# MATERIAL REPOSITORY

This section provides users with an overview of how the Hyperelastic Material Repository operates, detailing the structure of the input files in .yaml format and the adopted material classification. The repository is designed to gather, in an organized and standardized way, experimental data from the literature or provided by the user, enabling its direct use in hyperelastic model selection and parameter calibration. This structure allows the program to load experimental datasets dynamically and enables users to expand the repository without modifying the source code. When a .yaml data file is added to the appropriate directory, HyperSmart automatically recognizes it and makes it available for selection in the program. The adopted categorization facilitates the location and comparison of materials, covering different classes and subclasses, in order to meet the needs of research in mechanical, civil, biomedical engineering, and related fields.

## Input File Formatting

The Hyperelastic Material Repository uses input files in the YAML (.yaml) format to store experimental data in a structured and human-readable way. Each file corresponds to a single material dataset and contains both metadata (information about the material, source, and testing conditions) and the experimental data itself.

The main keys and their purposes are:

* **material\_class** – Main classification of the material. Must be one of: "Elastomers", "Foams", "Soft Biological Tissues", or "Gels & Hydrogels".
* **material\_subclass** – Subcategory within the selected material class (e.g., "Animal", "Human", "Polymeric Open-Cell Foams").
* **material** – Full name of the tested material.
* **publication\_title**, **author**, **year** – Bibliographic information of the study from which the data was obtained.
* **description** – Brief summary of the type of test performed and relevant conditions.
* **stress\_measure** – Stress type used in the dataset. Must be "Nominal" or "Cauchy".
* **deformation\_modes** – A dictionary indicating whether each pure deformation mode is available (true or false for "axial", "biaxial", "simple\_shear", and "pure\_shear").
* **unit\_of\_measure** – Unit of stress measurement for each mode (false if data is not available for that mode).
* **data\_source** – Indicates how the experimental data was obtained:
  + 1 – Data provided directly in the scientific article (tables or supplementary files).
  + 2 – Data extracted from figures using an image extraction tool.
* **citation**, **doi** – Full bibliographic citation and DOI link to the source.
* **note** – Additional remarks regarding the dataset (e.g., special test conditions, assumptions, limitations, or quality considerations).
* **data** – Nested dictionary containing numerical values for each available deformation mode. Each mode contains:
  + **stretch** – Array of stretch (λ) or shear parameter values.
  + **stress** – Corresponding stress values in the defined unit.

Below is shown a qualitative example of the formatting using part of a Dataset for Porcine Liver:

Interface gráfica do usuário, Aplicativo

O conteúdo gerado por IA pode estar incorreto.

## Material Categorization

To facilitate navigation and ensure that datasets can be quickly located and compared, all materials in the Hyperelastic Material Repository are organized into a hierarchical categorization system. At the top level, materials are grouped into four broad classes — *Elastomers*, *Foams*, *Soft Biological Tissues*, and *Gels & Hydrogels*. Within each class, materials are further subdivided into subclasses according to their composition, structure, or origin and examples are presented.

This categorization not only helps users filter and identify relevant datasets but also supports consistency when adding new entries to the repository. By following a standardized classification, researchers can efficiently compare materials with similar characteristics, regardless of their source or testing method.

The following sections describe each material class and its corresponding subclasses in detail.

### Elastomers

1. Natural Rubber (NR)
2. Synthetic Rubbers
   * *Hydrocarbon-Based*: SBR, BR, EPDM, IIR.
   * *Polar Rubbers*: NBR, CR.
   * *Specialty Synthetics*: FKM, ACM.
3. Silicone Rubbers (Q)
4. Polyurethanes (PU)
   * Thermoset PU.
5. Thermoplastic Elastomers (TPE)
   * TPO, TPV, SBS, SEBS, *and* Thermoplastic PU (TPU).
6. Others/Hybrid/Specialty Elastomers
   * TPSiV, Bio-Based Rubbers.

