The Cooper Union Department of Electrical Engineering
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ECE211 Signal Processing & Systems Analysis
Problem Set VIII: Random Signals
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This assignment all relates to a real WSS signal $x(n)$ that is modeled as:

$$x(n) = v(n) + 0.5v(n-1) + 0.4v(n-2) - 0.2x(n-1) - 0.8x(n-2)$$

where $v(n)$ is 0-mean white noise with $\sigma_v^2 = 4$.

1. Before you start coding in MATLAB:
   (a) Is x AR, MA or ARMA?
   (b) Is this filter (with v as input and x as output) the whitening or the innovations filter?
   (c) Write a difference equation for the inverse filter.
   (d) Write a formula for the transfer function $H(z)$ for the filter.
   (e) Write an explicit formula by hand (not simplified) for the PSD $S_x(\omega)$ of x.

2. Use MATLAB to compute the poles and zeros of H. Confirm that H is minimum-phase. Plot the poles and zeros.

3. Now you will generate random data and estimate the correlation.
   (a) In MATLAB, generate $N = 10000$ samples of v and then apply the filter to generate N samples of x.
   (b) Use time-averaging to estimate $r_x(m)$ for $0 \leq m \leq 5$. **Note:** Do not use MATLAB functions *xcorr* or *corr*. Write a *for* loop over the index m, and use the *dot* function.
   (c) Obtain a stem plot of $r_x(m)$ for $-5 \leq m \leq 5$ (remember, $r_x(m) = r_x(-m)$ since x is real).
   (d) Arrange your estimated correlation values in a 6×6 Toeplitz matrix R. This matrix represents the correlation of a block of samples of the form:

   $$x_M(n) = \begin{bmatrix} x(n) \\ \vdots \\ x(n-M+1) \end{bmatrix}$$

   What is M here?
   (e) Check that R is positive definite by computing its eigenvalues.

4. Now we are going to study the PSD. The MATLAB function *pwelch* can be used to estimate the power spectral density. The name of the method this function employs is called the *modified Welch periodogram.* This essentially means computing the magnitude-square of the Fourier transform (specifically, the DFT) on overlapping windowed data blocks and averaging.

   (a) Invoke *pwelch* as follows:

   $$[s\_est, w] = pwelch(x, \text{hamming}(512), 256, 512);$$

   This returns s_est as the estimated PSD and w is the frequency vector (in normalized digital radian units, from 0 to π). Compute and plot the estimated power spectral density (on a linear scale, not decibels).

   (b) The PSD estimate has a peak at some frequency ω_0. Find ω_0.

   (c) You will note that H has a poles near the unit circle. This is what is causing the strong peak in $S_x(\omega)$. Compute the pole angle and compare to ω_0.