Chapter 9:
Measuring the Stars

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AST 1002

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Astronomy vs. Cosmology

Dictionary

astronomy

as·tron·o·my

/əˈstränəmē/

noun

the branch of science that deals with celestial objects, space, and the physical universe as a whole.

Study of everything in the Universe

Study of large scale structure of the Universe

Dictionary

cosmology

cos·mol·o·gy

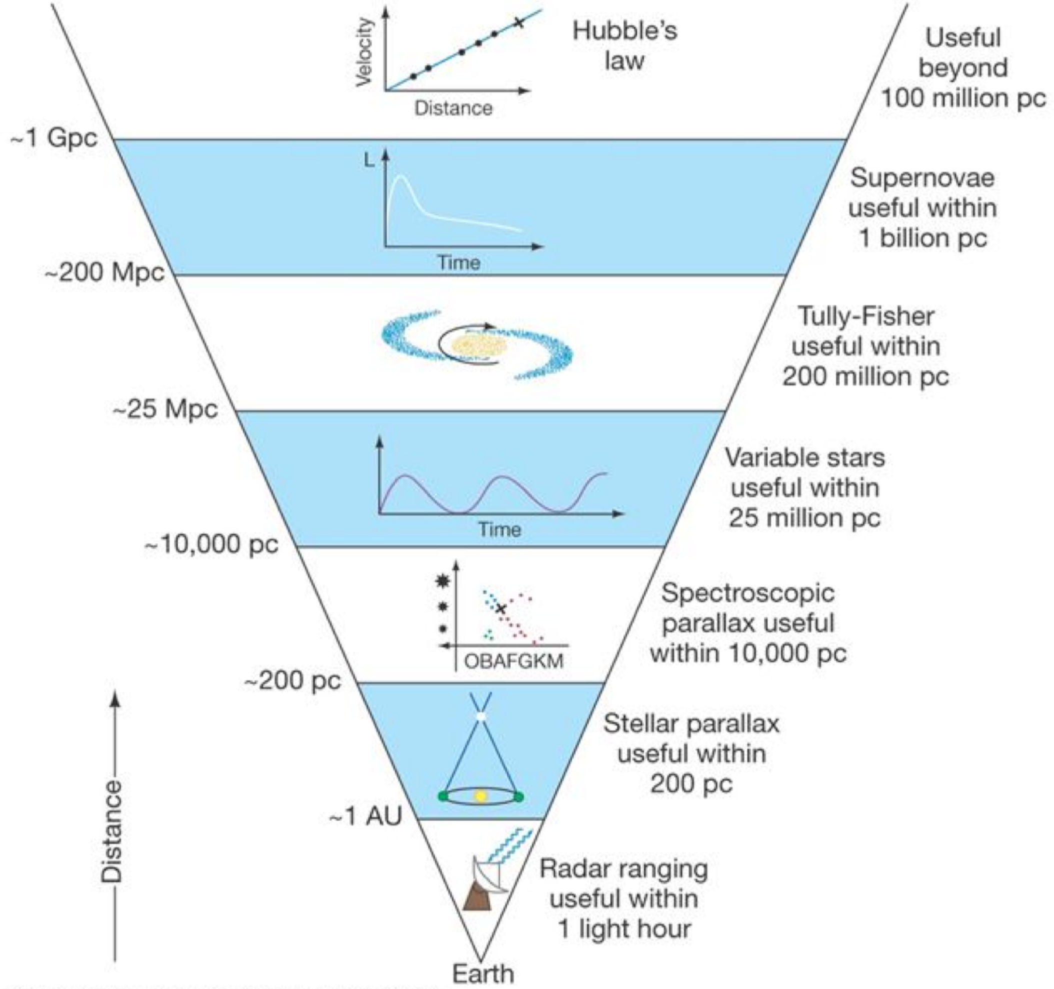
/käzˈmäləjē/

noun

the science of the origin and development of the universe. Modern astronomy is dominated by the Big Bang theory, which brings together observational astronomy and particle physics.

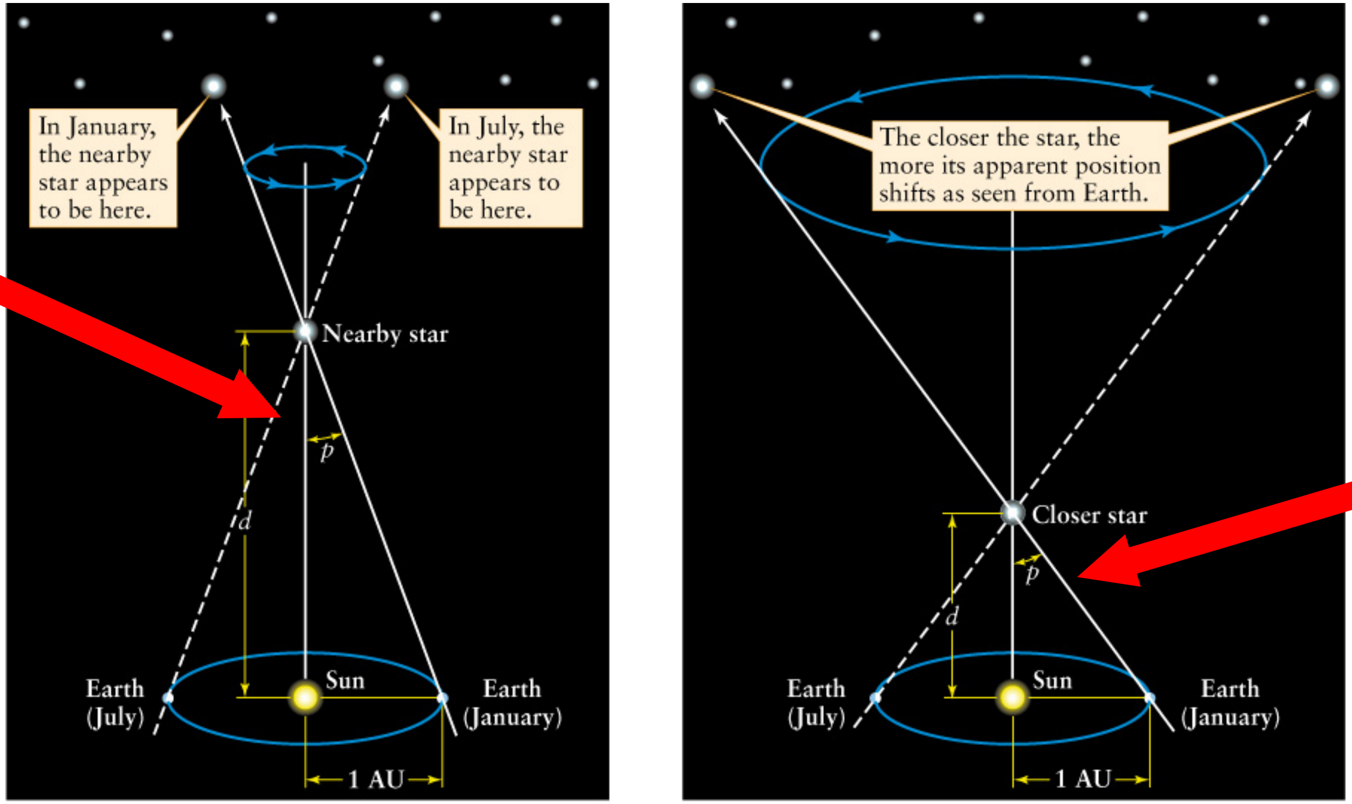
- an account or theory of the origin of the universe.
  plural noun: **cosmologies**

Cosmic Distance Ladder



Stellar Parallax

Smaller parallax
→ larger distance



(a) Parallax of a nearby star (b) Parallax of an even closer star

Larger parallax
→ smaller distance

Stellar Parallax Calculations

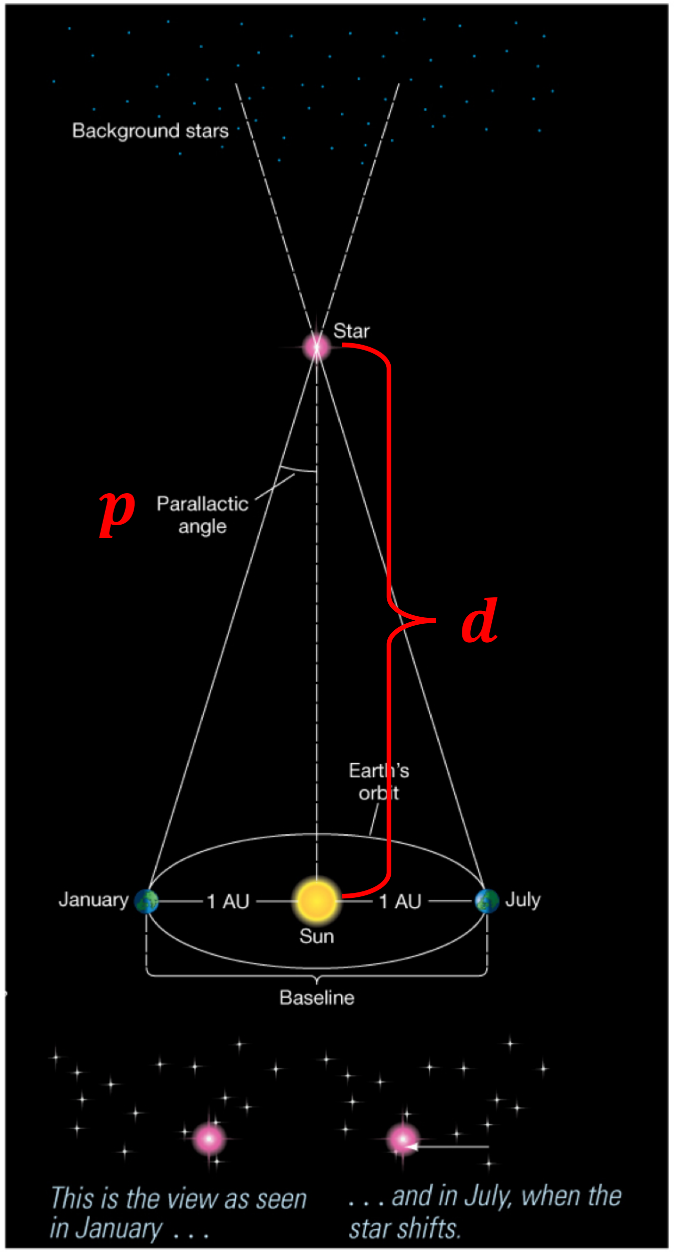
$$d = \frac{1}{p}$$

Distance in parsecs (pc)

Angle in arcseconds (")

$$1 \text{ pc} = 3.26 \text{ ly}$$

Same order of magnitude!

