarrow \quad \Delta y = -\frac{v_{0y}^2}{2a_y} = -\frac{(10.6)^2}{2(-9.8)} = 5.7\text{m}$$

We can't forget, though, that the projectile doesn't start on the ground; it starts at $y_0 = 12$m. So, the maximum height is:

$$H = \Delta y + y_0 = 5.7 + 12 \quad \Rightarrow \quad \boxed{H = 17.7\text{m}}$$

b) This part is solved by using all the same information as the previous part, since we're still dealing with the maximum height; the difference is that we're solving for time in this problem, so we need a different equation.

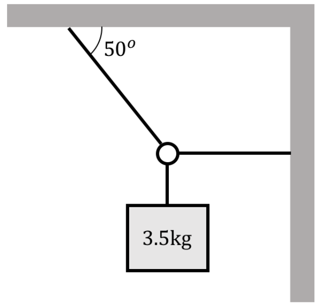
$$\cancel{v_y} = v_{0y} + a_y t \quad \Rightarrow \quad t = -\frac{v_{0y}}{a_y} = -\frac{(10.6)}{(-9.8)} \quad \Rightarrow \quad \boxed{t_{up} = 1.1\text{s}}$$

c) To find the time the projectile takes to fall to the ground, we know that $v_{0y} = 0$ (since we're starting at the peak) and $a_y = 9.8\text{m/s}^2$ (I changed the sign since this problem only involves moving down). In part a, we found that the maximum height is 17.7m off the ground, so in order to fall from the peak to the ground, $\Delta y = 17.7$m. So, finding the time is straightforward:

$$\Delta y = \cancel{v_{0y}}t + \frac{1}{2}a_y t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(17.7)}{(9.8)}} \quad \Rightarrow \quad \boxed{t_{down} = 1.9\text{s}}$$

d) To find the range, we simply multiply the velocity in the x direction (which isn't changing) by the time the ball is in the air. We know that it took 1.1s to reach the peak from the launch point, and 1.9s to fall back to the ground after, so the total time in the air is $t_F = 1.1 + 1.9 = 3$s. So, the range is:

$$R = v_{0x} t_F = (22.7)(3) \quad \Rightarrow \quad \boxed{R = 68.1\text{m}}$$



4. A 3.5kg mass hangs at rest from a ring, supported by two ropes, as shown in the figure above. Call the rope at an angle rope A and the horizontal rope B.

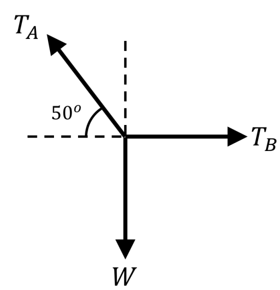
a) Draw the free body diagram of the ring.

b) Calculate the tension in rope A.

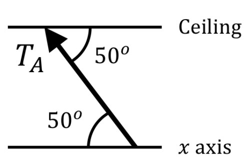
c) Calculate the tension in rope B.

d) Imagine if rope A was moved so that it was at a 90^o angle from the ceiling (straight up from the mass). What would the tension in rope B be in this case?

a)



Note that the angle between T_A and the x axis is 50^o. You can see this by using alternate interior angles:



b) Remember that tension is an adaptive force, so it can only be solved for by using Newton's second law on a free body diagram. The tension in rope A, T_A, will break into components:

$$T_{A,x} = T_A \cos(50)$$
$$T_{A,y} = T_A \sin(50)$$

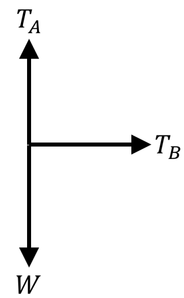
We don't know T_B, so Newton's second law in the x direction won't help us. But we do know the weight: $W = (3.5)(9.8) = 34.3$N, so we can use Newton's second law in the y direction. Since there is no y acceleration, the upward force balances the downward force, and:

$$T_{A,y} = W \quad \Rightarrow \quad T_A \sin(50) = W \quad \Rightarrow \quad T_A = \frac{W}{\sin(50)} = \frac{34.3}{\sin(50)} \quad \Rightarrow \quad \boxed{T_A = 44.7\text{N}}$$

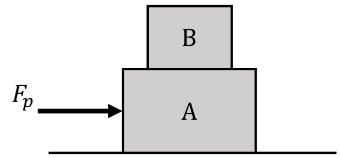
c) Now that we know T_A, we can use Newton's second law in the x direction to find T_B. Since there is no acceleration in the x direction, the force to the left balances the force to the right:

$$T_B = T_A \cos(50) = (44.7)\cos(50) \quad \Rightarrow \quad \boxed{T_B = 28.7\text{N}}$$

d) If rope A were moved so that it made a 90^o angle to the ceiling, then it would point straight upward, and the free body diagram would look like:



The key thing to remember here is that **tension is an adaptive force**; it will be whatever Newton's second law needs it to be. In this case, since there is no force to the left, $\boxed{T_B \text{ must be zero.}}$ To see this more clearly, let's think about the previous case. With rope A at an angle, the tension in rope A pulled the ring to the left, so rope B had to have a tension in it to counteract that pulling by rope A. However, with rope A pointing straight upward in this case, nothing is pulling the ring to the left, so rope B just hangs slack; it has no tension.



5. A 1.5kg box, B, is placed on top of a 4kg box, A, when a force F_p is applied to box A, as shown in the figure above. Between box A and the floor, the coefficients of friction are $\mu_{k1} = 0.25$ and $\mu_{s1} = 0.4$, and between box B and box A, the coefficients of friction are $\mu_{k2} = 0.32$ and $\mu_{s2} = 0.45$. Unless stated otherwise, assume that the boxes move together.

