

*Laboratory Manual*

*for*

***PHY2054L***

**June 2015**

*By*

***David Michael Judd***



*Laboratory Manual  
for  
PHY2054L*

**June 2015**

*By*

*David Michael Judd*

Broward College

Copyright © 2007 by David Michael Judd  
All rights reserved.



# ***Table of Contents***

<b><i>Experiment</i></b>	<b><i>Topic</i></b>
1	Introduction to Circuits and Circuit Elements
2	Current and Voltage Characteristics of Resistors
3	Resistors in Series and Parallel
4	Voltmeters and Ammeters
5	The Electric Field and Equipotential Lines
6	Measuring Capacitance
7	The Oscilloscope
8	The RC Time Constant
9	The Charge to Mass Ratio of the Electron
10	Measuring the Refractive Index
11	Measuring the Wavelength of Light
12	The Photoelectric Effect
	<b>Make-Up Lab:</b> Measuring the Earth's Magnetic Field

## ***Appendices:***

- Graphs
- Method of Least Squares
- The Greek Alphabet
- Physical Constants



# ***PHY2054L LABORATORY***

## *Experiment One*

### *An Introduction to Circuits and Circuit Elements*

I wish to identify some of the circuit elements we will be using in this lab and to acquaint you with the **schematic symbols** I will be using to signify these devices. In doing these labs, **you** will be responsible for getting the equipment and setting it up; and when you have finished your experiment, you will "tear down" the circuits and return the lab equipment to its proper place. So, on the list below, please make a note as to the location of the equipment listed.

Every circuit must have a source of energy. The energy is provided by a **power supply**. Power supplies are classified as either for direct current circuits, DC, or alternating current circuits, AC. The schematic symbols that I will be using for the power supplies are shown below.

## POWER SUPPLIES:

### DIRECT CURRENT POWER SUPPLY:

**ELECTRO INDUSTRIES  
MODEL 3002A  
REGULATED DC POWER SUPPLY**



The schematic I will be using for the DC Power supply is:

Direct Current



## ALTERNATING CURRENT POWER SUPPLIES:

**Elenco Electronics  
AC/DC Power Supply  
Model XP-800**

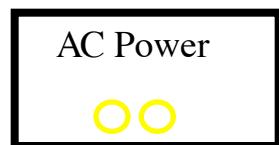


### WARNING:

The 28 V DC output is filtered by a very large capacitor. So, even if the DC voltage adjust knob is turned to zero, the capacitor will, for a time, maintain a voltage across the output terminals. So, do not connect the power supply to electrical equipment--or touch both posts of the output yourself--until this capacitor has been discharged by connecting a 10  $\Omega$  resistor to the output terminals.

The schematic I will be using for this AC Power supply is:

Alternating Current



## FUNCTION OR AUDIO GENERATOR:

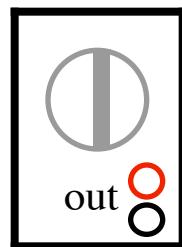
A second kind of AC Power source we will be using in this lab is the function or audio generator, represented below.

**BK Precision  
3001 Audio Generator**



The schematic I will be using for the audio generator is:

**Audio Generator**



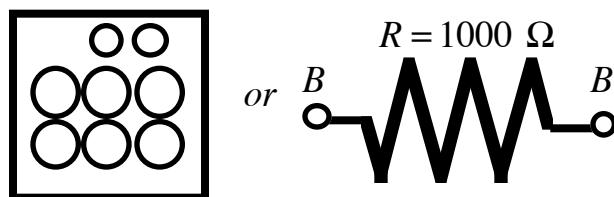
In general, circuits are designed to perform specific tasks; and, therefore, circuit elements are designed to help accomplish these tasks. Circuit elements are used to control currents and/or the electrical energy delivered. In this lab, we are going to be looking at a few of the more common electrical circuit elements. The first circuit element I want to mention is the **resistor**. The resistor has many uses. One of these uses is to help limit the current that is delivered to a specific branch of a circuit, as we shall soon see. In general, we will be using the Cenco decade resistor box. Usually, we will write the specific magnitude of the resistor next to its schematic symbol.

### THE RESISTOR:

Cenco Decade Resistance Box

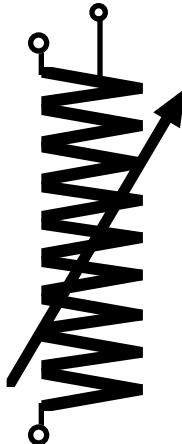


The resistor schematic I will be using is:

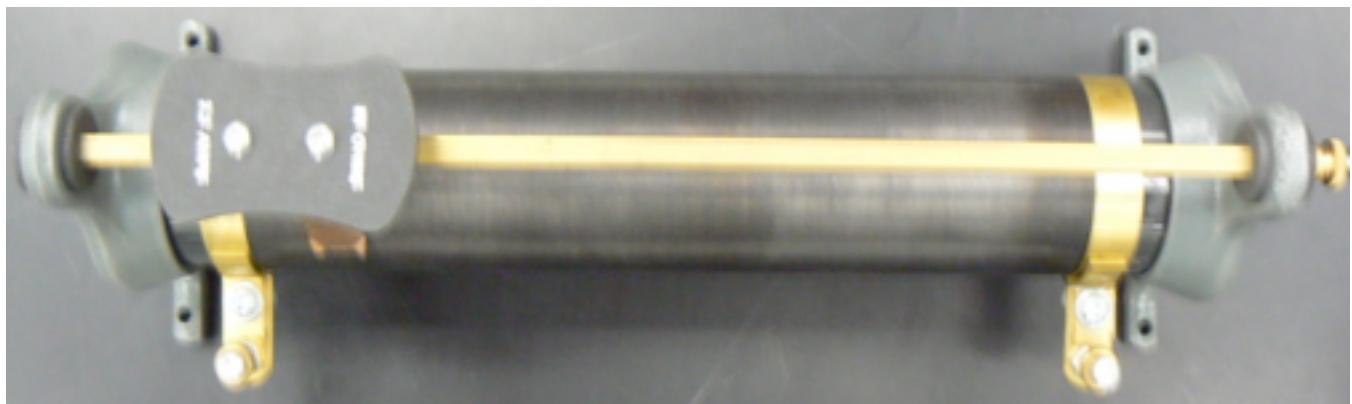


The next circuit element I want to mention is the **rheostat**. The rheostat, or potentiometer as it is sometimes called, is often used to insure a small potential difference, and, therefore, a small current in an isolated circuit branch. It is also commonly used as a light dimmer. The arrow through the "resistor-like" symbol indicates the resistance is variable. So, in essence, the rheostat is a variable resistor.

### THE RHEOSTAT:



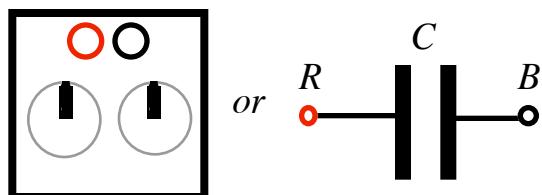
A Rheostat



In some circuits, there is a need to have electric potential energy stored for rapid access--say like when you start your car. **Capacitors** are devices designed to store electric potential energy. The symbol I will use for a capacitor is:

### THE CAPACITOR:

The schematic symbol I will use for the capacitor is:



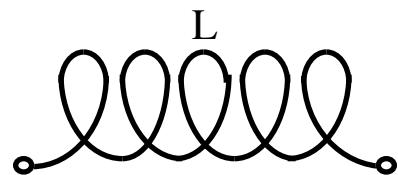
Most often in this lab we will be using a decade capacitance box like the one shown below

### A Decade Capacitor Box



### THE INDUCTOR:

A common circuit device we will be talking about in the lecture part of this class is called an inductor. In essence, an inductor is a winding of wire. Its schematic symbol is:



## METERS:

Meters are designed to measure specific physical quantities at a specific point in a circuit. The meters we will be using are:

### THE GALVANOMETER

The **galvanometer** is a device that can measure very weak electrical currents. We have two models of galvanometer that we use in this lab. Both are pictured below. I will refer to them as old or new. (This does not mean that the newer one is better!)

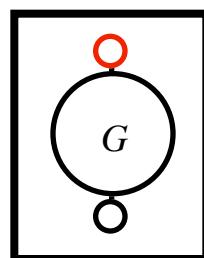
**Old Galvanometer**



**New Galvanometer**

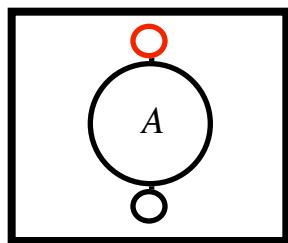


The schematic symbol I will use for the galvanometer is:



## THE AMMETER

An **ammeter** is a device used to measure larger currents than is generally the case with a galvanometer. Again, we have two models of ammeter we use most often in this lab. I will refer to one as the metal case ammeter and the other as the plastic case ammeter. You may feel free to use either. I will signify an ammeter with the following schematic symbol:



**Plastic Case Ammeter**



### Metal Case Ammeter



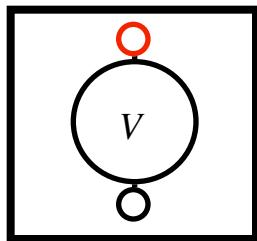
### THE VOLTMETER

A **voltmeter** is a device used to measure the electric potential difference between two points in a circuit. We also have a metal case voltmeter like the one shown below. Most often, however, we will use the Fluke Multimeter, about which I will have more to say later.

### Metal Case Voltmeter

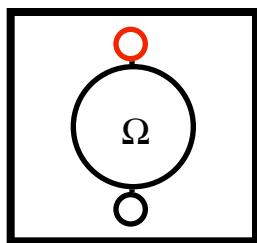


The schematic symbol for the voltmeter is given by



### THE OHMMETER

An ohmmeter is a device used to measure the resistance of a resistor. We will also use the Fluke Multimeter when we need an ohmmeter. The schematic symbol for the ohmmeter is:



### THE FLUKE MULTIMETER

**Multimeters** are devices that incorporate many meter functions in one handy instrument. We will often be using the Fluke Multimeter. Fluke multimeters are excellent for measuring the potential difference in both direct and alternating current circuits. They are also handy for measuring resistances, and capacitances. (We will not, as a rule, be using them as ammeters.) Below is pictured a Fluke multimeter. Probes--not shown--are connected to the meter and placed at the points in the circuit between which we wish to know the specific value of the physical quantity being measured.

## Fluke 179 Multimeter



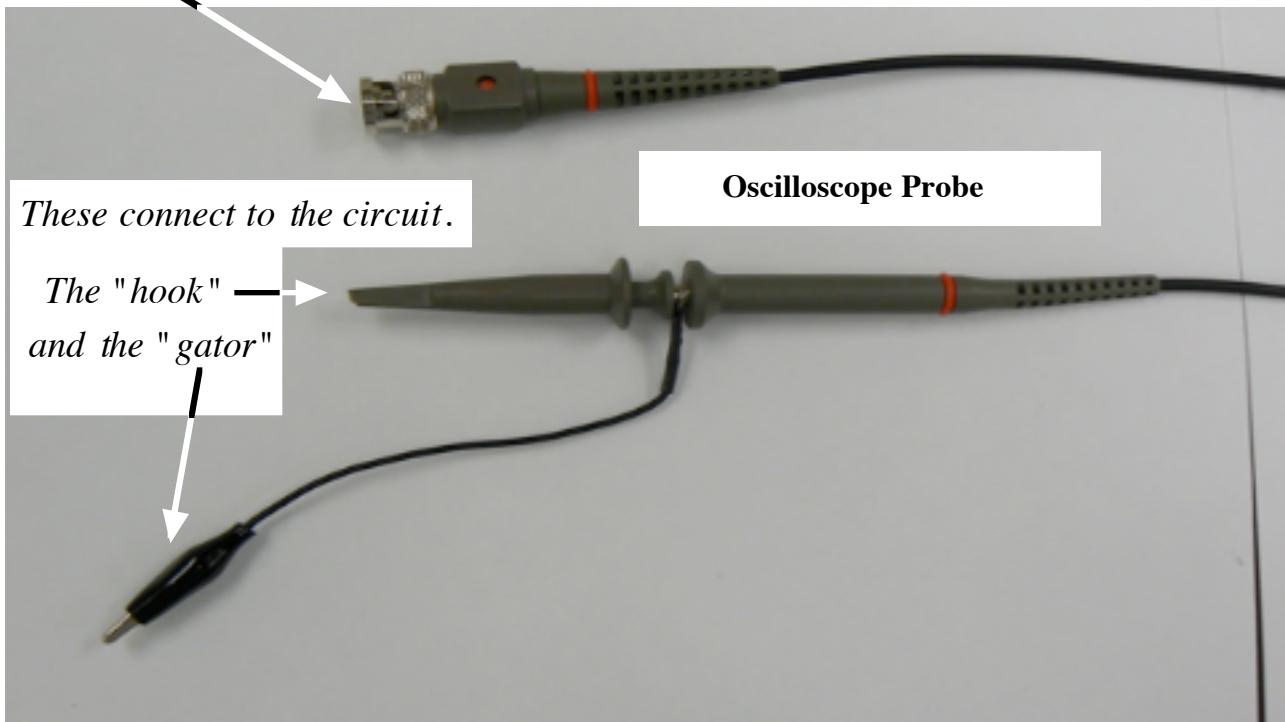
## THE OSCILLOSCOPE

There are circuits within which the conditions are changing very, very rapidly. **Oscilloscopes** are the best devices for getting information in such circumstances. Below is a picture of the oscilloscope we will be using in the lab.

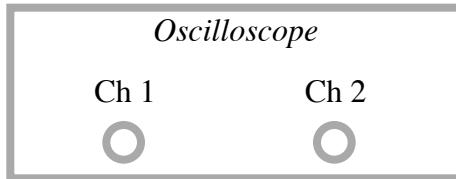
### An Oscilloscope and Probe



*connects to the oscilloscope post*



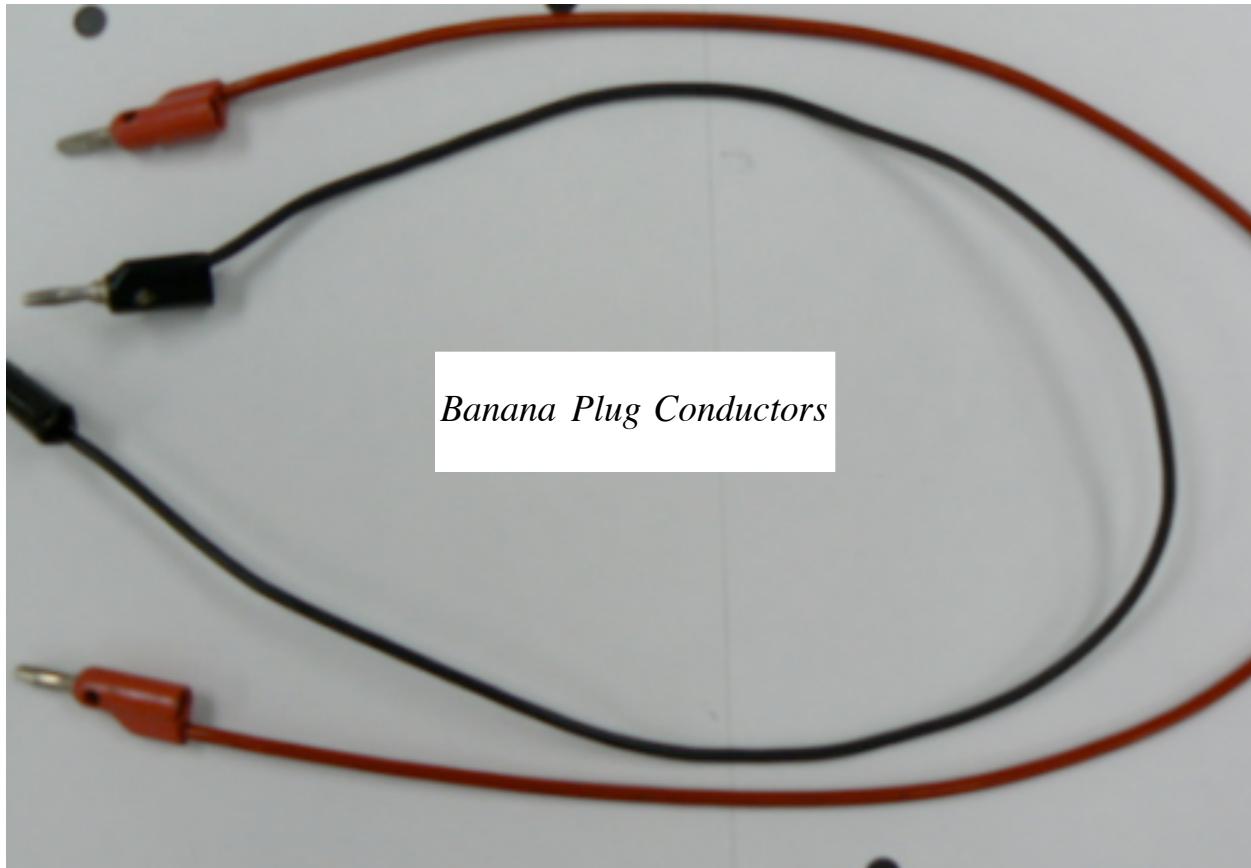
The funky schematic symbol I will use for the oscilloscope is:



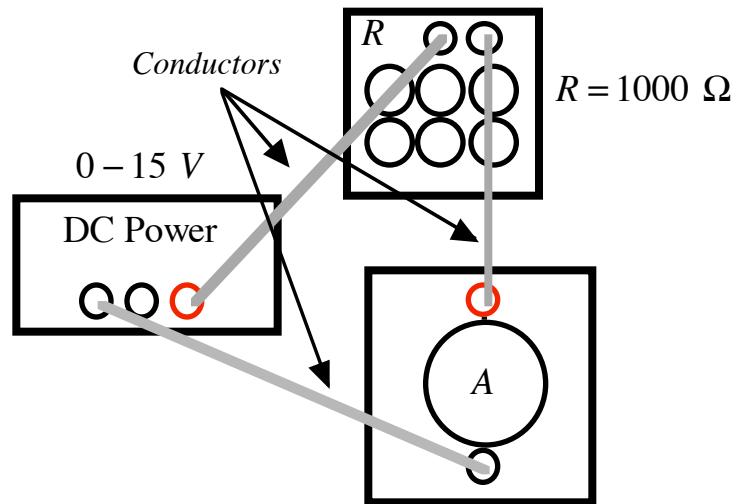
## THE CONDUCTORS

The last thing we will need for our circuits are devices to connect the components. These are called **conductors**. In general, conductors are represented by straight-line segments from component to component as shown in the simple circuit schematic below.

### Red and Black Banana Plug Conductors



### A Simple Circuit Schematic



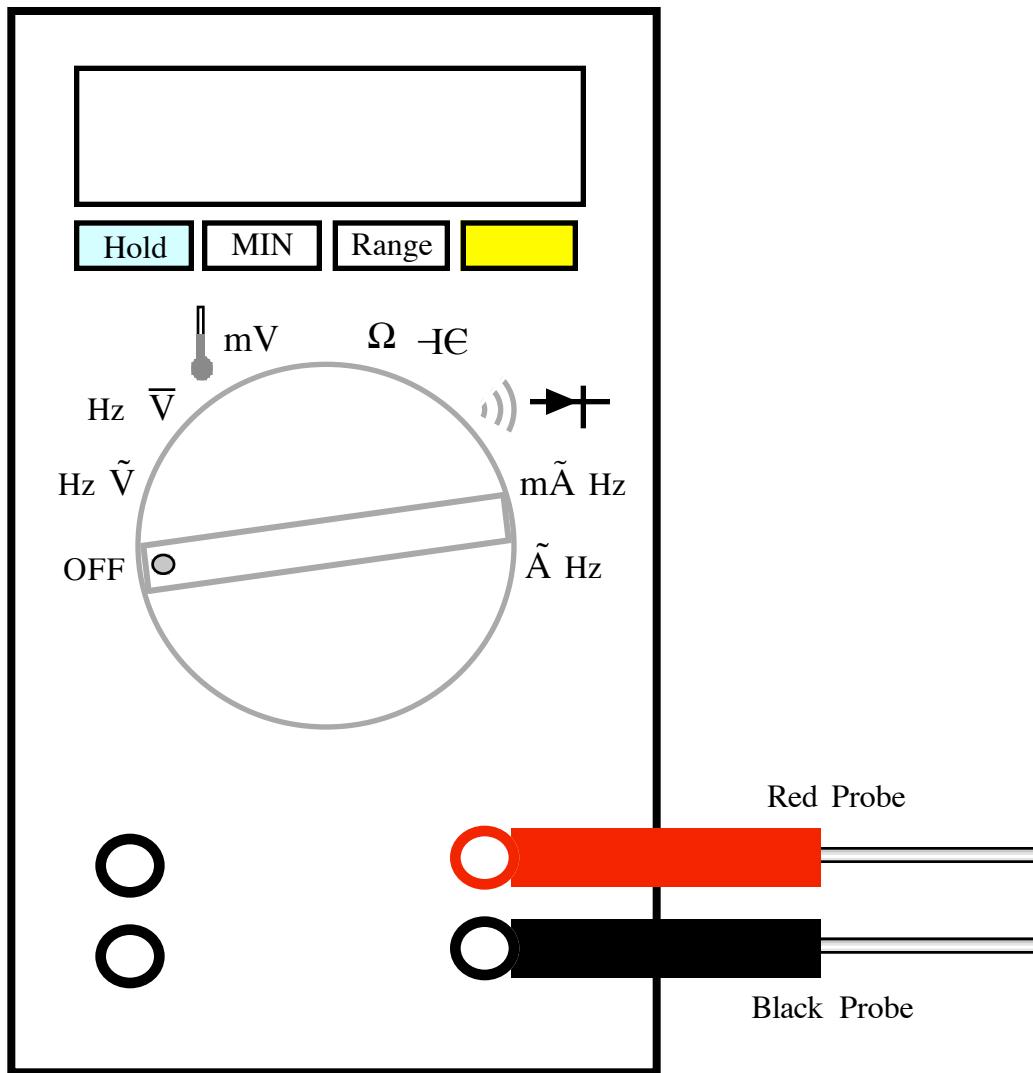


# **Troubleshooting 101**

It is a helpful skill to be able to determine if the equipment you have is working properly. I want to give you some information that might help you do some of the things you might otherwise be calling on me to do. (Unfortunately, or fortunately, as the case may be, there is only one of me and you will get bored if you simply sit around and wait for me to be able to get to your group before you can get started.)

## **I. Determine if the Fluke Multimeter is Working Properly!**

**Fluke 179 Multimeter**



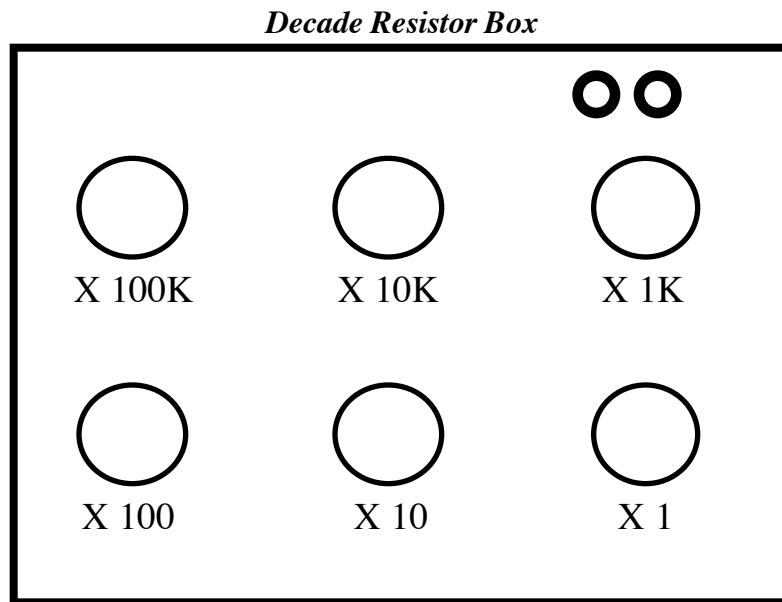
The Fluke multimeter is a very reliable device as long as its battery is OK. So, the first thing you want to do is turn on the Fluke and make sure the battery symbol is not showing. If it is, then we need to replace the battery.

Shown above is a representation of this multimeter and it is desirable to get to be able to use

this meter quickly. To that end, study the meter and become acquainted with its functions. When the dial is set on  $\tilde{V}$ , the meter is ready to measure electric potential differences in alternating current circuits. When the dial is set on  $\overline{V}$ , the meter is ready to measure electric potential differences in direct current circuits. When the dial is set on  $\Omega$ , the meter acts as an ohmmeter and can measure resistor values. And when the dial is set to  $C$  the meter is set to measure capacitance. In general, we will **not** use the Fluke as an ammeter.

## II. Determine if the Resistor Box is Working!

Many of the circuits we will be working with have resistors. Most often we will be using a **decade resistor box** for this purpose. (See the figure below.) Each dial has a setting from zero to ten. Under each dial is a multiplication factor signified by X. (The symbol K indicates one thousand, so  $10K\ \Omega \equiv 10,000\ \Omega \equiv 10^4\ \Omega$ .) One can determine if the resistor box is working by checking it with the Fluke Multimeter. Set the Fluke to measure resistances. Place the Fluke probes into the two black ports of the resistor box. Set the resistor box to the desired resistance and check its value with the Fluke. If the box is not working, set it aside and get another box.



## III. Setting Up the DC Power Supply to the Correct Allowable Current!

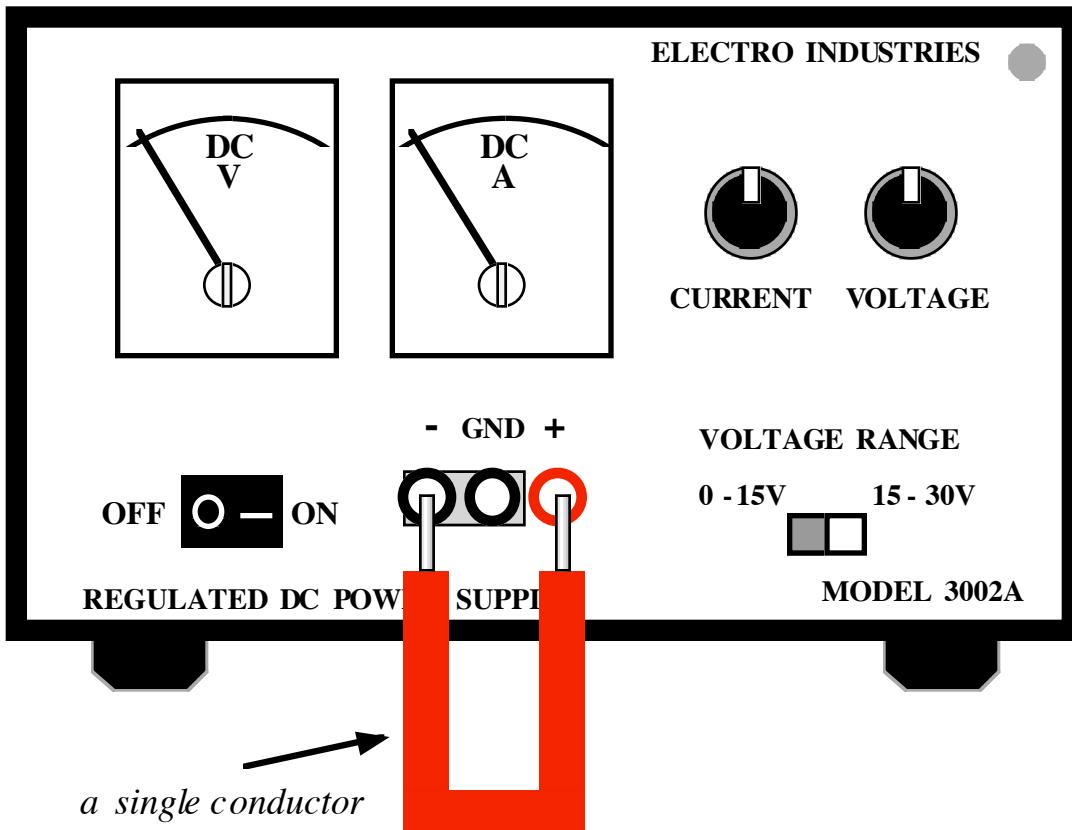
Often, we will be using the Electro Industries Model 3002A Regulated DC power supply when we work with DC circuits. It is said to be a regulated power supply because we can limit the amount of current the supply can deliver. (This, though it may seem to be a nuisance, is really a safety feature.) Every time before you use this power supply, you must set it to the correct limit on current. For most of the labs, we will not need more than 0.30 Amp. So, you need to follow the following instructions to set up the power supply properly.

Step One: With the power supply turned **off**, connect the power cable to the power supply. Plug in the cable to the nearest outlet. Turn on the supply and see if the light in the top right hand corner comes on. If it does, it means your power

supply is getting energy from FP&L and you can turn it off again. If the light does not go on, it usually means that the circuit the outlet is plugged into has been opened by a circuit breaker. Reset the circuit breaker and try again. Once the power supply is getting energy, turn it off and move on to step two.

Step Two: With the power turned **off**, take a single conductor and put one end into the black negative port and the other end into the red port. (This causes the two ports to have the same electric potential.) Turn the current knob and the voltage knob so that they are in the middle of their range--pointing directly upward. Turn on the power supply. The voltage meter should read zero. The current however should not be zero. Adjust the current knob until the current meter it reads **0.30 Amp**. Turn off the power supply and remove the conductor cable. (**For the remainder of the experiment, do not adjust the current knob!**) Your power supply is now ready.

### Short Circuit Configuration

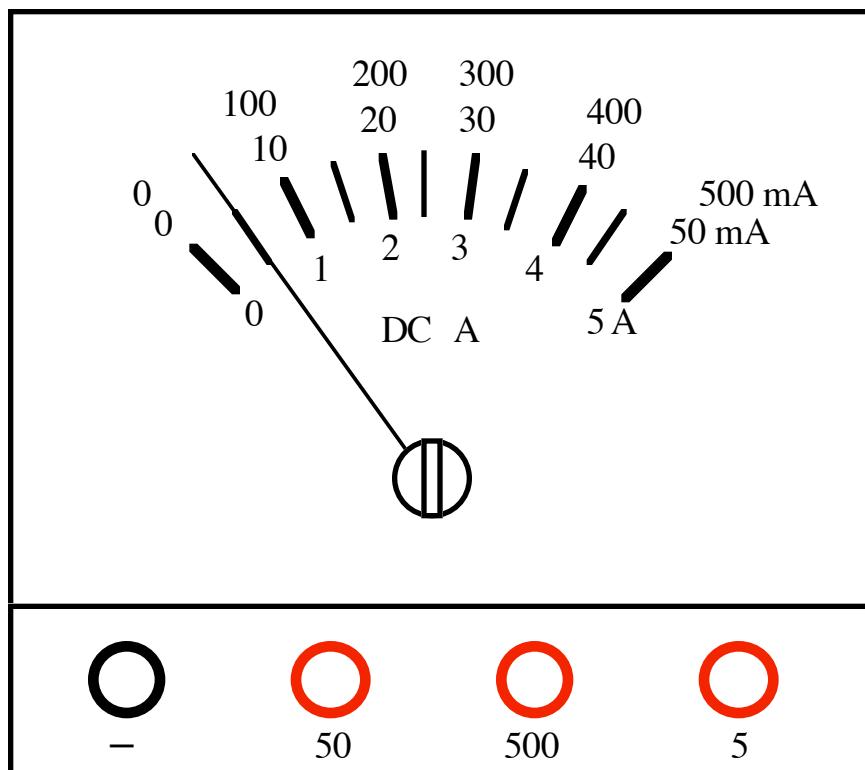


### IV. How to Determine if the Ammeter Works!

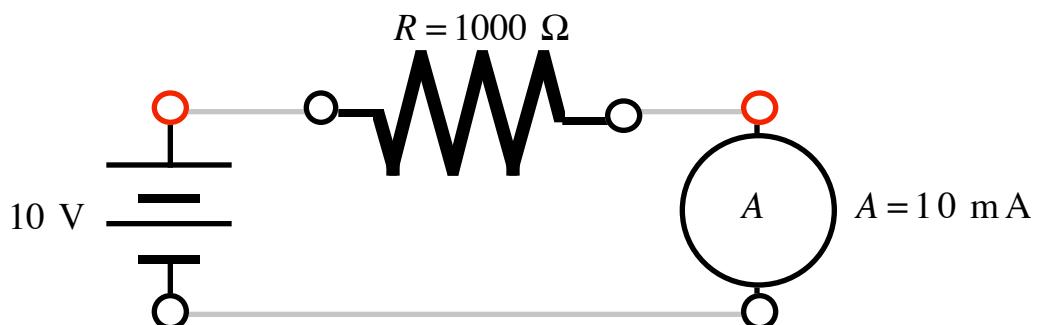
We will frequently be using an ammeter. In the diagram below is a representation of the most common ammeter in our lab. In the diagram, the dial is reading 5 mA, 50 mA, or 0.5 A depending on which red port is used. These ammeters are fairly cheap and so occasionally get damaged and need to be replaced. Now that we know how to set up a DC power supply and check a resistor box, we can set up the simple circuit, shown below, and use it to check the ammeter. We set the resistor to  $1000 \Omega$ . We run a conducting cable from the red port of the DC power supply to one of the black ports of the resistor. From the other black port of the resistor, run a second conductor

to the 50 mA red port of the ammeter. Finally, run a third conductor from the black port of the ammeter to the negative black port of the power supply. Without touching the current knob on the power supply, turn the voltage knob all the way down--completely counterclockwise. Take the Fluke Multimeter and set it so that it measures DC voltages. Put the red probe of the Fluke into the black port of the resistor that is connected to the power supply. Put the black probe of the Fluke into the other black port of the resistor--the one connected to the ammeter. Turn on the power supply and slowly increase the electric potential difference until the Fluke reads exactly 10.0 V . The ammeter is working properly if its value is 10 mA . After you have checked the ammeter, turn down the voltage knob of the power supply and then turn it off. (If your ammeter is not working, find one that is!)

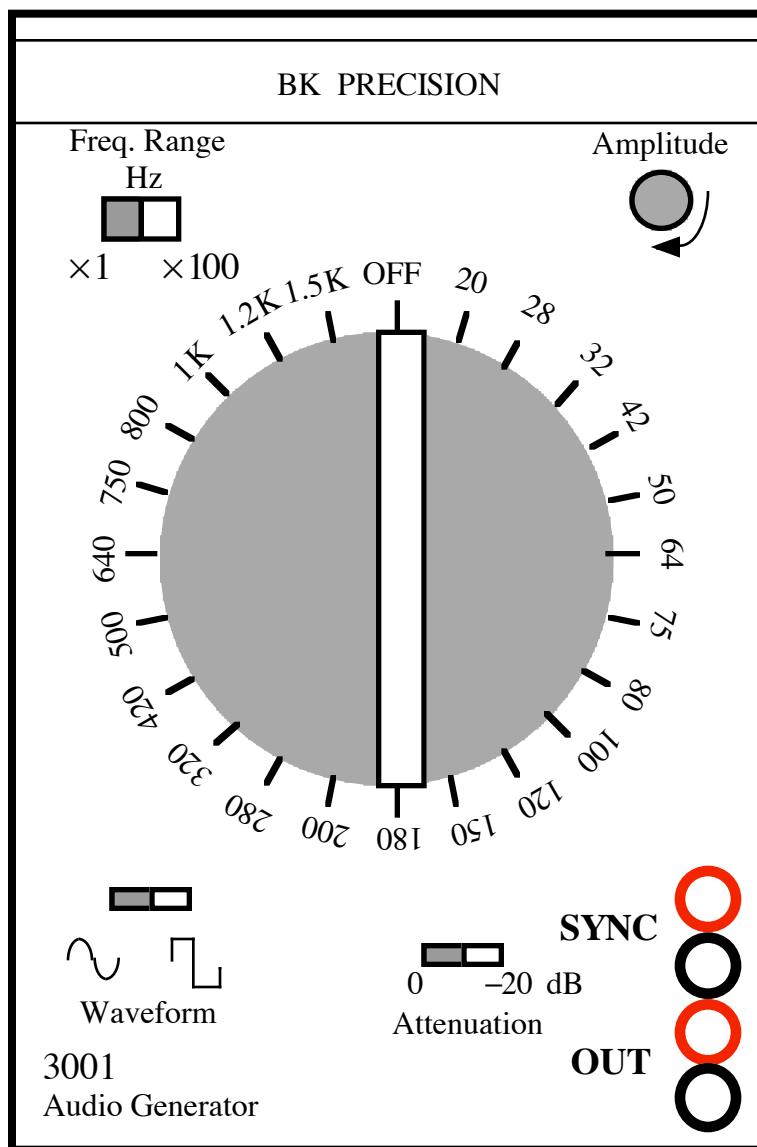
### *Direct Current Ammeter*



### *Simple Circuit for Checking a DC Ammeter*



## V. How to Determine if the Function Generator is Working Properly!



- 1) Set the function generator to sine wave mode at a frequency of 64 Hz. Turn the amplitude knob clockwise as far as possible.
- 2) Set the Fluke multimeter to AC voltage mode ( $\tilde{V}$ ). Put the red probe of the Fluke into the **red output port** of the function generator, and the black probe of the Fluke into the **black output port** of the function generator. Turn on the Fluke and measure the output voltage of the function generator. You probably need to change the function generator's battery if  $V_{out} < 1.00 \text{ Volt}$ .



***PHY2054L LABORATORY***

***Experiment One***

***An Introduction to Circuits  
and Circuit Elements***

**Name:**

---

**Date:**

---

**Day and Time:**

---



# ***PHY2054L LABORATORY***

## *Experiment Two*

### *Current and Voltage Characteristics of a Resistor*

## EQUIPMENT NEEDED

One Fluke Multimeter  
One Commercial Ammeter  
3 Conducting Cables

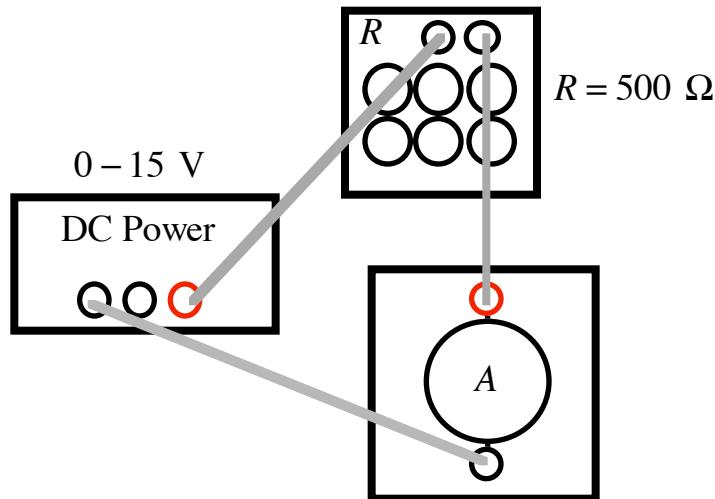
One Electro Model 3002A DC Power Supply  
One Decade Resistor Box

## PROCEDURE

- 1.) Get the equipment needed for this experiment.
- 2.) **Use the Troubleshooting Guide to do the following:**
  - a) Get a resistor box that works at the following resistor values:  
 $500 \Omega$ ,  $1000 \Omega$ ,  $1500 \Omega$ , and  $2000 \Omega$ .
  - b) Set up the Electro Industries Model 3002A DC power supply for a maximum current of  $0.30 \text{ A}$ .
  - c) Find an ammeter that reads  $10 \text{ mA}$  when the ammeter is series with a  $1000 \Omega$  resistor across which is applied a potential difference of  $10 \text{ Volts}$ .

### Measuring the Current and Voltage Characteristics of a Resistor

*Figure One*



3.) Once you have all of the necessary equipment in proper working order, set your resistor as close to  $500 \Omega$  as possible and record the value on the data sheet below. With the power supply **off**, set up the circuit shown above in Figure One.

4.) The power supply slide switch should be set in the  $0 - 15 \text{ V}$  position, and the voltage adjust knob should be at its **minimum** setting. (You should always start up electrical systems at minimum power settings! Later in the process when you need more energy, the slide switch will have to be moved to the  $15 - 30 \text{ V}$  position. When it is time to change to the  $15 - 30 \text{ V}$  setting, first turn the voltage adjust knob to its minimum setting!)

