PHY2048 Physics with Calculus I

Section 611806

Prof. Douglas H. Laurence

Exam 1 (Chapters 1 – 5)

February 25, 2019

Name: SOLUTIONS

Instructions:

This exam is composed of **10 multiple choice questions** and **4 free-response problems**. To receive a perfect score (100) on this exam, 3 of the 4 free-response problems must be completed. The fourth free-response problem **may not be answered for extra credit**. Each multiple choice question is worth 2.5 points, for a total of 25 points, and each free-response problem is worth 25 points, for a total of 75 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

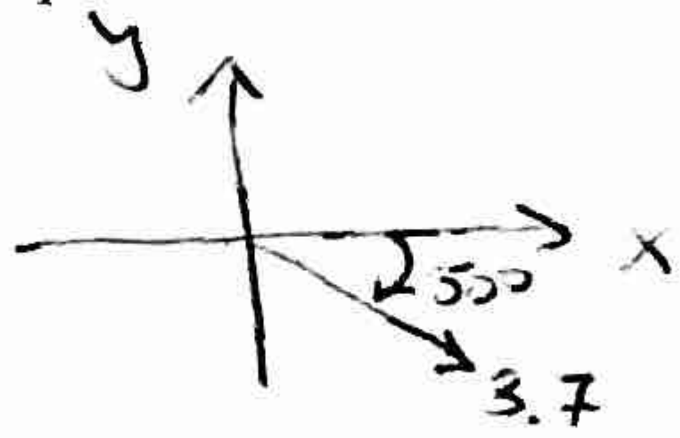
Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

1. A vector has a magnitude of 3.7 and makes an angle of 50^o clockwise from the x-axis. With the correct sign, what is the x-component of the vector?

   (a) 2.38
   (b) 2.83
   (c) −2.38
   (d) −2.83



x-comp: $3.7\cos 50 = \boxed{2.38}$
$\boxed{+}$ b/c it points in the $+x$ direction.

2. What should the magnitude of the cross product $\vec{A} \times (\vec{A} \times \vec{B})$ be? Assume that the angle between $\vec{A}$ and $\vec{B}$ is 90^o.

   (a) 0
   (b) A
   (c) B
   (d) A^2B

$|\vec{A}\times\vec{B}| = AB\sin\theta = AB$ | if θ between $\vec{A}$ & $\vec{B}$ is $90°$, then $\sin 90 = 1$

Now, $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ + $\vec{B}$, so θ between $\vec{A}$ & $\vec{A}\times\vec{B}$ is also $90°$, therefore:
$|\vec{A}\times(\vec{A}\times\vec{B})| = A^2B\sin 90 = \boxed{A^2B}$

3. The equations for kinematics can only be used under which condition(s)?

   (a) Constant velocity
   (b) Constant velocity and constant acceleration
   (c) Constant acceleration
   (d) They can be used under any condition

4. A car's velocity points in the $-x$ direction, and has an acceleration in the $-x$ direction. Which of the following is true?

   same direction ⇒ $\boxed{\text{speeding up}}$

   (a) The car moves in the $+x$ direction and speeds up
   (b) The car moves in the $+x$ direction and slows down
   (c) The car moves in the $-x$ direction and speeds up
   (d) The car moves in the $-x$ direction and slows down

Direction of velocity is direction of movement.
⇒ Moves in $\boxed{-x \text{ direction}}$

5. If a question were to state "A car is driving at 15 m/s when its brakes are applied..." the number 15 is what type of quantity?

   velocity

   (a) Displacement
   (b) Initial velocity
   (c) Final velocity
   (d) Acceleration

car drives at 15 m/s, then brakes
⇒ 15 m/s : $\boxed{\text{initial velocity}}$

6. Which of the following statements is always about projectile motion is always true?

   (a) The trajectory is always ~~parabolic, and~~ thus symmetric — only if $y_i = y_f$.
   (b) The velocity at ~~the~~ peak is zero — $V_y = 0$ at peak, $V_x \neq 0$
   (c) The acceleration is always due to gravity
   (d) The horizontal velocity ~~at the~~ peak is zero — $V_x \neq 0$ for a projectile to move horizontally.

a_x a_y

7. An object accelerates with $\vec{a} = (3\ \text{m/s}^2)\hat{\imath} + (4\ \text{m/s}^2)\hat{\jmath}$. If the object begins at rest, what is the object's vertical velocity after 1s?

