PHY2048 Exam 1 (12PM)

September 25, 2019

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Abstract

This exam consists of 28 multiple choice questions. **You must record your answers on a Scantron sheet.** Don't record your answers on this print-out; I will not accept it as a submission. Fill out the Scantron sheet in with a pencil, not a pen. **Don't forget to include your name, the course, and exam number on the Scantron sheet.** Please, **turn in this print-out** with your Scantron sheet.

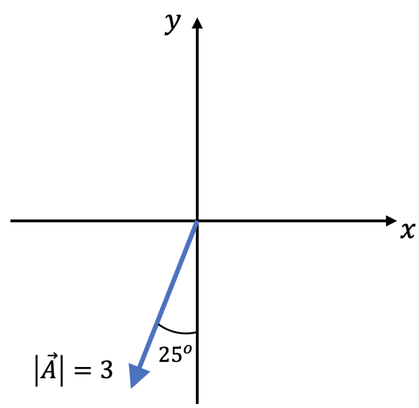


Figure 1: Figure for Problem 1

1. What is the x-component of the vector $\vec{A}$, shown in Figure 1 above?

   (a) -1.27
   (b) 1.27
   (c) -2.72
   (d) 2.72

2. In what direction does the position vector $\vec{r} = (5.1\text{m})\hat{i} - (3.2\text{m})\hat{j}$ point? Measure the angle **counter-clockwise from the $+x$-axis**.

   (a) 32^o
   (b) 58^o
   (c) 148^o
   (d) 328^o

3. Consider two vectors: $\vec{A}$ has a magnitude of 4.5 and direction of 57^o counter-clockwise from the $+x$-axis, and $\vec{B}$ has a magnitude of 2 and direction of 35^o clockwise from the $+x$-axis. What is the y-component of $2\vec{A} - 3\vec{B}$?

   (a) 2.6
   (b) 4.1
   (c) 4.9
   (d) 11.0

4. Consider the vectors

$$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$$
$$\vec{B} = -2\hat{\imath} + 5\hat{k}$$

   What is $\vec{A} \cdot \vec{B}$?

   (a) 4
   (b) 14
   (c) 16
   (d) 26

5. Consider a force $\vec{F} = -(2.5\text{N})\hat{\imath} + (3.7\text{N})\hat{\jmath}$ and a position $\vec{r} = (5.1\text{m})\hat{\imath} - (3.2\text{m})\hat{\jmath}$. What is the cross product $\vec{r} \times \vec{F}$?

   (a) $-(10.9\ \text{Nm})\hat{k}$
   (b) $(10.9\ \text{Nm})\hat{k}$
   (c) $-(26.9\ \text{Nm})\hat{k}$
   (d) $(26.9\ \text{Nm})\hat{k}$

6. Under what conditions can kinematics be used?

   (a) Kinematics can always be used
   (b) Only if the acceleration of an object is constant
   (c) Only if the speed of an object is constant
   (d) Only if the motion of an object is in a straight line

7. A runner jogs 100m east in 30s, then sprints 50m north in 10s, then jogs back to her starting point in an additional 50s. During her trip, what was the runner's average speed?

   (a) 0 m/s
   (b) 2.9 m/s
   (c) 3.3 m/s
   (d) 5 m/s

8. An airplane has a landing speed of 80 m/s, and can brake at a rate of 2 m/s^2. What is the shortest runway the airplane can land on safely?

   (a) 800m
   (b) 1600m
   (c) 2400m
   (d) 3200m

9. A Bugatti Veyron is well-known to accelerate from 0 to 60mph (27 m/s) in 2.5s. How far does the Veyron travel during these 2.5s, assuming its acceleration is constant?

   (a) 33.8m
   (b) 60m
   (c) 75m
   (d) 94.6m

10. Imagine you drop a rock down a well of unknown depth, and hear water splash after 1.2s. How deep would you conclude the well was?

