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PRODUCTION AND CONSUMPTION CREDIT IN A CONTINUOUS-TIME MODEL OF THE CIRCUIT OF CAPITAL

Paulo L. dos Santos*
Department of Economics, School of Oriental and African Studies

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ABSTRACT

This paper offers a characterization of the content of production and consumption credit based on the Marxian circuit of capital and a distinctive approach to credit relations. All credit allocations make the same contribution to demand, sales and profit flows, and, consequently to the pace of accumulation. Production credit uniquely contributes to investment by borrowers. Consumption credit thus effects a form of leveraging of social capital, boosting profitability while strengthening productive constraints and financial risks bearing on credit relations. Systems with higher allocations of consumption credit experience lower scopes for growth-enhancing credit extension, and face higher aggregate levels of credit risk than comparable economies.

1. INTRODUCTION

The dramatic recent increase in consumption credit across a range of economies poses a serious analytical challenge to all strains of political economy. In many economies it has rivalled or even displaced credit to private enterprises as the most significant credit allocation, creating new socioeconomic relationships between financial intermediaries and the mass of consumers, most of whom derive their incomes from wages. It has also played a historically unprecedented role in the current international crisis, in which the

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international financial system was brought to the edge of complete collapse by the unpaid debts of working-class, minority and poor borrowers in the USA.[1]

Mainstream political economy offers scant bases for interpreting these shifts and their possible macroeconomic consequences. Its conceptualizations of credit relations are chiefly microeconomic, centring on the existence of various transactional problems, ultimately stemming from the simple observation that borrowers may know more about themselves and their economic activities than a lender. The credit system appears as a collection of individual contracts, practices and institutions that help alleviate the resulting transactional problems, with no explicit grounding on the overall process of accumulation. Such stances leave little scope for analysis of differences between the character of credit allocations.

Yet there are reasons to believe that production and consumption credit have different effects on macroeconomic performance. Schumpeterian or neo-Austrian approaches would suggest bank lending to productive enterprises may contain the exercise of distinctive bank entrepreneurial capacities in the discovery of new potential avenues for profitable investment, directly fostering growth. In contrast, consumption loans, typically made to wage earners, only require knowledge and confidence about future borrower wage income. From a rather different methodological perspective, a recent cross-country statistical exercise by World Bank economists presents evidence that while credit allocated to enterprises exhibits a positive association with economic growth, credit allocated to finance consumption exhibits no such association.[2] Reflecting the absence of a satisfactory mainstream theoretical conceptualization of the macroeconomic content of credit and its allocation, the authors offer only general, tentative hypotheses from these significant findings.

Heterodox analyses have long offered integrated conceptualizations of credit and macroeconomic activity, centred on the role of credit in the determination of aggregate demand or in the resolution of the related 'problem of realization' of profits.[3] Despite the many insights that may be obtained from analysis grounded on these views, as hitherto pursued they offer no evident bases for characterizing the potential differences between consumption and production credit in the process of accumulation. This paper tackles the distinctive content of production and consumption credit with a framework

[1] See dos Santos (2009), Dymski (2009) and Lapavitsas (2009).
[2] Beck *et al*. (2008).
[3] Kalecki (1943), or as early as the debates between R Luxemburg and N Bukharin, available in Tarbuk (1972).

developed on the basis of the Marxian *circuit of capital* and of a distinctive Marxian approach to credit relations.

The circuit of capital is a systemic schematization of capitalist reproduction advanced by Marx in the second volume of *Capital*. It characterizes the capitalist economy as the continuous circular movement through which capital value self-expands. It starts with money being invested into input commodities, continuing with production of output commodities, and culminating with their sale, which realizes in the form of monetary profits the surplus value newly created through the exploitation of wage labour. By offering a systematic approach to these successive transformations of capital value, the circuit of capital affords a distinctive, integrated analysis of investment, and the production, sale and consumption of commodities. This allows characterizations of a capitalist economy's endogenous capacity to generate demand flows that may sustain the timely realization of monetary profits. As such, it offers a natural framework for the analysis of credit extension, its allocation to finance production or consumption, aggregate demand, and their impact on the pace and vicissitudes of the accumulation of capital.

In order to apply the framework of the circuit of capital to the analysis of the impact of credit allocation, it is necessary to characterize the constraints and contradictions bearing on credit relations. This is done by drawing on distinctive existing Marxian approaches to credit.[4] As in the Keynesian framework, Marxian analyses of credit relations may be grounded on the evolution of monetary hoards. In contrast with Keynesian approaches, the analytical emphasis is not placed on individual psychological or subjective preferences, but on the structural origins of idle money in the accumulation of capital and on the unique ability of money to serve as a store of wealth. In line with Marx's rejection of Say's Law, capitalist sales revenues, net of capitalist consumption, are understood not to be instantaneously committed to reinvestment. Past revenues are only committed to investment once capitalists have sufficient confidence in the prospects of profitability, and in their own future security and ability to meet all operational demands for payments and other outlays. As a result, dynamic pools or hoards of money develop spontaneously as accumulation continuously requires the social relations of capital to express themselves in the form of money hoards.

These money holdings are the foundation for the systematic creation of banking credit. They offer the bases for the circulation of bank liabilities as a form of money. Net credit extension throws new bank liabilities into circulation that function as means of payment and support demand.

[4] Founded on Uno (1980) and more recent contributions by Itoh and Lapavitsas (1999), and Lapavitsas (2000).

As borrowers use these liabilities to purchase commodities, capitalist sellers receiving them as payment may hold them as a form of their own capital. This willingness to hold bank liabilities sets the pace of their reflux back to banks demanding reserves, and consequently conditions bank lending behaviour. At the most general level, it hinges on capitalists' confidence that those liabilities themselves command value and are a suitable monetary expression for their capital. Concretely, this confidence is supported by guarantees that bank liabilities are convertible into a reserve money more generally commanding value, by the quality of banking system assets, and by stability in the exchange value of money in circulation.

By formally developing this analytical perspective the paper demonstrates a number of distinctive results concerning production and consumption credit and their content in the process of accumulation. All credit allocations contribute in the same manner to aggregate demand, sales and the realization of profits. As such they are shown to contribute identically to the determination of the economy's rate of accumulation. At the same time, different credit allocations make different contributions to investment and consequently to the total volume of capital value in circulation. All allocations contribute to the realization of profits by capitalists selling commodities to borrowers or to workers employed by borrowers. Some of those profits will be reinvested, adding to capital value in circulation. But credit allocated to production uniquely contributes to investment by borrowers themselves. The allocation of credit affects the balance between social stocks of debt and circulating capital, with consumption credit effecting a distinctive form of leveraging for social capital.

This leveraging effect is shown to heighten the aggregate profitability of social capital. At the same time it strengthens productive constraints and financial risks that limit the contribution credit may make to accumulation. Specifically, compared with otherwise equivalent economies, systems with higher shares of credit allocated to consumption loans will experience lower investment and output flows relative to demand flows. This diminishes the scope for growth-enhancing net credit extension as demand may more easily exhaust or tax productive capacity. Depending on monetary and institutional regimes, levels of net credit extension will be more narrowly constrained as heightened inflationary pressures either increase the pace at which convertible bank notes flow back to issuers demanding commodity money, or leads to tightening policy stances by the central monetary authority.

