PHY2053 General Physics I

Section 584771

Prof. Douglas H. Laurence

Exam 1 (Chapters 2-6) February 13, 2018

Name: Solutions

Instructions:

This exam is composed of **10** multiple choice questions and **5** free-response problems. To receive a perfect score (100) on this exam, 4 of the 5 free-response problems must be completed. The fifth free-response problem may <u>not</u> be answered for extra credit. Each multiple choice question is worth 2 points, for a total of 20 points, and each free-response problem is worth 20 points, for a total of 80 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please** do not write in the rubric below; it is for grading purposes only.

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. The formula sheet is attached to the end of the exam.

Exam Grade:

| Multiple Choice | |
|-----------------|--|
| Problem 1 | |
| Problem 2 | |
| Problem 3 | |
| Problem 4 | |
| Problem 5 | |
| Total | |

MULTIPLE CHOICE QUESTIONS

- 1. The equations for kinematics can only be used under which condition(s)?
 - (a) Constant velocity
 - (b) Constant velocity and constant acceleration
 - (c) Constant acceleration
 - (d) They can be used under any condition

Kinematics applies whenever the acceleration is constant. Option (a) isn't correct because it's too restrictive, as is option (b). Option (d) is incorrect because, as we know, there is a restriction to the use of kinematic equations. So option (c) is correct.

- 2. As you drive your car, you approach a curve in the road. Traveling around the curve, you see that your speedometer remains constant at 50mph. This means that the car is not accelerating.
 - (a) True
 - (b) False

Even though your speed is constant, as shown by the speedometer, your *velocity* is not constant because your direction changes as you go around the bend. Therefor you must be accelerating, and so option (b) is correct.

- 3. An athlete is working out by sprinting 10m forward, stopping, and then sprinting 10m back to the starting point. During one of these round trips:
 - (a) The distance traveled was 10m and the displacement was 10m
 - (b) The distance traveled was 20m and the displacement was 10m
 - (c) The distance traveled was 20m and the displacement was 0m
 - (d) The distance traveled was 0m and the displacement was 20m

If the athlete runs 10m forward and then 10m back to their starting spot, they run a total distance of 20m. However, since they begin and end at the same place during one of these round trips, their displacement is 0. So option (c) is correct.

- 4. A projectile moves through the air in the shape of a parabola. Which of the following statements is true about this motion?
 - (a) The motion is always symmetric, due to being parabolic
 - (b) The motion is symmetric, but only if the projectile starts and ends at the same height
 - (c) The motion is symmetric, but only if the projectile is launched at 45°
 - (d) The motion is never symmetric

A parabola is only symmetric if it begins and ends at the same height, so option (b) is correct.

- 5. A projectile is launched at 32 m/s at an angle of 25°. At the peak of its trajectory, what is its speed?
 - (a) 0 m/s
 - (b) 14 m/s
 - (c) 29m/s
 - (d) 32m/s

At the peak of the projectile's trajectory, it's y-velocity is zero. However, it still has its x-velocity, which doesn't change in projectile motion, since there's no x-acceleration. The initial velocity in the x-direction is $v_{0x} = v_0 \cos \theta = (32) \cos(25^\circ) = 29 \text{m/s}$. So option (c) is correct.

- 6. Out of the following options, choose the **best** description of a force:
 - (a) An influence on an object's motion due to a fundamental interaction between two objects
 - (b) Something that causes an object to move
 - (c) Something that causes an object to accelerate
 - (d) Something that satisfies at least Newton's first and second laws

What's important about this problem is we need to choose the **best** option. While a force is something that causes an object to accelerate, recall that there are things called "pseudoforces" like the Coriolis force that cause acceleration but are not forces. This is because they don't satisfy Newton's third law. So options (b) and (c) aren't a very complete description of a force. A real force will satisfy all three of Newton's laws, so option (d) is wrong. So the most correct option is option (a), because it is the only option that clearly illustrates Newton's third law, which none of the other options account for.