VERTICAL ⇒ Y DIRECTION: $V_{0y} = 0$, $a_y = 4\,\text{m/s}^2$, $t = 1\text{s}$

(a) 3 m/s

(b) 4 m/s

(c) 5 m/s

(d) 6 m/s

$\Rightarrow V_y = \cancel{V_{0y}} + a_y t = (4\ \text{m/s}^2)(1\text{s}) = \boxed{4\ \text{m/s}}$

8. A 5kg object is pushed forward with a force of 10N while a constant force pushes backwards against the object with a force of 6N. What is the acceleration of the object?

(a) The object moves at a constant velocity

(b) 0.8 m/s² forward

(c) 1.2 m/s² backward

(d) 2 m/s² forward

6N 10N $\rightarrow +$

$\Sigma F = (10N) - (6N) = 4N$

$\Sigma F = ma$

$\Rightarrow a = \frac{\Sigma F}{m} = \frac{4N}{5kg} = \boxed{0.8\ \text{m/s}^2}$ $\boxed{\text{FORWARD}}$ ↳ (+)

9. An 2kg object hangs at rest from a rope attached to the ceiling. The weight of the object pulls down at roughly 20N, while the tension in the rope pulls up on the object with an equal magnitude of roughly 20N. These forces form an action-reaction pair as defined by Newton's third law.

(a) True

(b) False

BOTH WEIGHT & TENSION ACT ON THE SAME OBJECT.

10. Newton's third law states:

(a) An object will remain in its current state of motion unless acted upon by a force (NEWTON'S 1ST)

(b) The net force on an object is equal to the mass of the object multiplied by its acceleration (NEWTON'S 2ND)

(c) For any force one object could put on another, the other must put an equal and opposite force back on the first

(d) None of the above

FREE-RESPONSE PROBLEMS

1. Consider two vectors:

$$\vec{A} = 3\hat{\imath} - 2\hat{\jmath}$$
$$\vec{B} = -\hat{\imath} + 4\hat{\jmath}$$

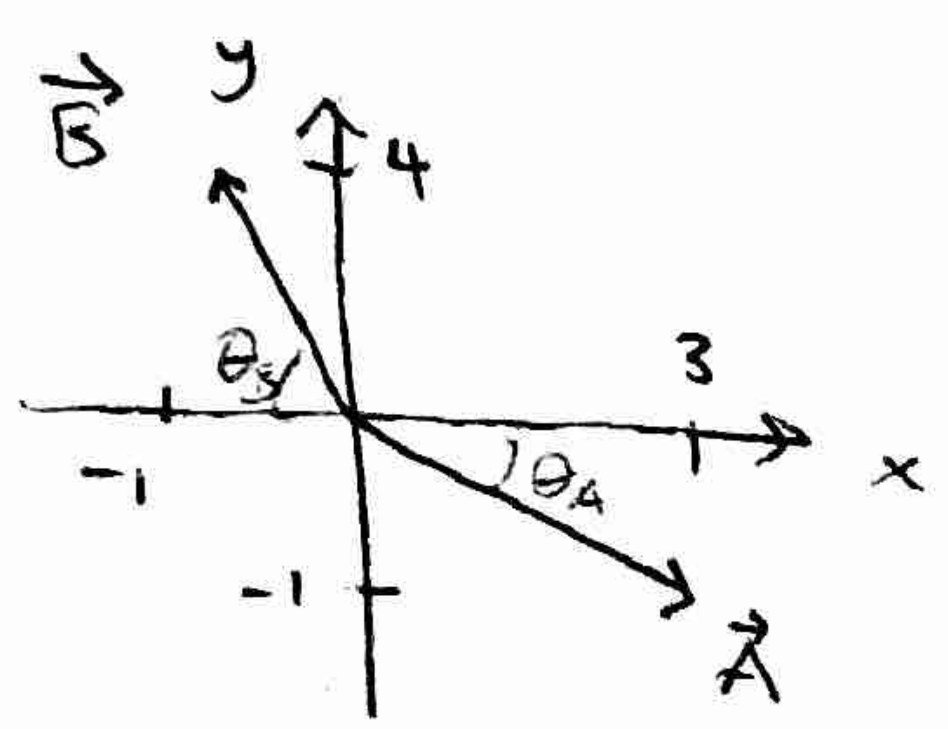
a) Find both the magnitude and direction of $\vec{A}$ and $\vec{B}$.

b) Find the vector $2\vec{A} - \vec{B}$. Give your result in unit vector notation (i.e. using $\hat{\imath}$ and $\hat{\jmath}$).

c) What is $\vec{A} \cdot \vec{B}$?