### Foams

1. Polymeric Open-Cell Foams
2. Polymeric Closed-Cell Foams
3. Others

### Soft Biological Tissues

1. Human
2. Animal
3. Others

### Gels & Hydrogels

1. Natural Polymer-Based
   * Collagen gels
   * Fibrin gels
   * Alginate hydrogels
   * Hyaluronic acid (HA) hydrogels
2. Synthetic Polymer-Based
   * PEG hydrogels
   * PAAm (Polyacrylamide) gels
   * PVA (Polyvinyl alcohol) hydrogels
3. Composite/Hybrid
   * Collagen-PEG hybrids
   * Nanocellulose-reinforced hydrogels
4. Others

# Hyperelastic models

Hyperelastic models are mathematical formulations used to describe the mechanical behavior of materials capable of undergoing large elastic deformations, such as elastomers, soft biological tissues, foams, and gels. These models differ in their theoretical foundations, functional forms, and range of applicability, but all share the same goal: to provide accurate stress–strain predictions for different deformation modes.

This section is divided into two main parts. The first (**Section 2.1**) presents the analytical expressions for the most important pure deformation modes across the main categories of hyperelastic models. The second (**Section 2.2**) provides an extensive literature review, cataloging a wide range of models along with their equations, sources, and classification.

## Pure Deformation Modes Expressions

A fundamental step in working with hyperelastic models is understanding how they express stress–strain relationships under the principal pure deformation modes:

* **Uniaxial** (axial tension or compression)
* **Equibiaxial** (equal stretches in two directions)
* **Simple shear**
* **Pure shear**

This subsection presents the analytical expressions for each of these deformation modes for the three major types of hyperelastic models:

* **Invariant-Based Models** (stress derived from invariants)
* **Stretch-Based Models** (stress expressed directly as a function of principal stretches)
* **Hookean-Type Models** (generalized Hookean formulations extended to large strains)

The aim is to provide a quick reference for the derivation and application of these expressions, serving as a link between theoretical model definitions and their practical use in material parameter calibration. These expressions are also essential for users who wish to expand the program’s hyperelastic model library, enabling the correct implementation of both existing models from the literature and custom-developed formulations.

### Invariant-Based Models

#### Uniaxial

#### Biaxial

#### Pure Shear

#### Simple Shear

### Stretch-Based Models

#### Uniaxial

#### Biaxial

#### Pure Shear

#### Simple Shear

### Hookean-type Models

#### Uniaxial

#### Biaxial

#### Pure Shear

#### Simple Shear

## Literature Review of Hyperelastic Models

This subsection compiles a comprehensive set of 106 hyperelastic models from the scientific literature, organized according to their theoretical basis and functional form. For each model, the equation, name, publication source and material parameters.

The models are grouped into three main categories and corresponding subcategories:

1 - Phenomenological Models

Principal Stretch-Based Models

Invariant-Based Models

Mixed Phenomenological Model

Series Function Models

Limiting Chain Extensibility Models

Power Law, Exponential or Logarithmic Function Models

2 - Micromechanical Network Models

Gaussian Chain Network Models

Non-Gaussian Chain Network Models

Mixed Micromechanical Network Models

3 - Hookean-type Models

This structured classification enables users to identify models relevant to their research focus, compare their mathematical complexity, and trace their origins in literature. It also supports the addition of new models to the repository by following consistent formatting and categorization.

### Phenomenological Models

Phenomenological models describe the macroscopic mechanical response of hyperelastic materials, as tested experimentally, using mathematical functions chosen for their ability to fit experimental data, without directly representing the material’s microstructure. They are widely used due to their flexibility, simplicity, and broad applicability across different material types.

#### Stretch-Based Models

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| --- | --- |
| **Model Name**: Valanis-Landel | **Source**: (Valanis; Landel, 1967) |
| **Equation**: | |
| **Parameters**: | |

|  |  |
| --- | --- |
| **Model Name**: Peng-Landel | **Source**: (Peng; Landel, 1972) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Ogden | **Source**: (Ogden, 1972) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Attard | **Source**: (Attard; Hunt, 2004) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Shariff | **Source**: (Shariff, 2000) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Arman-Narooei | **Source**: (Narooei; Arman, 2018) |
| **Equation**: | |
| **Parameters**: | |