The leveraging effect of consumption credit is also shown to increase the measure and significance of aggregate credit risk. Higher aggregate levels of borrowing relative to investment will, *ceteris paribus*, result in lower total wage and profit incomes relative to interest payments, and thus higher levels of credit risk in bank balance sheets. They will also increase the likelihood of corollary monetary disruptions. Economies with higher relative allocations of credit to consumption will be particularly susceptible to the latter, as they are shown to have more of social capital in monetary form, including bank liabilities. Heightened financial risks should lead to reductions in the overall pace of net credit extension, reducing rates of accumulation. They will otherwise require some combination of improvements in bank portfolio management or a general speculative perception of such improvements.

To characterize and demonstrate these effects, this paper applies the continuous-time model of accumulation, credit and its allocation of Foley (1982, 1986).[5] The model is structural, offering a tractable, mathematically rigorous framework for the integrated examination of production and exchange in the process of accumulation, and of the extension and allocation of credit. It characterizes the economy through a system of integral and differential equations which admits exponential solutions for its value flows and stocks. The resulting steady states of growth help illustrate and develop the paper's central points with the help of simple comparative-static analysis.

The rest of the paper proceeds as follows. Section 2 presents the broad conceptual and mathematical framework of the model. Section 3 identifies exponential solutions to the resulting system and discusses their implications for the rate of accumulation, profitability and the limits of credit extension. Section 4 discusses the credit and monetary issues posed by the allocation of credit. Section 5 concludes.

2. THE BASIC MACROECONOMIC AND CONCEPTUAL SETTING

The setting considered here is of a closed economy where accumulation proceeds as outlined in Marx (1917). The model is drawn from Foley's (1982, 1986) continuous-time framework of a single-department economy, in which the presence of time lags conditions the processes of commodity production, sales and reinvestment. In this setting credit relations are a *sine qua non*

[5] Discrete-time models of the circuit of capital have been robustly formulated, for instance in Kotz (1991). The central observation that consumption and production credit contribute identically to demand but differently to investment may be expressed within that framework. The distinctive advantage of Foley (1982, 1986) for current purposes lies in its more general treatment of time. It allows general analyses of the relationship between past sales, present demand flows, and the determination of rates of accumulation in circulation, as well as the evolution of commodity inventories, all of which are integral to the differences between consumption and production credit.

of capitalist accumulation, as without them investment and commodity demand, derived from past sales revenues, are insufficient to sustain consistently positive rates of growth.

In Marxian political economy the core of social reproduction consists of the expansion of capital, represented schematically by,

$$M - C(lp, mp) - P - C' - M' > M \tag{1}$$

Value in the form of money capital M is advanced by capitalists in order to purchase input commodities C, including labour power lp and means of production mp. Inputs are combined in the process of production yielding output commodities C', which are subsequently sold for a quantity of money M'. The expansion of capital is ensured so long as $M' > M$, which hinges on two factors. First and most fundamentally, it hinges on the existence of the wage–labour social relationship, which permits the systematic addition of value to produced commodities by employed labourers in excess of the value represented by their wages. Second, it hinges on the actual sale of the produced commodity at a mark-up over production costs. In Marxian terms, *surplus value* is created in production but only realized as monetary profits through sales.

2.1 Production, realization and investment lags

The circuit of capital is understood as a dynamic collection of social and technical processes. Total social capital will exist as an aggregation of myriad individual circuits simultaneously at every single stage of process (1). The journey through the circuit takes place over time, as various technical and social constraints delay the flow of capital value through the processes of investment in inputs, production and sale of outputs. These delays define the circuit's *turnover time*, most broadly understood as a measure of the average time it takes a quantum of value to traverse the entire process, which obviously constitutes a key driver of profitability over time. A simple continuous-time model of the visible money and commodity flows associated with process (1) illustrates these concepts.

Consider the process once capital value in the money form has purchased input commodities $C(t)$ in order to commence production,

$$C(t) = \kappa(t)C(t) + (1 - \kappa(t))C(t) \tag{2}$$

where $\kappa(t)$ denotes the share in total productive capital of variable capital or labour power. Once the decision to invest value into production is taken, its transformation into outputs takes place over time, as technological, social and economic constraints dictate the pace at which finished products emerge from production to become available for sale. This may be represented mathematically by a lag process function $x_p(t)$, denoting the proportion of the value of input commodities that emerges as finished commodities t units of time after their initial employment. Naturally,

$$\int_0^\infty x_p(t)\,\mathrm{d}t = 1$$

as eventually all value engaged as inputs emerges as outputs. The production lag obviously follows from technological conditions, as well as struggles in production over the pace of work. It may also be understood to follow from possible cyclical delays and congestions in securing inputs after the decision to invest and payments for inputs are made. Assuming for simplicity the case where the process is constant over time, the present flow of output commodities $P(t)$ may be expressed in relation to past quanta of value committed to investment $C(t')$,

$$P(t) = \int_{-\infty}^{t} C(t')\,x_p(t - t')\,\mathrm{d}t' \tag{3}$$

Once produced, the sale of finished commodities at a profit also takes place over time, as transportation, wholesale and retail activities are limited by technological, geographical and social constraints. Of greater economic significance is the fact that the pace for the realization of value through sales is conditioned by the state of demand. The pace of sales or the realization lag may be characterized endogenously from the relationship between commodity supply and demand flows. If prices reflect values, brisk demand will see quicker sales and diminishing inventories, with sluggish demand triggering opposite effects.

The flow of commodity supply given by (3) is confronted by aggregate demand $AD(t)$. If the realization lag process $x_r(t)$ is assumed to be a simple time delay T_r at any point in time,[6] and assuming commodities follow a simple first-in-first-out progression through inventories, it can be shown that,[7]

$$\dot{T}_r(t) = 1 - \frac{AD(t)}{(1+q)P(t - T_r)} \tag{4}$$

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[6] Formally $x_r(t) = \delta(T_r)$ or the Dirac Delta function, which takes the value of zero for all values except for $t = T_r$, at which it takes the value of positive infinity. The function integrates to one over any interval including T_r.

[7] By the Mean Value Theorem, Foley (1986).

Enhanced demand flows in relation to output flows will tend to shorten the realization delay period $T_r(t)$.

When commodities are sold they generate total sales revenue $S(t)$. Sales give surplus value its embodiment as a monetary mark-up q over production costs. In line with conventional Marxian analysis, the view taken here is that at any point in time $q(t) = \varepsilon(t)\kappa(t)$, the product of the share of labour power in total productive capital $\kappa(t)$, and the rate of $\varepsilon(t)$ exploitation,[8] both of which are determined prior to the sale of commodities.

It is useful to divide the funds in $S(t)$ between those representing the recovery of production costs, and those embodying surplus value, denoted respectively as,

$$S'(t) = \frac{S(t)}{1+q(t)} \tag{5}$$

$$S''(t) = \frac{q(t)S(t)}{1+q(t)} \tag{6}$$

Suppose capitalists can fully reinvest $S'(t)$ and a proportion $p(t)$ of profits $S''(t)$, so that value in the form of money equal to $S'(t) + p(t)S''(t)$ is set aside for reinvestment, and $(1 - p(t))S''(t)$ is allocated to capitalists' consumption. The transformation of these unconsumed revenues into investment also suffers from a range of important delays, ensuring pools of temporarily idle money accumulate alongside capital. Value set aside for future investment will remain in the form of idle money until opportunities for potentially profitable investment are identified. Delays will also reflect the holding of precautionary money reserves to ensure the continuity of purchases, prepare for price fluctuations, and meet demands for settlement, as well as the possible need to accumulate funds of sufficient size for lumpy investment from own funds.