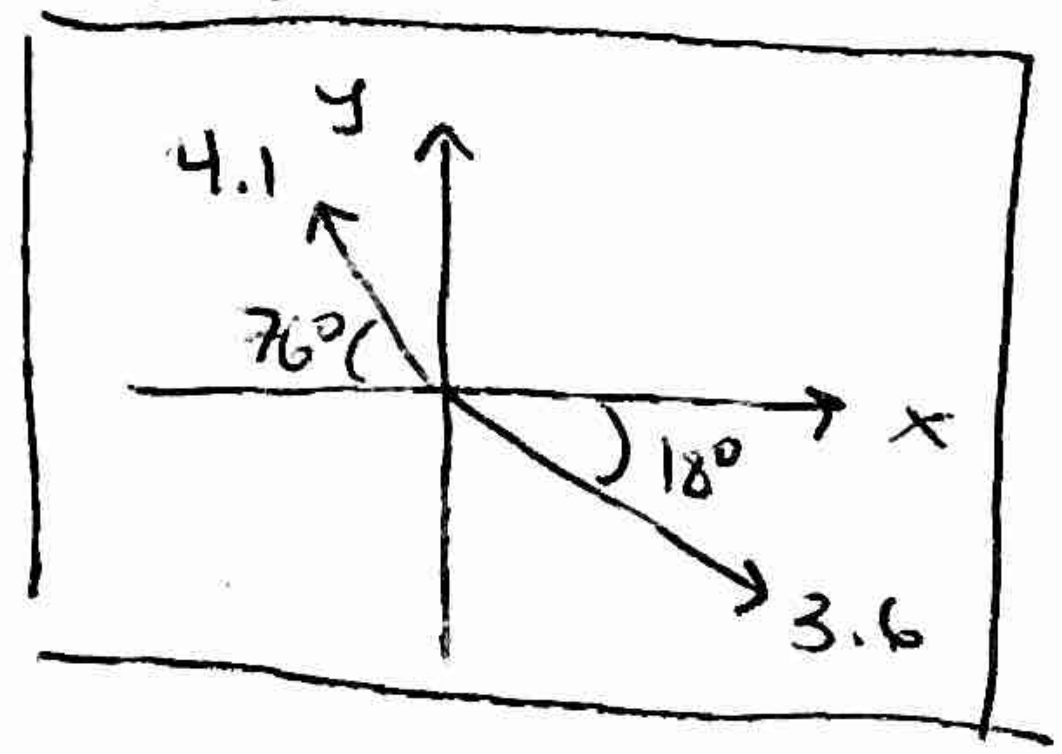
d) What is $\vec{A} \times \vec{B}$? Give your result in unit vector notation.

a)



$|\vec{A}| = \sqrt{(3)^2 + (2)^2} = 3.6$; $|\vec{B}| = \sqrt{(1)^2 + (4)^2} = 4.1$

$\theta_A = \tan^{-1}\left(\frac{1}{3}\right) = 18°$; $\theta_B = \tan^{-1}\left(\frac{4}{1}\right) = 76°$



b) $2\vec{A} - \vec{B}$

$= 2(3\hat{\imath} - 2\hat{\jmath}) - (-\hat{\imath} + 4\hat{\jmath}) = 6\hat{\imath} - 4\hat{\jmath} + \hat{\imath} - 4\hat{\jmath} = \boxed{7\hat{\imath} - 8\hat{\jmath}}$

c) $\vec{A} \cdot \vec{B} = (3)(-1) + (-2)(4) = -3 - 8 = \boxed{-11}$

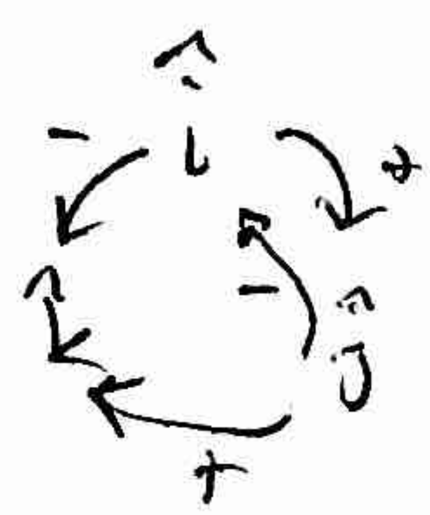
$\hat{\imath} \times \hat{\imath} = 0$ $\quad \hat{\jmath} \times \hat{\jmath} = 0$

d) $\vec{A} \times \vec{B} = (3\hat{\imath} - 2\hat{\jmath}) \times (-\hat{\imath} + 4\hat{\jmath})$