   (a) 6m
   (b) 7.2m
   (c) 12m
   (d) 14.4m

11. Your friend is on a 2nd story balcony, while you are 4m below on the ground. If he asks you to toss something up to him, with what minimum speed must you throw the object for it to reach him?

   (a) 4.5 m/s
   (b) 6.3 m/s
   (c) 7.7 m/s
   (d) 8.9 m/s

12. A car accelerates from rest at 8 m/s^2 for 3s, then brakes at a rate of 4 m/s^2 for 2s. What is the final velocity of the car?

   (a) 0 m/s
   (b) 8 m/s
   (c) 16 m/s
   (d) 24 m/s

13. A car accelerates from rest at 8 m/s^2 for 3s, then brakes at a rate of 4 m/s^2 for 2s. What is the average velocity of the car during the trip?

   (a) 12 m/s
   (b) 15.2 m/s
   (c) 18.4 m/s
   (d) 20 m/s

14. A car speeds up while driving to the left. If we choose positive to be to the right, the car's acceleration would be negative.

   (a) True
   (b) False

15. An object moves with the following acceleration:

$$a(t) = \alpha t + \beta t^3$$

with the constants $\alpha = -5$ m/s^3 and $\beta = 3$ m/s^5. If the object begins at rest, what is its velocity at $t = 2$s?

(a) 2 m/s
(b) 22 m/s
(c) 31 m/s
(d) 41 m/s

16. A projectile is launched at some speed v_0 and angle θ above the x-axis. At its peak, what is the direction of the projectile's velocity?

(a) Along the x-axis
(b) Along the y-axis
(c) θ above the x-axis
(d) No direction, because the velocity is zero at the peak

17. A projectile is launched at some speed v_0 and angle θ above the x-axis. If the projectile lands at the same height it was launched from, what speed does it hit the ground with?

(a) 0
(b) $v_0 \cos\theta$
(c) $v_0 \sin\theta$
(d) v_0

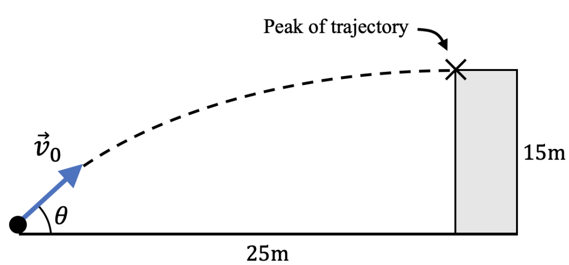


Figure 2: Figure for Problems 16 and 17

18. A projectile is launched at some speed v_0 and some angle θ so that it hits the roof of a 15m tall building 25m away *just* as it peaks, as shown in Figure 2 above. What is the launch angle θ? *Hint: treat the trajectory as symmetric, even though the "second half" is missing.*

(a) 40^o
(b) 50^o
(c) 60^o
(d) 70^o

19. For the same projectile in the previous problem, what is the initial speed v_0?

   (a) 12.6 m/s
   (b) 17.8 m/s
   (c) 22.6 m/s
   (d) 28.5 m/s

20. Which of the following statements is true regarding the trajectory of a projectile?

   (a) The trajectory is always symmetric
   (b) The trajectory is symmetric only if the projectile starts and ends at the same height
   (c) The trajectory is symmetric only if the projectile starts and ends at the same location
   (d) The trajectory is never symmetric

21. During uniform circular motion, which of the following quantities is constant?

   (a) Position
   (b) Speed
   (c) Velocity
   (d) Acceleration

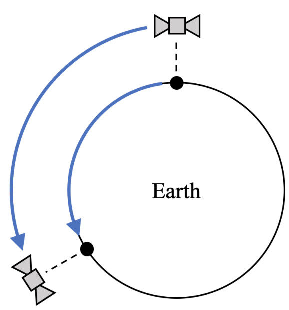


Figure 3: Figure for Problems 22 − 24

22. A geosynchronous orbit is one in which a satellite remains above the same point on the Earth at all times, as shown in Figure 3 above. What is the period of such an orbit?