In line with the aims of the current discussion, these various delays are taken together to be represented by the investment delay process $x_v(t)$ formally analogous to the lag processes above. This delay process clearly varies over the business cycle. In the upswing the pace at which money is recommitted to investment quickens and capitalists are generally willing to adopt less liquid positions. In the inevitable downswing, the pace at which surviving capitalists recommit revenues to investment slows significantly, and precautionary money hoards build up.

[8] The rate of exploitation is given by the ratio of surplus value to value paid in wages in a given period of time. The composition of capital is given by the ratio of the value paid in wages in a given period to total value advanced to acquire inputs. The mark-up rate $q(t)$ measures the ratio of surplus value to the value advanced to start production.

Abstracting for a moment from lending and debt servicing, investment expenditures will initially be given by,

$$C(t) = \int_{-\infty}^{t} (S'(t') + p(t')S''(t'))\, x_{\mathrm{v}}(t-t')\,\mathrm{d}t' \tag{7}$$

Investment draws on past capitalist revenues, funds demand for means of production and, via the purchase of labour power, demand for wage goods.

2.2 Borrowing, demand and investment

In order to complete the model it is necessary to locate borrowing, fully characterize aggregate demand, and describe the evolution of the various components of total stock of capital in circulation. If total debt outstanding is given by $B(t)$ and a fraction ζ of lending takes the form of consumption loans, then net borrowing by capitalist enterprises for purposes of production is given by $(1 - \zeta)\dot{B}(t)$, and net borrowing by workers to finance consumption is given by $\zeta\dot{B}(t)$.

Assuming capitalists spend the credit they obtain instantaneously, equation (7) becomes,

$$C(t) = \int_{-\infty}^{t} (S'(t') + p(t')S''(t'))\, x_{\mathrm{v}}(t-t')\,\mathrm{d}t' + (1-\zeta)\dot{B}(t) \tag{8}$$

Assuming for convenience that all consumption expenditures are instantaneous, and constant levels for $p(t)$ and $q(t)$, aggregate commodity demand will be given by this investment (which funds workers' demand for wage goods), capitalist consumption and the net extension of consumption credit,

$$AD(t) = C(t) + (1-p)S''(t) + \zeta\dot{B}(t) \tag{9}$$

Note that in this assumed setting of instantaneous consumption, the investment lag process provides an exhaustive measure of the velocity of money.

Simplifying and considering commodity–market equilibrium yields an expression for commodity sales,

$$S(t) = AD(t) = \int_{-\infty}^{t} (S'(t') + pS''(t'))\, x_{\mathrm{v}}(t-t')\,\mathrm{d}t' + (1-p)S''(t) + \dot{B}(t) \tag{10}$$

Equations (3)–(6) and (8)–(10) describe the evolution of all observable value flows in the economy and the evolution of the realization delay $T_{\mathrm{r}}(t)$. Equation (10) locates the source of money flows demanding commodities in relation to present and past sales, and net borrowing.

The realization delay $T_r(t)$ alongside the lag processes $x_p(t)$ and $x_v(t)$ define the circuit's total turnover time. They also ensure the existence of dynamic pools of value in the form of money and commodities sitting temporarily idle during the process of production, sales and investment. The evolution of these pools can be described in relation to the flows outlined above, respectively,

$$\dot{\Pi}(t) = C(t) - P(t) \tag{11}$$

$$\dot{N}(t) = P(t) - S'(t) \tag{12}$$

$$\dot{M}(t) = S'(t) + pS''(t) - \left[C(t) - (1-\zeta)\dot{B}(t)\right] \tag{13}$$

$\Pi(t)$ are pools of commodities undergoing transformation in production, which evolve as invested value $C(t)$ becomes input commodities and as value $P(t)$ emerges as finished output. $N(t)$ are inventories, fed by new output and depleted by sales (net of mark-ups).[9] $M(t)$ are money hoards formed as capitalists set aside their own unconsumed revenues for future reinvestment and sink their own capital into investment, given by investment $C(t)$ net of new borrowing, which draws on newly created (and thus not previously held) money.

It is easy to see that (8) and (10) ensure that (13) yields, as is to be expected,

$$\dot{M}(t) = \dot{B}(t) \tag{14}$$

The sum of these three stocks comprises total capital in circulation, $K(t) \equiv \Pi(t) + N(t) + M(t)$.[10] Its dynamic evolution is driven by net investment, which is funded by capitalized profits and production credit,

$$\dot{K}(t) \equiv pS''(t) + (1-\zeta)\dot{B}(t) \tag{15}$$

With the description of the economy's three autonomous stocks given by (11), (12) and (14), we have a complete model of the economy with nine equations and five unknown flows, $C(t)$, $P(t)$, $S(t)$, $S'(t)$, $S''(t)$, three unknown stocks, $\Pi(t)$, $N(t)$, $M(t)$, one unknown sales delay and an unspecified path for net borrowing $\dot{B}(t)$. The full system of integral and differential equations is reproduced in Appendix A.

[9] Inventories will typically be accounted with mark-ups in enterprise books, rendering (12) into the equivalent expression $N(t) = (1 + q)P(t) - S(t)$. This is not used in the model as surplus value is held here only to be realized in the money form after the sale of output commodities.

[10] The analysis here of necessity abstracts from consideration of fixed capital. Similarly, the view taken here is that inventories and idle sales revenues awaiting reinvestment are not to be understood as merchant and financial capital. In line with Itoh and Lapavitsas (1999), merchant and financial capital are advanced separately in order to facilitate the movement of value through those stages. Analysis here abstracts from both of these types of capital.

The core structure of the model may be understood by manipulating (10) and normalizing it by the scale of basic reproduction, $S'(t)$, which yields the central relationship governing the evolution of the system,

$$(1+pq)S'(t) = \int_{-\infty}^{t} (1+pq)S'(t')x_{\mathrm{v}}(t-t')\mathrm{d}t' + \dot{B}(t) \tag{16}$$

Mathematically, any given path in net borrowing $\dot{B}(t)$ will determine $S'(t)$, from which all flow and stock functions in the model may be identified. Economically (16) depicts the dynamic relationship between capitalist savings, investment, hoarding and the net extension of credit. It shows how net lending and investment help create new capitalist savings. It also shows that without net lending and borrowing, positive rates of accumulation are unsustainable, as in that case it becomes,

$$S'(t) = \int_{-\infty}^{t} S'(t')x_{\mathrm{v}}(t-t')\mathrm{d}t' \tag{17}$$

As long as the investment lag process is constant, the integral on the right-hand side of (17) is a weighted sum of past values of the scale of simple reproduction $S'(t)$. The only way in which the present scale of simple reproduction will be greater than a weighted sum of past values is if the investment lag process becomes less onerous. This is equivalent to an increase in the velocity of money, the pace of dishoarding, and degree of illiquidity faced by functioning capitalists. As these cannot rise indefinitely, this is not a foundation for sustainable growth. As in Keynes (1937), any growth in investment levels requires some combination of a reduction in the relative liquidity of capitalists and new net borrowing. But new money will only enter the economy through new lending by private banks (and its eventual accommodation by reserve suppliers), which supports demand and supplies new monetary expression for the growing mass of capital value resulting from accumulation. Characterizing the limits and contradictions of net credit extension is consequently an integral part of any characterization of economic growth.

