PHY2048 Physics with Calculus I

Section 584761

Prof. Douglas H. Laurence

Final Exam

May 2, 2018

Name: Solutions

Instructions:

This final exam is **a take home exam**. It will be posted online sometime around **noon of the exam date** and is **due at midnight of the exam date**. Be aware that **late submissions will not be accepted**! You will have roughly 12 hours to complete the exam and then **email it back to me**, so that I have a record of the time it was submitted. The submission can be typed, scanned, or (worse case scenario) **clear** photographs of your work can be submitted.

This exam is composed of **10 multiple choice questions** and **5 free-response problems**. To receive a perfect score (100) on this exam, 4 of the 5 free-response problems must be completed. The fifth free-response problem **may not be answered for extra credit**. Each multiple choice question is worth 2 points, for a total of 20 points, and each free-response problem is worth 20 points, for a total of 80 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total	

MULTIPLE CHOICE QUESTIONS

A 1. A basketball player is running "suicide sprints," which involve sprinting from one line to another line 40m away, turning around quickly, and sprinting back to the starting line. If the player can cover each of the 40m legs of the trip in 5s, what is his average velocity?

(a) 0
(b) 8 m/s
(c) 40 m/s
(d) 80 m/s

The average **velocity** depends upon the displacement, which is zero if you start and end at the same place. So the average velocity is **zero**.

C 2. Box B sits at rest on top of Box A. If you push Box A, causing it to accelerate, and Box B moves along on top of Box A undisturbed. Does Box B feel a friction force?

(a) No, because it's moving at a constant velocity
(b) No, because it's accelerating
(c) Yes, a static friction force
(d) Yes, a kinetic friction force

If Box B moves along with Box A, then it must be accelerating, which means there needs to be a force on B. Since B isn't sliding, it's static friction.

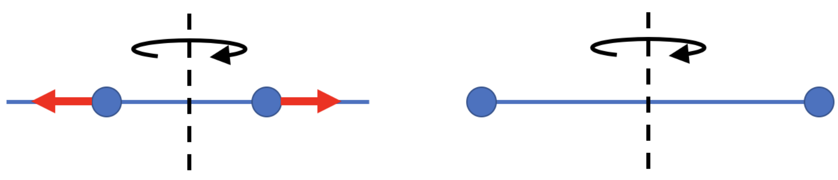
B 3. You're trying to move a heavy object, with a mass of 100kg, along a floor with friction. The coefficients of friction between the object and the floor are $\mu_s = 0.45$ and $\mu_k = 0.35$. If you push on the object with a force of 600 N, what is the magnitude of the friction acting on the object?

(a) 0N
(b) 343N
(c) 441N
(d) 600N

$f_{s,max} = u_s mg = (0.45)(100)(9.8) = 441N$

F applied exceeds the max static friction, so friction must be kinetic!

$f_k = u_k mg = (0.35)(100)(9.8) = 343N$



C 4. A rod with two identical masses spins with some angular speed ω, such that the masses are free to slide along the rod without friction. As the rod rotates, the masses slide out to the edge of the rod. As this is occurring, the angular speed ω is:

(a) Increasing
(b) Not changing
(c) Decreasing
(d) None of the above

Since I = sum(mr^2), as the masses move farther out along the rod, their r value increases, so I increases. Remember that angular momentum, L = Iw, is conserved, so if I increases, w must decrease.

B 5. A pulley is composed of a rope wound around a cylinder; as the rope is pulled out, the cylinder rotates. If at the end of the rope, there is some mass, and releasing this mass causes the pulley to rotate, what force is putting a torque on the pulley?

(a) There is no torque being put on the pulley
(b) The tension in the rope
(c) The weight of the mass
(d) The weight of the pulley

The rope is being pulled down by the weight, sure, but the rope itself is what's causing the pulley to rotate, so it's the force the rope puts on the pulley, the **tension**, that produces the torque.

C 6. A box is pushed along a path of some length, causing gravity to do work on the box. If the box were pushed along a path with a greater length, then:

(a) Gravity would do more work, because it is conservative
(b) Gravity would do more work, because it is non-conservative
(c) Gravity would do the same work, because it is conservative
(d) Gravity would do the same work, because it is non-conservative

Gravity is a conservative force, which means that the work is path-independent.