Keep Keep

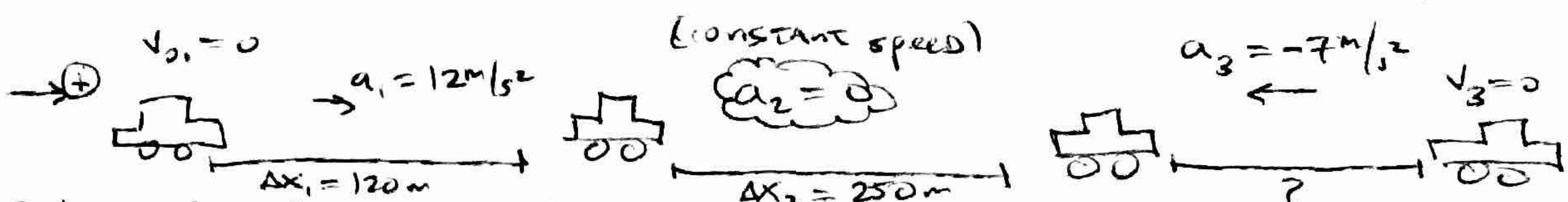
$= (3)(4)(\hat{\imath} \times \hat{\jmath}) + (-2)(-1)(\hat{\jmath} \times \hat{\imath})$

$= 12(\hat{k}) + 2(-\hat{k}) = \boxed{10\hat{k}}$



$\hat{\imath} \times \hat{\jmath} = +\hat{k}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$



2. A car accelerates from rest at 12 m/s^2 for a distance of 120m. After, the car travels at a constant speed for 250m, before decelerating at 7 m/s^2 until stopped.

a) How long was the car accelerating for during the first 120m?

b) What was the maximum speed of the car during the motion?

c) How long did it take the car to stop once it started decelerating?

d) How far did the car travel while it was decelerating?

a) IN MOTION 1 (AS DEFINED IN FIGURE ABOVE),

$V_{01} = 0, \ a_1 = 12 m/s^2, \ \Delta x_1 = 120 m$

$\Rightarrow \ \Delta x_1 = \cancel{V_{01} t_1}^{0} + \frac{1}{2} a_1 t_1^2 \ \Rightarrow \ t_1 = \sqrt{\frac{2 \Delta x_1}{a_1}} = \sqrt{\frac{2(120)}{(12)}} = \boxed{4.47 s}$

b) CAR SPEEDS UP DURING MOTION 1, MOVES AT CONSTANT SPEED DURING MOTION 2, THEN SLOWS DOWN DURING MOTION 3.

SO FASTEST SPEED IS SPEED AT END OF ACCELERATION IN MOTION 1:

$V_1 = \cancel{V_{01}} + a_1 t_1 = (12)(4.47) = \boxed{53.6 m/s}$

c) DON'T KNOW ENOUGH INFO ABOUT MOTION 3 (JUST ~~THAT~~ a_3 & V_3), BUT WE KNOW FINAL SPEED OF MOTION 1 = INITIAL SPEED OF MOTION 2, SO $V_{02} = V_1 = 53.6 m/s$. SINCE $a_2 = 0$, FINAL SPEED OF MOTION 2, WHICH EQUALS INITIAL SPEED OF MOTION 3, IS $53.6 m/s$, SO $V_{03} = V_2 = V_{02} = 53.6 m/s$. NOW WE KNOW 3 PIECES OF INFO:

$V_{03} = 53.6 m/s, \ a_3 = -7 m/s^2, \ V_3 = 0$

~~[illegible] EQUATION~~ $\Rightarrow \ \cancel{V_3}^{0} = V_{03} + a_3 t_3 \ \Rightarrow \ t_3 = \frac{-V_{03}}{a_3} = \frac{-(53.6)}{(-7)}$

$= \boxed{7.66 s}$

d) $\Delta x_3 = V_{03} t_3 + \frac{1}{2} a_3 t_3^2 = (53.6)(7.66) + \frac{1}{2}(-7)(7.66)^2$

$= \boxed{205 m}$

(NEGATIVE SIGN IS REALLY IMPORTANT!!)

3. A projectile is launched with an initial speed of 20 m/s at a launch angle of 50° from a building of height of 10m above the ground.

