PHY2049 Physics with Calculus II

Section 589357

Prof. Douglas H. Laurence

Exam 1 (Chapters 22 − 26)

June 13, 2018

Name: SOLUTIONS

Instructions:

This exam is composed of **10 multiple choice questions** and **4 free-response problems**. To receive a perfect score (100) on this exam, 3 of the 4 free-response problems must be completed. The fourth free-response problem **may not be answered for extra credit**. Each multiple choice question is worth 2.5 points, for a total of 25 points, and each free-response problem is worth 25 points, for a total of 75 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

ANSWER NOT AN OPTION ⇒ AUTOMATICALLY CORRECT FOR ALL.

1. Two charges are placed some distance apart, and exert a force F on each other. If the distance between the charges is halved, and each charge is doubled, what would the new force exerted between the charges be?

   (a) $F/8$

   (b) $F/4$

   (c) $4F$

   (d) $8F$

$$F = k\frac{q_1 q_2}{d^2} \rightarrow F' = k\frac{(2q_1)(2q_2)}{(d/2)^2} = \underbrace{\frac{2\cdot 2}{1/4}}_{8}\underbrace{\left(k\frac{q_1 q_2}{d^2}\right)}_{F} = 16F$$

2. Consider a charge q_A producing an electric field. A second charge q_B feels the electric field with a magnitude E. If the distance between q_A and q_B is doubled, and the charge q_B is halved, what is the new value of the electric field felt by q_B?

   (a) $E/8$

   (b) $E/4$

   (c) $E/2$

   (d) E

$$E = k\frac{q_A}{d^2} \rightarrow E' = k\frac{q_A}{(2d)^2} = \frac{1}{4}\underbrace{\left(k\frac{q_A}{d^2}\right)}_{E} = \frac{1}{4}E$$

3. Which of the following is true for conductors?

   (a) Charges will spread throughout a conductor and the electric field will be zero within the conductor.

   (b) Charges will move to the surface of a conductor and the electric field will be zero within the conductor.

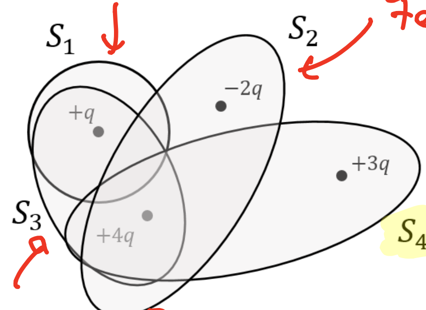
   (c) Charges will move to the surface of a conductor and the electric field will be non-zero within the conductor.

   (d) Charges will be spread throughout a conductor and the electric field will be non-zero within the conductor.

GAUSS' LAW: $\Phi_{tot} = \frac{q_{enc}}{\varepsilon_0}$

$q_{enc,1} = +q$

$q_{enc,2} = +2q$



$\boxed{q_{enc,4} = +7q}$

$q_{enc,3} = +5q$

LARGEST q_{enc} ⇒ LARGEST Φ

4. Which surface in the above figure has the largest flux passing through it?

   (a) S_1

   (b) S_2

   (c) S_3

   (d) S_4

5. A charge q can be placed at the center of 3 different Gaussian surfaces: Surface A is a cube with a side length L, Surface B is a sphere with a radius L, and Surface C is a cylinder with a height and radius of L. Which surface should the charge be placed inside to produce the maximum flux through the surface?

   (a) A
   (b) B
   (c) C
   (d) All Gaussian surfaces would have the same flux

$$\Phi_{tot} = \frac{q_{enc}}{\varepsilon_0}$$

All q_{enc} are the same, so all Φ_{tot} are the same!

6. A 1C charge moves from point A, which has a potential of 2V, to point B, which has a potential of 6V. What is the gain in the charge's kinetic energy from A to B?