   (a) 12 hours
   (b) 24 hours
   (c) 48 hours
   (d) There isn't enough information to solve the problem

23. What is the radius of a geosynchronous orbit if the orbital velocity is 3.07 km/s?

   (a) 11.7 m
   (b) 11.7 km
   (c) 42,416 km
   (d) 42,215,530 km

24. What is the radial acceleration of a satellite in a geosynchronous orbit?

    (a) 2.2×10^{-7} m/s^2
    (b) 2.2×10^{-4} m/s^2
    (c) 0.22 m/s^2
    (d) 2.2 m/s^2

25. A scale is essentially a platform placed on a spring: when an object is placed on the platform, it pushes down and compresses the spring, and the amount of compression indicates the weight of the object. Consider the force that an object places downward on the platform and the normal force the platform places on the object: the claim that these forces form an action/reaction pair under Newton's third law is

    (a) True
    (b) False

26. A 3.5kg box is pushed along a flat surface by the following forces:

$$\vec{F}_1 = (6\text{N})\hat{\imath} - (10\text{N})\hat{\jmath}$$
$$\vec{F}_2 = -(8\text{N})\hat{\imath} + (7\text{N})\hat{\jmath}$$

    What is the acceleration of the box? Consider the $+y$-direction to be upward.

    (a) 0.57 m/s^2
    (b) -0.57 m/s^2
    (c) 4 m/s^2
    (d) -4 m/s^2

27. A 3.5kg box is pushed along a flat surface by the following forces:

$$\vec{F}_1 = (6\text{N})\hat{\imath} - (10\text{N})\hat{\jmath}$$
$$\vec{F}_2 = -(8\text{N})\hat{\imath} + (7\text{N})\hat{\jmath}$$

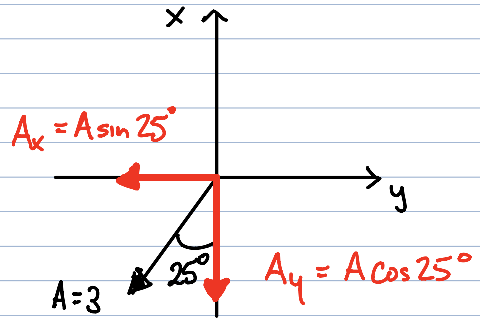
    What is the normal force on the box?

    (a) 18
    (b) 38
    (c) 45
    (d) 52

28. An elevator accelerates downward at 4 m/s^2. If a 8kg box sits on the floor of the elevator, what is the normal force on the box?

    (a) 0N
    (b) 48N
    (c) 80N
    (d) 112N

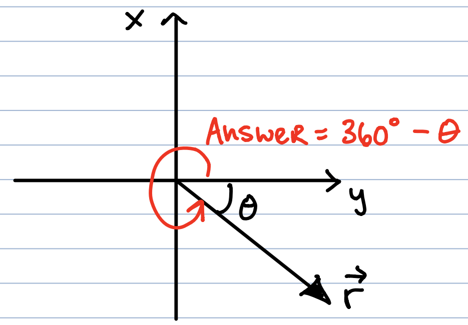
1.



$$A_x = -3\sin 25° = \boxed{-1.27}$$

a

2.



$$\theta = \tan^{-1}\left(\left|\frac{y}{x}\right|\right)$$

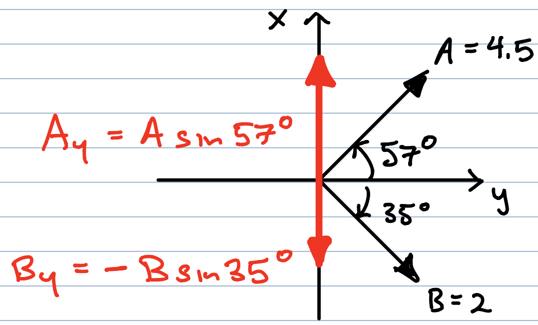
$$= \tan^{-1}\left(\frac{3.2}{5.1}\right)$$

$$= 32°$$

$$\Rightarrow \text{Ans} = 360° - 32° = \boxed{328°}$$

d

3.