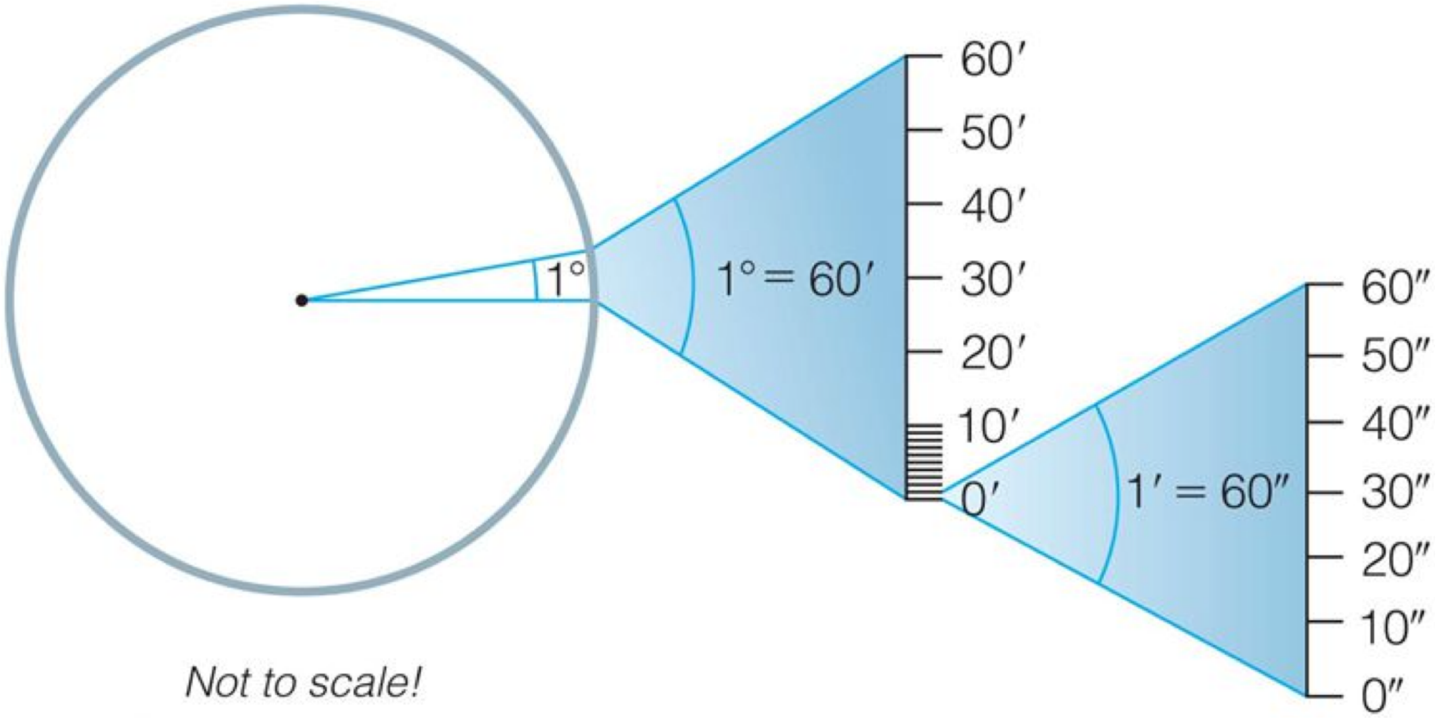


Arcminutes and Arcseconds

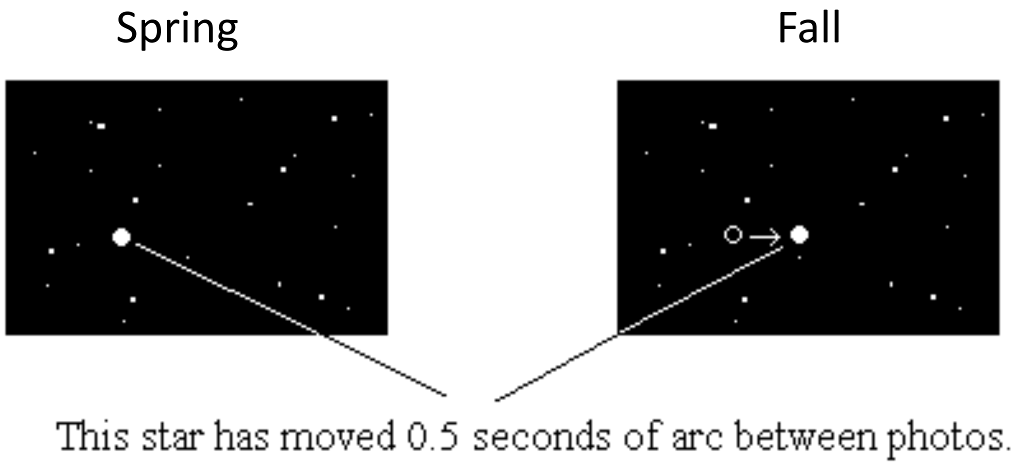
$$1 \text{ circle} = 360^o$$

$$1^o = 60'$$

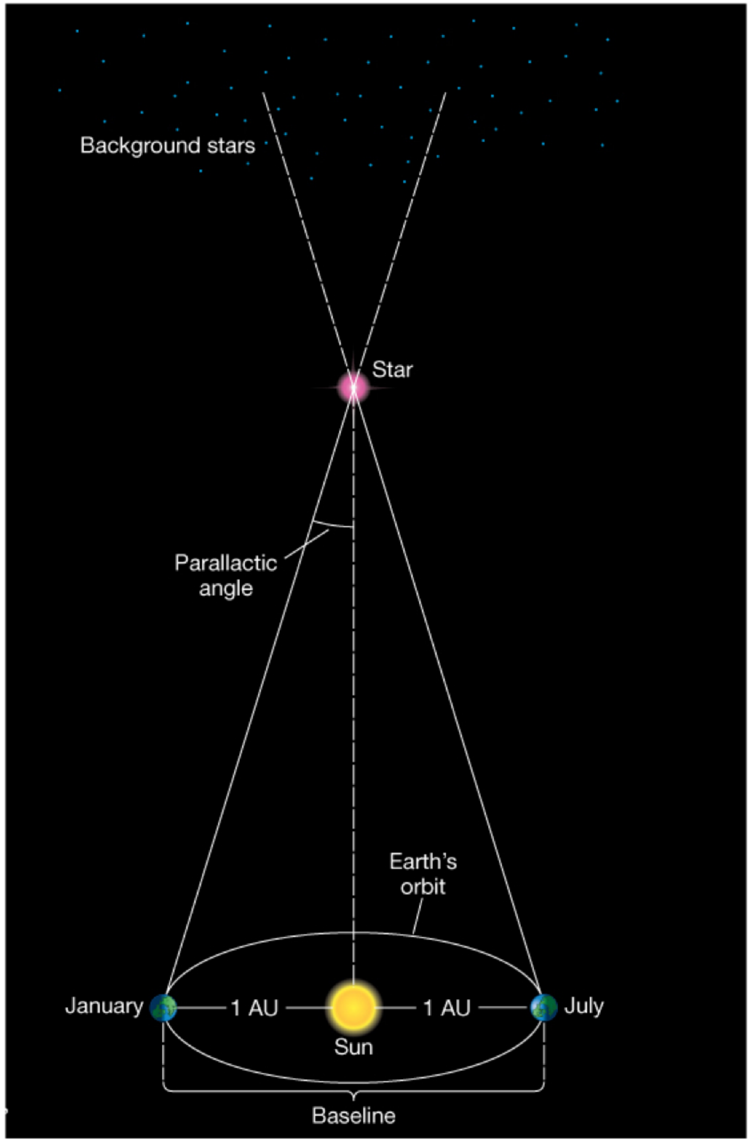
$$1' = 60''$$



Measuring Parallax



$$d = \frac{1}{p} = \frac{1}{0.25''} = 4\text{pc}$$



Limits of Parallax

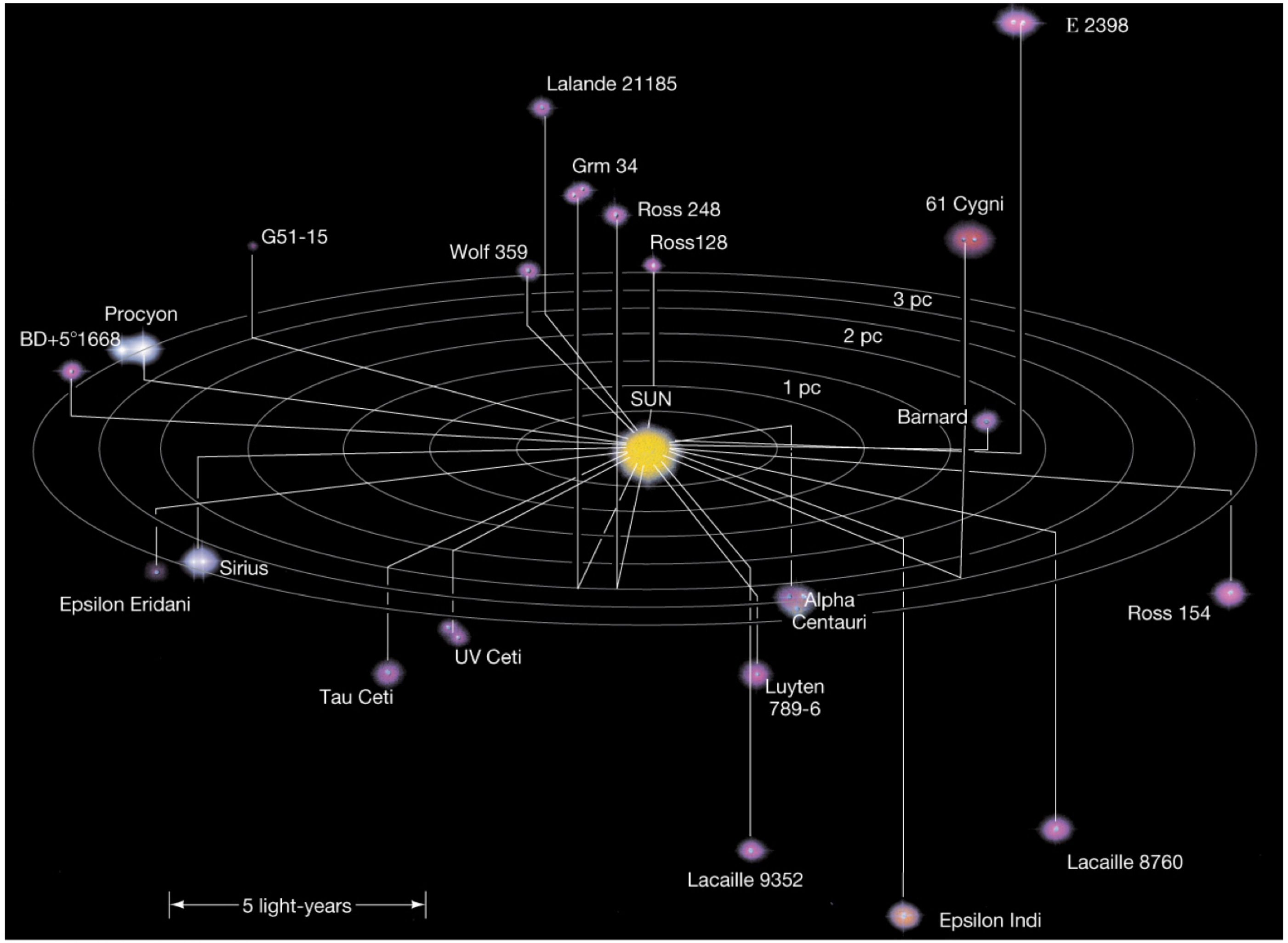
Angles smaller than 0.01’’ are hard to measure from Earth

$$d_{\max} = \frac{1}{0.01''} = 100\text{pc}$$

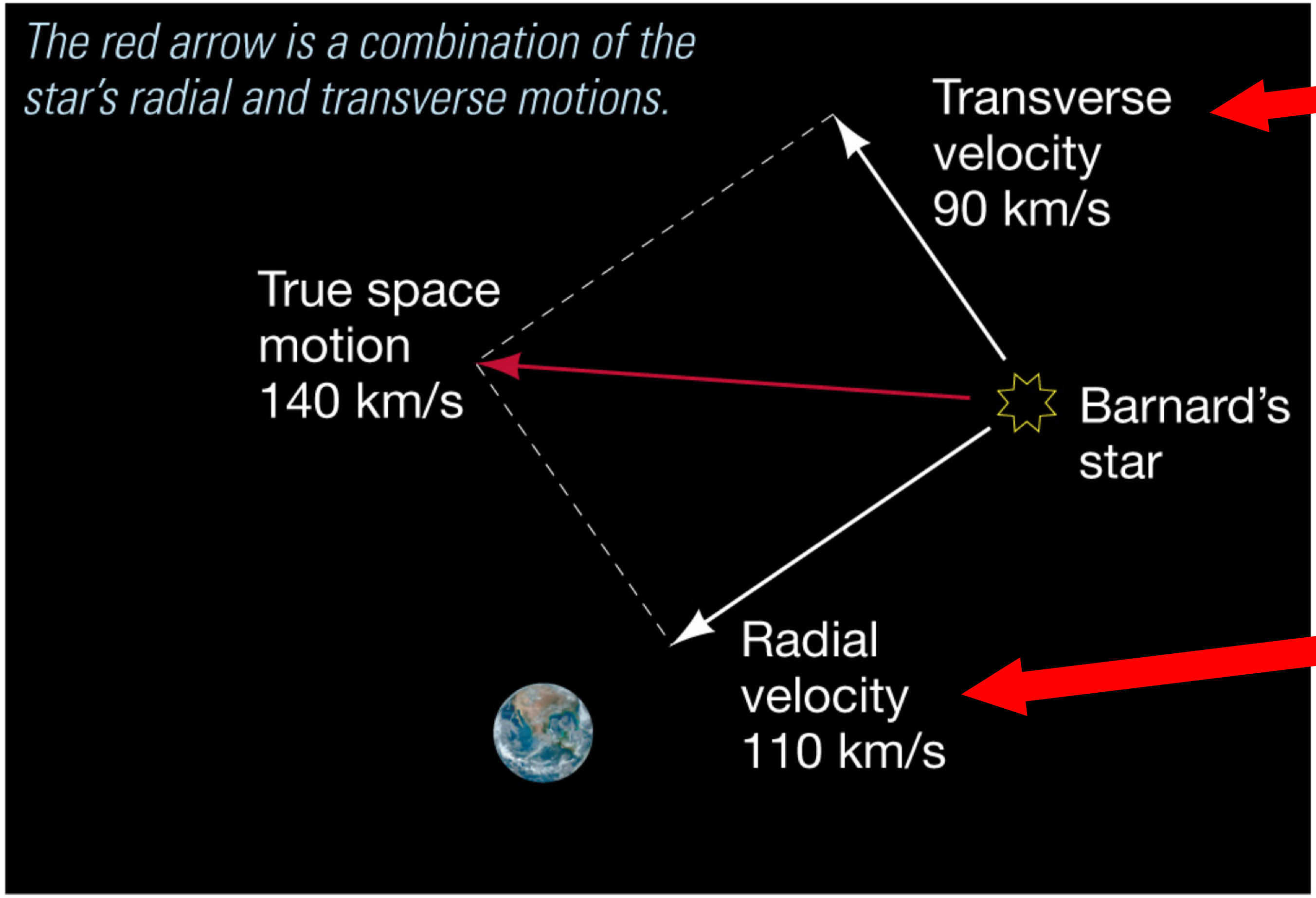
Angles smaller than 0.001’’ are hard to measure from Space

$$d_{\max} = \frac{1}{0.001''} = 1000\text{pc}$$

Nearest Neighbors



Motion of Stars



Found using geometry

Found using Doppler Effect

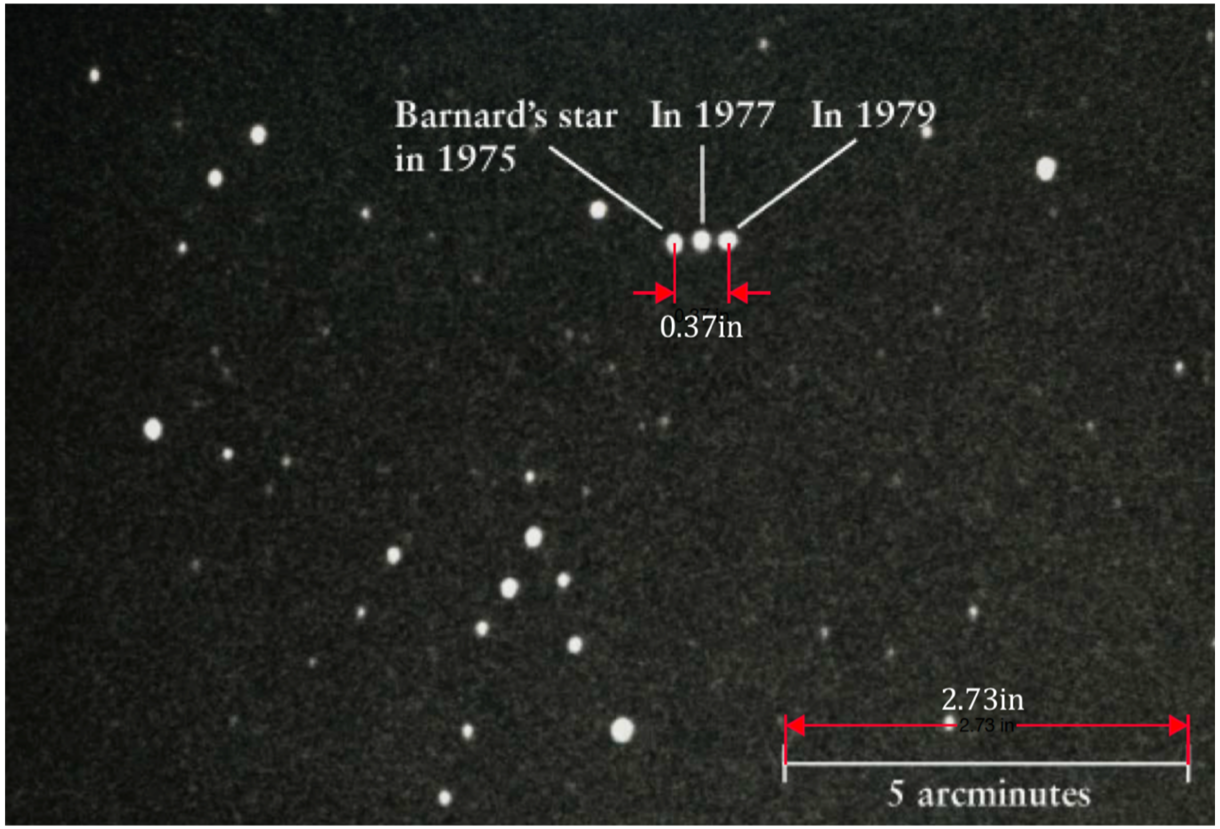
Translational Motion

Distance traveled (arcsec/year)

$$v_t = 4.74\mu d$$

Transverse velocity (km/s)

Distance to star (pc)



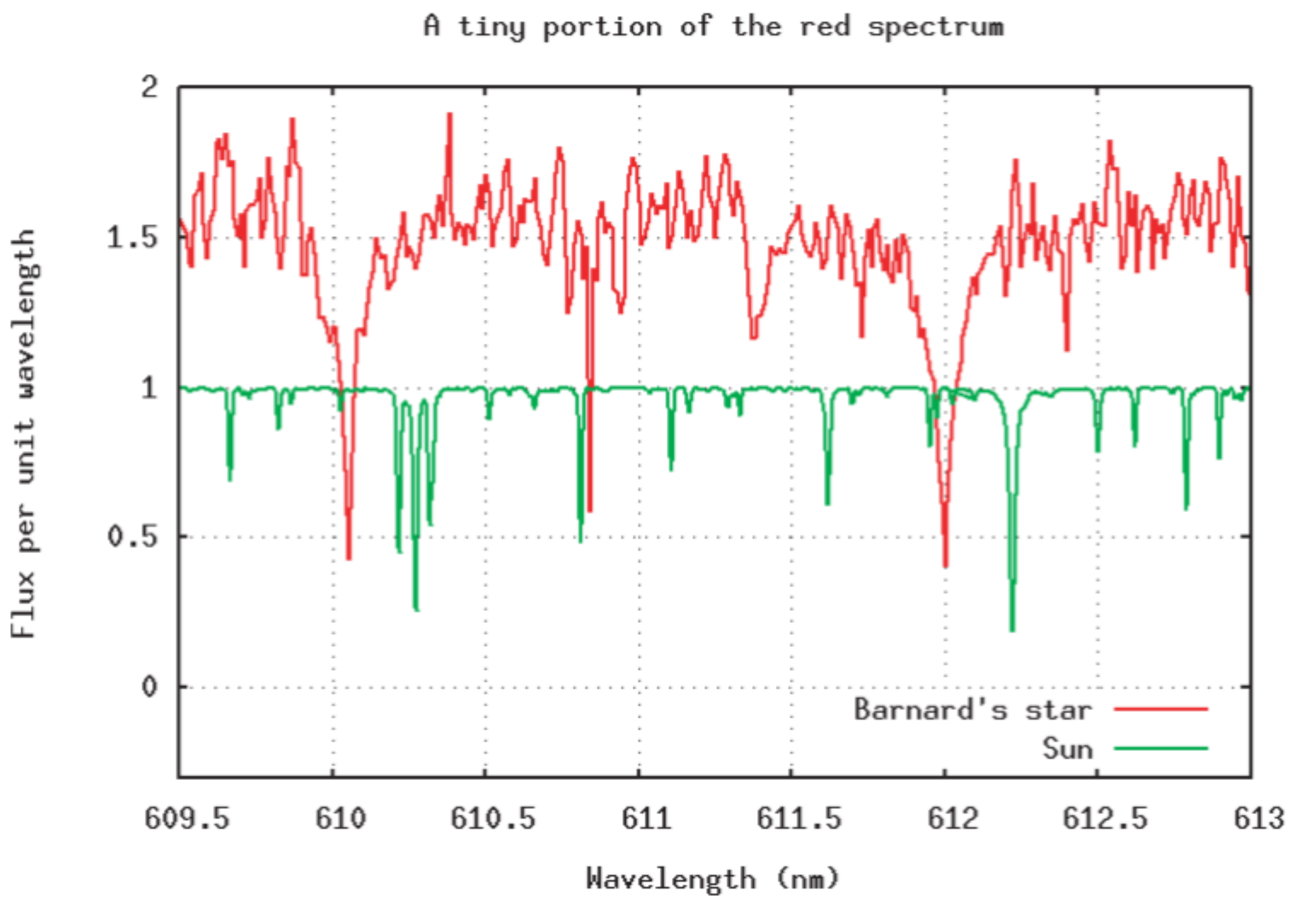
Radial Motion

Shifted wavelength

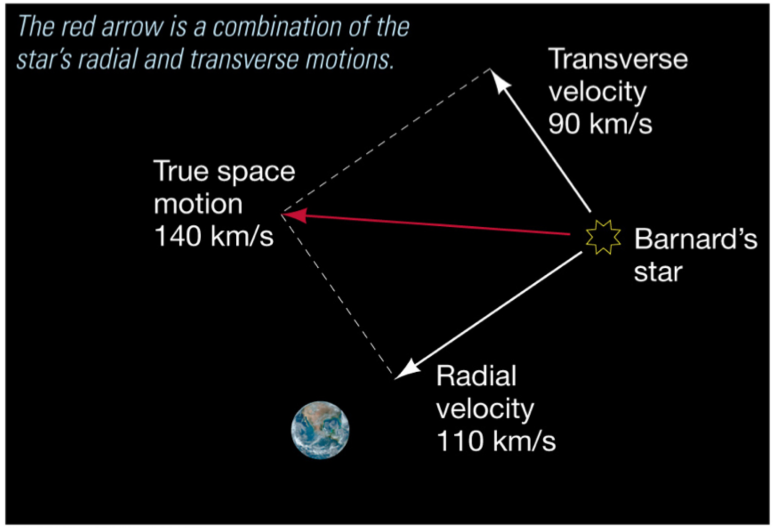
$$\frac{\lambda}{\lambda_0} - 1 = \frac{v_r}{c}$$