- 7. Box B sits at rest on top of Box A. If you push Box A, causing it to accelerate, and Box B moves along on top of Box A undisturbed, what force is causing the motion of Box B?
 - (a) No force; Newton's first law says that Box B can move under its own inertia
 - (b) The pushing force you apply on the boxes
 - (c) The upward interaction force that Box A puts on Box B
 - (d) The friction between Box A and Box B

Since Box B moves along with Box A, which is accelerating, there must be a force on Box B, so option (a) is wrong. Option (b) is wrong because your push is a contact force, so it only acts on Box A. While there is an upward force on Box B due to Box A, this force is perpendicular to the horizontal acceleration of Box B, so it can't be the force producing the motion on Box B, and therefore option (c) is wrong. It is in fact static friction that causes the motion of Box B, so option (d) is correct.

- 8. A man stands at rest on a flat surface. This is because:
 - (a) His weight and the normal force acting on him cancel out, due to Newton's first law
 - (b) His weight and the normal force acting on him cancel out, due to Newton's second law
 - (c) His weight and the normal force acting on him cancel out, due to Newton's third law
 - (d) His weight and the normal force acting on him cancel out, but not for any of the above reasons His weight and normal force are equal and opposite, but not because of Newton's third law, as most students might think at first. Newton's third law applies to a pair of forces acting on **different** objects, whereas both of these forces are acting on the same object. The normal force and weight are equal and opposite because the man isn't accelerating vertically, so the upward force must cancel the downward force. This is Newton's second law. The first law has nothing to do with the magnitude of forces; it just says that an object will change its motion only under the influence of a force. So the correct answer is option (b).

- 9. You're trying to move a heavy object, with a mass of 100kg, along a floor with friction. The coefficients of friction between the object and the floor are $\mu_s = 0.45$ and $\mu_k = 0.3$. If you push on the object with a force of 250 N, what is the magnitude of the friction acting on the object?
 - (a) 0N
 - (b) 250N
 - (c) 294N
 - (d) 441N

If you're trying to move an object, you first need to break static friction. The maximum static friction on this object is:

$$f_{s,max} = \mu_s N = (0.45)(100 * 9.8) = 441$$
N

This is **not** the correct answer, however, because you aren't pushing on the object with 441N. You push on the object with 250N, which isn't enough force to overcome static friction. Since static friction is an adaptive force, it will simply cancel out the force you apply so that the object remains at rest. So the correct answer is $f_s = 250$ N, which is option (b).

- 10. A table is said to "have a breaking point of 980N." Using your knowledge to interpret this statement, if the table already has a 20kg block resting on it, how many 10kg blocks can be added to the table before it breaks? *Hint: Recall that we talked about why a surface wouldn't normally break in class*.
 - (a) 2
 - (b) 5
 - (c) 8
 - (d) 10

The phrase "have a breaking point of 980N" should be interpreted as "the maximum normal force is 980N." So, how much weight can you place on the table without exceeding the maximum normal force, and thus breaking the table? 980N, of course. This corresponds to:

$$m = \frac{W}{g} = \frac{980}{9.8} = 100$$
kg

There is already 20kg on top of the table, so you can only add 100 - 20 = 80kg to the table. If each block has a mass of 10kg, then you can only add 80/10 = 8 blocks without breaking the table. Thus, option (c) is correct.

FREE-RESPONSE PROBLEMS

1. Consider two vectors: \vec{A} has a magnitude of 6 and makes an angle of 42° counter-clockwise from the x-axis, and \vec{B} has a magnitude of 8 and makes an angle of 60° clockwise from the x-axis.

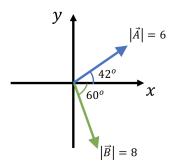
a) Draw the vectors \vec{A} and \vec{B} on an x-y graph.

b) Find the x and y components of \vec{A} and \vec{B} .

c) Consider a third vector, $\vec{C} = \vec{A} + \vec{B}$. What is the magnitude of \vec{C} ?

d) What is the angle of \vec{C} , measured counter-clockwise from the x-axis.

a)



b) Both the angles for \vec{A} and \vec{B} are made with respect to the x axis, so their x components use cosine and their y components use sine:

$$A_x = A\cos\theta = 6\cos(42) \Rightarrow \boxed{A_x = 4.5}$$

$$A_y = A\sin\theta = 6\sin(42) \Rightarrow \boxed{A_y = 4}$$

$$B_x = B\cos\theta = 8\cos(60) \Rightarrow \boxed{B_x = 4}$$

$$B_y = B\sin\theta = 8\sin(60) \Rightarrow \boxed{B_y = 6.9 \text{ or } B_y = -6.9}$$