5.) Turn on the power supply and slowly increase the power until the potential difference across the ports of the resistor reads  $4.0 \text{ Volts}$ . Record the value of the current, as indicated by the ammeter, on the data sheet below. Measure and record current values for the potential

differences indicated on the data sheet. Remember, after the measurement at 12.0 *Volts*, you need to change the power supply slide switch to the 15 – 30 V position.

6.) Next, repeat step 5.) for measured resistances of  $1000\ \Omega$ ,  $1500\ \Omega$ , and  $2000\ \Omega$ . Be sure to record your data on the data sheets below.

## THINGS TO DO

- I. Answer the questions in the **PROLEGOMENA** section below.
- II. On the **single sheet** of graph paper provided, graph the data for **each** specified resistance. The potential difference  $\Delta V$  is on the vertical axis and in units of *Volts*, while the current  $I$  is on the horizontal axis in units of *millamps*. **Draw** a straight line estimate of the best fit of the data for **each** specified resistance. From your graph, calculate the slope of **each** best fit line and record these values on the data sheet.
- III. Also on the data sheet below, answer the following question:  
Your calculated slope values represent what physical quantity?
- IV. See if you can determine an appropriate equation relating the potential difference across the resistor to the current passing through the resistor and the value of the resistor's resistance.



***PHY2054L LABORATORY***

***Experiment Two***

***Current and Voltage  
Characteristics of a Resistor***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## PROLEGOMENA

Consider the following equation:

$$y = m x + b , \quad (1)$$

where  $x$  is the independent variable,  $y$  is the dependent variable, and  $m$  and  $b$  are constants. The graph of this equation would be

- a) a circle.
- b) an ellipse.
- c) a straight line.
- d) a hyperbola.
- e) a parabola.

Using equation (1), note that

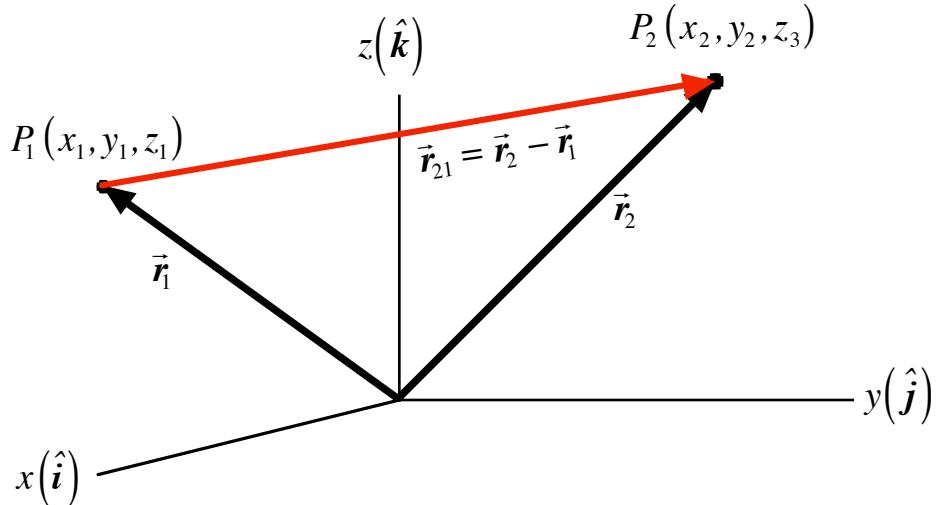
$$\Delta y = y_L - y_E = [mx_L + b] - [mx_E + b] = m[x_L - x_E] = m \Delta x . \quad (2)$$

What is the **ratio** of  $\Delta y$  to  $\Delta x$ ?

Exactly what information about the line does  $m$  give us?

Exactly what information about the line does  $b$  give us?

Using the information given in the diagram below, write expressions for the magnitude  $|\vec{r}_{21}|$  and the unit vector  $\hat{\vec{r}}_{12}$  that represents the direction of vector  $\vec{r}_{21}$ , where  $\vec{r}_{21} = |\vec{r}_{12}| \hat{\vec{r}}_{12} = \vec{r}_2 - \vec{r}_1$ .



## Data Sheets

### *Trial One*

$$R = 500 \text{ } \Omega$$

$$R_{\text{measured}} = \underline{\hspace{10mm}} \Omega$$

$\Delta V(\text{V})$	$I(\text{mA})$
4	
6	
8	
10	
12	
14	
16	
18	
20	

*Slope obtained from your graph?*

---

*What is the physical significance of this slope?*

***Trial Two***

$$R = 1000 \Omega$$

$$R_{measured} = \underline{\hspace{5cm}} \Omega$$

$\Delta V(V)$	$I(mA)$
4	
6	
8	
10	
12	
14	
16	
18	
20	

*Slope obtained from your graph?*

---

*What is the physical significance of this slope?*

***Trial Three***

$$R = 1500 \Omega$$

$$R_{measured} = \underline{\hspace{10mm}} \Omega$$

$\Delta V(V)$	$I(mA)$
4	
6	
8	
10	
12	
14	
16	
18	
20	

*Slope obtained from your graph?*

---

*What is the physical significance of this slope?*

***Trial Four***

$$R = 2000 \Omega$$

$$R_{measured} = \underline{\hspace{10mm}} \Omega$$

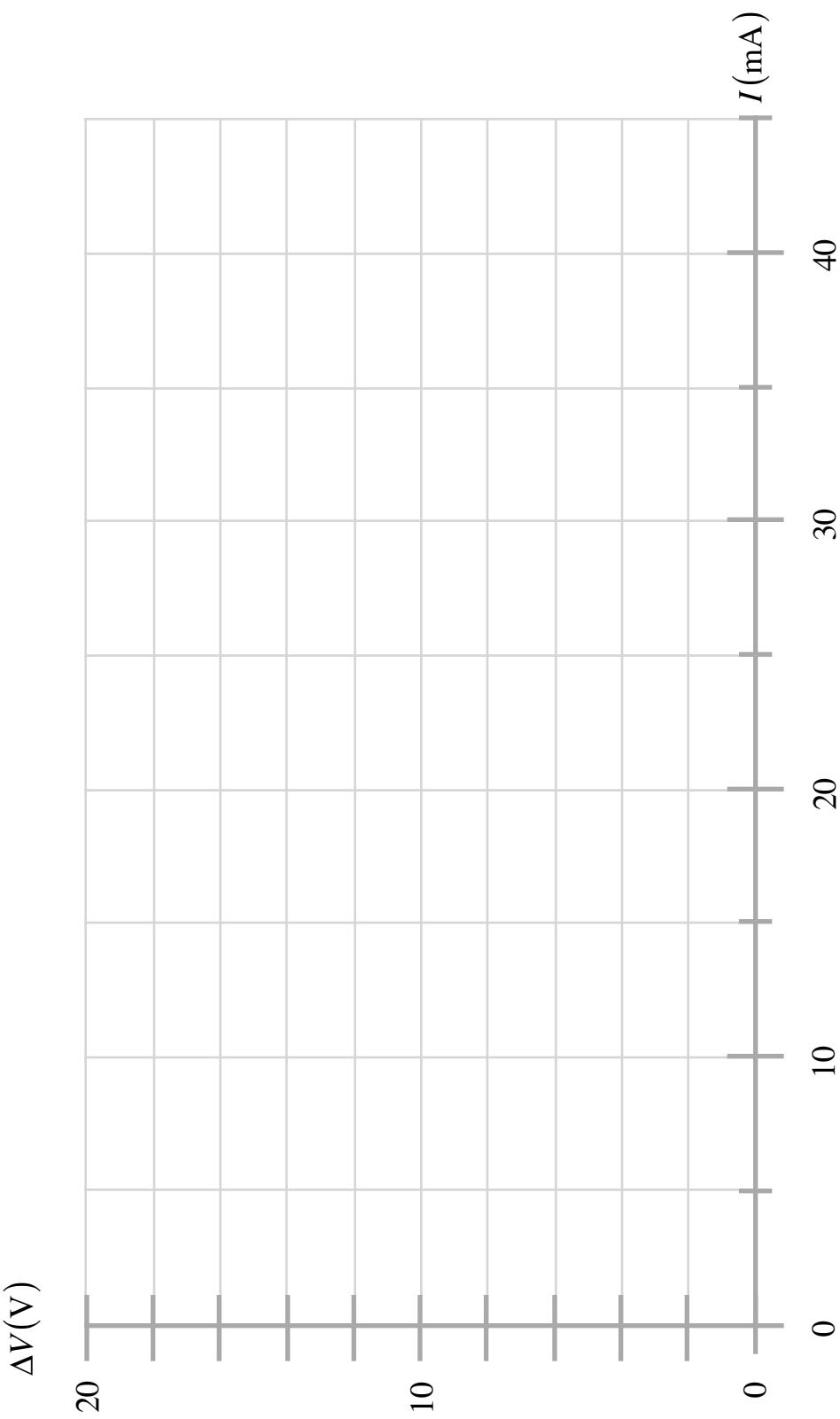
$\Delta V(V)$	$I(mA)$
4	
6	
8	
10	
12	
14	
16	
18	
20	

*Slope obtained from your graph?*

---

*What is the physical significance of this slope?*

*The Current through a Resistor as a Function of the Voltage across the Resistor*





# ***PHY2054L LABORATORY***

## *Experiment Three*

### *Resistors in Series and Parallel*



***PHY2054L LABORATORY***

***Experiment Three***

***Resistors in Series and Parallel***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## EQUIPMENT NEEDED

One Fluke Multimeter  
Two Decade Resistor Boxes  
6 Conducting Cables

One Electro Model 3002A DC Power Supply  
Two Ammeters

## REVIEW

In our last experiment, we found that there is a simple relationship between the potential difference across a resistor and the current passing through the resistor. This relationship is called **Ohm's Law** and can be written as

$$\Delta V = IR , \quad (1)$$

where  $\Delta V$  is the potential difference across the resistor,  $I$  is the current passing through the resistor, and  $R$  is the resistance of the resistor. In fact, physicists define the **resistance** of any circuit element as the ratio of the voltage to the current, that is

$$R = \Delta V / I . \quad (2)$$

You also found out last time, that this ratio is constant over a wide range of operating conditions for the resistor. A circuit element is said to be **ohmic** if its current-voltage characteristics are accurately described by equation one. There are, however, other types of circuit elements for which the ratio expressed in equation two is not constant. These circuit elements are said to be **nonohmic**. Regardless of the circuit element, however, physicists and electrical engineers are very much interested in the so-called current-voltage characteristics of circuit elements. These characteristics are represented with a graph of  $\Delta V$  versus  $I$ , just as you did for a few simple resistors last week. (Even ordinary resistors do not always "obey" Ohm's Law. For most metals, there is a temperature at which they become super conductors and all resistance vanishes.)

In this experiment, we want to use this information and look more closely at what happens to the current and the voltage across a resistor when the resistor is placed in a circuit with other resistors. The two most common configurations for resistors are the **series** circuit and the **parallel** circuit. Of course, one can have more complicated configurations also. These more complicated configurations usually can be reduced to combinations of series and parallel circuits.

## PROCEDURE

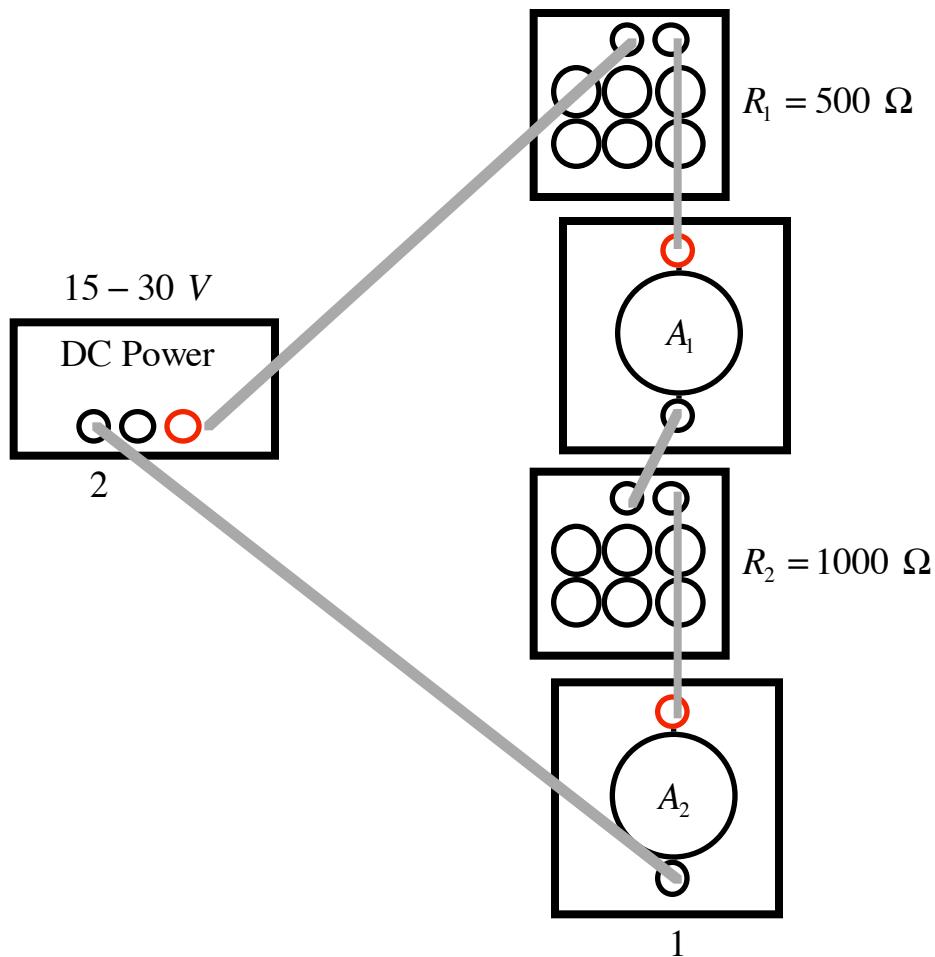
### Resistors in Series

#### *Part One: Measuring the Current and Voltage Characteristics of Resistors in Series*

- 1.) Get the equipment needed for this experiment.
- 2.) **Use the Troubleshooting Guide to do the following:**
  - a) Get two resistor boxes that work at the following resistor values:  
 $500 \Omega$  and  $1000 \Omega$  .
  - b) Set up the Electro Industries Model 3002A DC power supply for a maximum current of  $0.30 \text{ A}$  .
  - c) Find two ammeters that read  $10 \text{ mA}$  when the ammeter is in series with a  $1000 \Omega$  resistor across which is applied a potential difference of  $10 \text{ Volts}$ .
- 3.) With the power **off**, set up the circuit represented in Figure One below. (Resistor values always are to be measured using the Fluke multimeter when the resistor is isolated.) After you have set up the circuit, try to understand what will happen in the circuit once the power is turned on. **Before you turn on the power, I want you to predict what you think will happen in the circuit once the power is turned on.** (Please remember that it is OK to be mistaken!)

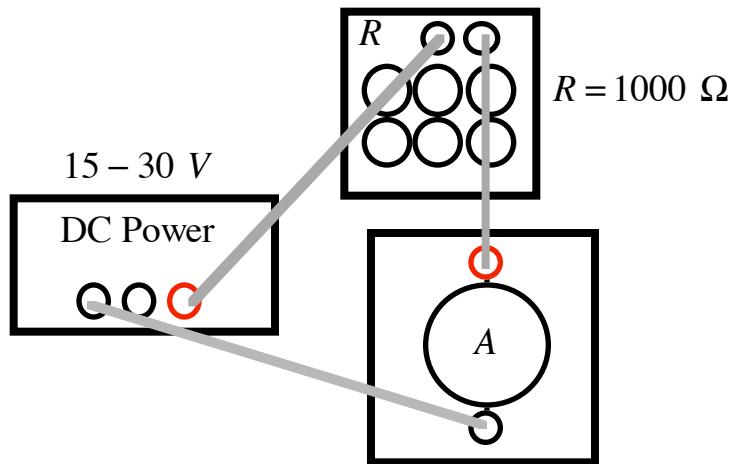
- (1) The current will flow
  - a) from point 1 to point 2.
  - b) from point 2 to point 1.
- (2) For a constant potential difference  $\Delta V$  across the power supply, the ammeter values will be
  - a)  $A_1 > A_2$ .
  - b)  $A_1 < A_2$ .
  - c)  $A_1 = A_2$ .
- (3) The potential differences across the resistors will be
  - a)  $\Delta V_1 > \Delta V_2$ .
  - b)  $\Delta V_1 < \Delta V_2$ .
  - c)  $\Delta V_1 = \Delta V_2$ .
- (4) Exactly what does an ammeter measure?

**Figure One**  
**Two Resistors in Series**



- 4.) Now that you have made your predictions, turn on the power and increase it until the potential difference **across the power supply** is 20 V . Once at this value, do not change it for the remainder of this part of the experiment.
- 5.) On the data sheet, record the values of  $A_1$  , and  $A_2$  .
- 6.) Measure and record the value of the potential difference across **each** resistor,  $\Delta V_1$  and  $\Delta V_2$  . Turn off the power when you have finished this step.
- 7.) Compare the actual measurements of the current through each resistor with your prediction in step 3.) part (2). What did you find out?
- 8.) Compare the measurements of the potential differences across each resistor with your prediction in step 3.) part (3). What did you find out?
- 9.) Next, take the product of  $A_1 R_1$  and  $A_2 R_2$  and record these values in the  $A_i R_i$  column.
- 10.) Compare the  $A_i R_i$  products with the measured potential differences  $\Delta V_1$  and  $\Delta V_2$  . Do these products confirm or contradict Ohm's Law:  $\Delta V = IR$  ?
- 11.) Add  $\Delta V_1$  and  $\Delta V_2$  . Record this value on the data sheet. How does this sum compare to the potential difference setting of the power supply?
- Part Two: Determining the Equivalent Resistance for Resistors in Series***
- In part one, you should have found that  $A_1 = A_2 \approx 13.3$  mA and that  $\Delta V_1 < \Delta V_2$  while  $\Delta V_1 + \Delta V_2 \approx 20$  V. We are now going to find out what resistance a single resistor must have so that if we were to replace  $R_1$  and  $R_2$  with this resistor, then at 20 V we would have a current passing through it of 13.3 mA. Such a resistor is said to be an **equivalent resistor**.
- 12.) Using the 1000  $\Omega$  resistor, set up the circuit shown below in Figure Two with a potential difference of 20 V across the power supply.
- 13.) **Adjust the resistance** until the value of  $A$  is the **same** as you had in part one.
- 14.) **Turn off the power.** Measure the resistance of the resistor and record this value as  $R_{eq, series}$  .
- 15.) Identify what measured values were the **same** in the circuit in part two and the circuit in part one.
- 16.) How does  $R_{eq, series}$  compare with ,  $R_1$  and  $R_2$  of part one? Can you quantify the relationship in an equation?

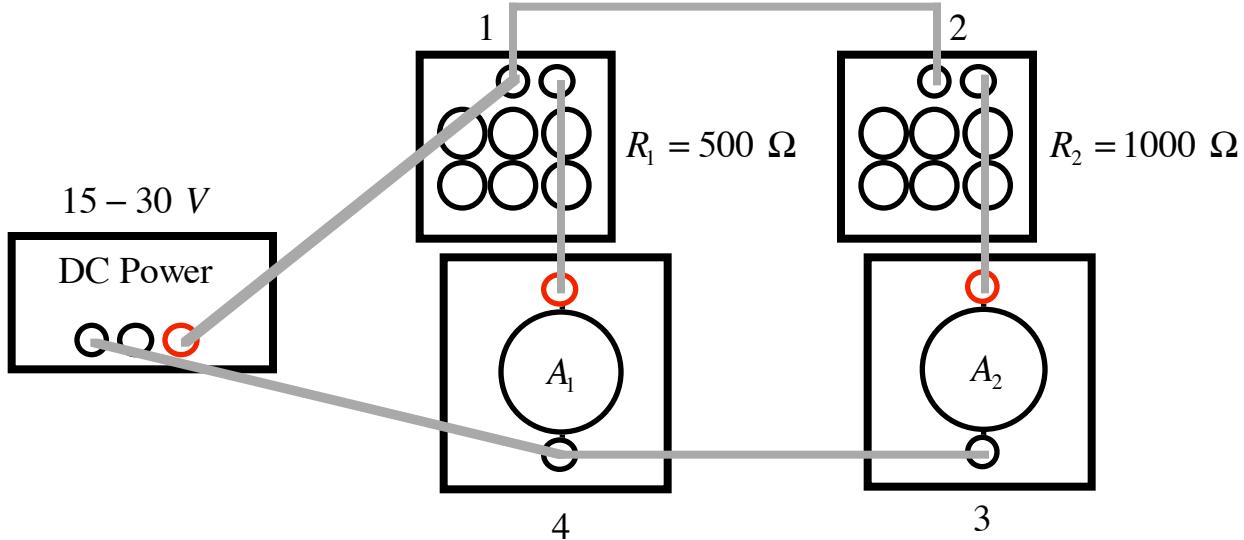
**Figure Two**  
*Circuit for Finding the Equivalent Resistance for Resistors in Series*



### Resistors in Parallel

*Part Three: Measuring the Current and Voltage Characteristics of Resistors in Parallel*

**Figure Three**  
*Two Resistors in Parallel*



- 17.) With the power turned off, set up the circuit shown above in Figure Three.
- 18.) Again, consider this circuit carefully trying to understand what is going to happen when the power is turned on.
  - (1) The current will flow from
    - a) point 1 through the resistor to point 4 and from point 2 to point 3 .
    - b) point 4 through the resistor to point 1 and from point 3 to point 2 .
    - c) If you do not like choices a) or b), identify what you think the path will be:

- (2) The total amount of current in the circuit:
- $A_{tot} = A_1 + A_2$
  - $A_{tot} = A_1 - A_2$
  - $A_{tot} = A_1 A_2$
  - None of the above.
- (3) For a fixed output potential difference  $\Delta V$ , the ammeter values will be such that
- $A_1 > A_2$ .
  - $A_1 < A_2$ .
  - $A_1 = A_2$ .
- (4) You expect the potential differences across the resistors to be such that
- $\Delta V_1 > \Delta V_2$ .
  - $\Delta V_1 < \Delta V_2$ .
  - $\Delta V_1 = \Delta V_2$ .
- (5) Exactly what does a voltmeter measure?

19.) Now that you have made your predictions, turn on the power supply and set the potential difference across the power supply to 20 V .

20.) Record the values of  $A_1$  , and  $A_2$  in the table on the data sheet below.

21.) Measure and record the value of the following potential differences:

The potential difference across resistor one  $\Delta V_1$

The potential difference across resistor two  $\Delta V_2$

The potential difference across points one and two  $\Delta V_{12}$

The potential difference across points one and four  $\Delta V_{14}$

The potential difference across points two and three  $\Delta V_{23}$

The potential difference across points three and four  $\Delta V_{34}$

(Note, for potential differences of the form  $\Delta V_{ab}$  , the red probe goes at point  $a$  and the black probe at point  $b$  .)

22.) **Turn off** the power.

23.) Compare the measurements of the currents through the ammeters with your prediction in step 18.) part (3). What did you find out?

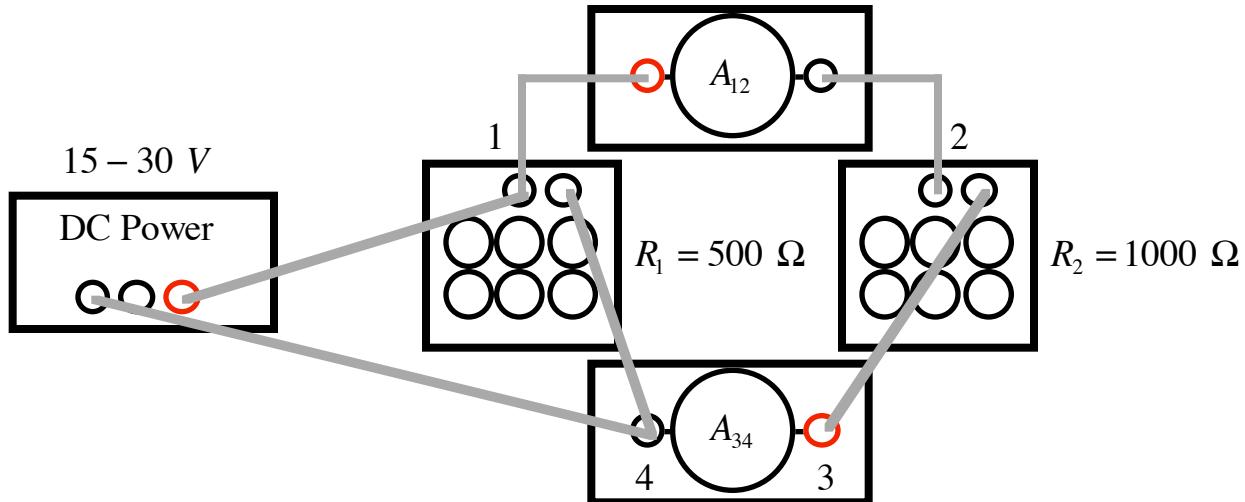
24.) Compare the measurements of the potential differences across the resistors with your prediction in step 18.) part (4.). What did you find out?

25.) Take the product of the corresponding currents and resistances and record the results in the  $A_i R_i$  column on the data sheet below.

26.) Compare the  $A_i R_i$  products with the measured potential differences across the resistors  $\Delta V_1$  and  $\Delta V_2$ . What did you find out?

27.) Are your potential difference measurements consistent with Ohm's Law:  $\Delta V = IR$  ?

**Figure Four**



28.) Now I want you to look at the potential differences  $\Delta V_{12}$  and  $\Delta V_{34}$ . These values should be close to zero. This would seem to imply that there is very little or no current passing from point one to point two, and from point three to point four. I want you to set up the circuit shown above in Figure Four to measure the currents in these two paths.

29.) Turn on the power supply and set the output potential difference to 20 V .

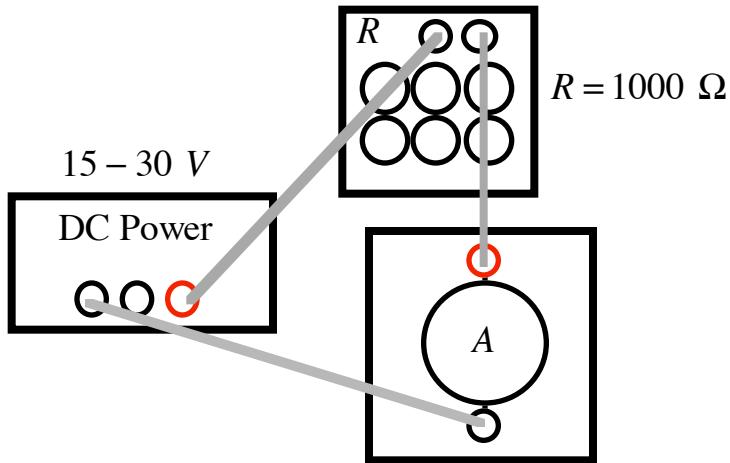
30.) Record the values of  $A_{12}$  and  $A_{34}$  on the data sheet.

31.) Ohm's Law requires  $\Delta V_{12} = I_{12} R_{12}$  and  $\Delta V_{34} = I_{34} R_{34}$  . How, then, can one explain  $A_{12}$  and  $A_{34}$  ? (Hint, consider what is providing the resistances  $R_{12}$  and  $R_{34}$  .)

#### **Part Four: The Equivalent Resistance of Resistors in Parallel**

32.) In part three, you should have found that  $\Delta V_1 \approx 20$  V and  $A_1 \approx 40$  mA , while  $\Delta V_2 \approx 20$  V and  $A_2 \approx 20$  mA . These currents join together at point four and the total in the circuit is  $A_{tot} = A_1 + A_2 \approx 60$  mA . We now want to determine what the equivalent resistance is for two resistor in series. To that end, with the power turned off, set up the circuit shown below in Figure Five.

**Figure Five**  
*Circuit for Finding the Equivalent Resistance for Resistors in Parallel*



- 33.) Turn on the power supply and increase it until there is a potential difference of 20 V across the power supply.
- 34.) Now, adjust the resistor value  $R$  until the current in the circuit is  $A_{tot} = A_1 + A_2$  .
- 35.) **Turn the power off.** Measure the resistance and record the value of  $R$  as  $R_{eq, parallel}$  .
- 36.) Identify what values were the **same** in this circuit and the circuit of part three.

- 37.) How does  $R_{eq, parallel}$  compare with  $R_1$  , and  $R_2$  of the circuit in part three?

It can be shown that the equivalent resistance for  $N$  resistors in series is given by

$$\frac{1}{R_{eq,parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} . \quad (3)$$

- 38.) Use equation (3) to calculate the theoretically expected value of the equivalent resistance for resistors in parallel and record the value below.
- 39.) Calculate the percent difference between the theoretical value and the value you found in step 35.) and record this value.

## Data Sheet

### *Resistors in Series*

#### *Part One*

$$\Delta V_{powersupply} = 20 \text{ V}$$

$i$	$R_i$	$A_i$	$A_i R_i$	$\Delta V_i$
1	500 $\Omega$			
2	1000 $\Omega$			

$$\sum_{i=1}^2 \Delta V_i = \underline{\hspace{10mm}}$$

#### *Part Two*

$$\sum_{i=1}^2 R_i = \underline{\hspace{10mm}}$$

$$R_{eq, series} = \underline{\hspace{10mm}}$$

## ***Resistors in Parallel***

### ***Part Three***

$$\Delta V_{powersupply} = 20 \text{ V}$$

$i$	$R_i$	$A_i$	$A_i R_i$	$\Delta V_i$
1	500 $\Omega$			
2	1000 $\Omega$			

$$\Delta V_{14} = \underline{\hspace{100pt}}$$

$$\Delta V_{23} = \underline{\hspace{100pt}}$$

$$\Delta V_{12} = \underline{\hspace{100pt}}$$

$$\Delta V_{34} = \underline{\hspace{100pt}}$$

For the circuit of Figure Four:

$$A_{12} = \underline{\hspace{100pt}}$$

$$A_{34} = \underline{\hspace{100pt}}$$

### ***Part Four***

$$A_{tot} = A_1 + A_2 = \underline{\hspace{100pt}}$$

Measured value for the equivalent resistance:

$$R_{eq,parallel} = \underline{\hspace{100pt}}$$

Calculated value for the equivalent resistance:

$$R_{eq,parallel,cal} = \underline{\hspace{100pt}}$$

$$\% Difference = \underline{\hspace{100pt}}$$

# ***PHY2054L LABORATORY***

## *Experiment Four*

### *Voltmeters and Ammeters*

## **THEORY**

The galvanometer is an instrument that can detect very small electric currents. The analog galvanometer consists of a pivoted coil of thin wire in the field of a permanent magnet. (There is also an indicator needle attached to the coil.) When there is a current in the coil, it interacts with the field of the permanent magnet producing a torque on the coil that is proportional to the current passing through the coil. This torque is opposed by a spring similar to the hairspring on the balance wheel of a watch. The spring exerts a restoring torque proportional to the angular displacement. The angular deflection of the indicator needle is directly proportional to the current in the coil and can be calibrated to measure current.

As the galvanometer coil is made of very thin wire, it can carry safely only very small currents. In this experiment, we are going to design a voltmeter and an ammeter.

## **EQUIPMENT NEEDED**

One Electro Model 3002A DC Power Supply  
One Fluke Multimeter  
Two Decade Resistor Boxes

One Commercial Ammeter  
**One New Galvanometer**  
6 Connecting Cables

## **PROCEDURE**

**Using the Troubleshooting Guide, set the power supply so that it will not deliver a current greater than  $0.30\text{ A}$ . Test the ammeter to make sure that it is working properly. It should read  $10\text{ mA}$  when connected to a resistor of  $1000\text{ }\Omega$  across which is a potential difference of  $10\text{ V}$ .**

### ***Part One: Measuring the Coil Resistance***

#### **Use the Fluke**

1.) The coil of the galvanometer is a winding of wire and has a resistance we will signify with  $R_C$ . With the galvanometer isolated, use the Fluke to measure the coil resistance and record this value on the data sheet below.

#### **Use a Special Circuit**

2.) Now, I wish to show you a way to measure the resistance of a circuit element if you do not have an ohmmeter. With everything turned off, set up the circuit shown below in Figure One (a). The  $R_l = 30\text{ k}\Omega = 30,000\text{ }\Omega$  is in the circuit to protect the galvanometer.

3.) (If you must use an **old galvanometer**, you must depress the *Top Left Key* of the galvanometer and twist it clockwise so that it remains down. In this mode, the galvanometer needle will fully deflect whenever there is a current passing through the coil of magnitude  $I_C = 500\mu\text{A}$ .)

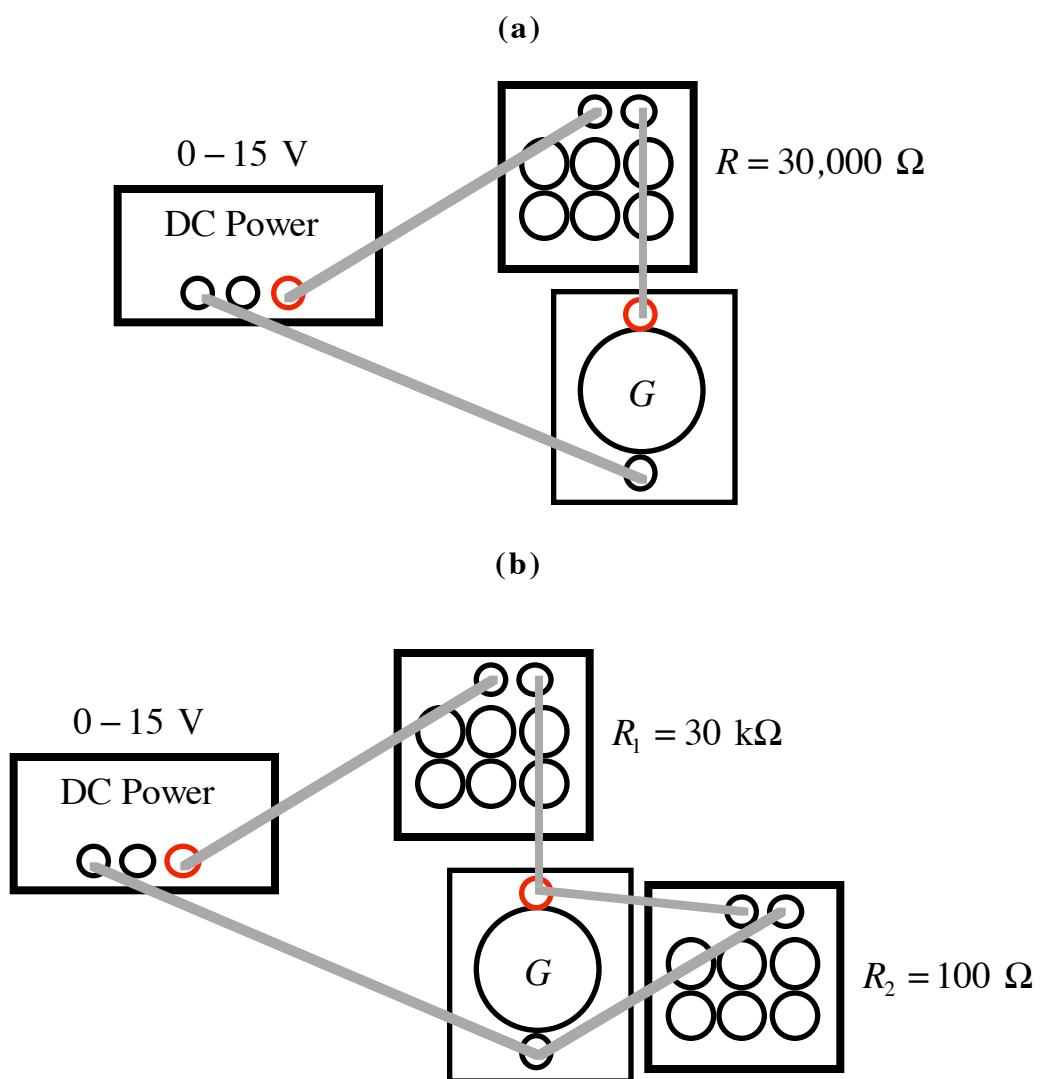
If you are using a new galvanometer, it also fully deflects at  $I_C = 500\text{ }\mu\text{A} \equiv 0.0005\text{ A}$ .

4.) **Turn on the power** and adjust it until the galvanometer needle is **fully deflected**. (Fully deflected means the needle is aligned with the very last hash mark on the dial.) Whenever the dial is fully deflected, then  $0.0005\text{ A}$  is passing through the galvanometer.

5.) Without changing the voltage knob of the power supply, turn off the power supply.

Now, connect a second resistor with an initial resistance value of  $R_2 = 100 \Omega$  in parallel with the galvanometer, as shown below if Figure One (b). Once the second resistor is connected, turn the power supply back on. **Adjust the second resistor** until the galvanometer is at **half-scale deflection** and then **turn off the power supply**. (Half-scale deflection implies that the current passing through the galvanometer is the same as the current passing through resistor two, and, therefore, that they have the same resistance.) Isolate the resistor box two and use the Fluke to measure the value of  $R_2$  and record this value on the data sheet. Hopefully,  $R_2 \approx R_C$  as measured by the Fluke.

**Figure One**  
*Circuit for Measuring the Resistance of the Coil Inside the Galvanometer*



## Part Two: Construction of a Voltmeter That Fully Deflects at 20 V

We are now going to pretend we are an electrical engineer. Our design objective here is to use the galvanometer as a voltmeter that has a full-scale deflection whenever placed across a potential difference of 20 V.

We know that the galvanometer fully deflects when a current of  $500 \mu\text{A}$  passes through it. Using Ohm's Law, we can calculate the current that would pass through the galvanometer coil if a potential difference of 20 V were placed across it. We have, as an example calculation, where we have assumed a coil resistance of  $200 \Omega$ :

$$I = \frac{\Delta V}{R_C} = \frac{20 \text{ V}}{200 \Omega} = 0.100 \text{ A} .$$

Of course, this current is two hundred times larger than the galvanometer can handle safely. So, if we are to use the galvanometer as a voltmeter that can measure up to twenty volts, we have to protect the galvanometer by putting the coil in series with a second resistor  $R_S$ .

We require then

$$\Delta V_{design} = I_C R_C + I_C R_S , \quad (1)$$

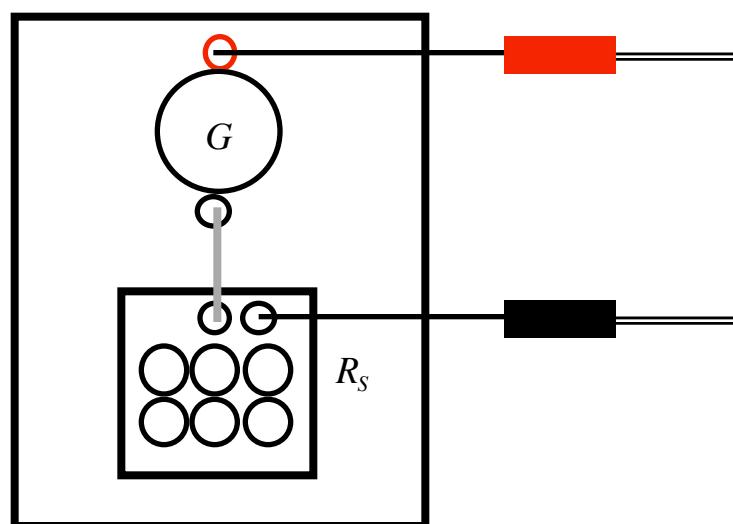
and, solving for  $R_S$  we have

$$R_S = \left[ \frac{\Delta V_{design}}{I_C} \right] - R_C . \quad (2)$$

Using equation (2), your measured value for  $R_C$  (the Fluke value), and with  $\Delta V_{design} = 20 \text{ V}$ , and  $I_C = 0.0005 \text{ A}$ , calculate a value for  $R_S$  and record it on the data sheet.

Basically, we can build an analog voltmeter by putting an appropriate resistor in series with the galvanometer. Below, is a diagram representing how we might configure the voltmeter.