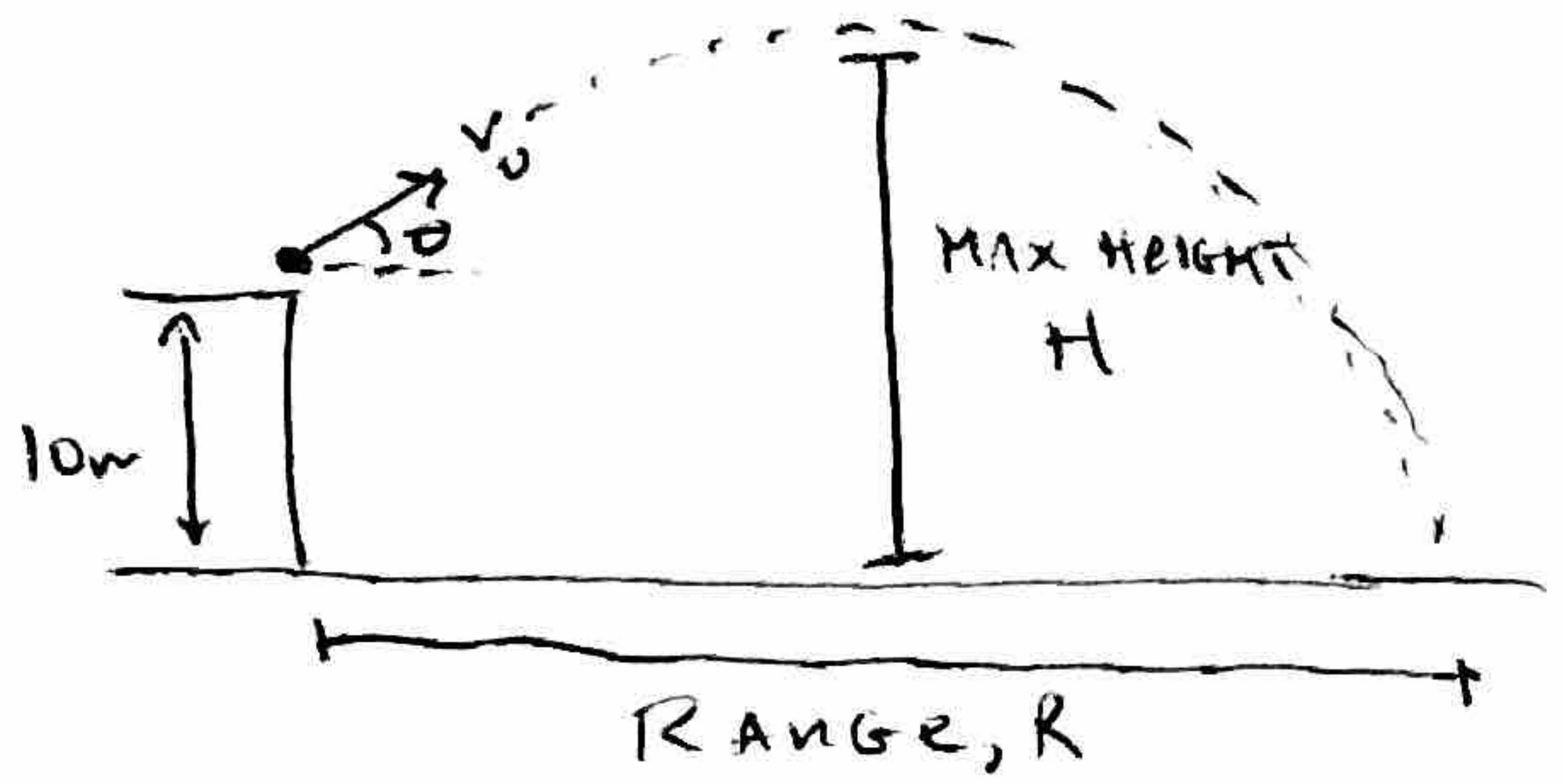
a) What is the maximum height of the projectile above the ground?

b) How long does it take the projectile to reach the maximum height?

c) How long is the projectile in the air for?

d) What is the range of the projectile?

