PHYS2350 General Physics I/Lab

Section EV1

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Exam 2 (Chapters 5 – 7)

October 26, 2017

Name: Solutions

ID:

Instructions: This exam is composed of **5 multiple choice questions** and 4 free-response problems. To receive a perfect score (100) on this exam, three of the four free-response problems must be completed. The fourth free-response problem **may be answered for extra credit**.

Each multiple choice question is worth 5 points, and each free-response problem is worth 25 points, so there will be 25 points from the multiple choice questions and 75 points from the 3 free-response problems, totalling 100 points. The fourth free-response problem, worth 25 points of extra credit, pulls the maximum score possible to 125 points.

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

1) A satellite orbits the Earth at an altitude four times the radius of the Earth. If the satellite weighed 10,000N on Earth, what is the gravitational force on the satellite in orbit?

a) $\boxed{\text{400N}}$

b) 625N

c) 2,000N

d) 2,500N

First, it's important to recognize the word **altitude** used in the problem; this means the distance above the surface of the Earth, not the center-to-center distance needed in the equations for gravity and circular motion. The center-to-center distance is going to be the altitude **plus** the radius of the Earth. Since the altitude is four times the radius of the Earth, the center-to-center distance will be five times the radius of the Earth. Next, since we're talking about weight, it's important to know that **weight is just the magnitude of the gravitational force**, which is given by:

$$F = G\frac{mM_E}{R_E^2}$$

where M_E and R_E are the mass and radius of the Earth, respectively, and m is the mass of the satellite. If the center-to-center distance is increased by a factor of 5, notice that the denominator is increased by a factor of 25:

$$F = G\frac{mM}{r^2} \quad \rightarrow \quad F = G\frac{mM}{(5r)^2} = G\frac{mM}{25r^2} = \frac{1}{25}\left(G\frac{mM}{r^2}\right)$$

Everything in parentheses is the weight at the surface of the Earth, which is 10,000N, so the weight at an altitude of four times the radius of the Earth is:

$$F = \frac{1}{25}(10,000) = \boxed{\text{400N}}$$

which is **option (a)**.

2) A 2kg mass undergoing circular motion at a constant speed of 15m/s in an orbit of 5m. During one-quarter of a revolution, how much work is done on the mass?

a) $\boxed{\text{0J}}$

b) 353.5J

c) 707J

d) 1414J

This problem is easily solved using the **work-energy theorem**:

$$W_{tot} = \Delta K$$

Since the mass is undergoing motion at a **constant speed**, there is **no change in kinetic energy** and so $\boxed{W_{tot} = 0\text{J}}$, which is **option (a)**. It's important not to simply plug in numbers, because there are enough numbers to find a force and a distance (1/4 of the circumference), but since the centripetal force is **always perpendicular to the motion**, the angle is always 90^o. Since $\cos(90) = 0$, the **work is always zero**.

3) A 3kg object slides down a slope with an unknown curvature and friction. If the object starts at a height of 1.2m, how much work must friction by the time the object reaches the bottom in order for the object to move at a constant speed?

a) 35J

b) $\boxed{-35\text{J}}$

c) 70J

d) -70J

Since the object moves at a constant speed, the **work-energy theorem** says:

$$W_{tot} = \Delta K = 0$$

The total work is going to be the sum of the work due to each force: gravity will do work on the object, friction will do work on the object, but the normal force won't do any work because it acts at a 90^o angle to the displacement. So, the **total work is**:

$$W_{tot} = W_{grav} + W_f = 0 \quad \Rightarrow \quad W_f = -W_{grav}$$

Gravity is a conservative force, which means that its work is **always equal to**:

$$W_{grav} = -\Delta U_g$$

as per the formula sheet. Plugging this into the work-energy theorem, we see that:

$$W_f = -W_{grav} = +\Delta U_g = U_f - U_i = mgy_f - mgy_i$$

Setting $y = 0$ at the bottom of the incline, the initial height is $y_i = 1.2$m and the final height is $y_f = 0$, so the **work done by friction is**:

$$W_f = \underbrace{mgy_f}_{=0} - mgy_i = -(3)(9.8)(1.2) = \boxed{-35\text{J}}$$

making **option (b)** the correct choice.