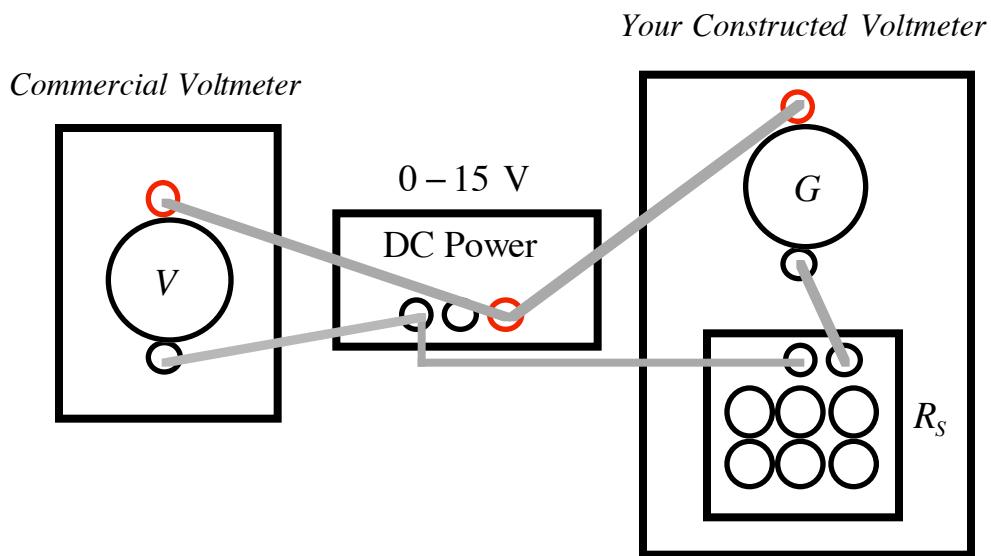
**Galvanometer as Voltmeter**



6.) With everything turned off, set up the circuit shown below in Figure Two. The shaded box on the right in the figure is intended to help you visualize the galvanometer and the series resistor as a voltmeter. The box on the left represents the commercial voltmeter--your Fluke. Note that voltmeters are always connected in parallel with the points the potential difference of which they are intended to measure. (In this case, the electric potential difference across the output of the power supply.)

- 7.) Set the series resistor in the circuit shown below to the value  $R_S$  that you calculated above.
- 8.) Turn on the power and set the power supply so that the commercial voltmeter measures 4.0 V across the terminals of the power supply. (As with any meter, make sure that you have the meter in the proper measurement range to reduce the risk of damage to the meter!) Record the value in *Volts* of your constructed "voltmeter". (Remember, **full deflection is equal to 20 V!**)
- 9.) Repeat step 8.) for 8.0 V, 12.0 V, 16.0 V, and 20.0 V, and recording the corresponding galvanometer "voltages". (Remember, after 12 V, you will have to change the slide position to 15-30 V on the power supply.)
- 10.) **Turn off the power** after you have made and recorded your measurements.

**Figure Two**  
*Circuit for Testing a Voltmeter that Fully Deflects at 20 V*



### **Part Three: Construction of an Ammeter That Fully Deflects at 5 mA**

Now, we wish to continue our masquerade as an electrical engineer by attempting to design an ammeter that fully deflects when placed in a circuit were the current is *five millamps*.

Obviously, 5 mA is more current than the galvanometer can carry safely. So, to protect the galvanometer, we will connect it in parallel to a **shunt resistor**,  $R_{SH}$ . (In this context, shunt means "to bypass." So, some of the current will bypass the galvanometer.) Carefully consider the circuit shown below in Figure Three. Since the coil and the shunt resistor are in parallel, we can write

$$\Delta V = I_C R_C = I_{SH} R_{SH}, \quad (3)$$

$$R_{SH} = [I_C / I_{SH}] R_C. \quad (4)$$

Now, when the scale is fully deflected, we require

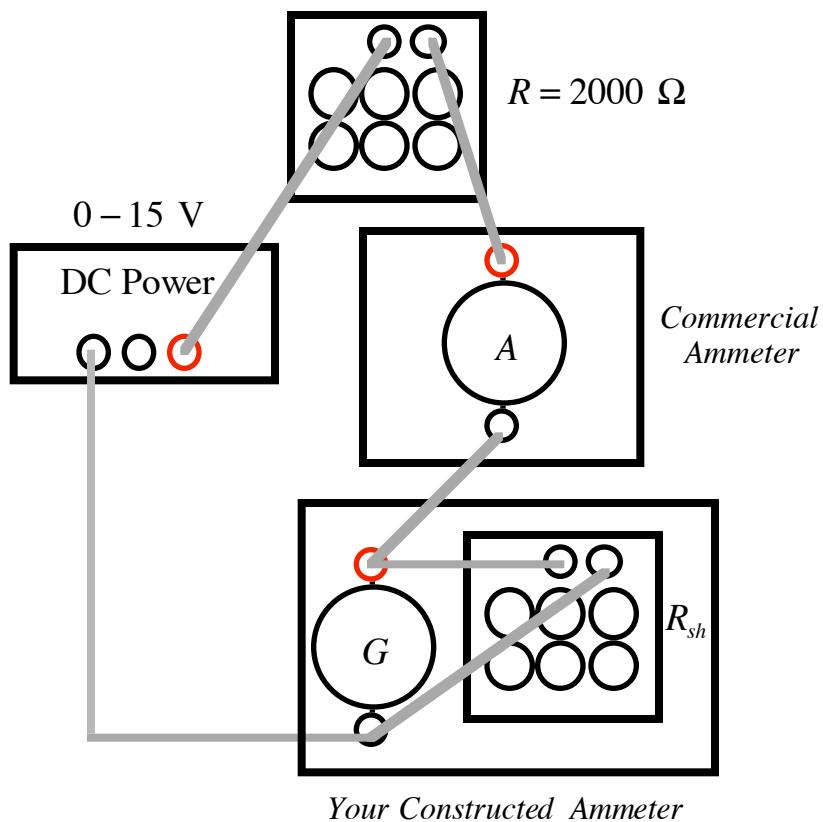
$$I_{design} = I_C + I_{SH}, \quad (5)$$

where  $I_C = 0.0005 \text{ A}$ , and  $I_{design} = 0.005 \text{ A}$ . So, equation (4) can be written as

$$R_{SH} = \left[ \frac{I_C}{I_{SH}} \right] R_C = \left[ \frac{I_C}{I_{design} - I_C} \right] R_C. \quad (6)$$

11.) Using equation (6) and the design criteria, **calculate the value of the shunt resistor  $R_{SH}$**  and record this value on the data sheet.

**Figure Three**  
*Circuit for Testing an Ammeter that Fully Deflects at 5 mA*



12.) Using your multimeter as an ohmmeter, set up your decade resistor box to the value calculated in step 11.)

13.) Connect the resistor parallel to the galvanometer. When you have finished, your circuit should look like the circuit in Figure Three above.

14.) **Turn on the power and adjust** it until you have a value of 5 mA indicated on the constructed ammeter. Record the corresponding value of your commercial ammeter on the Data Sheet. (Remember, when the constructed ammeter it is fully deflected, you have 5 mA passing through!) When you have done this for all of the values listed on the data sheet, **turn off the power** supply.

***PHY2054L LABORATORY***

***Experiment Four***

***Voltmeters and Ammeters***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

### Part One: Measuring the Coil Resistance

Fluke Measurement:

$$R_C = \underline{\hspace{2cm}} \Omega$$

Measurement using the circuit:

$$R_2 = \underline{\hspace{2cm}} \Omega$$

### Part Two: Constructing a 20 V Voltmeter

$$R_S = \underline{\hspace{2cm}} \Omega$$

<i>Commercial Voltmeter (V)</i>	<i>Constructed Voltmeter (V)</i>	<i>% Difference</i>
4.0 V		
8.0 V		
12.0 V		
16.0 V		
20.0 V		

### Question:

What would one have to do if one wanted to use the galvanometer to construct a 50 V full-scale deflection voltmeter?

**Part Three: Constructing an Ammeter Which Fully Deflects at 5 mA**

$$R_{SH} = \underline{\hspace{100pt}} \Omega$$

<i>Commercial Ammeter (mA)</i>	<i>Constructed Ammeter (mA)</i>	<i>% Difference</i>
	1 mA	
	2 mA	
	3 mA	
	4 mA	
	5 mA	

**Question:**

What would one have to do to use the galvanometer to construct a 10.0 A full-scale deflection ammeter?



# ***PHY2054L LABORATORY***

## *Experiment Five*

### *Electric Fields and Equipotential Lines*

## THEORY

**Electric charge** is that **property** of the physical world which gives rise to the **electromagnetic interaction**. In ordinary matter, this charge manifests itself through the electrically charged particles electrons and protons. On the macroscopic level, it is an excess count of these particles that causes an object to be electrically "charged".

One interesting problem that confronts the physicist concerns how it is that charges that are separated spatially can influence each other at all. Intuitively, it seems reasonable that objects that come into contact with each other might influence each other physically. But how is an electromagnetic interaction possible between charged bodies that are **not** in contact? (A similar question could be asked of the mass interaction which, of course, gives rise to the gravitational force!) It is the notion of a **field** which physicists use to deal with this problem. (We will leave it to the philosophers to determine if a "field" actually solves the problem or only "displaces" it.) Let us consider how this might work with electric charge.

Assume you can observe another universe in which the laws of physics are formally like those in our own universe. Next, assume that this second universe is populated by a solitary **proton**. Finally, assume you have the power to create other particles if you so wish! (Let us also agree to suspend consideration of the interesting theological and axiological ramifications of such power! Not to mention, the equally interesting epistemic question as to how one might be able to "observe" such a universe. Can one observe a universe? Does not observation itself involve interaction of some kind?)

Now, as long as there is only one proton, only one electrically charged body in this fantasy universe, there will be *no physical interaction*. Experiment indicates that the proton has "point symmetry." As long as one is at the same radial distance from the proton, its influence is of the same magnitude and has the same relative orientation. (Since the proton has point symmetry, we model it as a sphere.) For a solitary proton, the **possibility** for an electromagnetic interaction is ever present, and that possibility exists at **each** and **every** point in the space around the proton. All that is necessary to give rise to an interaction is to have another charged object show up on the scene! It is as if the mere existence of our proton turns all the space around it into something physically different, into a space of electromagnetic-interaction-possibility. (Wow! What a positively Heideggerian construction!) In some very profound sense, the charge of the proton affects the space around it. We call this artifact, this transformed space, an **electric field**. Please realize that this field is not a mere mental construct. It has real, physically measurable properties. It is, if you will, in some important sense "physical."

Next, we create a **test** particle of positive charge  $q_t$  and constrain it and the proton to remain separated some distance  $r$  from each other. (How we constrain the particles is not now of interest. We only want to avoid the complications of moving charges, that is, we want an **electrostatic** configuration! Not to mention that constraints require more charged objects than two.) We know that the test particle  $q_t$  will "feel" a repulsive electrical force the magnitude of which is given by Coulomb's law. The magnitude of the electrical force exerted on the test charge by the proton is given by

$$F_{q_t p}^E = \frac{k q_t e}{r^2} = q_t \left[ \frac{ke}{r^2} \right] = q_t E . \quad (1)$$

In writing Coulomb's law we have separated out that part which is "contributed" by the proton from that "contributed" by the test particle  $q_t$ . Now,  $E$  represents the magnitude of the electric field generated by the proton at the point in space occupied by the test charge  $q_t$ . It must be pointed out that the electric field is a vector quantity. The direction of the electric field is the same as the direction of the force that is felt by our positive test charge  $q_t$ . Using this

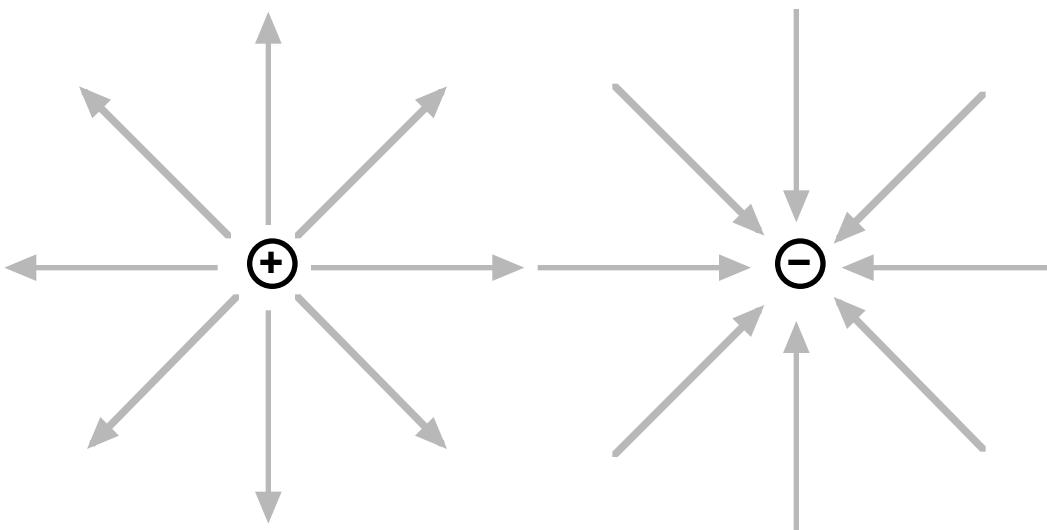
convention, we note that in general, electric field lines leave positive charges while they approach negative charges, as represented below in Figure One. (The field lines aid us in visualizing the direction of the electric field.)

Back to the two charges. If we were to remove the constraints, the repulsive electrical force would cause the particles to accelerate away from each other. Also, if we were to “force” the test charge to move **closer** to the proton, we would have to do work against the repulsive electrical force. Since the electrical force is a conservative force, this work is equal to the **negation of the change in the electrical potential energy**. The work done in moving our test charge  $q_t$  **from infinity**, where the electrical potential energy is zero, to some point  $P$  a distance  $r$  away from the proton is given by

$$W_E = -\Delta U^E = U_o^E - U_f^E = 0 - \frac{kq_t e}{r} = -\frac{kq_t e}{r}. \quad (2)$$

Be sure you understand why the initial potential energy term has vanished! Make sure to ask if you do not! You should also understand why, in this case, the work done by the electric field is negative.

**Figure One**  
*Pictorial Representation of Electric Field Lines*



If we have a sphere of radius  $r$  that is centered on the proton, then the exact same amount of work would have to be done to place the test charge at any point on the surface of the sphere! This leads us to introduce the following notion.

If we now take the work done by the repulsive electric force and divide it by the test charge, then we will have a measure of the amount of **work per unit charge** done by the repulsive electric force in putting the test charge a distance  $r$  from the proton. This new quantity we call the **electric potential**,  $V$ .

$$\frac{W_E}{q_t} = -\frac{\Delta U^E}{q_t} = -\Delta V = -\frac{ke}{r}. \quad (3)$$

Be sure you understand why equations (2) and (3) are negative! Ask if you do not! Also, it is very important that you understand **the difference between the electrical potential energy and the electric potential!**

If we consider again the spherical surface of radius  $r$ , we recognize that this is a surface of equal electric potential, that is, an **equipotential surface**. The charge of the proton, then, not only gives rise to a vector field--the electric field--but also to a scalar field--the electric potential. At each point in space around the proton, we can assign two quantities: 1) a vector quantity of magnitude and direction associated with the **electric field**, and, 2) a scalar quantity associated with the **electrical potential**.

## EQUIPMENT NEEDED

One Electro Model 3002A DC Power Supply	Digital Multimeter
Two Sheets of Conduction Paper; Each Sheet With A Different Pattern	
Steel Pins	Cork Board
Connecting Cables	Tracing Paper

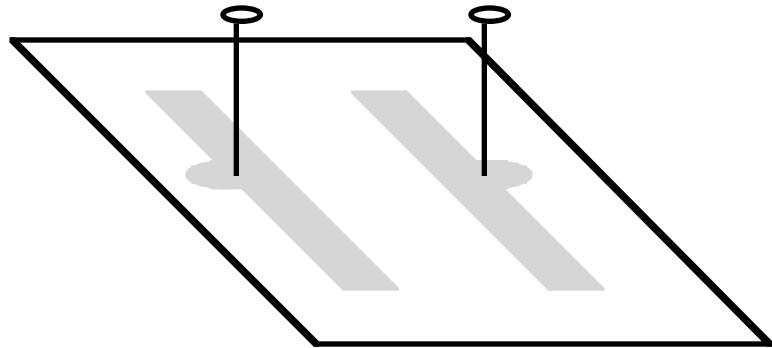
## EXPERIMENTAL PROCEDURE

Your fundamental task in this experiment is to map several equipotential lines for two different charge configurations. One configuration models two point-like charges of opposite sign, and the other configuration models a parallel plate capacitor.

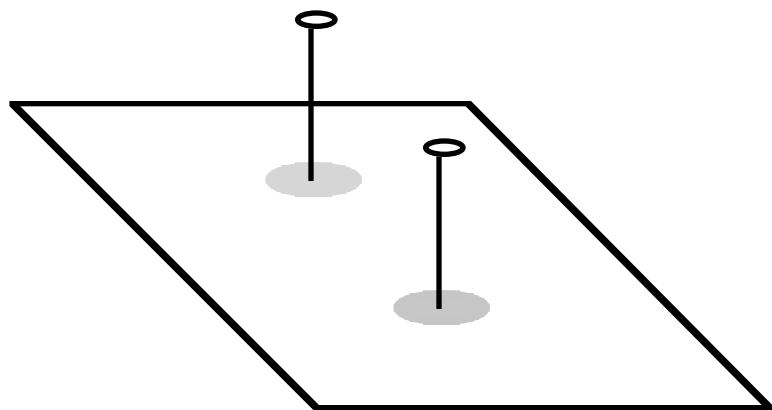
- 1.) Take one sheet of tracing paper for each person in your group. Place the sheets of tracing paper under a single sheet of conduction paper. Fix all of these sheets onto the cork board by punching one steel pin into the center of each painted region on the conduction paper. (See Figure Two below.)
- 2.) Use the Electro Model 3002 power supply. Make sure the **power is off!** Set the voltage slide switch to the 15 – 30 V position. Using the procedure described in Troubleshooting 101, set up the power supply to deliver a current no greater than 0.30 A . Set the DC voltage adjustment knob all the way counterclockwise to its **minimum** output position while the power supply is **still off**!
- 3.) Connect a cable from the negative (black) port of the power supply to the leftmost pin in the conduction paint pattern. (Use an alligator clip for this pin connection.) Connect a cable from the positive (red) port of the power supply to the rightmost pin in the conduction paint pattern. (Also use an alligator clip for this pin connection and see Figure Three below.)
- 4.) Get a Fluke multimeter. Set the meter to the DC Voltage setting by turning the dial to the appropriate function, namely  $[\bar{V}]$  . I recommend setting the scale to 00.0 Volts. This will help reduce unwanted fluctuations. Feel free to play around with this to suit yourself. Connect a cable with an alligator clip from the black port of the multimeter to the leftmost pin. Connect a cable with a pointed probe to the red port of the multimeter.
- 5.) **Turn on** the DC source. By placing the probe on the leftmost pin, you can measure the potential difference between the pins. Turn up the power until it reads 18.0 V . **This value should not be changed during the experiment.**
- 6.) Locate on the conduction paper with the probe a point where the potential is 3.0 V . Punch a hole in the paper. Continue in this manner to identify at least nine other such points on the paper where the potential is 3.0 V . Add more such points until you have a pattern to the punched holes that you can recognize. (There should be at least ten holes punched for each voltage value. They should be about an inch apart and half on each side of the symmetry line. See Figure Four below for a sense of the spacing--not a revelation of the actual pattern. That is for you to find out.)
- 7.) Repeat step six for the 6.0 V , 9.0 V , 12.0 V , and 15.0 V equipotential lines.
- 8.) Turn off the power. Place the second conduction sheet on the board and repeat the above procedure.

*Figure Two*

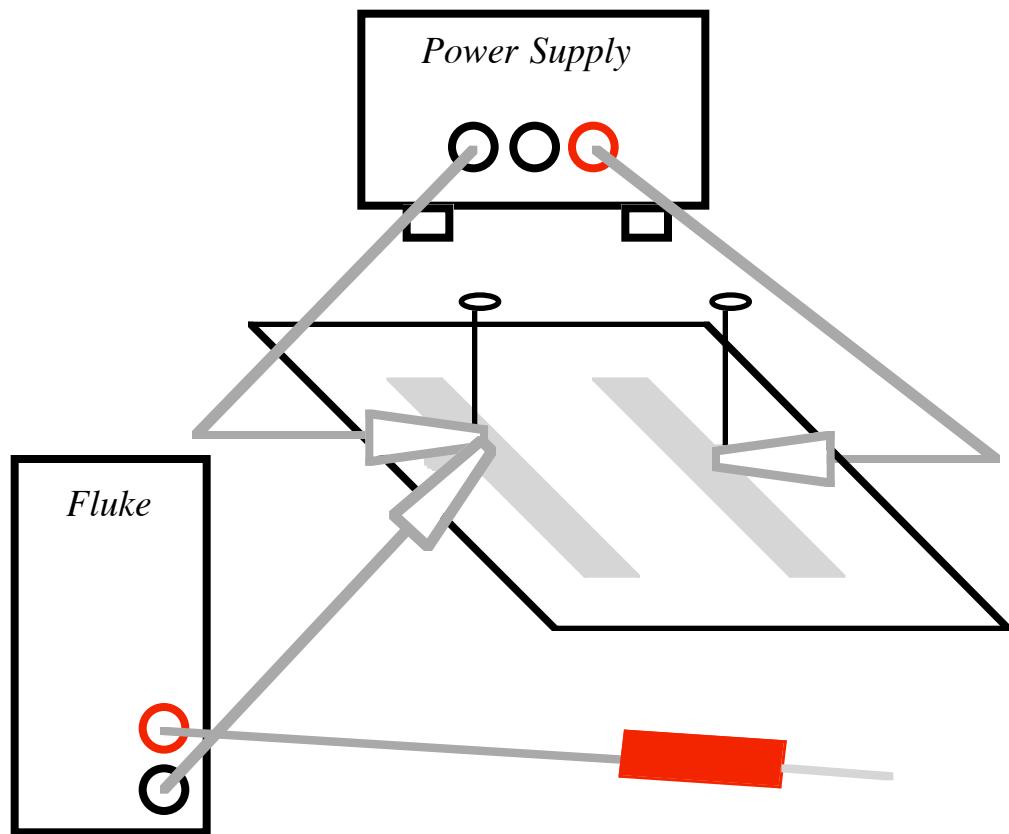
*Configuration 1*



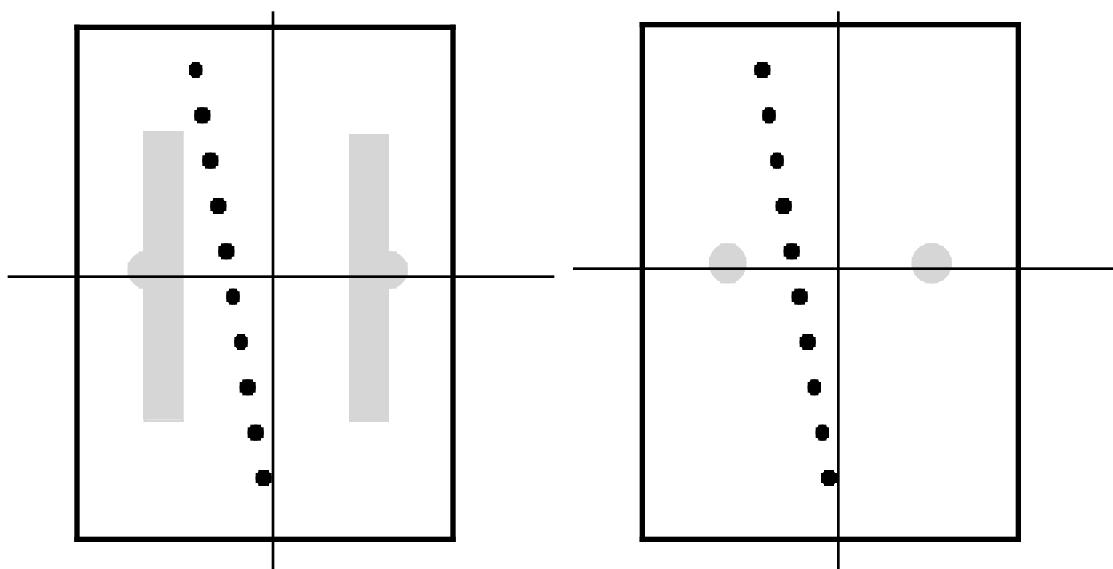
*Configuration 2*



*Figure Three*



*Figure Four*





***PHY2054L LABORATORY***

***Experiment Five***

***Electric Fields and Equipotential Lines***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## THINGS TO DO

I. On your tracing paper, using a pencil, lightly connect the dots of each set of equal potential points, forming a continuous, smooth line. Mark the electric potential value of each such line.

II. Consider one of the equipotential lines that you have just mapped. What can you say about the electric potential of each of the points on that line?

III. Now, if there were a small positive test charge  $q_t = 1.0 \text{ C}$  at some point A on the 3.0 V equipotential line, then how much work would be done by the electric field to move it to another point B on the same equipotential line?

IV. If however, our positive test charge  $q_t$  were at some point A on the 3.0 V equipotential line, the how much work would have to be done by an outfit like FP&L to move the test charge to another point B on the 15.0 V equipotential line?

V. Remember how the work done by the electric force is defined

$$W_{A \rightarrow B} = -\Delta U^E = \int_{r_A}^{r_B} \vec{F}^E \bullet d\vec{r} , \quad (4)$$

where

$$\vec{F}^E \bullet d\vec{r} = (F^E dr) \hat{\vec{F}}^E \bullet \hat{d\vec{r}} = (F^E dr) \cos \angle_{bet} . \quad (5)$$

Using the definition, what **must be the orientation** of the electric field lines to the equipotential lines? (This is not necessarily a trivial mental exercise!)

VI. What does it mean physically to say that an equipotential line is at 120 V ?

# ***PHY2054L LABORATORY***

## *Experiment Six*

### *Measuring Capacitance*

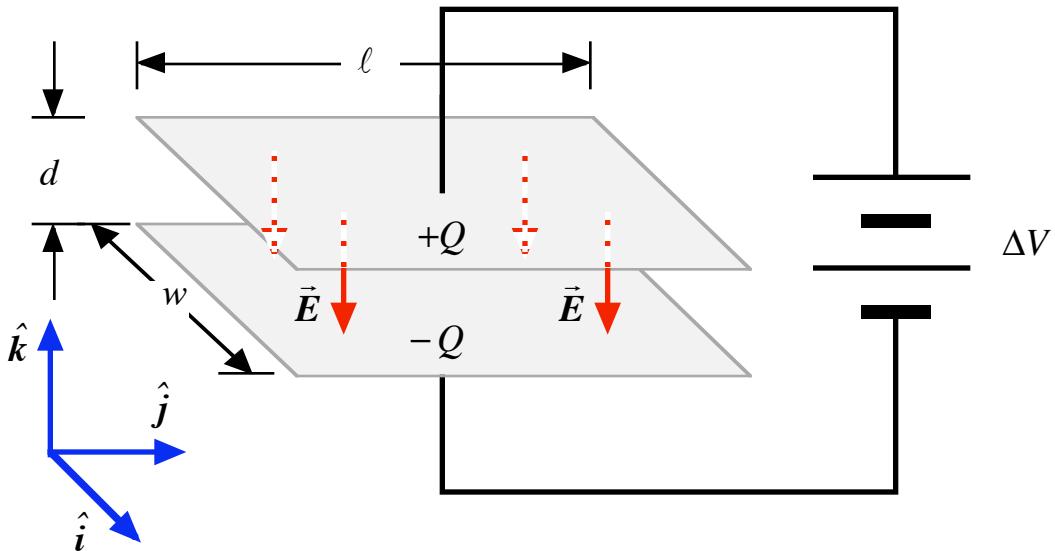
## THEORY

The amount of charge  $Q$  that can be put on a conductor at electric potential  $V$ , relative to some point in space where the electric potential is zero, is given by

$$Q = CV, \quad (1)$$

where  $C$  is a constant of proportionality called the **capacitance**. A **capacitor** is a device that is used to store electrical potential energy. It is common for the capacitor to have two conductors that carry equal and opposite electric charges. The conductors are separated by air or some other nonconducting material. This nonconducting material is called a **dielectric**. In our experiment today, we will be using parallel-plate capacitors with air between the plates. A parallel-plate capacitor is represented in Figure One below.

*Figure One  
A Parallel-plate Capacitor*



Using equation (1), we can write

$$C = \frac{Q}{V}. \quad (2)$$

Using Gauss' law, the magnitude of the electric field interior to a parallel-plate capacitor--not too close to the edges--is given by

$$E = \frac{\sigma}{\epsilon_0}, \quad (3)$$

where  $\sigma$  is the surface charge density on the positive plate and given by

$$\sigma = \frac{Q}{A} = \frac{Q}{lw}. \quad (4)$$

The electric field is directed from the positively charged plate to the negatively charged plate.

Substitution of equation (4) into equation (3), we have

$$E = \frac{Q}{\epsilon_0 A} . \quad (5)$$

Recall, the electric potential  $V$  is given by

$$\Delta V = V_+ - V_- = - \int_0^d \vec{E} \bullet d\vec{r} , \quad (6)$$

where we are assigning the negatively charged plate a zero potential and noting that  $d\vec{r}$  is directed from the negatively charged plate to the positively charged plate. So, we can write equation (6) as

$$V = - \int_0^d \left[ -\frac{Q}{\epsilon_0 A} \hat{k} \right] \bullet [dz \hat{k}] = \frac{Q}{\epsilon_0 A} \int_0^d dz (\hat{k} \bullet \hat{k}) = \frac{Qd}{\epsilon_0 A} . \quad (7)$$

Substitution of equation (7) into equation (2) gives us

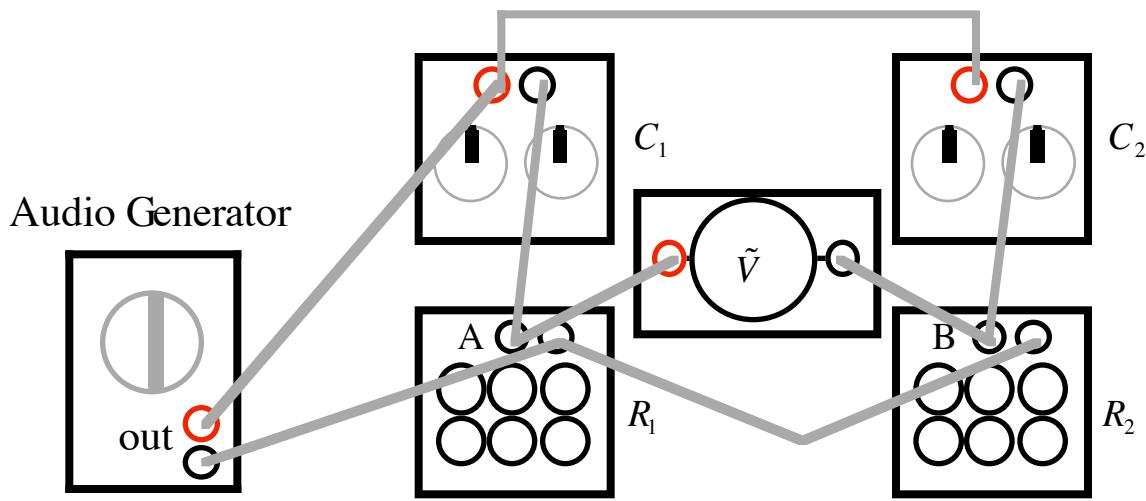
$$C = \frac{Q}{Qd / \epsilon_0 A} = \frac{\epsilon_0 A}{d} = \left[ \frac{1}{4\pi k} \right] \frac{A}{d} . \quad (8)$$

Equation (8) indicates that the capacitance of a parallel-plate capacitor with air between the plates depends on the “geometry” of the parallel plates. (Strictly speaking, the  $\epsilon_0$  is associated with vacuum. However, using  $\epsilon_0$  for air introduces only a very, very small error.)

As we have discussed earlier, the measurement process involves a kind of comparison and a kind of counting. The comparison is between two physical objects or systems which **both** possess the **same physical property**; namely, the property that we wish to measure. In turn, the comparison made with the help of some agreed upon standard unit. In research labs, one can use equation (8) in constructing a parallel-plate capacitor with the area of the plates and the separation distance precisely measured. In this experiment, we are going to use a capacitor with “known” capacitance to measure a capacitor with an “unknown” capacitance. **The known capacitance will act as our standard.**

The specific circuit we will use to measure the capacitance is known as a **capacitance bridge**, and it is represented in Figure Two below.

*Figure Two  
A Capacitance Bridge*



Consider carefully the circuit represented schematically in Figure Two above. We are going to use a function generator as an AC source for our capacitance bridge. Between points **A** and **B** we will use an AC-voltmeter to indicate when we have a "balanced" condition in the bridge. In a balanced condition, the potential difference across each capacitor is the same. When this condition exists, the potential difference between points **A** and **B** must be zero!

If the potential difference across the capacitors is the same, then

$$Q_1 = C_1 \Delta V \quad \text{and} \quad Q_2 = C_2 \Delta V , \quad (9)$$

which implies that

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} . \quad (10)$$

In a similar manner we can see that the potential differences across the resistors must also be the same. Using Ohm's law we have

$$I_1 R_1 = I_2 R_2 , \quad (11)$$

and

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} . \quad (12)$$

Now, even though we have an alternating current, we know that the charge on each capacitor must be proportional to the current through that branch of the circuit. From this we conclude that

$$\frac{Q_1}{Q_2} = \frac{I_1}{I_2} . \quad (13)$$

Using equations (10), (11) and (12) we write

$$C_2 = \left[ \frac{R_1}{R_2} \right] C_1 . \quad (14)$$

Now, we can set up our bridge so that  $C_2$  is the "unknown" capacitance and  $C_1$  is the "known" capacitance. We can alter equation (14) to reflect this approach. We have

$$C_u = \left[ \frac{R_k}{R_u} \right] C_k , \quad (15)$$

where the subscript *u*--unknown--is intended to convey not only epistemic information, but, also to identify the branch of the bridge to which it belongs. It must be remembered that equation (15) is true if and only if the potential difference between points **A** and **B** is **zero**. Under these conditions:

$$\text{If } C_u = C_k , \text{ then } \frac{R_k}{R_u} = 1 , \text{ and } R_k = R_u ; \quad (16)$$

$$\text{or if } C_u > C_k , \text{ then } \frac{R_k}{R_u} > 1 , \text{ and } R_k > R_u ; \quad (17)$$

$$\text{while if } C_u < C_k , \text{ then } \frac{R_k}{R_u} < 1 , \text{ and } R_k < R_u . \quad (18)$$

## EQUIPMENT NEEDED

BK Precision 3001 Audio Generator (Function Generator)  
Two decade resistor boxes  
8 Connecting Cables

Fluke Multimeter  
Three capacitor boxes

## THINGS TO DO

### *First Things First*

- 1.) To do this experiment, we need three capacitors that work! Our first task is to find them! To that end, set the Fluke Multimeter to measure capacitance. The Fluke symbol for capacitance is represented below.



- 2.) Set three capacitors so that their dial settings indicate  $1.00 \mu\text{F}$ . Use the Fluke to measure the capacitance of each capacitor. Adjust each capacitor dial setting so that its value is as close to  $1.00 \mu\text{F}$  as possible. Record their final measured values on the data sheet.
- 3.) Also, to do this experiment we need two decade resistor boxes that work. Set the Fluke Multimeter to measure resistors. The Fluke symbol for resistance is  $\Omega$ .
- 4.) Using the Fluke, set two decade resistor boxes to the same resistance. Make this value as close to  $1000 \Omega$  as possible. Record their final measured value on the data sheet.

### *Testing the Bridge Circuit*

- 5.) In Figure Two above, we have the bridge circuit we are going to be using. Study the circuit closely. If the circuit is set up properly, then when  $C_1 = C_2$  and  $R_1 = R_2$ , then the bridge should be balanced and  $V_{AB} = 0$ . To test the circuit, place two capacitors with equal measured values in the positions represented in Figure Two. Place the equal valued resistors in their proper place.
- 6.) Set the BK Precision 3001 Audio Generator to a frequency of 28 Hz. Turn the amplitude knob clockwise as far as possible for maximum output. Use the sine wave setting. Turn on the audio generator.
- 7.) Measure the AC potential difference between points A and B in the circuit. If the bridge is working properly, then the potential difference should be very close to zero; record the measured value. (It should be at least less than 0.006 V.) To make sure the zero reading is not an “accident,” change the value of one of the resistors and see if this makes the potential difference increase significantly. If it does not, then your bridge is not working properly and you need to call me over. If, however, there is a significant change, then your bridge is probably working properly and you can turn the audio generator off.
- 8.) Once your bridge is working properly, let capacitor one and resistor one be the so-called “known” capacitor and resistor. You are now ready to proceed.

### *Using the Bridge to Measure a Single “Unknown” Capacitance*

- 9.) Capacitor two is going to act as the so-called “unknown” capacitor. Change the dial on capacitor two so that its capacitance is no longer the same as capacitor one.

10.) Turn the audio generator on and measure the AC potential across points **A** and **B**. It should not be close to zero. Adjust the “unknown” resistor value until the potential difference between points **A** and **B** is very close to zero again. Turn the audio generator off. Use the Fluke to measure the “unknown” resistance and capacitance. Record these values. (Before you attempt to measure the resistance and capacitance with the Fluke, make sure the resistor and the capacitor are no longer connected to the circuit.)

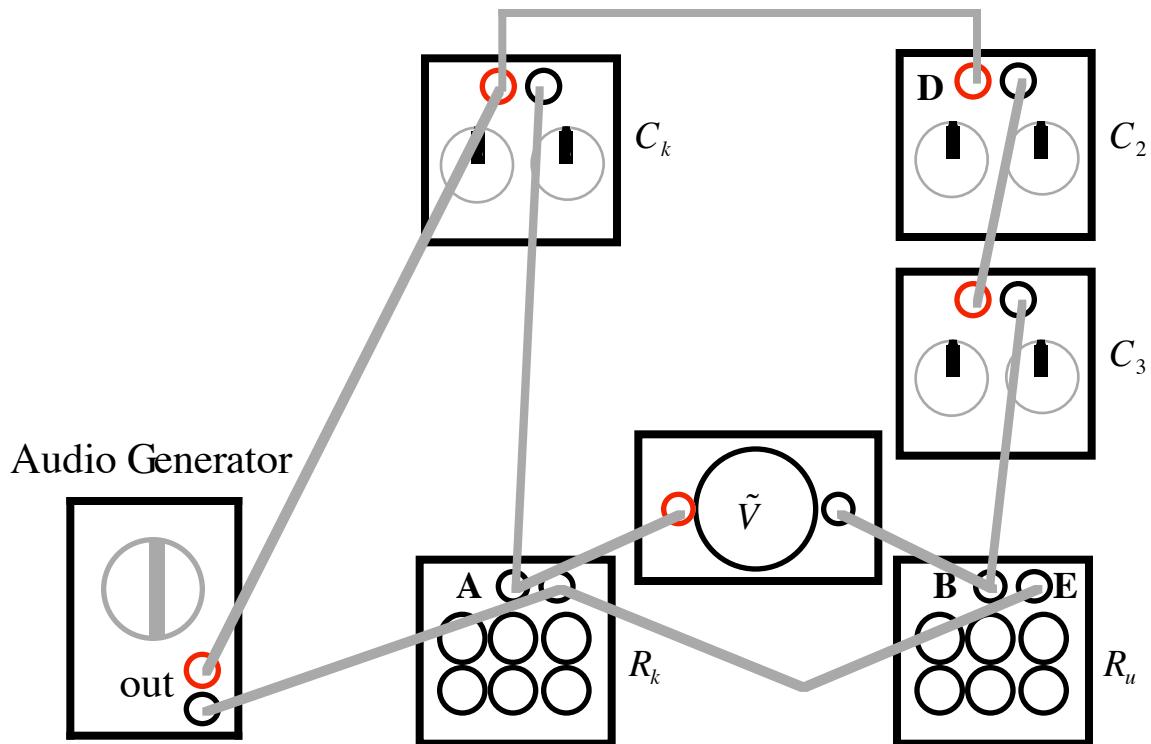
11.) Use equation (15) to calculate the “unknown” capacitance and record this value. Compare this measured value to the calculated value and calculate the percent difference.

12.) Repeat steps nine through eleven for two other different dial settings for capacitor two.

### ***Measuring the Equivalent Capacitance of Two Capacitors in Series***

We now want to place two capacitors in series into the “unknown” branch of the bridge. The beauty of the bridge is that it does not know there are two capacitors. So, the bridge will allow us to determine the value of the single capacitor that is equivalent to these two. We call this single capacitance value the **equivalent capacitance**.