Since \vec{B} dips below the x axis, it's y component is technically negative. This isn't as important in this question as it will be in the next part, so it's fine if it's given either as a positive or a negative.

c) To add vectors, the components of the sum will be the sum of the components:

$$C_x = A_x + B_x = 4.5 + 4 = 8.5$$

 $C_y = A_y + B_y = 4 + (-6.9) = -2.9$

This is what I meant at the end of the last part: the sign of B_x is very important here. Without including the negative sign, you won't get the right answer. By looking at the vectors, you can tell that their vertical components should, in fact, compete against each other when you add them: \vec{A} points up while \vec{B} points down. Now, to find the magnitude of \vec{C} , we just use the Pythagorean theorem:

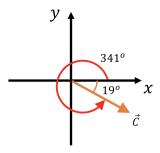
$$|\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{8.5^2 + 2.9^2} \implies |\vec{C}| = 9$$

d) To find the angle, we'll begin where we always do: using the tangent, we can find the angle to the nearest x axis:

$$\tan \theta = \left(\left| \frac{C_y}{C_x} \right| \right)$$

$$\Rightarrow \quad \theta = \tan^{-1} \left(\left| \frac{C_y}{C_x} \right| \right) = \tan^{-1} \left(\frac{2.9}{8.5} \right) = 19^o$$

To find the correct angle – the one counter-clockwise from the x axis – we should draw \vec{C} . It has an x component of 8.5 and a y component of -2.9, so it points below the x axis, in the fourth quadrant.



So, the correct angle is clearly 360° minus what we found, or $\boxed{341^{\circ}}$

- 2. A car starts from rest and accelerates at 6m/s^2 for 8s, then drives at a constant speed for 5s, and finally decelerates at a rate of 12m/s^2 until coming to rest.
 - a) What total distance did the car travel during this trip?
 - b) During the braking, how long does it take the car to come to a stop?
 - c) Graph the velocity vs. time of the car, with the correct velocities at t = 8s and t = 13s.
 - d) Graph the position vs. time of the car, with the correct position at t = 8s and t = 13s. Assume that the car starts at x = 0.

This is a kinematics problem that is broken up into three steps, each of which has its own constant acceleration: step 1 has an acceleration $a_1 = 6\text{m/s}^2$, step 2 has $a_2 = 0$ because the speed is constant, and step 3 has $a_3 = -12\text{m/s}^2$. I chose a_3 to be negative because I'm going to assume that the direction the car is initially moving in is positive, meaning that a deceleration would be negative (in order to decrease the speed).

Now, the general approach to these problems is that the final speed of one step is the initial speed of the next, so:

$$v_{02} = v_1$$

$$v_{03} = v_2$$

We'll need these in a moment.

a) For the first step, we know $a_1 = 6\text{m/s}^2$, $v_{01} = 0$ (because the car starts from rest), and $t_1 = 8s$. So, the distance during step 1 can be found:

$$\Delta x_1 = y_{\text{ol}} t_1 + \frac{1}{2} a_1 t_1^2 = \frac{1}{2} (6)(8)^2 = 192 \text{m}$$

To solve for the distance in step 2, we need to know the final speed in step 1:

$$v_1 = v_{01} + a_1 t_1 = (6)(8) = 48 \text{m/s}$$

So, $v_{02} = 48$ m/s. We also know that $a_2 = 0$ and $t_2 = 5$ s. So, the distance traveled during step 2 is:

$$\Delta x_2 = v_{02}t_2 + \frac{1}{2}g_2t_2^2 = (48)(5) = 240$$
m

Since the car doesn't accelerate during step 2, the final speed will be the same as the initial speed, so we know that during step 3, $v_{03} = v_{02} = 48$ m/s. We also know that $a_3 = -12$ m/s² and $v_3 = 0$ (the car comes to a stop during step 3). Since we don't know time during step 3, we can't use the same equation, but we can still easily find the distance:

$$y_3^2 = v_{03}^2 + 2a_3\Delta x_3 \implies \Delta x_3 = -\frac{v_{03}^2}{2a_3} = -\frac{(48)^2}{2(-12)} = 96$$
m

Thus, the total distance traveled is just the sum of the distances during each of the steps:

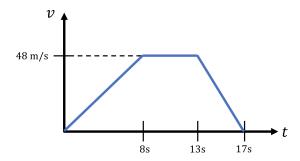
$$\Delta x_T = \Delta x_1 + \Delta x_2 + \Delta x_3 = 192 + 240 + 96 \implies \boxed{\Delta x_T = 528m}$$

b) "During the braking" means during step 3, so we need to find the time during step 3 for the car to slow to a stop. We know from the previous part that $v_{03} = 48 \text{m/s}$, $a_3 = -12 \text{m/s}^2$, and $v_3 = 0$, so:

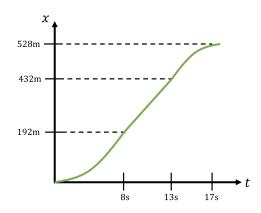
$$y_3 = v_{03} + a_3 t_3 \implies t_3 = -\frac{v_{03}}{a_3} = -\frac{48}{12} \implies \boxed{t_3 = 48}$$

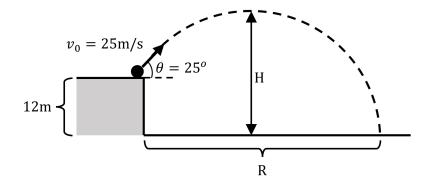
c) We should approach drawing the velocity vs. time graph in two steps. First, let's label specific points on the graph, where we know the velocity at particular times. At t=0, we know v=0 (the car starts at rest); at t=8s, we know v=48m/s (the speed of the car during step 2); at t=13s, we know v=48m/s (because the car doesn't accelerate during step 2); and, finally, at t=17s, we know that v=0 once again, because the car comes to a stop 4s after step 2 ends (this was the t_3 that we found).

Now that we know those specific points, we need to think about the lines that connect those points. The slope of velocity vs. time is acceleration, and since acceleration is always constant, the lines will be straight everywhere. During step 1, the line is pointing up, due to a positive acceleration; during step 2, the line is flat, due to a constant acceleration (zero slope); and during step 3, the line is pointing down, due to a negative acceleration.



d) The approach to the position vs. time graph is the same. We know after step 1 that $x=192\mathrm{m}$ (the distance traveled during step 1); after step 2, $x=432\mathrm{m}$ (the distance traveled during step 1 plus the distance traveled during step 2); and after step 3, $x=528\mathrm{m}$ (the total distance traveled). The question is: what do the lines look like? Remember that for position vs. time, velocity is the slope. We know that during step 2 the velocity is constant, so that will be a straight line, but during steps 1 and 3, the velocity changes and so the slope changes. During step 1, the velocity starts at zero, so the line starts flat, and then the velocity increases, so the slope progressively increases. This is an upward facing parabola. During step 3, the velocity starts non-zero and positive, so the line starts pointing up, but then the velocity decreases, so the slope decreases, eventually becoming flat at the end when velocity is zero. This is a downward facing parabola.





- 3. A projectile is launched with an initial speed of 25m/s at a launch angle of 25° from a height of 12m above the ground, as shown in the figure above.
 - a) What is the maximum height of the projectile, H? Hint: find the maximum height above the launch point, and use that to find the maximum height above the ground.
 - b) How long does it take the projectile to reach the maximum height?
 - c) How long does it take the projectile to drop from the maximum height to the ground?
 - d) What is the range R of the projectile?

First, we should break the initial velocity up into components; this is always how we want to start 2-dimensional problems.

$$v_{0x} = v_0 \cos \theta = 25 \cos(25) = 22.7 \text{m/s}$$

$$v_{0y} = v_0 \sin \theta = 25 \sin(25) = 10.6 \text{m/s}$$

Now we can start answering the problem.

a) We know that $v_{0y} = 10.6 \text{m/s}$, $a_y = -9.8 \text{m/s}^2$, and $v_y = 0$ at the maximum height (this is the defining characteristic of the maximum height). So, the distance from the launch point to the peak is:

$$y_y^2 = v_{0y}^2 + 2a_y \Delta y \implies \Delta y = -\frac{v_{0y}^2}{2a_y} = -\frac{(10.6)^2}{2(-9.8)} = 5.7$$
m

We can't forget, though, that the projectile doesn't start on the ground; it starts at $y_0 = 12$ m. So, the maximum height is:

$$H = \Delta y + y_0 = 5.7 + 12 \implies H = 17.7 \text{m}$$

b) This part is solved by using all the same information as the previous part, since we're still dealing with the maximum height; the difference is that we're solving for time in this problem, so we need a different equation.