4) Two cars undergo a head-on collision in which they stick together after. If one car is 1200kg, moving to the left at 24m/s, and the other car is 1700kg moving at 15m/s, what is the speed and direction of the wreckage?

a) $\boxed{\text{1.14m/s, to the left}}$

b) 1.14m/s, to the right

c) 1.94m/s, to the left

d) 1.94m/s, to the right

Since this is a collision, we're going to use momentum conservation to solve the problem. Specifically, this is a **perfectly inelastic collision**, since the objects stick together post-collision. This means that we can treat the final object as a single mass, with a total mass of the combined objects, moving at a single velocity v_f. Our equation for momentum conservation will then be:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \quad \Rightarrow \quad v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Something that's very important to remember is that the v's in this equation are **vectors, not scalars**, which means that the direction (i.e. the sign) of the quantity is very important. While it's not explicitly stated in the problem, if the 1200kg car moves to the left, the other car must move to the right in order for the cars to collide head-on. If we choose the left as the positive direction, and m_1 to be the 1200kg car, then the final velocity of the wreckage is:

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(1200)(+24) + (1700)(-15)}{(1200) + (1700)} = \boxed{+1.14\text{m/s}}$$

Since the final velocity is positive, the direction of the wreckage is **to the left**, making **option (a)** the correct answer.

5) A 145g baseball is thrown at a 40m/s. A bat hits the ball, sending it at 30 m/s in the opposite direction. If the collision took 100ms, what is the average force put on the ball?

a) 14.5N

b) 58N

c) 43.5N

d) $\boxed{\text{101.5N}}$

During the collision with the bat, the ball will undergo some change in momentum Δp, and the **force due to a change in momentum** is equal to:

$$F_{av} = \frac{\Delta p}{\Delta t}$$

if the collision took place over some time Δt. We already know $\Delta t = 100$ms (miliseconds), so we just need to calculate the change in momentum. For this, it's really important to remember that **momentum is a vector**, and so the change in momentum depends upon the **direction** of the initial and final momenta. Choosing the positive direction to be the direction along the final velocity of the ball, meaning the initial velocity is **negative**, the **change in momentum is**:

$$\Delta p = m v_f - m v_i = m(v_f - v_i) = (0.145)((+30) - (-40)) = 10.15\text{Ns}$$

So, the **average force on the ball** during the collision is (noting that 100ms = 0.1s):

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{10.15}{0.1} = \boxed{101.5\text{N}}$$

making **option (d)** the correct choice.

FREE-RESPONSE PROBLEMS

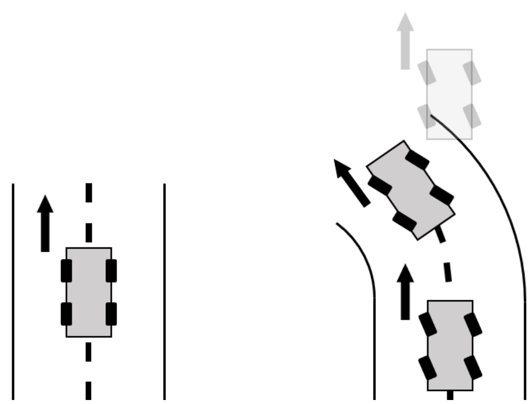
1) A 2000kg car is driving in a straight line at 20m/s when it approaches a turn with a radius of 100m. The car has a coefficient of static friction $\mu_s = 0.5$ between the tires and the road.

a) What is/are the force/forces responsible for the car's ability to turn?

b) What is the centripetal acceleration of the car during the turn?

c) What is the magnitude of each force acting on the car during the turn?

a) Let's start by listing the forces acting on the car, in general. There will be the weight of the car, pointing downwards, the normal force on the car, point upwards, and static friction on the wheels, pointing in an (as of now) unknown direction. Why is there static friction on the wheels? Think about what the car is doing during a turn, as illustrated in the figure below.