***Figure Three***  
***The Circuit for Measuring***  
***The Equivalent Capacitance of Two Capacitors in Series***



13.) Set up the circuit in Figure Three above with all three capacitors as close as possible to the same capacitance, namely,  $C_k = C_2 = C_3 = 1.00 \mu\text{F}$ . Turn the audio generator back on and measure the potential difference between points A and B; it should **not** be near zero. Adjust the “unknown” resistor until the value is very close again to zero. Turn off the audio generator and measure the “unknown” resistance and record the value.

14.) Using equation (15) and your measurement of the “unknown” resistance, calculate the **measured** equivalent capacitance for your two capacitors in series.

15.) Theory suggests that the equivalent capacitance for  $N$  capacitors in series is given by

$$\frac{1}{C_{eq,series}^T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}. \quad (19)$$

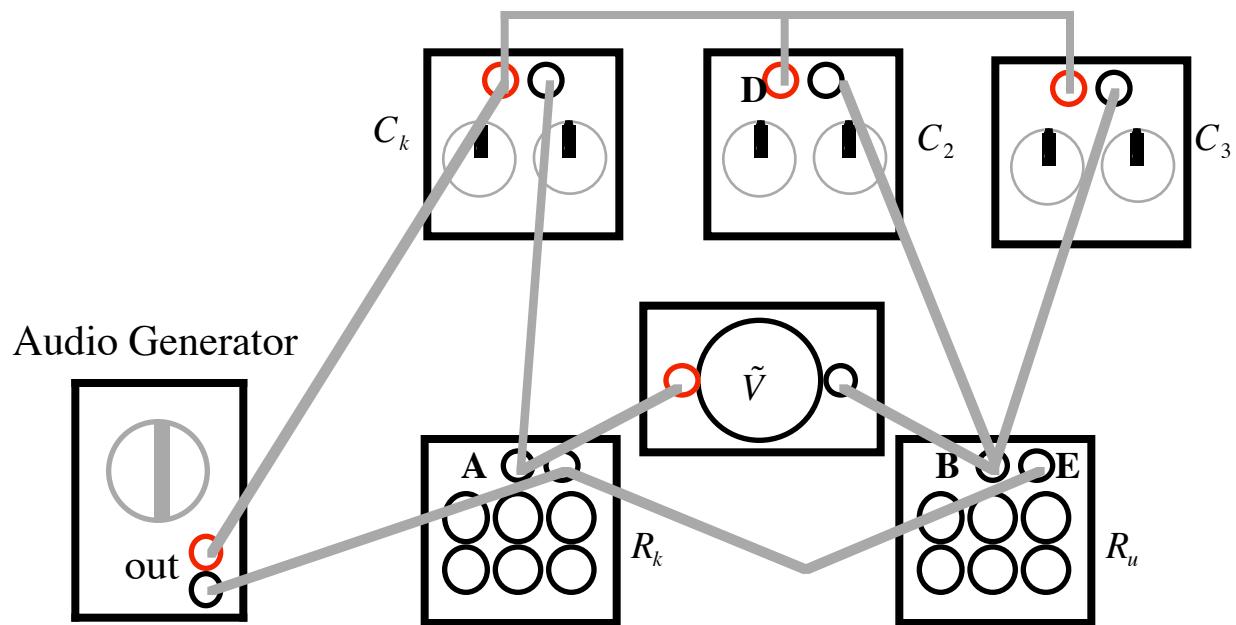
Use equation (19) to calculate the **theoretical value** for the equivalent capacitance of your two capacitors.

16.) Calculate the percent difference between the measured value and theoretical value for the equivalent capacitance of your two capacitors in series.

### ***Measuring the Equivalent Capacitance of Two Capacitors in Parallel***

Now, we configure things so that capacitors  $C_2$  and  $C_3$  are **in parallel** between the points **D** and **B**, as represented below in Figure Four.

**Figure Four**  
**The Circuit for Measuring**  
**The Equivalent Capacitance of Capacitors in Parallel**



17.) Set up the circuit in Figure Four above with all three capacitors as close as possible to the same capacitance, namely,  $C_k = C_2 = C_3 = 1.00 \mu\text{F}$ . Turn the audio generator back on and measure the potential difference between points A and B; it should **not** be near zero. Adjust the “unknown” resistor until the value is very close again to zero. Turn off the audio generator and measure the “unknown” resistance and record the value.

18.) Using equation (15) and your measurement of the “unknown” resistance, calculate the **measured** equivalent capacitance for your two capacitors in parallel.

19.) Theory suggests that the equivalent capacitance for  $N$  capacitors in parallel is given by

$$C_{eq,parallel}^T = C_1 + C_2 + C_3 + \dots + C_N . \quad (20)$$

Use equation (20) to calculate the **theoretical value** for the equivalent capacitance of your two capacitors in parallel.

20.) Calculate the percent difference between the measured value and the theoretical value for the equivalent capacitance of your two capacitors in parallel.

***PHY2054L LABORATORY***

***Experiment Six***

***Measuring Capacitance***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

**First Things First**

$C_1$ ( $\mu\text{F}$ )	$C_2$ ( $\mu\text{F}$ )	$C_3$ ( $\mu\text{F}$ )	$R_1$ ( $\Omega$ )	$R_2$ ( $\Omega$ )

**Testing the Bridge Circuit:**

$C_1$ ( $\mu\text{F}$ )	$C_2$ ( $\mu\text{F}$ )	$R_1$ ( $\Omega$ )	$R_2$ ( $\Omega$ )	$\Delta V_{AB}$ (V)

**A Single Unknown Capacitor:**

$C_k$ ( $\mu\text{F}$ )	$R_k$ ( $\Omega$ )	$\Delta V_{AB}$ (V)	Measured $R_u$ ( $\Omega$ )	Measured $C_u$ ( $\mu\text{F}$ )	Calculated $C_u$ ( $\mu\text{F}$ )	% Difference

### Measuring the Equivalent Capacitance of two Capacitors in Series:

Measured:

$$C_{eq,series}^M = \left[ \frac{R_k}{R_u} \right] C_k . \quad (15)$$

Theoretical Prediction:

For  $N$  capacitors in series, the equivalent capacitance is given by

$$\frac{1}{C_{eq,series}^T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N} . \quad (19)$$

$C_k$ ( $\mu\text{F}$ )	$C_2$ ( $\mu\text{F}$ )	$C_3$ ( $\mu\text{F}$ )	$R_k$ ( $\Omega$ )	$\Delta V_{AB}$ (V)	$R_u$ ( $\Omega$ )

<i>Measured</i> $C_{eq,series}^M$ ( $\mu\text{F}$ )	<i>Theoretical</i> $C_{eq,series}^T$ ( $\mu\text{F}$ )	<i>% Difference</i>

### Measuring the Equivalent Capacitance of two Capacitors in Parallel:

Measured:

$$C_{eq,parallel}^M = \left[ \frac{R_k}{R_u} \right] C_k . \quad (15)$$

Theoretical Prediction:

For  $N$  capacitors in parallel, the equivalent capacitance is given by

$$C_{eq,parallel}^T = C_1 + C_2 + C_3 + \dots + C_N . \quad (20)$$

$C_k$ ( $\mu\text{F}$ )	$C_2$ ( $\mu\text{F}$ )	$C_3$ ( $\mu\text{F}$ )	$R_k$ ( $\Omega$ )	$\Delta V_{AB}$ (V)	$R_u$ ( $\Omega$ )

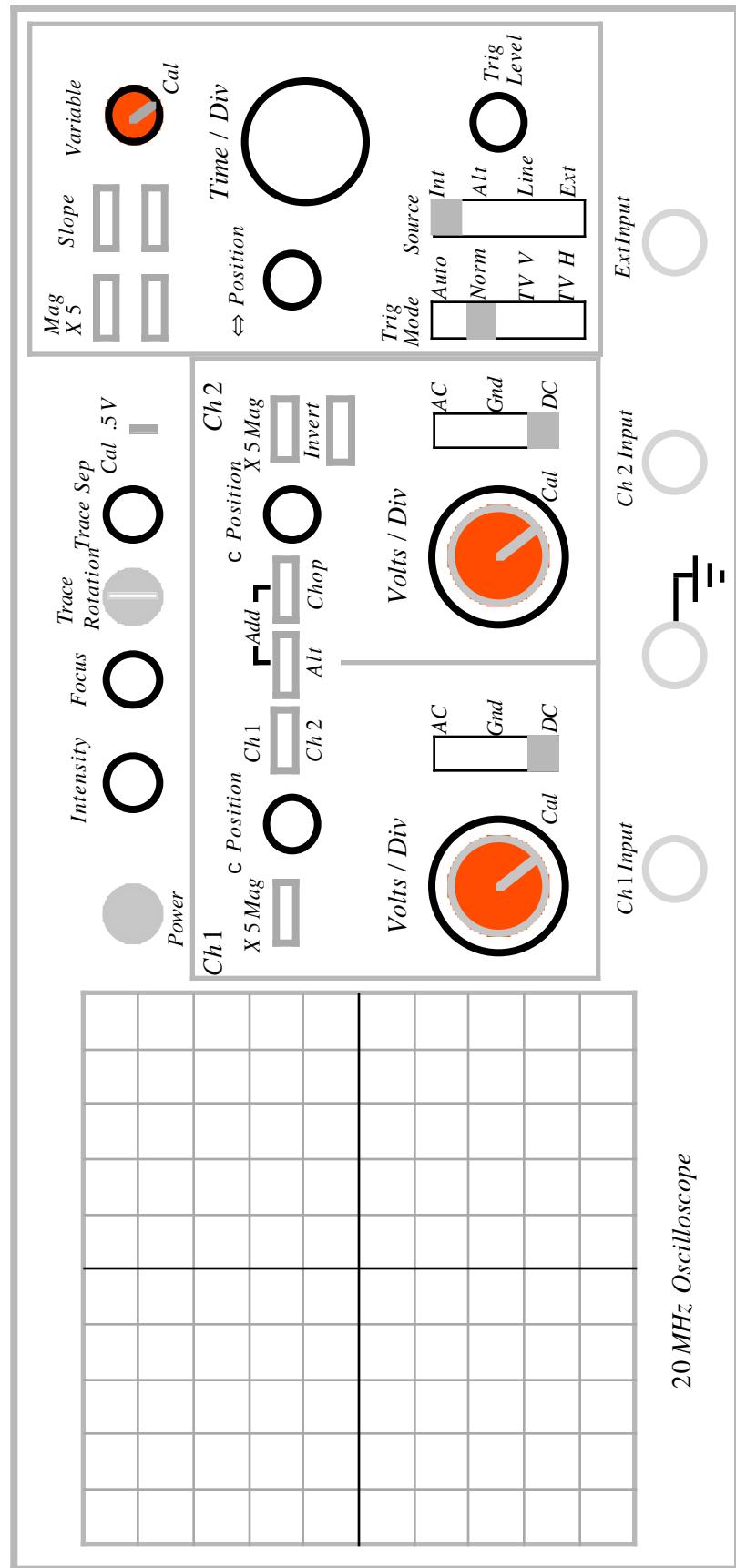
<i>Measured</i> $C_{eq,parallel}^M$ ( $\mu\text{F}$ )	<i>Theoretical</i> $C_{eq,parallel}^T$ ( $\mu\text{F}$ )	<i>% Difference</i>

# ***PHY2054L LABORATORY***

## *Experiment Seven*

### *The Oscilloscope*

## Oscilloscope Control Panel



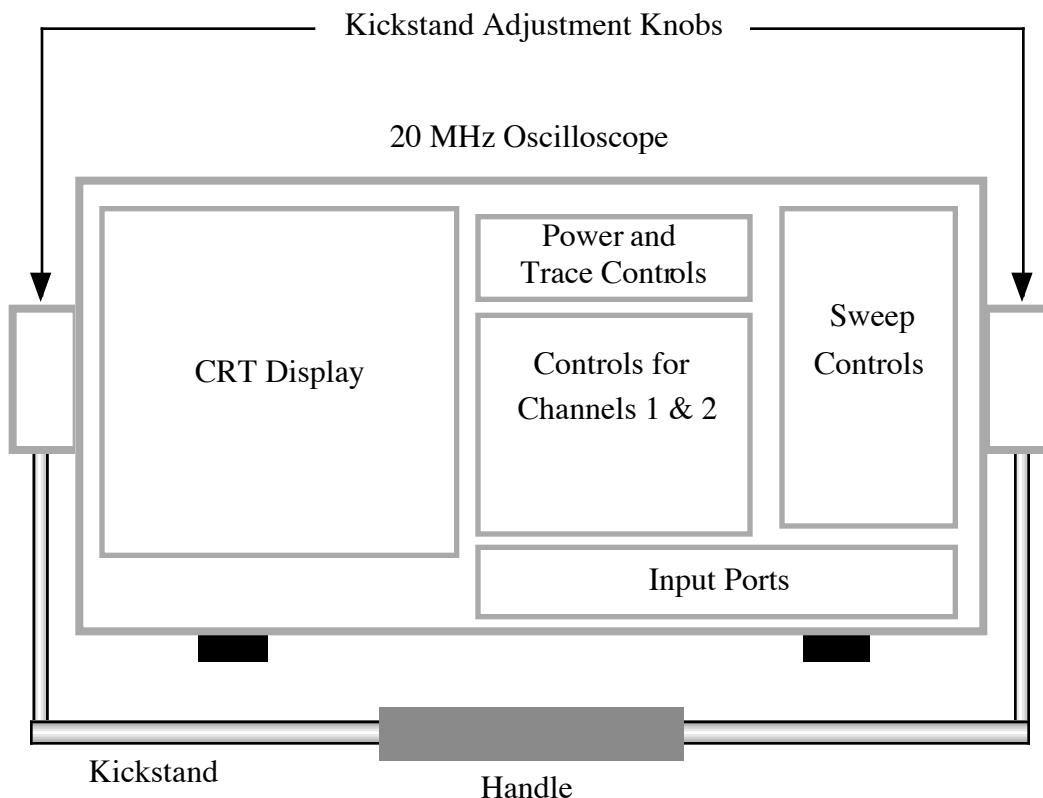
## EQUIPMENT NEEDED

One 20 MHz Oscilloscope  
One Fluke Multimeter  
1 Oscilloscope probe

One Electro Industries DC Power Supply  
2 Connecting Cables  
BK Precision 3001 Audio Generator (Function Generator)

## INTRODUCTION TO THE OSCILLOSCOPE

*Figure One*  
*“Anatomy of an Oscilloscope”*



## PROCEDURE

Please take a few moments to familiarize yourself with the general layout of the oscilloscope you will be using in this experiment--see Figure One above. An oscilloscope is a very sophisticated “multimeter” used for analyzing DC circuits and, more importantly, AC circuits which change very rapidly in time. This particular scope can work with circuits for which the frequency is up to 20 million Hz. (**Hertz** is the standard MKS unit for frequency--the old standard was *cycles per second*.) Probes are used to make measurements and provide a means for the “circuit information” to be transferred electrically to the scope. The “information” enters the scope through the input ports. If a probe is connected to the port for channel one, that information can be displayed on the CRT Display using the controls for channel one. If the probe is connected to the port for channel two, one uses the controls for channel two to display that information. (There are ways to “combine simultaneously” information input into both ports.)

In Figure Four below, is a diagram representing the power button and the trace controls.

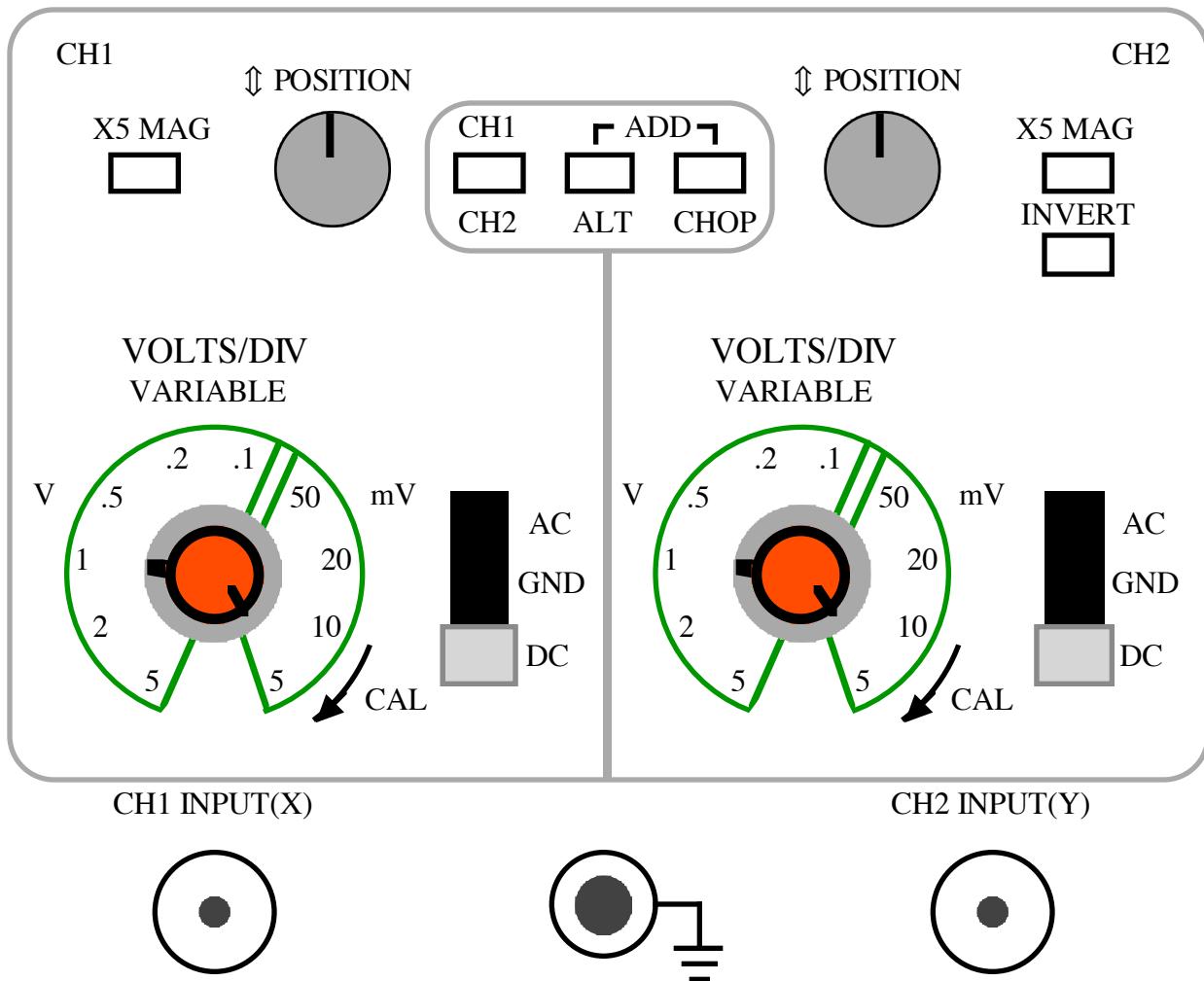
(The trace is the “line image” formed on the CRT display. CRT stands for **cathode ray tube**.) Figure Two below depicts the input ports and the controls for channels one and two. What I call the “sweep control center” is represented below in Figure Three. These functions primarily control how the image sweeps from left to right on the display screen. The CRT display is represented below in Figure Five. Voltage values are displayed on the vertical axis, while time values are displayed on the horizontal axis of the CRT display. Time increases from left to right. (The amount of time needed for a signal to “sweep” across the screen from left to right is controlled by the TIME/DIV knob found in the “sweep control center.”)

### ***Getting a Trace on the Scope***

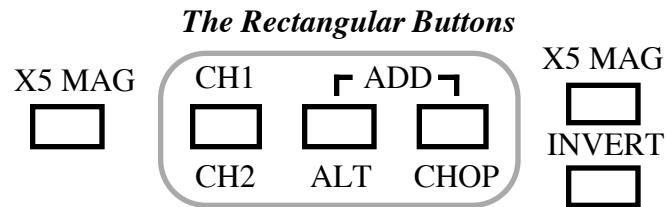
The first thing we want to do is get a trace on the scope. If we can, then the scope is probably in good working order. To that end, I am going to walk you through how the scope should be set up before you turn on the power.

- 1.) First, plug in the power cord.

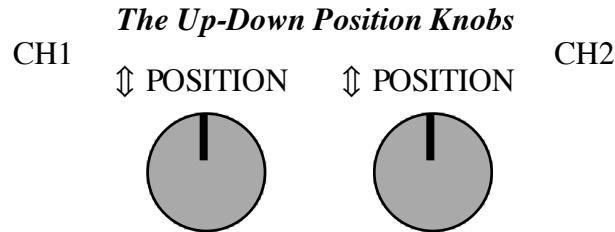
***Figure Two  
Input Ports and the Controls for Channels One and Two***



- 2.) Initially, **all of the rectangular buttons should be in their “out” position**. Push these buttons in and then out to assure that they are indeed out.



Set the knobs so the indicator is pointed up.



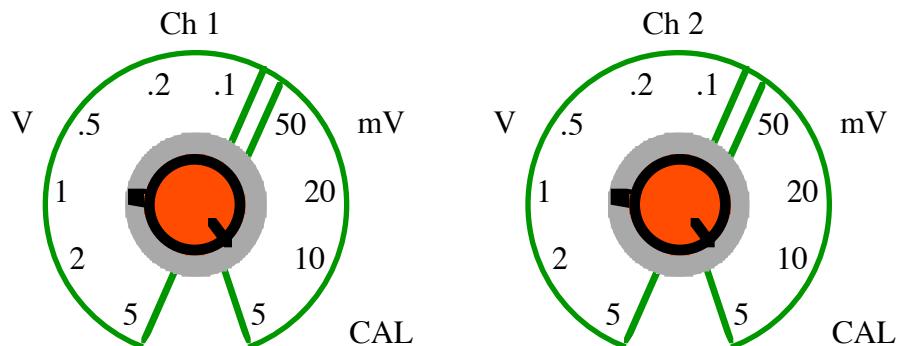
Set the three-position toggle switch for each channel to the DC position.

*Three-Position Toggle Switches*

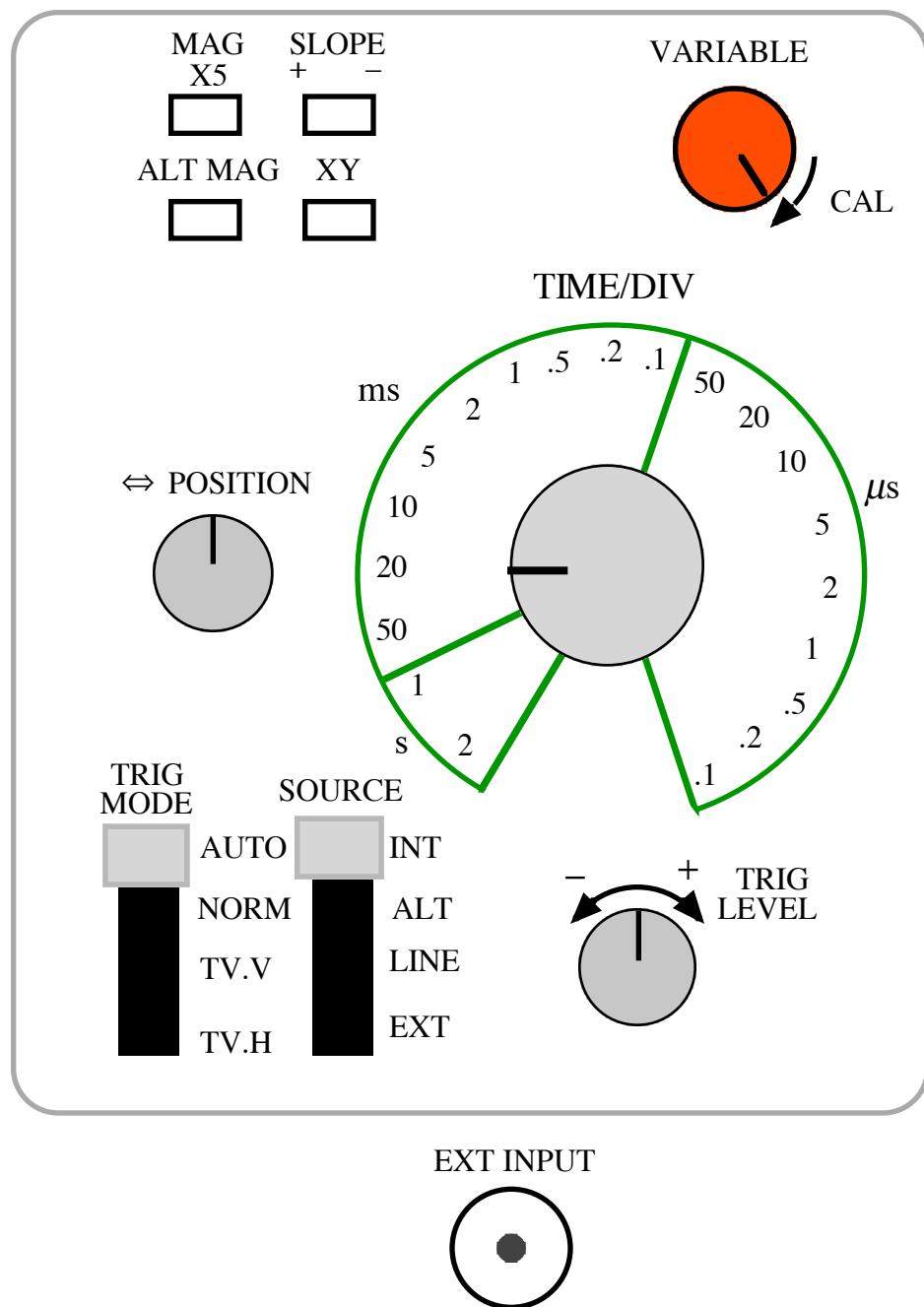


Next, note that each VOLTS/DIV knob is really two knobs. The outside sleeve controls the scale factor for the vertical grid on the CRT display. Turn this portion of each knob to the **one volt per division** position. (Voltages are plotted vertically on the CRT.) The inner knob, orange in the diagram, is for calibration. These need to be turned as far clockwise as possible and they must stay in that position to ensure that the scope is calibrated.

*Volts/Division and Calibration Knobs*



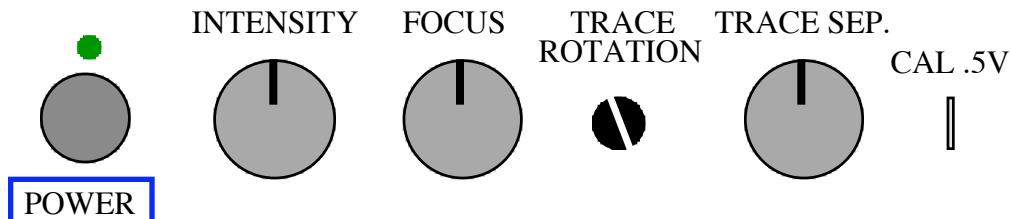
*Figure Three*  
The “Sweep Control” Center



- 4.) Initially, all four of the rectangular buttons in the “Sweep Control” center should be in the “out” position. Also, the calibration knob, orange in the diagram, needs to be turned completely clockwise and kept in that position throughout the experiment. The left-right position knob should be pointed up. Set the trigger mode four-position toggle switch to **auto**. Set the source four-position toggle switch to **internal**. The trigger level knob should be pointed up. Set the TIME/DIV knob to 20 ms/div .

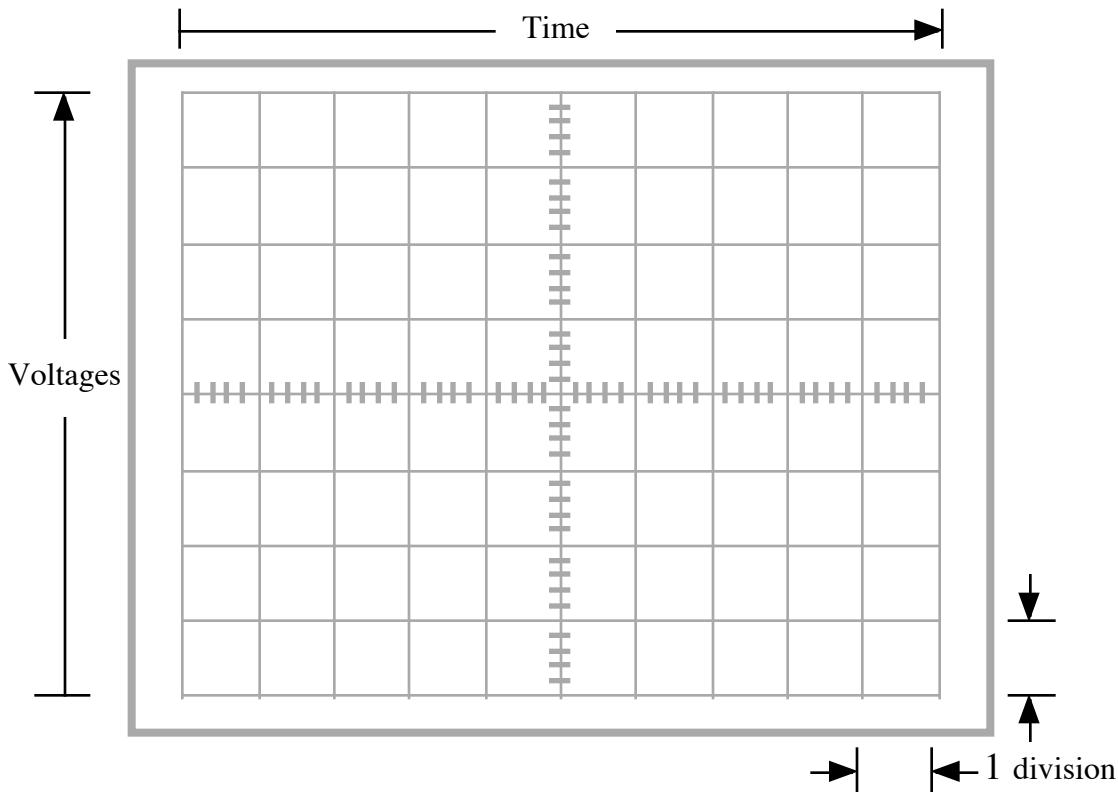
- 5.) Locate the trace controls. Set the intensity, focus and trace separation knobs so that they point up. The scope should now be ready to go.

**Figure Four**  
**The Power Button and the Trace Controls**



- 6.) Locate the power button and push it in. A small green light indicates the scope is on. An intermittent horizontal green dot should appear on the CRT display screen moving from left to right. Now, adjust the TIME/DIV knob to 0.10 ms/div and the trace should become a continuous line. If there is no such line, call the instructor over.  
 7.) Once you have a trace, turn the intensity knob to see how this affects the brightness of the trace line. Also turn the focus knob to get the line into focus.  
 8.) If the trace is not horizontal, use a small blade screwdriver in the trace rotation slot to adjust the line. Once this is accomplished, you are good to go.

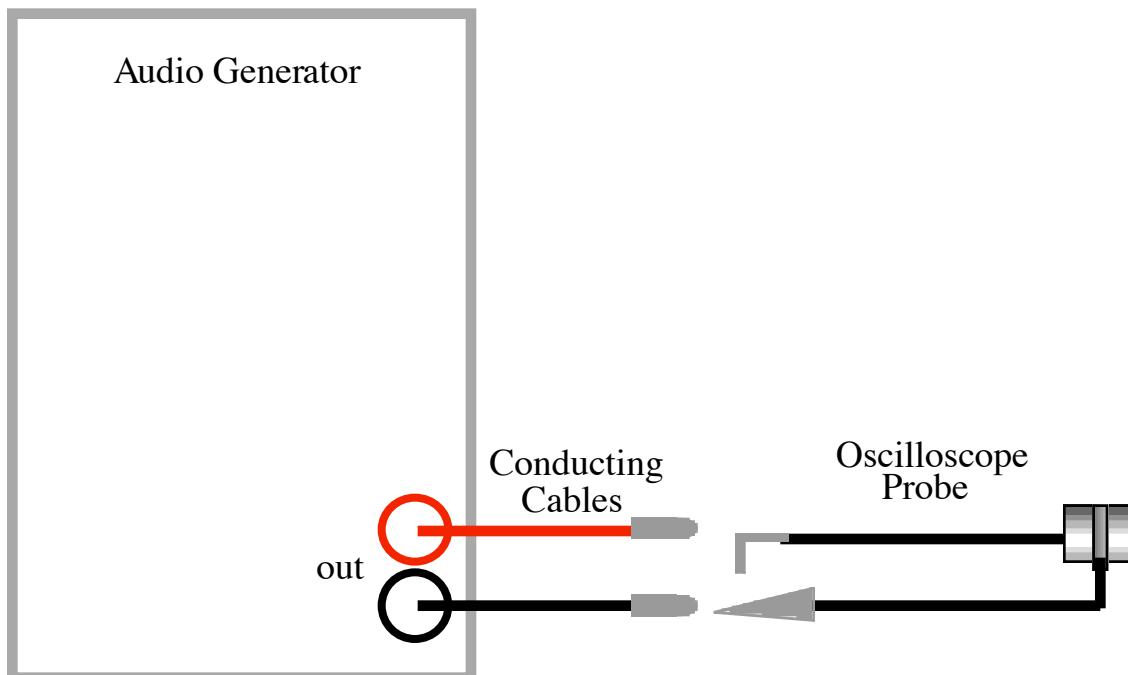
**Figure Five**  
**The CRT Display**



## *Getting a Sine Wave Trace*

Our next task is to get a sine wave on the CRT display. We are going to use the BK 3001 Audio Generator as our sine wave source.

- 9.) First, we must determine if the audio generator has power. Set your Fluke to measure AC voltage. Turn the generator on by turning the frequency knob to 20 Hz and turn the amplitude knob all the way in the clockwise sense. Place the red probe of the Fluke into the **red output port** of the generator and the black probe into the **black output port**. The potential difference should be  $\geq 1.5$  V. The DC voltage on the battery itself--should be close to 9 V. If it is not, you need a new battery. Once you have the necessary voltage, turn off the generator and the Fluke and move on to the next step.
- 10.) Set up the audio generator in the following way:  
Set the frequency range slide switch to X1, which means **times one**.  
The amplitude knob should be at the maximum amplitude position.  
Set the wave form slide switch to the **sine wave** position and **not** the **square wave** position. Set the attenuation slide switch to 0 dB .
- 11.) Put a red conducting cable into red output port of the generator. Put a black conducting cable into the black output port of the generator.
- 12.) Connect an oscilloscope probe to the channel one input port. (The channel one toggle should still be in the DC position.) Take the “hook” part of the probe and stick it into the free end of the red conducting cable that is attached to the generator. Connect the “gator” clip to the free end of the black conducting cable that is attached to the generator. (The probe multiplier should be at X1.)
- 13.) Set the scope’s TIME/DIV knob to 10 ms . Turn on the scope and set the audio generator to 20 Hz . A sine wave should be displayed on the CRT screen.
- 14.) To see a square wave form, slide the wave form switch on the generator to the square wave position.
- 15.) Turn everything off.



## ***Measuring the Period of a Sine Wave***

Now, we want to see how one can use the oscilloscope to measure the period of a wave form. The period,  $\tau$ , is the amount of time it takes the wave to make one complete cycle. The speed of the wave,  $v$ , is the speed at which the wave transfers energy. This speed is given by

$$v = f\lambda, \quad (1)$$

where  $f$  is the frequency of the wave and  $\lambda$  is the wavelength. For a single cycle, we have

$$v = \lambda / \tau. \quad (2)$$

Equations one and two imply that

$$\tau = 1 / f, \quad (3)$$

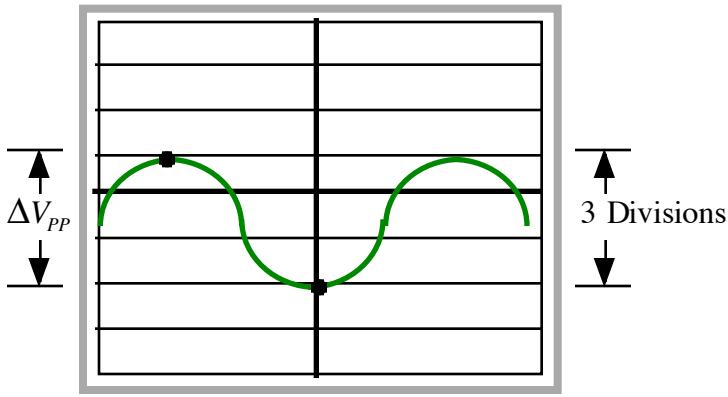
Let us now see if this is the case.

- 15.) Set the audio generator wave form slide switch back to the sine wave position. Set the frequency of the generator to 100 Hz. Turn on the audio generator and the oscilloscope.
- 16.) Turn the TIME/DIV knob to 2 ms. This should give you a clear trace of the sine wave. Using the up-down position knob of channel one, move the “peaks” of the sine wave down so that they “touch” the center horizontal axis. Use the  $\leftarrow\rightarrow$  position knob to get the left peak on a vertical line. **Count the number of horizontal divisions** between any two adjacent peaks. Record this number on the data sheet. (Adjacent peaks represent one complete cycle.)
- 17.) Repeat this process for frequencies of 5,000 Hz, 10,000 Hz, and 20,000 Hz. For each new frequency, you will have to change the TIME/DIV setting to smaller intervals. **Turn off the scope and the audio generator when you have done all of these.**
- 18.) Next, multiply the number of divisions for each specific frequency by the TIME/DIV setting, respectively, and you will get the measured period of each specific sine wave,  $\tau_{mea}$ .
- 19.) According to equation (3), the period is the reciprocal of the frequency. Using equation (3) calculate the period of a sine wave with frequency 100 Hz, 5,000 Hz, 10,000 Hz, and 20,000 Hz and record these values on the data sheet. These are the calculated periods,  $\tau_{cal}$ .
- 20.) Calculate the percent difference between your measured value of the period and your calculated value for the period and record these results.

## ***Measuring AC Voltages***

We now want to see how we can use the oscilloscope as an AC voltmeter.

- 21.) The audio generator should still be connected to channel one of the oscilloscope via the probe. Set the audio generator to a frequency of 20 Hz. Set the VOLTS/DIV knob on the oscilloscope to 0.50 Volt/Div and set the TIME/DIV knob to 10 ms. Turn the oscilloscope on. The trace should look like a normal sine wave, like that in the diagram below.
- 22.) Adjust the amplitude of the audio generator until the trace **spans three vertical divisions**. The span from the top of the curve to the bottom of the curve represents what is called the peak-to-peak voltage  $\Delta V_{PP}$ .



- 23) With the Fluke in AC voltage mode, measure the AC voltage across the red and black output ports of the audio generator. Record this value on the data sheet as  $\Delta V_{Fluke}$ .  
 (The range of the volt meter should be to two decimal places. For example, 1.41 V.) Turn off the scope, the Fluke, and the audio generator.
- 24.) Remember that voltages are plotted on the vertical axis of the CRT display. The span of the extremes of the trace represent what is called the peak-to-peak voltage,  $\Delta V_{PP}$ .  
 The Fluke, however, measures what is called the root-mean-square voltage, RMS.  
 The RMS voltage is related to the peak-to-peak voltage by

$$\Delta V_{RMS} = \left( \sqrt{2} / 4 \right) \Delta V_{PP} = (0.35355) \Delta V_{PP}, \quad (4)$$

where

$$\Delta V_{PP} = N_{divisions} (\Delta V / division). \quad (5)$$

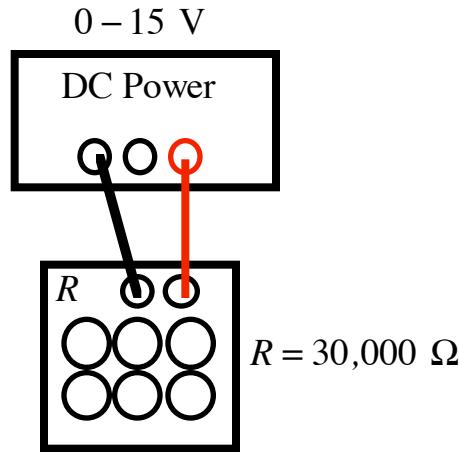
Using equation (4), calculate the RMS voltage and record this value on the data sheet as  $\Delta V_{scope}$ .