3. SOLUTIONS AND PROPERTIES OF EXPONENTIAL STEADY STATES

It is possible to solve the system of integral and differential equations developed above explicitly. Solutions include complex-valued exponential paths for all stocks and flows in the economy. Along such paths, accumulation is most immediately constrained by its own ability to generate funds demanding commodities at sufficiently quick paces. The extension of credit mediates this relationship, and is shown in this section to sustain positive rates of accumulation independently of its allocation across production and consumption loans. But this ability is constrained by production capacities as well as by the credit and monetary risks credit poses, all of which are shaped by the allocation of credit. This section also discusses the relationship between credit allocation and productive capacities, and how it conditions credit creation and growth. The latter credit and monetary mechanisms are taken up in section 4 below.

3.1 *Demand, exchange and the first determination of growth rates*

Equation (16) is a Volterra equation of the second type with a difference kernel, known as the *renewal equation*. It admits explicit solutions in a number of functional forms, including exponential evolutions for $\dot{B}(t)$ and $S'(t)$, leading to similar solutions for all stocks and flows in the model. An equivalent approach to solving the model's system of equations that lends itself more easily to economic interpretation is to normalize net borrowing by the scale of simple reproduction,

$$\dot{B}(t) = h(t)S'(t) \tag{18}$$

This simply amounts to measuring the scale of current net borrowing in relation to sales needed to cover production costs, which turns (16) into,

$$(1 + pq - h(t))S'(t) = \int_{-\infty}^{t} (1 + pq)S'(t')x_{\mathrm{v}}(t - t')\,\mathrm{d}t' \tag{19}$$

Consideration of a simple setting in which the scale of net borrowing remains constant relative to the scale of simple reproduction yields significant insights into the structural differences between consumption and production credit.[11] In this case, $h(t) = h$ and the resulting system of integral and differential equations has exponential solutions, given explicitly in Appendix B. These are characterized by the Laplace Transforms of the lag processes in the economy for the system's complex-valued rate of growth g,

$$x_{\mathrm{i}}^{*}(g) \equiv \mathcal{L}\{x_{\mathrm{i}}(t), g\} = \int_{0}^{\infty} e^{-gt} x_{\mathrm{i}}(t)\,\mathrm{d}t \tag{20}$$

[11] More complex paths for net borrowing are in fact typical, often exhibiting important business-cycle variations. But as (15) and (16) make clear, along any path for net borrowing, consumption credit, while contributing to aggregate demand in the same way as production credit, makes a weaker contribution to the growth of total capital value in circulation. All results discussed below on the basis of a steady-state abstraction follow from this difference, and will thus have applicability to all credit expansion paths.

The factors $x_i^*(g)$ effectively provide discounted present-value measures of the severity of the corresponding lag processes, with the system's rate of accumulation g as the discount factor. In this case (19) is solved when,

$$(1+pq-h)S'(t)=(1+pq)S'(t)x_v^*(g) \tag{21}$$

or,

$$\frac{1}{x_v^*(g)}=\frac{1+pq}{1+pq-h} \tag{22}$$

Further, in a steady-state growth abstraction, the relationship between demand and supply flows is stable, so that the endogenous realization lag is constant. This amounts to a requirement that (4) be equal to zero, which together with (3) and (10) yield,

$$\frac{1}{x_r^*(g)x_p^*(g)}=1+pq-\zeta h \tag{23}$$

Equations (22) and (23) characterize the economy's exponential paths, and together yield the system's characteristic equation,

$$\frac{1}{x_v^*(g)x_r^*(g)x_p^*(g)}=\frac{(1+pq-\zeta h)(1+pq)}{1+pq-h} \tag{24}$$

Equation (22) embodies the model's first significant result: that in an economy constrained by its own ability to generate demand flows that allow for sales flows that realize profits, all credit extension, regardless of its allocation, directly contributes to growth by creating means of purchase and a monetary embodiment for capital values. Formally (22) offers the simplest expression of the relationship between the financial parameters (h, ζ) and the system's rate of growth, as determined in the sphere of commodity exchange. It yields an endogenous definition of the economy's rate of accumulation, which can be seen to be independent depend on the credit allocation share ζ. As a result $g=g(h)$, taken here to be real-valued, with the particular form of this function defined implicitly by the right-hand side of (22) and the actual shape and severity of the investment lag process.

Further, the properties of the Laplace transform ensure $x_v^*(g)$ is decreasing on g,[12] so that the left-hand side of (22) is increasing on g. Given that the right-hand side of the equation is increasing h on it follows that, higher

[12] Except for the aberrant case of infinite money velocity, or $x_v(t)=\delta(0)$, in which case $x_v^*(g)=1$, $\forall g$.

levels of net credit extension relative to the scale of simple reproduction lead to systems with higher steady-state rates of accumulation, as expected. Note also that $g(0) = 0$. Steady growth, in a setting where the investment lag process remains constant, may only occur with positive net extensions of credit.

The second important result is evident by considering the evolution of the stocks of debt and total capital in circulation in steady-state growth. Using the solutions provided in Appendix B it is evident that the economy's social level of gearing or leveraging, defined as the stock of debt measured relative to total capital in circulation, will be given by,

$$\Gamma(h,\zeta) \equiv \frac{B(t)}{K(t)} = \frac{h}{(1-\zeta)h + pq} \tag{25}$$

As expected, $\Gamma_h(h, \zeta) \geq 0$ as higher overall levels of net credit extension increase the relative size of overall indebtedness. Furthermore, it is obvious that $\Gamma_\zeta(h, \zeta) \geq 0$, formally demonstrating that consumption credit effects a distinctive form of leverage, over and above that effected by overall net credit extension. One important consequence of this effect relates to the economy's social rate of profit, defined as profit flows relative to total capital in circulation,

$$\rho = \frac{S''(t)}{K(t)} = \frac{qg(h)}{(1-\zeta)h + pq} \tag{26}$$

It is clear that this rate will have a positive relationship to the relative allocation of credit to consumption loans. Systems with higher credit allocations to consumption loans will experience the same sales and profit flows as otherwise comparable systems. But they will have lower volumes of capital in circulation. The consequent heightened profitability of aggregate social capital can be understood to support the existence of microeconomic incentives for increasing credit allocation towards consumption loans. At the same time, such systems will face stronger constraints on the ability of credit relations to sustain positive rates of accumulation. The productive constraints responsible for these restrictions are taken up immediately below, while constraints arising as a result of heightened credit and monetary risks are tackled in section 4.

3.2 Production and output constraints on credit

While higher relative levels of overall credit extension permit higher real rates of accumulation, they cannot do so indefinitely. Net credit enhances demand flows, which must be met by new commodity supply flows if sustainable, real accumulation is to take place. Here an important difference between each allocation of credit arises. While all credit buttresses demand flows, production and consumption credit contribute differently to investment and, thus, to total productive capacity. All credit contributes to future investment by capitalists selling to borrowers and to workers employed by borrowers. But only production credit contributes to investment by borrowers themselves. The allocation of credit will shape dynamic supply flows and consequently the existing scopes for overall credit extension in the economy.

Within the terms of the model, the ability of credit to enhance real accumulation is limited by the requirement that the endogenous realization delay should never fall below zero. That is, if demand flows grow so much in relation to supply flows that inventories begin to vanish, additional demand will not secure commodities but will instead bid up commodity prices, starting to lower the exchange value of money. In a commodity monetary system with convertibility, the spectre of falls in the exchange value of credit money tends to increase the pace at which bank liabilities flow back to issuers demanding the money commodity. This will constrain the pace of credit extension by private banks. In a contemporary setting of inconvertible central-bank liabilities supporting broader monetary circulation, the possibility of an inflationary fall in the value of money will typically trigger monetary tightening, similarly constraining the pace of credit extension.

