PHYS2350 General Physics I/Lab

Section EV1

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Exam 3 (Chapters 8, 10 – 13)

November 30, 2017

Name: Solutions

ID: ____________________

Instructions: This exam is composed of **5 multiple choice questions** and 4 free-response problems. To receive a perfect score (100) on this exam, three of the four free-response problems must be completed. The fourth free-response problem **may be answered for extra credit**.

Each multiple choice question is worth 5 points, and each free-response problem is worth 25 points, so there will be 25 points from the multiple choice questions and 75 points from the 3 free-response problems, totalling 100 points. The fourth free-response problem, worth 25 points of extra credit, pulls the maximum score possible to 125 points.

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

1) Consider a cylinder rolling down a hill. If we consider the rotational axis to be the point of contact with the ground, which force puts a torque on the cylinder and causes it to rotate?

a) The normal force

b) $\boxed{\text{Gravity}}$

c) Static friction

d) A combination of forces

If the rotational axis is at the point of contact with the ground, then the force putting a torque on the cylinder will be $\boxed{\text{gravity}}$, option (b). The only other 2 forces acting on the cylinder, the normal force and the static friction force, both act at the point of contact with the ground, so their distance from the rotational axis $r = 0$, and therefore $\tau = 0$.

2) A 3kg object with a density of 1200 kg/m^3 rests on a scale in a vat of water. What does the scale read?

a) 1N

b) $\boxed{\text{4.9N}}$

c) 24.5N

d) 29.4N

If the object rests on a scale in a vat of water, then it must sink in water. (We can also tell this because its density is greater than that of water.) This means that the weight W is greater than the buoyant force B, so if it's at rest, there must be a third force, pointing upward, that balances the other two forces. This force is just the normal force N, since it's the supporting force that the scale puts on the object so the object won't continue to sink. So we have two forces pointing upward, B and N, and one force pointing downward, W, and the acceleration is zero. So, Newton's second law says:

$$B + N = W$$

Recall that a scale measures the *apparent weight*, which is the normal force N. Also recall that the buoyant force is $B = \rho_f g V_{obj}$, since the object is fully submerged and that $\rho_f = 1000$ kg/m^3 for water. So, in order to find the buoyant force, we need to solve for the volume of the object:

$$\rho_{obj} = \frac{m}{V_{obj}} \quad \Rightarrow \quad V_{obj} = \frac{m}{\rho_{obj}} = \frac{3}{1200} = 0.0025 \text{ m}^3$$

So, solving for the normal force N in Newton's second law above:

$$N = W - B = mg - \rho_f g V_{obj} = (3)(9.8) - (1000)(9.8)(0.0025) = \boxed{4.9 \text{ N}}$$

3) Standing waves are produced on a 1kg string of length 20cm fixed at both ends. What is the fifth longest wavelength possible for such a standing wave?

a) 1cm

b) 2.2cm

c) 4cm

d) 5cm

If the string is fixed at both ends, we have node-node standing waves, which means that the harmonic number n can be any integer. The fifth integer is, obviously, going to be 5, so the fifth largest wavelength will just be the wavelength when $n = 5$:

$$\lambda = \frac{2L}{n} = \frac{2(20)}{5} = \boxed{8 \text{ cm}}$$

Since I made a mistake in this problem, and 8 cm isn't an option, everyone will **automatically get this question correct**. I forgot to multiply the 2 in the numerator when I wrote the answers to this question, so I got $20/5 = 4$cm, which is *not* correct.

4) Two sources emit sound: one at 1000Hz and one at an unknown frequency. If the known sound is changed from 1000Hz to 1100Hz, the beat frequency remains the same. What is the unknown frequency?

a) 1000Hz

b) $\boxed{\text{1050Hz}}$

c) 1100Hz

d) 1150Hz

Remember that the beat frequency is the **absolute value** of the difference in frequencies, $f_b = |f_1 - f - 2|$. So if there is a change in the known frequency, but the beat frequency doesn't change, it must be true that the unknown frequency must be halfway between the change in the known, i.e. $\boxed{1050 \text{ Hz}}$. You can see that $|1000 - 1050| = 50$ Hz and $|1100 - 1050| = 50$ Hz, so in both cases, the beat frequency is the same, exactly as the problem required.