During the turn, the car wants to keep moving along a straight line, according to Newton's first law. What causes the car to turn? The fact that the **wheels can't slide**, which is due to static friction. So the $\boxed{\text{static friction is the force that causes the car to turn}}$, and thus points towards the center of the turn (to the left in the figure above).

b) The centripetal acceleration during the turn **is determined solely by the motion of the car**, not by any force acting on the car:

$$a_c = \frac{v^2}{r} = \frac{(20)^2}{(100)} = \boxed{4\text{m/s}^2}$$

c) The magnitude of the weight and normal force **are the same**, since they are the only forces acting in the vertical direction:

$$W = N = mg = (2000)(9.8) = \boxed{19600\text{N}}$$

But, more importantly, the magnitude of the **static friction** force is equal to the mass times the centripetal acceleration, according to Newton's second law:

$$f_s = ma_c = (2000)(4) = \boxed{8000\text{N}}$$

The static friction is **not guaranteed to be maximal**, and so we can't say $f_s = \mu_s N$. In this problem, the maximum static friction is 9800N which is larger than the actual static friction of 8000N.

2) A 800kg satellite orbits the Earth in a uniform, circular motion. Note that the mass of the Earth is 5.97×10^{24}kg and the radius is 6.37×10^{6}m.

a) What speed would the satellite need to orbit the Earth with in order to maintain an altitude of 10,000km?

b) What is the period of the satellite's orbit?

c) If the satellite were to be moved to a further orbit, at 15,000km, what would the new speed of the satellite need to be?

d) How much work would have to be done on the satellite to move it from the lower orbit to the outer orbit?

a) In order to maintain an altitude, the force of gravity **must be equal** to the mass times the centripetal acceleration of the orbit. If the gravitational force were, say, less than the mass times the centripetal acceleration of the orbit, then the force of gravity would be too weak to turn the satellite, and the radius of the orbit would get larger. If the gravitational force were greater than the mass times the centripetal acceleration, the gravitational force would be too strong and the satellite would turn too quickly, causing it to orbit at a smaller radius. Only when they are equal will the satellite orbit at the chosen radius. So,

$$F_g = ma_c \quad \Rightarrow \quad G\frac{mM_E}{r^2} = m\frac{v^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM_E}{r}}$$

Note that m is the mass of the satellite, and this is the mass that appears in Newton's second law (as in $F = ma$), because **the satellite is the mass that accelerates**, not the Earth. r is the center-to-center distance, as always, which is going to be the altitude of the orbit h **plus** the radius of the Earth R_E (note that 10,000km $= 1 \times 10^7$m):

$$r = h + R_E = 1 \times 10^7 + 6.37 \times 10^6 = 1.64 \times 10^7\text{m}$$

So, the **speed required** to maintain the orbit is:

$$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(1.64 \times 10^7)}} = \boxed{4928\text{m/s} = 4.93\text{km/s}}$$

b) There are a **couple of equations** we can use to calculate this:

$$v = \frac{2\pi r}{T} \qquad \text{or} \qquad T^2 = \frac{4\pi^2}{GM}a^3$$

where a is equal to the radius for a circular orbit. I'll choose the less complicated equation $v = 2\pi r/T$:

$$v = \frac{2\pi r}{T} \quad \Rightarrow \quad T = \frac{2\pi r}{v} = \frac{2\pi(1.64 \times 10^7)}{(4928)} = \boxed{20,909\text{s} = 5.81\text{hr}}$$

Using Kepler's third law, the answer should be something like 20,912s. The difference is just due to a rounding error.