Mathematically, the requirement that $T_r \geq 0$ is equivalent to the requirement that $x_r^*(g) \leq 1$.

Together with (23) this yields,

$$\frac{1}{x_p^*(g)} \leq 1 + pq - \zeta h \tag{27}$$

Given that $g = g(h)$, inequality (27) defines sets of parameter values (h, ζ) under which demand will not exhaust commodity inventories. As can be seen from (22) and (27) the shape of these sets is conditioned by pq, $x_p(t)$, as well as by $x_\upsilon(t)$, which conditions the shape of $g(h)$. The intuition behind this is straightforward, as the capitalization of profits and production lags directly condition dynamic flows of commodity supply, while investment lags directly condition demand flows.

Condition (27) defines maximum levels of net credit extension h_m, and consequently maximum rates of accumulation $g_m = g(h_m)$. The maximum rate of growth has a negative relationship with the relative allocation of credit to consumption loans ζ. Systems with higher relative credit allocation to consumption loans will exhibit smaller supply flows, and thus smaller scopes for overall net credit extension, than otherwise comparable economies.

To see this, consider the case where $h = 0$, under which $g(h) = 0$ and $x_{\mathrm{p}}^*(0) = 1$, which ensures that for all $pq \geq 0$, (27) is satisfied, independently of the allocation of credit. The existence of a unique h_{m} is guaranteed by the fact that as h increases from zero, the left-hand side of (27) increases monotonically, while the right-hand side falls monotonically, ensuring a unique intersection between both functions at h_{m}.[13] Further, as the left-hand side of (27) and its derivatives are independent of ζ and the derivative of the right-hand side of (27) is $-\zeta$, higher relative credit allocations to consumption loans ensure this intersection occurs at lower values for h_{m}. Formally, this ensures

$$\frac{\mathrm{d}}{\mathrm{d}\zeta}h_{\mathrm{m}}, \frac{\mathrm{d}}{\mathrm{d}\zeta}g_{\mathrm{m}} < 0$$

It is possible to illustrate these points graphically by assuming specific functional forms for the economy's lag processes. This permits the identification of explicit solutions of the endogenous rate of accumulation, and explicit identification of financial parameter values that do not exhaust inventories along steady states with given parameter values.

Consider the case in which the lag processes are described by exponential decays. The investment and production lag functions become $x_j(t) = je^{-jt}$, where the parameters $j = \upsilon, \pi$ respectively represent the rates of decay of the investment and production lag processes, and offer positive measures of the respective paces of investment and production. In this case the definition of the Laplace Transform yields,

$$x_{\mathrm{v}}^*(g(h)) = \frac{v}{v + g(h)} \tag{28}$$

$$x_{\mathrm{p}}^*(g(h)) = \frac{\pi}{\pi + g(h)} \tag{29}$$

Equation (22) then offers an explicit solution for the economy's rate of growth,

$$g(h) = \frac{hv}{1 + pq - h} \tag{30}$$

[13] Formally $\frac{\mathrm{d}}{\mathrm{d}h}\left[\frac{1}{x_{\mathrm{p}}^*(g(h))}\right] = g'(h)\frac{\mathcal{L}\{tx_{\mathrm{p}}(g(h))\}}{x_{\mathrm{p}}^*(g(h))^2} > 0$, and $\frac{\mathrm{d}}{\mathrm{d}h}(1 + pq - \zeta h) = -\zeta \leq 0$.

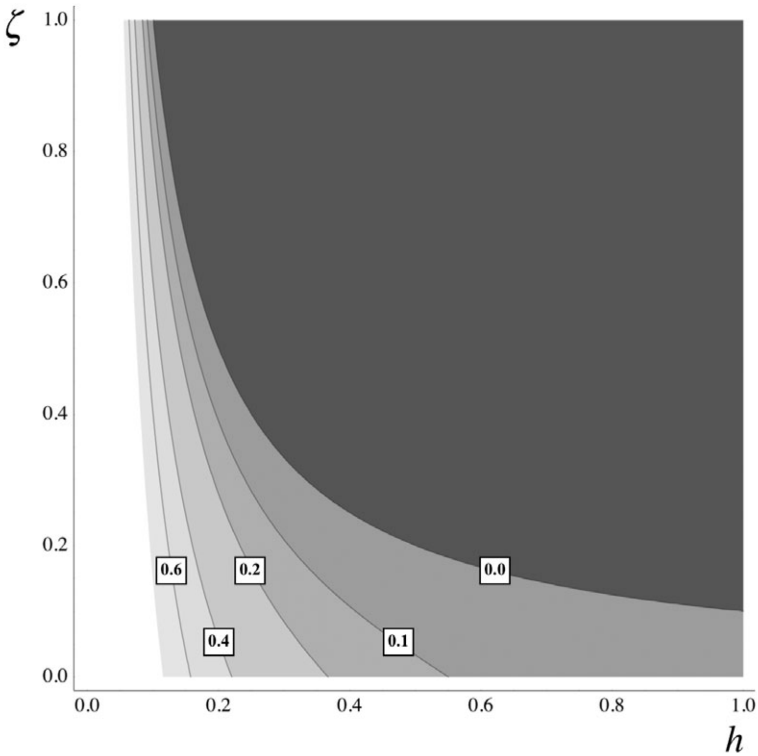

Figure 1. Contours of financial parameter sets with positive steady-state inventories, decay process. $pq = 0.1$, $v/\pi = 0.0, 0.1, 0.2, 0.4, 0.6$.

It is now possible to characterize the values for the financial parameters (h, ζ) that will not exhaust commodity inventories along steady state paths. This can be done by substituting definition (30) into (29), and using the resulting expression for $x_{\mathrm{p}}^{*}(g(h))$ in inequality (27). After some simplification this yields a description of such financial parameter values,

$$\frac{v}{\pi} \leq \frac{(pq - \zeta h)(1 + pq - h)}{h} \tag{31}$$

The sets of (h, ζ) that maintain positive inventories are thus characterized by the rate at which own investment is undertaken through the capitalization of profits, and by the pace of investment (in this context the velocity of money) relative to the pace of production (the time-productivity of labour).

Inequality (31) may be represented graphically. Figure 1 shows the sets of parameter values (h, ζ) that obey (31) for an assumed rate of own investment of $pq = 0.1$. Each value of v/π defines one such set, whose boundary is defined by the parameter values that satisfy (31) with equality. The figure shows the boundaries for situations where v/π takes the values 0.0, 0.1, 0.2, 0.4 and 0.6. Points to the left of each boundary represent values for (h, ζ) that obey (31) strictly for the corresponding measure of the velocity of money relative to the time-productivity of labour.

As described in Appendix C, the same procedure may be undertaken for situations where the lag processes take the form of discrete time delays T_i, which offer negative measures of the pace of value through the relevant phase of the circuit of capital.

As is evident from these figures, economies with higher levels of labour time-productivity relative to the pace of investment will enjoy greater scopes for growth-enhancing credit extension. The figures also illustrate the general finding that $\frac{d}{d\zeta} g_m < 0$, as the set boundaries are downward sloping. Economies with higher proportions of credit allocated to consumption will experience smaller scopes for growth-enhancing credit extension, *ceteris paribus*. Finally, equation (22) ensures that the locus of financial parameter values yielding the same rate of accumulation are vertical lines in figures 1 and 2, with rising rates for higher levels of h. As a result, the maximum possible real rate of accumulation any given economy may reach requires that all credit be allocated to production.