D 7. Block A, with a mass of 0.25kg, moving at 20 m/s in the x-direction, collides with Block B, of mass 0.5kg, moving at 10 m/s in the $-x$-direction. If they collide such that they stick together after the collision, in what direction do they move after?

(a) x-direction
(b) $-x$-direction
(c) y-direction
(d) They don't move post-collision

$m_1v_{1i} + m_2v_{2i} = (0.25)(20) + (0.5)(-10) = 0$

Since the total momentum is zero, after they stick together, $v_f = 0$, so they don't move.

C 8. A pendulum's bob (the mass at the end of the pendulum) is released from rest, at some angle. As it falls towards equilibrium, can you use kinematics to determine the bob's speed?

(a) Yes, because acceleration is constant
(b) Yes, because energy is conserved
(c) No, because acceleration isn't constant
(d) No, because energy is not conserved

We can only use kinematics if a is constant. As the bob falls, a = gsin(theta), so it's not constant, so we can't use kinematics.

D 9. Imagine some planet X existed, with a mass $M_X = 2M_E$ and a radius $R_X = R_E/\sqrt{2}$, where M_E and R_E are the mass and radius of the Earth, respectively. What would the gravitational acceleration be at the surface of X?

(a) 2.45 m/s^2
(b) 4.9 m/s^2
(c) 19.6 m/s^2
(d) 39.2 m/s^2

$a = GM/R^2 = G(2M_E)/(R_E/\text{sqrt}(2))^2 = G(2M_E)/(R_E^2/2) = 4GM_E/R_E^2$

So, the acceleration is four times that on Earth, or 39.2 m/s^2

C 10. A man in a boat drops from peak to trough of the waves he is riding every 4s. What is the amplitude of the water waves?

(a) 2s
(b) 4s
(c) 8s
(d) 12s

If the boat drops from a peak to a trough, it completed **half** a cycle (a cycle is peak to peak), so the time taken is half the period, which means that the period is 2*(4s) = 8s.

FREE-RESPONSE PROBLEMS

1. An object is dropped from the roof of a building of unknown height. Assume that $t = 0$ when the object is dropped.

5pt (a) How far does the object drop from $t = 1.5s$ to $t = 2.7s$?

5pt (b) If you placed a device to measure impact speed on the ground, and you measured an impact speed of 60m/s, how tall was the building?

5pt (c) How long, then, did it take the object to hit the ground?

5pt (d) If the building were four times as tall, what would the measured impact speed of the object be?

(a) The distance dropped from t = 0 to t = 1.5s is:

$y = v_0t + (1/2)at^2 = 0 + (1/2)(9.8)(1.5)^2 = 11.0m$

The distance dropped from t = 0 to t = 2.7s is:

$y = v_0t + (1/2)at^2 = 0 + (1/2)(9.8)(2.7)2 = 35.7m$

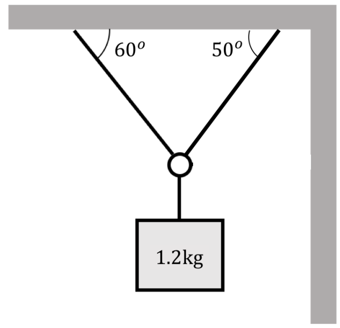
So, the distance dropped from t = 1.5s to t = 2.7s is: D = 35.7m - 11.0m = **24.7m**

(b) $v^2 = v_0^2 + 2aH \text{ --> } H = v^2/2a = (60)^2/2(9.8) =$ **184m**

(c) $v = v_0 + at \text{ --> } t = v/a = (60)/(9.8) =$ **6.1s**

(d) (since $v_0 = 0$) $v^2 = 2aH \text{ --> } v^2 = 2a(4H) = 4(2aH) \text{ --> } v = sqrt(4(2aH)) = 2sqrt(2aH)$

The original final speed was v = sqrt(2aH), but by quadrupling the height, the new final speed is v = 2sqrt(2aH), so the final speed **doubles**, or v = **120 m/s**.



