Static Games of Complete Information-Lecture 1

Sanxi LI

Renmin University

Text

- Robert Gibbons, 1992, *Game Theory for Applied Economists*, Princeton University Press

What is game theory

- Game Theory is a method of studying strategic situations
- What is not a strategic situation
  - Perfect competition: firms/consumers are price taker
  - Monopolist: No competitors
- For a strategic situation
  - There are at least two rational players
  - Each player has more than one choices
  - The outcome depends on the strategies chosen by all players; there is strategic interaction

What is game theory

- Game Theory has applications
  - economics/politics/sociology/law/biology

Classic Example: Prisoners' Dilemma

- Two suspects held in separate cells are charged with a major crime. However, there is not enough evidence.
- Both suspects are told the following policy:
  - If neither confesses then both will be convicted of a minor offense and sentenced to one month in jail.
  - If both confess then both will be sentenced to jail for six months.
  - If one confesses but the other does not, then the confessor will be released but the other will be sentenced to jail for nine months.

	Mum	*Confess*
Mum	$-1, -1$	$-9, 0$
Confess	$0, -9$	$-6, -6$

Classic Example: Prisoners' Dilemma

- Lesson: do not play a strictly dominated strategy
- Dominated strategy: there exists another strategy which always does better regardless of other players' choices
- By playing a strategy that dominates it, one can do better in every case.

Classic Example: Prisoners' Dilemma

- Another arguement: if me and my pair both reason in this way and confess, we'll both get -6. However, if we reason in a different way and choose mum, we'll both get -1. So I should choose mum, since -1 is better than -6.
- What's wrong with this arguement?
- Lesson: rational choice can lead to inefficiency.

Examples of Prisoners' Dilemma in reality

- Price war between firms
- Tournament between employees
- Take the courses by XDF for GRE/TOEFL

Remedies

- Collusion (illegal)
- Repeat
- To change the payoffs (-1,-6 is actually the outcomes, not payoffs)
- Experimental: 30% choose mum, 70% choose confess.

The formal description of a game (Static (or simultaneous-move) games of complete information)

- The normal-form (or strategic-form) representation of a game G specifies:
  - A set of players (at least two players) : {Player 1, Player 2, ... Player n}
  - For each player, a set of strategies/actions: $\{S_1$, S_2, ..., $S_n\}$
  - Payoffs received by each player for the combinations of the strategies, or for each player, preferences over the combinations of the strategies. $u_i\left(s_1, \ldots, s_n\right) : S_1 \times S_2 \times \ldots S_n \rightarrow R$.
- We denote this game by $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$.
- For convenient, we write (s_i, s_{-i}) instead of $(s_1, \ldots, s_n)$.

Static (or simultaneous-move) games of complete information

- Simultaneous-move
  - Each player chooses his/her strategy without knowledge of others' choices.
- Complete information (on game's structure)
  - Each player's strategies and payoff function are common knowledge among all the players.
- Assumptions on the players
  - Rationality
    - Players aim to maximize their payoffs
    - Players are perfect calculators
  - Each player knows that other players are rational

Normal-form representation of the P-D game

- Show this on the blackboard (payoff matrix)

Formal definition of strictly dominated strategy

Definition

A strategy s_i' is strictly dominated by s_i'' if

$$u_i\left(s_i', s_{-i}\right) < u_i\left(s_i'', s_{-i}\right), \forall s_{-i} \in S_{-i}.$$

Another example

- Two firms, Mengniu and Yili, share some market
- Each firm earns \$60 million from its customers if neither do advertising
- Advertising costs a firm \$20 million
- Advertising captures \$30 million from competitor

Iterated elimination of strictly dominated strategies

	L	M	R
U	$1, 0$	$1, 2$	$0, 1$
D	$0, 3$	$0, 1$	$2, 0$
- Can the reasoning "a rational player never play a dominated strategy" predict the outcome of this game?
- No. Why? Play 1 does not have a dominated strategy.

Iterated elimination of strictly dominated strategies

- However, player 2 have a dominated strategy R. And he never play it.
- Player 1 knows player 2 is rational. So he eliminates the strategy R from player 2's strategy space. It is as if the game were the following:

$$\begin{array}{ccc} & L & M \\ U & 1,0 & 1,2 \\ D & 0,3 & 0,1 \end{array}.$$

- Now player 1 has a dominated strategy: D. He never play it.

Iterated elimination of strictly dominated strategies

- We can go deeper. If player 2 knows that player 1 is rational, and player 2 knows that player 1 knows that player 2 is rational (so player 2 knows that player 1 is actually playing the second game), then he can eliminate D from player 1's strategy space. The game were the following

	L	M
U	$1, 0$	$1, 2$

- Now L is dominated by M, so player 2 never play it. The outcome of this game is (U, M).
- This process is called "iterated elimination of strictly dominated strategies".