5) A brass rod is heated from 400K to 500K. If it increases its length by 0.1cm, what was the initial length of the rod? The expansion coefficient of brass is 19×10^{-6} K^{-1}.

a) 0.53cm

b) 5.3cm

c) $\boxed{\text{53cm}}$

d) 530cm

The equation for thermal expansion is:

$$\Delta l = \alpha l_0 \Delta T$$

Solving for the initial length of the rod is a simple matter of manipulating the above formula:

$$l_0 = \frac{\Delta l}{\alpha \Delta T} = \frac{.001}{(19 \times 10^{-6})(500 - 400)} = 0.53 \text{ m} = \boxed{53 \text{ cm}}$$

FREE-RESPONSE PROBLEMS

1) A 5kg disc with a radius of 0.4m rotates with an initial speed of 100 rad/s.

a) What is the initial kinetic energy of the disc?

b) If a brake is applied and the disc stops in 4s, what torque did the brake apply on the disc? Assume the deceleration was constant.

c) How much heat was released during the braking process?

a) The rotational kinetic energy of an object depends on the moment of inertia of that object, so we should calculate that first. For a disc, the moment of inertia is:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(5)(0.4)^2 = 0.4 \text{ kgm}^2$$

Now that we know the moment of inertia, the initial kinetic energy is:

$$K_i = \frac{1}{2}I\omega_0^2 = \frac{1}{2}(0.4)(100)^2 = \boxed{2000 \text{ J}}$$

b) Since the deceleration was constant, we can use rotational kinematics. Knowing the initial angular velocity, the time, and the final angular velocity (which is zero), the best equation to use is:

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = -\frac{\omega_0}{t} = -\frac{(100)}{(4)} = -25 \ \frac{\text{rad}}{\text{s}^2}$$

The negative sign is just to indicate that this is a deceleration, and isn't relevant to the answer. (Another way of thinking about this is that angular acceleration, and torque, are vectors, so the negative sign indicates a direction, which we don't care about.) We can use the rotational form of Newton's second law to find the torque required to produce this angular acceleration:

$$\tau = I\alpha = (0.4)(25) = \boxed{10 \text{ Nm}}$$

An equivalent way of doing this is to find the initial angular momentum, $L = I\omega = 40$ Js, and then using torque as the rate of change of angular momentum to find the torque applied, $\tau = \frac{\Delta L}{\Delta t} = \frac{40}{4} = 10$ Nm.

c) The amount of heat released is simply the amount of kinetic energy lost during this breaking process. Since the disc started out with 2000J of kinetic energy, and ended with zero, the amount of heat emitted was:

$$\boxed{Q = 2000 \text{ J}}$$

2) Water is pumped through a horizontal pipe. At one point in the pipe, where the radius is 10cm, the water moves with a pressure of 2×10^5 Pa at a speed of 10m/s.

a) How many cubic meters of water move through the piper per second?

b) At a second point in the pipe, the radius decreases to 5cm. What is the speed of the water in this section of pipe?

c) What is the pressure of the water in the thinner section of pipe?

a) The flow rate of a fluid, f, is the amount of volume per unit time of a fluid that moves through a pipe. If the pipe has an area A, and a fluid moves some distance x in a time t, the flow rate is:

$$f = \frac{\text{volume}}{\text{time}} = \frac{Ax}{t} = Av$$

For an incompressible fluid, i.e. a liquid, the flow rate is going to be the same everywhere in a pipe; if we know it at one point in the pipe, we know it at all points in the pipe. We know that when the pipe has a radius of 10cm, the water moves at a speed of 10m/s, so the flow rate of the water is:

$$f = Av = \pi r^2 v = \pi(0.1)^2(10) = \boxed{0.31\ \frac{\text{m}^3}{s}}$$

Or, you could say 0.31 m^3 moves through the pipe in 1s.