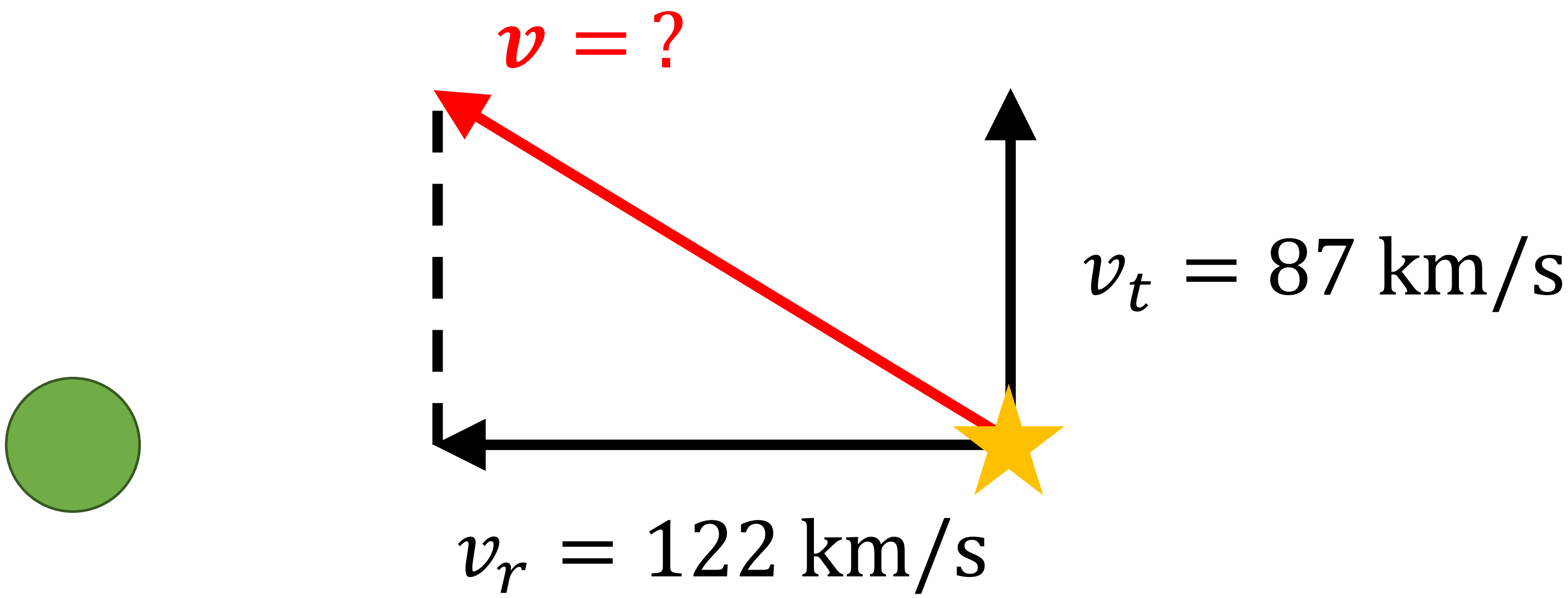
Proper wavelength

Figure 1: Barnard Star and Sun's Spectrum



Total Motion



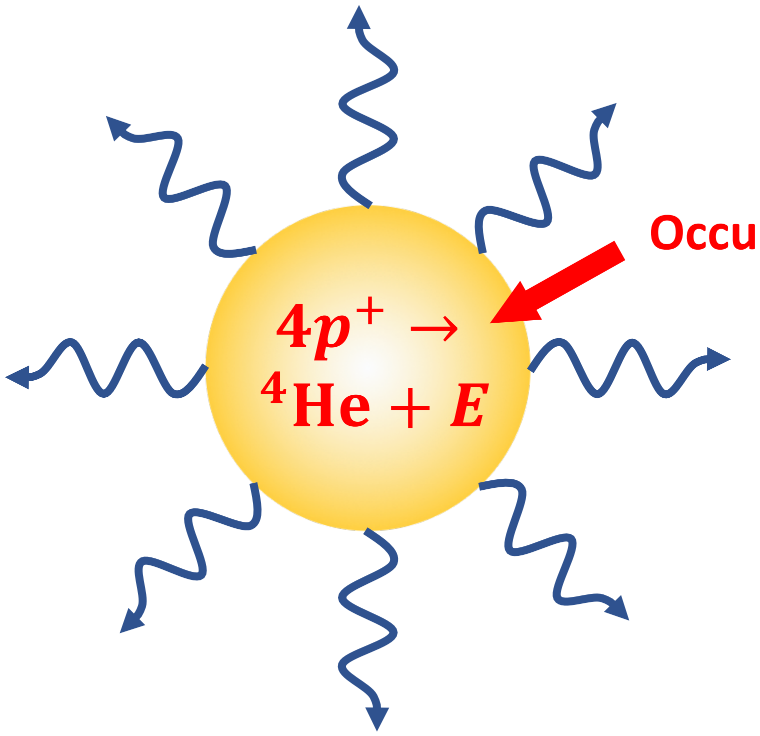


Luminosity

Brightness is what we call **Luminosity**, L

→ It's the amount of energy released per second (Watts, W)

→ It's an **inherent quantity** of a star



Occurs 9.58×10^{37} times every second

$$L_\odot = 3.828\times10^{26}\ \mathrm{W}$$

$\odot$ = Sun

Flux

We don’t measure the brightness of a star, but it’s **apparent brightness**

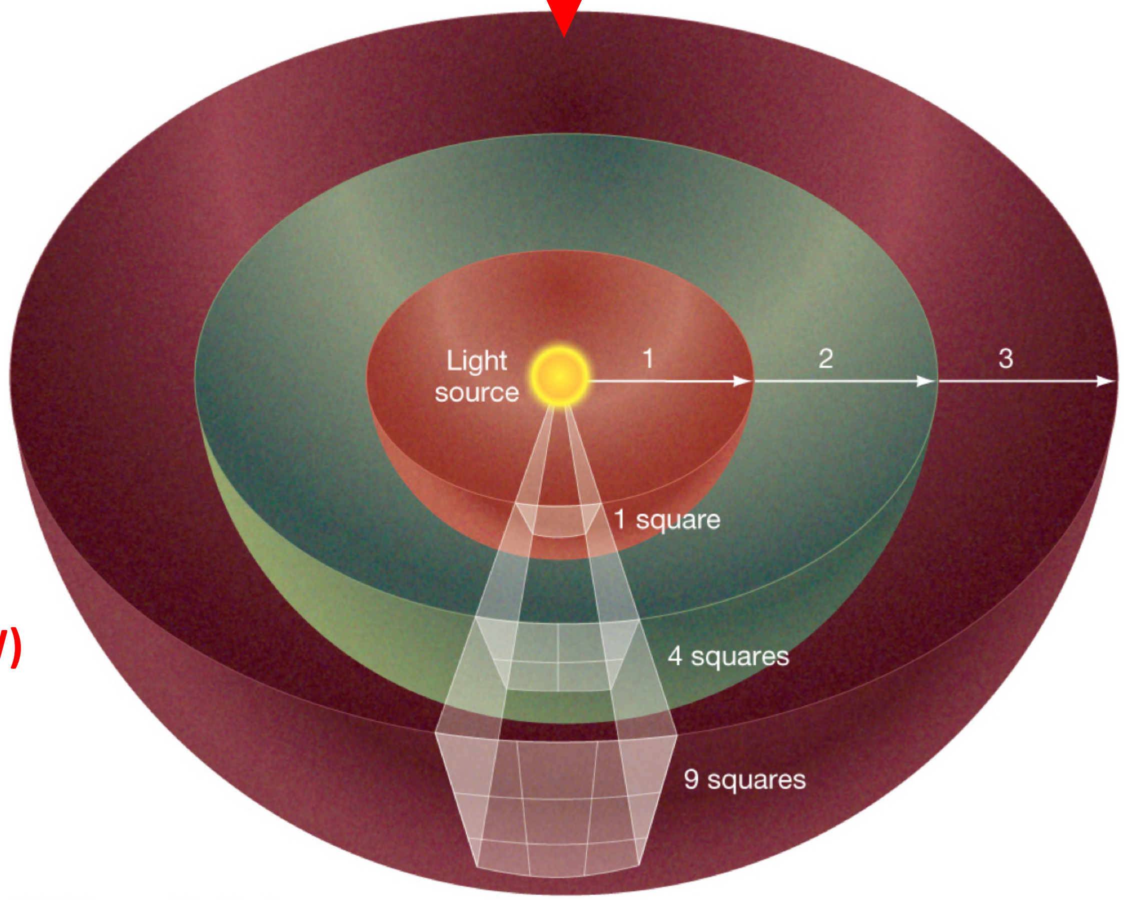
- This is known as **Flux**, F (W/m^2)
- Flux measures how the luminosity spreads out over a sphere for isotropic emission

$$F = \frac{L}{\underbrace{4\pi r^2}_{\text{Surface area of sphere (m}^2\text{)}}}$$

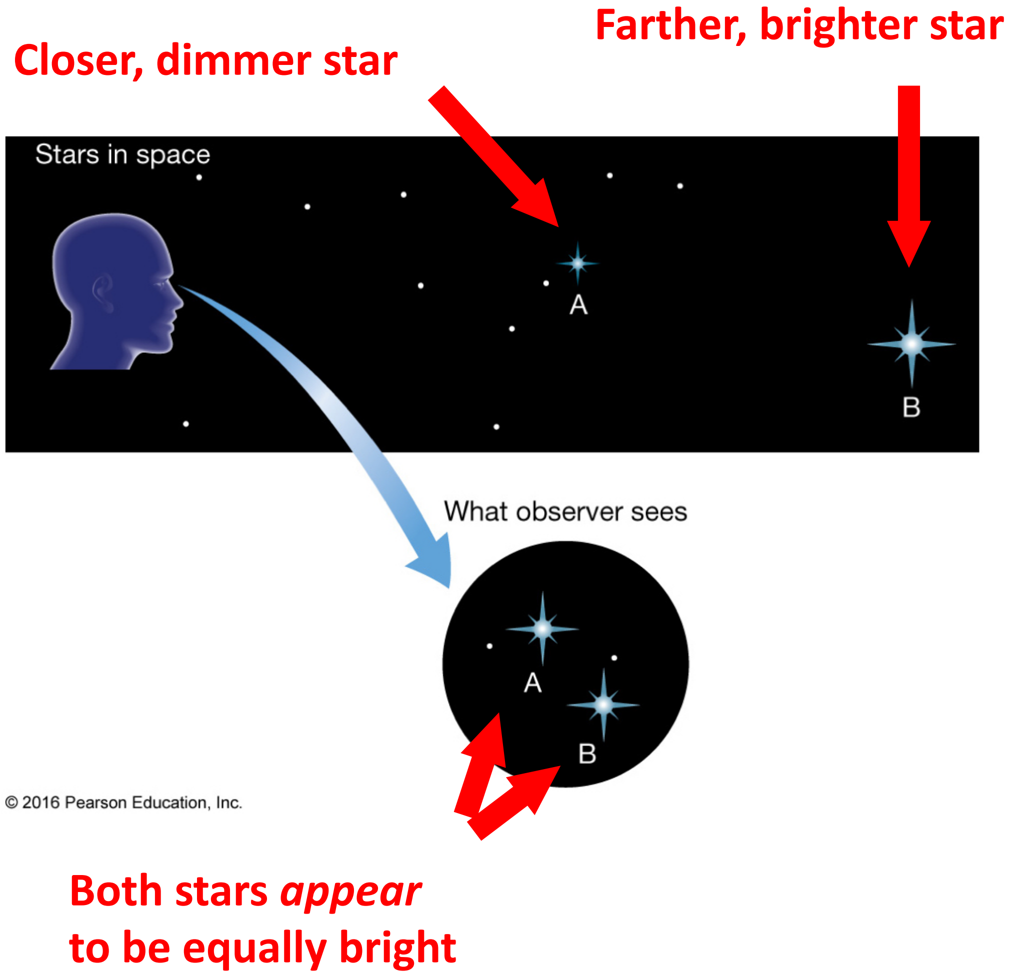
Luminosity (W)

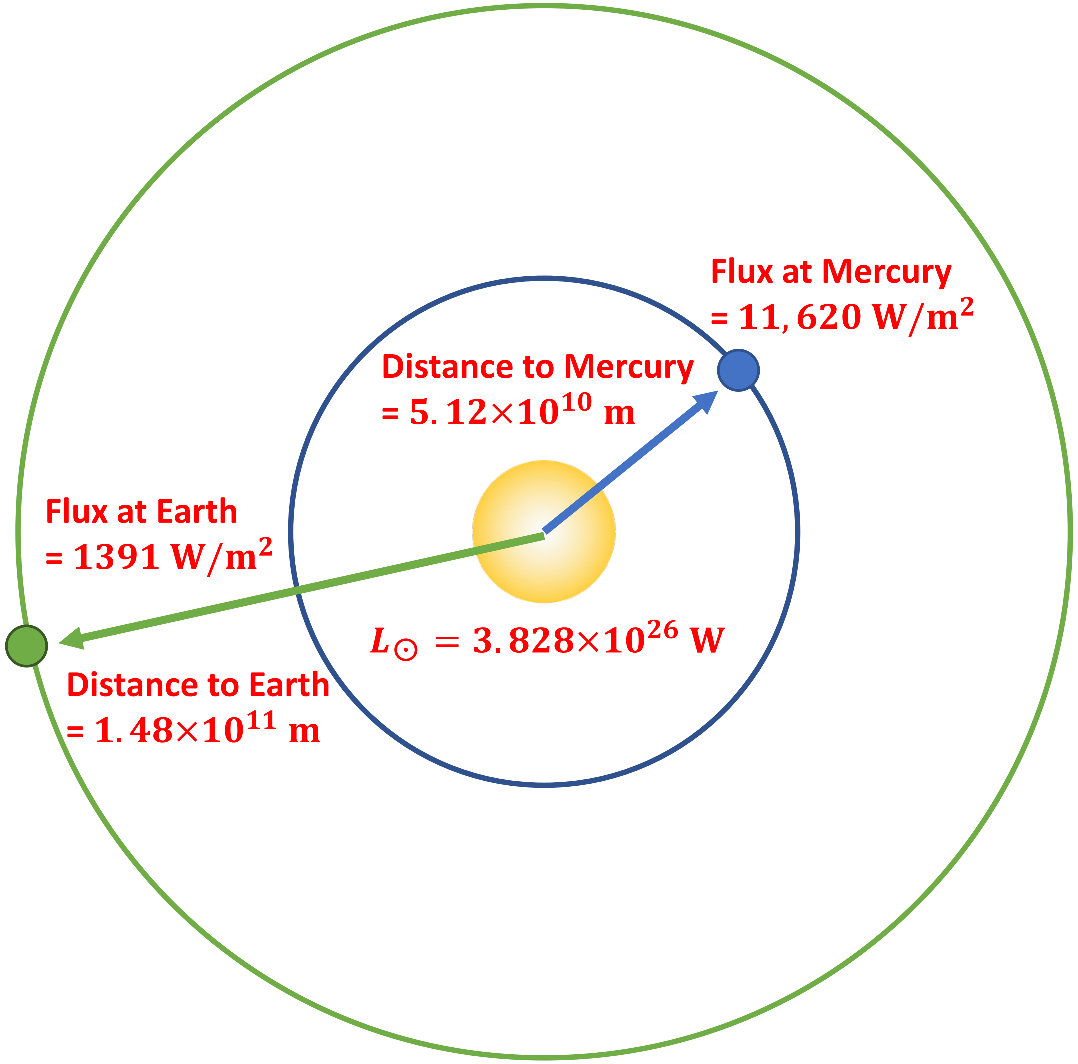
Flux (W/m^2)

Isotropic Emission!



Flux and Distance





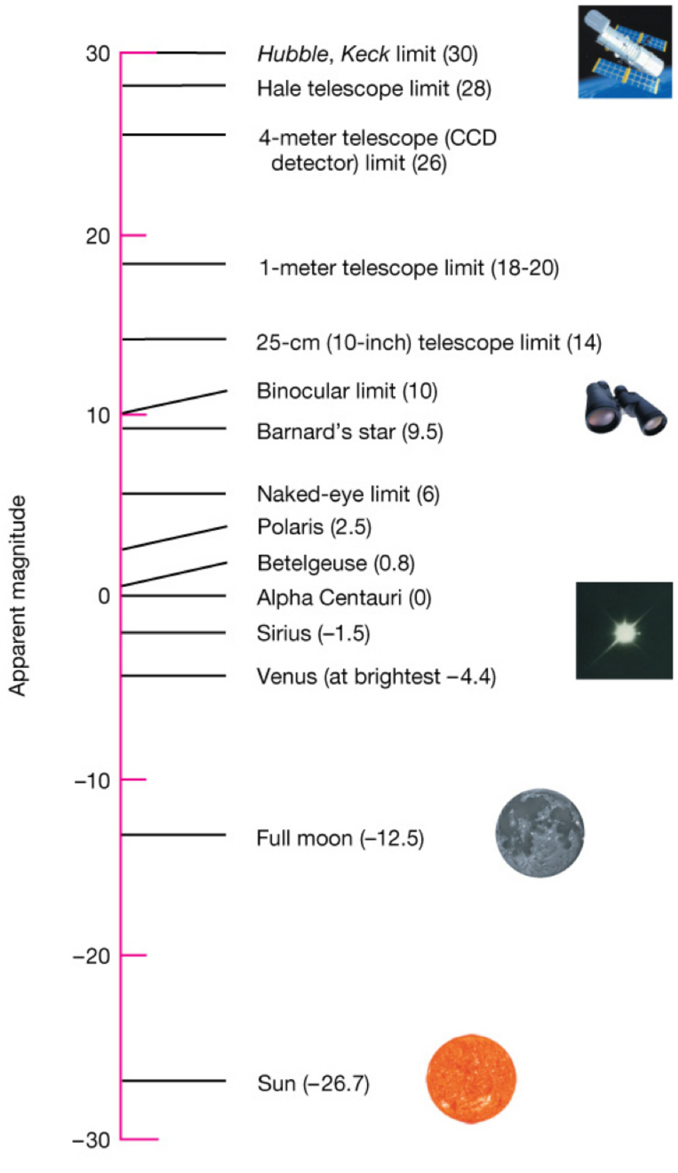
Magnitude (of Brightness)

A common way to measure brightness is not in terms of luminosity or flux, but in terms of **magnitude** (of brightness)

- ➢ The **absolute magnitude (M)** replaces luminosity
- ➢ The **apparent magnitude (m)** replaces flux

Magnitude runs “in reverse” – the larger the magnitude, the dimmer the object is.

- ➢ 0 is the magnitude of Vega
- ➢ 6 is the dimmest object the human eye can see
- ➢ Negative magnitude are objects brighter than Vega
- ➢ The Sun is the brightest object in the sky at -26.7



Brighter objects have smaller apparent magnitudes.