#### Invariant-Based Models

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| --- | --- |
| **Model Name**: Mooney-Rivlin | **Source**: (Mooney, 1940) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Neo-Hookean | **Source**: (Treloar, 1943b) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Isihara | **Source**: (Isihara; Hashitsume; Tatibana, 1951) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Biderman | **Source**: (Biderman, 1958) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: James-Green-Simpson | **Source**: (James; Green; Simpson, 1975) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: James-Green-Simpson | **Source**: (Haines; Wilson, 1979) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Yeoh | **Source**: (Yeoh, 1990) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Lion | **Source**: (Lion, 1997) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Haupt-Sedlan | **Source**: (Haupt; Sedlan, 2001) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Hartmann-Neff | **Source**: (Hartmann; Neff, 2003) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Carroll | **Source**: (Carroll, 2011) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Nunes | **Source**: (Nunes, 2011) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Bahreman-Darijani | **Source**: (Bahreman; Darijani, 2015) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Zhao | **Source**: (Zhao; Mu; Du, 2019) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Arnoux | **Source**: (Arnoux, 2000; Arnoux et al., 2002) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Warner | **Source**: (Warner, 1972) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Kilian | **Source**: (Kilian, 1981) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Van der Waals | **Source**: (Kilian, 1980) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Gent | **Source**: (Gent, 1996) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Takamizawa-Hayashi | **Source**: (Takamizawa; Hayashi, 1987) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Yeoh-Fleming | **Source**: (Yeoh; Fleming, 1997) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: 3 Parameters Gent | **Source**: (Gent, 1999) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Pucci-Saccomandi | **Source**: (Pucci; Saccomandi, 2002) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Horgan-Saccomandi | **Source**: (Horgan; Saccomandi, 2004) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Beatty | **Source**: (Beatty, 2008) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Horgan-Murphy | **Source**: (Horgan; Murphy, 2007) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: bin Othman & Gregory | **Source**: (Othman; Gregory, 1990) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Knowles | **Source**: (Knowles, 1977) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Swanson | **Source**: (Swanson, 1985) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Yamashita-Kawabata | **Source**: (Yamashita; Kawabata, 1992) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Davis-De-Thomas | **Source**: (Davies; De; Thomas, 1994) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Gregory | **Source**: (Gregory; Muhr; Stephens, 1997) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: modified Gregory | **Source**: (He et al., 2022) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: modified Gregory | **Source**: (He et al., 2022) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Beda | **Source**: (Beda, 2005) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Amin | **Source**: (Amin et al., 2006) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Lopez-Pamies | **Source**: (Lopez-Pamies, 2010) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: gen-Yeoh | **Source**: (Hohenberger et al., 2019) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Hart-Smith | **Source**: (Hart-Smith, 1966) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Veronda-Westmann | **Source**: (Veronda; Westmann, 1970) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Fung-Demiray | **Source**: (Demiray, 1972; Fung, 1967) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Vito | **Source**: (Vito, 1973) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Humphrey-Yin | **Source**: (Humphrey; Yin, 1987) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: modified Yeoh | **Source**: (Yeoh, 1993) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Martins | **Source**: (Martins et al., 1998) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Chevalier-Marco | **Source**: (Gornet et al., 2012) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Mansouri-Darijani | **Source**: (Mansouri; Darijani, 2014) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Gent-Thomas | **Source**: (Gent; Thomas, 1958) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Alexander | **Source**: (Alexander, 1968) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Lambert Diani-Rey | **Source**: (Lambert-Diani; Rey, 1999) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Hoss-Marczak-I | **Source**: (Hoss; Marczak, 2010) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Hoss-Marczak-II | **Source**: (Hoss; Marczak, 2010) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Exp-Ln | **Source**: (Khajehsaeid; Arghavani; Naghdabadi, 2013) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: modified Demiray | **Source**: (Demiray et al., 1988) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Holmes-Mow | **Source**: (Holmes; Mow, 1990) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Raghavan-Vorp | **Source**: (Raghavan; Vorp, 2000) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Singh | **Source**: (Singh; Katiyar; Singh, 2013) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Tang | **Source**: (Tang et al., 2009) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: van Dam | **Source**: (Van Dam et al., 2008) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: modified exp-ln | **Source**: (Dwivedi et al., 2022) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Fu | **Source**: (Van Dam et al., 2008) |
| **Equation**: | |
| **Parameters**: | |

#### Mixed Phenomenological Model

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| --- | --- |
| **Model Name**: Continuum Hybrid | **Source**: (Beda; Chevalier, 2003) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Bechir-4term | **Source**: (Bechir et al., 2006) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: WFB | **Source**: (Korba; Barkey, 2017) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Volokh-Vorp | **Source**: (Volokh; Vorp, 2008) |
| **Equation**: | |
| **Parameters**: | |

### Micromechanical Network Models

Micromechanical network models are based on the statistical mechanics of polymer chains, relating macroscopic behavior to the material’s underlying microstructure. These models provide a physical interpretation of parameters and are especially useful when microstructural insights are important for design or analysis.