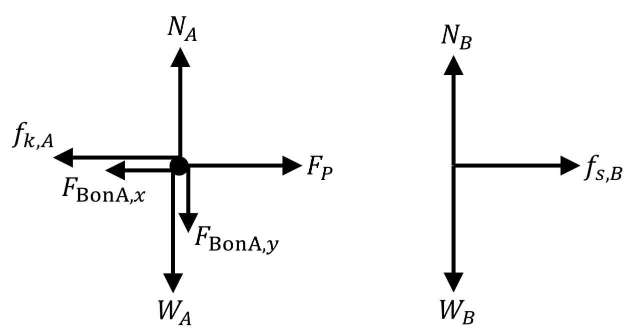
a) Draw a free body diagram for each box **independently**.

b) If the boxes are moving at a constant speed, what amount of friction is acting on each box?

c) If the boxes are accelerating at 1.5m/s^2, what amount of friction is acting on each box?

d) What is the maximum force F_p that can be placed on box A without box B falling off?

a) There are two acceptable ways of giving the free body diagram, because the question is a bit ambiguous: the boxes could be accelerating, in which case the free body diagrams would look like:



Or the boxes are moving at a constant speed, in which case the free body diagrams look the same, except there is no $F_{BonA,x}$ acting on A and no $f_{s,B}$ acting on B; those forces are only present if the boxes are accelerating.

b) If the boxes are moving at a constant speed, then as I said above, there is no friction acting on box B; Newton's first law says B is content moving at a constant velocity on its own, and doesn't need a force to push it along. However, box A is sliding, and therefor it will experience kinetic friction. The normal force on A can be found by solving Newton's second law in the vertical direction for A:

$$N_A = F_{BonA,y} + W_A$$

W_A is easy to find, $(4)(9.8) = 39.2$N, but we don't know $F_{BonA,y}$ right off the bat. To find it, we need to recognize that **the normal force on B is the reaction force to** $F_{BonA,y}$, so they are equal in magnitude. Solving Newton's second law in the y direction for box B gives N_B:

$$N_B = W_B = (1.5)(9.8) = 14.7\text{N}$$

So $F_{BonA,y} = N_B = 14.7$N, and therefor the normal force on A is:

$$N_A = F_{BonA,y} + W_A = 14.7 + 39.2 = 53.9\text{N}$$

Now that we know the normal force on A, we can find the friction force by simply multiplying it by the coefficient μ_{k1} (the coefficient between A and the surface): $(0.25)(53.9) = 13.5$N. So, the answer to the question is:

$$\boxed{\begin{aligned} &f_A = 13.5\text{N} \\ &f_B = 0 \end{aligned}}$$

c) Even though the boxes are now accelerating, this acceleration is in the horizontal direction, so nothing in the vertical direction for either box changes. This means that the normal force on B is the same, so the normal force on A is the same, so the friction force on A is the same (kinetic friction is a constant!). What changes is the friction on B. Now that B is accelerating, it needs that static friction force to pull it along. Solving Newton's second law in the x direction for B, we see that the static friction is $f_s = m_B a = (1.5)(1.5) = 2.3$N. Note that we **cannot** use $f_s = \mu_{s2} N_B$ because this problem doesn't imply that static friction is a constant! So, the answer is:

$$\boxed{\begin{aligned} &f_A = 13.5\text{N} \\ &f_B = 2.3\text{N} \end{aligned}}$$

d) In this problem, the statement "without B falling off" implies that we are looking at the point at which the static friction on B becomes maximal: if F_p is lower than the value we're looking for, f_s on B can get large enough to accelerate B, but if F_p is too big, then f_s on B would have to exceed its maximum allowed value to accelerate B along; this means that B would fall off of A. So, the first thing we need to do is find the maximum static friction on B. To do so, we need to know the normal force on B, which we found in part b: 14.7N. So, the maximum static friction on B is:

$$f_{s,B,max} = \mu_{s2} N_B = (0.45)(14.7) = 6.6\text{N}$$

To find F_p, we need to solve Newton's second law on box A. **Note that both boxes are accelerating, so $a \neq 0$!** Newton's second law in the x direction for A is:

$$F_p - F_{BonA,x} - f_{k,A} = m_A a \quad \Rightarrow \quad F_p = F_{BonA,x} + f_{k,A} + m_A a$$

What is $F_{BonA,x}$? Recall that it's the reaction force to the friction on B, so we know that it's 6.6N (the maximum static friction on B). We also know $f_{k,A} = 13.5$N, because that doesn't ever change in this problem. However, to solve for F_p, we need to know a. This can't be found from box A, but we can find it by considering box B again. Not only do we know the maximum static friction that B can produce, but knowing that will tell us the maximum acceleration that B can have, using Newton's second law:

$$f_{s,B,max} = m_B a_{max} \quad \Rightarrow \quad a_{max} = \frac{f_{s,B,max}}{m_B} = \frac{6.6}{1.5} = 4.4\text{m/s}^2$$

So, the value of F_p at this acceleration, when the static friction on B is a maximum, is:

$$F_p = F_{BonA,x} + f_{k,A} + m_A a = 6.6 + 13.5 + (4)(4.4) \quad \Rightarrow \quad \boxed{F_p = 37.7\text{N}}$$

FORMULA SHEET

- Constants:

$$g = 9.8\text{m/s}^2$$

- Vectors:

$$\begin{aligned}\vec{A}\cdot\vec{B} &= AB\cos\theta \\ &= A_xB_x + A_yB_y + A_zB_z \\ \left|\vec{A}\times\vec{B}\right| &= AB\sin\theta\end{aligned}$$

- Kinematics:

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum \vec{F} = m\vec{a}$$

$$W = mg$$

$$f_{s,max} = \mu_s N$$

$$f_k = \mu_k N$$