2. A 1.2kg mass hangs at rest from a ring, supported by two ropes, as shown in the figure above. Call the rope at a 60^o angle rope A and the rope at a 50^o angle rope B.

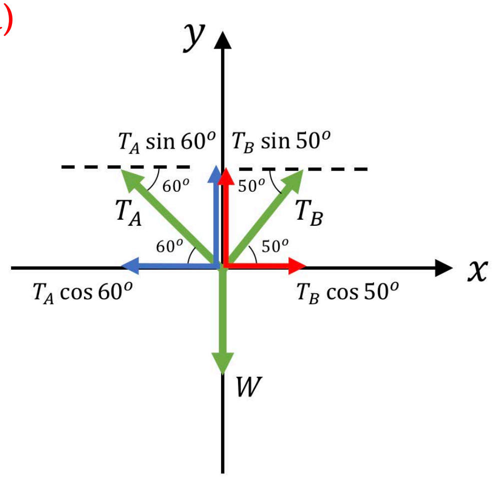
2pt (a) Draw the free body diagram of the ring.

10pt (b) Calculate the tension in rope A.

3pt (c) Calculate the tension in rope B.

5pt (d) Imagine if rope A was moved so that it was at a 90^o angle from the ceiling (straight up from the mass). What would the tension in rope B be in this case?

(a)



(b) We have two equations from Newton's second law: the x equation an the y equation.

The x equation is: $T_A\cos60 = T_B\cos50$
The y equation is: $T_A\sin60 + T_B\sin50 = W$

Using the x equation: $T_B = (\cos60/\cos50)T_A = 0.778T_A$
Plugging this into the y equation:

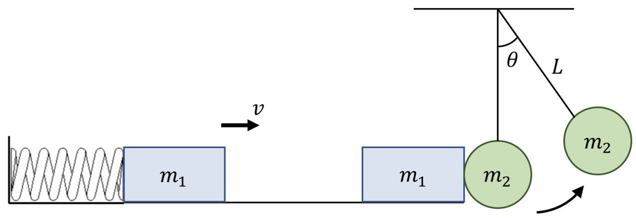
$T_A\sin60 + (0.778T_A)\sin50 = 1.46T_A = W = (1.2)(9.8) = 11.8$N

Solving gives: $T_A = 11.8/1.46 =$ **8.08N**

(c) Using our (modified) x equation from Newton's second law:

$T_B = 0.778T_A = 0.778(8.08) =$ **6.29N**

(d) If T_A was brought straight upwards, then we can see from the above free body diagram that the x-component of T_B would be zero, meaning that T_B would be zero. So T_A would simply balance out the weight, or $T_A =$ **11.8N**.



3. A block of mass $m_1 = 0.85$kg is pressed against a spring with a force constant $k = 150$ N/m, which is initially compressed by 20cm. When the spring is let go, it launches the mass at some speed v towards a pendulum, with a length $L = 10$cm and a mass $m_2 = 0.5$kg. After the block collides with the pendulum, m_2 rises to a maximum angle θ, as depicted in the above figure.

6pt (a) At what speed v is the block launched from the spring?

8pt (b) If the block and pendulum collide elastically, with what speed does the pendulum leave the collision?

6pt (c) To what maximum angle θ does the pendulum rise?

(a) The block gains its speed from the conversion of the spring potential energy into kinetic energy. Since there's no mention of friction, we assume that there isn't any, meaning energy is conserved. Thus:

$U_{sp} = K$ --> $(1/2)kx^2 = (1/2)m_1v^2$ --> $v = \text{sqrt}(k/m_1)*x = \text{sqrt}((150)/(0.85))*(0.2) =$ **2.66 m/s**

(b) If the collision is elastic, then we have two equations to solve:

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ and $v_{1i} - v_{2i} = v_{2f} - v_{1f}$ --> $v_{1f} = v_{2f} - v_{1i} + v_{2i} = v_{2f} - 2.66$

Plugging this into the first equation, we get:

$(0.85)(2.66) + 0 = (0.85)(v_{2f} - 2.66) + (0.5)v_{2f} = 1.35v_{2f} - 2.26$ --> $1.35v_{2f} = 4.52$ --> $v_{2f} =$ **3.35 m/s**