c) In order to maintain an orbit of 15,000km, the gravity on the satellite still needs to be equal to the mass of the satellite times the centripetal acceleration, **by the same logic** as part (a). This means we can **use the same equation** as well. First, the new **center-to-center distance** would be:

$$r = h + R_E = 1.5 \times 10^7 + 6.37 \times 10^6 = 2.14 \times 10^7\text{m}$$

and so the **speed of the satellite** must be:

$$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(2.14 \times 10^{7})}} = \boxed{4314\text{m/s} = 4.31\text{km/s}}$$

d) The total work on the satellite is going to be given by the **work-energy theorem**:

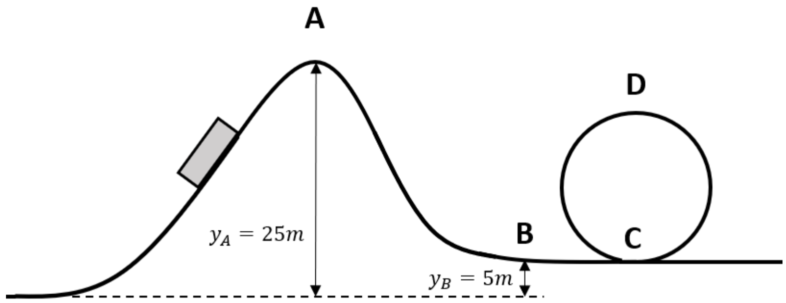
$$W_{tot} = \Delta K$$

We know the mass of the satellite, the initial speed (in the 10,000km orbit) and the final speed (in the 15,000km orbit), so **we can calculate the work**:

$$\begin{aligned} W_{tot} &= \Delta K = K_f - K_i \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) \\ &= \frac{1}{2}(800)\left[(4314)^2 - (4928)^2\right] \\ &= \boxed{-2.27 \times 10^9\text{J}} \end{aligned}$$

Note that the total work **should be negative** because the satellite loses speed going from the smaller orbit to the larger orbit.

3) A roller coaster is set up as shown in the figure below. The car has a mass of 600kg, including its passengers. Assuming it's carried up to point A by some device, such that it drops at rest from point A, answer the following questions. Ignore any effects due to friction or air resistance.



a) What is the car's speed at point B?

b) During the car's travel from point B to point C, the roller coaster is designed to deliver an impulse of $\Delta p = 3000$Ns to the car. What would its speed be at point C, entering the loop?

c) What would the car's speed be at the top of the loop, point D, if the radius of the loop were 8m?

d) Do you think this ride is safe? Provide a calculation and a reasoning to support your answer.

a) Moving from point A to point B, energy is conserved because there's no friction along the track. If we set $y = 0$ at **point B, not the ground**, then the initial height at point A is:

$$y_A = 25 - 5 = 20\text{m}$$

So, the roller coaster car has a potential energy at point A, but no kinetic energy (it drops from rest), and has a kinetic energy at point B, but no potential energy (we set $y_B = 0$). So, the **energy conservation** equation is:

$$\underbrace{K_A}_{=0} + U_A = K_B + \underbrace{U_B}_{=0} \quad \Rightarrow \quad mgy_A = \frac{1}{2}mv_B^2 \quad \Rightarrow \quad v_B = \sqrt{2gy_A} = \sqrt{2(9.8)(20)} = \boxed{19.8\text{m/s}}$$

b) Recall that an impulse is just a **change in momentum**; we can also deduce this by the fact that an impulse is denoted by Δp explicitly in the problem. Going into this accelerating mechanism, at point B, the car has a momentum of:

$$p_B = mv_B = (600)(19.8) = 11,880\text{Ns}$$

The momentum coming out of the accelerating mechanism, at point C, is going to be the momentum at point B **plus** the gain in momentum Δp:

$$p_C = p_B + \Delta p = (11800) + (3000) = 14,880\text{Ns}$$

So, the **speed at point C** is just the momentum at point C divided by the mass of the roller coaster car:

$$v_C = \frac{p_C}{m} = \frac{14880}{600} = \boxed{24.8\text{m/s}}$$

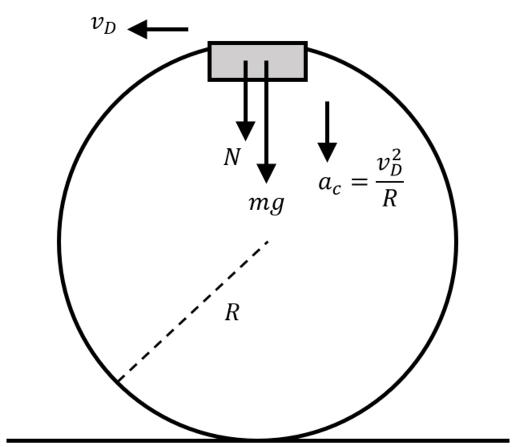
c) To find the speed at the top of the loop, all we need to consider is **conservation of energy**, since there is no friction. Once again, considering $y_B = 0$, which means $y_C = 0$ as well, the potential energy at C is zero. Note that the height at point D is **twice the radius of the loop**. So, the equation for energy conservation becomes:

$$K_C + \underbrace{U_C}_{=0} = K_D + U_D \quad \Rightarrow \quad \frac{1}{2}mv_C^2 = \frac{1}{2}mv_D^2 + mg(2R)$$

$$\Rightarrow \quad v_D = \sqrt{v_C^2 - 4gR} = \sqrt{(24.8)^2 - 4(9.8)(8)} = \boxed{17.4\text{m/s}}$$

Note that this answer has **nothing to do with the centripetal motion** of the car; it only has to do with conservation of energy. The centripetal motion will figure into the answer for the next question.

d) How can we determine if the ride is safe? Obviously, we can only consider things we have learned. There is nothing about the drop from A to B or the acceleration from B to C that indicates to us the ride might be unsafe, but we definitely know how to check if the ride is unsafe as it moves through the loop from C to D: if the normal force drops to zero at any point, the people in the roller coaster car would lose contact with their seats and could fall out of the car. Consider the free body diagram drawn in the figure below.



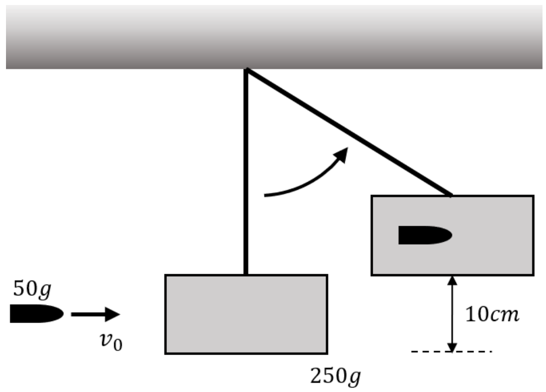
Applying Newton's second law to the roller coaster car, considering downward to be the positive direction, we get the equation:

$$N + mg = ma_c = m\frac{v_D^2}{R} \quad \Rightarrow \quad N = m\frac{v_D^2}{R} - mg = m\left(\frac{v_D^2}{R} - g\right)$$

If the normal force is greater than zero for the numbers given for this roller coaster, then the ride is **safe**; if the normal force is zero or negative, then the ride is **unsafe**. Plugging in our numbers:

$$N = m\left(\frac{v_D^2}{R} - g\right) = (600)\left(\frac{(17.4)^2}{(8)} - (9.8)\right) = \boxed{(600)(37.8 - 9.8) > 0}$$

So, the ride is **safe**.