Converting Brightnesses

Apparent magnitude

$$M = m - 5\log d + 5$$

Absolute magnitude

Distance to star (in pc)

Absolute magnitude

$$L = 2.51^{4.83-M}$$

Luminosity (in $L_{\odot}$)

Example: Sirius

$$m = -1.46$$
$$d = 2.64 \text{ pc}$$

$$M = (-1.46) - 5\log(2.64) + 5$$
$$= 1.43$$

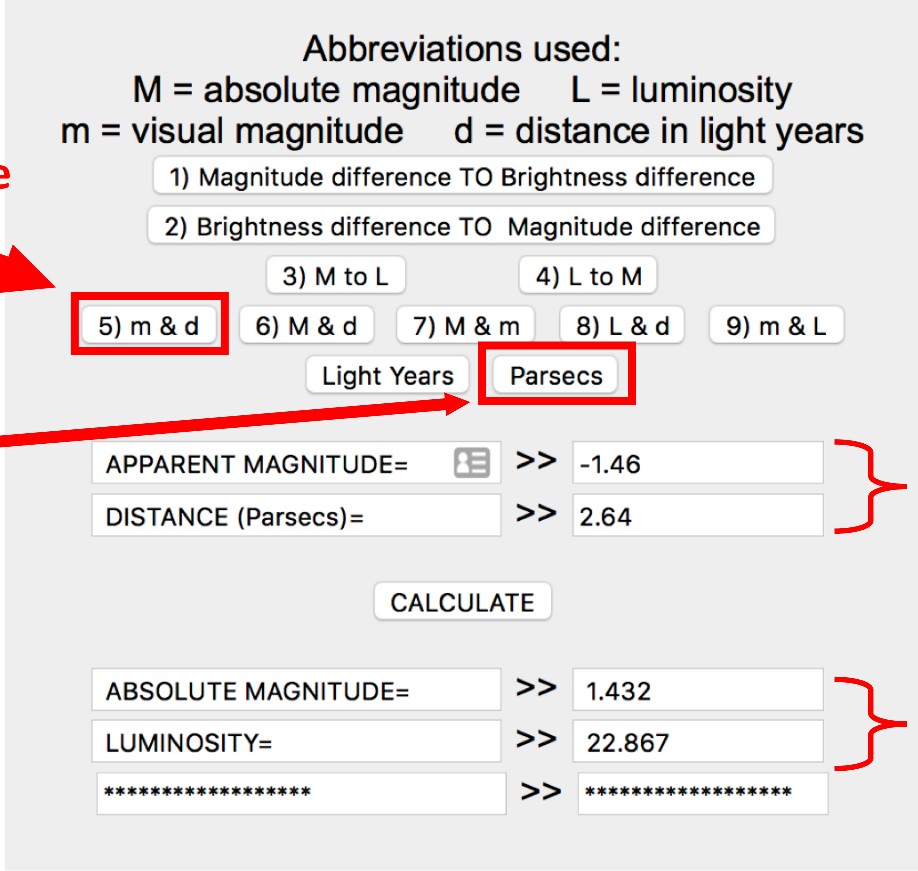
$$L = 2.51^{4.83-(1.43)} = 22.8L_{\odot}$$

Brightness Calculator

http://www.1728.org/magntudj.htm

Use apparent magnitude (m) and distance (d)

Use units of parsecs for distance



Input your apparent magnitude (m) and distance (d)

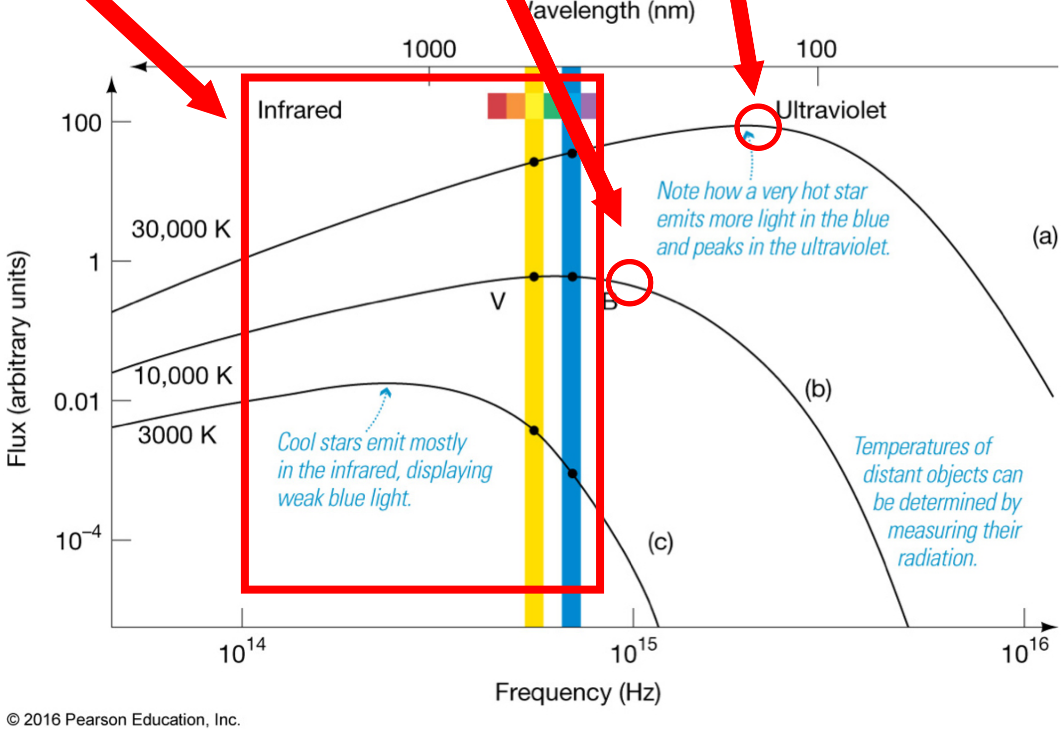
Output absolute magnitude (M) and luminosity ($L_\odot$)

Stellar Temperature

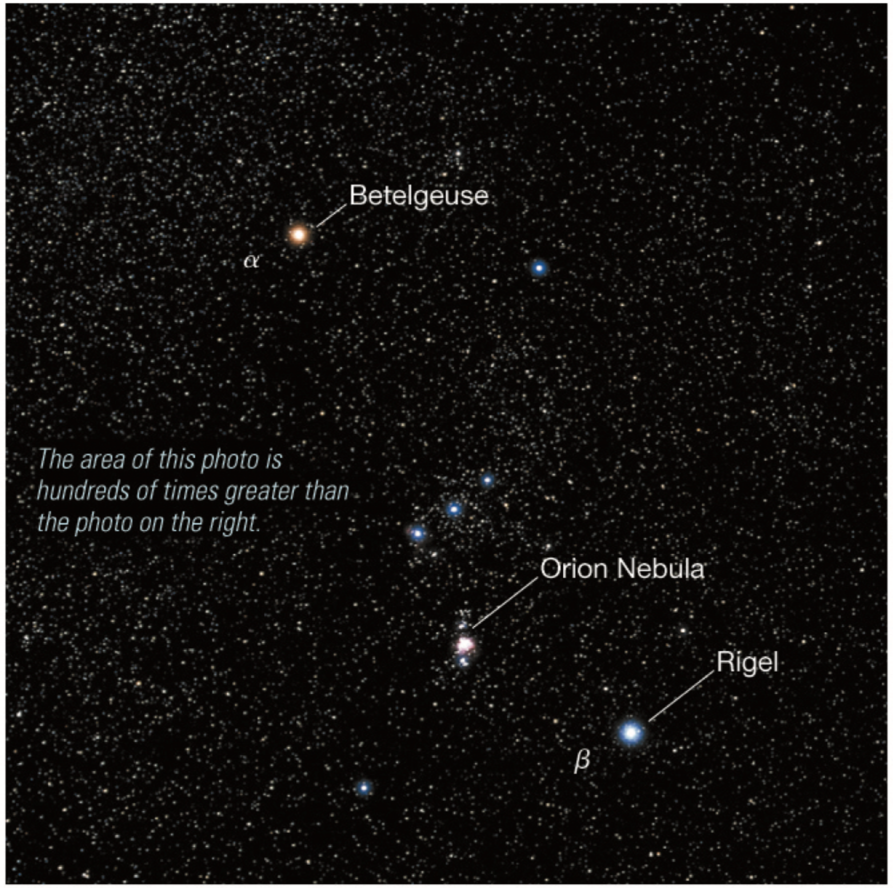
Wein's Law: $T = \dfrac{b}{\lambda_{\max}}$

Typical range to measure

What to do about peaks outside of this range?

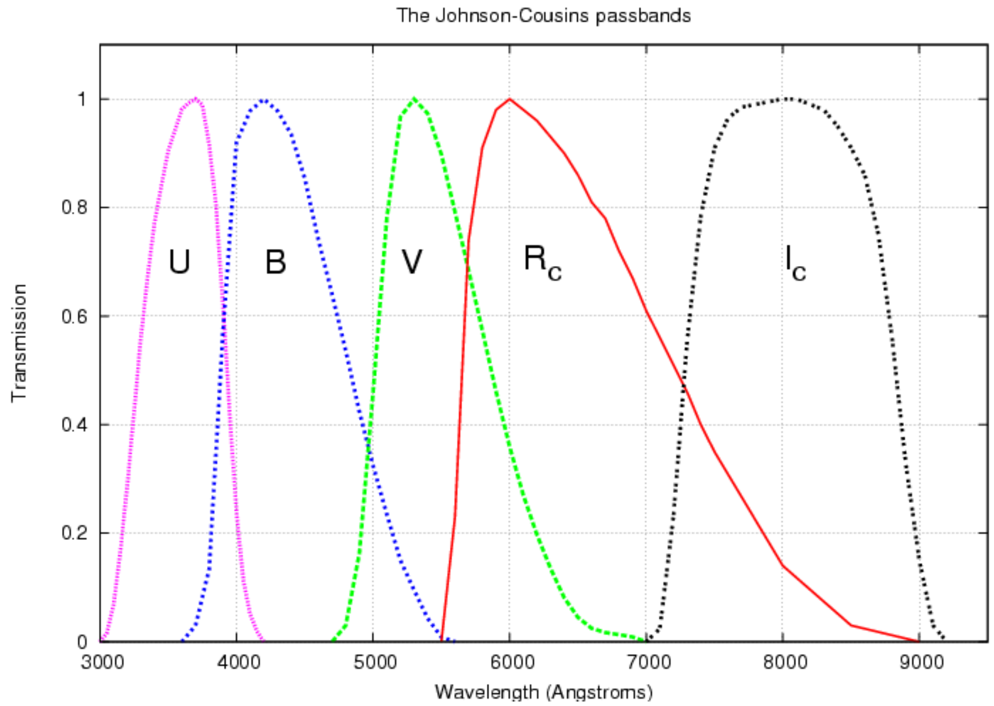


Need a method that doesn't depend on entire spectrum!



Temperature-Color Relationship

Johnson telescope filters, UBVRI



Apparent magnitude in B

Apparent magnitude in V

Color Index of star

$$B - V = m_B - m_V$$

Temperature-Color relationship of stars

$$T \approx \frac{9000\mathrm{K}}{(B - V) + 0.93}$$

Works for stars with $4000\mathrm{K} \leq T \leq 11000\mathrm{K}$, or $-0.1 \leq B - V \leq 1.4$

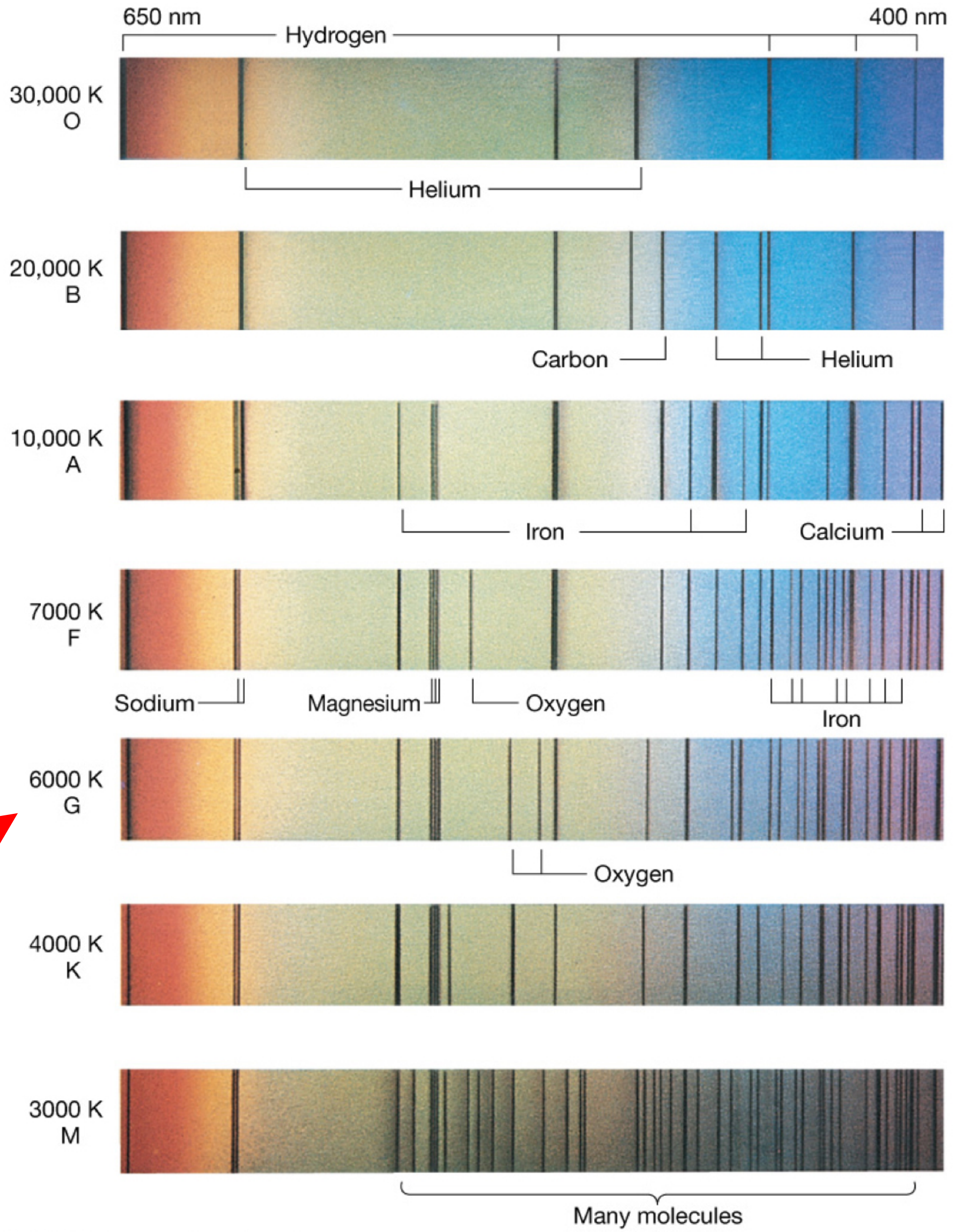
Stellar Classification

Stars are classified by their **emission spectra**, which varies with temperature. The common classification system is the **Harvard system**.

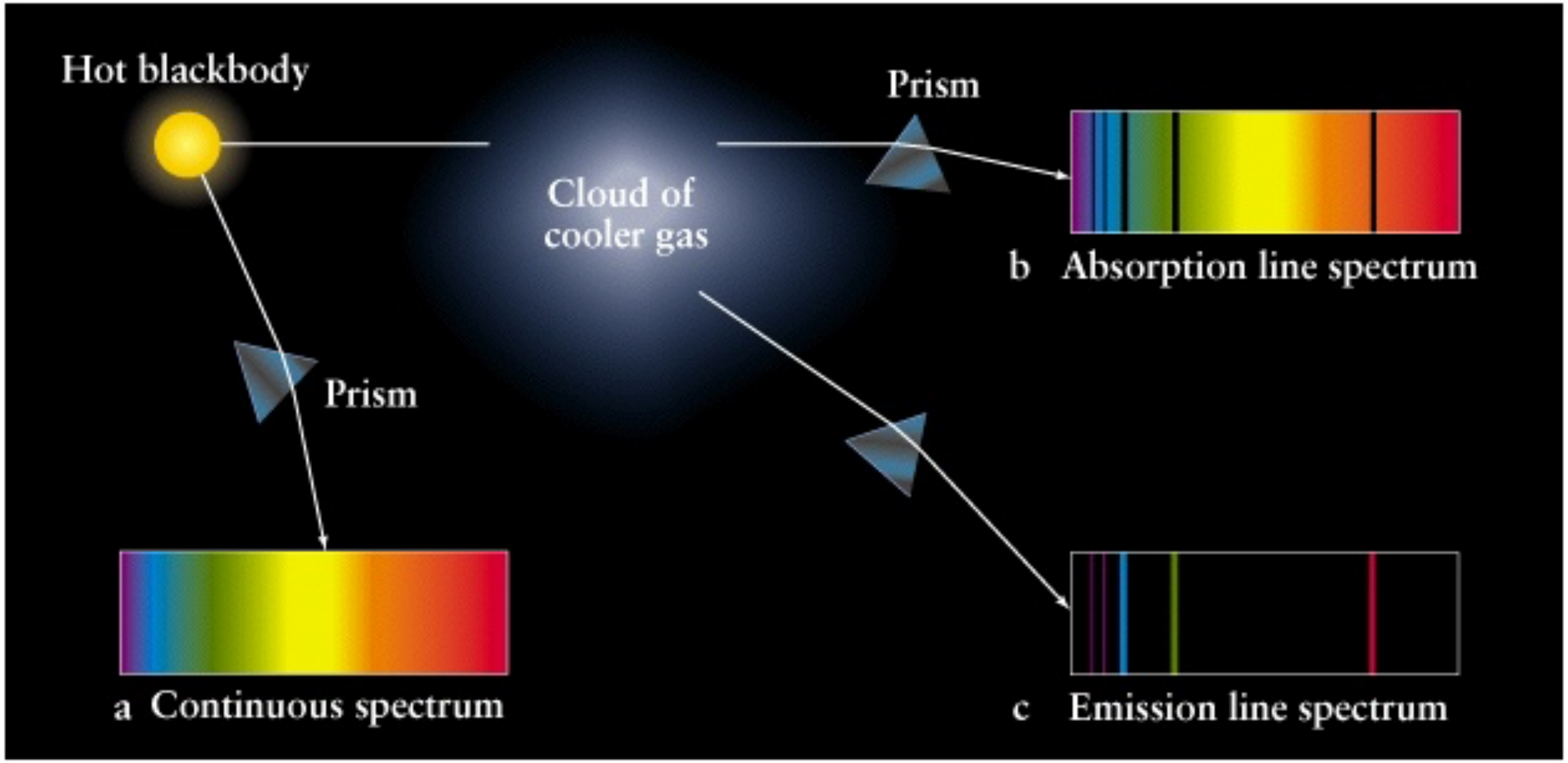
From **hottest-to-coldest**, the classes are:

- **O** (Oh)
- **B** (Be)
- **A** (A)
- **F** (Fine)
- **G** (Guy/Girl)
- **K** (and Kiss)
- **M** (Me)

Sun in a G-type star, with $T = 5800$ K

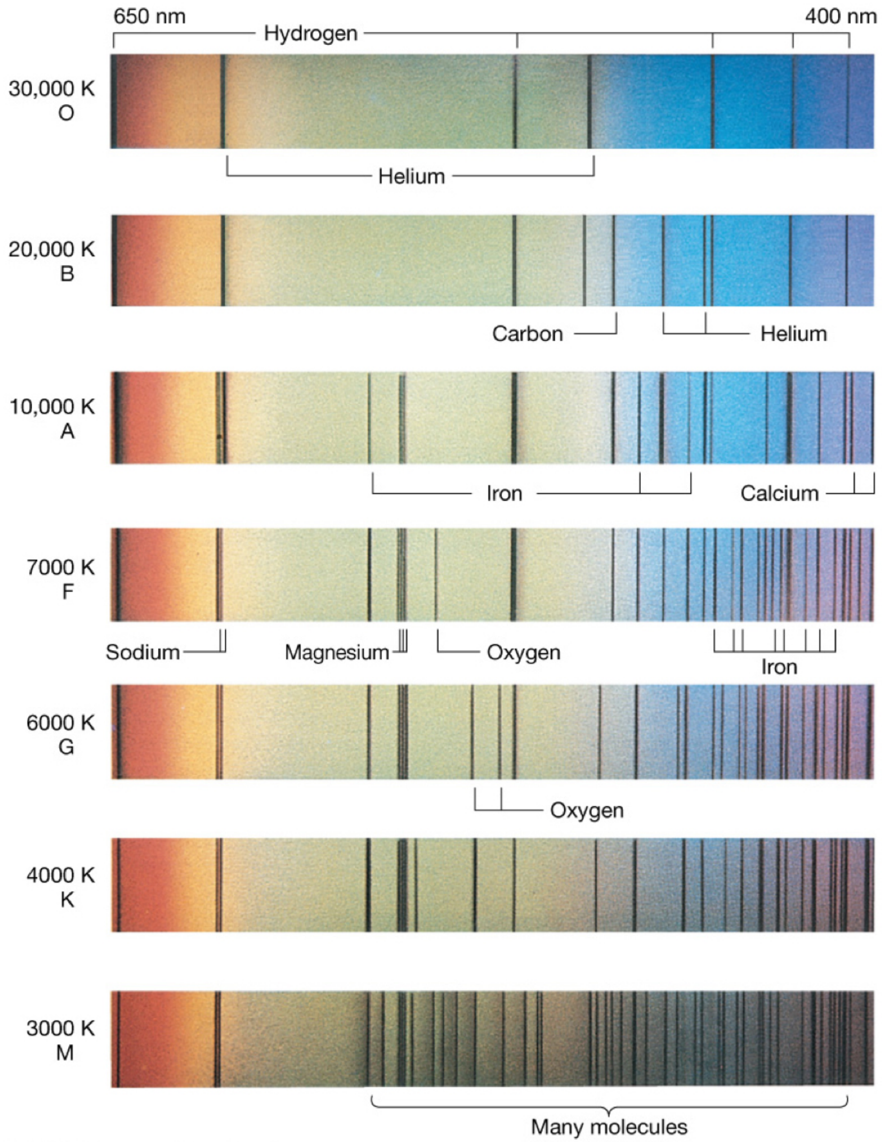


Reminder: Emission vs. Absorption Spectra

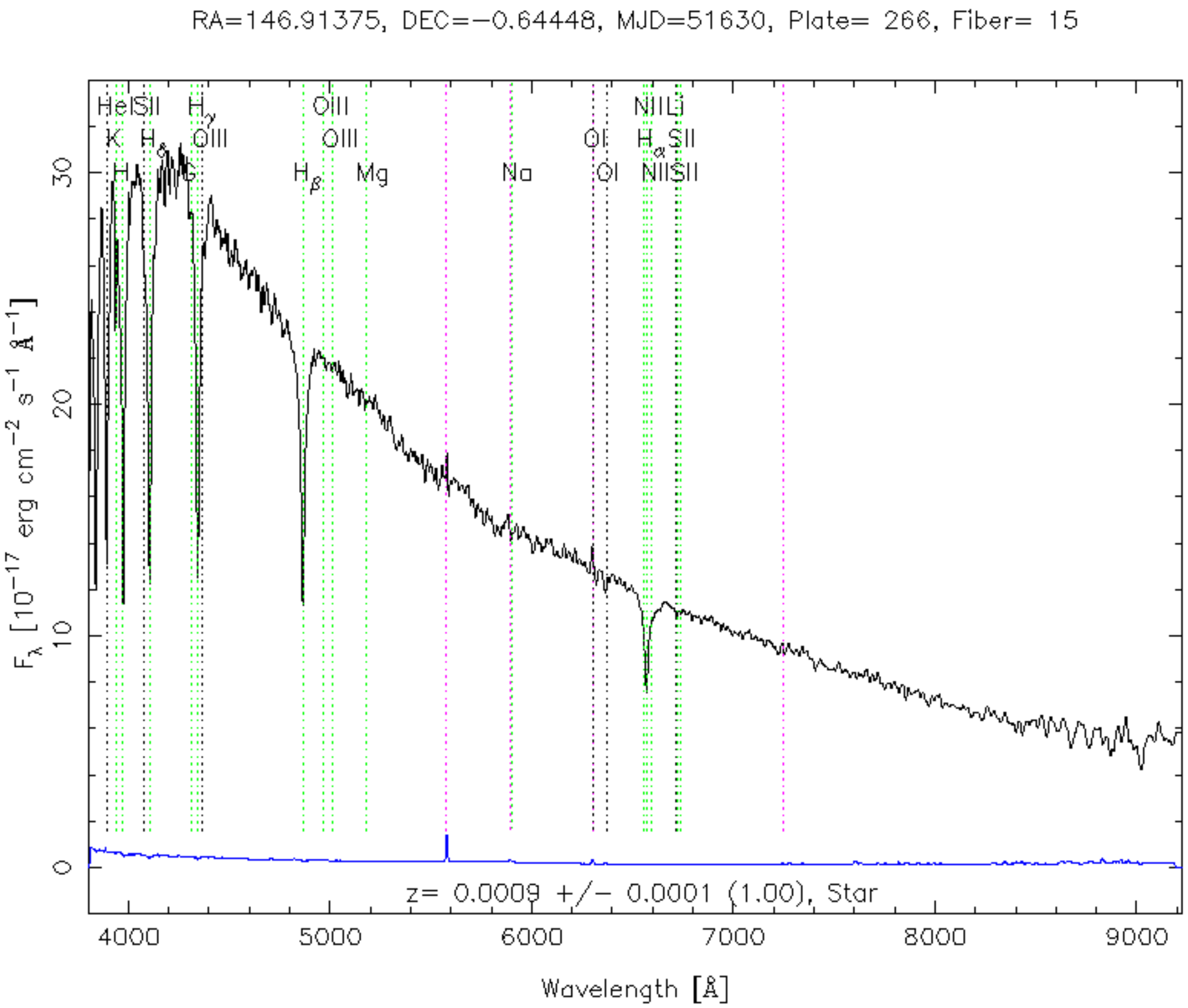


Spectral Class Characteristics

Spectral Type	Temperature (Kelvin)	Spectral Lines
O	28,000 - 50,000	Ionized helium
B	10,000 - 28,000	Helium, some hydrogen
A	7500 - 10,000	Strong hydrogen, some ionized metals
F	6000 - 7500	Hydrogen, ionized calcium (labeled H and K on spectra) and iron
G	5000 - 6000	Neutral and ionized metals, especially calcium; strong G band
K	3500 - 5000	Neutral metals, sodium
M	2500 - 3500	Strong titanium oxide, very strong sodium



Example 1: Spectrum of Unknown Star



Look for hydrogen lines first!

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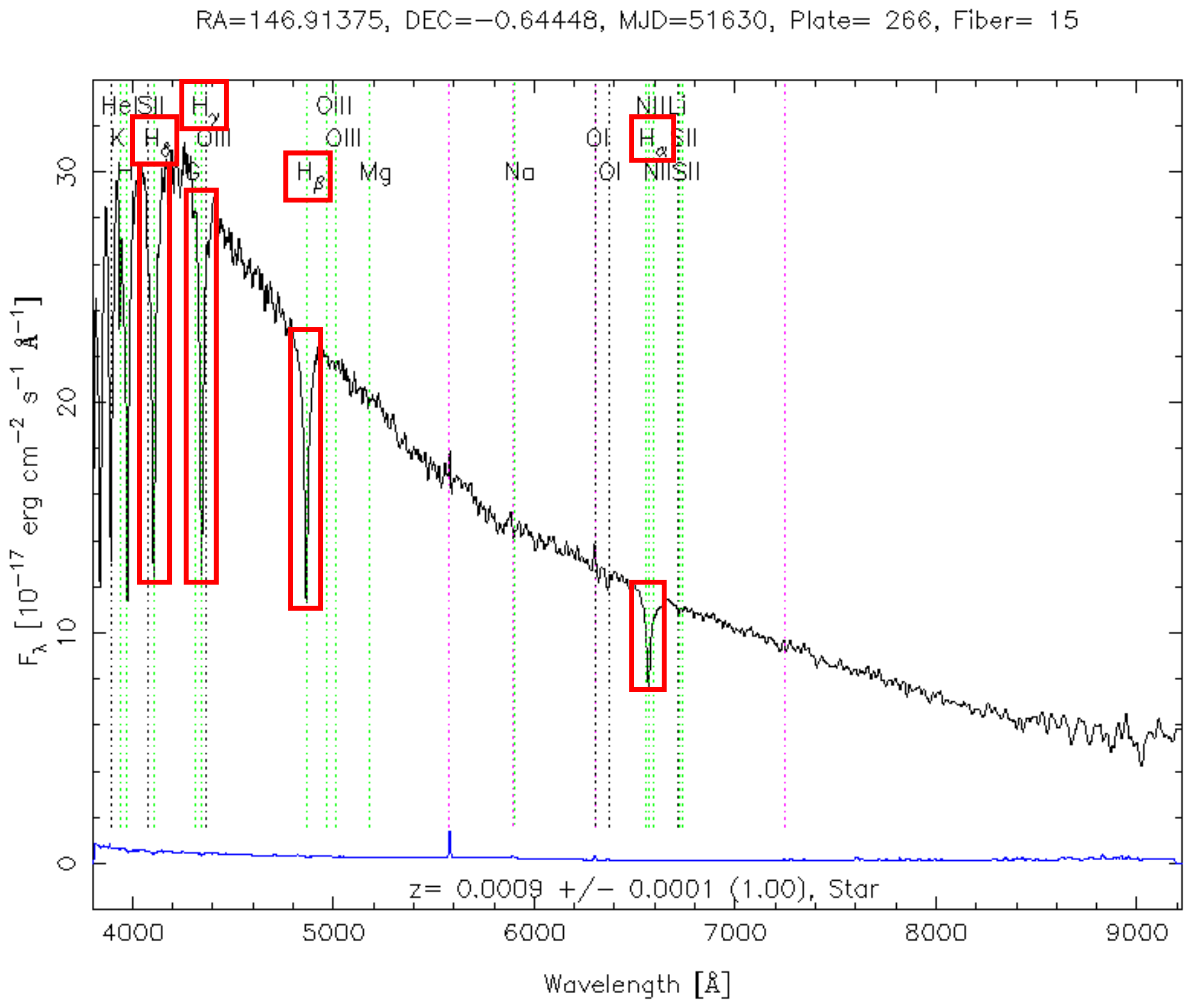

Hydrogen lines
→ B, A, or F type star

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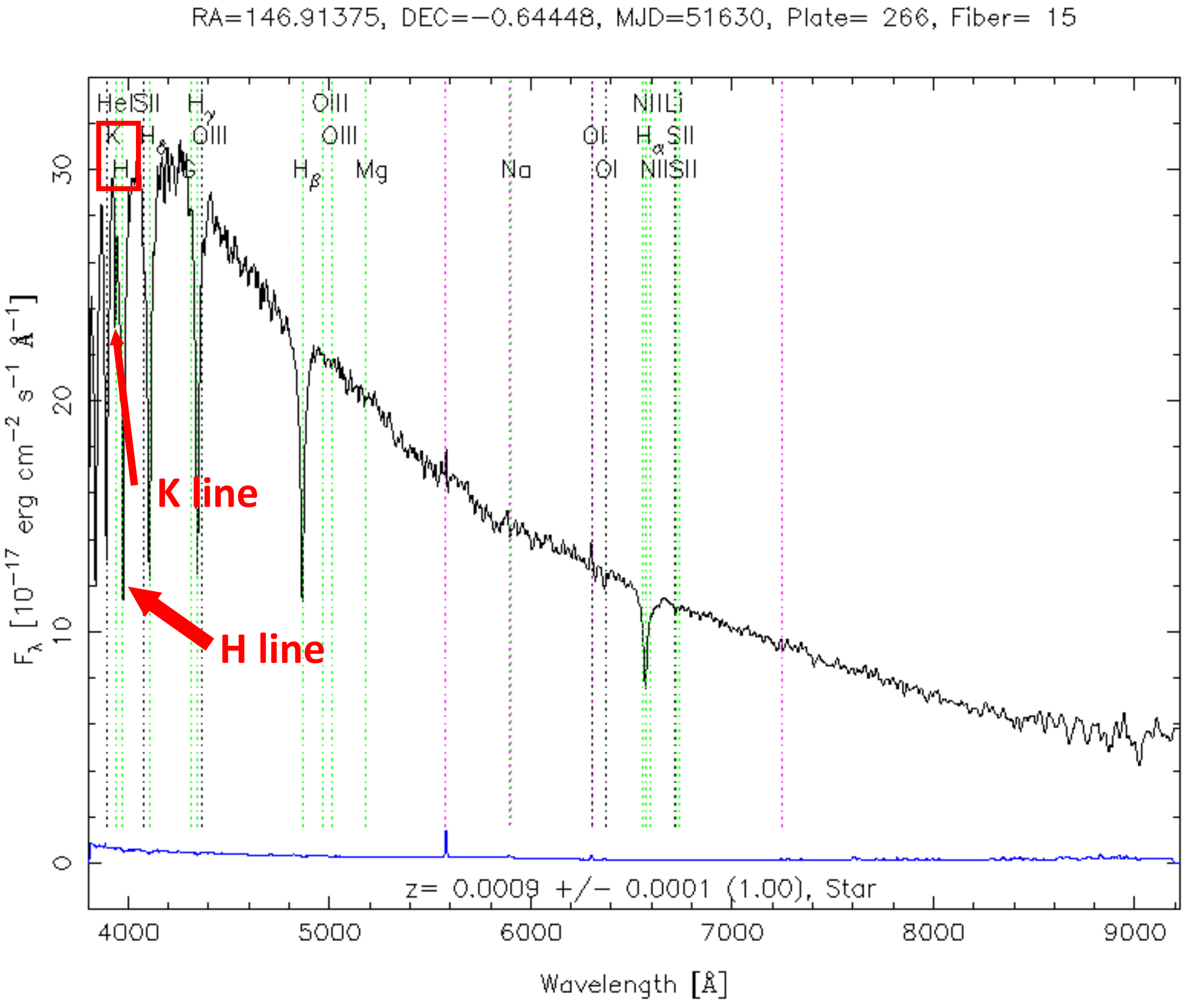

H and K lines present
→ F type star!