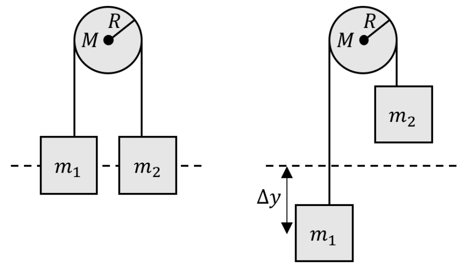
(c) The kinetic energy of the pendulum gained by the collision with the block is converted into gravitational potential energy as the pendulum rises. It will eventually rise to some height h where the kinetic energy goes to zero. By analyzing the geometry (we did this multiple times in class, so consult your notes if you don't remember), the relationship between h and the angle is:

$h = L(1 - \cos(\text{theta}))$

So:

$K_i = U_f$ --> $(1/2)m_2v^2 = m_2gh$ --> $(1/2)v^2 = gL(1 - \cos(\text{theta}))$ --> $1 - \cos(\text{theta}) = v^2/2gL$

--> $\cos(\text{theta}) = 1 - v^2/2gL = 1 - (3.35)^2/2(9.8)(1) = 0.427$ --> theta = **64.7 degrees**



4. Consider two masses, $m_1 = 1.5$kg and $m_2 = 0.6$kg, attached to the ends of a light rope wrapped around a solid cylinder of mass $M = 2.5$kg and radius $R = 17$cm, as shown in the figure above. m_1 and m_2 are released from rest and allowed to move such that m_1 drops a distance $\Delta y = 1.2$m.

1pt (a) How much work, total, is done on the cylinder due to friction?

10pt (b) What is the final angular speed of the cylinder?

5pt (c) How long did it take for m_1 to drop the 1.2m? *Hint: assume that acceleration is constant.*

4pt (d) What was the net torque acting on the cylinder during this motion?

(a) There is no mention or implication of friction in the problem, so we need to assume that it's zero, as we would normally do in any other problem. So, the work due to friction is also **zero**.

(b) To find the final angular speed, we should use energy conservation:

$K_{1i} + K_{2i} + K_{3i} + U_{1i} + U_{2i} + U_{3i} = K_{1f} + K_{2f} + K_{3f} + U_{1f} + U_{2f} + U_{3f}$

where object 3 is the cylinder. First, every object starts at rest, so all initial kinetic energies are zero. Next, note that no matter where we set y = 0 for the potential energy, the height of the cylinder doesn't change, so $U_{3i} = U_{3f}$ and we can cancel it from both sides. Now, if we set y = 0 at the initial height of each block, then $U_{1i} = U_{2i} = 0$. Note that this means that y_{1f} is **negative** because it's below y = 0. Plugging all of this in, our energy conservation equation becomes:

$0 = (1/2)m_1v_1^2 + (1/2)m_2v_2^2 + (1/2)I_3w_3^2 + m_1g(\text{-Delta y}) + m_2g(\text{Delta y})$

Since the rope is inelastic (as we always assume), both blocks have to be moving at the same speed. Also, since the rope pulls on the cylinder without slipping (which we also assume), we know that v = Rw, where v is the speed of **both** blocks. Since we want to solve for w of the cylinder, we should re-write all v's in terms of w. Then our above equation becomes:

$0 = (1/2)m_1R^2w^2 + (1/2)m_2R^2w^2 + (1/2)I_3w^2 + (m_2 - m_1)g^*\text{Delta y}$

$= (1/2)(m_1R^2 + m_2R^2 + I_3)w^2 + (m_2 - m_1)g^*\text{Delta y}$

(Continued on next page)

Let's start plugging in our known numbers. First, we don't actually know I_3 yet, but we can easily solve for it. Since the pulley is a solid cylinder, we know that:

$$I_3 = (1/2)MR^2 = (1/2)(2.5)(0.17)^2 = 0.036 \text{ kgm}^2$$

So, our energy conservation equation becomes:

$$0 = (1/2)((1.5)(0.17)^2 + (0.6)(0.17)^2 + (0.036))w^2 + ((0.6) - (1.5))(9.8)(1.2) = 0.0483w^2 - 10.6$$

$$\text{-->}\ 0.0483w^2 = 10.6 \text{ -->}\ w = \text{sqrt}(10.6/0.0483) = \mathbf{14.8\ rad/s}$$