#### Gaussian Chain Network Models

|  |  |
| --- | --- |
| **Model Name**: Gaussian | **Source**: (Treloar, 1943a) |
| **Equation**: | |
| **Parameters**: | |

|  |  |
| --- | --- |
| **Model Name**: Affine | **Source**: (Flory, 1976; Kuhn, 1946; Wall; Flory, 1951) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Phantom | **Source**: (Flory, 1976; Kuhn, 1946; Wall; Flory, 1951) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Edwards-Tube | **Source**: (Edwards, 1967) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Slip-Link | **Source**: (Ball et al., 1981) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Constrained Junctions | **Source**: (Erman; Flory, 1982; Flory; Erman, 1982) |
| **Equation**: | |
| **Parameters**: | |

|  |  |
| --- | --- |
| **Model Name**: Edwards-Vilgis | **Source**: (Edwards; Vilgis, 1986) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: MCC | **Source**: (Erman; Monnerie, 1989) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Tube | **Source**: (Heinrich; Kaliske, 1997) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Nonaffine-Tube | **Source**: (Rubinstein; Panyukov, 1997) |
| **Equation**: | |
| **Parameters**: | |

#### Non-Gaussian Chain Network Models

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| --- | --- |
| **Model Name**: Three-Chain | **Source**: (James; Guth, 1943) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Four-Chain | **Source**: (Flory, 1944) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Arruda-Boyce | **Source**: (Arruda; Boyce, 1993; Cohen, 1991) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: modified Flory-Erman | **Source**: (Boyce; Arruda, 2000; Edwards, 1967) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Extended-Tube | **Source**: (Kaliske; Heinrich, 1999) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Meissner-Matějka | **Source**: (Meissner; Matějka, 2003, 2004) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Micro-Sphere | **Source**: (Miehe; Göktepe; Lulei, 2004) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Bootstrapped-8-Chain | **Source**: (Miroshnychenko; Green, 2009; Miroshnychenko; Green; Turner, 2005) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Davidson-Goulbourne | **Source**: (Davidson; Goulbourne, 2013) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Network Averaging Tube | **Source**: (Khiêm; Itskov, 2016) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: SpT | **Source**: (Xiang et al., 2018) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Dobrynin & Carrillo | **Source**: (Dobrynin; Carrillo, 2011) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Simplified Tube | **Source**: (Plagge et al., 2020) |
| **Equation**: | |
| **Parameters**: | |

#### Mixed Micromechanical Network Models

|  |  |
| --- | --- |
| **Model Name**: Wu-Giessen (Full Network) | **Source**: (Treloar; Riding, 1997; Wu; Van der Giessen, 1992; Wu; Van Der Giessen, 1993) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Model Name**: Zuniga-Beatty | **Source**: (Elı́as-Zúñiga; Beatty, 2002) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Lim | **Source**: (Elı́as-Zúñiga; Beatty, 2002) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Lim | **Source**: (Lim, *[S.d.]*) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Bechir-Chevalier | **Source**: (Bechir; Chevalier; Idjeri, 2010) |
| **Equation**: | |
| **Parameters**: | |

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| **Model Name**: Davies | **Source**: (Bechir; Chevalier; Idjeri, 2010) |
| **Equation**: | |
| **Parameters**: | |

### Hookean-type Models

Every Hookean-type hyperelastic model have mechanical behavior intrinsically linked to the used strain measure. The linear relationship between the conjugated pair of stress and strain, as well as the Strain Energy Function the represent the hyperelastic model are presented below. As one can see, every Hookean-type model has at least two material parameters: Young’s modulus () and Poisson’s ratio (), which are used to define the Lamé Parameters ( and ), so any other parameter necessary to define the strain measure is added to those two.

|  |  |
| --- | --- |
| **Strain Measure**: Generalized Hyperbolic-Sine (GHS) | **Source**: (Peixoto, 2024; Peixoto; Greco; Vasconcellos, 2024) |
| **Equation**:  \* and can be any given strain measure in its uniaxial and multiaxial format, respectively. | |
| **Parameters**: | |