Consideration of the impact of credit allocation on capital in circulation and productive capacities thus allows the identification of important differences between consumption and production credit in this simple model of accumulation. The model also allows consideration of the requirements and contradictions of credit relations, which points to further constraints on credit extension and rates of accumulation that are affected by credit allocation.

4. THE CREDIT AND MONETARY CONSEQUENCES OF CONSUMPTION CREDIT

The extension of credit in a capitalist economy is significantly constrained by the credit and liquidity risks inherent to credit relations, the capacity of the credit system to manage those adequately, and the corollary possibilities of monetary crises in which capitalists seek to abandon holdings of all but the most social forms of money. This section discusses the mechanisms through which the allocation of credit affects these financial constraints on credit extension, considering first the distribution, burden and risks of indebtedness, and second the monetary circulation in the economy.

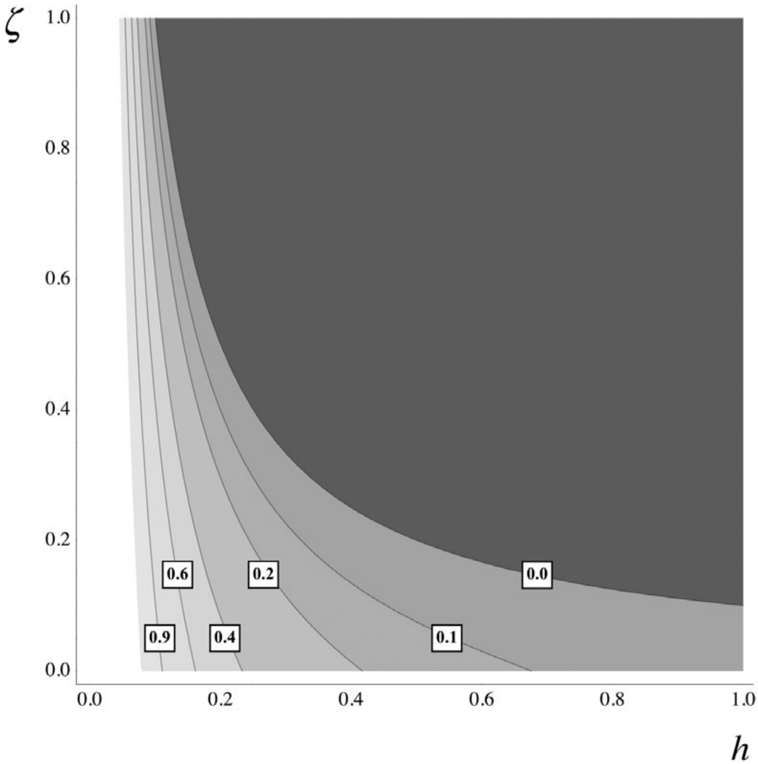

Figure 2. Contours of financial parameter sets with positive steady-state inventories, delay process. $pq = 0.1$, $T_p/T_v = 0.0, 0.1, 0.2, 0.4, 0.6, 0.9$.

In both cases systems with higher relative allocations of consumption credit are shown to be more fragile, as lower relative levels of total capital in circulation provide narrower bases for servicing debt and for sustaining the circulation of credit money. Unless the credit system's capacity for managing risks is better in those economies, those systems will experience either lower levels of credit extension and rates of accumulation, or heightened levels of credit and monetary risks.

4.1 Distribution and debt burden

Consumption credit effects redistributions of income through interest payments.[14] In the simple model developed here, workers make gross debt

[14] For discussions of these transfers, their theoretical and empirical significance in contemporary banking, see dos Santos (2009).

payment flows of $\zeta B(t)i$. Assuming a constant interest rate, any increase in consumption debt will naturally increase the size of these gross transfers. A change in the allocation of credit in favour of consumption loans will, *ceteris paribus*, also shift the debt burden away from capitalists and towards workers. As shown in section 3, the reallocation will not change profit flows, but will reduce the debt burden on capitalists. The capitalist sector will enjoy greater financial robustness. In contrast, workers will face higher debt payments. They also face smaller wage flows as a result of the reduction in investment ushered in by the reallocation of credit. As a result, their collective financial fragility increases.

While clearly effecting redistributions of income and debt burden in ways that are disadvantageous to workers, it is unclear at this level of abstraction what the net effect of such a reallocation of credit on the economy will be. To examine the net effect on financial fragility consider the ratio of total interest payments to total wage and profit incomes along the steady-state paths in Appendix B,

$$F_0 = \frac{iB(t)}{\kappa C(t) + S''(t)} = \frac{ih}{g(h)[\kappa(1+pq-\zeta h)+q)]} \tag{32}$$

from which it is trivial to see that $\frac{\mathrm{d}}{\mathrm{d}\zeta}F_0 \geq 0$. Steady states with more consumption credit have smaller total income flows relative to total debt payment flows. They will as a result be more susceptible at the aggregate level to the possibility that disruptions to incomes will lead to credit disruptions than otherwise identical economies. Bank balance sheets will correspondingly be weaker and more fragile.

Unless those economies experience either improvements in banking system risk management, or a spread of general speculative perceptions of such improvements, the overall pace of net credit extension will face stronger constraints than in comparable economies with lower levels of consumer lending. Banks may be less willing to bear more illiquid positions, capitalists may be less willing to hold fractions of their money in the form of banking system liabilities, and the central monetary authority may deliberately seek to curtail the pace of credit extension.

4.2 Monetary circulation and behaviour

The exponential solutions to the model describe how credit allocation shapes the monetary requirements of accumulation. As evident from Appendix B, capitalist money holdings as a fraction of total capital will, for a given pace of own investment, be a function of the financial parameters,

$$\frac{M(t)}{K(t)} = \frac{h}{(1-\zeta)h + pq} \tag{33}$$

Evidently, this ratio is rising on financial parameters (h, ζ). A number of important consequences follow.

Sustaining any given rate of accumulation requires a given level of overall credit expansion h. But equation (33) indicates that in systems with more consumption credit, supporting any given level of credit extension structurally requires the circulation of higher volumes of money in relation to total capital value in circulation. This poses a number of related difficulties.

In contemporary monetary systems, in which monetary circulation consists of private credit-system liabilities supported by liabilities of a central monetary authority controlled by the state, all money is ultimately supported by credit-system assets. These consist of the loans of private banks, the reserves held by the central bank, and the creditworthiness of the state. Albeit in mediated ways, the quality of these assets and promises ultimately draws on volumes of total capital in circulation. Higher levels of monetary circulation relative to total capital in circulation thus require higher levels of confidence in the credit system's ability to manage its balance sheet. Without this higher confidence, systems with higher relative levels of consumption credit will be unable to sustain the same relative level of net credit extension as otherwise comparable systems.