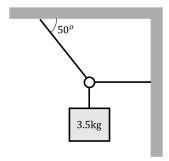
$$y_y = v_{0y} + a_y t \implies t = -\frac{v_{0y}}{a_y} = -\frac{(10.6)}{(-9.8)} \implies \boxed{t_{up} = 1.1s}$$

c) To find the time the projectile takes to fall to the ground, we know that $v_{0y} = 0$ (since we're starting at the peak) and $a_y = 9.8 \text{m/s}^2$ (I changed the sign since this problem only involves moving down). In part a, we found that the maximum height is 17.7m off the ground, so in order to fall from the peak to the ground, $\Delta y = 17.7 \text{m}$. So, finding the time is straightforward:

$$\Delta y = y_{\text{OV}} t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(17.7)}{(9.8)}} \quad \Rightarrow \quad \boxed{t_{down} = 1.9s}$$

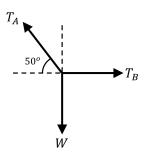
d) To find the range, we simply multiply the velocity in the x direction (which isn't changing) by the time the ball is in the air. We know that it took 1.1s to reach the peak from the launch point, and 1.9s to fall back to the ground after, so the total time in the air is $t_F = 1.1 + 1.9 = 3s$. So, the range is:

$$R = v_{0x}t_F = (22.7)(3) \Rightarrow \boxed{R = 68.1 \text{m}}$$

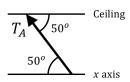


- 4. A 3.5kg mass hangs at rest from a ring, supported by two ropes, as shown in the figure above. Call the rope at an angle rope A and the horizontal rope B.
 - a) Draw the free body diagram of the ring.
 - b) Calculate the tension in rope A.
 - c) Calculate the tension in rope B.
 - d) Imagine if rope A was moved so that it was at a 90° angle from the ceiling (straight up from the mass). What would the tension in rope B be in this case?

a)



Note that the angle between T_A and the x axis is 50° . You can see this by using alternate interior angles:



b) Remember that tension is an adaptive force, so it can only be solved for by using Newton's second law on a free body diagram. The tension in rope A, T_A , will break into components:

$$T_{A,x} = T_A \cos(50)$$

$$T_{A,u} = T_A \sin(50)$$

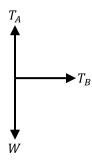
We don't know T_B , so Newton's second law in the x direction won't help us. But we do know the weight: W = (3.5)(9.8) = 34.3N, so we can use Newton's second law in the y direction. Since there is no y acceleration, the upward force balances the downward force, and:

$$T_{A,y} = W \Rightarrow T_A \sin(50) = W \Rightarrow T_A = \frac{W}{\sin(50)} = \frac{34.3}{\sin(50)} \Rightarrow \boxed{T_A = 44.7N}$$

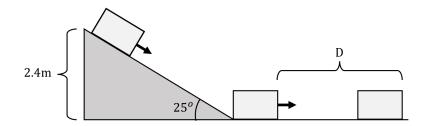
c) Now that we know T_A , we can use Newton's second law in the x direction to find T_B . Since there is no acceleration in the x direction, the force to the left balances the force to the right:

$$T_B = T_A \cos(50) = (44.7)\cos(50) \implies \boxed{T_B = 28.7N}$$

d) If rope A were moved so that it made a 90° angle to the ceiling, then it would point straight upward, and the free body diagram would look like:

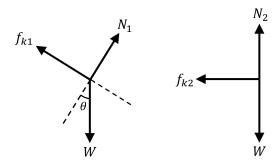


The key thing to remember here is that **tension is an adaptive force**; it will be whatever Newton's second law needs it to be. In this case, since there is no force to the left, T_B must be zero. To see this more clearly, let's think about the previous case. With rope A at an angle, the tension in rope A pulled the ring to the left, so rope B had to have a tension in it to counteract that pulling by rope A. However, with rope A pointing straight upward in this case, nothing is pulling the ring to the left, so rope B just hangs slack; it has no tension.