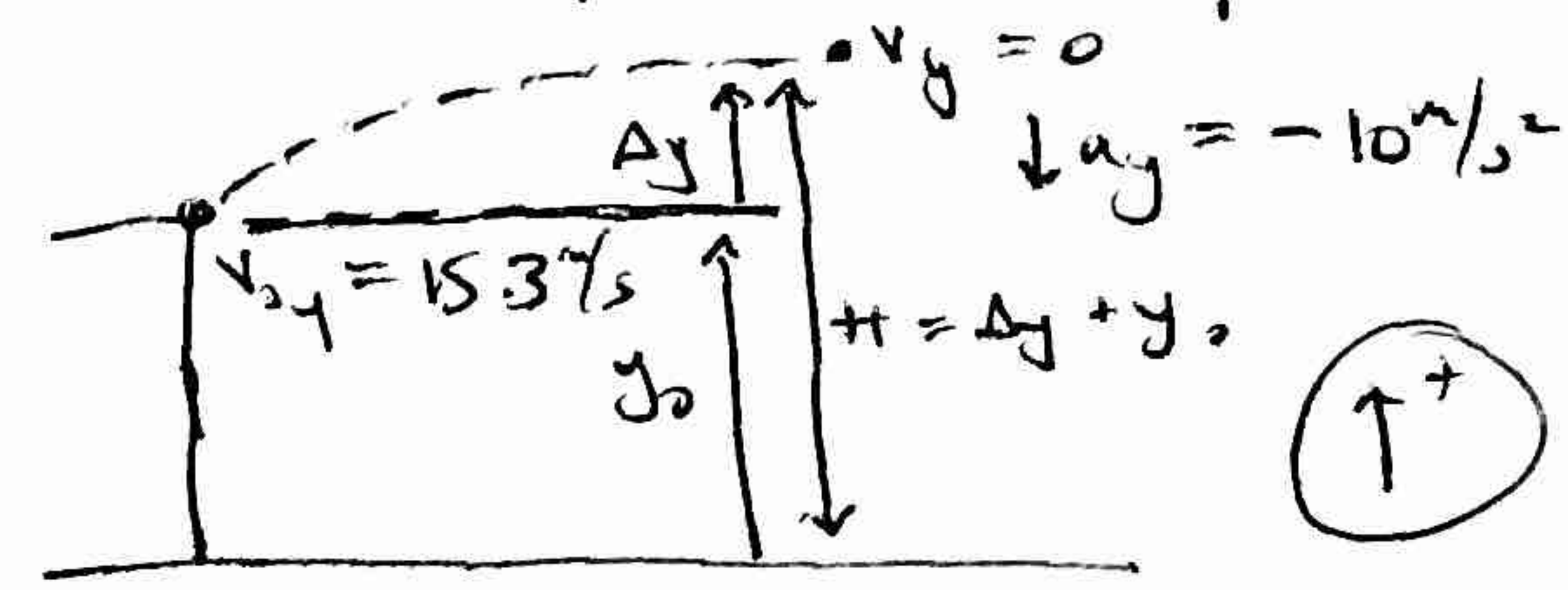
NOTE! I'M USING $g = 10\,m/s^2$; YOUR #'S WILL BE SLIGHTLY DIFFERENT FOR $g = 9.8\,m/s^2$



$V_{0x} = V_0 \cos\theta = (20)\cos 50 = 12.9\,m/s$

$V_{0y} = V_0 \sin\theta = (20)\sin 50 = 15.3\,m/s$

a) ONLY CONSIDER THE MOTION GOING UP TO THE PEAK:



$\cancel{V_y^2}^{0} = V_{0y}^2 + 2a_y \Delta y$

$\Rightarrow \Delta y = \frac{-V_{0y}^2}{2a_y} = \frac{-(15.3)^2}{2(-10)} = 11.7\,m$

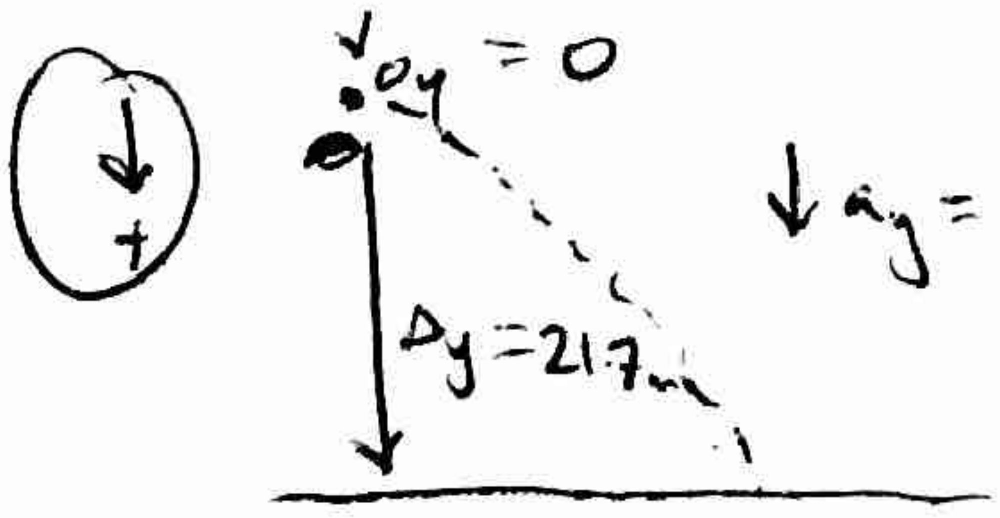
$\Rightarrow H = y_0 + \Delta y = 10 + 11.7 = \boxed{21.7\,m}$

b) TO FIND TIME TO GO UP, ONCE AGAIN, ONLY CONSIDER UPWARD MOTION:

$V_{0y} = 15.3\,m/s$, $a_y = -10\,m/s^2$, $V_y = 0$

$\Rightarrow \cancel{V_y}^{0} = V_{0y} + a_y t \Rightarrow t_{up} = \frac{-V_{0y}}{a_y} = \frac{-(15.3)}{(-10)} = \boxed{1.53\,s}$

c) TOTAL TIME = TIME UP + TIME DOWN. FOR DOWNWARD MOTION ONLY,



$\downarrow a_y = 10\,m/s^2$

$\Rightarrow \Delta y = \cancel{V_{0y} t}^{0} + \frac{1}{2} a_y t^2$

$\Rightarrow t_{down} = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(21.7)}{(10)}} = 2.08\,s$

$\Rightarrow t_{total} = t_{up} + t_{down} = 1.53 + 2.08$

$= \boxed{3.61\,s}$

d) RANGE = HORIZONTAL DISTANCE. SINCE $a_x = 0$,

$$\Delta x = v_{ox} t \Rightarrow R = v_{ox} t_{TOTAL} = (12.9)(3.61)$$

$$= \boxed{46.6\,m}$$

4. A 1.2kg ball attached to a 50cm string is moved in a circle in the horizontal plane. If the maximum tension the string can withstand without breaking is 1500N,

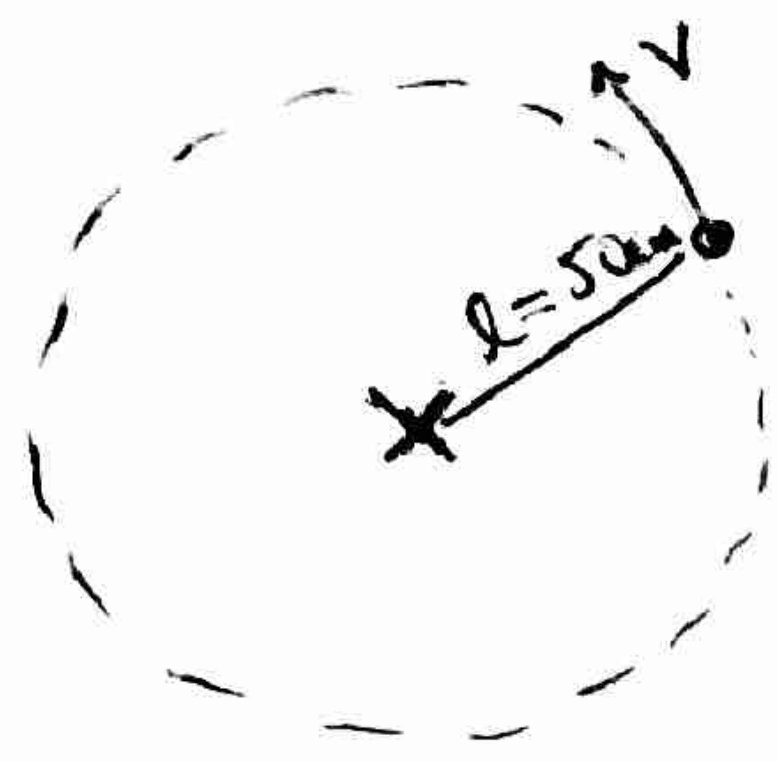
a) What is the maximum angular speed of the ball?

b) What is the maximum linear speed of the ball?