b) As mentioned, flow rate is conserved for an incompressible fluid, so:

$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_2 = \frac{A_1}{A_2} v_1 = \frac{r_1^2}{r_2^2} v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{0.1}{0.05}\right)^2 (10) = \boxed{40\ \frac{\text{m}}{\text{s}}}$$

c) To find the pressure in this second section of pipe, we need to use the Bernoulli equation. The three variables in the Bernoulli equation are pressure, speed, and height. The height of the water through a horizontal pipe doesn't change, so we can just set $y_1 = y_2 = 0$; we know the speed in the first section of the pipe, $v_1 = 10$ m/s, and we calculated the speed through the second section of the pipe, $v_2 = 40$ m/s, in part b; and we're told that the pressure in the first section of pipe is $P_1 = 2 \times 10^5$ Pa, so we just need to solve for the pressure in the second section of pipe P_2:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\Rightarrow P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = (2 \times 10^5) + \frac{1}{2}(1000)(10)^2 - \frac{1}{2}(1000)(40)^2 = \boxed{-550,000 \text{ Pa}}$$

As a side note, pressure is actually due to a *net* force, not due to a single force. What a positive pressure means is that the fluid inside the tube is pushing harder on the tube's interior that the pressure outside the tube. What a negative pressure would mean is that the pressure outside the tube is pushing harder on the tube's exterior than the fluid inside the tube. A large positive pressure would cause the tube to *ex*plode, while a large negative pressure would cause the tube to *im*plode.

3) A 400g mass oscillates on a spring with a force constant of 150N/m. The maximum speed of the oscillations is 10m/s.

a) What is the total energy of the mass?

b) What is the period of the mass' oscillation?

c) What is the amplitude of the oscillation?

a) The total energy is calculated by knowing the speed of the mass at any point in the oscillation. If we know that the maximum speed is 10m/s, which occurs at the equilibrium position $x = 0$, then the total energy of the oscillation is:

$$E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.4)(10)^2 = \boxed{20 \text{ J}}$$

b) The period of an oscillation is related to the angular frequency of the oscillation by the equation:

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

Using the equation for the angular frequency of a spring, the period of the oscillation is:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.4}{150}} = \boxed{0.324 \text{ s}}$$

c) The amplitude of the oscillation can be found from the total energy. Remember that the total energy is equal to the kinetic energy plus the potential energy at any point in the oscillation, because energy is conserved. At the amplitude, the speed is zero, so the kinetic energy is also zero. So, we have:

$$E = \frac{1}{2}kA^2 \quad \Rightarrow \quad A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(20)}{(150)}} = \boxed{0.52 \text{ m}}$$

4) A gas made of particles with a mass of 5×10^{-26} kg is stored at a volume of 0.005m^3 and a temperature of 350K.

a) What is the rms speed of the gas?

b) If the gas is compressed at a constant pressure, what would the temperature be at a volume of 0.001m^3?

c) Using the work-energy theorem, estimate the amount of work this compression would take.

a) The rms speed of the gas is given by the formula:

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(350)}{(5 \times 10^{-26})}} = \boxed{538\ \frac{\text{m}}{\text{s}}}$$

b) If the gas is compressed at a constant pressure, we can re-arrange the ideal gas law such that:

$$PV = Nk_BT \quad \Rightarrow \quad \frac{V}{T} = \frac{Nk_B}{P} = \text{constant} \quad \Rightarrow \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Solving for T_2:

$$T_2 = \frac{V_2}{V_1}T_1 = \frac{0.001}{0.005}(350) = \boxed{70\ \text{K}}$$

c) Now, as we learned in chapters 14 and 15, calculating work as the change in kinetic energy of the gas is **not correct**, because it ignores heat transfer (i.e. it ignores the first law of thermodynamics). However, this is an *estimate*, so we'll just go with it and see what it says. The (average) kinetic energy is going to be $\frac{1}{2}mv_{rms}^2$, or by the equipartition theorem $\frac{3}{2}k_BT$, which is easier to calculate:

$$K_i = \frac{3}{2}k_BT_i = \frac{3}{2}(1.38 \times 10^{-23})(350) = 7.24 \times 10^{-21}$$

$$K_f = \frac{3}{2}k_BT_f = \frac{3}{2}(1.38 \times 10^{-23})(70) = 1.45 \times 10^{-21}$$

The work energy theorem says $W = \Delta K$, so:

$$W = K_f - K_i = 1.45 \times 10^{-21} - 7.24 \times 10^{-21} = \boxed{-5.79 \times 10^{-21}\ \text{J}}$$

This is actually a really *terrible* estimate because the sign isn't even correct. The work to compress a gas at a constant pressure is $W = -P\Delta V$; since $\Delta V < 0$, the work should be positive. This illustrates why the work-energy theorem is no good for thermodynamic processes, and we need to use the first law of thermodynamics, which is like a more complete form of the work-energy theorem because it allows for heat transfer.

FORMULA SHEET

- Vectors:

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

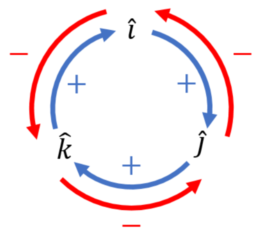

Figure 1: Cyclic permutations for cross product

- Kinematics:

$$g = 9.8\text{m/s}^2$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

- Forces:

$$\sum \vec{F} = m\vec{a}$$

$$W = mg$$

$$f_{s,max} = \mu_s N$$

$$f_k = \mu_k N$$

- Circular Motion:

$$a_c = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

- Gravity:

$$G = 6.67 \times 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$$

$$F_g = G\frac{mM}{r^2}$$

$$a_g = G\frac{M}{r^2}$$

$$T^2 = \frac{4\pi^2}{GM}a^3 \quad \text{(Kepler's third law)}$$

- Work & Energy:

$$
\begin{aligned}
&K = \frac{1}{2}mv^2\\
&U_g = mgy\\
&W = F\Delta x\cos\theta\\
&W_{tot} = \Delta K\\
&W_{cons} = -\Delta U\\
&W_{other} = \Delta E\\
&K_i + U_i + W_{nc} = K_f + U_f\\
&P = \frac{\Delta E}{\Delta t}\\
&P = Fv \;\; \text{(at constant velocity)}
\end{aligned}
$$

- Linear Momentum:

$$
\begin{aligned}
&\vec{p} = m\vec{v}\\
&\sum \vec{F}_{ext,sys} = \frac{\Delta \vec{p}_{sys}}{\Delta t}\\
&m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi} = m_A\vec{v}_{Af} + m_B\vec{v}_{Bf}\\
&v_{1i} - v_{2i} = v_{2f} - v_{1f} \;\; \text{(elastic collisions)}\\
&\vec{J} = \vec{F}_{av}\Delta t \;\; \text{(impulse)}
\end{aligned}
$$

- Rotational Motion:
  - Rotational Kinematics:

$$
\begin{aligned}
&\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2\\
&\omega = \omega_0 + \alpha t\\
&\omega^2 = \omega_0^2 + 2\alpha\Delta\theta
\end{aligned}
$$

  - Rolling without slipping:

$$
\begin{aligned}
&\Delta x = R\Delta\theta\\
&v = R\omega\\
&a = R\alpha
\end{aligned}
$$