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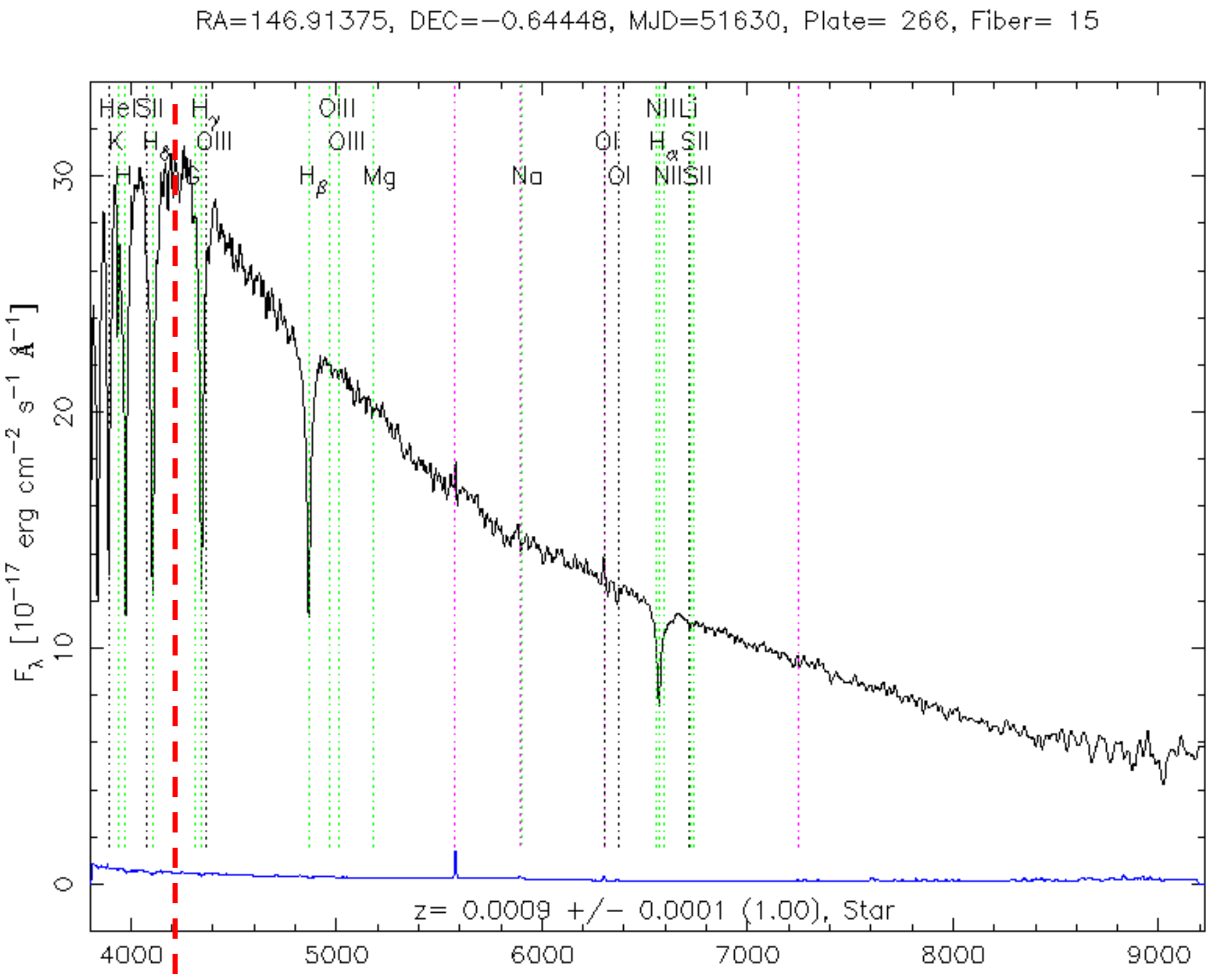

$\lambda_{max} \approx 420$ nm

Confirmation with Wein's Law:

$$T = \frac{b}{\lambda_{max}} = \frac{2.9\times10^{6}\ \text{nm}\cdot\text{K}}{420\ \text{nm}} = \boxed{6905\ \text{K}}$$

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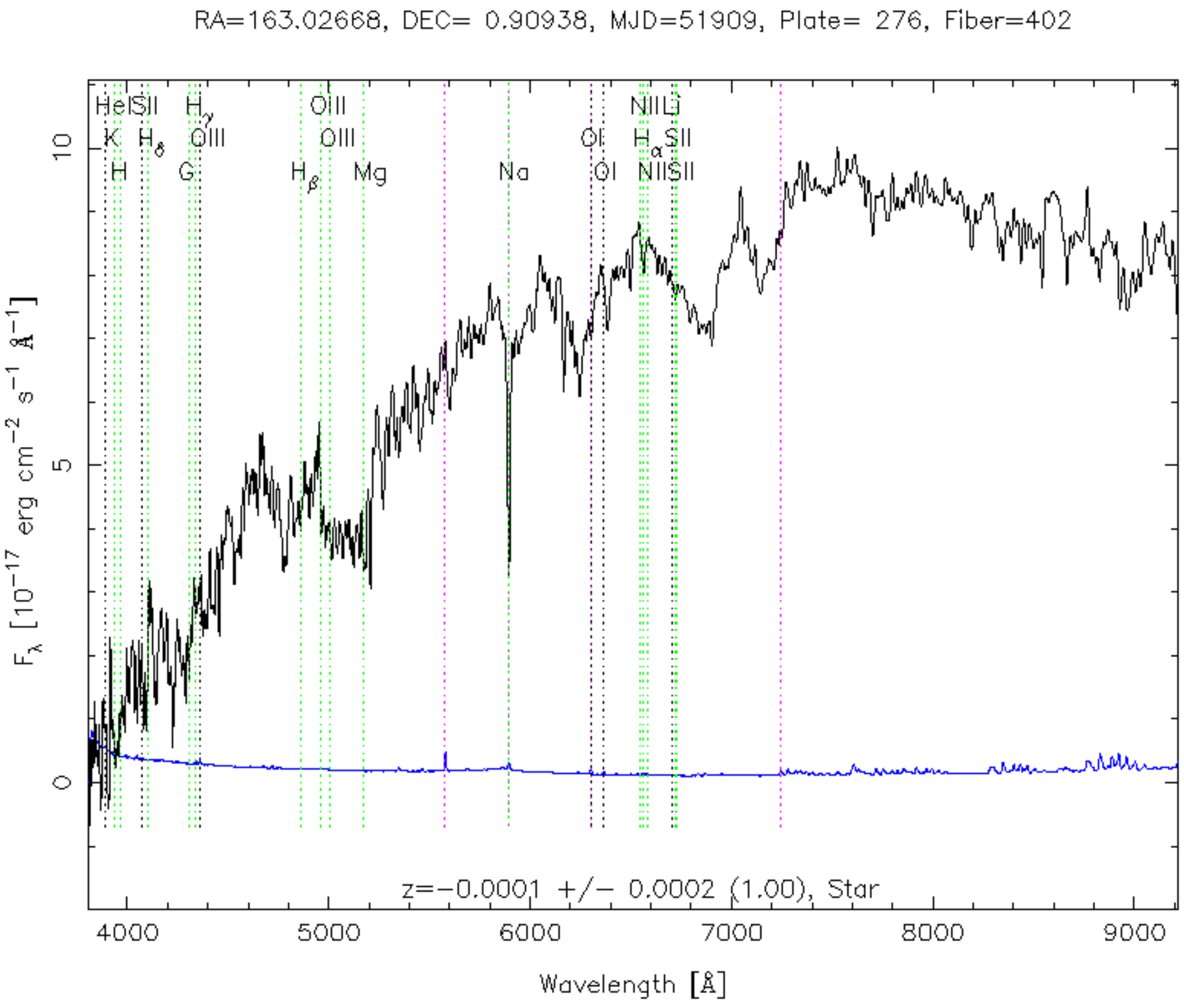
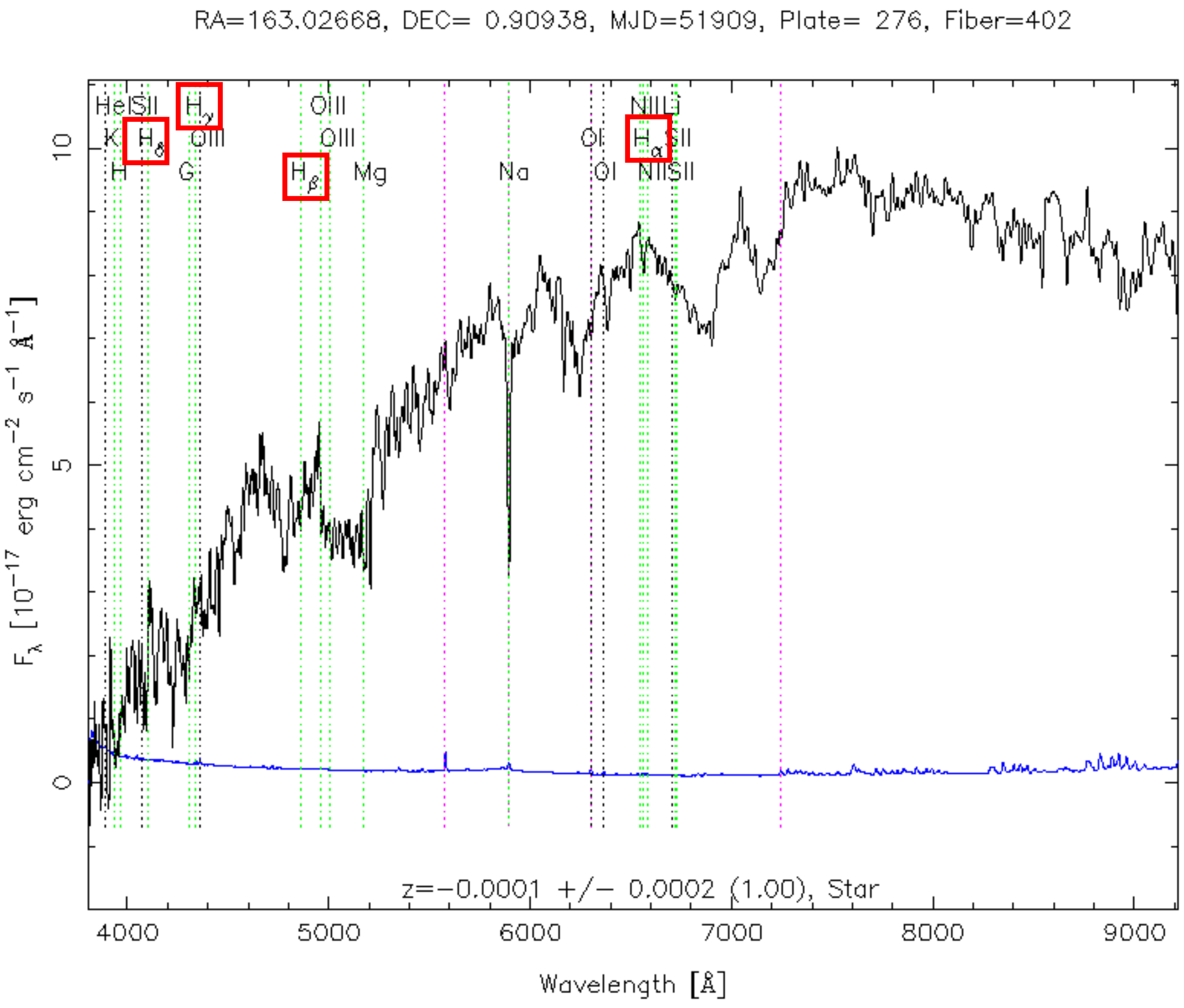


Look for hydrogen lines first!

Example 2: Spectrum of Unknown Star



No hydrogen lines

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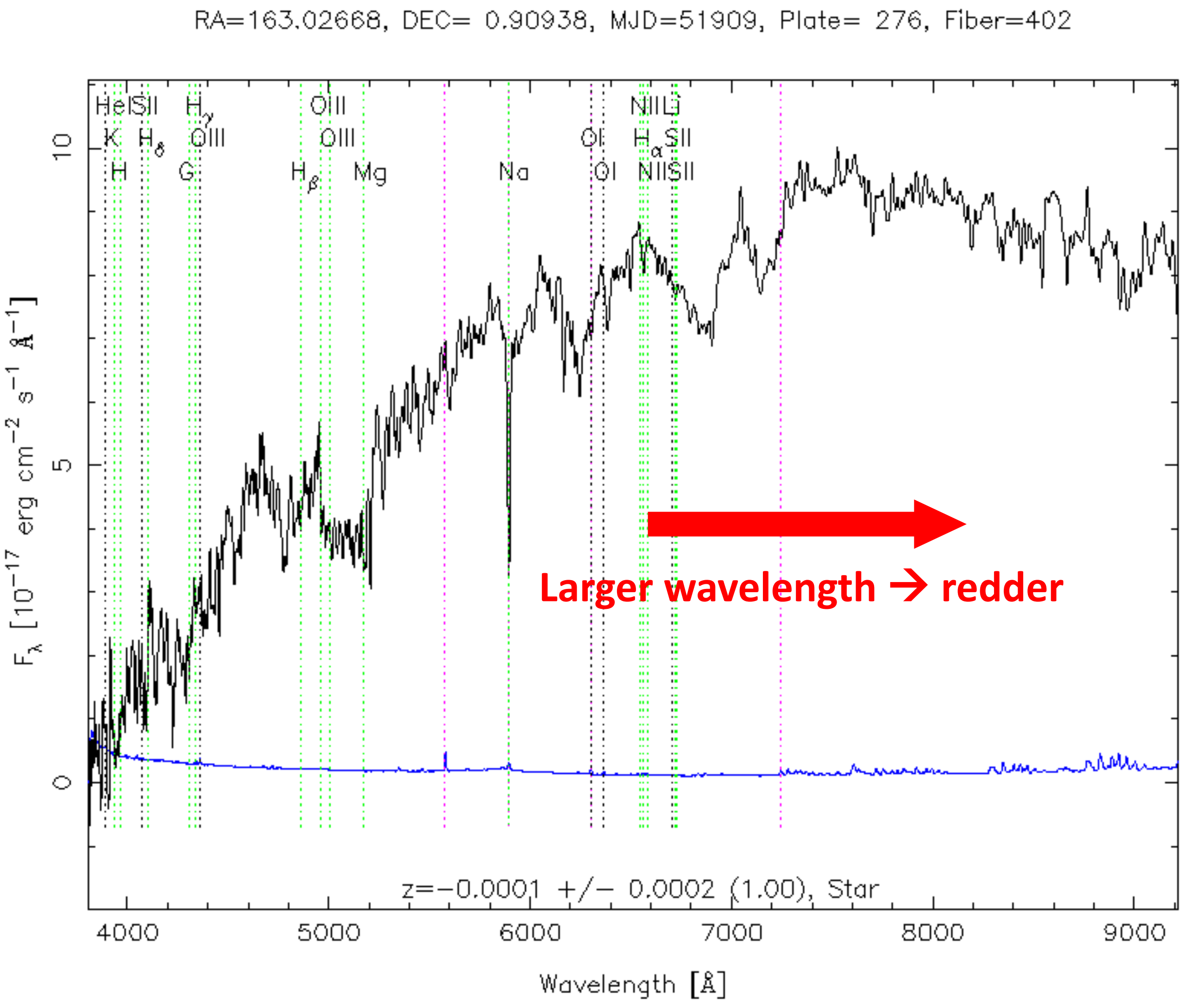

Redder than previous F-type star → colder than F-type!

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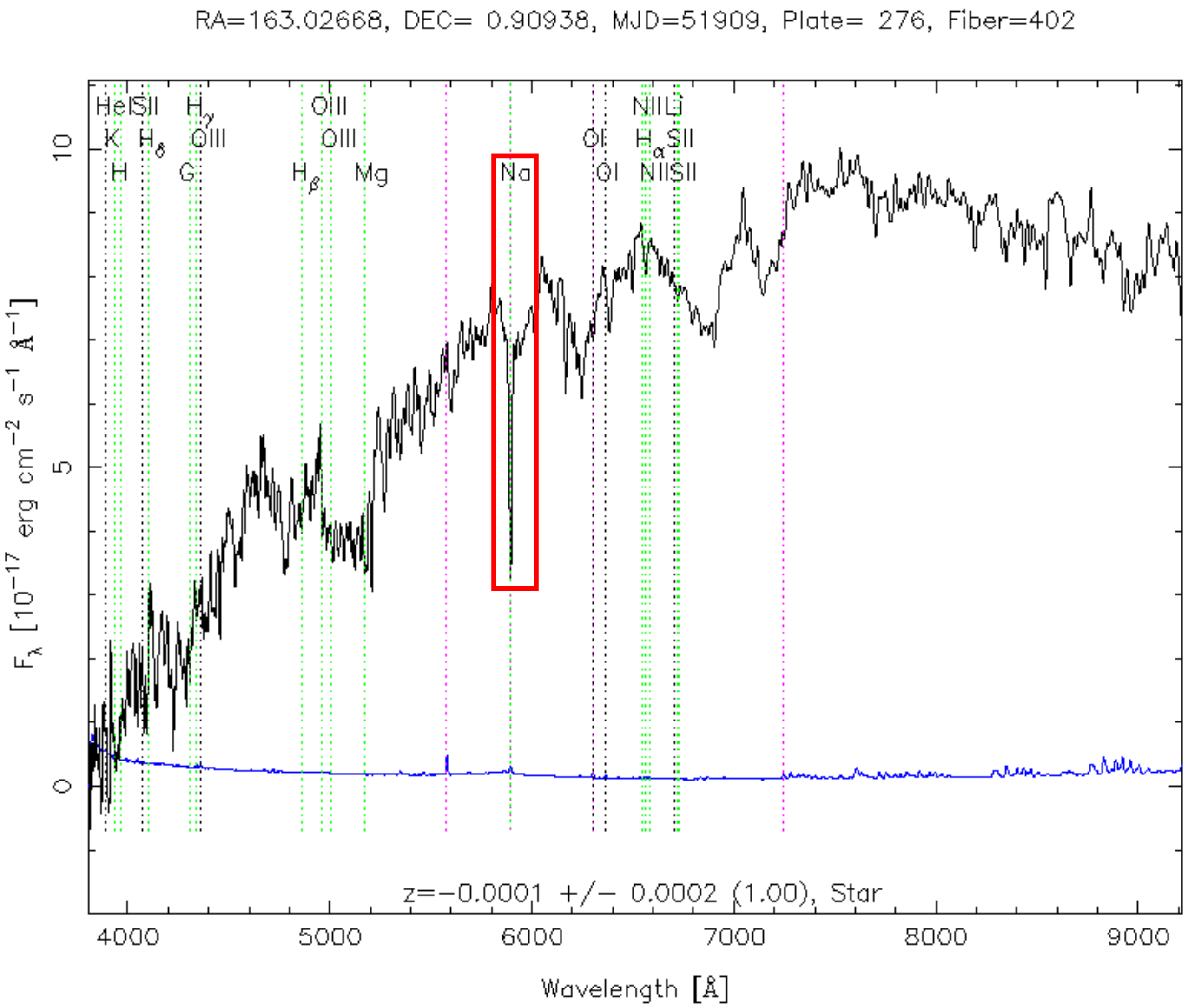

Prominent sodium (Na) line → K or M type star

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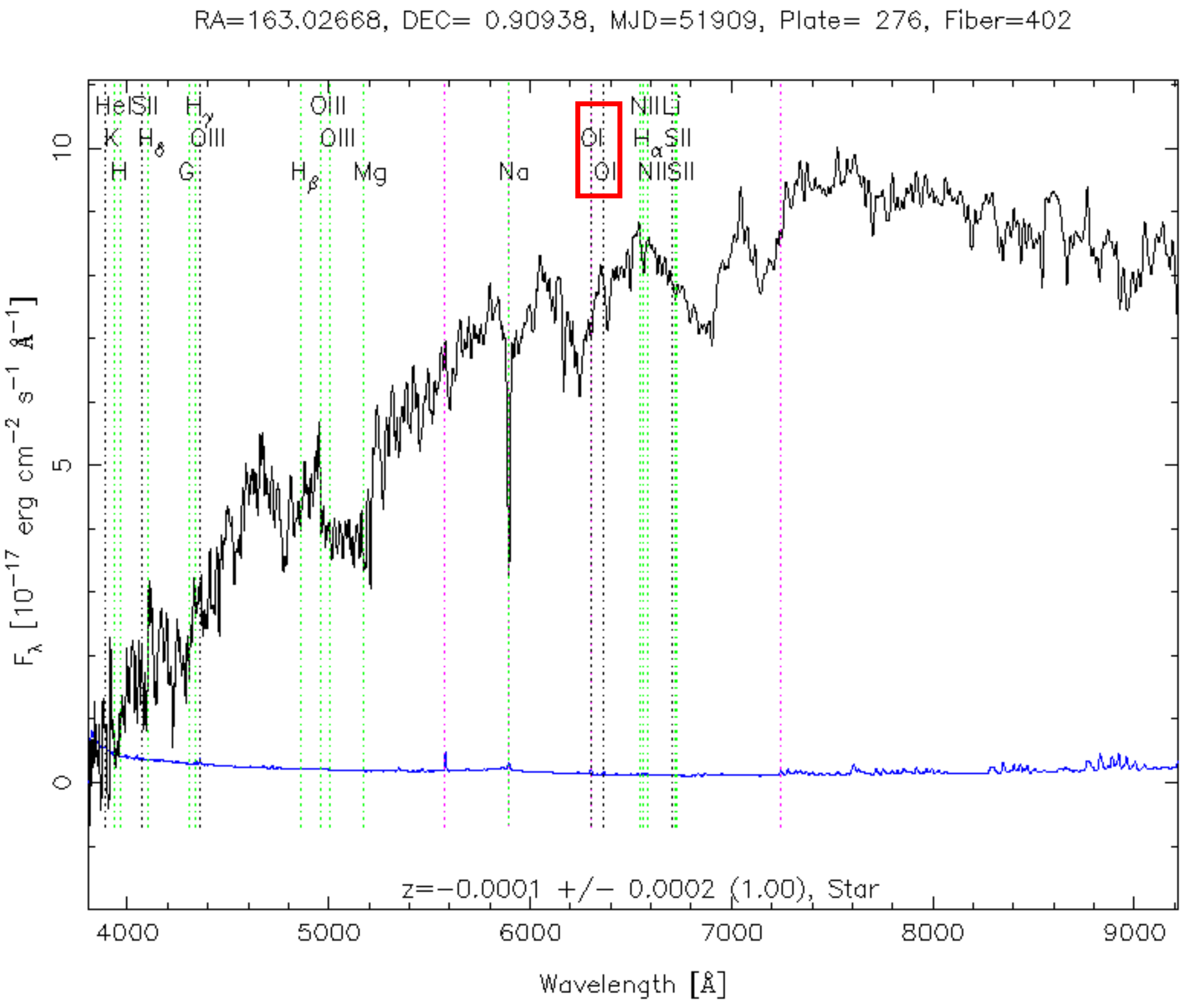

No titanium-oxide (TiO) lines → K-type star!