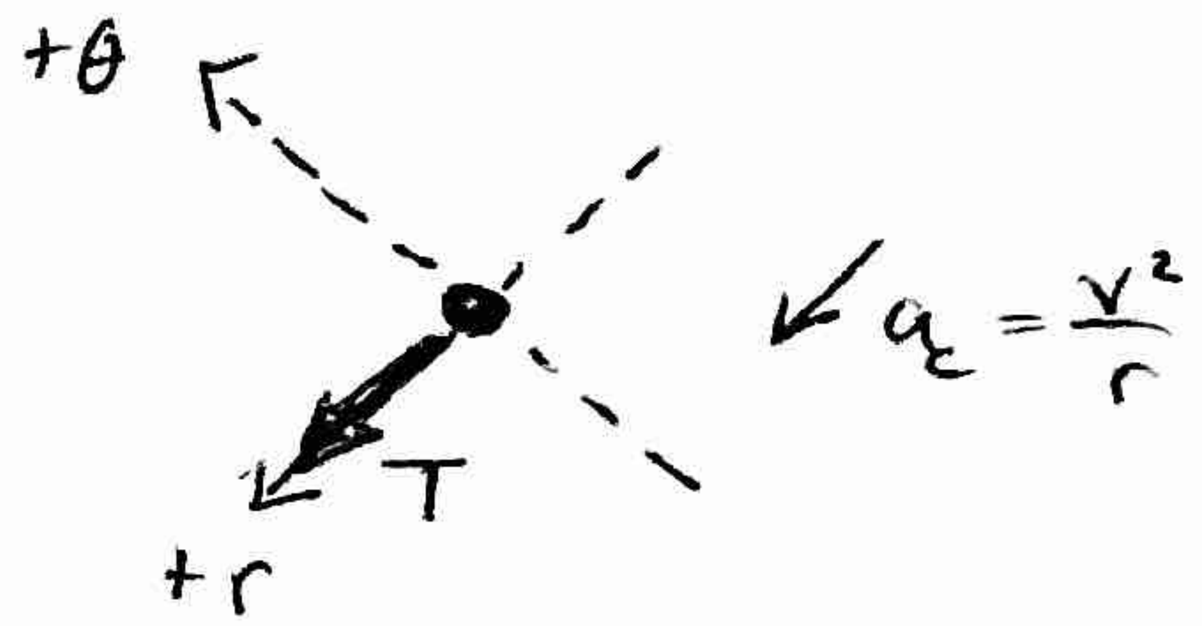
c) What is the minimum period of the ball's motion?

HORIZONTAL PLANE

(GRAVITY IS PERPENDICULAR)



FREE BODY DIAGRAM



$$\Sigma F_r = T$$ (NO OTHER RADIAL FORCES)

I ACCIDENTALLY DID (b) FIRST

b) Newton's 2ND LAW:

$$\Sigma F_r = m a_r \Rightarrow T = m a_c = m \frac{V^2}{r}$$ (RECALL 50cm = 0.5m)

$$\Rightarrow V_{MAX} = \sqrt{\frac{r T_{MAX}}{m}} = \sqrt{\frac{(0.5)(1500)}{(1.2)}} = \boxed{25 \text{ m/s}}$$

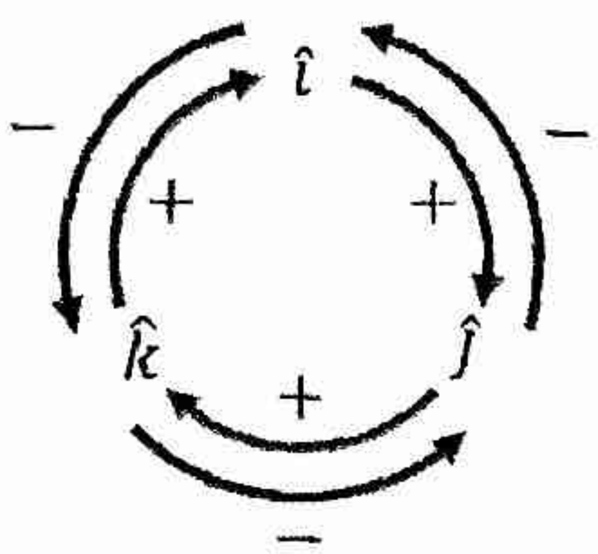
a) $$\omega_{MAX} = \frac{V_{MAX}}{r} = \frac{(25)}{(0.5)} = \boxed{50 \text{ RAD/s}}$$

c) $$\omega = \frac{2\pi}{T} \Rightarrow T_{MIN} = \frac{2\pi}{\omega_{MAX}} = \frac{2\pi}{(50)} = \boxed{0.126 \text{ s}}$$

FORMULA SHEET

- Vectors:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$
$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$



- Kinematics:

$$g = 9.8\mathrm{m/s}^2$$
$$\vec{v}_{av} = \frac{\Delta\vec{x}}{\Delta t};\ \vec{v}(t) = \frac{d\vec{x}}{dt}$$
$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t};\ \vec{a}(t) = \frac{d\vec{v}}{dt}$$
$$\Delta x = v_0t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a\Delta x$$

- Circular motion:

$$a_c = \frac{v^2}{r} = \omega^2 r$$
$$v = \omega r$$
$$\omega = \frac{2\pi}{T}$$

- Forces:

$$\sum\vec{F} = m\vec{a}$$
$$W = mg$$