- 5. A 5kg block starts from rest at the top of a slope, as shown in the figure above. As it slides down the slope, it gains speed, reaching its maximum speed at the bottom of the slope. After reaching the bottom of the slope, it then slides for a distance D across a horizontal surface before coming to rest. Both the slope and the horizontal surface have the same coefficients of friction: $\mu_k = 0.3$ and $\mu_s = 0.4$.
 - a) Draw a free body diagram of the block on the slope and on the horizontal surface.
 - b) What is the acceleration of the block down the slope?
 - c) What is the speed of the block at the bottom of the slope?
 - d) What is the distance D traveled by the block before coming to a rest?

a)



b) To find the acceleration down the slope, we need to use Newton's second law in the direction parallel to the slope. We'll choose down the slope to be positive, since that's the direction of the acceleration. Don't forget to break the weight up into its component down the slope, which is the sine component, and its component perpendicular to the slope, which is the cosine component (since the angle touches the perpendicular axis).

$$\sum F_{\parallel} = W \sin \theta - f_{k1} = ma \quad \Rightarrow \quad a = \frac{W \sin \theta - f_{k1}}{m}$$

In order to solve for a, we need to know f_{k1} . We know the equation for kinetic friction depends on the normal force, so we'll need to solve for that first. The normal force is going to be found by using Newton's second law in the perpendicular direction, in which the acceleration is zero:

$$N_1 = W \cos \theta = (5)(9.8) \cos(25) = 44.4$$
N

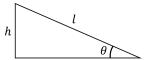
Now we can calculate the friction down the slope:

$$f_{k1} = \mu_k N_1 = (0.3)(44.4) = 13.3$$
N

Now we can go back to our equation down the slope and solve for a:

$$a = \frac{W \sin \theta - f_{k1}}{m} = \frac{(5)(9.8)\sin(25) - (13.3)}{(5)} \Rightarrow \boxed{a = 1.48 \text{m/s}^2}$$

c) To find the speed of the block at the bottom of the slope, we can use 1-dimensional kinematics, since the acceleration is constant. We know that, down the slope, $a = 1.48 \text{m/s}^2$ and $v_0 = 0$; all we need to know is how far down the slope the block travels, which we can solve for with a big of trigonometry.



The height, which we know, is the opposite edge and the length down the slope is the hypotenuse, so we can use sine:

$$\sin \theta = \frac{h}{l} \implies l = \frac{h}{\sin \theta} = \frac{2.4}{\sin(25)} = 5.7$$
m

Now all we need to do is use our kinematic formulas:

$$v^2 = y\sigma^2 + 2a\Delta x \Rightarrow v = \sqrt{2a\Delta x} = \sqrt{2(1.48)(5.7)} \Rightarrow v = 4.1 \text{m/s}$$

d) To find out how far the block travels before stopping, we'd like to use kinematics. We know $v_0 = 4.1 \text{m/s}$ (as we just found in the last part) and v = 0 (since it stops), but we don't know a. However, we can find a by using Newton's second law on the free body diagram from part a for the block on the horizontal surface. This tells us that it's the friction which is producing the deceleration, so we need to find the normal force before we can find the friction. Since this is a horizontal surface, the normal force is just equal to the weight:

$$N = W = (5)(9.8) = 49N$$

So the friction is (note that the coefficient along the horizontal surface is the same as down the slope):

$$f_k = \mu_k N = (0.3)(49) = 14.7N$$

So, using Newton's second law:

$$f_k = ma \implies a = \frac{f_k}{m} = \frac{14.7}{5} = 2.94 \text{m/s}^2$$

The last thing to consider before solving the problem is that the answer above is just the **magnitude** of the acceleration. The direction is given in the free body diagram, but we didn't consider the direction when solving Newton's second law; we only calculated the magnitude of a. Since the block is slowing down, a must be negative, so a = -2.94m/s². Thus, the distance traveled is:

$$p^2 = v_0^2 + 2a\Delta x \implies \Delta x = -\frac{v_0^2}{2a} = -\frac{(4.1)^2}{2(-2.94)} \implies \boxed{D = 2.9\text{m}}$$

FORMULA SHEET

• Constants:

$$g = 9.8 \text{m/s}^2$$

• Vectors:

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$\left| \vec{A} \times \vec{B} \right| = AB \sin \theta$$

• Kinematics:

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

• Forces:

$$\sum \vec{F} = m\vec{a}$$

$$W = mg$$

$$f_{s,max} = \mu_s N$$

$$f_k = \mu_k N$$