- 25.) Calculate the percent difference between  $\Delta V_{scope}$  and  $\Delta V_{Fluke}$ . Record this value on the data sheet.
- 26.) Turn the oscilloscope and the audio generator back on. Now adjust the generator amplitude so that it spans only **four vertical divisions**. Do steps 23.) through 25.) for this amplitude.
- 27.) Also, do this for **five** vertical divisions and then for **six** vertical divisions.

### ***Measuring DC Voltages***

We want now to see how we can use the oscilloscope as a DC voltmeter.

- 28.) For safety, set up the Electro Industries DC Power Supply so that it does not deliver more than  $0.40 \text{ A}$ . Once this task is completed, turn the power supply off. Set up the circuit shown below, with the resistor set to  $R = 30,000 \Omega$ .
- 29.) With the power supply **off**, attach the “hook” of the probe to the resistor post that has the red conductor in it. Attach the “gator” clip of the oscilloscope probe to the other resistor post. Attach the probe to the input port for channel one of the oscilloscope.
- 30.) With power supply off, turn on the oscilloscope and set the VOLTS/DIV knob to one *volt/div*. Adjust the trace so that it is aligned on the bottom horizontal line of the CRT display. (The toggle should still be at DC not GRD or AC.) Turn the scope off!



- 31.) Set the Fluke to  $\overline{V}$  mode, and adjust the display to 0.0 V. Put the red probe of the Fluke into the resistor port with the red conducting cable and the black probe into the port with the black conducting cable. Turn the voltage knob of the power supply to its minimum value position. (The slide should be on the 0-15 V position.) Turn the power supply on. Using only the voltage control knob of the power supply, adjust it until the Fluke reads 3.0 V. Now, if everything works properly, when you turn on the scope, the trace line should “jump” up to the third horizontal line above the zero! Turn on the scope and see what happens. The scope measured DC voltage is given by

$$\Delta V_{scope} = N_{divisions} (\Delta V / \text{division}) .$$

Record the calculated scope voltage, using the above equation.

To convince yourself that the change in voltage is causing the trace line to move, slowly change the voltage output of the power supply and watch the trace line move in response.

- 32.) Assuming everything is working properly, record the  $\Delta V_{scope}$  values for the following power supply values measured by the Fluke:

$$\begin{aligned} & 6.0 \text{ V}, \\ & 12.0 \text{ V}, \\ & 20.0 \text{ V}. \end{aligned}$$

The last two values will require a different setting for the VOLTS/DIV!

- 33.) Turn off all of the equipment and return the equipment to its proper storage location!



***PHY2054L LABORATORY***

***Experiment Seven***

***The Oscilloscope***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

### *Measuring the Period of a Sine Wave*

<i>Generator Frequency (Hz)</i>	<i>Scope Time / Div</i>	<i>Number of Divisions</i>	<i>Period <math>\tau_{mea}</math> (s)</i>	<i>Period <math>\tau_{cal}</math> (s)</i>	<i>% Difference</i>
100					
5,000					
10,000					
20,000					

### *Measuring AC Voltages*

<i>Audio Amplitude</i>	<i>Scope Volt / Div</i>	<i>Number of Divisions</i>	$\Delta V_{PP}$ (V)	$\Delta V_{scope}$ (V)	$\Delta V_{Fluke}$ (V)	<i>% Difference</i>
3 Divisions	0.50	3	1.50			
4 Divisions	0.50	4	2.00			
5 Divisions	0.50	5	2.50			
6 Divisions	0.50	6	3.00			

### ***Measuring DC Voltages***

<i>Scope Volt / Div</i>	<i>Number of Divisions</i>	$\Delta V_{scope}$ (V)	$\Delta V_{Fluke}$ (V)	<i>% Difference</i>
1			3	
1			6	
2			12	
5			20	



# ***PHY2054L LABORATORY***

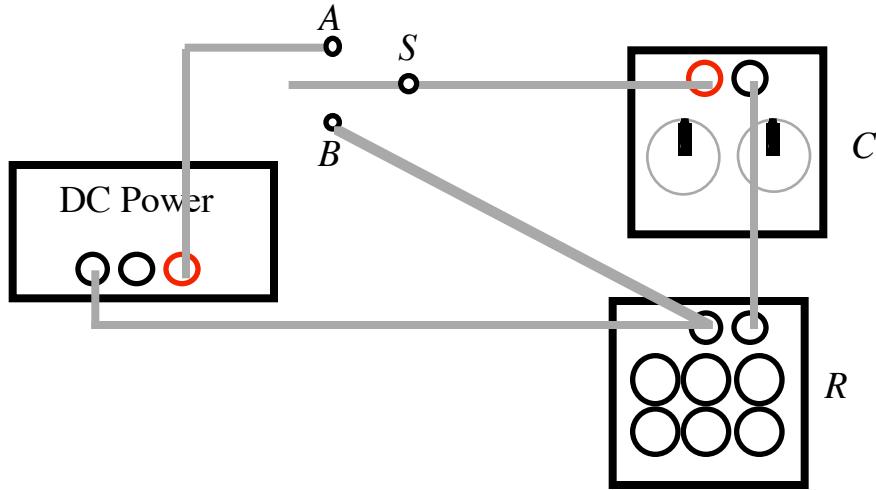
## *Experiment Eight*

### *The RC Time Constant*

## THEORY

Carefully consider the circuit shown below in Figure One. **Note the switch.** When the switch is open--neither in position A nor B--we assume there is no charge on the capacitor. When the switch is closed at position A, the capacitor begins to charge.

*Figure One  
A Resistor and Capacitor in Series*



During the **charging phase**, positive charge is increasing on one plate of the capacitor. It turns out that during this phase, the amount of positive charge on this plate is given by:

$$q = CV \left[ 1 - e^{-t/RC} \right], \quad (1)$$

where  $C$  is the capacitance of the capacitor,  $V$  is the potential difference across the power supply, and  $R$  is the resistance of the resistor.

If you look carefully at equation (1) you can see that at time  $t = t_0 = 0$  the charge on the capacitor is given by

$$q(t=0) = CV \left[ 1 - e^{-0} \right] = 0. \quad (2)$$

At some later time, we will have  $t = RC$ , the charge on the capacitor is given by

$$q(t=RC) = CV \left[ 1 - e^{-RC/RC} \right] = CV \left[ 1 - e^{-1} \right] = (0.63)CV. \quad (3)$$

Finally, as the time becomes very, very large,  $t \rightarrow \infty$ , the charge on the capacitor becomes

$$q(t=\infty) = CV \left[ 1 - e^{-\infty} \right] = CV \left[ 1 - 0 \right] = CV. \quad (4)$$

Now, as the capacitor is being charged, there must be a current in the circuit. The current during the charging phase is given by

$$I = \frac{dQ}{dt} = \frac{V}{R} e^{-t/RC}. \quad (5)$$

To understand what is happening to the current in the circuit during the charging phase, we use equation (5) to determine some representative values of the current: Initially, when  $t = 0$ , the starting current is given by

$$I(t=0) = (V/R) e^{-0/RC} = (V/R). \quad (6)$$

Also, when  $t = RC$ , we find

$$I(t = RC) = \frac{V}{R} e^{-RC/RC} = \frac{V}{R} e^{-1} = (0.37) \frac{V}{R} . \quad (7)$$

And finally, after a long time, as  $t \rightarrow \infty$ , we have

$$I(t = \infty) = \frac{V}{R} e^{-\infty/RC} = \frac{V}{R}(0) = 0 . \quad (8)$$

So, during the charging phase, the charge on the capacitor increases exponentially while the current in the circuit decreases exponentially.

Next, if one were to move the switch to position **B** **after** the capacitor had been completely charged, then the capacitor would begin to discharge. For the **discharge phase** we would have

$$Q = CVe^{-t/RC} . \quad (9)$$

Using equation (9), initially at time  $t = 0$ , the charge is

$$Q(t = 0) = CVe^{-0/RC} = CV(1) = CV . \quad (10)$$

When time  $t = RC$ , the charge on the capacitor has decreased to

$$Q(t = RC) = CVe^{-RC/RC} = CVe^{-1} = (0.37)CV , \quad (11)$$

about 37% of the initial value. Finally, after a long time,

$$Q(t = \infty) = CVe^{-\infty/RC} = CV(1/e^{\infty}) = 0 . \quad (12)$$

Exponential functions occur so frequently in the natural sciences that it is important for you to be able to calculate their values and graph them. An understanding of the exponential function is essential to doing quantitative scientific work.

Consider now just the exponential factor of equation (9). Note that at time  $t = RC$  we have

$$e^{-RC/RC} = e^{-1} = \frac{1}{e} = 0.367879441.... \quad (13)$$

In physics, and all of the natural sciences for that matter, when we have **exponential functions** involving  $t$ , the specific time  $t_c$  that makes the exponential term equal to one is important enough to give it a name. It is called the **characteristic time**,  $t_c$ . For our specific situation involving circuits with capacitors and resistors in series, the characteristic time  $t_c$  is called the "RC time constant" as **RC is the value of  $t$**  that makes the exponent equal to one. Characteristic times give us useful information on the time scale for the growth and/or the decay of the exponential.

## EQUIPMENT NEEDED

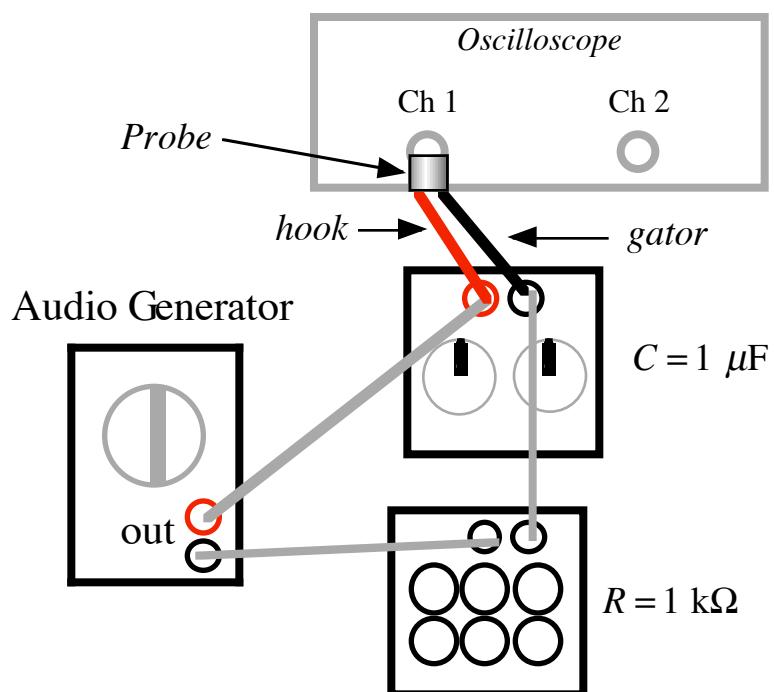
Oscilloscope and Probe  
One Decade Resistor Box  
3 Connecting Cables

Function Generator (Use the **BK 3001.**)  
One Capacitor Box  
One Fluke Multimeter

## PROCEDURE

We are going to use the oscilloscope to study the response of an *RC* circuit to a change in the applied potential difference. This will be accomplished by using a square wave which will, in essence, act like changing the switch position from **A** to **B** to **A** to **B** and so on. The capacitor will go through successive periods of charging and discharging and we can represent the graph of this on the screen of the oscilloscope.

*Figure Two*



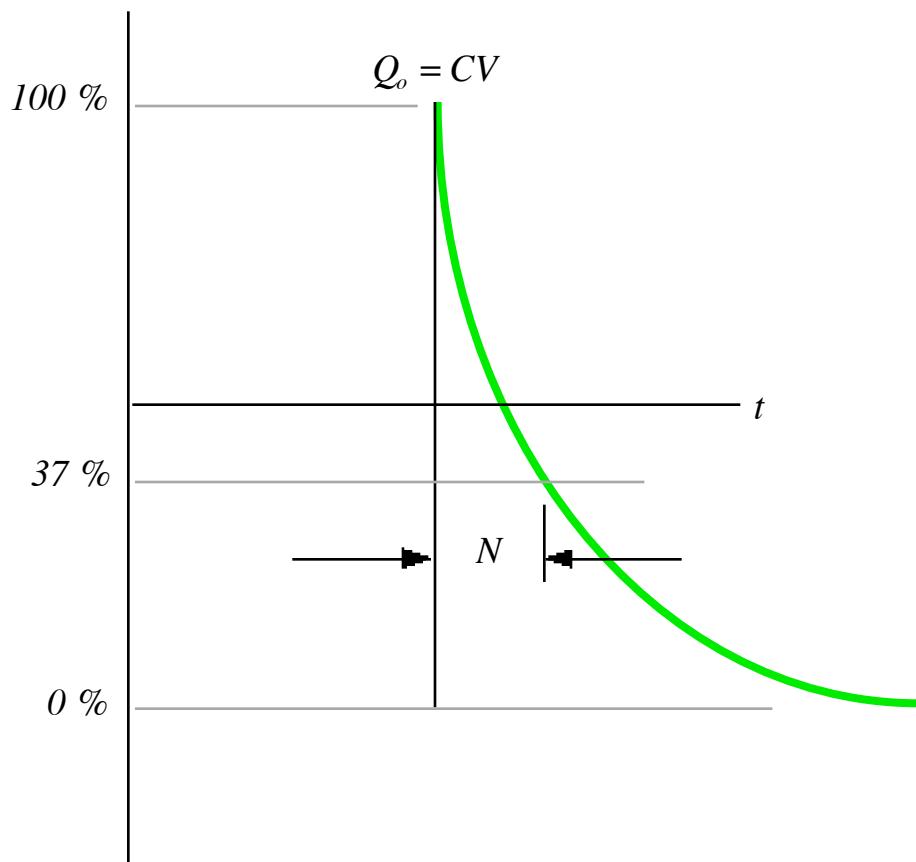
- 1.) With **everything turned off**, set up the circuit shown above in Figure Two.
- 2.) Turn on the oscilloscope and get a trace.
- 3.) Initially, set the **vertical amplifier** on the scope to 0.1 Volt/Div . (Set to DC mode.)
- 4.) Set the resistance box to  $1,000 \Omega$  and check with the Fluke multimeter.
- 5.) Set the capacitance box to  $1.00 \mu\text{F}$  .
- 6.) Set the frequency of the **square wave** to 28 Hz. Turn on the function generator and increase its amplitude until the pattern covers from 0 to 100 % of the grid on the oscilloscope screen.
- 7.) Adjust the **sweep rate** on the scope until you get one full exponential curve on the screen. It should look like the one of the graphs you sketched. You should have either the charging or discharging phase displayed clearly.
- 8.) Consider the discharging phase represented in Figure Three below.

9.) Identify the intersection point where the discharging begins. Measure off of the scope, the time taken to go from this starting point to the 37% point, and record this value. You do this by multiplying  $N$ --the number of divisions--by the sweep time  $t$ --the time/division--on the scope.

Also record the total resistance  $R$  of the circuit (be sure to add the **output resistance** of the function generator;  $850 \Omega$  for the BK 3001 function generator), and the  $C$  of the circuit.

10.) Do steps 7 through 9 with another value of  $R$  and another value of  $C$ . Do a total of three such measurements.

**Figure Three**  
**Representation of the Discharge Phase**  
**Measuring  $\tau$  on the Oscilloscope**





***PHY2054L LABORATORY***

***Experiment Eight***

***The RC Time Constant***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Sketches

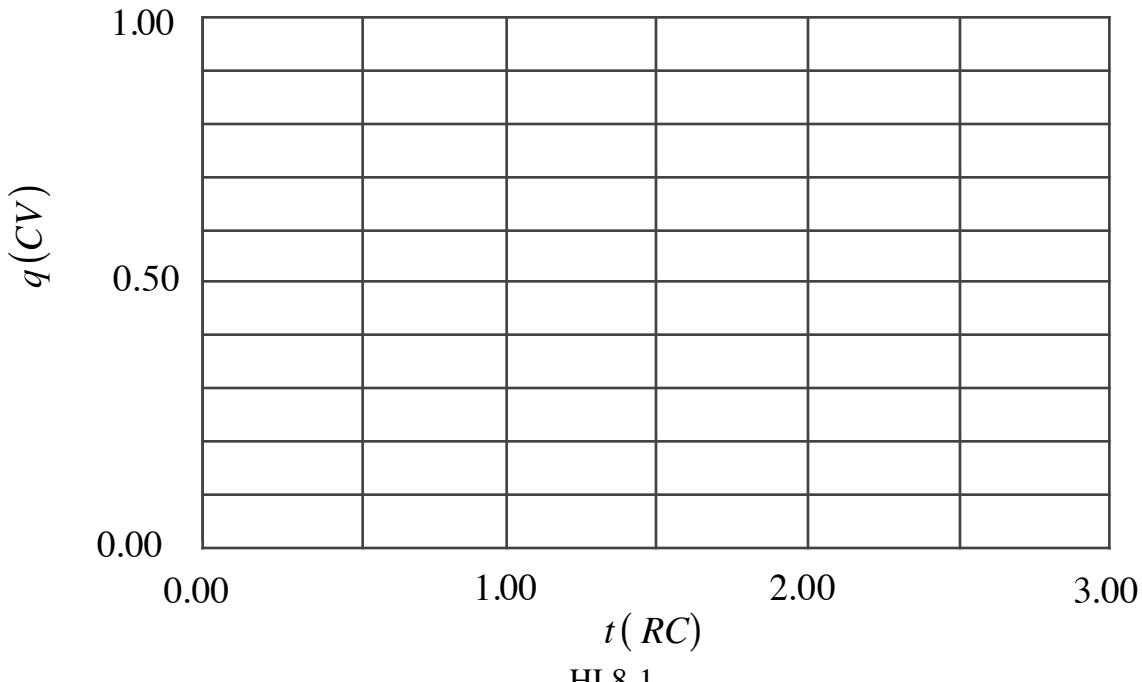
**Charging Phase:**

**Graph Table**

$t(RC)$	$e^{-t/RC}$	$1 - e^{-t/RC}$	$q(CV)$
0	$e^{-0/RC} = e^{-0} = 1.0000$	0.00	0.00
1/2	$e^{-(1/2)RC/RC} = e^{-1/2} = 0.6065$	0.39	0.39
1	$e^{-RC/RC} = e^{-1} = 0.3679$	0.63	0.63
3/2	$e^{-(3/2)RC/RC} = e^{-3/2} = 0.2231$	0.78	0.78
2	$e^{-2RC/RC} = e^{-2} = 0.1353$	0.86	0.86
5/2	$e^{-(5/2)RC/RC} = e^{-5/2} = 0.0821$	0.92	0.92
3	$e^{-3RC/RC} = e^{-3} = 0.0498$	0.95	0.95

On the grid given below, plot the coordinates  $(q, t)$  and sketch the smooth curve that represents the charge on the capacitor as a function of time as the capacitor charges.

**Charge on the Capacitor Versus Time in the Charging Phase**

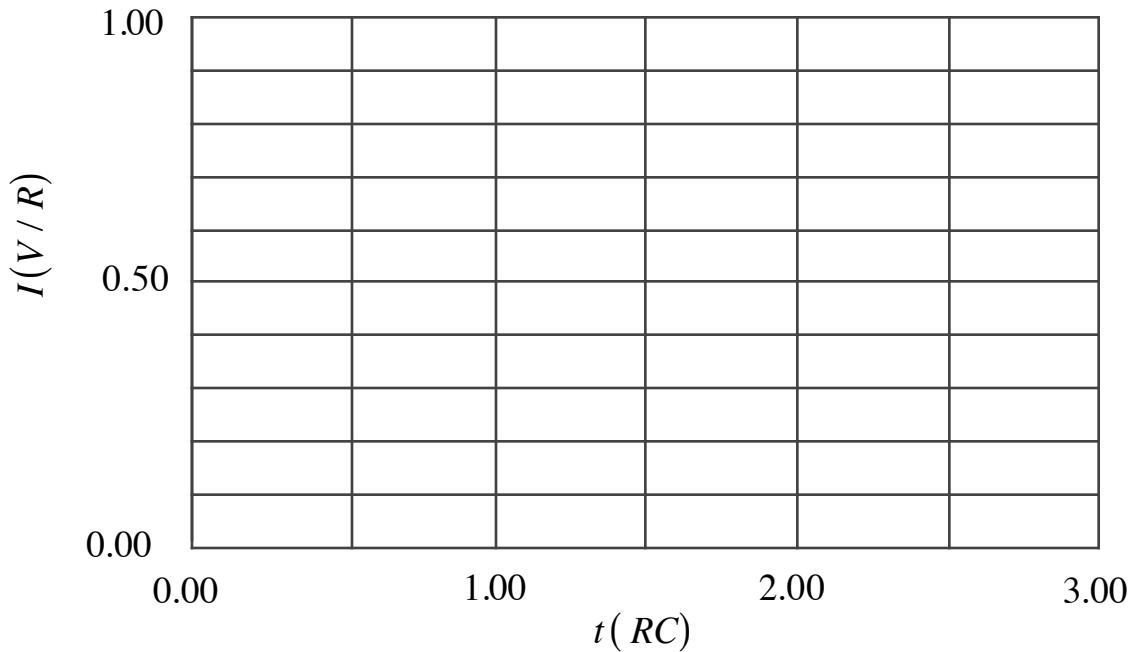


**Graph Table**

$t(RC)$	$e^{-t/RC}$	$I(V / R)$
0	$e^{-0/RC} = 1.0000$	1.00
1/2	$e^{-(1/2)RC/RC} = e^{-1/2} = 0.6065$	0.61
1	$e^{-RC/RC} = e^{-1} = 0.3679$	0.37
3/2	$e^{-(3/2)RC/RC} = e^{-3/2} = 0.2231$	0.22
2	$e^{-2RC/RC} = e^{-2} = 0.1353$	0.14
5/2	$e^{-(5/2)RC/RC} = e^{-5/2} = 0.0821$	0.08
3	$e^{-3RC/RC} = e^{-3} = 0.0498$	0.05

On the grid given below, plot the coordinates  $(I, t)$  and sketch the smooth curve that represents the current through the resistor as a function of time as the capacitor charges.

**Current Through the Resistor Versus Time in the Charging Phase**



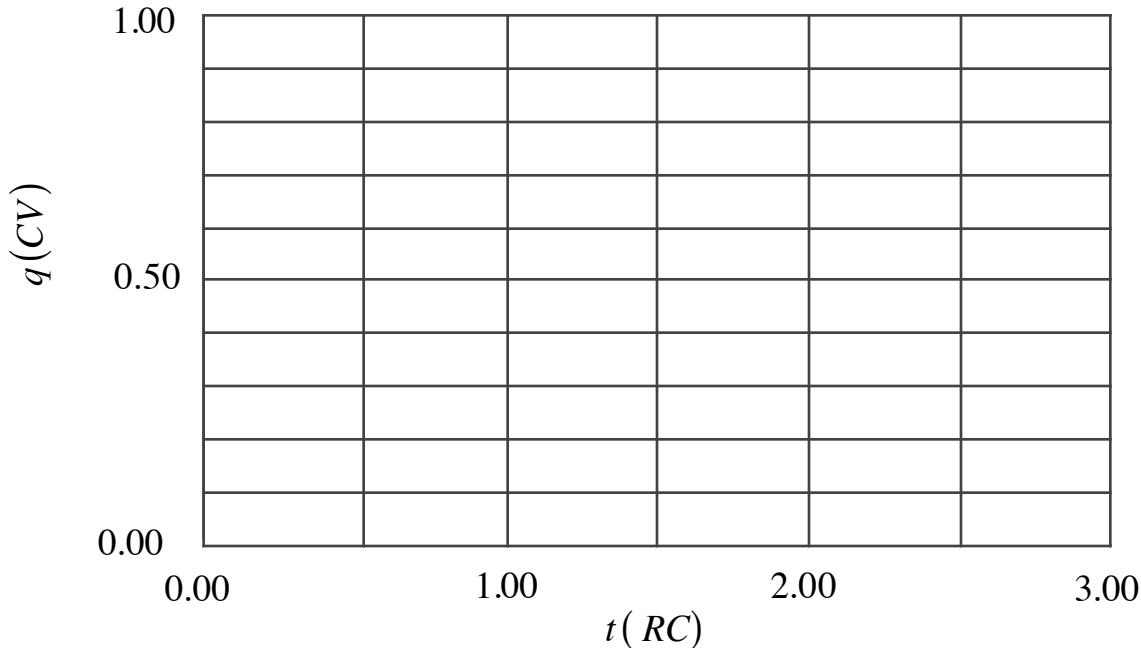
## Discharging Phase

**Graph Table**

$t(RC)$	$e^{-t/RC}$	$Q(CV)$
0	$e^{-0/RC} = 1.0000$	1.00
1 / 2	$e^{-(1/2)RC/RC} = e^{-1/2} = 0.6065$	0.61
1	$e^{-RC/RC} = e^{-1} = 0.3679$	0.37
3 / 2	$e^{-(3/2)RC/RC} = e^{-3/2} = 0.2231$	0.22
2	$e^{-2RC/RC} = e^{-2} = 0.1353$	0.14
5 / 2	$e^{-(5/2)RC/RC} = e^{-5/2} = 0.0821$	0.08
3	$e^{-3RC/RC} = e^{-3} = 0.0498$	0.05

On the grid given below, plot the coordinates  $(q, t)$  and sketch the smooth curve that represents the charge on the capacitor as a function of time as the capacitor discharges.

**Charge on the Capacitor Versus Time in the Discharge Phase**



## Data Sheet

$R_{total}$ ( $\Omega$ )	$C$ (F)	$\tau_{theory}$ (s)	<i>Number of Divisions</i>	<i>Time / Div</i> (s)	$\tau_{measured}$ (s)	<i>% Difference</i>



# ***PHY2054L LABORATORY***

## *Experiment Nine*

### ***Measuring the Charge to Mass Ratio of the Electron***

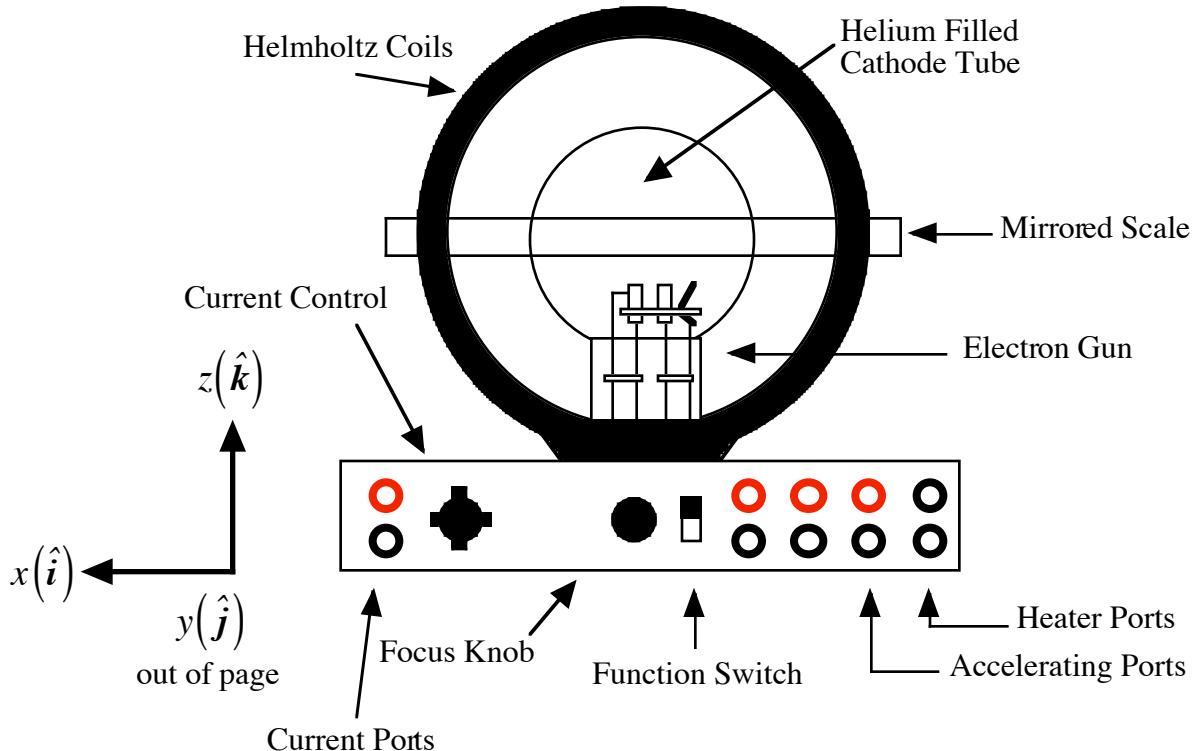
## THEORY

If a metal is given enough thermal energy, the metal will emit electrons. This process is called thermionic emission. The cathode electrode is just such an emitter in a cathode ray tube. The electrons can then be directed to selected targets by accelerating them through appropriately oriented electromagnetic fields. This is the basic operating strategy used in a cathode ray tube, CRT for short.

Consider a CRT that is set up so that the free electrons are accelerated horizontally through an electric field running from the anode to the cathode; we will call this the negative  $x$ -axis. The  $y$ -axis is also horizontal and parallel to the major axis of symmetry of the Helmholtz coils, as represented below in Figure 9-1.

Since the electric field is directed from the anode to the cathode, it will accelerate the electrons from the cathode to the anode. The electrons will be moving in the negative  $x$  direction.

**Figure 9-1**  
 **$e/M$  Apparatus**



The electrons are collimated by placing a metal target with a small horizontal slit in the path of the moving electrons. If we assume that the average initial kinetic energy of the emitted electrons is very small compared to the kinetic energy they have when they reach the anode, then, using the conservation of mechanical energy, we can write

$$\frac{1}{2}M_{e^-}v^2 = e(\Delta V), \quad (1)$$

where  $M_{e^-}$  is the mass of the electron,  $v$  is the speed of the electron as it passes through the slit,  $e$  is the magnitude of the electric charge on the electron, and  $\Delta V$  is the **magnitude** of the potential

difference between the anode and the cathode. Equation (1) implies that

$$v^2 = 2(\Delta V) \left[ \frac{e}{M_{e^-}} \right]. \quad (2)$$

After the electron passes through the slit, it enters a region between two large Helmholtz coils that are aligned along the  $y$ -axis with the plane of the coils parallel to the  $x$ - $z$  plane. If a direct current  $I$  passes through these coils, the current will produce a magnetic field which, at the center of the coils, will be aligned along the  $y$ -axis. So, the electron, since it is moving with respect to the magnetic field, will experience a magnetic force. The magnetic force, recall, is given by

$$\vec{F}_{mag} = q[\vec{v} \times \vec{B}]. \quad (3)$$

Since the velocity and the magnetic field are perpendicular to each other, the magnitude of the magnetic force exerted on an electron will be given by

$$F_{mag} = evB. \quad (4)$$

Furthermore, since the magnetic force is perpendicular to the direction of motion, it will only change the direction of the motion and not the speed of the electron. In essence, the electron will move on a circular path of radius  $R$ . Consequently, the magnetic force is a centripetal force and we can write

$$F_{mag} = evB = \frac{M_{e^-} v^2}{R}. \quad (5)$$

Solving equation (5) for  $v$  gives us

$$v = RB \left[ \frac{e}{M_{e^-}} \right]. \quad (6)$$

Squaring  $v$  and also using equation (2) gives us

$$R^2 B^2 \left[ \frac{e}{M_{e^-}} \right]^2 = 2(\Delta V) \left[ \frac{e}{M_{e^-}} \right]. \quad (7)$$

We are now in a position to write an expression for the ratio of  $e$  to  $M_{e^-}$ :

$$\left[ \frac{e}{M_{e^-}} \right] = \frac{2(\Delta V)}{(RB)^2} \quad (8)$$

The accelerating potential difference  $\Delta V$  is easy to measure. We now turn our attention to experimentally measuring the magnetic field strength  $B$  and the radius of the curved path on which the electron moves,  $R$ .

Although the calculation is not trivial, it can be shown that for a single hoop of radius  $a$ , the center of which is at the origin of a Cartesian coordinate system, the plane of which lies on the  $x$ - $z$  plane--see Figure 9-2 below--and through which a steady state current  $I$  passes in a counterclockwise sense, that the magnetic field produced at a point on the  $y$ -axis is given by

$$\vec{B}_{single\ hoop} = \frac{2\pi k' I a^2}{(y^2 + a^2)^{3/2}} \hat{j}. \quad (9)$$

Now, if we tightly wind a coil of wire so that we have, in essence,  $N$  such hoops, then the magnetic

field generated would be

$$\vec{B}_{N \text{ hoops}} = \frac{2\pi k' NIa^2}{(y^2 + a^2)^{3/2}} \hat{j} . \quad (10)$$

Now, Helmholtz coils are arranged so that they are the same distance apart as their radius. This suggests that a point directly between the two coils corresponds to setting  $y = a/2$ . Substituting this into equation (10) and noting that there are two such coils, we get

$$\vec{B} = \frac{4\pi k' NIa^2}{((a/2)^2 + a^2)^{3/2}} \hat{k} = \frac{4\pi k' NIa^2}{((5/4)a^2)^{3/2}} \hat{k} = \frac{4(4/5)^{3/2} \pi k' NI}{a} \hat{k} . \quad (11)$$

The Helmholtz coils used in this experiment have a radius

$$a = 0.150 \text{ m} , \quad (12)$$

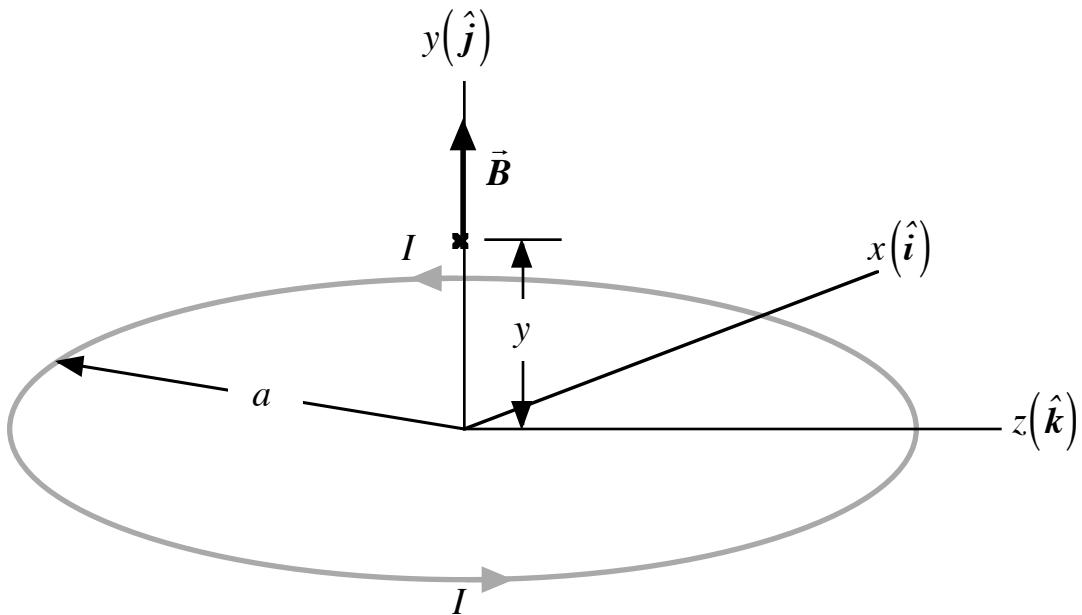
and a total of

$$N = 130 , \quad (13)$$

turns per coil. So, equation (11) implies that the magnitude of the magnetic field at the center of the Helmholtz coils is

$$B = \frac{4(4/5)^{3/2} \pi k' (130) I}{(.15 \text{ m})} = (7.79 \times 10^{-4} \text{ T} \cdot \text{A}^{-1}) I . \quad (14)$$

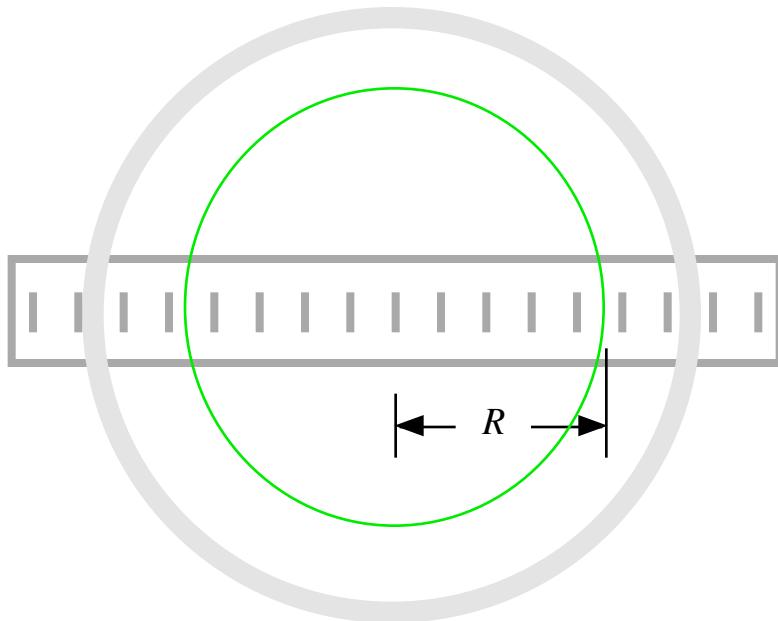
**Figure 9-2**  
**Magnetic Field of a Single Hoop**



When the electron beam is established in the tube, we will increase the current to the Helmholtz coils which, in turn, will increase the magnitude of the magnetic field, increasing the magnetic force exerted on the electron. We will continue to increase the current until we get the electron to form a **circular spiral**. We will be able to measure the radius of this circle by viewing it as it is superimposed on a mirrored scale in the back of the  $e / M_{e^-}$  apparatus. This state of

affairs is represented below in Figure 9-3. Remember, the radius can be measured from a point of view where the circle is centered on the “background” mirrored ruler.

*Figure 9-3*  
*Determining the Radius of the  
Circular Path of the Electron*



## EQUIPMENT NEEDED

One Pasco Model TG-13  $e / M_{e^-}$  Apparatus

One Pasco Model SF-9585A High Voltage Power Supply

One Pasco Model SF-9584A Low Voltage Power Supply

6 High Voltage Connecting Cables

## PROCEDURE

- 1.) With the power turned off, set up the configuration shown below in Figure 9-4.

### The High Voltage Power Supply:

The AC voltage knob should be set at 6 V AC. **Do NOT change this value!**

One cable should run from one yellow AC port to one of the black heater ports.

A second cable should run from the second yellow AC port to the second black heater port. (This energy source is used to heat the cathode; to emit the electrons.)

Connect a cable from the black 500 V DC port of the high voltage power supply to the red port of the e/m apparatus as shown. Connect another cable from the black 0 V DC port of the high voltage power supply to the black port of the e/m apparatus as shown. (This connection will provide the accelerating potential  $\Delta V$  for the electrons.)

Make sure the slide switch of the e/m apparatus is set for e/m. Also, make sure the cloth hood is over the apparatus to cut down on outside light.

### The Low Voltage Power Supply:

Turn the voltage and current knobs on the low voltage power supply till they point straight up. Connect a cable from the red 24 V DC port of the low voltage power supply to the red port of the e/m apparatus as shown. Connect another cable from the black 0 V DC port of the low voltage power supply to the black port of the e/m apparatus as shown. (This connection will provide the energy to move a current in the Helmholtz coils and, thereby, produce the magnetic field needed to "bend" our electron beam.)

- 2.) When you have set up the configuration shown, have your instructor check it.
- 3.) After you get the OK from your lab instructor, turn on the high voltage supply. Adjust the **DC** voltage--**do not touch the AC voltage**--until your meter indicates 200 V. Wait a time and you should eventually see a short, green horizontal beam in the lower right portion of the bulb. (If your e/m apparatus is not aligned so that the horizontal beam is running north-south, turn the apparatus so that is so aligned.)
- 4.) Turn on the low voltage power supply. Set the DC voltage to **nine volts**. Turn the current knob on the e/m apparatus until the ammeter on the low voltage power supply indicates that you have **two amps** moving through the coils. By now, your beam should have taken on the shape of a circular helix.
- 5.) Adjust the focus knob on the e/m apparatus until your beam is as well defined as you can get it.
- 6.) Find an observation point in front of the bulb so that the circle appears centered; as much of the circle to the right of center as to the left. Use the mirrored scale to measure the radius of

the circle. Record this value on the data sheet. (Make sure you record the radius in *meters*.)