   (a) 2J
   (b) 4J
   (c) 6J
   (d) 12J

$$\underbrace{\Delta K}_{\text{Kinetic gained}} = \underbrace{\Delta U}_{\text{Potential lost}} = q\,\Delta\phi = (1C)(4V) = 4J$$

$\Delta\phi = \phi_A - \phi_B$
$= 6V - 2V = 4V$

7. Protons will always move towards:

   (a) High potential energy and high potential
   (b) High potential energy but low potential
   (c) Low potential energy but ~~high potential~~
   (d) Low potential energy and low potential

$\underset{(-)}{\Delta U} = \underset{(+)}{q}\,\underset{(-)}{\Delta\phi}$ for protons

$\boxed{\text{+ charges to low } \phi}$

E from + to −

8. Charges are separated such that all positive charges accumulate at point A and all negative charges accumulate at point B, as shown in the figure above. Which of the following statements is true?

   (a) The electric field points from A to B, with A being the point of low potential
   (b) The electric field points from A to B, with B being the point of low potential
   (c) The electric field points from B to A, with A being the point of low potential
   (d) The electric field points from B to A, with B being the point of low potential

$q > 0$

+ charge to low ϕ!

9. Capacitance is a quantity that should depend upon:

   (a) The charge stored on the capacitor
   (b) The voltage across the capacitor
   (c) The geometry of the capacitor
   (d) Any number of these factors

Doesn't depend on dynamic variables like Q & V.

10. A capacitor with a capacitance of 4F is connected to a battery outputting a voltage of 2V. What is the charge on the capacitor?

   (a) 2C
   (b) 4C
   (c) 8C
   (d) 12C

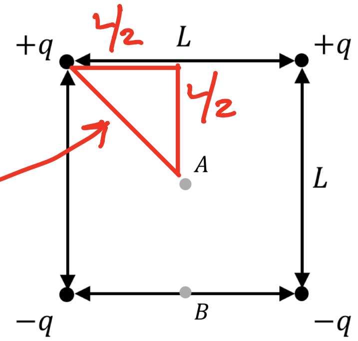
$$Q = CV = (4F)(2V) = 8C$$

FREE-RESPONSE PROBLEMS

$r = \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}$

$= \sqrt{5^2 + 5^2}$

$= 7.1\text{ cm}$



$q = e = 1.6 \times 10^{-19}$

$L = 10\text{ cm}$

1. Imagine four charges arranged in the corners of an $L \times L$ square, as shown in the figure above.

   (a) What is the electric field at the center of the square, marked as point A?

   (b) If a fifth charge of $+q$ was placed at the center of the square, what would the electric force on that charge be?

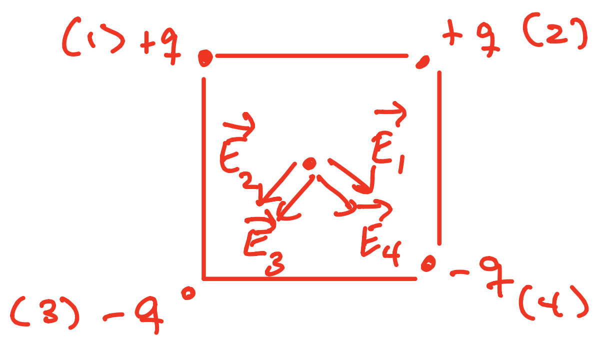
   (c) If the fifth charge moves from the center of the square downwards until it reached the center of the bottom edge, from point A to B that is, how much work would the electric force do on the charge?

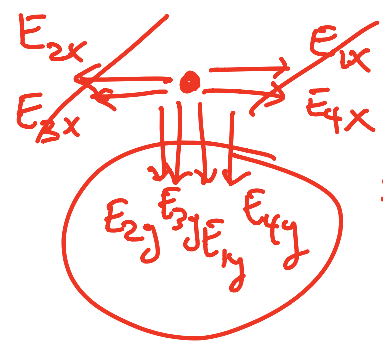
   (d) If the fifth charge started at rest at point A, how fast would it be moving at point B? Consider it to have a mass $m = 2.7 \times 10^{-10}$ kg.