(c) Since we know w, we can also calculate v of the blocks: v = Rw = (0.17)(14.8) = 2.52 m/s. Now, since we're told to assume acceleration is constant, we can use kinematics. We know that the initial speed is 0, the final speed is 2.52 m/s, and the distance traveled was 1.2m. Unfortunately, none of our three kinematic equations can solve this in one step, but we can solve for a first and then use that to solve for t:

$$v^2 = v_0^2 + 2a\text{*Delta } y \text{ -->}\ a = v^2/2\text{*Delta } y = (2.52)^2/(2)(1.2) = 2.65 \text{ m/s}^2$$

which means that the time taken is:

$$v = v_0 + at \text{ -->}\ t = v/a = (2.52)/(2.65) = \mathbf{0.95s}$$

(d) The torque on the cylinder can be found by using the rotational form of Newton's second law, which means we need to find the angular acceleration. Once again, since the rope pulls on the cylinder without slipping, we know that a = R*alpha, so:

$$\text{alpha} = a/R = (2.65)/(0.17) = 15.6 \text{ rad/s}^2$$

and so the torque acting on the cylinder is:

$$\text{torque} = I_3\text{*alpha} = (0.036)(15.6) = \mathbf{0.56\ Nm}$$

5. Halley's comet is a very famous comet in the sky that passes by Earth roughly every 75 years. In reality, Halley's comet has a fairly elliptical orbit, but for the purposes of this question, we'll consider the orbit to be circular. As a comet, it orbits the Sun, not the Earth. Note that $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6400$ km, $M_S = 1.99 \times 10^{30}$ kg, $R_S = 700,000$ km, and the distance between the Earth and the Sun is 1.5×10^{11} m.

6pt (a) What is the radius of the comet's orbit?

2pt (b) What is the closest the comet gets to Earth?

6pt (c) What is the angular speed of the comet?

6pt (d) What is the angular acceleration of the comet?

(a) To find the radius of the comet's orbit, we need to study its circular motion. The force of gravity is what produces the circular motion, and circular motion requires a centripetal acceleration, so by Newton's second law:

$$F_G = ma_c \text{ --> } GmM/r^2 = mv^2/r \text{ --> } GM/r = v^2$$

Now, we don't know the speed of the comet in orbit, but we do know that $v_{orb} = 2pi*r/T_{orb}$, so:

$$GM/r = 4pi^2r^2/T^2 \text{ --> } r^3 = (GM/4pi^2)T^2 \text{ --> } r = \text{cuberoot}((GM/4pi^2)T^2)$$

We know the period T = 75 yr, as given in the problem. After converting, $T = 2.36\text{x}10^9$ s. Note that Halley's comet orbits the **Sun**, not the Earth, so M (the attracting body) is the mass of the Sun. So:

$$r = \text{cuberoot}[((6.67\text{x}10^{-11})(1.99\text{x}10^{30})/4pi^2)(2.36\text{x}10^9)^2] = \mathbf{2.66x10^{12}\ m}$$

(b) The closest that Halley's comet approaches Earth is simply the difference in their orbital radii:

$$D = 2.66\text{x}10^{12} - 1.5\text{x}10^{11} = \mathbf{2.51x10^{12}\ m}$$

(c) The angular speed can be found with the usual v = rw equation, but since we don't know v, we should substitute in the equation for T instead:

$$w = v/r = (2pi*r/T)/r = 2pi/T = 2pi/(2.36\text{x}10^9) = \mathbf{2.66x10^{-9}\ rad/s}$$

(d) The angular acceleration is going to obey the usual equation a = r*alpha. However, it's **very** important to remember which a appears in that equation! The acceleration here is the **tangential** acceleration; the acceleration that changes the orbital speed. Since the comet has a constant period, it has a constant orbital speed, so a = 0 and therefore **alpha = 0**.