4) A 50g bullet is fired into a 250g block of wood, as shown in the figure above. After the bullet embeds in the block, the block rises by a height of 10cm.

a) How fast would the block have to be moving after the bullet struck it?

b) How fast must the bullet have been traveling before it struck the block of wood?

c) How much energy is lost as heat when the bullet embeds itself in the wood?

a) **After** the bullet struck the block, the bullet+block combo rises with its energy conserved. If we set $y = 0$ to be the lowest position of the block, then the initial potential energy is zero. The final kinetic energy is going to be zero because the bullet+block combo should stop moving at its peak height, once all of its kinetic energy has been converted to potential energy. So, the equation for **conservation of energy** becomes:

$$K_i + \underbrace{U_i}_{=0} = \underbrace{K_f}_{=0} + U_f \quad \Rightarrow \quad \frac{1}{2}(m_{\text{bullet}} + M_{\text{block}})v_i^2 = (m_{\text{bullet}} + M_{\text{block}})gh$$

$$\Rightarrow \quad v_i = \sqrt{2gh} = \sqrt{2(9.8)(0.1)} = \boxed{1.4\text{m/s}}$$

b) The bullet embedding itself into the block is an instance of a **perfectly inelastic collision**, so we'll use conservation of momentum to solve this problem. The key feature of a perfectly inelastic collision is that the two objects colliding stick together and move as one post-collision. The **momentum conservation equation** for this collision is:

$$m_{\text{bullet}}v_{\text{bullet},i} + M_{\text{block}}\underbrace{v_{\text{block},i}}_{=0} = (m_{\text{bullet}} + M_{\text{block}})v_f$$

$$\Rightarrow \quad v_{\text{bullet},i} = \frac{(m_{\text{bullet}} + M_{\text{block}})v_f}{m_{\text{bullet}}} = \frac{((0.05) + (0.25))(1.4)}{(0.05)} = \boxed{8.4\text{m/s}}$$

c) The amount of energy lost due to heat is equal to the amount of **kinetic energy lost** in the collision:

$$E_{\text{heat}} = -\Delta K = \left(\frac{1}{2}m_{\text{bullet}}v_{\text{bullet},i}^2 + \frac{1}{2}M_{\text{block}}\underbrace{v_{\text{block},i}^2}_{=0}\right) - \left(\frac{1}{2}(m_{\text{bullet}} + M_{\text{block}})v_f^2\right)$$

$$= \frac{1}{2}(0.05)(8.4)^2 - \frac{1}{2}((0.05) + (0.25))(1.4)^2 = \boxed{1.47\text{J}}$$

FORMULA SHEET

- Vectors:

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

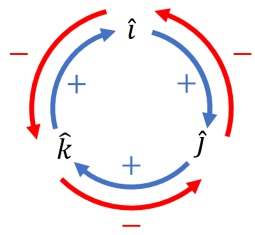


Figure 1: Cyclic permutations for cross product

- Kinematics:

$$g = 9.8\text{m/s}^2$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum \vec{F} = m\vec{a}$$

$$W = mg$$

$$f_{s,max} = \mu_s N$$

$$f_k = \mu_k N$$

- Circular Motion:

$$a_c = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

- Gravity:

$$G = 6.67 \times 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$$

$$F_g = G\frac{mM}{r^2}$$

$$a_g = G\frac{M}{r^2}$$

$$T^2 = \frac{4\pi^2}{GM}a^3 \quad \text{(Kepler's third law)}$$

- Work & Energy:

$$K = \frac{1}{2}mv^2$$
$$U_g = mgy$$
$$W = F\Delta x \cos\theta$$
$$W_{tot} = \Delta K$$
$$W_{cons} = -\Delta U$$
$$W_{other} = \Delta E$$
$$K_i + U_i + W_{nc} = K_f + U_f$$
$$P = \frac{\Delta E}{\Delta t}$$
$$P = Fv \text{ (at constant velocity)}$$

- Linear Momentum:

$$\vec{p} = m\vec{v}$$
$$\sum \vec{F}_{ext,sys} = \frac{\Delta \vec{p}_{sys}}{\Delta t}$$
$$m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = m_A\vec{v}_{Af} + m_B\vec{v}_{Bf}$$
$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \text{ (elastic collisions)}$$
$$\vec{J} = \vec{F}_{av}\Delta t \text{ (impulse)}$$