(a) ALL CHARGES HAVE SAME q & r, SO ALL HAVE SAME E:

$$E = k\frac{q}{r^2} = (8.99 \times 10^9)\frac{(1.6 \times 10^{-19})}{(0.071)^2} = \boxed{2.85 \times 10^{-7}\ \frac{N}{C}}$$

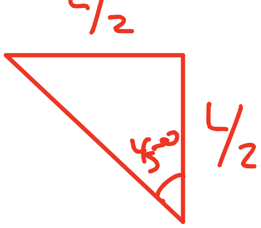
BY SYMMETRY, THE HORIZONTAL COMPONENTS CANCEL:





SUM OF 4 y-COMPONENTS

BREAK $\vec{E}$ INTO ITS y-COMPONENT, THEN MULTIPLY BY 4 (SINCE $E_{1y} = E_{2y} = E_{3y} = E_{4y}$):



$$E_y = E\cos(45°) = (2.85 \times 10^{-7})\cos(45°)$$

$$= \boxed{2.02 \times 10^{-7}\ \frac{N}{C}}$$

So, $\sum E_x = 0$

& $\sum E_y = 4E_y = 4(2.02\times10^{-7})$

$\Rightarrow \sum E_y = 8.08\times10^{-7}\ \frac{N}{C}$

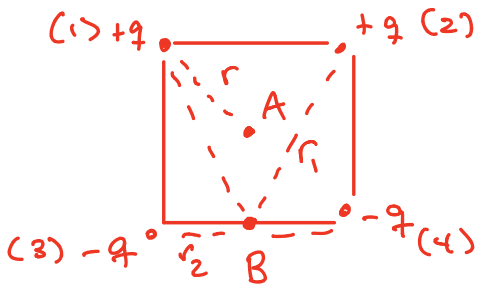
OR, in vector notation:

$\vec{E} = -\left(8.08\times10^{-7}\ \frac{N}{C}\right)\hat{j}$

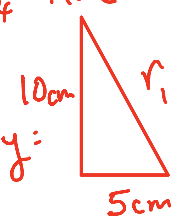
(Negative b/c $\vec{E}$ points down.)

(b) $\vec{F} = q\vec{E} = (1.6\times10^{-19})\left[-\left(8.08\times10\ \frac{N}{C}\right)\hat{j}\right] = -\left(1.29\times10^{-25} N\right)\hat{j}$

(c) $W = -\Delta U = U_i - U_f$



Initially, all q same distance from A, r = 7.1 cm. At B, q_1 & q_2 are at a new distance r_1 while q_3 & q_4 are at r_2.



$r_1 = \sqrt{10^2+5^2}$
$= 11.2\text{ cm}$

Clearly, $r_2 = 5\text{cm}$, and r_1 is given by:

$U_i = U_{1i} + U_{2i} + U_{3i} + U_{4i}$ ← (each charge gives the 5th q an energy.)

$$= k\frac{(q)(q)}{r} + k\frac{(q)(q)}{r} + k\frac{(-q)(q)}{r} + k\frac{(-q)(q)}{r} = 0$$

$U_f = U_{1f} + U_{2f} + U_{3f} + U_{4f}$

$$= k\frac{(q)(q)}{r_1} + k\frac{(q)(q)}{r_1} + k\frac{(-q)(q)}{r_2} + k\frac{(-q)(q)}{r_2}$$

$$= 2k\frac{q^2}{r_1} - 2k\frac{q^2}{r_2} = 2kq^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 2(8.99\times10^9)(1.6\times10^{-19})^2\left[\frac{1}{(0.112)^2} - \frac{1}{(0.05)^2}\right]$$

$$= -1.47\times10^{-25}\ J$$

$$W = U_i - U_f = 0 - (-1.47 \times 10^{-25})$$

$$\Rightarrow \boxed{W = 1.47 \times 10^{-25}\ J}$$

(d) ELECTRIC FORCE IS CONSERVATIVE, SO:

$$\cancel{K_i}^{\,0 \text{ (STARTS @ REST)}} + U_i = K_f + U_f$$

$$\Rightarrow K_f = \underbrace{U_i - U_f}_{\text{FROM (c)}} = 1.47 \times 10^{-25}\ J$$

SINCE $K_f = \frac{1}{2} m v_f^2$,

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(1.47 \times 10^{-25})}{(2.7 \times 10^{-10})}}$$

$$\Rightarrow \boxed{v_f = 3.30 \times 10^{-8}\ \frac{m}{s}}$$

2. Consider an infinite sheet of charge, with some surface charge density σ.

   (a) In what direction would the electric field at some height z above the plate point? Make sure to use arguments from symmetry to justify your claim.