$$(2\vec{A} - 3\vec{B})_y = 2A_y - 3B_y = 2(4.5\sin 57°) - 3(-2\sin 35°)$$

$$= \boxed{11.0}$$

d

4. $$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$= (3)(-2) + (4)(0) + (2)(5) = \boxed{4}$$

a

5. $\vec{r}\times\vec{F} = \left(5.1\hat{i} - 3.2\hat{j}\right)\times\left(-2.5\hat{i} + 3.7\hat{j}\right)$

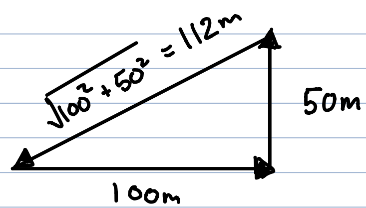
$\hat{i}\times\hat{i}=0$ $\hat{j}\times\hat{j}=0$

b

$= (5.1)(3.7)\underbrace{\hat{i}\times\hat{j}}_{+\hat{k}} + (-3.2)(-2.5)\underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} = \boxed{(10.9\,Nm)\hat{k}}$

6. b

7.



Speed = Distance / Time

Distance: $\Delta x = 100 + 50 + 112 = 262m$

Time: $\Delta t = 30 + 10 + 50 = 90s$

b

$\Rightarrow v = \frac{\Delta x}{\Delta t} = \frac{262}{90} = \boxed{2.9\ m/s}$

8. $\cancel{v^2} = v_0^2 + 2a\Delta x$

b

$\Rightarrow \Delta x = \frac{-v_0^2}{2a} = \frac{-(80)^2}{2(-2)} = \boxed{1600m}$

9. Note: No kinematics equation is independent of a, so first we find a & then we find Δx.

$v = \cancel{v_0} + at \Rightarrow a = \frac{v}{t} = \frac{27}{2.5} = 10.8\ m/s$

a

$\Delta x = \cancel{v_0 t} + \frac{1}{2}at^2 = \frac{1}{2}(10.8)(2.5)^2 = \boxed{33.8m}$

10. $\Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$

b

$= \frac{1}{2}(10)(1.2)^2 = \boxed{7.2m}$

11. $\cancel{v^2} = v_0^2 + 2a\Delta x$

d

$\Rightarrow v_0 = \sqrt{-2a\Delta x} = \sqrt{-2(-10)(4)} = \boxed{8.9\ m/s}$

12. $V_1 = \cancel{V_{01}} + a_1 t_1 = (8)(3) = 24\,\text{m/s}$ (FINAL OF 1 = INITIAL OF 2)

$V_2 = V_{02} + a_2 t_2 = (24) + (-4)(2) = \boxed{16\,\text{m/s}}$ c

13. $V_{AV} = \dfrac{\Delta x}{\Delta t}$

$X_1 = \cancel{V_{01} t_1} + \frac{1}{2} a_1 t_1^2 = \frac{1}{2}(8)(3)^2 = 36\text{m}$

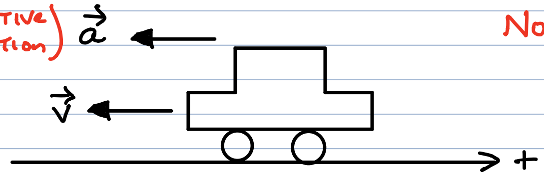
$X_2 = V_{02} t_2 + \frac{1}{2} a_2 t_2^2 = (24)(2) + \frac{1}{2}(-4)(2)^2 = 40\text{m}$