Example: Tourists & Natives

- Only two bars (bar 1, bar 2) in a city
- Can charge price of \$2, \$4, or \$5
- 6000 tourists pick a bar randomly
- 4000 natives select the lowest price bar
- Example 1: Both charge \$2
  - each gets 5,000 customers and \$10,000
- Example 2: Bar 1 charges \$4, Bar 2 charges \$5
  - Bar 1 gets 3000+4000=7,000 customers and \$28,000
  - Bar 2 gets 3000 customers and \$15,000

Example: Tourists & Natives

-

	\$2	\$4	\$5
\$2	10, 10	14, 12	14, 15
\$4	12, 14	20, 20	28, 25
\$5	15, 14	15, 18	25, 25

Example: pick a number

- Everyone in the classroom write down a number between 1 and 100
- The winner is the one whose number is closest to two thirds times the average of the class.
- What's the outcome of this game?

Drawbacks

- Each step requires a further assumption about what the player know about each other's rationality.
- For the process to be applied for arbitrage numbers, we need to assume that it is "common knowledge" that each player is rational.
- Common knowlege: not only all players are rational, but also all players know that all players are rational, and all players know that all players know that all players are rational, and so on...
- An example of common knowlege: "Rubinstein's email game".
- Lesson: while applying game th into practice, not only you need to put yourself in other people's shoes to think about other people's payoffs, but also you need to think about how sophisticated are they, and how sophisiticated that they think you are, and so on...The Level of knowlege leads to differenct outcome of the game.

Drawbacks

- It always leads to imprecise prediction of a game. See the following example

	L	M	R
T	$0,4$	$4,0$	$5,3$
M	$4,0$	$0,4$	$5,3$
B	$3,5$	$3,5$	$6,6$

- Hence, we need an other appoach that can produce much tighter predictions—Nash Equilibrium.

Best Response

Definition

A strategy s_i^* is player i's best response to other players strategy s_{-i}^*, i.e., $s_i^* \in BR_i\left(s_{-i}^*\right)$, if

$$u_i\left(s_i^*, s_{-i}^*\right) \geq u_i\left(s_i, s_{-i}^*\right), \ \forall s_i \in S_i;$$

that is, s_i^* solves

$$\max_{s_i' \in S_i} u_i\left(s_i', s_{-i}^*\right).$$

Definition

Player i's best response correspondence is the set-valued mapping from S_{-i} to S_i: $BR_i\left(s_{-i}\right) = \left\{s_i \middle| s_i \in \arg\max_{s_i \in S_i} u_i\left(s_i, s_{-i}\right)\right\}$. Let $BR : S \mapsto S$ be defined by

$$BR\left(s\right) = \Pi_{i \in I} BR_i\left(s_{-i}\right).$$

Nash Equilibrium

Definition

In the normal form game $\{S_i, S_{-i}; u_i, u_{-i}\}$, strategies $\{s_i^*, s_{-i}^*\}$ are Nash Equilibrium if, for each player i, s_i^* is player i"s best response to the strategies specified for other players, s_{-i}^*:

$$u_i\left(s_i^*, s_{-i}^*\right) \geq u_i\left(s_i, s_{-i}^*\right), \forall s_i \in S_i;$$

that is, s_i^* solves

$$\max_{s_i' \in S_i} u_i\left(s_i', s_{-i}^*\right).$$

Motivation of Nash Equilibrium

- No regret.
- Self-enforcing
- The idea of convention: the strategies speficied by convention must be a NE.

Find The NE

	L	M	R
T	$0, \underline{4}$	$\underline{4}, 0$	$5, 3$
M	$\underline{4}, 0$	$0, \underline{4}$	$5, 3$
B	$3, 5$	$3, 5$	$\underline{6}, \underline{6}$

Relation of NE and itereted elimination of dominated strategies

- NE is a strong concept: if iterated elimination eliminates all but the strategies $\left(s_i^*, s_{-i}^*\right)$, then it must be NE.
- The reverse is not true.
- If the strategies $\left(s_i^*, s_{-i}^*\right)$ are a NE, then they survive itereted elimination of dominated strategies, but there can be strategies that survive iterated elimination of dominated strategies, but are not part of any NE.

Existence of NE

- Is NE too strong? Can we be sure the existence of NE?
- Nash (1950): any finite game has at least one NE (including mixed strategies).

Battle of Sex

	opera	*fight*
opera	2, 1	0, 0
fight	0, 0	1, 2

- There can be multiple equilibrium
- Focal point a la Thomas Schelling: Culture; Some other salient features