No.

Date Tsaqif Jalaluddin Ahmad

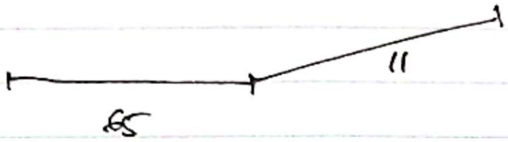
24 / 537665 / TK / 59611

Servo 1 dan Servo 2 dipetar : 40° dan 30°

α B

537665 59611

65 11



$$H^0 = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H^1 = \begin{bmatrix} 1 & 0 & 65 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H^2 = \begin{bmatrix} \cos B & -\sin B & 0 \\ \sin B & \cos B & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H^3 = \begin{bmatrix} 1 & 0 & 11 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H^4 = H^0 H^1 H^2 H^3$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 65\cos\alpha \\ \sin\alpha & \cos\alpha & 65\sin\alpha \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos B & -\sin B & 11\cos B \\ \sin B & \cos B & 11\sin B \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{matrix} \cos(\alpha+B) & -\sin(\alpha+B) & 11\cos(\alpha+B) + 65\cos\alpha \\ \sin(\alpha+B) & \cos(\alpha+B) & 11\sin(\alpha+B) + 65\sin\alpha \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{matrix} 11\cos(70) + 65\cos(40) \\ 11\sin(70) + 65\sin(40) \end{matrix} \approx \begin{matrix} 53.5[illegible]6 \\ 52.117 \end{matrix}$$

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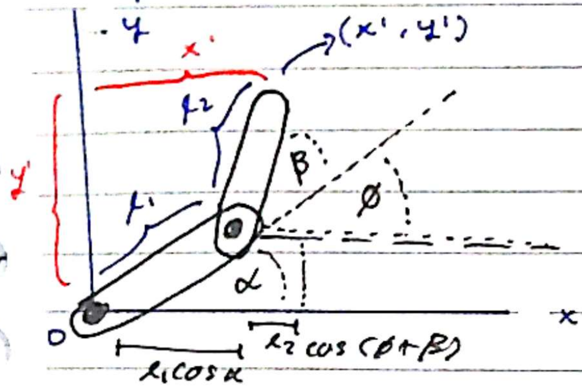
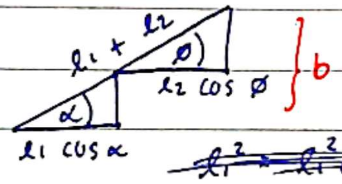
2 Dof



$y' = l_1 \sin(\alpha) + l_2 \sin(\beta + \phi)$

$x' = l_1 \cos(\alpha) + l_2 \cos(\beta + \phi)$



$\cancel{l_1^2 = l_1^2\cos^2\alpha + l_1^2\sin^2\alpha}$

$\cancel{1 = \cos^2\alpha + \sin^2\alpha}$

$\tan\alpha = \tan\phi$

karena kemiringan l_1 dan l_2 sama

saat kondisi tersebut. $(l_1 + l_2)\cos(\alpha) = a$

$\alpha = \phi$ $\qquad l_1\cos\alpha + l_2\cos\phi = a$

$$\boxed{\begin{aligned} x' &= l_1\cos(\alpha) + l_2\cos(\alpha + \beta) \\ y' &= l_1\sin(\alpha) + l_2\cos(\alpha + \beta) \end{aligned}}$$ //

melewati titik :

$(0,0)$, (a,b)

$$\frac{y-0}{b-0} = \frac{x-0}{a-0}$$

$$y = \frac{b}{a}x \qquad \frac{dy}{dx} = \frac{b}{a}$$

$\tan\alpha =$

$$\frac{b}{a} = \frac{(l_1 + l_2)\sin\alpha}{(l_1 + l_2)\cos\alpha} = \frac{l_1\sin\alpha + l_2\sin\phi}{l_1\cos\alpha + l_2\cos\phi}$$

- $$\frac{l_1\sin\alpha + l_2\sin\alpha}{l_1\cos\alpha + l_2\cos\alpha} = \frac{l_1\sin\alpha + l_2\sin\phi}{l_1\cos\alpha + l_2\cos\phi}$$