   (b) What choice of Gaussian surface would be the best choice to evaluate the amount of electric flux being emitted from this sheet?

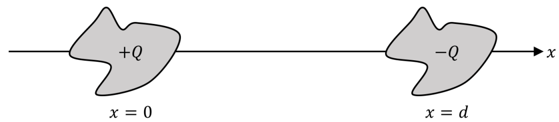
   (c) With your choice of Gaussian surface, use Gauss' law to prove that the electric field at some height z above the infinite plate has a magnitude of:

$$E = \frac{\sigma}{2\epsilon_0}$$

   (d) Would you be able to make the same arguments from symmetry in order to evaluate the electric field at any point above a *finite* sheet of charge? Make sure to justify your answer.

THIS WAS DONE IN CLASS MULTIPLE TIMES. I'M ONLY GOING TO INCLUDE THE ANSWER FOR PART (d):

(d) THE ONLY POINT ABOVE THE FINITE SHEET WHERE THIS WILL STILL WORK IS AT THE <u>CENTER</u>, WHERE THE SYMMETRIES AREN'T BROKEN.



3. Consider two arbitrary conductor, each with a charge $\pm Q$, separated by a distance d along the x-axis as shown in the above figure. The electric field between the two plates is measured to be:

$$\vec{E}(x) = Q\left(Ax + Bx^2\right)\hat{\imath}$$

with $A = 200\ \mathrm{N/mC^2}$, $B = 5000\ \mathrm{N/m^2C^2}$, $Q = 4$ C, and $d = 5$ cm.

(a) If a charge $q = 0.5$ C were to move from the $+Q$ conductor, at $x = 0$, to the $-Q$ conductor, at $x = d$, what change in potential would it undergo?

(b) How much potential energy does q lose as it moves? How much kinetic energy does q gain?

(c) What is the capacitance of these two conductors? Recall that any two conductors which can produce a separation of charges can act as a capacitor.

(d) In order to charge this capacitor, it must be connected to some sort of voltage source to provide the charge separation for the plates. Imagine this source was a battery. What voltage would the battery have to output to charge this capacitor to $Q = 4$C?

(a) $$\Delta\phi = \int_{x_i}^{x_f} E_x\,dx = \int_0^d Q\left(Ax + Bx^2\right)dx$$

$$= QA\int_0^d x\,dx + QB\int_0^d x^2\,dx = Q\left(\tfrac{1}{2}Ad^2 + \tfrac{1}{3}Bd^3\right)$$

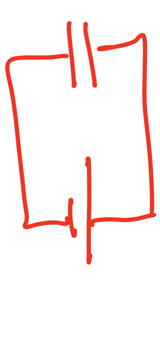
$$= (4)\left[\tfrac{1}{2}(200)(0.05)^2 + \tfrac{1}{3}(5000)(0.05)^3\right] = 1.83\,\mathrm{V}$$

(SIGN DOESN'T MATTER)

(b) $$\Delta U = q\Delta\phi = (0.5)(1.83) = 0.92\,\mathrm{J}$$

(c) $$C = \frac{Q}{\Delta\phi} = \frac{(4\mathrm{C})}{(1.83\mathrm{V})} = 2.19\,\mathrm{F}$$

(d)



CAPACITOR'S VOLTAGE EQUALS BATTERY'S VOLTAGE. TO PRODUCE $Q = 4$C CHARGE ON CAPACITOR REQUIRES A VOLTAGE OF 1.83V $\Rightarrow$ $V_{BAT} = 1.83\,\mathrm{V}$

4. A parallel plate capacitor has an area $A = 2.5 \times 10^{-4}$ m^2 and a plate separation $d = 0.7$mm. This capacitor is then connected to a voltage source of $V = 5$V.