7.) Repeat steps 3.) through 6.) with an accelerating potential difference of 300 V.

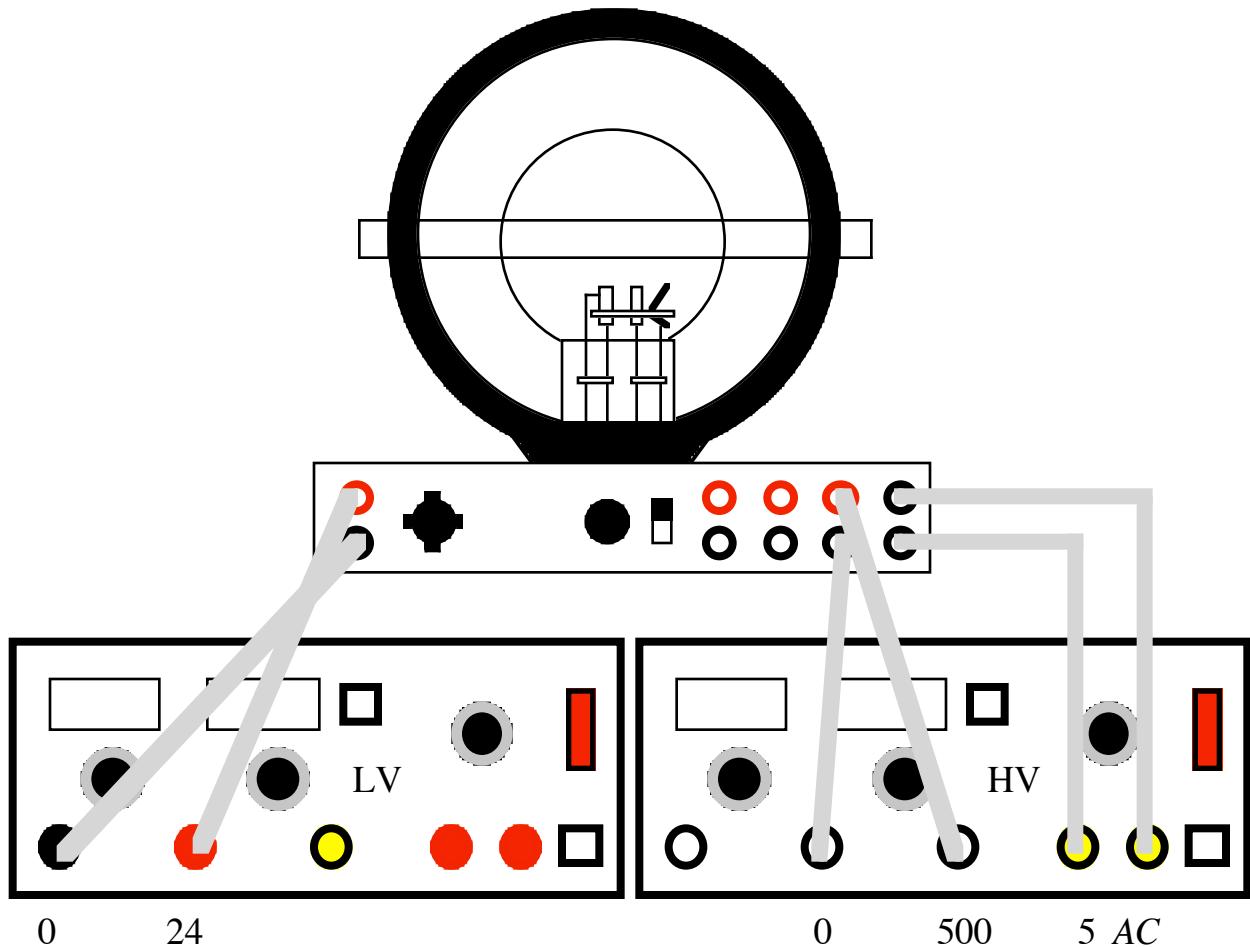
8.) Calculate the average value for  $e / M_{e^-}$  from the values found in your two trials.

9.) Calculate the experimental error of your average value compared with the accepted value given below in equation (15).

**The Accepted Value of  $e / M_{e^-}$  :**

$$\left[ \frac{e}{M_{e^-}} \right]_{\text{accepted}} = \frac{1.60218 \times 10^{-19} \text{ C}}{9.10939 \times 10^{-31} \text{ kg}} = 1.7588 \times 10^{11} \text{ C} \cdot \text{kg}^{-1} \quad (15)$$

*Figure 9-4*





***PHY2054L LABORATORY***

***Experiment Nine***

***Measuring the Charge to  
Mass Ratio of the Electron***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

### Trial One

$$\begin{aligned}\Delta V &= 200 \text{ V} \\ I &= 2.0 \text{ A}\end{aligned}$$

$$B = (7.79 \times 10^{-4} \text{ T} \cdot \text{A}^{-1})I = \underline{\hspace{10cm}} \text{ T}$$

$$R = \underline{\hspace{10cm}} \text{ m}$$

Your measured value for  $e/m$  for trial one:

$$\left[ \frac{e}{M_{e^-}} \right]_{trial \ one} = \frac{2(\Delta V)}{(RB)^2} = \underline{\hspace{10cm}} \text{ C} \cdot \text{kg}^{-1}$$

### Trial Two

$$\Delta V = 300 \text{ V}$$

$$I = 2.0 \text{ A}$$

$$B = (7.79 \times 10^{-4} \text{ T} \cdot \text{A}^{-1})I = \underline{\hspace{10cm}} \text{ T}$$

$$R = \underline{\hspace{10cm}} \text{ m}$$

Your measured value for  $e/m$  for trial two:

$$\left[ \frac{e}{M_{e^-}} \right]_{trial \ one} = \frac{2(\Delta V)}{(RB)^2} = \underline{\hspace{10cm}} \text{ C} \cdot \text{kg}^{-1}$$

Your average value for  $e/m$  for the two trials is:

$$\left[ \frac{e}{M_{e^-}} \right]_{average} = \underline{\hspace{10cm}} \text{ C} \cdot \text{kg}^{-1}$$

$$\% Error = \underline{\hspace{10cm}}$$

# ***PHY2054L LABORATORY***

## *Experiment Ten*

### *Measuring the Refractive Index*

## THEORY

Light of any wavelength has a speed  $c$  when traveling in vacuum, where

$$c = 2.99792458 \times 10^8 \text{ m} \cdot \text{s}^{-1}. \quad (1)$$

However, when light travels in a material medium, its speed changes. Materials in which the speed of the light depends on the wavelength--or color, if you will--are said to be **dispersive**. The glass used in prisms is one such dispersive material.

Light incident on a boundary surface, is either reflected or transmitted through the boundary. For an individual photon, there is a probability  $P_{\text{reflection}}$  for being reflected and a probability for passing through  $P_{\text{transmission}}$ . The only requirement is that the photon do one or the other, that is, that

$$P_{\text{reflection}} + P_{\text{transmission}} = 1. \quad (2)$$

If, for example, visible light is incident on a mirrored surface, the probability is much greater that it will be reflected than if the same light is incident on a pane of window glass. (These probabilities also are usually dependent on the wavelength of incident light.)

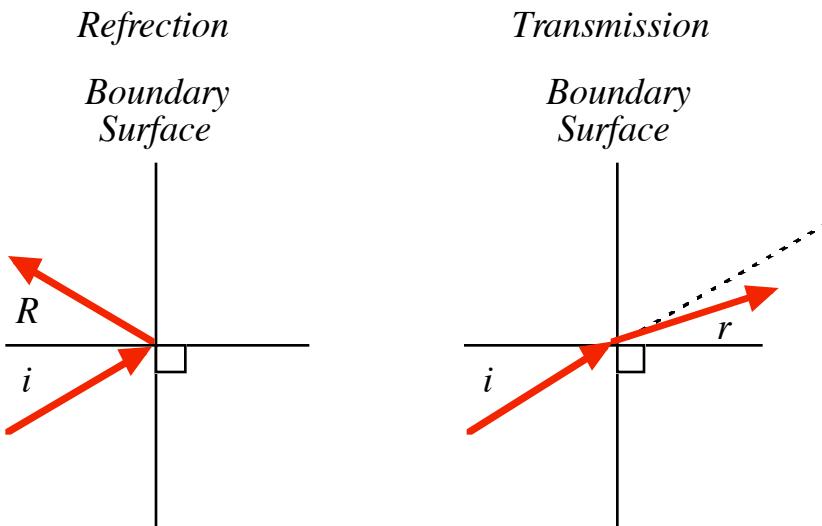
If the incident light is reflected, the angle at which the light is reflected will be the same as the incident angle, as represented in Figure One below, we would have

$$i = R. \quad (3)$$

(Note that the angles are measured from the path to a line segment that is perpendicular to the boundary surface, and all lie in the same plane.)

Light transmitted through the boundary will have the direction of its path of motion changed. When the direction of the motion of the light changes, it is said to be **refracted**. We measure that change with angle  $r$ . The amount the light is refracted depends on the material through which the light moves and the wavelength of the light.

**Figure One**  
**The Reflection or Transmission of Light at a Boundary Surface**



When light is transmitted across a boundary, its direction is not the only thing changed. The speed with which the light moves is also changed. A number called the **refractive index**, signified by  $n$ , is used to describe the refractive properties of materials. The refractive index of vacuum is

assigned the value of one. The index of refraction is defined by

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} = \frac{c}{c_n}, \quad (4)$$

where

$$c_n \leq c, \quad (5)$$

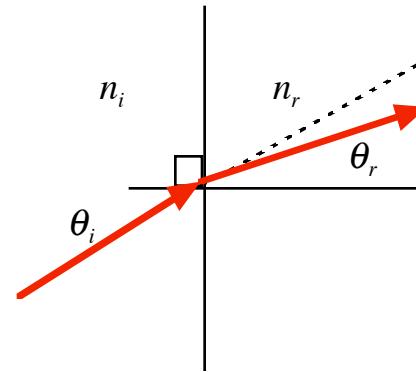
being equal only for the case of light traveling in vacuum.

Another important property of incident light on a boundary surface is described by Snell's Law. The notation we will use is represented in Figure Two below. We have

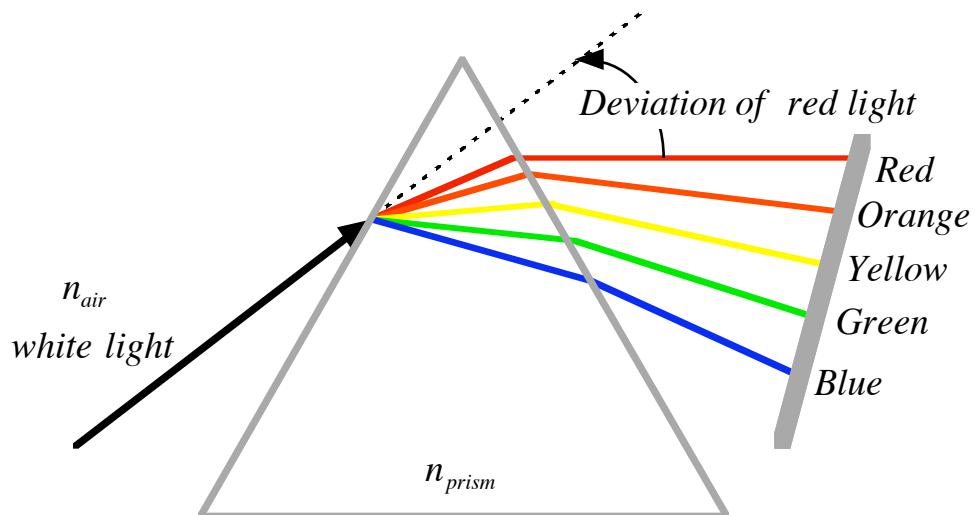
$$n_i \sin \theta_i = n_r \sin \theta_r. \quad (6)$$

**Figure Two**  
**Snell's Law**

*Boundary  
Surface*



**Figure Three**  
**The Dispersion of White Light by a Glass Prism**



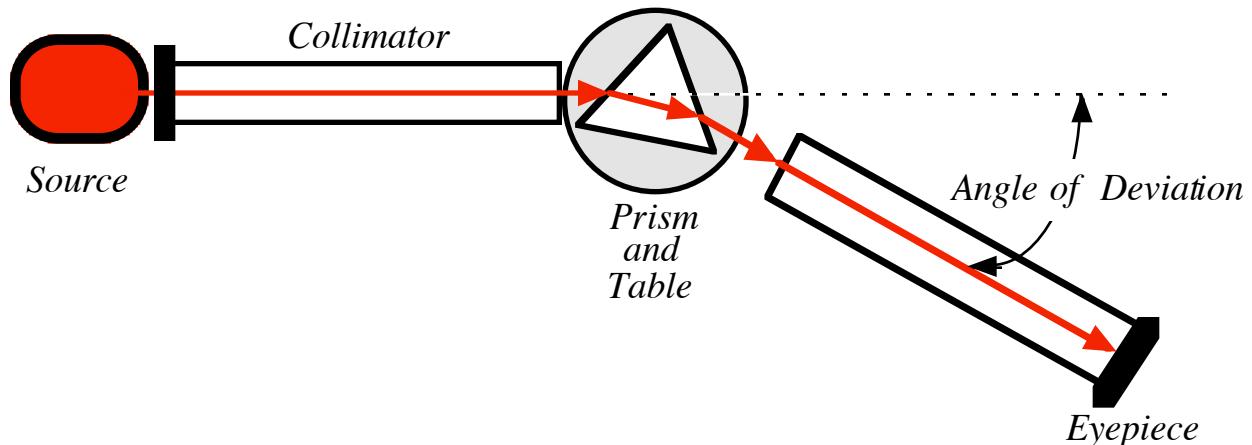
In this experiment, we will have light moving in air incident on a glass prism. For the light that is transmitted through the prism, Snell's law requires

$$n_{\text{prism}} = \left[ \frac{\sin \theta_i}{\sin \theta_r} \right] n_{\text{air}} . \quad (7)$$

It is, of course, the refractive index of the glass prism that we wish to measure. The task facing us as experimenters is to determine how to measure these quantities.

In Figure Four below, we have a schematic representation of the prism spectrometer that we will use in this experiment. A source of light emits electromagnetic radiation that passes through a collimator and is incident on a glass prism. The light is bent by the prism and emerges from the prism and enters a small movable telescope that can be used to accurately measure the angle of deviation of the light. The remaining theoretical question has to do with how the angle of deviation is related to the refractive index of the prism.

**Figure Four**  
**Prism Spectrometer**



Please spend some time looking carefully at Figure Five. First, note that  $D$ , the angle of deviation, measures the angle between the incident path direction of the light and the emergent path direction. Here is the crux, this angle will be a minimum,  $D_{\min}$ , when the light ray passes symmetrically throughout the prism; that is, when

$$i = r', \text{ and } r = i' . \quad (8)$$

Let us look closely now at the minimum deviation.

First, we have

$$D_{\min} + \varphi = 180^\circ = \alpha + \delta + \varphi , \quad (9)$$

and, therefore,

$$D_{\min} = \alpha + \delta . \quad (10)$$

Further, we can see that

$$\delta = i - r , \quad (11)$$

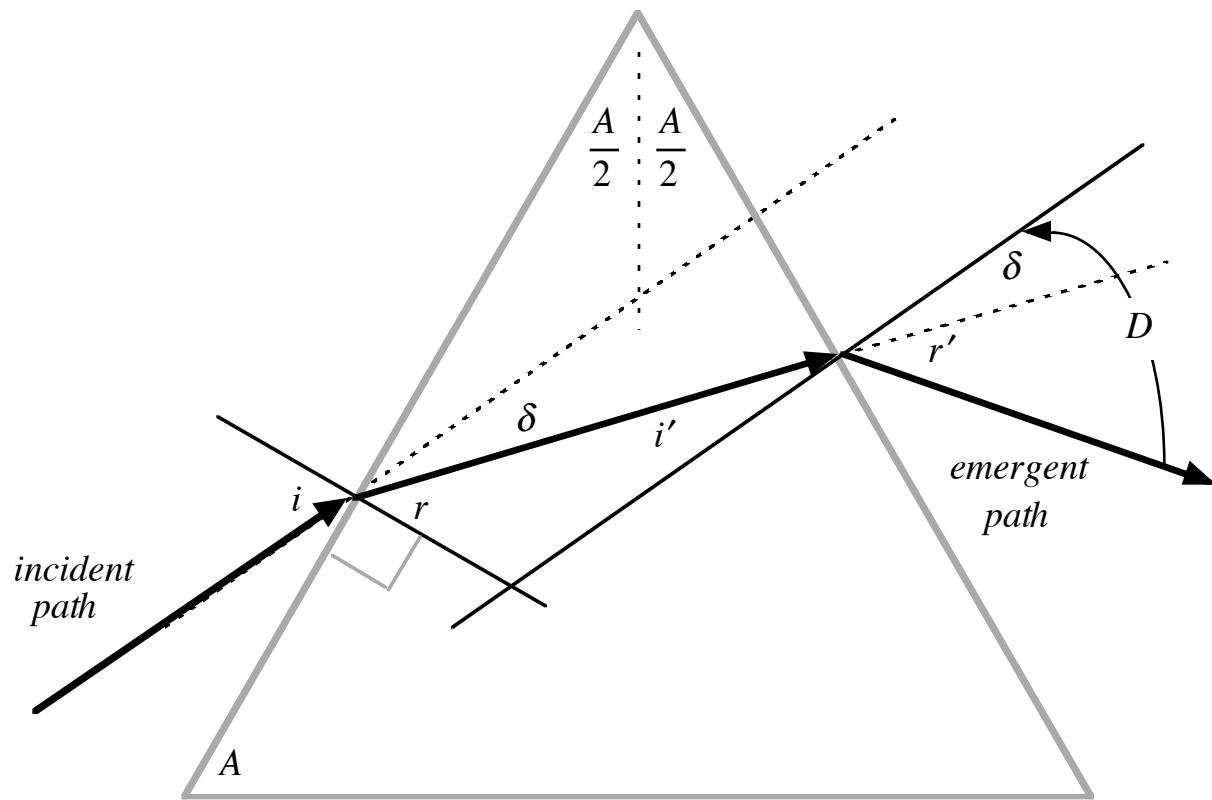
while

$$\alpha = r' - i' . \quad (12)$$

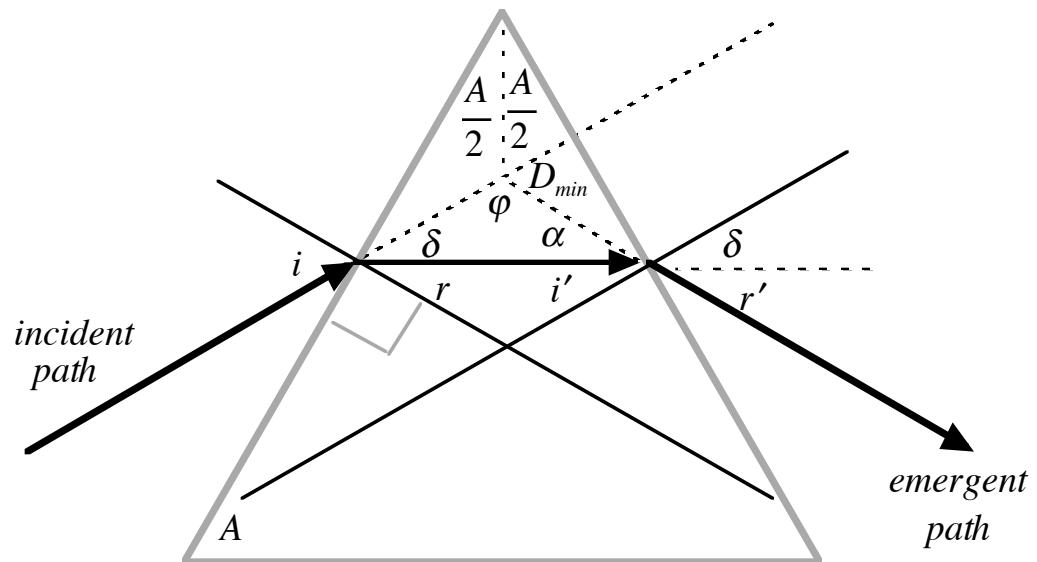
So,

$$D_{\min} = (i - r) + (r' - i') = 2(i - r) = 2i - A , \quad (13)$$

**Figure Five**  
**A Closer Look at Incident Light on a Glass Prism**  
**The General Case**



*At Minimum Deviation*



where we have used the fact that

$$r = \frac{A}{2} . \quad (14)$$

Finally, we can solve for  $i$ . We get

$$i = \frac{1}{2}(D_{min} + A) . \quad (15)$$

From Snell's law, we can write that the index of refraction of the prism is

$$n_{prism} = \left[ \frac{\sin i}{\sin r} \right] n_{air} = \left\{ \frac{\sin \left[ \frac{1}{2}(D_{min} + A) \right]}{\sin \left[ \frac{A}{2} \right]} \right\} n_{air} . \quad (16)$$

As the triangular faces of the prism are equilateral,  $A = 60^\circ$ , equation (16) reduces to

$$n_{prism} = \left\{ 2 \sin \left[ \frac{1}{2}(D_{min} + 60^\circ) \right] \right\} n_{air} . \quad (17)$$

Waves propagate at speeds that are related to the frequency and the wavelength by

$$v_{propagation} = f \lambda . \quad (18)$$

So, for electromagnetic radiation propagating in vacuum, we have

$$c = f \lambda , \quad (19)$$

while the speed of light in a medium of refractive index  $n$  is given by

$$c_n = f \lambda_n . \quad (20)$$

For air, we have

$$c_{air} = f \lambda_{air} = c / n_{air} , \quad (21)$$

and

$$n_{air} = \left[ \frac{c}{f} \right] \left[ \frac{1}{\lambda_{air}} \right] . \quad (22)$$

Using a formally similar argument, for the prism, we can write

$$n_{prism} = \left[ \frac{c}{f} \right] \left[ \frac{1}{\lambda_{prism}} \right] . \quad (23)$$

From equations (23) and (22), we can write

$$n_{prism} = \left[ \frac{\lambda_{air}}{\lambda_{prism}} \right] n_{air} . \quad (24)$$

Comparison of equations (24) and (17) implies that

$$\frac{\lambda_{air}}{\lambda_{prism}} = 2 \sin \left[ \frac{1}{2}(D_{min} + 60^\circ) \right] , \quad (25)$$

and that

$$\lambda_{air} = \left\{ 2 \sin \left[ \frac{1}{2} (D_{min} + 60^\circ) \right] \right\} \lambda_{prism} . \quad (26)$$

For this experiment, we are going to use 1.0003 as the refractive index for air. So, we can now write the refractive index of the prism as a function of the minimum deviation for a specific wavelength of light as

$$\begin{aligned} n_\lambda &= \left\{ 2 \sin \left[ \frac{1}{2} (D_{min} + 60^\circ) \right] \right\} (1.0003) \\ &= (2.0006) \left\{ \sin \left[ \frac{1}{2} (D_{min} + 60^\circ) \right] \right\} . \end{aligned} \quad (27)$$

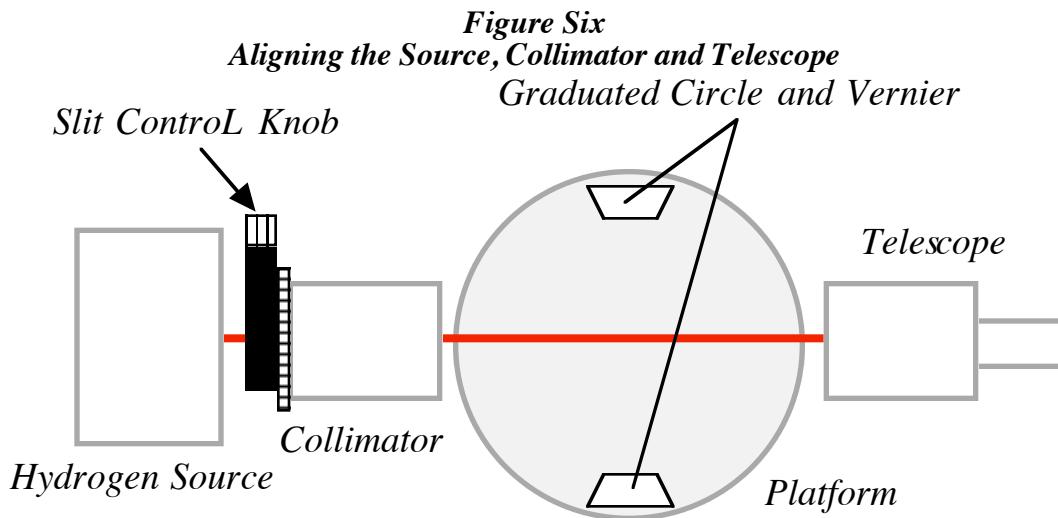
## EQUIPMENT NEEDED

One Prism Spectrometer  
One Discharge Tube of Hydrogen  
One Tensor Lamp for Light in the Dark

One 5 kV Power Supply  
Paper Towels or Cloth to Handle the Hot Tube

## PROCEDURE

Please do not handle the discharge tube with your bare hands. Always use a cloth or several paper towels. We want to avoid two things: 1) Burning your fingers. 2) Getting “stuff” on the tube. To increase the useful life of the tube, try not to leave it on for more than a couple of minutes at a time.



- 1.) With the power supply unplugged, using a cloth or paper towels as described above, place the hydrogen discharge tube into the five *kilovolt* power supply.
- 2.) Plug in the power supply and turn the switch on. The gas in the tube should glow a reddish hue if you have hydrogen and everything is in working order. If everything is working correctly turn off the power supply. If everything is not working properly, turn off the power supply and seek out your instructor.
- 3.) **Familiarize yourself with the prism spectrometer. (See Figure Eight below.) Note especially:**

**The Telescope Lock and the Telescope Fine Adjustment Knobs  
The Platform Lock and the Platform Fine Adjustment Knobs**

At one end of the collimator is a slit. The slit acts as an aperture through which the light will pass. Set the width of the slit so that it appears as a vertical line with a width that looks like the one represented below.



Unlock both the telescope and the platform table. When unlocked, they should be free to swing around. Now, lock the platform table. The telescope should still be free to swing around. Turn the platform fine adjustment knob and note that it rotates the platform through very small angles. Next, lock the telescope. You should be able to get the telescope to rotate through small angles by turning

the telescope fine adjustment knob. **You will need to know how to lock and unlock these components.**

4.) Move the tube very close to the collimator without the two touching. Center the tube in the slit. (See Figure Six above.)

5.) With the prism **off of the table**, **unlock** the telescope and turn it until it is aligned with the collimator. Look through the eyepiece and focus the eyepiece on the slit. Turn on the power supply and make sure you have a bright, clear, vertical line of red light in focus. **Lock the telescope** and use the **telescope fine adjustment** to center the cross hairs on the slit.

6.) Put the prism on the table and configure the system like the one represented in the Figure Seven below.

7.) Look into the prism directly--not through the telescope--and move your head until you see the "hydrogen lines." Once you know where they are, **unlock the telescope** and rotate it until it points at the lines.

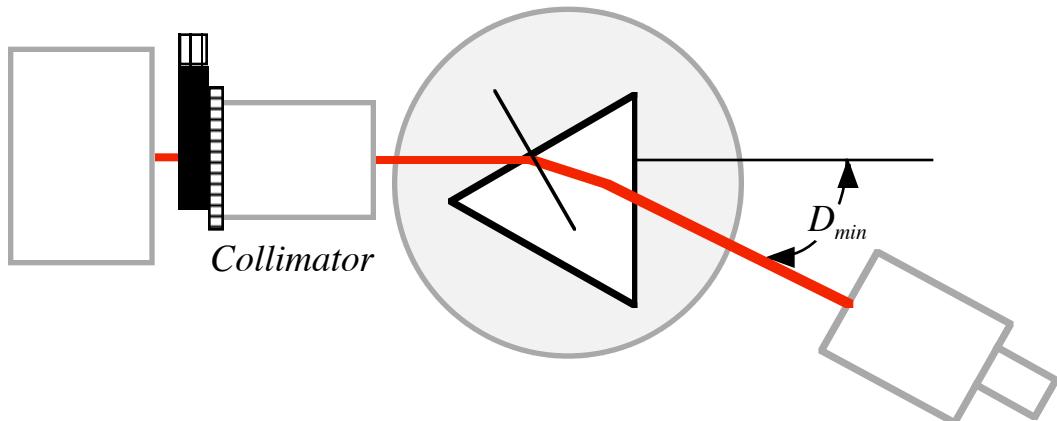
8.) Focus the cross-hairs on the center of the blue-green line and **lock the telescope**. Without moving the prism relative to the table on which it sits, very carefully use the platform fine adjustment to rotate the platform through small angles first one direction and then in the opposite direction. Eventually, you should find minimum deviation--the point at which a continued increase in fine adjustment causes the lines to appear to change their direction of motion in the field of view.

9.) Center the cross hairs on the red line and record  $\angle_{f,min}$  on the data sheet--the value signified by the graduated circle. Use the Vernier to estimate the nearest tenth of a degree.

10.) Measure  $\angle_{f,min}$  for each of the other lines of color. (It is possible that you will have only three lines that are bright enough to measure.)

11.) Rotate the telescope until it is aligned again at the center. Record this angle,  $\angle_o$ , on the data sheet and **turn off the power supply!**

*Figure Seven  
Orientation of the Prism*



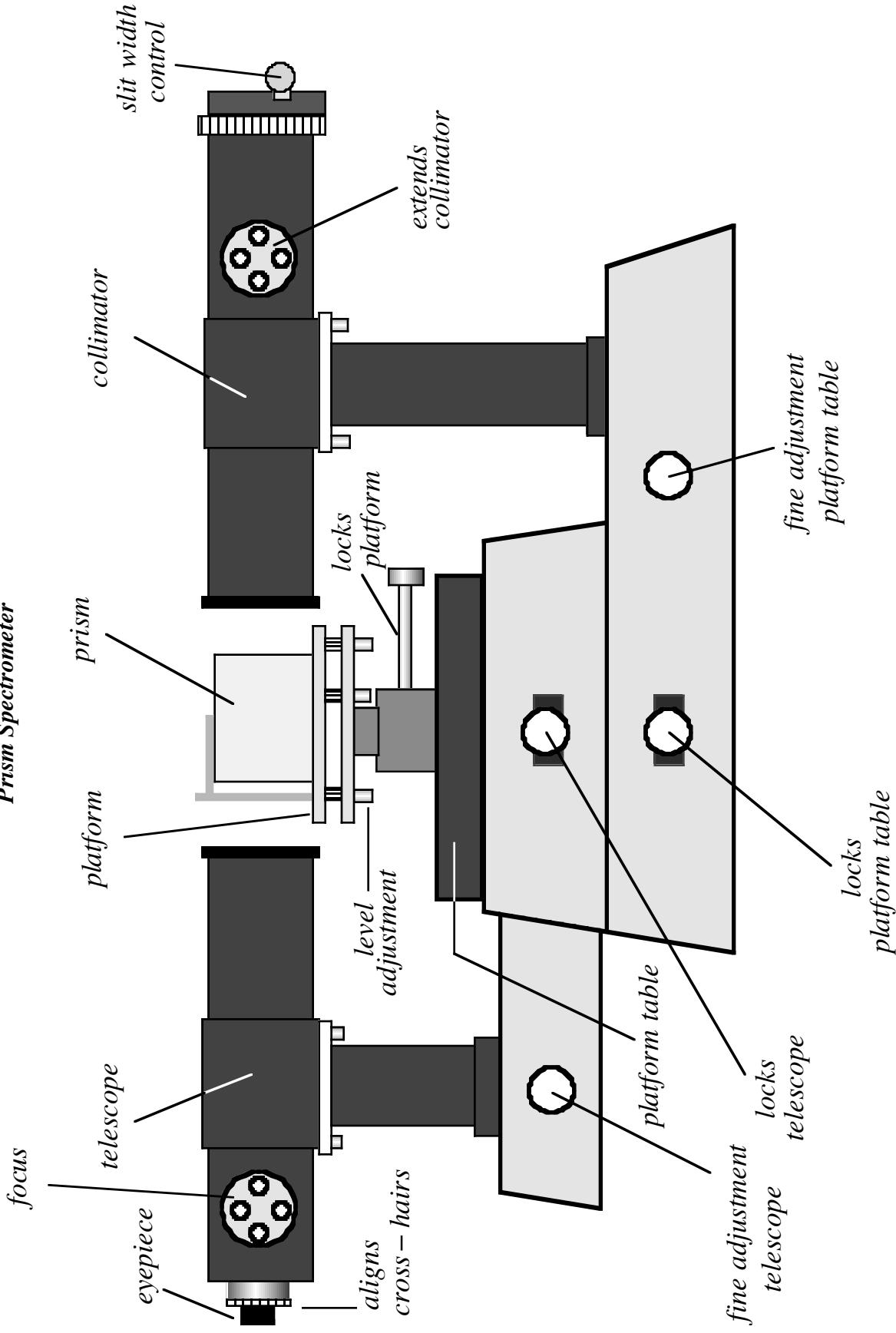
### THINGS TO DO

1.) Calculate  $D_{min} = |\angle_{f,min} - \angle_o|$  for each color and record this value on the data sheet.

2.) Calculate  $n_\lambda$ , the refractive index of glass prism for each color wavelength, and record this value on the data sheet.

3.) Plot your data points on the graph paper. Each point has coordinates  $P(\lambda, n_\lambda)$ .

*Figure Eight*  
Prism Spectrometer



***PHY2054L LABORATORY***

***Experiment Ten***

***Measuring the Refractive Index***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

Recall:

$$n_\lambda = (2.0006) \left\{ \sin \left[ \frac{1}{2} (D_{min} + 60^\circ) \right] \right\}. \quad (27)$$

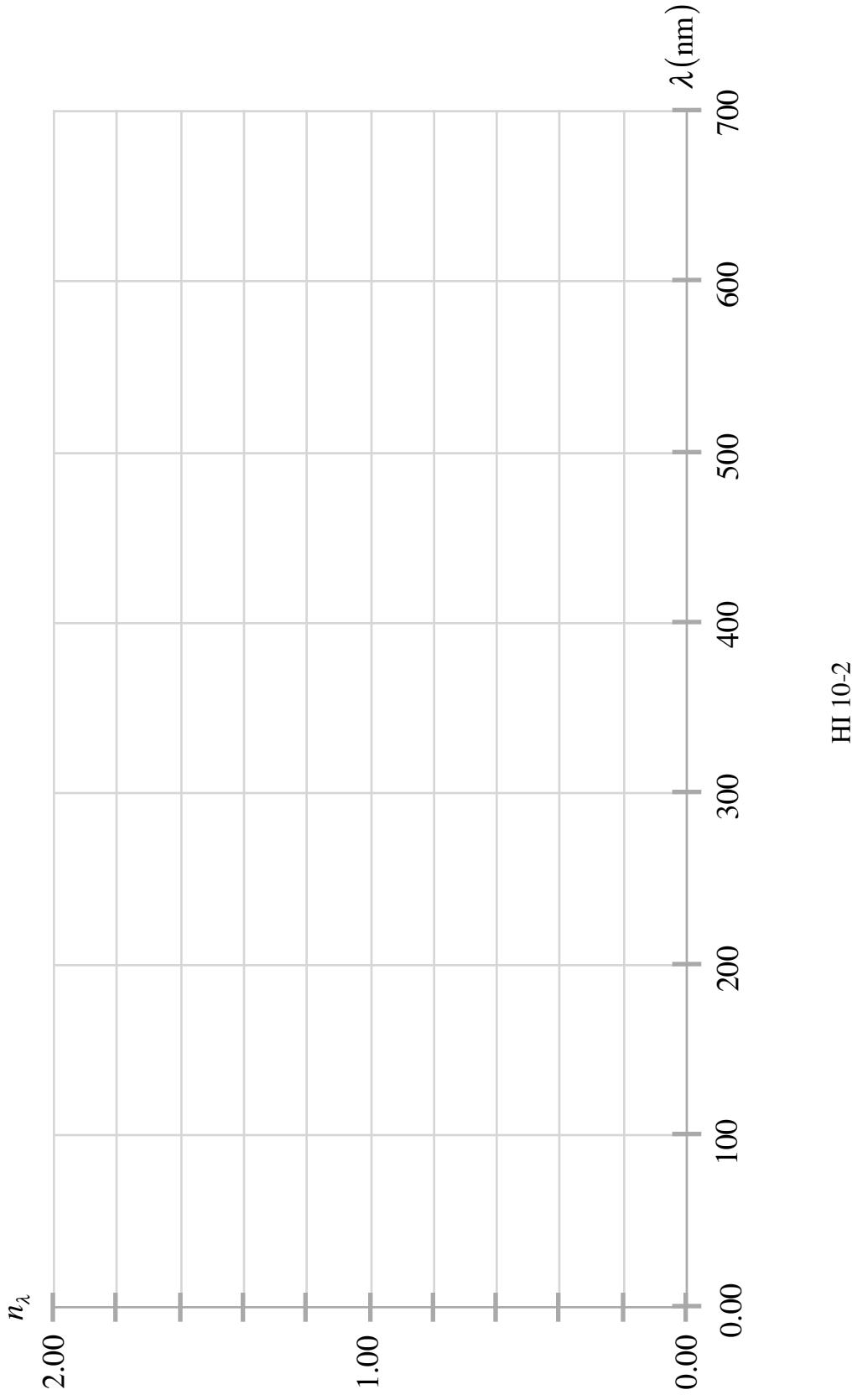
Color	$\lambda$ (nm)	$\angle_{f,min}$	$\angle_o$	$D_{min}$	$n_\lambda$
Red	656.5				
Blue-Green	486.3				
Blue	434.2				
Violet	410.3				

The manufacturer states that the refractive index of the prism is

$$n = 1.52,$$

but did not specify at what wavelength.

### The Refractive Index as a Function of Wavelength





# ***PHY2054L LABORATORY***

## *Experiment Eleven*

### ***Measuring the Wavelength of Light***

## THEORY

### *The Bohr Model of the Atom*

At the beginning of the twentieth century, the physical evidence for the existence of the atom was formidable. One of the major tasks confronting physicists and chemists was the construction of a viable model of the structure of the atom. Monatomic hydrogen is the simplest system with which to begin. We assume a solitary electron of mass  $M_{e^-}$  orbiting a fixed proton of mass  $M_p$  on a circular path of radius  $R_n$ , as represented in Figure One below.

Initially, the idea was that the electron was held in its orbit by the electromagnetic force exerted on it by the proton; analogous to the planets orbiting the Sun under the influence of the Sun's gravitational force. Assuming a circular path and using a classical mechanical analysis, we can write for the force

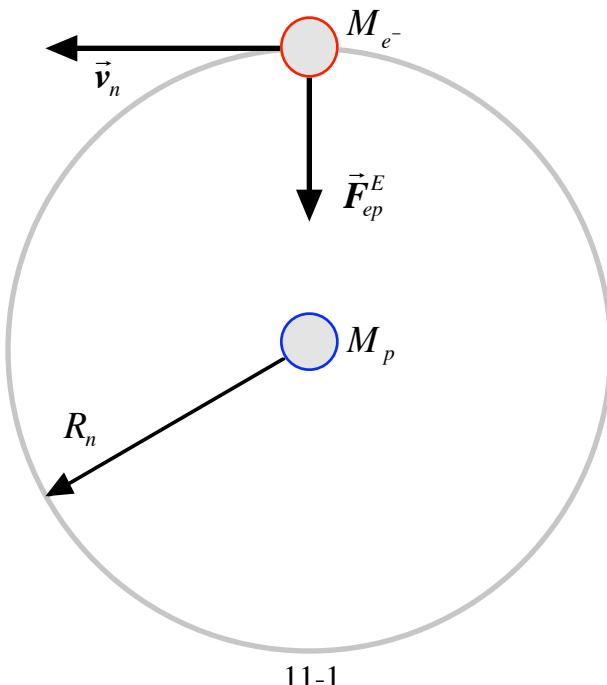
$$F_{ep}^E = \frac{ke^2}{R_n^2} = \frac{M_{e^-}v_n^2}{R_n}, \quad (1)$$

and for the angular momentum

$$L_n = R_n M_{e^-} v_n. \quad (2)$$

The problem with the "miniature solar system" model was that there was abundant experimental evidence that when electric charges are accelerated, they give off energy in the form light, electromagnetic waves. The kinetic energy is the only source for that energy. So, over a very short period of time, the orbit of the electron should decay. How, then, could such a process give rise to a stable atomic structure? Another problem was related to the nature of the spectrum. When a glass tube filled with a dilute hydrogen gas was subjected to a large electrical potential difference, the spectrum generated was not continuous, but consisted of a finite set of lines of different colors

**Figure One**  
**Monatomic Hydrogen**



that is today called a **line spectrum**. The wavelengths of the spectral lines were measured, and Johann Balmer discovered a formula for calculating the wavelengths of the visible line spectrum of hydrogen. The model of the hydrogen atom needs also to be able to account for the spectral lines.