Consider first the capacity of the central monetary authority to accommodate the greater relative need for credit and money in circulation. In the closed economy abstraction pursued in the model, this ability will be more tightly constrained by attempts to manage the exchange value of money given the heightened inflationary pressures systems with high relative levels of consumption credit face.[15] The ability of private credit-system liabilities to fill the gap will be conditioned by the willingness of banks to bear illiquidity, and by the related willingness of capitalists to hold fractions of their money

[15] Parenthetically, in an open economy whose state does not issue a reserve currency, this capacity will also be conditioned by the extent to which central bank liabilities circulate before flowing back and demanding reserves. For those economies international capital mobility and the possibility of degrees of domestic currency substitution will constrain the circulation of central-bank liabilities, which will be conditioned by the quality of central bank assets—claims on domestic banks, international reserves and the creditworthiness of the state. These assets all ultimately draw on total capital in circulation, which has been shown to be relatively smaller in systems with high levels of consumption credit.

holdings as private credit-system liabilities. This can be shown in reference to two structural identities. Total volumes of outstanding central-bank liabilities at any point in time will be divided into private bank reserves and volumes held by capitalists,

$$H(t) = H^R(t) + H^C(t) \tag{34}$$

Capitalist money holdings will also be divided into holdings of central-bank liabilities and holdings of private credit-system liabilities,

$$M(t) = H^C(t) + D(t) \tag{35}$$

The required circulation of private credit-system liabilities can be characterized in relation to volumes of central-bank liabilities in the economy via a monetary 'multiplier' taken here to be neither stable nor indicative of any given causal relation,

$$\frac{M(t)}{H(t)} = \frac{D(t) + H^C(t)}{H^R(t) + H^C(t)} \tag{36}$$

Letting $d(t) = \frac{D(t)}{M(t)}$ denote the fraction of capitalists' total money holdings they are willing to keep in the form of private credit-system liabilities, and $l(t) = \frac{D(t) - H^R(t)}{D(t)}$ denote the level of illiquidity banks are willing to bear at any given point in time (36) becomes,

$$\frac{M(t)}{H(t)} = \frac{1}{1 - l(t)d(t)} \tag{37}$$

Increases in the circulation of private credit-system liabilities relative to central-bank liabilities thus require a combination of increases in bank tolerance for illiquidity $l(t)$ and a higher willingness by capitalists to hold credit-system liabilities $d(t)$.

For any given pace of investment, sustaining a given rate of accumulation in systems with higher relative credit allocations to consumption loans requires more net lending, and greater injections of new money, relative to total capital value in circulation. The central monetary authority may accommodate higher relative paces of net lending, but it will generally face stronger inflationary pressures. Sustaining the circulation of greater volumes of bank liabilities relative to liabilities of the central monetary authority requires a combination of higher levels of bank illiquidity and greater capitalist willingness to hold money balances in the form of private credit-system liabilities. Yet this economy also faces heightened credit risks, which should generally lead to lower levels of $d(t)$ and $l(t)$.

Three broad possible outcomes follow. First, this economy may simply exhibit lower levels of net credit extension and lower rates of accumulation. In this case consumption lending may be understood directly to contribute to lower rates of growth. Second, if the system enjoys superior credit-system management of balance sheet risks it may be possible to maintain comparable levels of net credit extension supported by higher $d(t)$ and $l(t)$. Even in this somewhat contrived case the economy will still face the heightened inflationary pressures discussed in section 3.2 above, which will eventually limit the scope for net credit extension.

And third, these economies may simply see a cyclical or speculative increase in confidence in the liquidity and credit-risk position of the credit system. In those cases higher levels of $l(t)$ and $d(t)$ are speculative developments, possibly compounded by an overly accommodative stance by the central monetary authority. In this scenario capitalists are more strongly exposed to credit and monetary risks and potential crises. By leading to heightened levels of effective social leveraging, consumption credit makes a distinctive contribution to heightened credit and monetary fragility in a capitalist economy.

5. CONCLUSIONS

This paper has offered a new analytical approach to the macroeconomic content of consumption credit based on the Marxian circuit of capital, Foley's (1982, 1986) interpretation and formalization of the circuit, and a distinctive understanding of credit relations in capitalist reproduction. On those bases, a continuous-time model of accumulation yielded a novel, systemic account of the structural differences between production and consumption credit. These were shown both in relation to the general structure of the circuit of capital, and through comparative-static analysis of the properties of exponential steady-state evolutions for all of the economy's stocks and flows.

As in Foley (1982, 1986), net credit extension was shown to be a requirement for sustainable positive rates of accumulation. This simple finding is common to a range of heterodox approaches, including Kalecki (1943) and both French and Italian Circuitism.[16] The circuit of capital does however offer a unique,

[16] See, respectively, the canonical expositions in Lavoie (1992) and Graziani (2003).

systemic perspective on this requirement. Capitalist reproduction requires capital value successively to take the form of money, input commodities and finished output. Accumulation thus imposes growing requirements not only on labour and non-labour inputs but also on money. As noted by Kotz (1991), the centrality of credit relations to accumulation may be understood to follow from the need for systematic growth of money in circulation and, relatedly, in aggregate demand.

The contribution of any given pace of net credit extension to the rate of accumulation was found to be independent of the allocation of credit. This finding emerges distinctively from the application of Foley's (1982, 1986) continuous-time model offered by the paper. Accumulation was held to be most immediately constrained by its own ability to generate flows of money demanding commodities. As such, the model abstracted from labour or resource restrictions on accumulation. In this setting the rate of accumulation is directly conditioned by demand flows, which are ultimately driven by investment and set the pace for sales that allow the monetary realization of profits. Since all credit allocations contribute identically to demand, sales and profit flows, they make the same contribution to the pace of accumulation.

Different credit allocations were shown to contribute differently to the accumulation of total capital value in circulation. While both consumption and production credit contribute to accumulation by capitalists selling to the borrower or to workers employed by the borrower, only production credit leads to accumulation by borrowers themselves. If the rate of accumulation is understood as defined and determined exclusively by capitalist investment with no explicit account of its dependence on sales, money and aggregate demand,[17] this conclusion is equivalent to a statement that production credit leads to higher rates of growth. In contrast, the approach taken here suggests that while systems with higher relative allocations of credit to production exhibit higher levels of investment and capital value in circulation, they will experience the same sales and profit flows and debt stocks as equivalent systems with more credit financing consumption.

As a result, the paper uniquely identifies consumption credit as effecting a distinctive form of leverage for aggregate social capital. As with all leverage, this was shown to enhance the profitability of social capital. This finding has potentially far reaching consequences, as heightened aggregate profitability likely supports an array of microeconomic incentives to increase consumption relative to production lending. At the same time, this leveraging effect

[17] As is done, for instance, in the standard formulation of the neoclassical Solow-Swan model of accumulation.

was shown to exacerbate a series of productive and financial constraints bearing upon credit relations.

In the sphere of production, investment and time-productivity constraints ensure credit relations will only boost real accumulation up until the point where demand flows exhaust or overly challenge commodity inventories. This requirement defined a set of steady-state financial parameter values for which aggregate demand will not eliminate inventories. The shape of this set was shown to depend on the mark-up and profit reinvestment rates, and the weight of investment lags relative to lags in production. Put differently, they are shaped by the dynamic impact new credit has on supply, which is defined by rate of own investment and the time-productivity of labour, and the dynamic impact it has on demand, which is given by the velocity of money.

Consumption credit makes a weaker contribution to investment, ensuring that for any given overall pace of net credit extension, systems with higher relative levels of such lending will have smaller inventories. As net lending rises, those economies will face greater demand-pull inflationary pressures, which, depending on the institutional setting, may create greater pressures to moderate the pace of lending, reducing growth. At the limit, those systems face lower maximal rates of growth for any given pace of own investment, time-productivity of labour and velocity of money.

Credit allocation also affects the distribution, overall measure and significance of credit risk in the economy. Systems with high relative allocations of credit to consumption place more of the aggregate credit burden on wage earners. Further, those economies exhibit lower levels of investment relative to net borrowing flows. As a result, for a given interest rate they face higher overall debt burdens relative to total profit and wage income flows. They will generally be more susceptible at the aggregate level to the possibility that disruptions to income flows will trigger defaults.