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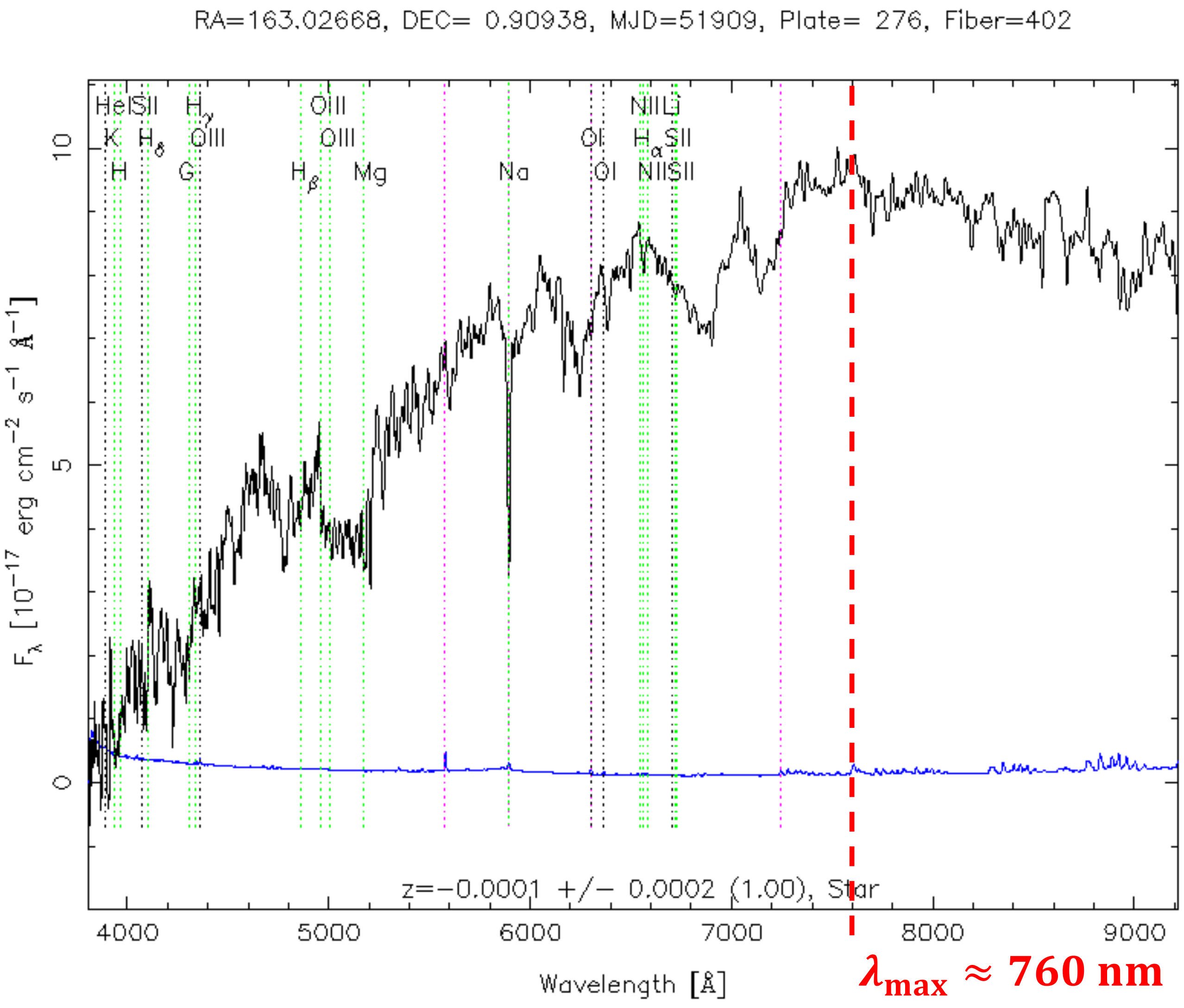

Confirmation with Wein's Law:

$$T = \frac{b}{\lambda_{max}} = \frac{2.9\times10^{6}\ \mathrm{nm}\cdot\mathrm{K}}{760\ \mathrm{nm}} = \boxed{3816\ \mathrm{K}}$$

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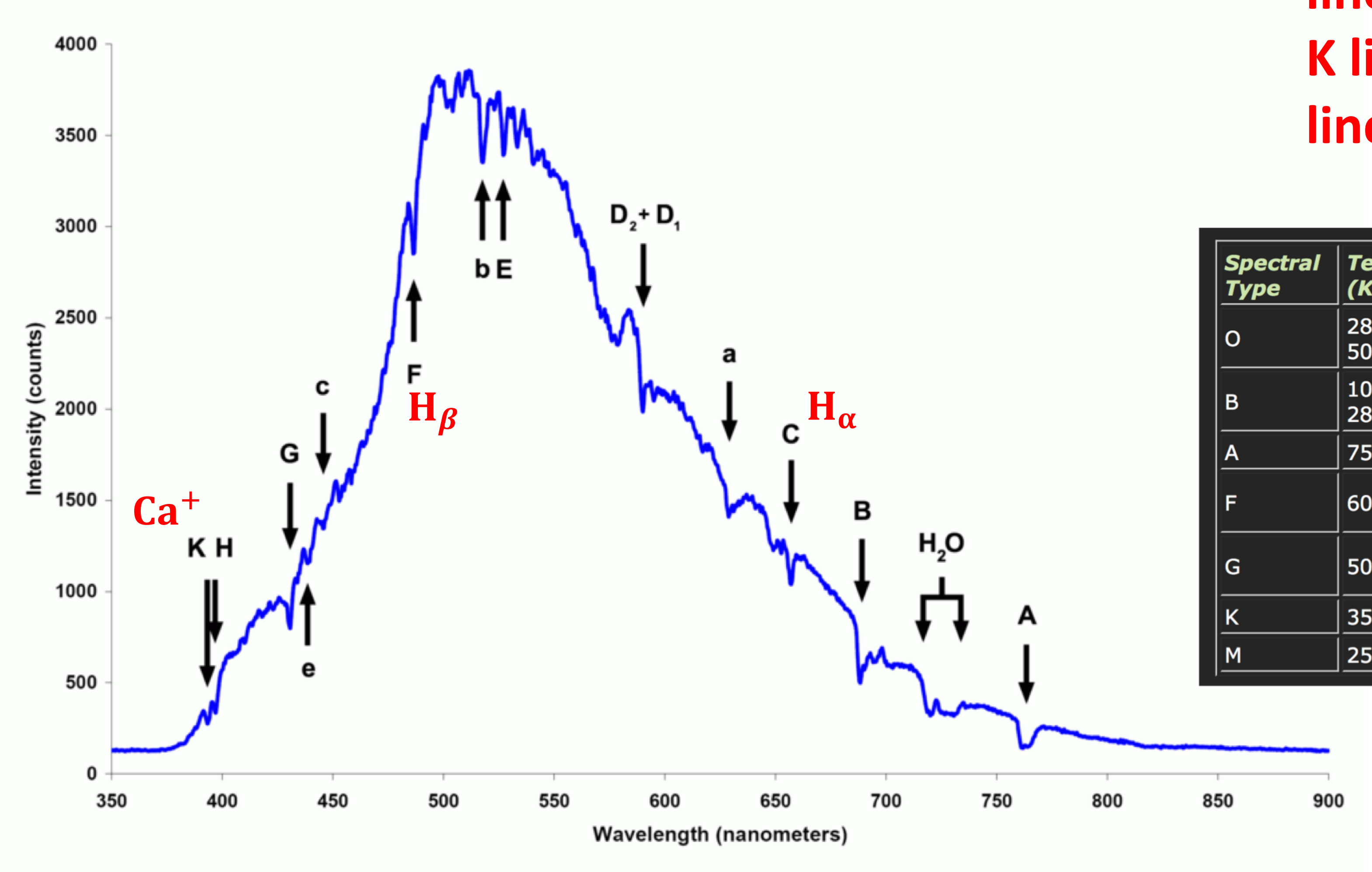
Example 3: The Sun



Weak/missing hydrogen lines, ionized calcium(H & K lines) present, strong G line → G-type star!

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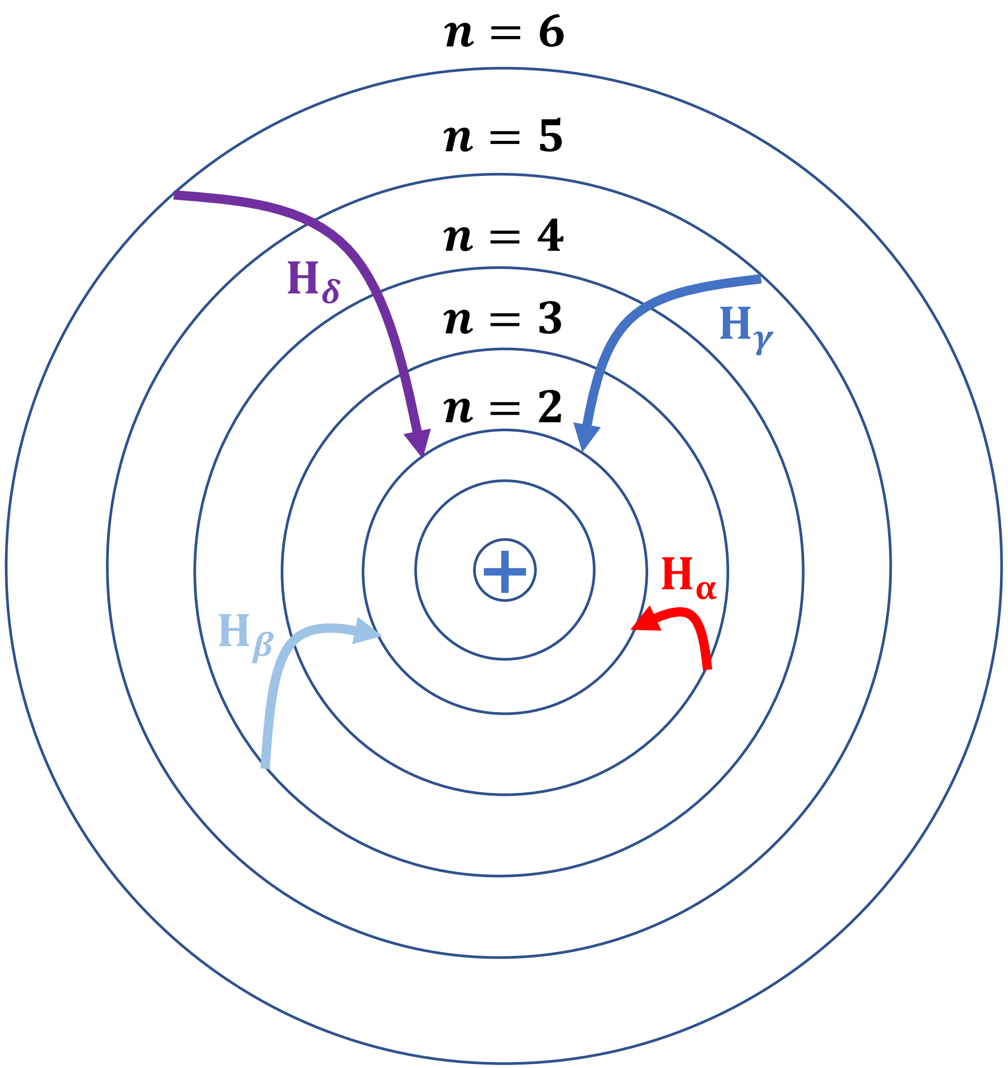
Why do Spectra Depend on Temperature?



The visible lines produced by hydrogen are known as the **Balmer lines**, and are the 4 electron transitions shown. For an electron to be "happy" in the $n = 2$ state, $T \approx 10{,}000$ K.

- At temperatures near this value, hydrogen emission is strong
- At temperatures much below this, electrons hang out in the $n = 1$ state, so you don't get Balmer lines
- At temperatures much above this, electrons typically escape from hydrogen, and so can't produce any emission lines

Reminder: Stefan-Boltzmann Law

A blackbody with a surface temperature T and has a **surface brightness $\boldsymbol{B}$** given by:

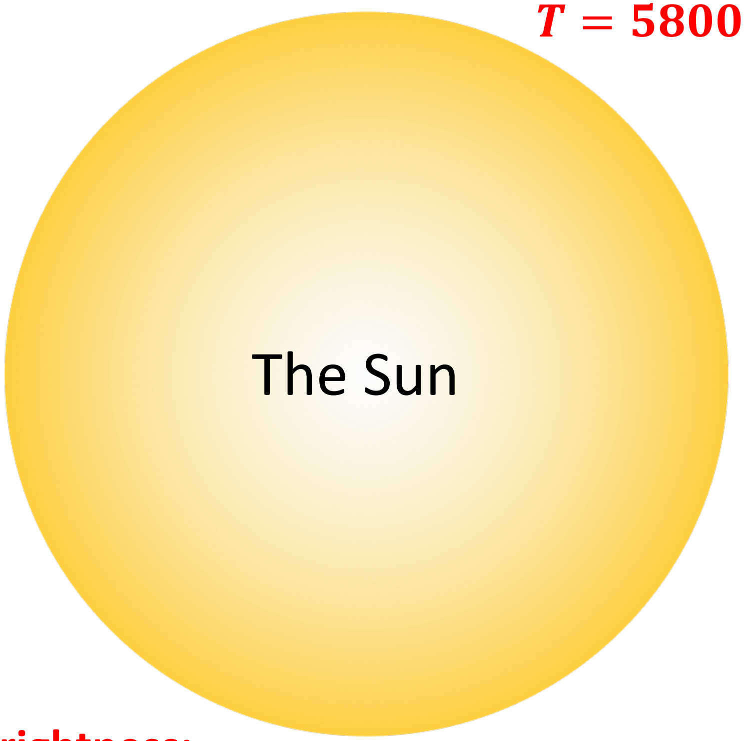
Surface brightness (W/m²)

Temperature (K)

$$B = \sigma T^4$$

$$\boldsymbol{\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}}$$

$\boldsymbol{T = 5800\ \mathrm{K}}$



Surface brightness:

$$\boldsymbol{B = \left(5.67 \times 10^{-8}\right)(5800)^4 = 6.42 \times 10^{7}\ \mathrm{W/m^2}}$$

Stellar Radii

If we can find the **luminosity** and the **surface brightness (B)**, a type of flux, of a star, then we can figure out its **radius**:

Stefan-Boltzmann law

Definition of flux

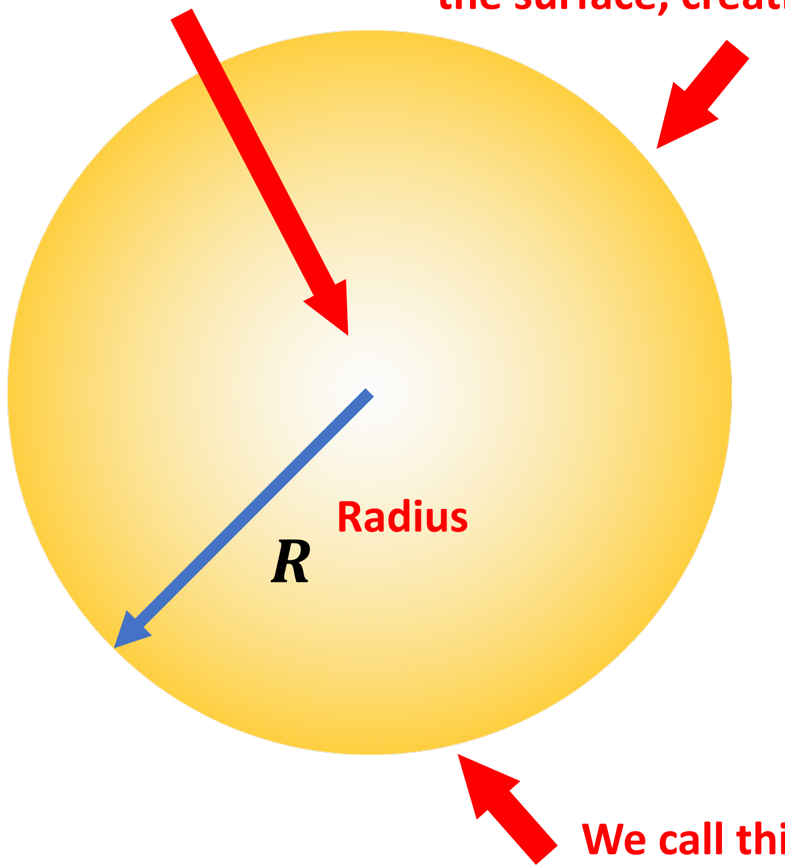
$$B = \boxed{\sigma T^4} = \boxed{\frac{L}{4\pi R^2}}$$

$$\Rightarrow \quad R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

This is sometimes called the "radius-luminosity-temperature" (RLT) relationship, for lack of a better name.