Bohr attempted to deal with these problems by making the following assumptions:

- 1) The electron can orbit the proton in certain “allowable” orbits only.
- 2) When in one of these allowed orbits, the electron, even though accelerating, **does not** give off energy in the form of light.
- 3) An electron only gives off light when it spontaneously transitions from an orbit that is further from the proton to one closer to the proton.
- 4) The angular momentum also has certain allowed values given by

$$L_n = n \left[ \frac{h}{2\pi} \right] = n\hbar , \quad (3)$$

where  $n$  is an integer value  $n = 1, 2, 3, 4, \dots$ , and  $\hbar$  is Planck’s constant.

The symbol  $\hbar$ , pronounced “h bar”, is given by

$$\hbar = h / 2\pi . \quad (4)$$

Now, if we look at the mechanical energy of the electron in the  $n^{th}$  orbit, we can write

$$E_n = K + U^E = \frac{1}{2} M_{e^-} v_n^2 + \left[ \frac{k(-e)e}{R_n} \right] = \frac{1}{2} M_{e^-} v_n^2 - \frac{ke^2}{R_n} . \quad (5)$$

Using equation (1), we can write

$$\frac{1}{2} R_n \left( \frac{ke^2}{R_n^2} \right) = \frac{1}{2} R_n \left( \frac{M_{e^-} v_n^2}{R_n} \right) = \frac{ke^2}{2R_n} = \frac{1}{2} M_{e^-} v_n^2 . \quad (6)$$

Substitution of equation (6) into equation (5) gives us

$$E_n = \frac{ke^2}{2R_n} - \frac{ke^2}{R_n} = -\frac{ke^2}{2R_n} . \quad (7)$$

Next, if we use equations (2) and (3), we can write

$$n\hbar = R_n M_{e^-} v_n , \quad (8)$$

and

$$v_n = \frac{n\hbar}{R_n M_{e^-}} , \quad (9)$$

and

$$v_n^2 = \frac{n^2 \hbar^2}{R_n^2 M_{e^-}^2} . \quad (10)$$

If we substitute equation (10) into equation (1), we find

$$\frac{ke^2}{R_n^2} = \frac{M_{e^-}}{R_n} \left[ \frac{n^2 \hbar^2}{R_n^2 M_{e^-}^2} \right] = \frac{n^2 \hbar^2}{R_n^3 M_{e^-}} . \quad (11)$$

We can solve equation (11) for the radius of the  $n^{th}$  orbit. We find

$$R_n = \frac{n^2 \hbar^2}{ke^2 M_{e^-}} = n^2 \left[ \frac{\hbar^2}{ke^2 M_{e^-}} \right]. \quad (12)$$

The electron orbit closest to the proton is called the **ground state orbit**, and its radius is the **Bohr radius**, and it is given when  $n = 1$ . The Bohr radius is

$$\begin{aligned} R_{Bohr} &= \left[ \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s} / 2\pi)^2}{(8.99 \times 10^9 \text{ Nm}^2 / \text{kg}^2)(1.602 \times 10^{-19} \text{ C})^2 (9.109 \times 10^{-31} \text{ m})} \right] \\ &= 5.29 \times 10^{-11} \text{ m}. \end{aligned} \quad (13)$$

So, the radius of the  $n^{th}$  allowed orbit of the electron in monatomic hydrogen, according to the Bohr model, is given by

$$R_n = n^2 R_{Bohr}. \quad (14)$$

We can use equation (14) in equation (7) to write

$$E_n = -\frac{ke^2}{2[n^2 R_{Bohr}]} = -\frac{1}{n^2} \left[ \frac{ke^2}{2R_{Bohr}} \right]. \quad (15)$$

If we set  $n = 1$ , we can calculate the **ground state energy** of the electron. We find

$$\begin{aligned} E_1 &= -\left[ \frac{(8.99 \times 10^9 \text{ Nm}^2 / \text{kg}^2)(1.602 \times 10^{-19} \text{ C})^2}{2(5.29 \times 10^{-11} \text{ m})} \right] \\ &= -2.18 \times 10^{-18} \text{ J} \equiv -13.6 \text{ eV}, \end{aligned} \quad (16)$$

where  $\text{eV}$  is a common unit for orbital energies called an *electron-volt*. In general, then, the energy of the electron in the  $n^{th}$  orbit of monatomic hydrogen according to the Bohr model is

$$E_n = \frac{1}{n^2} E_1. \quad (17)$$

One of Einstein's major contributions to modern physics was his assertion that electromagnetic radiation, light, is a particle--we now call a photon. Each and every photon carries an energy given by

$$E_\gamma = hf, \quad (18)$$

where  $f$  is the frequency of the photon, and we use  $\gamma$  as a subscript to signify a photon. The energy of a wave always propagates at a speed given by the product of the frequency and the wavelength. So, for a photon, we have

$$f\lambda = c, \quad (19)$$

where  $\lambda$  signifies the wavelength and  $c$  signifies the speed of light in vacuum. So, we can use equations (15) and (16) to express the energy of a photon in terms of the wavelength. We have

$$E_\gamma = hf = h \left[ \frac{c}{\lambda} \right] = \frac{hc}{\lambda}. \quad (20)$$

Bohr postulated that the photon is emitted when an electron spontaneously transitions from an orbit further from the proton to an orbit closer to the proton. (Electrons transition to orbits further from the proton only when they absorb energy.) The conservation of energy implies that the

energy of the initial orbital state must equal the sum of the energy of the later orbital state plus the energy of the photon. So, we can write

$$E_{n_o} = E_{n_L} + E_\gamma , \quad (21)$$

and

$$E_\gamma = E_{n_o} - E_{n_L} = \left[ \frac{1}{n_o^2} E_1 \right] - \left[ \frac{1}{n_L^2} E_1 \right] = \left[ \frac{1}{n_o^2} - \frac{1}{n_L^2} \right] E_1 = \frac{hc}{\lambda} . \quad (22)$$

Also, the wavelength of the photon emitted in the transition of an electron from an orbit  $n_o$  to an orbit  $n_L$  is given by

$$\begin{aligned} \lambda &= \frac{hc}{\left[ \frac{1}{n_o^2} - \frac{1}{n_L^2} \right] E_1} = \frac{hc}{\left[ \frac{n_L^2 - n_o^2}{n_o^2 n_L^2} \right] E_1} = \left[ \frac{n_L^2 n_o^2}{n_L^2 - n_o^2} \right] \left[ \frac{hc}{E_1} \right] \\ &= \left[ \frac{n_L^2 n_o^2}{n_L^2 - n_o^2} \right] \left[ \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m} / \text{s})}{(-2.18 \times 10^{-18} \text{ J})} \right] \\ &= - \left[ \frac{n_L^2 n_o^2}{n_L^2 - n_o^2} \right] \left[ 9.112 \times 10^{-8} \text{ m} \right] = \left[ \frac{n_L^2 n_o^2}{n_o^2 - n_L^2} \right] (91.12 \text{ nm}) . \end{aligned} \quad (23)$$

### *The Balmer Series*

Four of the electron transitions which produce the visible spectrum of monatomic hydrogen terminate at orbit  $n = 2$ . For the transition from  $n = 6$  to  $n = 2$ , we get a “violet” photon of wavelength

$$\lambda_{6 \rightarrow 2} = \left[ \frac{(2)^2 (6)^2}{(6)^2 - (2)^2} \right] [91.12 \text{ nm}] = 410 \text{ nm} . \quad (24)$$

For the transition from  $n = 5$  to  $n = 2$ , we get a “blue” photon of wavelength

$$\lambda_{5 \rightarrow 2} = \left[ \frac{(2)^2 (5)^2}{(5)^2 - (2)^2} \right] [91.12 \text{ nm}] = 434 \text{ nm} . \quad (25)$$

For the transition from  $n = 4$  to  $n = 2$ , we get a “blue-green” photon of wavelength

$$\lambda_{4 \rightarrow 2} = \left[ \frac{(2)^2 (4)^2}{(4)^2 - (2)^2} \right] [91.12 \text{ nm}] = 486 \text{ nm} . \quad (26)$$

For the transition from  $n = 3$  to  $n = 2$ , we get a “red” photon of wavelength

$$\lambda_{3 \rightarrow 2} = \left[ \frac{(2)^2 (3)^2}{(3)^2 - (2)^2} \right] [91.12 \text{ nm}] = 656 \text{ nm} . \quad (27)$$

The predictions made by the Bohr model are in very good agreement with the values found experimentally. Even better agreement is obtained if we model the orbits as elliptical, as was first

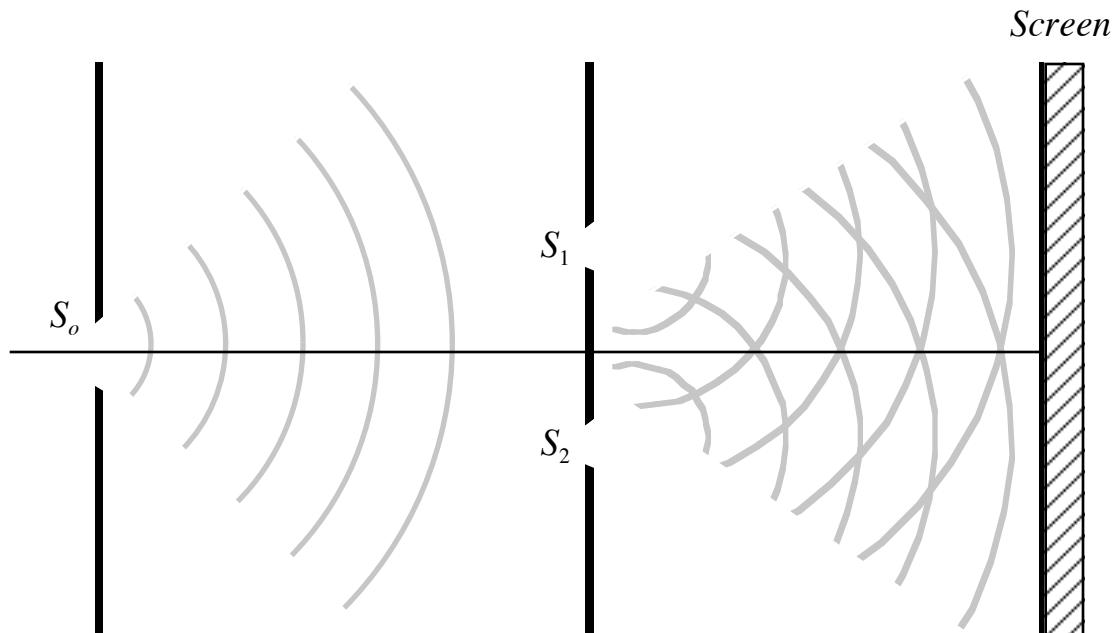
done by Arnold Sommerfeld.

### ***Diffraction***

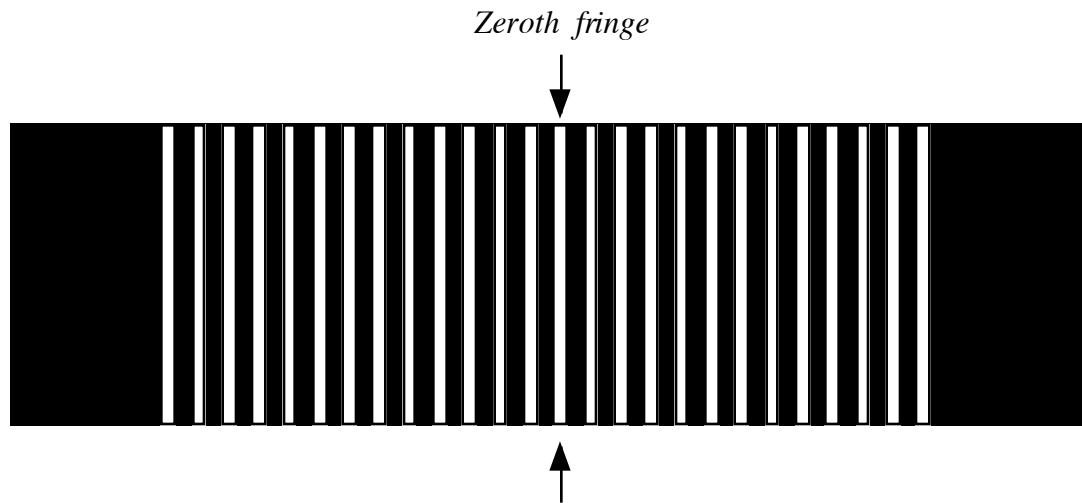
In the beginning of the nineteenth century, Thomas Young, an English scientist, performed a crucial experiment in which he demonstrated that light interferes. This provided more evidence that light really has a wave nature. In Figure Two below, a single slit  $S_o$  lets light of wavelength  $\lambda$  pass through. The light then passes through two more slits,  $S_1$  and  $S_2$ , before striking a screen. The interfering waves produce what we call interference fringes, as represented below in Figure Three. In Figure Four, represented is the geometry used by Young to analyze the interference pattern.

Young reasoned that wavelets leave  $S_1$  and  $S_2$  at the same time. If the waves arrive in phase at some point  $P$  on the screen, then there will be constructive interference and a bright fringe will be produced. If the wavelets arrive out of phase, they will destructively interfere and produce a dark fringe.

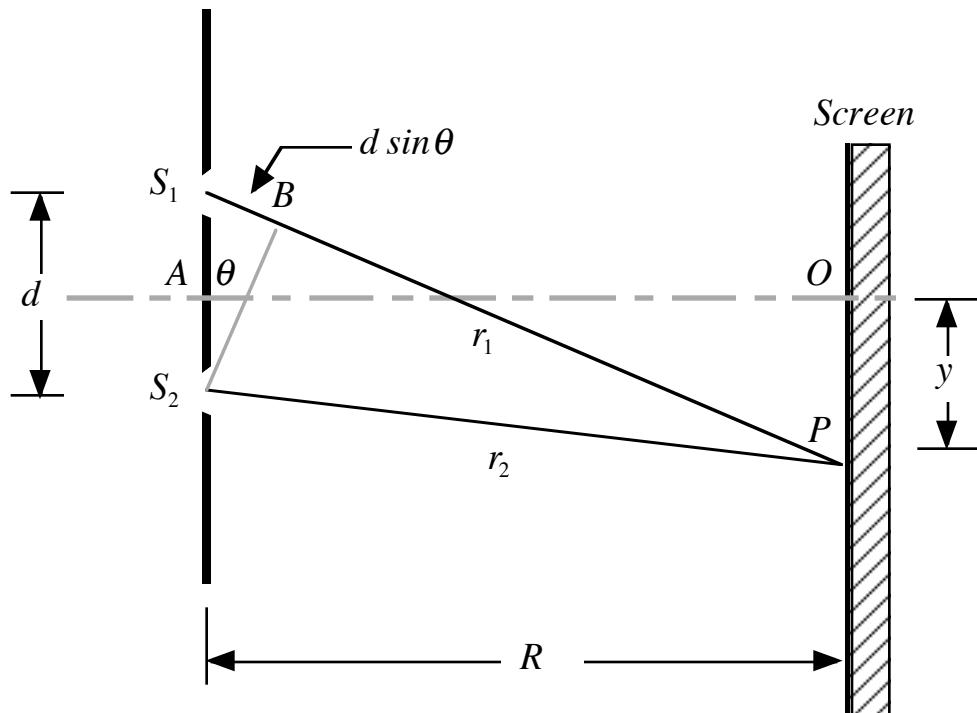
***Figure Two***  
***Interference of Light Waves***



*Figure Three*  
*Representation of the Interference Fringes*  
*Produced by Young's Double-slit Interferometer*



*Figure Four*  
*Young's Analysis*



Imagine a circle centered on point  $P$  a radius of which would be the line segment connecting point  $P$  and  $S_2$ . Also, arc  $S_2B$  would lie on the circle. Now, if the perpendicular distance from the slits to the screen is very much greater than the distance between the slits,  $R \gg d$ , then points  $B$  and  $S_2$  are essentially the same distance from point  $P$ , and arc  $S_2B$  becomes line segment  $S_2B$  which is perpendicular to line segment  $S_1P$ . This means that in general,

$$|\Delta r| = |r_2 - r_1| = d \sin \theta . \quad (28)$$

Finally, Young argued that constructive interference, regions of bright fringes, must occur when the difference in path lengths is equal to some integer multiple of the wave length, that is, when

$$d \sin \theta_m = |\Delta r| = m \lambda , \quad (29)$$

where  $m$  is called the order of the fringe and can take on values

$$m = 0, 1, 2, 3, \dots . \quad (30)$$

When  $m = 0$ , then  $|\Delta r| = 0 = \theta_m$  and we have the so-called zeroth fringe, which, occurs at point  $O$  in the center of the screen. Please note that there are regions of constructive interference distributed symmetrically on either side of point  $O$ .

**One of the most important theoretical consequences of all of the experimental work done on light is that in some situations, light behaves as if it were a particle. However, in other situations it behaves as if it had wave properties. We call this the *wave-particle duality!***

### ***Diffraction Grating***

If we take a piece of glass and etch a large number  $N$  of closely spaced parallel lines, we will have, in essence, created  $N$  slits. Such an optical device is called a diffraction grating. Glass plate gratings are expensive so we will use gratings made with mylar. As the light passes on through the grating, as opposed to being reflected, it is called a transmission diffraction grating.

## EQUIPMENT NEEDED

One Spectrometer  
One Discharge Tube of Helium  
One High Voltage Power Supply

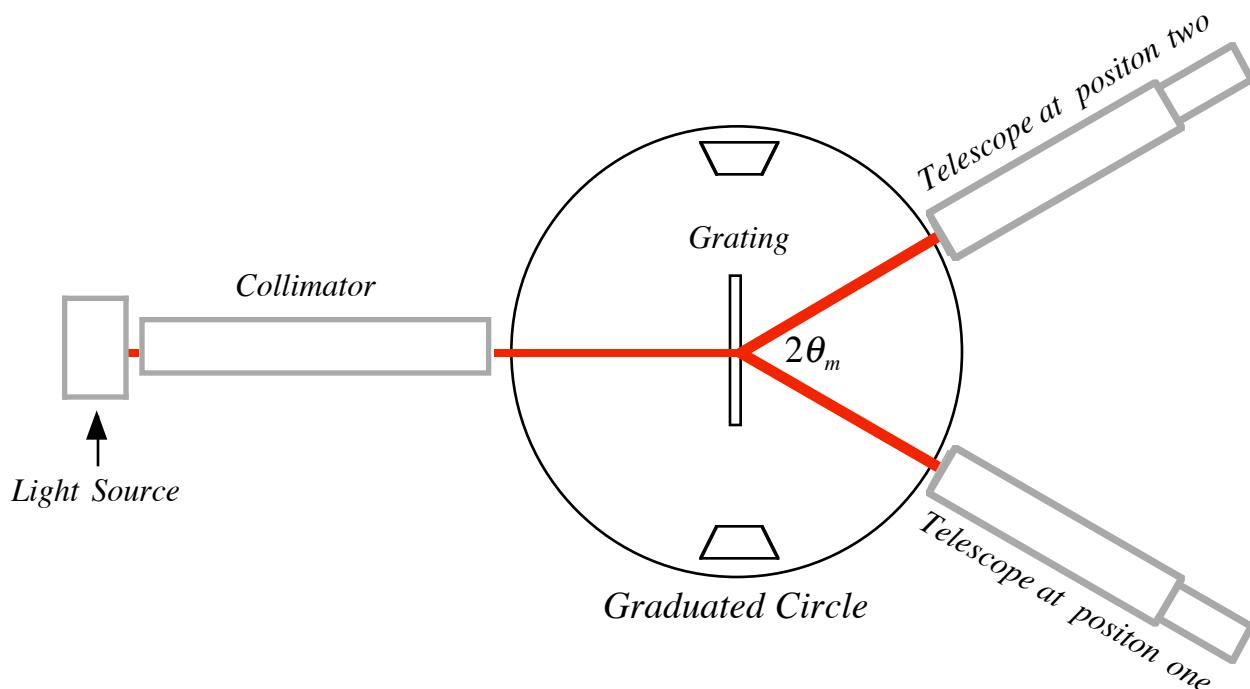
One Transmission Diffraction Grating and Holder  
One Discharge Tube of Mercury  
Paper Towels or Cloth to Handle Hot Tubes

## PROCEDURE

**Please do not handle the discharge tubes with your bare hands. Always use a cloth or several paper towels. We want to avoid two things: 1) Burning your fingers. 2) Getting “stuff” on the glass. To increase the useful life of the tube, try not to leave it on for more than 30 seconds at a time.**

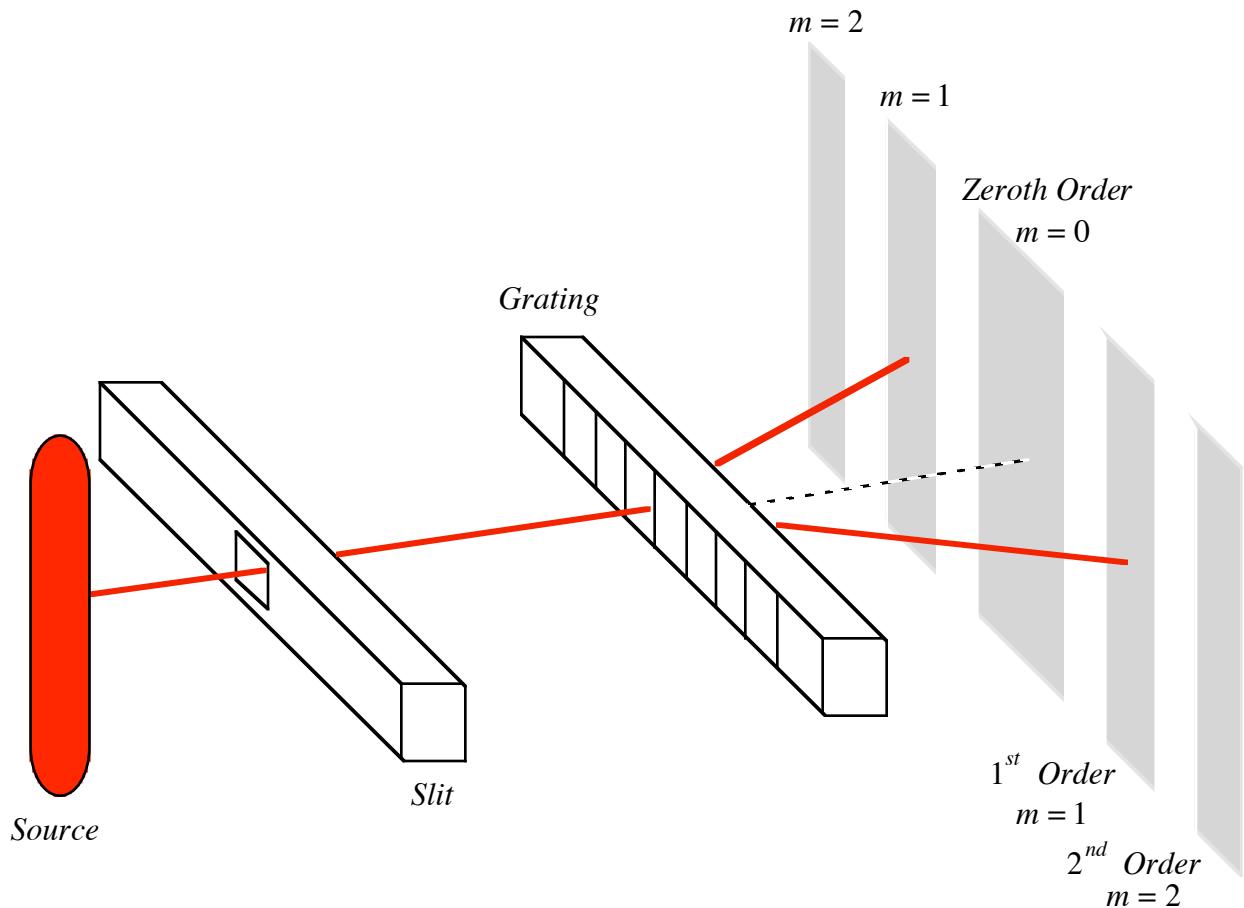
- 1.) Familiarize yourself with the spectrometer. (See Figure Five below.) There is a fixed cylinder that is called a collimator one end of which has a slit. The slit will act as our aperture and let light into the collimator. The width of the slit you can change by turning the knob at the end. Work the slit until it is vertical and open approximately 1/16 of an inch. There is a second cylinder that you can move around the perimeter of a graduated circle that is used to measure angles. This cylinder is the telescope. At the end furthest from the circular table is an eyepiece the focus of which you can change by turning the knob at the end.
- 2.) With the power unplugged and the on-off switch in the off position, put a helium discharge tube into power supply. Move the discharge tube right up next to the slit on the collimator.
- 3.) Plug in the power supply and turn on the power. The plasma should have a yellowish hue to it. Look at the discharge glow through the diffraction grating and orient the grating until the spectral lines appear vertical in the grating. Turn off the power. Fix the transmission diffraction grating into its holder and place it at the center of the circular table such that the rulings are parallel to the slit in the collimator and the plane of the grating is perpendicular to the axis of symmetry.

**Figure Five**



- 4.) Turn on the power. Swing the telescope around until you are looking directly into the slit. (A straight line running from the source through the collimator through the grating and into the telescope. You will then be looking at the light from the zeroth order.) Focus the telescope on the center of the slit. Now, move the telescope to left of center and find the first order spectral lines. Now, move the telescope to the right of center and find the first order spectral lines. (Figure Six below is a rough schematic representation of a first order bright red line of helium on both sides of the center line.)
- 5.) Now, center the cross hairs on the first bright purple line of the helium spectrum that is left of center. Record its angle measure on the data sheet below. Repeat this measuring process for the following lines: Blue, Faint Blue-Green, Green, Yellow, and Bright Red.
- 6.) Repeat the process described in 5.) for the same helium spectral lines that are right of center.
- 7.) Using your data, determine the wavelength of each line measured.
- 8.) Calculate the per cent error in your measurement and the accepted wavelength values.
- 9.) If time permits, repeat steps two through eight for the mercury discharge tube. (There are two yellow lines that are very close together; so do not make the slit too wide!)

**Figure Six**



***PHY2054L LABORATORY***

***Experiment Eleven***

***Measuring the Wavelength of Light***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

The diffraction grating you will use has 13,400 lines / inch , or 600 lines / millimeter . So, the distance between the lines would be given by

$$d = \frac{0.0254 \text{ m}}{13,400 - 1} = 1.896 \times 10^{-6} \text{ m ,}$$

or

$$d = \frac{0.001 \text{ m}}{600 - 1} = 1.670 \times 10^{-6} \text{ m .}$$

Recall, the wavelength for first order is given by

$$\lambda = d(\sin \theta) .$$

## HELIUM DISCHARGE TUBE

### *First Order Helium Spectrum*

Line Color	Left	Right	$2\theta$	$\theta$	$\lambda_m (\text{nm})$	% Error
<b>Bright Purple 447 nm</b>						
<b>Blue 471 nm</b>						
<b>Faint Green 492 nm</b>						
<b>Green 502 nm</b>						
<b>Yellow 588 nm</b>						
<b>Bright Red 668 nm</b>						

## Data Sheet

The diffraction grating you will use has 13,400 lines / inch , or 600 lines / millimeter . So, the distance between the lines would be given by

$$d = \frac{0.0254 \text{ m}}{13,400 - 1} = 1.896 \times 10^{-6} \text{ m ,}$$

or

$$d = \frac{0.001 \text{ m}}{600 - 1} = 1.670 \times 10^{-6} \text{ m .}$$

Recall, the wavelength for first order is given by

$$\lambda = d(\sin \theta) .$$

## MERCURY DISCHARGE TUBE

### *First Order Mercury Spectrum*

Line Color	Left	Right	$2\theta$	$\theta$	$\lambda_m$ (nm)	% Error
<b>Faint Purple 436 nm</b>						
<b>Green 546 nm</b>						
<b>1st Yellow 577 nm</b>						
<b>2nd Yellow 579 nm</b>						



# ***PHY2054L LABORATORY***

## *Experiment Twelve*

### ***The Photoelectric Effect***

## THEORY

Below, I have listed Maxwell's Equations in integral form. These equations assume the electric and magnetic fields propagate in vacuum. Although I do not expect you to completely understand the equations, they, along with the Lorentz force, represent the crowning achievement of classical electromagnetism.

### *Maxwell's Equations*

$$\oint \vec{E} \bullet d\vec{A} = \frac{1}{\epsilon_0} \int \rho_{charge} dV_{volume} , \quad (1)$$

$$\oint \vec{B} \bullet d\vec{A} = 0 , \quad (2)$$

$$\oint \vec{E} \bullet d\vec{l} = - \frac{d}{dt} \left[ \int \vec{B} \bullet d\vec{A} \right] , \quad (3)$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left\{ \int \vec{J} \bullet d\vec{A} + \epsilon_0 \frac{d}{dt} \left[ \int \vec{E} \bullet d\vec{A} \right] \right\} . \quad (4)$$

Equation three asserts that there is a close relationship between ***changing magnetic fields*** and electric fields. On the other hand, equation four asserts that there is also a close relationship between ***changing electric fields*** and magnetic fields. Also, the permittivity of free space,  $\epsilon_0$ , and the permeability of free space,  $\mu_0$ , are related to the speed of light in vacuum by:

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}} = c , \quad (5)$$

where  $c$  is the speed of light in vacuum.

It was the genius of James Clerk Maxwell to recognize that these phenomena indicate that light is an **electromagnetic wave**. A light wave propagates by the mutual interaction of its electric and magnetic fields. An extraordinarily bold idea, especially in light of the fact--pun intended--that light is itself **not electrically charged**--so why electric and magnetic fields?. Maxwell also predicted that electromagnetic waves--light--should be produced whenever electric charges are accelerated.

In 1888, Heinrich Hertz performed an historic experiment which demonstrated conclusively the correctness of Maxwell's prediction that accelerating electric charges produce **electromagnetic waves**. This work made Hertz famous worldwide. Unfortunately, his fame was short lived; he died of blood poisoning in 1894. However, a year earlier while working in his lab, Hertz observed that electrical discharges between electrodes took place at lower voltages if the electrodes were exposed to ultraviolet light. In related findings, investigators found that a freshly polished zinc plate, when negatively charged, would lose charge if exposed to ultraviolet light. A positively charged zinc plate, on the other hand, did not lose charge when exposed to ultraviolet light.

After he discovered the electron in 1897, J. J. Thomson was able to show that light striking a metallic surface could, under certain circumstances, cause the emission of electrons from the metal. This phenomenon became known as **the photoelectric effect**.

Three observations about the photoelectric effect could not be explained by the newly triumphant wave theory of light:

- 1.) Electrons are emitted **only** when the **frequency** of the light is above some threshold value, regardless of the **intensity** of the light.
- 2.) The maximum kinetic energy of the emitted electrons depends **only** on the frequency of the light.

3.) The electrons are emitted at the instant light of sufficient frequency strikes the metallic surface, there is no time delay.

Max Planck had already shown that the electromagnetic radiation emitted by a so-called **black body** could be explained by assuming that the individual atoms in the black body could oscillate **only** at certain discrete frequencies with possible energies directly proportional to the "allowed" discrete frequencies, where

$$E_{\text{oscillators}} = nhf , \quad (6)$$

where  $f$  is the frequency of oscillation,  $h$  is called Planck's constant, in honor of his important work in this matter, and where  $n$  can take on only certain discrete integer values,

$$n = 1, 2, 3, 4, \dots . \quad (7)$$

It was, however, Albert Einstein who boldly suggested that not only is the equilibrium exchange of energy in a black body quantized, but that electromagnetic radiation itself is composed of discrete particles--today called **photons**--that have energies proportional to their frequency

$$E_\gamma = hf . \quad (8)$$

Einstein used this to "explain" the photoelectric effect. Einstein's reasoning went something like this. Consider a smooth, flat metallic plate at some potential  $V_P$ . Surrounding the plate is a conducting collector at some potential  $V_C$ . The collecting cup has a small aperture through which light can pass. (See Figure One below.) Light of a specific frequency is allowed to fall on the plate. If this light is thought of as stream of individual photons of energy  $E_\gamma$ , then when a photon strikes the surface of the plate it will either be reflected or absorbed.

Photons which are absorbed will collide with, or **scatter** off of, valence electrons in the atoms near the surface. The photons will give up their energy to these electrons. Under normal circumstances, electrons are held in a metal by local electrostatic forces, and, as a result, have a kind of **binding energy**. However, if the electrons receive enough energy, that is if

$$E_\gamma > E_{\text{binding}} , \quad (9)$$

the electrons can escape from the metal with some kinetic energy  $E_K$ . The conservation of energy requires that

$$E_\gamma = E_K + E_{\text{binding}} . \quad (10)$$

Electrons closer to the surface of the plate have binding energies less than electrons in the bulk of the material. As a result, electrons which escape from a metal have a spectrum of kinetic energies. We can measure the **maximum** kinetic energy of the electrons by doing the following. If we make the potential of the collector negative with respect to the plate, that is if  $V_C < V_P$ , then the electrons will be repelled by the collector and only the most energetic electrons will reach the collector to produce a current in it. If we continue to increase the negative potential on the collector there will come a time when the potential difference between the plate and collector will preclude any electrons from reaching the collector and the current created by the escaping electrons will stop. We can measure this "stopping" potential difference and it is signified by  $V_0$ . This means that the maximum kinetic energy of the electrons is given by

$$E_{K,\max} = eV_0 . \quad (11)$$

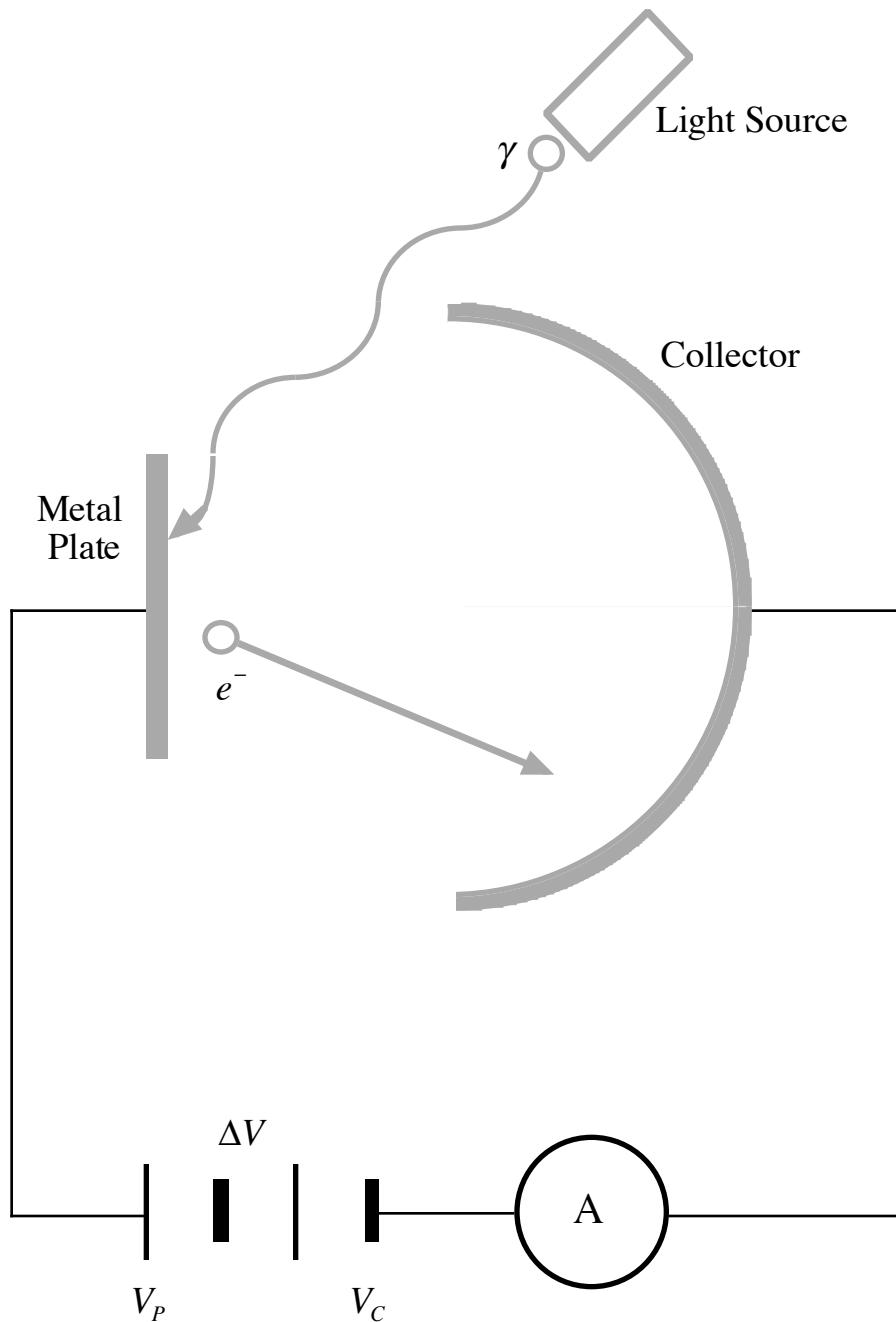
For all types of waves, the product of the frequency and the wavelength yields the speed at which the energy propagates. We have, for photons,

$$f\lambda = c \equiv \text{speed of light in vacuum} . \quad (12)$$

For photons in air, the speed is approximately that of vacuum and is also represented by  $c$ . Substituting equation (12) into equation (8) yields

$$E_\gamma = hf = \frac{hc}{\lambda} . \quad (13)$$

**Figure One**  
*A Photon Striking a Metal Plate Ejecting an Electron*



We are now in a position to write an energy equation that looks like

$$E_{K,max} = E_\gamma - E_{binding,surface} , \quad (14)$$

12-3

where we have made explicit the fact that the electrons with maximum kinetic energies will be those which have escaped from the surface of the plate and strike the collector. Substituting equations (11) and (13) into equation (14), gives us

$$eV_0 = \frac{hc}{\lambda} - \varphi , \quad (15)$$

where we have introduced  $\varphi$ , the so-called work function for the surface binding energy. The value of  $\varphi$  will depend on the type of metal being used in the plate. If we solve equation (15) for the stopping potential we get

$$V_0 = \left[ \frac{hc}{e} \right] \left[ \frac{1}{\lambda} \right] - \left[ \frac{\varphi}{e} \right] , \quad (16)$$

where we have written it in terms of the reciprocal of the wavelength, what is sometimes called **the wave number**.

In the *Festschrift* for Einstein's seventieth birthday, Robert A. Millikan revealed--with his usual candor--that he had spent ten years working experimentally on the photoelectric effect fully expecting to demonstrate Einstein wrong. Millikan believed that the overwhelming experimental evidence for the so-called wave nature of light meant that Einstein's model could not be correct. However, in 1916, he presented the results of his experiments that indeed confirmed the correctness of Einstein's analysis. Also, Millikan was able to determine a precise value for Planck's constant  $h$ .

It is worth noting that Einstein won his Nobel Prize for his work on the photoelectric effect and not for his special or general theories of relativity. Further, Millikan received a Nobel Prize for his experimental work on the photoelectric effect as well as that of measuring the charge on the electron. It is of some historical interest that Einstein's work on the photoelectric effect and the specific heats of metals proved very important in the early development of the most important physical theory of this century, the so-called quantum theory. In spite of his contribution to the development of the quantum theory, Einstein was never able to embrace the quantum theory. Einstein believed the philosophical consequences of the quantum theory were simply unacceptable.