In addition, sustaining any pace of accumulation in those economies requires higher relative levels of lending and money creation. This requires some combination of more accommodative stances by the central monetary authority, higher bank illiquidity and higher holdings of bank liabilities relative to total capital in circulation. Yet all these developments pose difficulties for those economies given the greater inflationary pressures and credit risks they face. Unless they experience real improvements in the management of credit and liquidity risks, they will experience either growth-diminishing falls in overall levels of credit extension, or move towards positions of heightened risks of credit and monetary crises.

From these results it is clear that even economies operating well below capacity, represented here by economies with ample inventory stocks, will react differently to expansions in the pace of lending, depending on the allocation of credit. A small increase in the pace of net consumption lending will, *ceteris paribus*, lead to a steady state with higher profit flows and a quicker pace of accumulation, boost the aggregate profitability of social capital, and change levels of credit and monetary risk.[18] In this sense consumption credit may make a positive contribution to accumulation, so long as credit risk is well managed.

An alternative, equivalent increase in the pace of net production lending would make a different contribution. Profit flows and the pace of accumulation would increase in exactly the same measure. But investment and wage flows would enjoy a greater boost, ensuring aggregate income flows relative to debt repayments would be higher than under the original credit expansion. Structurally, this path of credit expansion leads to lower levels of credit risk. But it also leads to lower increases in aggregate profitability. Even at this high level of abstraction, this suggests the presence of incentives tending to favour the former type of credit expansion, at the expense of wage flows and financial stability.

The paper's conclusions raise important concerns regarding the significance of the reorientation of credit in favour of loans financing consumption (and private residential investment) in the USA, the UK and across a range of advanced and middle-income economies. As has been discussed and documented elsewhere,[19] this development started as a mediated expression of important secular tendencies in the conditions and financial behaviour of non-financial enterprises and wage earning households in the USA. It has been conditioned over the past 30 years by increased reliance on private provision for retirement, education, housing and health, as well as stagnant wage incomes and rising inequality.

As a result of its high microeconomic profitability and compatibility with large-scale international banking, consumption credit has spread aggressively across the world since the early 1990s. As credit has been actively reoriented by banks in favour of such loans, the resulting indebtedness by wage-earning households has effected growing transfers of value from them to the credit system in the form of bank profits. As this paper suggests these processes may not just be seen to be iniquitous, but may also be understood as a new instance in which individual profitability throws up distinctive obstacles to sustainable and equitable economic development.

[18] See Foley (1986) for a discussion on the transitional dynamics between steady states in this type of analysis.
[19] dos Santos (2009), Lapavitsas (2009).

APPENDIX A

The model's full system of equations

Flows

$$P(t)=\int_{-\infty}^{t} C(t')\,x_{\mathrm{p}}(t-t')\,\mathrm{d}t' \tag{A1}$$

$$S'(t)=\frac{S(t)}{1+q} \tag{A2}$$

$$S''(t)=\frac{qS(t)}{1+q} \tag{A3}$$

$$C(t)=\int_{-\infty}^{t} (S'(t')+pS''(t'))\,x_{\mathrm{v}}(t-t')\,\mathrm{d}t'+(1-\zeta)\,\dot{B}(t) \tag{A4}$$

$$S(t)=C(t)+(1-p)\,S''(t)+\zeta\dot{B}(t) \tag{A5}$$

Stocks

$$\dot{\Pi}(t)=C(t)-P(t) \tag{A6}$$

$$\dot{N}(t)=P(t)-S'(t) \tag{A7}$$

$$\dot{M}(t)=S'(t)+pS''(t)-\left[C(t)-(1-\zeta)\,\dot{B}(t)\right]=\dot{B}(t) \tag{A8}$$

Which add up to

$$\dot{K}(t)=pS''(t)+(1-\zeta)\,\dot{B}(t) \tag{A9}$$

The endogenous realization lag

$$\dot{T}_{\mathrm{r}}(t)=1-\frac{AD(t)}{(1+q)\,P(t-T_{\mathrm{r}})} \tag{A10}$$

APPENDIX B

Full exponential solutions

Exponential growth paths for all stocks and flows in the model can be normalized to the scale of simple reproduction $S'(t)$ yielding

$$S'(t) = S'(0)e^{g(h)t} \tag{B1}$$

Total sales

$$S(t) = S'(t)(1+q) \tag{B2}$$

Profits

$$S''(t) = qS'(t) \tag{B3}$$

Investment

$$C(t) = (1+pq-\zeta h)S'(t) \tag{B4}$$

Output

$$P(t) = (1+pq-\zeta h)x_{\mathrm{p}}^{*}(g)S'(t) \tag{B5}$$

Money hoards and debt outstanding

$$M(t) = \frac{(1+pq)}{g(h)}(1-x_{\mathrm{v}}^{*}(g))S'(t) = h\frac{S'(t)}{g(h)} = B(t) \tag{B6}$$

Unfinished outputs

$$\Pi(t) = \frac{(1+pq-\zeta h)}{g(h)}(1-x_{\mathrm{p}}^{*}(g))S'(t) \tag{B7}$$

Inventories

$$N(t) = \frac{[(1+pq-\zeta h)x_{\mathrm{p}}^{*}(g)-1]}{g(h)}S'(t) \tag{B8}$$

Total capital in circulation

$$K(t) = \frac{[(1-\zeta)h+pq]}{g(h)}S'(t) \tag{B9}$$

APPENDIX C

Derivation of shape of feasible financial parameter sets for discrete delays

Discrete delays

If the system's production and investment lag processes take the form of discrete delays, the lag functions are given $x_{\mathrm{i}}(t) = \delta(t - T_{\mathrm{i}})$ by or the Dirac delta function for the delay in question. In this case the system's endogenous rate of growth may be derived explicitly from (22), which becomes,

$$e^{gT_{\mathrm{v}}} = \frac{1+pq}{1+pq-h} \tag{C1}$$

yielding,

$$g(h) = \frac{1}{T_{\mathrm{v}}}\ln\left(\frac{1+pq}{1+pq-h}\right) \tag{C2}$$

Inequality (27) then becomes,

$$\frac{1}{x_{\mathrm{p}}^{*}(g)} = e^{g(h)T_{\mathrm{p}}} = e^{\frac{T_{\mathrm{p}}}{T_{\mathrm{v}}}\ln\left(\frac{1+pq}{1+pq-h}\right)} \leq 1+pq-\zeta h \tag{C3}$$

or,

$$\frac{T_{\mathrm{p}}}{T_{\mathrm{v}}}\ln\left(\frac{1+pq}{1+pq-h}\right) \leq \ln(1+pq-\zeta h) \tag{C4}$$

Since we are only interested in $h \in (0, 1+pq)$,[20] this can be expressed as,

$$\frac{T_{\mathrm{p}}}{T_{\mathrm{v}}} \leq \frac{\ln(1+pq-\zeta h)}{\ln\left(\frac{1+pq}{1+pq-h}\right)} \tag{C5}$$

from which figure 2 is derived in the same way as figure 1.

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[20] The lower bound follows from the fact that $h = 0$ is always feasible, $h < 0$ is irrelevant to this particular issue, and (19) makes clear that in a closed economy $h < 1 + pq$.

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Paulo L. dos Santos
Department of Economics
School of Oriental and African Studies
Thornhaugh Street, Russell Square
London WC1H 0XG
UK
E-mail: ps45@soas.ac.uk