$\Delta x = 36 + 40 = 76\text{m}$

$\Delta t = 3 + 2 = 5\text{s}$

$\Rightarrow V_{AV} = \dfrac{76}{5} = \boxed{15.2\,\text{m/s}}$ b

14. (NEGATIVE DIRECTION) $\vec{a}$



NOTE: SPEEDING UP MEANS $\vec{a}$ & $\vec{v}$ POINT IN THE SAME DIRECTION.

a IS NEGATIVE $\Rightarrow$ $\boxed{\text{TRUE}}$ a

15. $a = \dfrac{dv}{dt} \Rightarrow v(t) = \int a(t)\,dt = \int (\alpha t + \beta t^3)\,dt$

$= \frac{1}{2}\alpha t^2 + \frac{1}{4}\beta t^4 + C$ ← INTEGRATION CONSTANT

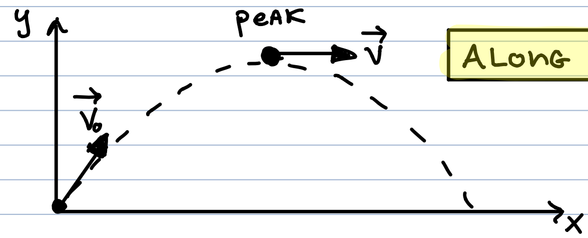
SINCE OBJECT STARTS FROM REST,

$V(0) = \frac{1}{2}\alpha(0)^2 + \frac{1}{2}\beta(0)^4 + C = \underline{C = 0}$

SO, $v(t) = \frac{1}{2}\alpha t^2 + \frac{1}{4}\beta t^4$

$\Rightarrow v(t=2\text{s}) = \frac{1}{2}(-5)(2)^2 + \frac{1}{4}(3)(2)^4 = \boxed{2\,\text{m/s}}$ a

16.

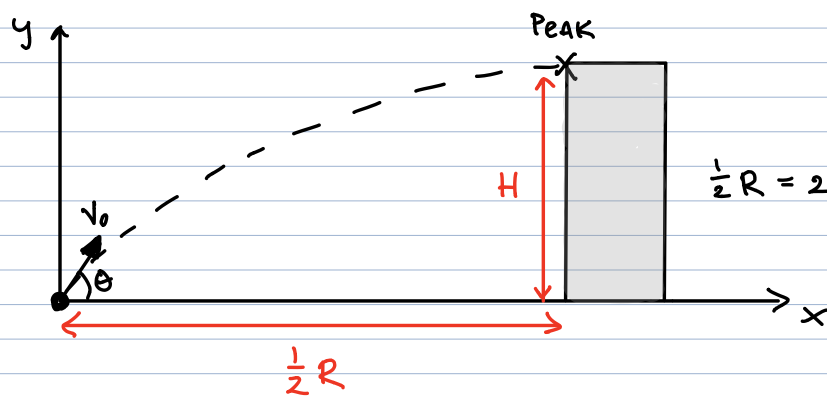


$\boxed{\text{ALONG THE X-AXIS}}$ a

17. IF INITIAL & FINAL HEIGHT ARE THE SAME, THEN THE TRAJECTORY IS SYMMETRIC, MEANING IF IT LEAVES THE GROUND WITH A SPEED OF v_0, IT HITS THE GROUND WITH A SPEED OF

$\boxed{v_0}$ d

18.



$H = 15m$

$\frac{1}{2}R = 25m \Rightarrow R = 50m$

IF PROJECTILE PEAKS AS IT HITS BUILDING, THEN THE HEIGHT OF THE BUILDING IS THE HEIGHT OF THE TRAJECTORY, H, & DISTANCE TO THE BUILDING IS HALF THE RANGE, R, OF THE PROJECTILE. SO:

$$\frac{H}{R} = \frac{\frac{\cancel{v_0^2} \sin^{\cancel{2}}\theta}{2\cancel{g}}}{\frac{2\cancel{v_0^2}\cancel{\sin\theta}\cos\theta}{\cancel{g}}} = \frac{\sin\theta}{4\cos\theta} = \frac{1}{4}\tan\theta = \frac{15}{50}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4(15)}{50}\right) = \boxed{50^\circ} \quad b$$

