Assignment 1

tldr: Perform linear regression of a noisy sinewave using a set of gaussian basis functions with learned location and scale parameters. Model parameters are learned with stochastic gradient descent. Use of automatic differentiation is required. Hint: note your limits!

Problem Statement Consider a set of scalars $\{x_1, x_2, \ldots, x_N\}$ drawn from $\mathcal{U}(0, 1)$ and a corresponding set $\{y_1, y_2, \ldots, y_N\}$ where:

$$y_i = \sin(2\pi x_i) + \epsilon_i \tag{1}$$

and ϵ_i is drawn from $\mathcal{N}(0, \sigma_{\text{noise}})$. Given the following functional form:

$$\hat{y}_i = \sum_{j=1}^{M} w_j \phi_j (x_i \mid \mu_j, \sigma_j) + b \tag{2}$$

with:

$$\phi(x \mid \mu, \sigma) = \exp \frac{-(x-\mu)^2}{\sigma^2} \tag{3}$$

find estimates $\hat{b}$, $\{\hat{\mu}_j\}$, $\{\hat{\sigma}_j\}$, and $\{\hat{w}_j\}$ that minimize the loss function:

$$J(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2 \tag{4}$$

for all (x_i, y_i) pairs. Estimates for the parameters must be found using stochastic gradient descent. A framework that supports automatic differentiation must be used. Set $N = 50, \sigma_{\text{noise}} = 0.1$. Select M as appropriate. Produce two plots. First, show the data-points, a noiseless sinewave, and the manifold produced by the regression model. Second, show each of the M basis functions. Plots must be of suitable visual quality.

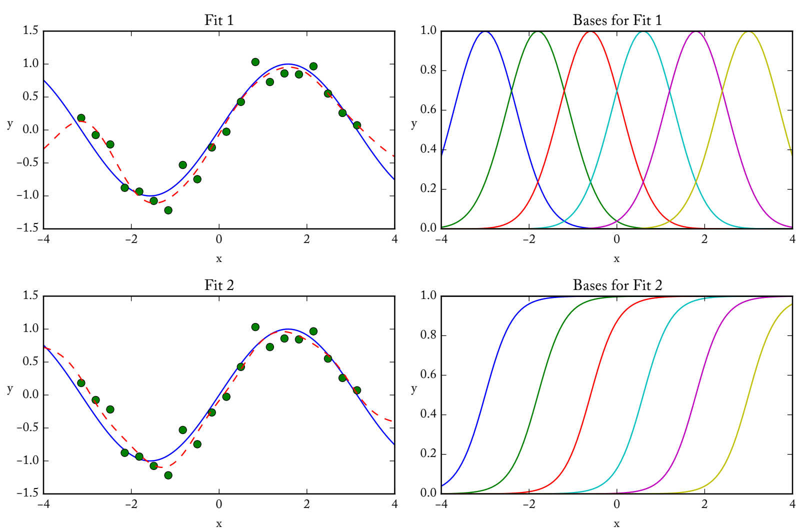

Figure 1: Example plots for models with equally spaced sigmoid and gaussian basis functions.