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| --- | --- |
| **Strain Measure**: Seth-Hill (Doyle-Ericken or Classical) | **Source:**  (Doyle; Ericksen, 1956; Hill, 1968; Seth, 1961) |
| **Equation**: | |
| **Parameters**: | |

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| --- | --- |
| **Strain Measure**: GHS–Seth-Hill | **Source**: (Peixoto, 2024; Peixoto; Greco; Vasconcellos, 2024) |
| **Equation**: | |
| **Parameters**: | |

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| **Strain Measure**: Curnier-Zysset Metric Family | **Source**: (Curnier; Zysset, 2006) |
| **Equation**: | |
| **Parameters**: | |

|  |  |
| --- | --- |
| **Strain Measure**: Curnier-Rakotomanana Family (Darijani-Naghdabadi Power Family) | **Source**: (Curnier; Rakotomanana, 1991; Darijani; Naghdabadi, 2013) |
| **Equation**: | |
| **Parameters**: | |

|  |  |
| --- | --- |
| **Strain Measure**: Darijani-Naghdabadi Exponencial Family | **Source**: (Darijani; Naghdabadi, 2013) |
| **Equation**: | |
| **Parameters**: | |

|  |  |
| --- | --- |
| **Strain Measure**: Beex Family | **Source**: (Beex, 2019) |
| **Equation**:  \*is a set*:* and are the Seth-Hill strain measures. | |
| **Parameters**: | |

## Use of Hyperelastic Models in HyperSmart

In HyperSmart, hyperelastic models are stored in YAML (.yaml) files, a format chosen for its readability, flexibility, and ease of editing. This structure allows the program to load model definitions dynamically and enables users to expand the model library without modifying the source code.

Users can add new models from the literature—or create their own—by preparing a .yaml file following the same structure as the existing examples. To derive the mathematical expressions, the formulae presented in the “2.1 Pure Deformation Modes Expressions” subsection can be used.

A model .yaml file contains:

* Basic Information: model\_name, model\_class, and description, summarizing the model and its category.
* References: Complete bibliographic citations for traceability.
* Mathematical Definition: The strain energy function or strain measure, depending on the model type.
* Parameters: Number of material parameters, their names, display formats, and bounds.
* Deformation Mode Expressions: Analytical formulas for each pure deformation mode (uniaxial, biaxial, pure shear, and simple shear) in both nominal and Cauchy stress forms, when applicable.
* Additional Information: Optional constraints or notes relevant to the model.

**Example – Mooney–Rivlin Model**

Texto

O conteúdo gerado por IA pode estar incorreto.

When a .yaml model file is added to the appropriate directory, HyperSmart automatically recognizes it and makes it available for selection in the program. By following this structure, users can:

* Integrate models from the literature into the software’s library.
* Implement custom models for specialized applications.
* Share model files with other researchers, ensuring reproducibility and consistency.

# Numerical Methods of Calibration

This section describes how numerical methods are implemented in HyperSmart to calibrate hyperelastic material parameters from experimental data, and explain the extensible architecture used that makes it easy to add new methods. Rather than hard-coding solvers, the application exposes a plugin-style interface: each numerical method is defined by a pair of files in *numerical\_methods/* folder: a small YAML manifest (name + description) and a Python module that exports a unified *MethodWindow(...)* class. At runtime the app discovers methods dynamically by scanning the YAML files, lists them in the UI, and loads the corresponding Python module by stem (no changes to the main program). This separation of concerns – core UI/navigation vs. self-contained method plugins – lets users add, replace, or compare methods simply by dropping in new files (e.g., our current Enumeration and BUS implementations), ensuring the system can grow without refactoring the application core.

## Calibration methods

Below the two methods already avaiable in the program are presented.

### Enumeration (grid search)

Enumeration performs a brute-force search over a user-defined grid in parameter space. For each material parameter ​, the user specifies a lower bound, an upper bound, and a step size ​. The algorithm evaluates the model on the Cartesian product of these grids and returns the parameter set that minimizes the error metric.

* Strengths: simple, transparent, and guarantees the exact optimum if it lies on the grid. Useful for low-dimensional problems, coarse sweeps, and diagnostics.
* Limitations: computational cost grows exponentially with the number of parameters and with finer step sizes.