  - Rotational Dynamics:

$$
\begin{aligned}
&\tau = rF\sin\theta\\
&\Delta\tau = I\alpha = \frac{\Delta L}{\Delta t}\\
&K_{rot} = \frac{1}{2}I\omega^2\\
&L = I\omega = rp
\end{aligned}
$$

- Rotational Motion (continued):
  - Moment of inertia:
$$I = mr^2 \text{ (point mass)}$$

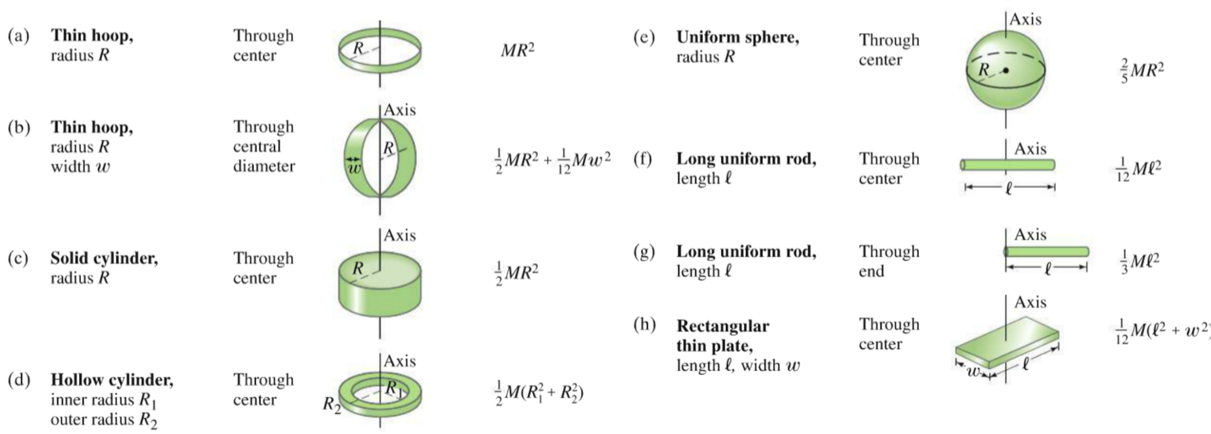

Figure 2: Moments of Inertia of Rigid Objects

- Fluids:

$$\rho_{H_2O} = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$P = \frac{F}{A}$$

$$P_f = \rho_f g D$$

$$B = \rho_f g V_{sub}$$

$$\frac{V_{sub}}{V_{obj}} = \frac{\rho_{obj}}{\rho_f}$$

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

- Oscillations:

$$F_{sp} = -kx$$

$$U_{sp} = \frac{1}{2}kx^2$$

$$E = U_{max} = K_{max}$$

$$\omega_{sp} = \sqrt{\frac{k}{m}}$$

$$\omega_{pend} = \sqrt{\frac{g}{l}}$$

- Waves and Sound:

$$v_{\text{sound}} = 350\text{m/s}$$

$$I_0 = 10^{-12}\frac{\text{W}}{\text{m}^2}$$

$$v = \lambda f$$

$$v = \sqrt{\frac{T}{m/L}} \;\; \text{(mechanical wave on a string)}$$

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\beta = (10\text{dB})\log\left(\frac{I}{I_0}\right)$$

$$\lambda_n = \frac{2L}{n},\; f_n = \frac{nv}{2L},\; n = 1, 2, 3, ... \;\; \text{(node-node)}$$

$$\lambda_n = \frac{4L}{n},\; f_n = \frac{nv}{4L},\; n = 1, 3, 5, ... \;\; \text{(node-antinode)}$$

$$f_{obs} = \frac{v \pm v_{obs}}{v \mp v_s} f_s \;\; \text{("top is towards")}$$

- Temperature and Heat:

$$k_B = 1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}$$

$$\Delta l = l_0 \alpha \Delta T$$

$$PV = N k_B T$$

$$K_{av} = \frac{3}{2} k_B T$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$