## EQUIPMENT NEEDED

Incandescent Light Source  
Red, Green and Blue Filters  
One Fluke Multimeter  
Tensor Lamp

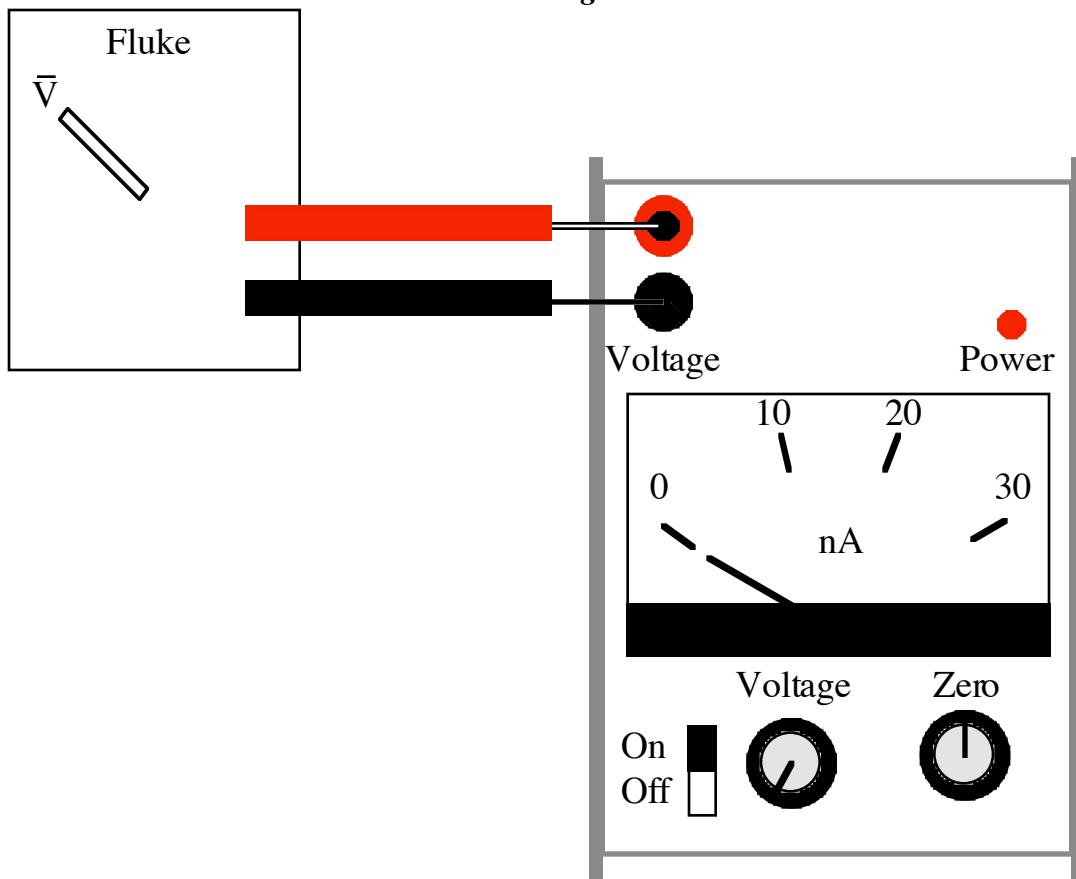
Mercury Light Source  
Photoelectric Effect Apparatus (**PEA**)  
Connecting Cables

## PROCEDURE

**Safety Notes:** 1) Before you plug in any light source, inspect the power cord and make sure there are no cracks or exposed wire that might lead to a short! If there are, let me know so that we can get a new source and I can discard the old one. 2) Set up the tensor lamp to work by. (We will soon turn off the overhead lights.)

### Zeroing the Photoelectric Effect Apparatus

*Figure Two*



1.) Turn on the Fluke Multimeter and set it to measure DC voltage. Put the red probe into the red port of the **PEA** and the black probe into the black port of the **PEA**, as represented in Figure Two above. Cover the aperture of the **PEA** with a piece of cardboard. Turn on the **PEA** and adjust the "voltage adjust" knob to a maximum setting--completely clockwise. Now, adjust the "zero adjust" knob until the ammeter reads zero. After you have zeroed the meter, be sure **not** to accidentally turn this knob thinking it is the "voltage adjust" knob.

2.) Once the PEA is zeroed, turn the "voltage adjust" knob completely in the counterclockwise direction; the voltmeter should now read zero. We are now ready to measure the

stopping potentials for various light sources. (It is okay to redo the zeroing process at any time.)

### Measuring the Stopping Potential for Specific Wavelengths

3.) Let us begin with the incandescent source. Remove the cardboard from the aperture and put in the **red** filter. Turn on the light source and position it until you get the highest reading possible on the ammeter. (It does not damage the meter if the needle goes off scale.) Try to make sure that only the light from your source is going in the **PEA**. (You may put a “hood” over the aperture to help reduce “stray” light.) Once the source is so positioned, **do not** move it until you have finished collecting data. Slowly increase the potential and note the corresponding decrease in the ammeter value. Determine the **minimum value of the potential** needed to stop the current. Record this minimum value as the stopping potential  $V_0$ .

- 6.) Repeat the above procedure with the incandescent source using the green and blue filters.
- 7.) Repeat the above procedure with the mercury source using the green and blue filters only.
- 8.) Turn everything off when you have finished.

### THINGS TO DO

- 9.) On graph paper, plot the stopping potentials  $V_0$  versus  $\frac{1}{\lambda}$ .
- 10.) From your graph, determine the slope and calculate an experimental value for  $h$ , and compare your value with the accepted value;  $h_{accepted} = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$ .
- 11.) From your graph, determine the work function  $\varphi$  for the metal in this particular photo diode.



***PHY2054L LABORATORY***

***Experiment Twelve***

***The Photoelectric Effect***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

<i>Source</i>	$\lambda$ (nm $\equiv 10^{-9}$ m)	$1/\lambda$ ( $10^6 \text{ m}^{-1}$ )	$V_0$ (V)
<i>Incandescent Red</i>	685		
<i>Mercury Green</i>	546		
<i>Incandescent Green</i>	540		
<i>Incandescent Blue</i>	490		
<i>Mercury Blue</i>	436		

Slope: \_\_\_\_\_

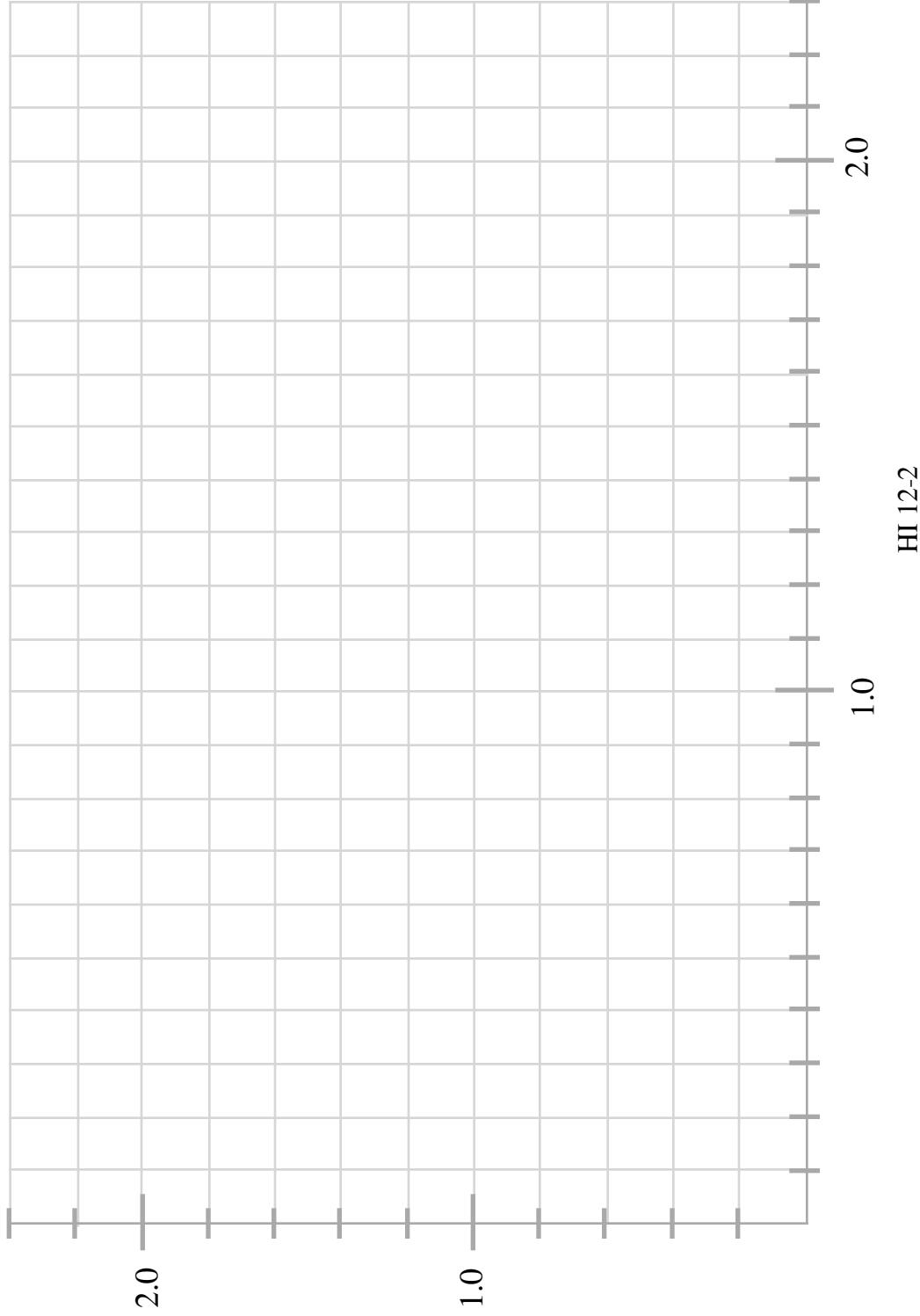
$h_{experimental} =$  \_\_\_\_\_

% Error: \_\_\_\_\_

$\varphi =$  \_\_\_\_\_

*The Stopping Potential as a Function of the Reciprocal of the Wavelength*

$V_o$  (V)





# ***PHY2054L LABORATORY***

*Make-Up Lab*

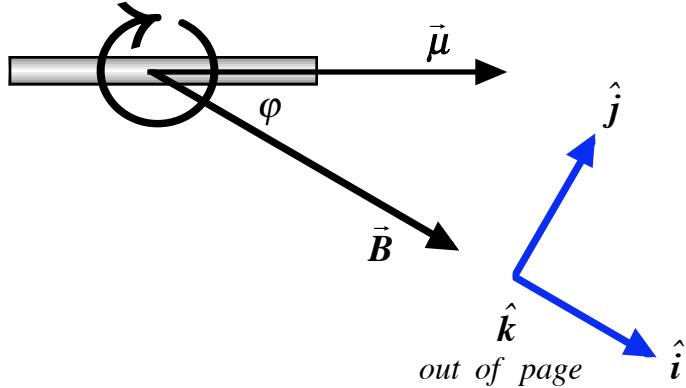
***Measuring the Earth's  
Magnetic Field***

## THEORY

If we place a bar magnet with magnetic moment  $\vec{\mu}$  in an external magnetic field  $\vec{B}$ , as represented below in Figure One, the magnet will experience a torque  $\vec{\Gamma}$  given by

$$\vec{\Gamma} = \vec{\mu} \times \vec{B} = -\mu B \sin \varphi \hat{k}. \quad (1)$$

*Figure One*



We also know that for systems where the moment of inertia is constant and the rotational axis is fixed in space, then

$$\Gamma = I\alpha = I \frac{d^2\varphi}{dt^2}. \quad (2)$$

Combining equations (1) and (2) gives us the differential equation

$$\frac{d^2\varphi}{dt^2} = -\frac{\mu B}{I} \sin \varphi. \quad (3)$$

The solution to differential equation (3) would require an infinite series. To avoid that complication, we will use the so-called small angle approximation, and assume that the angle  $\varphi$  is never very large. If so, then

$$\sin \varphi \approx \varphi, \quad (4)$$

while

$$\frac{d^2\varphi}{dt^2} = -\left[\frac{\mu B}{I}\right] \varphi, \quad (5)$$

and a solution of the form

$$\varphi = \varphi_o \cos(\omega' t), \quad (6)$$

where  $\omega'$  is a constant called the angular frequency and not to be confused with the angular speed  $\omega$ . The angular speed is given by

$$\omega = \frac{d\varphi}{dt} = -\omega' \varphi_o \sin(\omega' t), \quad (7)$$

while the angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = -\omega'^2 \varphi_o \cos(\omega't) . \quad (8)$$

Substitution of equations (6) and (8) into (5) yields

$$-\omega'^2 \varphi_o \cos(\omega't) = -\left[\frac{\mu B}{I}\right] \varphi_o \cos(\omega't) . \quad (9)$$

Of course, equation (9) is true if and only if

$$\omega'^2 = \left[\frac{\mu B}{I}\right] = \frac{4\pi^2}{\tau^2} , \quad (10)$$

where  $\tau$  is the period of the oscillation of the bar magnet in the external magnetic field. Now, note

$$\frac{1}{\tau^2} = \left[\frac{\mu}{4\pi^2 I}\right] B = CB , \quad (11)$$

where we have introduced the constant  $C$  and

$$C = \frac{\mu}{4\pi^2 I} . \quad (12)$$

(Please remember that here,  $I$  represents the moment of inertia of the bar magnet.) Since we do not know  $\mu$  or  $I$  we are going to need a way to determine the value of  $C$ .

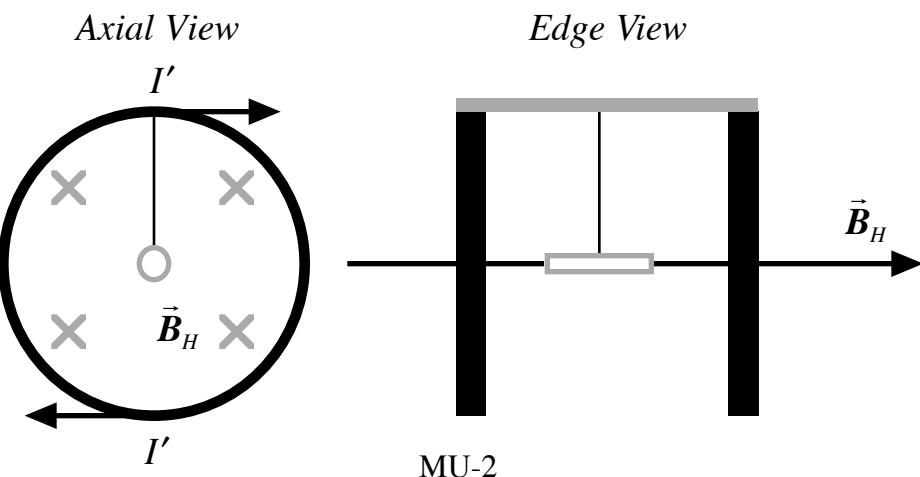
Recall that when we measured the ratio of the charge to the mass of the electron, we made use of Helmholtz coils. We are going to use them in this experiment to provide us with a known magnetic field. In Figure Two below, we illustrate how this can be done. We orient the Helmholtz coils and run the current in such a way as to produce a magnetic field directed north, in the same direction as the horizontal component of the Earth's magnetic field, the physical quantity we wish to measure. So, we can write

$$B_H = C_1 I' , \quad (13)$$

where  $C_1$  is a constant that depends on the specific Helmholtz coils used. For our set we have,

$$C_1 = 7.79 \times 10^{-4} \text{ T} \cdot \text{A}^{-1} . \quad (14)$$

**Figure Two**  
*Axial and Edge Views of the Helmholtz Coils*



The magnetic field in equation (11) is the net magnetic field and can be written as

$$B = B_H + B_{\oplus,h} = C_1 I' + B_{\oplus,h} . \quad (15)$$

Substitution of equation (15) into equation (11) allows us to write

$$\frac{1}{\tau^2} = CC_1 I' + CB_{\oplus,h} . \quad (16)$$

We can find  $C$  and  $B_{\oplus,h}$  by measuring the period of oscillation of the magnet for various values of the current  $I'$  in the Helmholtz coils. We graph this data with  $(1/\tau^2)$  versus  $I'$ . This will graph as a straight line the slope of which is  $CC_1$  and the vertical axis intercept will allow us to find  $B_{\oplus,h}$ .

## EQUIPMENT NEEDED

One Pasco Model TG-13 $e/M_e$ Apparatus	One Stopwatch
One Pasco Model SF-9584A Low Voltage Power Supply	
High Voltage Connecting Cables	One Small Bar Magnet

## PROCEDURE

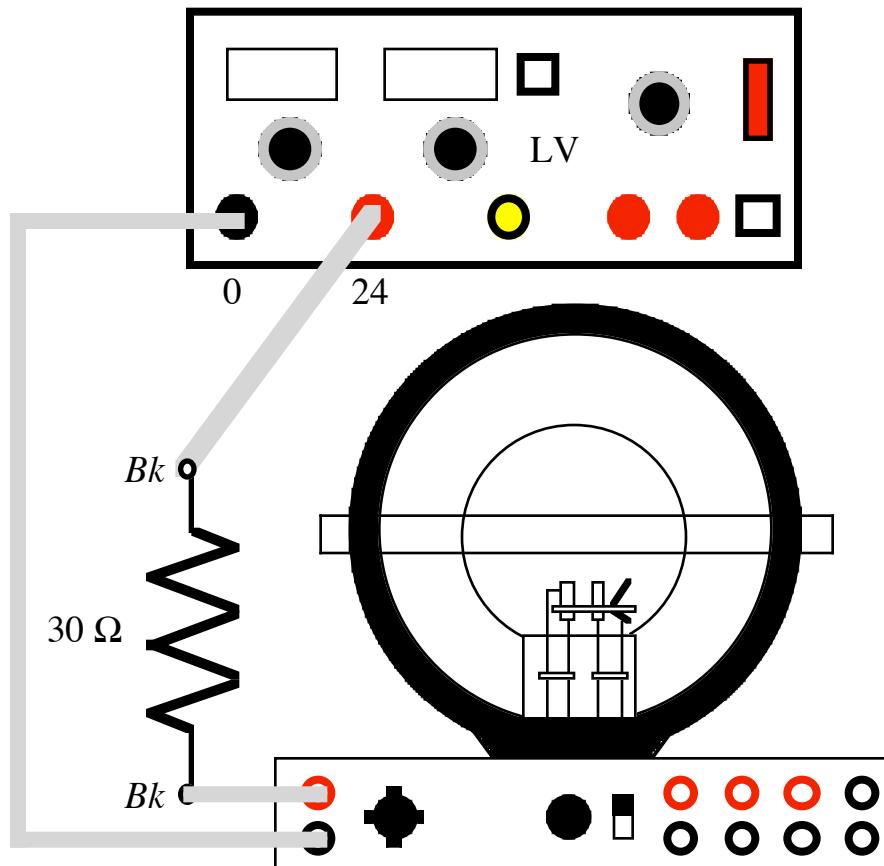
- 1.) With the power turned off, set up the configuration shown below in Figure Three. Make sure the axis of the Helmholtz coils is directed parallel to a north-south line.

### The Low Voltage Power Supply:

Turn the voltage and current knobs on the low voltage power supply till they point straight up. Connect a cable from the red 24 V DC port of the low voltage power supply to the red port of the  $e/m$  apparatus as shown. Connect another cable from the black 0 V DC port of the low voltage power supply to the black port of the  $e/m$  apparatus as shown. (This connection will provide the energy to move a current in the Helmholtz coils and, thereby, produce the magnetic field  $B_H$ .)

- 2.) Turn on the low voltage power supply. Set the DC voltage to **sixteen volts**. Turn the current knob on the  $e/m$  apparatus until the ammeter on the low voltage power supply indicates that you have 0.10 A moving through the coils.
- 3.) Measure the total time it takes the magnet to oscillate for **twenty** complete oscillations. That time divided by twenty will get the period of oscillation. Record the period on the data sheet.
- 4.) Repeat step 3.) for current values of:  
0.20 A , 0.30 A , 0.40 A , 0.50 A , 0.60 A , 0.70 A .

*Figure Three*



### THINGS TO DO

- 1.) Using equations (13) and (14), calculate the Helmholtz magnetic field for each current value and record this value on the data sheet.
- 2.) Square each of the period values measured record these values as well as the reciprocal of the period squared on the data sheet.
- 3.) On the graph paper provided, graph the seven points with coordinates  

$$P[(1/\tau^2), I'];$$
 the reciprocal of the period squared on the vertical axis and the current on the horizontal axis.
- 4.) Draw a best fit straight line through the plotted data points and then calculate the slope of that line and record this value on the data sheet.
- 5.) Use the slope to find the value of our missing constant  $C$  and record this value on the data sheet.
- 6.) Calculate from your graph a value for the intercept of the vertical axis. From this value, calculate the magnitude of the horizontal component of the Earth's magnetic field. Record this value on the data sheet.
- 7.) A reasonable estimate of the horizontal component of the Earth's magnetic field at this latitude is  $2.5 \times 10^{-5}$  T. How do your two estimates compare to this one? (I want something quantitative.)



***PHY2054L LABORATORY***

***Make-Up Lab***

***Measuring the Earth's Magnetic Field***

**Name:**

---

**Date:**

---

**Day and Time:**

---

## Data Sheet

$I'$	$B_H$ (T)	$\tau$ (s)	$\tau^2$ (s <sup>2</sup> )	$(1 / \tau^2)$ (s <sup>-2</sup> )
0.10 A				
0.20 A				
0.30 A				
0.40 A				
0.50 A				
0.60 A				
0.70 A				

Slope: \_\_\_\_\_

$C =$  \_\_\_\_\_

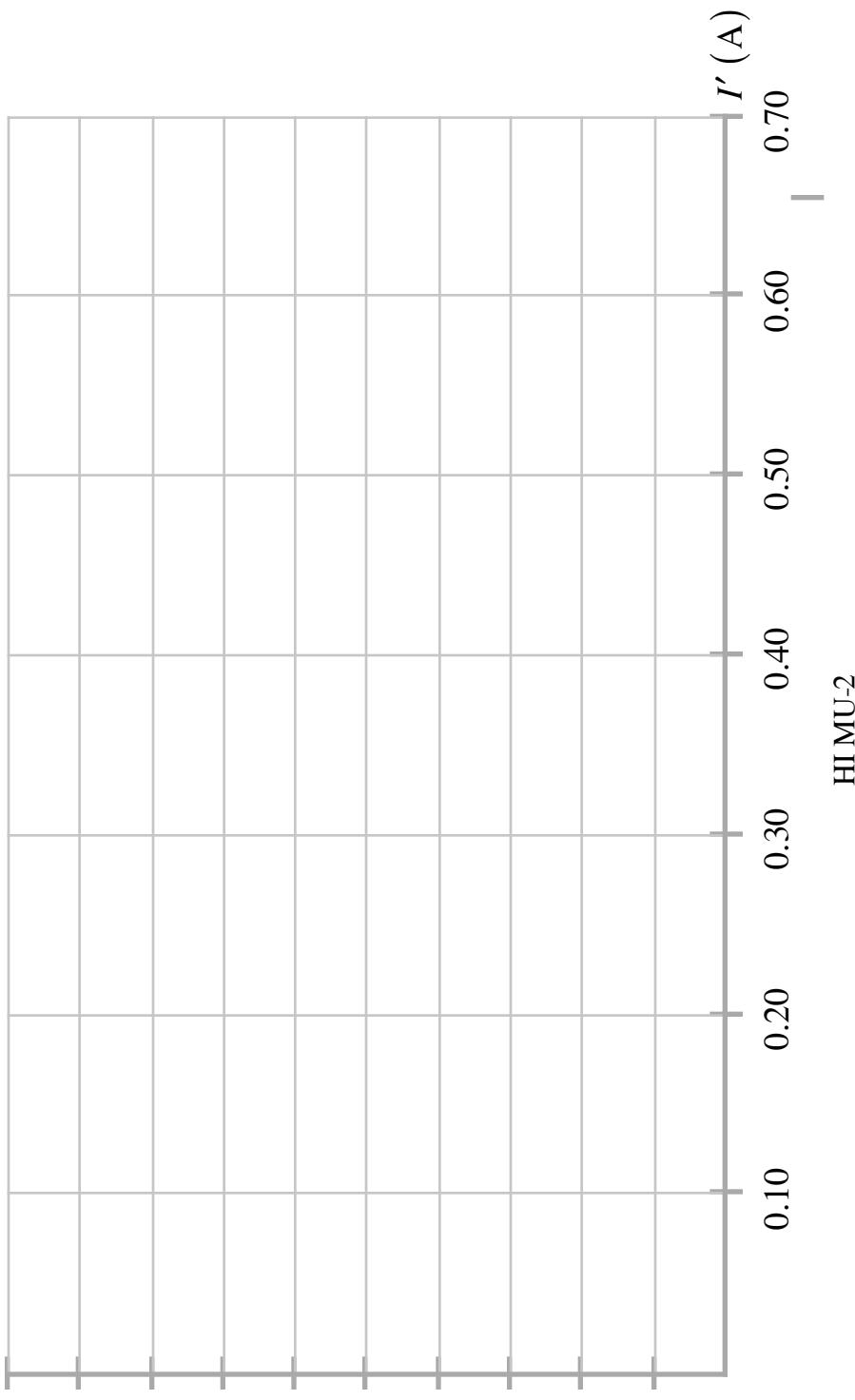
Vertical Intercept: \_\_\_\_\_

$B_{\oplus,h} =$  \_\_\_\_\_

Comparison of measured values and the stated estimate:

*The Reciprocal of the Period as a Function of the Current*

$$\frac{1}{\tau^2} \left( \text{s}^{-2} \right)$$





## *Appendices for PHY2054L*

## Graphs

You will on occasion be asked to graph your data. Graphs are an extremely important part of the scientific process. I want to give you some idea of what I expect to see on the graphs that you do for this lab.

First, I will ask you to create a graph of something **versus** something else. Whatever quantity appears first--the something--is to be graphed on the **vertical axis**, while the second quantity--the something else--is to be graphed on the horizontal axis. For example, if I were to ask you to graph the period squared **versus** mass, then on the vertical axis would be the period squared, while on the horizontal axis would be the mass.

I think maybe the best way for me to explain what it is that I want is to do an example. If one suspends a mass from the end of vertical spring, it will stretch the spring. If one were to pull this mass down a little bit further and then release the mass, it would oscillate up and down. There would be a pattern, however, to this motion, and it would take the same amount of time--each time--to go up and down. This constant time is called the period of the motion and I signify it with the lowercase Greek letter *tau*:  $\tau$ . Assume that in doing an experiment where we measure the period of an elastic spring stretched by a mass  $M$  we get the following data:

$M$ (kg)	$\tau$ (s)	$\tau^2$ (s <sup>2</sup> )
0.200	0.726	0.527
0.300	0.889	0.790
0.400	1.026	1.053
0.500	1.147	1.316
0.600	1.257	1.580

It is highly likely that I would ask you to graph the period squared **versus** the mass. I have done such a graph below. Note the salient features of this graph that you, of course, will incorporate into your graphs. First, there should be a title to the graph. Also, each axis should be labeled telling what is graphed along that axis and the units of the measured quantity--represented in the parentheses. The scale on each axis should also be clearly indicated.

Usually, the graphs will involve straight lines. As the data is not exact, the data points do not form an exactly straight line. You are to try and draw a straight line that "best fits" the data plotted. This is not an exact process, so do the best you can. A best fit will try to be as close to as many of the data points as possible.

The reason we are interested in straight lines is that we understand them well. For example, we know that a straight line has the mathematical form

$$y = mx + b , \quad (1)$$

where  $y$  is plotted on the vertical axis,  $x$  is plotted on the horizontal axis,  $m$  is the slope of the line, and  $b$  is the value at which the line crosses the  $y$ -axis, the so-called  $y$ -intercept. For the graph that I have shown below, note that the slope of this line can be found by using

$$m \equiv \frac{\text{change in vertical}}{\text{change in horizontal}} = \frac{\Delta(\tau^2)}{\Delta(M)} . \quad (2)$$

For this “massaged” data,

$$\Delta(\tau^2) = 1.580 \text{ s}^2 - .527 \text{ s}^2 = 1.053 \text{ s}^2, \quad (3)$$

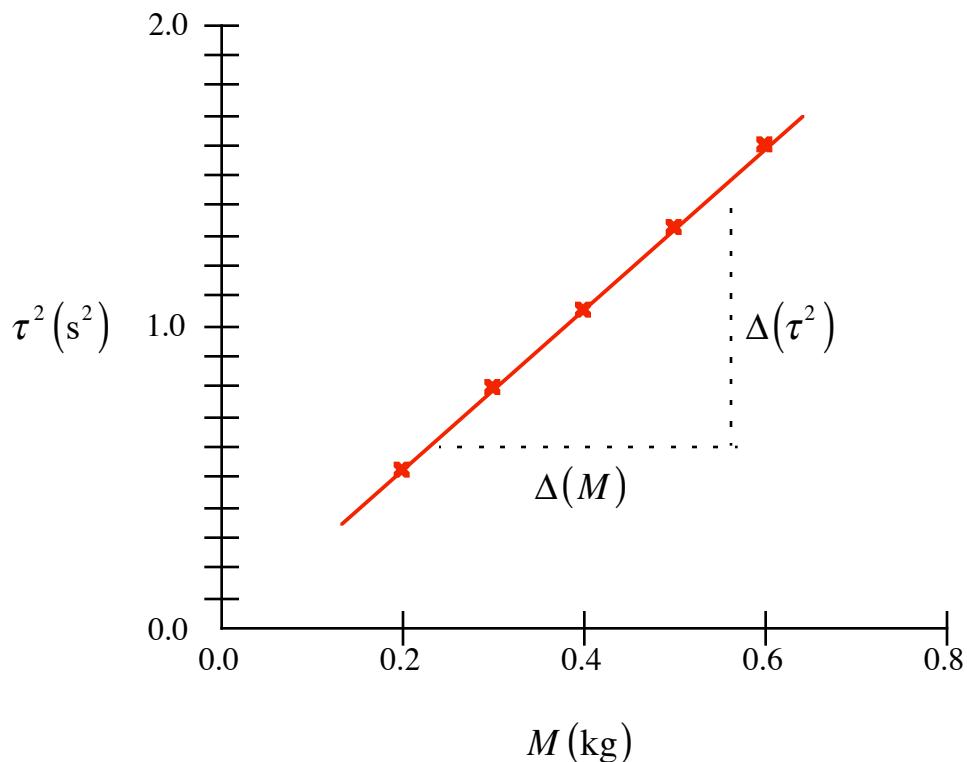
and

$$\Delta(M) = 0.6 \text{ kg} - 0.2 \text{ kg} = 0.4 \text{ kg}. \quad (4)$$

So, the slope would be given by

$$m = \frac{\Delta(\tau^2)}{\Delta(M)} = \frac{1.053 \text{ s}^2}{0.4 \text{ kg}} = 2.633 \frac{\text{s}^2}{\text{kg}}. \quad (5)$$

*The Square of the Period Versus the Mass  
For a Mass Oscillating on a Spring*



It turns out that the relationship between the period of a mass on a spring and the mass itself is given by

$$\tau = 2\pi \sqrt{M / k_{sp}}, \quad (6)$$

where  $k_{sp}$  is the so-called spring constant that tells us how stiff the spring is. Note, however that if we graphed the period versus the mass, we would not get a straight line. So, we graph the period squared versus the mass and do get a straight line.

$$\tau^2 = (4\pi^2 / k_{sp})M. \quad (7)$$

In this form, the  $\tau^2$  acts like the  $y$  value, while the  $M$  acts like the  $x$  value, with  $b = 0$ , and the slope is equal to

$$slope \equiv 4\pi^2 / k_{sp} . \quad (8)$$

However, as we saw with equation (2) we can also measure the slope directly off of the graph. So, we can use this to find the spring constant  $k_{sp}$ . We have

$$k_{sp} = \frac{4\pi^2}{slope} = \frac{4\pi^2}{2.633 \text{ s}^2 \cdot \text{kg}^{-1}} = 15 \text{ N} \cdot \text{m}^{-1} . \quad (9)$$

There are many values in physics that can be measured indirectly like this--using the slope of a graph of **other** directly measured values.

### **Method of Least Squares**

Although one can use a graph to determine the slope of a line, this method is only as good as the “eye” of the person constructing the “best fit” of the data. There is another, better way. It is called the method of least squares.

Recall that the slope-intercept form for the equation of a straight line is given by

$$y = mx + b . \quad (10)$$

Assume we have made  $N$  measurements of  $y$  and  $x$ . Then we will have  $N$  equations of the form

$$\begin{aligned} y_1 &= mx_1 + b \\ y_2 &= mx_2 + b \\ y_3 &= mx_3 + b \\ &\vdots \\ y_N &= mx_N + b \end{aligned} \quad (11)$$

Adding the equations listed in equation (11) gives us

$$\sum_{i=1}^N y_i = m \sum_{i=1}^N x_i + Nb , \quad (12)$$

Now, if we multiply each of the equations listed in (11) by its  $x$  value, we have

$$\begin{aligned} x_1 y_1 &= mx_1^2 + bx_1 \\ x_2 y_2 &= mx_2^2 + bx_2 \\ x_3 y_3 &= mx_3^2 + bx_3 \\ &\vdots \\ x_N y_N &= mx_N^2 + bx_N \end{aligned} \quad (13)$$

Adding the equations listed in (13) gives us

$$\sum_{i=1}^N x_i y_i = m \sum_{i=1}^N x_i^2 + b \sum_{i=1}^N x_i . \quad (14)$$

First, if we solve equation (12) for  $b$ . We have

$$b = \frac{1}{N} \left[ \sum_{i=1}^N y_i - m \sum_{i=1}^N x_i \right] . \quad (15)$$

Substitution of equation (15) into equation (14) yields

$$\begin{aligned}
\sum_{i=1}^N x_i y_i &= m \sum_{i=1}^N x_i^2 + \left\{ \frac{1}{N} \left[ \sum_{i=1}^N y_i - m \sum_{i=1}^N x_i \right] \right\} \sum_{i=1}^N x_i \\
&= m \sum_{i=1}^N x_i^2 + \frac{1}{N} \left\{ \sum_{i=1}^N y_i \sum_{i=1}^N x_i - m \sum_{i=1}^N x_i \sum_{i=1}^N x_i \right\} \\
&= m \sum_{i=1}^N x_i^2 + \frac{1}{N} \sum_{i=1}^N y_i \sum_{i=1}^N x_i - \frac{m}{N} \left[ \sum_{i=1}^N x_i \right]^2.
\end{aligned} \tag{16}$$

Isolating the terms with the slope  $m$ , we have

$$\sum_{i=1}^N x_i y_i - \frac{1}{N} \sum_{i=1}^N y_i \sum_{i=1}^N x_i = m \sum_{i=1}^N x_i^2 - \frac{m}{N} \left[ \sum_{i=1}^N x_i \right]^2, \tag{17}$$

so that

$$\begin{aligned}
m &= \frac{\sum_{i=1}^N x_i y_i - \frac{1}{N} \sum_{i=1}^N y_i \sum_{i=1}^N x_i}{\sum_{i=1}^N x_i^2 - \frac{1}{N} \left[ \sum_{i=1}^N x_i \right]^2} \\
&= \frac{N \sum_{i=1}^N x_i y_i - \sum_{i=1}^N y_i \sum_{i=1}^N x_i}{N \sum_{i=1}^N x_i^2 - \left[ \sum_{i=1}^N x_i \right]^2}.
\end{aligned} \tag{18}$$

Now that we have the slope, we can find the intercept. Using equation (15), we have

$$b = \frac{1}{N} \sum_{i=1}^N y_i - \frac{m}{N} \sum_{i=1}^N x_i. \tag{19}$$

To recapitulate,

$$m = \frac{N \sum_{i=1}^N x_i y_i - \sum_{i=1}^N y_i \sum_{i=1}^N x_i}{N \sum_{i=1}^N x_i^2 - \left[ \sum_{i=1}^N x_i \right]^2}, \tag{20}$$

and

$$b = \frac{1}{N} \sum_{i=1}^N y_i - \frac{m}{N} \sum_{i=1}^N x_i. \tag{21}$$

### *The Greek Alphabet*

A	$\alpha$	alpha	N	$\nu$	nu
B	$\beta$	beta	$\Xi$	$\xi$	xi
$\Gamma$	$\gamma$	gamma	O	$o$	omicron
$\Delta$	$\delta$	delta	$\Pi$	$\pi$	pi
E	$\varepsilon$	epsilon	P	$\rho$	rho
Z	$\zeta$	zeta	$\Sigma$	$\sigma, \varsigma$	sigma
H	$\eta$	eta	T	$\tau$	tau
$\Theta$	$\theta$	theta	Y	$\upsilon$	upsilon
I	$\iota$	iota	$\Phi$	$\varphi, \phi$	phi
K	$\kappa$	kappa	X	$\chi$	chi
$\Lambda$	$\lambda$	lambda	$\Psi$	$\psi$	psi
M	$\mu$	mu	$\Omega$	$\omega$	omega

## ***Values of Some Physical Constants***

### **Universal Physical Constants:**

<b>Quantity</b>	<b>Symbol</b>	<b>Value</b>	<b>MKS Units</b>
Speed of light in vacuum	$c$	$2.99792458 \times 10^8$	$\text{m} \cdot \text{s}^{-1}$
Permittivity of free space	$\epsilon_0$	$8.854187187 \times 10^{-12}$	$\text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}$
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{N} \cdot \text{s}^2 \cdot \text{C}^{-2}$
Gravitational Constant	$G$	$6.674 \times 10^{-11}$	$\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$
Planck's Constant	$h$	$6.626076 \times 10^{-34}$	$\text{J} \cdot \text{s}$
Charge on an electron	$-e$	$-1.602177 \times 10^{-19}$	$\text{C}$
Mass of an electron	$M_{e^-}$	$9.10939 \times 10^{-31}$	$\text{kg}$
Charge on a proton	$e$	$1.602177 \times 10^{-19}$	$\text{C}$
Mass of the proton	$M_p$	$1.67262 \times 10^{-27}$	$\text{kg}$
Mass of the neutron	$M_n$	$1.67493 \times 10^{-27}$	$\text{kg}$

### **Some Other Useful Physical Values:**

<b>Quantity</b>	<b>Symbol</b>	<b>Value</b>	<b>MKS Units</b>
Mass of the Sun	$M_\odot$	$1.989 \times 10^{30}$	$\text{kg}$
Radius of the Sun	$R_\odot$	$6.960 \times 10^8$	$\text{m}$
Luminosity of the Sun	$L_\odot$	$3.847 \times 10^{26}$	$\text{W}$
Mass density of the Sun	$\rho_\odot$	1,408	$\text{kg} \cdot \text{m}^{-3}$
Mass of the Earth	$M_\oplus$	$5.974 \times 10^{24}$	$\text{kg}$
Radius of the Earth	$R_\oplus$	$6.378 \times 10^6$	$\text{m}$
Mass density of the Earth	$\rho_\oplus$	5,497	$\text{kg} \cdot \text{m}^{-3}$
Mean Distance of the Earth from the Sun		$1.496 \times 10^{11}$	$\text{m}$ ( $\equiv 1 \text{ AU}$ )
Mass of the Moon	$M_M$	$7.35 \times 10^{22}$	$\text{kg}$
Radius of the Moon	$R_M$	$1.738 \times 10^6$	$\text{m}$
Mass density of the Moon	$\rho_M$	3,340	$\text{kg} \cdot \text{m}^{-3}$
Mean Distance of the Moon from the Earth		$3.844 \times 10^8$	$\text{m}$
Avogadro's Number	$N_A$	$6.02214 \times 10^{23}$	$\text{mol}^{-1}$
Atomic mass unit	$u$	$1.66054 \times 10^{-27}$	$\text{kg}$

<b>Quantity</b>	<b>Symbol</b>	<b>Value</b>	<b>MKS Units</b>
Boltzmann constant	$k_B$	$1.3807 \times 10^{-23}$	$\text{J} \cdot \text{K}^{-1}$
Stefan-Boltzmann constant	$\sigma$	$5.6705 \times 10^{-8}$	$\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$
Atmospheric pressure	atm	$1.013 \times 10^5$	$\text{N} \cdot \text{m}^{-2}$
Density of dry air at $(0^\circ\text{C}, 1\text{atm})$	$\rho_{air}$	1.29	$\text{kg} \cdot \text{m}^{-3}$
Speed of sound in air at $(0^\circ\text{C}, 1\text{atm})$	$v_{air}$	331	$\text{m} \cdot \text{s}^{-1}$
Density of water	$\rho_{H_2O}$	1000	$\text{kg} \cdot \text{m}^{-3}$
Earth's surface gravity	$g$	9.806	$\text{m} \cdot \text{s}^{-2}$