### BUS (Bayesian Updating with Subset Simulation)

BUS casts calibration as Bayesian inference. Starting from user-defined priors on the parameters, it constructs an augmented probability space and uses Subset Simulation to sample progressively from higher-likelihood regions until reaching the posterior. It returns:

* a MAP (maximum a posteriori) estimate, and
* a sampled posterior for uncertainty quantification and parameter correlations.
* **Strengths:** uncertainty quantification; often more efficient than plain MCMC for multi-modal or high-dimensional posteriors; natural credibility intervals.
* **Limitations:** require evaluating the forward model at every sample; results depend on the choice of priors, noise model (likelihood), and BUS hyperparameters (samples per level , target conditional probability ​).

## Error metric and multi-mode calibration

We use a normalized root-mean-square error tailored for simultaneous calibration across multiple deformation modes:

where is the total number of experimental points across all selected modes. The normalization by the mean absolute experimental stress prevents modes with larger stress magnitudes from dominating the fit. REQMN is well-defined even when stresses include negative values; only the (rare) case of a near-zero denominator requires caution.

* **Enumeration** Method minimizes REQMN over the grid.
* **BUS** uses a Gaussian likelihood on residuals (equivalent to minimizing the sum of squared errors) and reports REQMN for diagnostics.

## Data handling and model evaluation

Experimental data are carried in a single ExperimentalData object:

* deformation-mode arrays (e.g., uniaxial, biaxial, pure shear, simple shear)
* stress measure flag (nominal vs. Cauchy)
* unit metadata (MPa/kPa)
* optional pre-assigned constants

For each selected mode, the program maps experimental independent variables to the symbolic names used by the model expressions (e.g., *lamb* for *stretch*, *gamma* for *shear*). The stress measure is respected by selecting the matching expression (e.g., *expression\_nominal* or *expression\_cauchy*).

Some models include conditional expressions (e.g., the Generalized Hyperbolic Sine Hookean-tye model has one formula for and another for ). Each model’s YAML can therefore provide either a single string formula or a list of {condition, formula} entries. A small evaluator selects the formula whose condition is true for the current parameter values, ensuring the correct branch is used during calibration.

## Plugin architecture for numerical methods

To enable new methods without changes to the main code, we use a lightweight plugin pattern:

* Each method lives in two files inside *src/numerical\_methods/*:
  + a YAML metadata file: *method\_stem.yaml*
  + a Python implementation: *method\_stem.py*
* The YAML must define:
  + *method\_name* – display name shown in the UI
  + description – short explanation shown in the info panel
* The Python module must expose a class named *MethodWindow* with the unified constructor:

*“MethodWindow(root, material, model\_data, proceed\_callback\_back)”*

This class is responsible for rendering its own UI (bounds/steps for Enumeration; priors and BUS settings for BUS) and for running the method.

* The application discovers methods dynamically:

1. It scans *numerical\_methods/\*.yaml*, displays their *method\_name* in the selection window, and remembers each file’s stem (e.g., enumeration).
2. When the user clicks *Next*, the app imports *numerical\_methods.<stem>* and instantiates *MethodWindow(...)*. No changes to the main script are needed to support new methods.

Benefits of this design

* Extensibility: users can add new calibration strategies by dropping in a .yaml + .py pair; the main app remains untouched.
* Separation of concerns: the core UI manages discovery and navigation; each method owns its own inputs, solver, and result reporting.
* Consistency: all methods adhere to the same constructor, making the user experience uniform.
* Reproducibility & transparency: the YAML summaries document method intent and options; Enumeration grids and BUS priors/hyperparameters are all explicit.

## Typical Workflow

1 Choose a model and review its description/equations.

2 Pick a calibration method:

* *Enumeration:* set parameter ranges and step sizes.
* *BUS:* set priors, noise level , samples/level N, and ​.

3 Select deformation modes to include (one or multiple).

4 Run calibration:

* Enumeration searches the grid and reports the parameter set with the lowest REQMN.
* BUS estimates the posterior, reports MAP parameters, and provides posterior samples for uncertainty analysis.

5 Inspect results: calibrated parameters, REQMN, and plots comparing experimental vs. model stress for each mode.

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