- $$\cancel{l_1^2\sin\alpha\cos\alpha} + l_1 l_2(\cancel{\cos\alpha\sin\alpha} + \sin\alpha\cos\phi) + l_2^2\sin\alpha\cos\phi$$

$$=$$

$$\cancel{l_1^2\sin\alpha\cos\alpha} + l_1 l_2(\sin\phi\cos\alpha + \cancel{\sin\alpha\cos\alpha}) + l_2^2\sin\phi\cos\alpha$$

$$l_1 l_2 \sin\alpha\cos\phi + l_2^2\sin\alpha\cos\phi = l_1 l_2\sin\phi\cos\alpha + l_2^2\sin\phi\cos\alpha$$

$$l_1 \cancel{l_2}(\sin\alpha\cos\phi - \sin\phi\cos\alpha) = l_2^{\cancel{2}}[(\sin\phi\cos\alpha) - \sin\alpha\cos\phi]$$

$$l_1(\sin(\alpha - \phi)) = l_2(\sin(\phi - \alpha))$$

karena $l_1 \neq l_2$ "l_1 tidak pasti sama dengan l_2"

$$\sin(\alpha - \phi) = \sin(\phi - \alpha) = 0$$

$$\phi - \alpha = 0$$

$$\alpha = \phi$$ //

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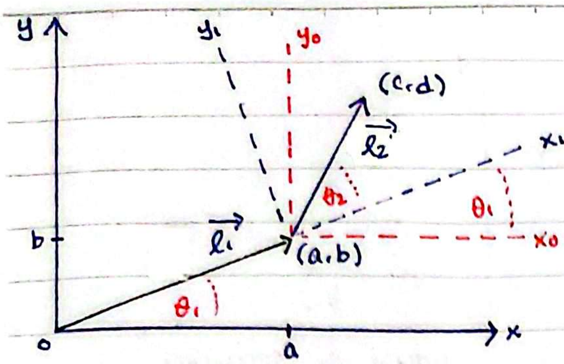

Seperti melakukan rotasi $\vec{l_2}$ dengan pusat (a,b)

pusat (a,b) didapat dari $\vec{l_1}$ dirotasikan θ_1 terhadap $(0,0)$ sumbu x,y.

Untuk Rotasi $\vec{l_1}$:

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Tambahkan translasi:

$$\underbrace{\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}}_{\text{identik}}$$

- Bisa digabungkan dalam 1 matrix:

$$\begin{bmatrix} x_1 \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & tx \\ \sin\theta_1 & \cos\theta_1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta_1 & -y\sin\theta_1 & +tx \\ x\sin\theta_1 & +y\cos\theta_1 & +ty \\ 0 & 0 & 1 \end{bmatrix}$$

Untuk mencapai titik (a,b), $\vec{l_1}$:

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & l_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & l_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & l_1\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$l_1\cos\theta_1 \rightarrow$ ~~ke~~ posisi a

$l_1\sin\theta_1 \rightarrow$ posisi b

rotasi terhadap x,y tidak pelu translasi

untuk merotasi sumbu x_0, y_0 sejauh θ_1

Rotasi $\vec{l_2}$:

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & l_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & l_2\sin\theta_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Tetapi operasi ini dilakukan di pusat (a,b) terhadap sumbu x_1, y_1.
Kalikan dengan matrix sebelumnya untuk merotasi $x_0, y_0 \xrightarrow[\theta_2]{} x_1, y_1$ serta menetapkan pusatnya (a,b)

$$\underbrace{\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \\ 0 & 0 \end{bmatrix}}_{\text{rotasi sumbu}} \underbrace{\begin{bmatrix} l_1\cos\theta_1 \\ l_1\sin\theta_1 \\ 1 \end{bmatrix}}_{\text{translasi}} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & l_2\sin\theta_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2 & l_1\sin\theta_1 + l_2\cos(\theta_1+\theta_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 = \cos(\theta_1+\theta_2)$$
$$-\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 = -\sin(\theta_1+\theta_2)$$
$$\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1 = \sin(\theta_1+\theta_2)$$
$$-\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 = \cos(\theta_1+\theta_2)$$

$$\begin{bmatrix} c \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) \\ l_1\sin\theta_1 + l_2\cos(\theta_1+\theta_2) \\ 1 \end{bmatrix} //$$