ALTERNATIVELY: $H = \frac{v_0^2 \sin^2\theta}{2g} \Rightarrow v_0^2 = \frac{2gH}{\sin^2\theta}$ (SAME AS ABOVE)

$$\Rightarrow R = \frac{2\cancel{\sin\theta}\cos\theta}{\cancel{g}}\left(\frac{2\cancel{g}H}{\sin^{\cancel{2}}\theta}\right) = 4H\frac{\cos\theta}{\sin\theta} = \frac{4H}{\tan\theta} \Rightarrow \tan\theta = 4\frac{H}{R}$$

19. Now that we know θ, we can use H to find V_0:

$$H = \frac{V_0^2 \sin^2\theta}{2g} \Rightarrow V_0^2 = \frac{2gH}{\sin^2\theta}$$

$$\Rightarrow V_0 = \frac{\sqrt{2gH}}{\sin\theta} = \frac{\sqrt{2(10)(15)}}{\sin 50} = \boxed{22.6 \text{ m/s}}$$

c

20. b

21. b

22. b

23. Note that for angular velocity: $\omega = \frac{2\pi}{T}$ & $\omega = \frac{V}{r}$

(convert km to m) (convert days to s)

$$\Rightarrow \frac{2\pi}{T} = \frac{V}{r} \Rightarrow r = \frac{VT}{2\pi} = \frac{(3070)(86400)}{2\pi} = 42,215,530 \text{ m}$$

$$= \boxed{42,216 \text{ km}}$$

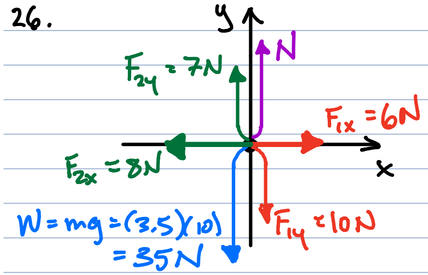
Answers are in km!

24. Radial acceleration (aka centripetal acceleration) is:

$$a_c = \frac{V^2}{r} = \frac{(3070)^2}{(42,215,530)} = \boxed{0.22 \text{ m/s}^2}$$

25. a

26.



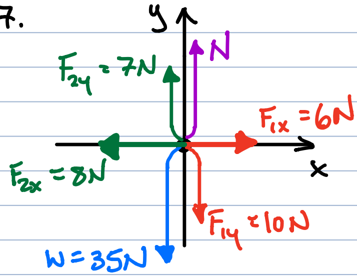
The forces are balanced in the y-direction since the box moves along a flat surface. So, Newton's 2nd law in the x-direction yields:

$$\sum F_x = F_{1x} - F_{2x} = ma$$

$$\Rightarrow a = \frac{F_{1x} - F_{2x}}{m} = \frac{6 - 8}{3.5} = \boxed{-0.57 \text{ m/s}^2}$$

b

27.

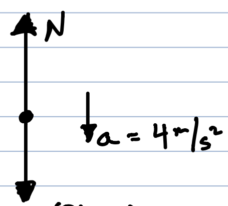


NO ACCELERATION IN THE y-DIRECTION MEANS THAT THE y-AXIS IS BALANCED, SO THE SUM OF UPWARD FORCES EQUALS THE SUM OF DOWNWARD FORCES:

$$\Rightarrow N + F_{2y} = W + F_{1y}$$

$$\Rightarrow N = W + F_{1y} - F_{2y} = 35 + 10 - 7 = \boxed{38N}^{\,b}$$

28.



$W = mg = (8)(10)$
$= 80N$

Choose the positive direction to be <u>Down</u>, in the direction of a.

$$\Rightarrow \sum F_y = W - N = ma$$

$$\Rightarrow N = W - ma = 80 - (8)(4)$$

$$= \boxed{48N}^{\,b}$$