FORMULA SHEET

- Constants:

$$g = 9.8 \text{m/s}^2$$

- Vectors:

$$\begin{aligned} \vec{A} \cdot \vec{B} &= AB\cos\theta \\ &= A_x B_x + A_y B_y + A_z B_z \end{aligned}$$

$$\left| \vec{A} \times \vec{B} \right| = AB\sin\theta$$

- Kinematics:

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum \vec{F} = m\vec{a}$$

$$W = mg$$

$$f_{s,max} = \mu_s N$$

$$f_k = \mu_k N$$

- Work & Energy:

$$W = \vec{F} \cdot \Delta\vec{x} \quad \text{or} \quad W = \int \vec{F} \cdot d\vec{x}$$

$$W_{tot} = \Delta K$$

$$W_{cons} = -\Delta U$$

$$K = \frac{1}{2} m v^2$$

$$U_g = mgy$$

$$K_i + U_i + W_{nc} = K_f + U_f$$

$$\vec{F} = -\vec{\nabla} U$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

- Momentum & Collisions:

$$\vec{p} = m\vec{v}$$

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$\vec{v}_{1i} - \vec{v}_{2i} = \vec{v}_{2f} - \vec{v}_{1f}$$

- Rotational Mechanics

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\tau = rF\sin\theta$$

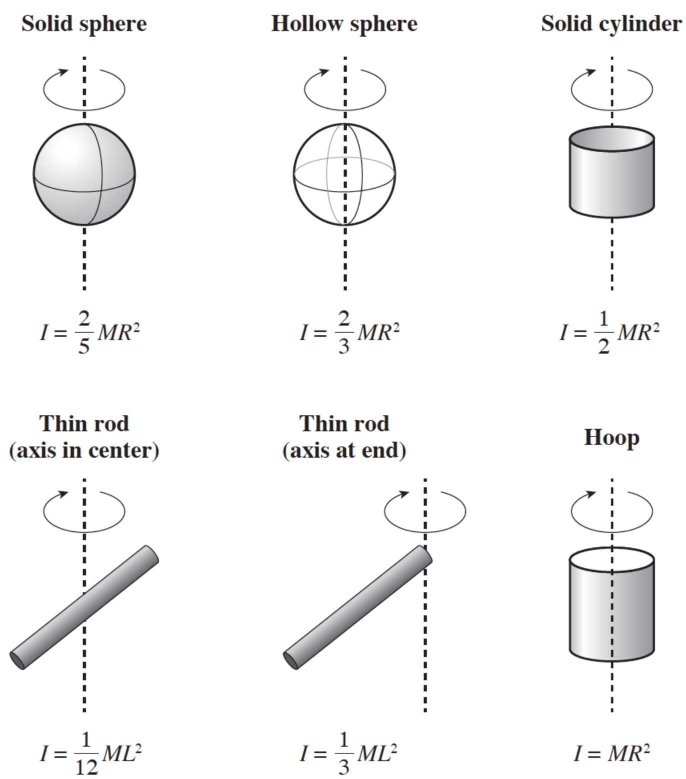
$$\sum \tau = I\alpha \quad \text{or} \quad \sum \tau = \frac{dL}{dt}$$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$L = I\omega \quad \text{or} \quad L = rp$$

$$I = \int r^2 dm$$

$$I_{new} = I_{cm} + md^2$$



- Gravity:

$$G = 6.67 \times 10^{-11}\ \frac{\mathrm{N}m^2}{\mathrm{kg}^2}$$

$$F_G = G\frac{mM}{r^2}$$

$$a_G = G\frac{M}{r^2}$$

$$U_G = -G\frac{mM}{r}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$v_{orb} = r\omega_{orb} = \frac{2\pi r}{T_{orb}}$$

- Oscillations:

$$F_{sp} = kx$$

$$U_{sp} = \frac{1}{2}kx^2$$

$$\omega_{sp} = \sqrt{\frac{k}{m}}$$

$$\omega_{pend} = \sqrt{\frac{g}{l}}$$

$$f = 1/T$$

$$\omega = 2\pi f$$

- Waves:

$$v = \lambda f$$

$$f_{beat} = |f_1 - f_2|$$