We treat the luminosity as coming from the center

Which spreads out over the surface, creating a flux

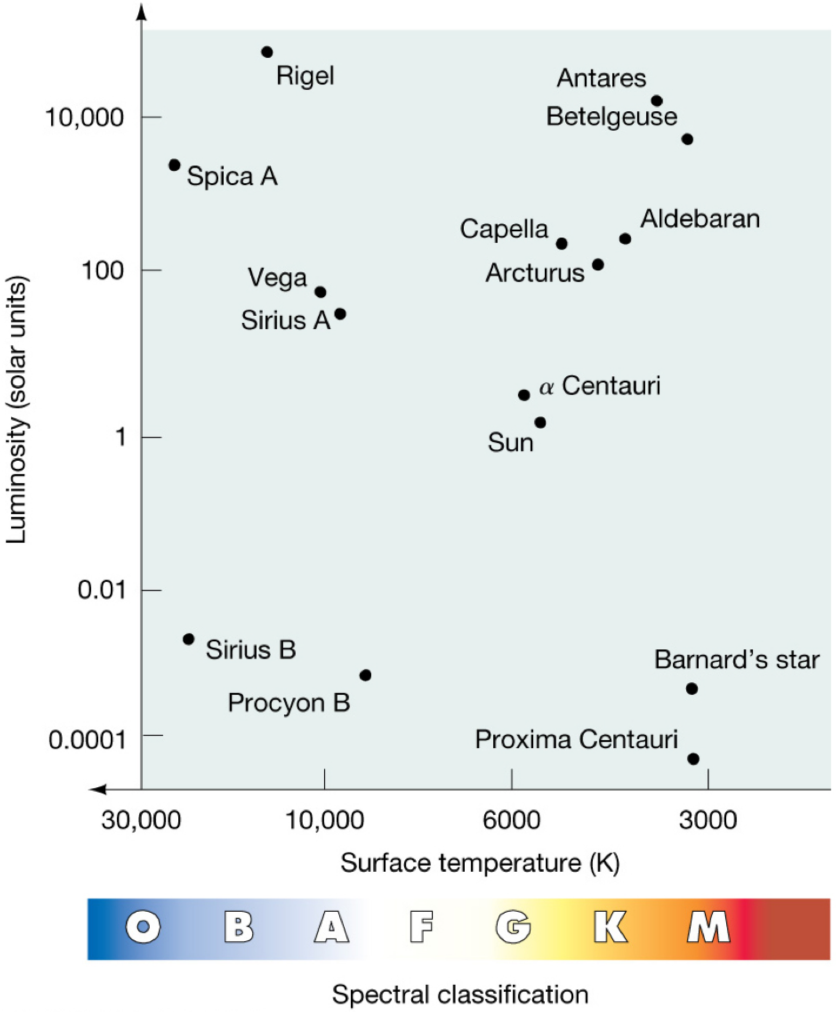


We call this flux the surface brightness B

Trends in Stellar Characteristics

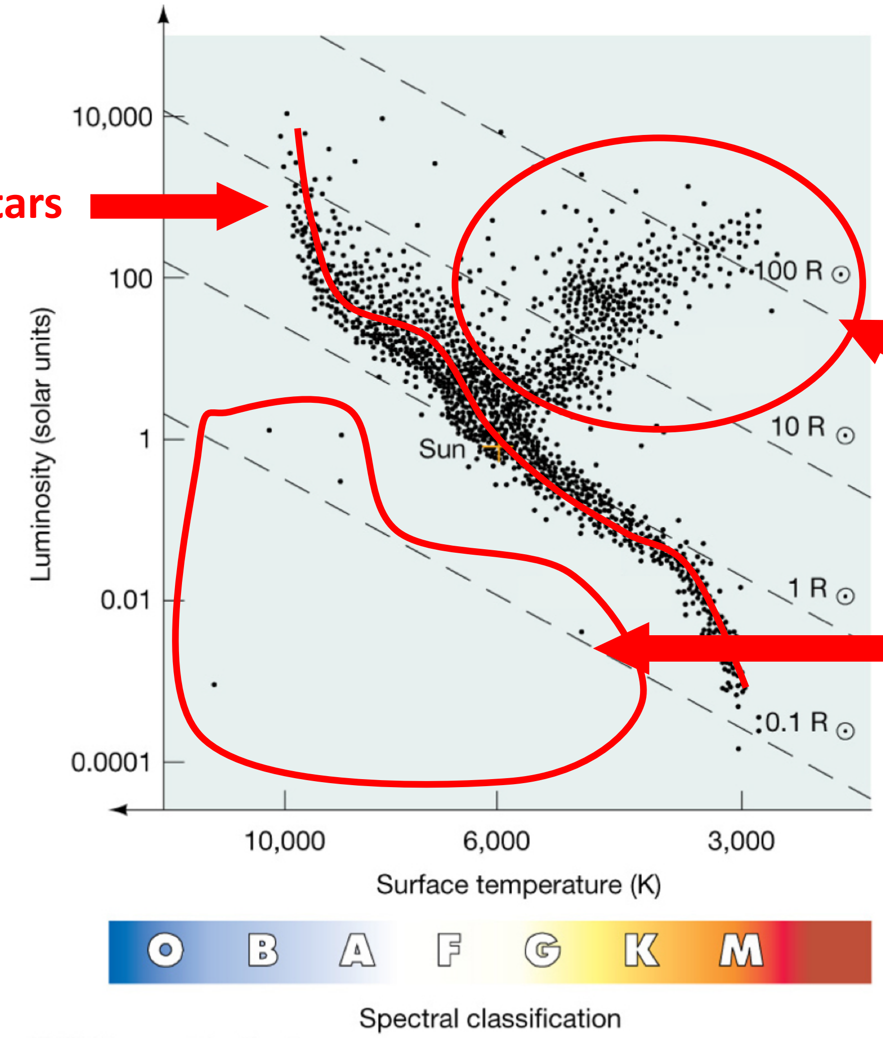
Plotting **luminosity vs. temperature** for our nearest neighbors doesn't look like much, as you can see in the figure. But if we expand this to farther stars, and stars at different points in their evolution, we find very clear trends in these characteristics.

- Note that while we're plotting luminosity vs. temperature, we're actually plotting against spectral class, so temperature increases to the **left** of the diagram, not to the right.
- Stars are going to be, essentially, of two types: **main sequence stars**, which are healthy stars burning hydrogen, and **off sequence stars**, which are at other points in their evolution.



The Hertzsprung-Russell (H-R) Diagram

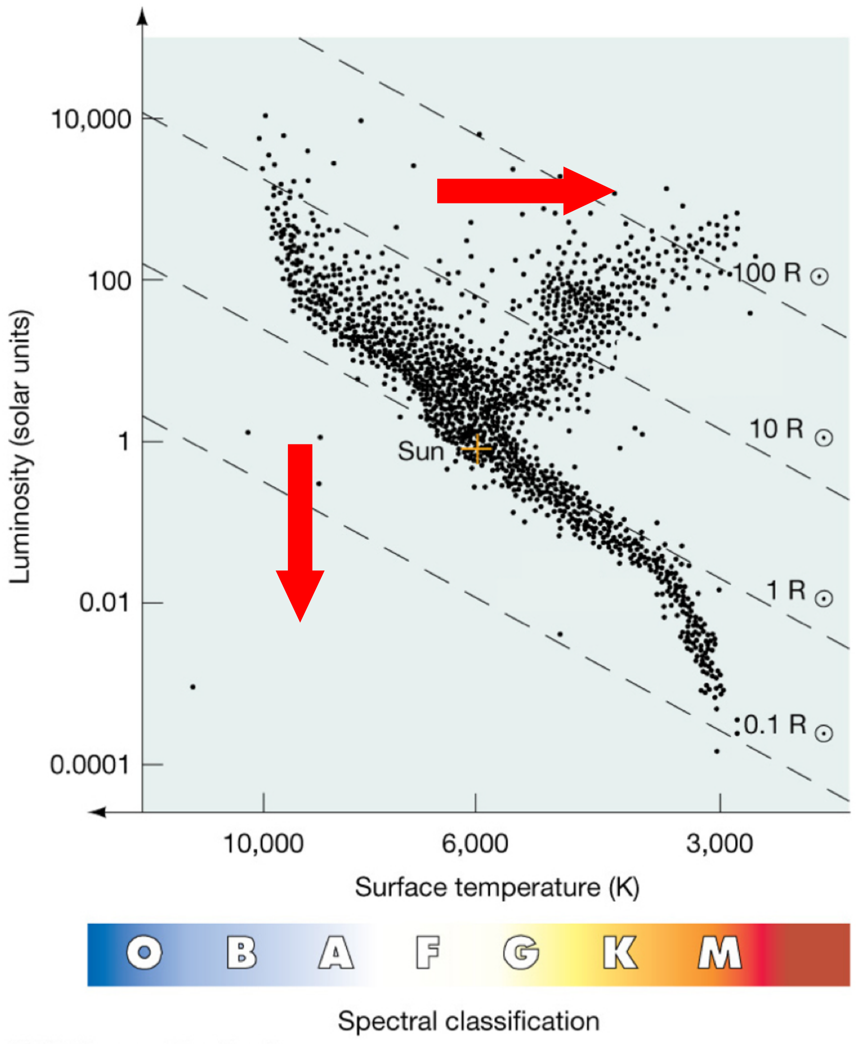
Healthy, main sequence stars



Two regions of off-sequence stars, which are at different points in their evolution after they run out of hydrogen to burn

Off-Sequence Regions

RLT Relationship: $R = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

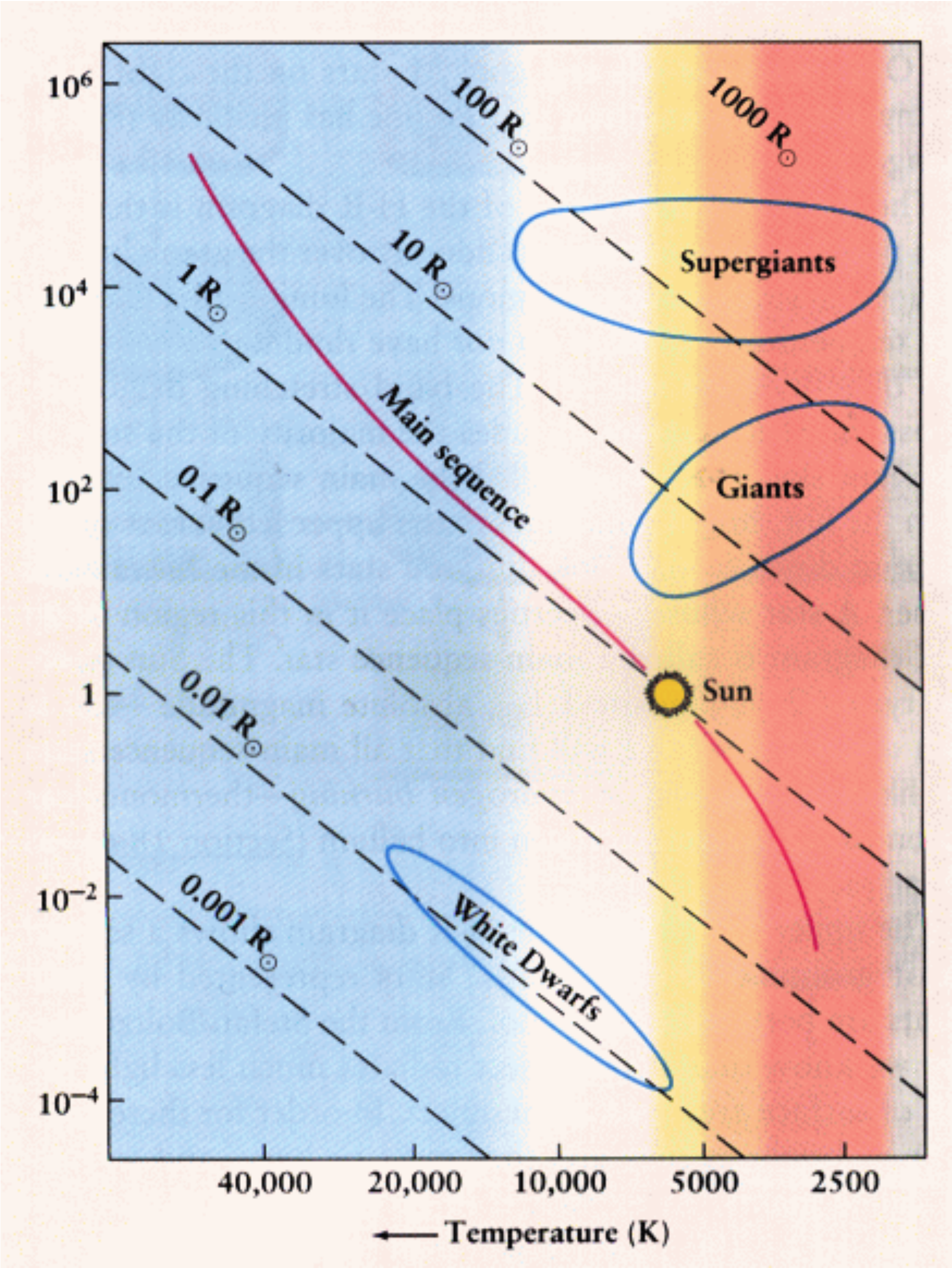


Moving to the right means decreasing T while keeping L constant. This means that R is going to increase. So, this region is known as the giant region, and stars within it are either giants or supergiants.

Moving down means keeping T constants while decreasing L. This means that R is going to decrease. Thus, this region is known as the dwarf region.

H-R Diagram in Detail

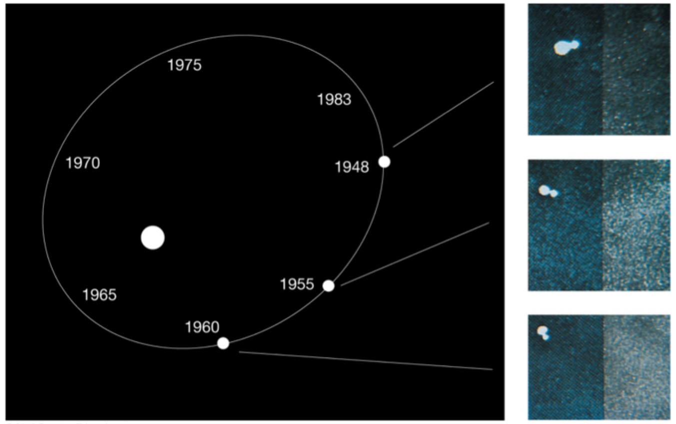
- 91% of stars are main sequence stars
- Along the main sequence, the highest percentage of stars are red, and the percentage drops as you move to the blue stars, which have the lowest percentage
- Giants and supergiants tend to be red and compose less than 1% of all stars
- The remaining 8% of stars are dwarfs, which can be blue, white, or red
- Because of the RLT relationship, knowing luminosity and temperature allows us to find the radius. The diagonal lines represent lines of constant radius on the diagram.

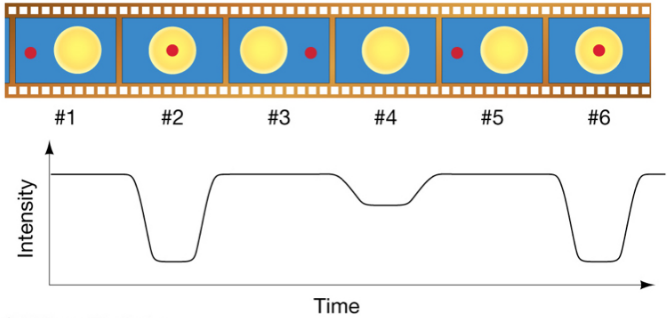


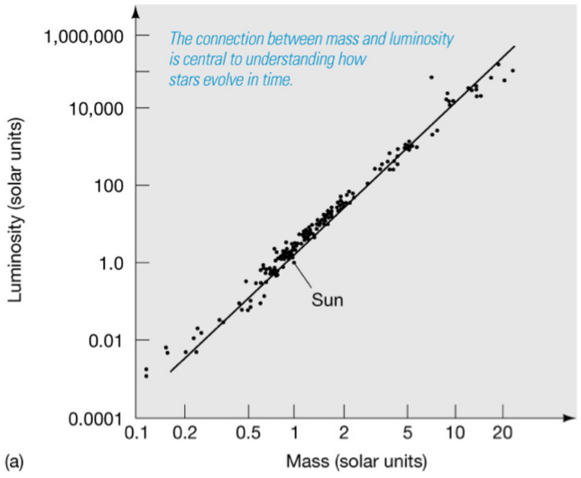
Stellar Mass

There are 3 main ways we'll try to find a star's mass:

1. **Stellar binaries**. Two stars that are orbiting a common center (a **barycenter**). These are typically called **visual binaries**.
2. **Stars with satellite**. Many stars have satellites like our Sun does; they could be planets like our Sun, or other stars. These are usually called **eclipsing binaries**.
3. **The Mass-Luminosity Relationship**. This is an empirical relationship that is different for different masses of stars, so before you can apply it, you have to have a good guess as to what the mass probably is.







Mass-Luminosity Relationship

$$L = M^4$$

Luminosity in units of $L_\odot$

Mass in units of $M_\odot$