(a) What is the capacitance of this capacitor? *Note: you only need to use the formula for capacitance, you don't need to prove it.*

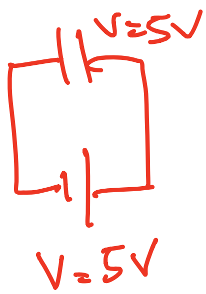
(b) What would the potential difference across the capacitor plates be? Make sure to include a justification for why you know this to be true.

(c) What is the charge stored by the capacitor?

(d) What is the electric field between the capacitor plates?

(a) $C = \varepsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}) \frac{(2.5 \times 10^{-4})}{(0.7 \times 10^{-3})} = \boxed{3.16 \times 10^{-12}\ \text{F}}$

(b) IF VOLTAGE SOURCE IS 5V, THEN V = 5V FOR CAPACITOR.



LOOP RULE! BOTH ELEMENTS HAVE THE SAME VOLTAGE!

(c) $Q = CV = (3.16 \times 10^{-12})(5) = \boxed{1.58 \times 10^{-11}\ \text{C}}$

(d) $E = \frac{Q}{A\varepsilon_0} = \frac{(1.58 \times 10^{-11})}{(2.5 \times 10^{-4})(8.85 \times 10^{-12})} = \boxed{7141\ \frac{\text{N}}{\text{C}}}$

FORMULA SHEET

- Vectors:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

- Physics I Formulae:

$$\sum \vec{F} = m\vec{a}$$

$$W = \vec{F} \cdot \Delta\vec{x} \quad \text{or} \quad W = \int \vec{F} \cdot d\vec{x}$$

$$W_{tot} = \Delta K$$

$$W_{cons} = -\Delta U$$

$$K = \frac{1}{2}mv^2$$

$$K_i + U_i = K_f + U_f$$

$$\vec{F} = -\vec{\nabla} U \quad \text{where} \quad \vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

- Electric Forces and Fields:

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$k = 8.99 \times 10^9 \ \frac{\text{Nm}^2}{\text{C}^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \ \frac{\text{F}}{\text{m}}$$

$$Q = \Delta N e$$

$$F = k\frac{q_1 q_2}{r^2}$$

$$E = k\frac{q}{r^2} \quad \text{or} \quad E = \int k\frac{dq}{r^2}$$

$$dq = \text{density} * \text{space element}$$

$$\vec{F} = q\vec{E}$$

$$\Phi_E = \vec{E} \cdot \vec{A} \quad \text{or} \quad \Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi_{tot} = \frac{q_{enc}}{\epsilon_0}$$

$$\lambda = \frac{Q}{L} \quad \text{or} \quad \sigma = \frac{Q}{A} \quad \text{or} \quad \rho = \frac{Q}{V}$$

- Electric Potential Energy and Electric Potential:

$$U = k\frac{q_1 q_2}{r}$$

$$\phi = k\frac{q}{r} \quad \text{or} \quad \phi = \int k\frac{dq}{r}$$

$$dq = \text{density} * \text{space element}$$

$$U = q\phi \quad \text{and} \quad \Delta U = q\Delta\phi$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\vec{F} = -\vec{\nabla} U$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$U = -\int \vec{F} \cdot d\vec{x} \quad \text{or} \quad \Delta U = -\int_S \vec{F} \cdot d\vec{x}$$

$$\phi = -\int \vec{E} \cdot d\vec{x} \quad \text{or} \quad \Delta\phi = -\int_S \vec{E} \cdot d\vec{x}$$

$$V = \Delta\phi$$

- Capacitance and Dielectrics:

$$Q = CV$$

$$\left.\begin{aligned} C &= \epsilon_0 \frac{A}{d} \\ E &= \frac{\sigma}{\epsilon_0} \end{aligned}\right\} \text{Parallel plate capacitors}$$