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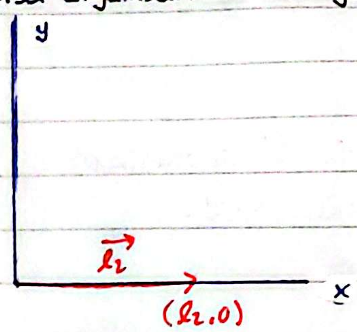
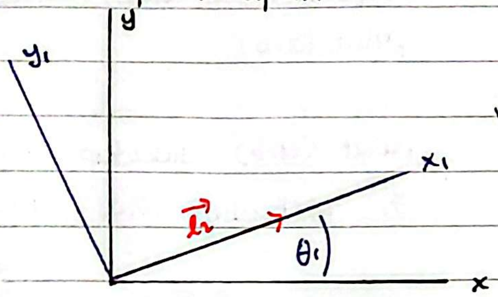
Bisa digambarkan sebagai beberapa tahapan :

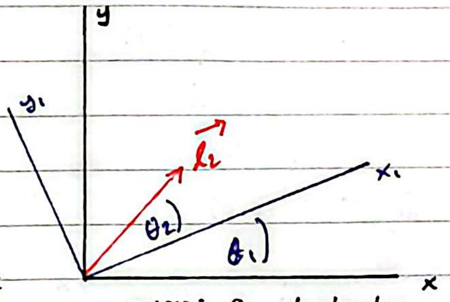


$$\begin{bmatrix} 1 & 0 & l_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

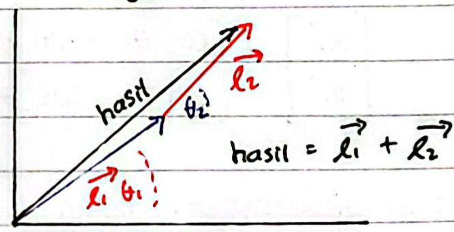


Sumbunya dirotasi oleh

$$\begin{matrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \\ 0 & 0 \end{matrix}$$



rotasi θ_2 terhadap origin



hasil = $\vec{l_1} + \vec{l_2}$

translasi agar ~~sama~~ ekor vektor di (a,b)

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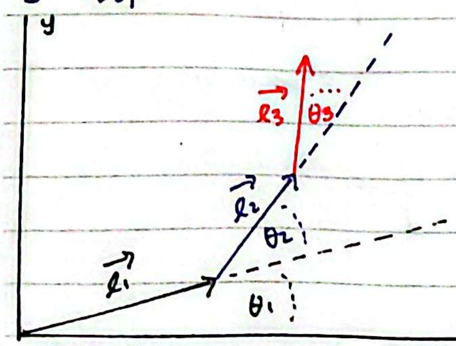

Pada kasus ini, relasi antara $\vec{l_3}$ dengan $\vec{l_2}$ juga bisa dipandang sebagai kasus 2 degrees of freedom sebelumnya.

rotasi sumbu, translasi, dan rotasi vektor yang sama bisa diaplikasikan

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & l_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & l_1\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & l_2\sin\theta_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & l_3\sin\theta_3 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & l_1\cos(\theta_1) + l_2(\cos(\theta_1+\theta_2)) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & l_1\sin(\theta_1) + l_2(\sin\theta_1+\theta_2) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & l_3\sin\theta_3 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\theta_1+\theta_2)\cos(\theta_3) - \sin(\theta_1+\theta_2)\sin\theta_3 & -\cos(\theta_1+\theta_2)\sin\theta_3 - \sin(\theta_1+\theta_2)\cos\theta_3 & \cdots \\ \sin(\theta_1+\theta_2)\cos\theta_3 + \cos(\theta_1+\theta_2)\sin\theta_3 & -\sin(\theta_1+\theta_2)\sin\theta_3 + \cos(\theta_1+\theta_2)\cos\theta_3 & \cdots \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos([\theta_1+\theta_2]+\theta_3) & -\sin([\theta_1+\theta_2]+\theta_3) & \boxed{l_3\cos(\theta_1+\theta_2+\theta_3) + l_1\cos(\theta_1) + l_2\cos(\theta_1+\theta_2)} \\ \sin([\theta_1+\theta_2]+\theta_3) & \cos([\theta_1+\theta_2]+\theta_3) & \boxed{l_3\sin(\theta_1+\theta_2+\theta_3) + l_1\sin(\theta_1) + l_2\sin(\theta_1+\theta_2)} \\ 0 & 0 & 1 \end{bmatrix} //$$

Nilai x dan y akhir pada baris 1 dan 2 berturut-turut.

