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ECE478 Financial Signal Processing
Problem Set I: Portfolios
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General Comments

I have posted sample Python code you may find useful, including code that will pull down stock prices and what we can use as "risk-free interest rate" based on 3 month US T-bills. Feel free to obtain the data you need other ways, including through MATLAB.
You will also need S&P500 data. I haven't provided explicit code for you to obtain it– you should figure out on your own how to get that.
Note that in all cases we will be working with *daily adjusted closing prices.*[1] Other data that is often used is "open, closing, high, low" for one day, plus volume (which can be measured, for example, by number of traded shares, or by total value (in dollars) traded), but we won't be using that data here.
You can consider three issues to deal with: get data; analyze it; display results. Although the analysis part is probably easiest with MATLAB, the other two may be easier with Python. Feel free to mix and match among the languages as you like.
Note that here we are dealing with real, not simulated data. [In other assignments in this course, you will generate simulated data according to certain models we will develop]
Also note sometimes below I am not particularly specific. In particular, it is not my intention that you look at 1000 permutations of the data! Do some exploration to address some of the questions I bring up. I am being somewhat vague on purpose.

Data Acquisition and Pre-Processing

Select 10 stocks. Try to select them in two groups of 5, from two different sectors. For each stock, plus S&P500, pull down the daily closing price for year 2018, plus the last day of the 2017 trading year. From this, compute the daily returns for 2018.
You will also need a reference risk-free interest rate. For this, pull down US 3 month T-bill rates. These rates will need to be converted to daily rates; see the sample Python code for more explanation (even if you don't use Python to accomplish this).
In order to simplify the analysis (since the risk-free interest rate is actually time-varying), subtract the daily risk-free interest rate from your stock returns, so now everything is *excess* returns. That means the μ parameter below will be, in all cases, *excess* expected return!

[1] The adjusted closing price is determined through a somewhat complicated process, to remove the effects of actions such as stock splits. For example, if a stock price was $50 last Monday, but is then "split" by 2 (one stock now becomes two stocks), the price Tuesday would be $25. The way this is "fixed" would be to indicate the adjusted Monday price as $25. This means the adjusted price on date X, when viewed at a later date Y, will VARY depending on Y. In any case, here we will always be using ADJUSTED prices. BE CAREFUL!!

Data Analysis

1. First let's work on the full year of data. All returns here are excess returns!

   (a) Compute the (σ, μ) point for the S&P 500, the expected returns and covariance matrix for the full set of stocks. By pulling out submatrices, obtain the covariance matrices of each of the two subsets of stocks (corresponding to each sector).

   (b) First examine the covariance matrices. Computer the condition number of each. Also look for correlations: positive versus negative, high values ($|\rho|$ big) and small ($|\rho|$ small). Are the correlations within one sector stronger in general than across the two sectors?

   (c) Compute the single factor model using S&P500. That is, model the return of each stock k as:

   $$r_k = \alpha_k + \beta_k r_{\text{S\&P 500}} + \varepsilon_k$$

   Compute the correlation matrix of the residuals ε_k and comment whether the diagonal model is reasonable here. Again, are there stronger correlations within a sector than across sectors?

   (d) Compute the MVP and MP of the full set of stocks, and graph the efficient frontier, with markers at the MVP and MP, and the CML for this set. Repeat this for the two subsectors. On your graph(s) you should also place a marker for the S&P 500. **Note:** It may look better to superimpose these graphs, or do them separately- up to you.

   (e) Look at the MP portfolios (full set, and each subset). Is there significant short-selling? Do they seem to be anywhere "close" to S&P 500 (view it subjectively).

   (f) Compute the β for each individual stock relative to the MP of the full set, and again relative to the MP of the sector subset.

2. Now let's look at the daily returns within a month. Start with the full set of stocks. Take a month, say January, compute the expected daily return and β for each stock relative to the S&P 500 [i.e., based on January data only] Now compute the average daily return for each stock and S&P 500 for the month of February. Obtain a graph of the security market line based on data from February: you should have 11 points- each of the 10 stocks, and the S&P 500; the horizontal axis is the β computed from January data ($\beta = 1$ for S&P 500), the vertical axis is the actual returns from February data. The SML itself should be the line from $(0, 0)$ (remember, we are working with excess returns, effectively $r_{\text{rf}} = 0$) to $(1, \mu_{\text{S\&P 500}})$. Now do this again for each sector, separately. [Here we are drawing the SML using the S&P 500 as a reference, not the numerically computed MP]

3. Take the January data to find the MP for the set of 10 stocks, and the separate MP's for each sector. Imagine three investors each put in $1 in the respective MP's after the end of the last day of trading in January, and collect the returns through February. At the last day of February, the portfolios are updated to the MP's based on February data (using a self-financing approach). Here we will neglect transaction costs. Continue through December, and each investor will have a certain amount of money as of the

last trading day in December. As a reference point, imagine someone invests \$1 in the S&P 500 at the end of the last trading day of January, and just holds it until end of December. Compare the end of year results. You may also want to track the monthly progress of each portfolio (i.e., show graphs). **Note:** If stock prices are always positive, it may still be possible for the value of your portfolio to go negative; how could that happen? Check- does it ever happen? Assume you have enough cash reserves to sustain negative portfolio value if it occurs. **Remark:** Here we are looking at overall returns over one month periods. You can either go back to the original stock prices to find this, or reconstruct it from the daily returns; in any case, remember we are looking at the actual return for one MONTH, say from Jan 31 to Feb 28 (or whatever the final trading days are), not the AVERAGE daily return!