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Inverse Kinematics

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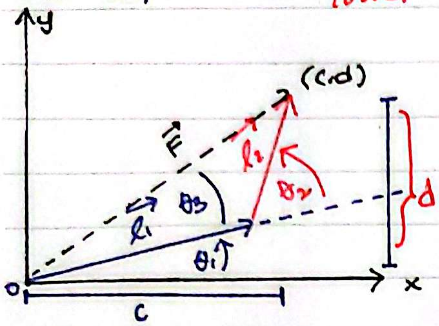
lower - Elbow



titik akhir dari sistem : (c, d)

$$c = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

$$d = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

$$c^2 = l_1^2 \cos^2(\theta_1) + 2 l_1 l_2 \cos(\theta_1)\cos(\theta_1+\theta_2) + l_2^2 \cos^2(\theta_1+\theta_2)$$

$$d^2 = l_1^2 \sin^2(\theta_1) + 2 l_1 l_2 \sin(\theta_1)\sin(\theta_1+\theta_2) + l_2^2 \sin^2(\theta_1+\theta_2)$$

$$\sin^2(\theta_1) + \cos^2(\theta_1) = 1 \qquad \sin^2(\theta_1+\theta_2) + \cos^2(\theta_1+\theta_2) = 1$$

$$c^2 + d^2 = l_1^2 + l_2^2 + 2 l_1 l_2 (\cos\theta_1 \cos(\theta_1+\theta_2)) + \sin\theta_1 (\sin(\theta_1+\theta_2)))$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta_1 - (\theta_1 + \theta_2))$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 \cos(-\theta_2)$$

$$\cos(-\theta_2) = \cos(\theta_2)$$

$$\cos(\theta_2)\, 2 l_1 l_1 = c^2 + d^2 - (l_1^2 + l_2^2)$$

$$\cos(\theta_2) = \frac{c^2 + d^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$$\theta_2 = \cos^{-1}\left[\frac{c^2 + d^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right] //$$

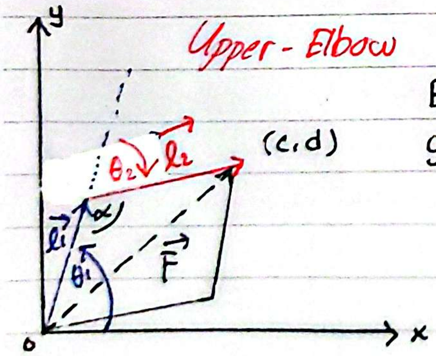
$$\tan(\theta_3) = \frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}$$

$$\tan(\theta_3 + \theta_1) = \frac{d}{c}$$

$$\theta_3 + \theta_1 = \tan^{-1}(d/c)$$

$$\theta_1 = \tan^{-1}(d/c) - \tan^{-1}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2\cos\theta_2}\right) //$$

Upper - Elbow



Berdasarkan sifat penjumlahan vektor dengan jajar genjang / Parallelogram :

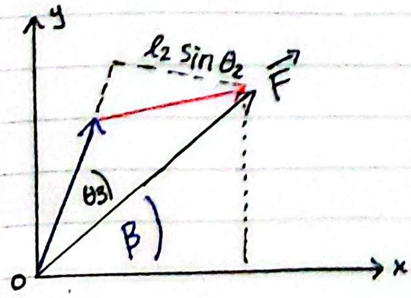
terdapat 2 solusi untuk membentuk $\vec{F}$ (resultan)

Hukum Cosinus :

$$c^2 + d^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\alpha)$$

$$\cos(\alpha) = \frac{l_1^2 + l_2^2 - c^2 - d^2}{2 l_1 l_2}$$

$$\alpha = \cos^{-1}\left[\frac{l_1^2 + l_2^2 - c^2 - d^2}{2 l_1 l_2}\right]$$



$$\theta_2 = \pi - \alpha$$

$$= \pi - \cos^{-1}\left[\frac{l_1^2 + l_2^2 - c^2 - d^2}{2 l_1 l_2}\right]$$

sifat sudut trigonometri

$$\cos(\pi - \theta) = -\cos(\theta)$$

$$= -\cos^{-1}\left[\frac{c^2 + d^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right]$$

$$\theta_1 = \theta_3 + \beta$$

sudut θ_2 negatif karena berputar clockwise

$$= -\tan^{-1}\left[\frac{l_2 \sin(\theta_2)}{l_1 + l_2 \cos(\theta_2)}\right] + \tan^{-1}\left[\frac{d}{c}\right]$$