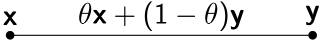
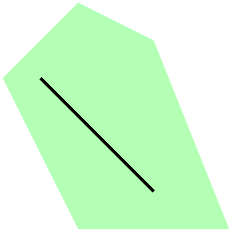
Convex Combination / Line Segment

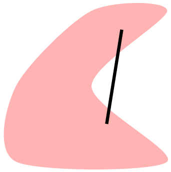


Convex Sets

Convex Set



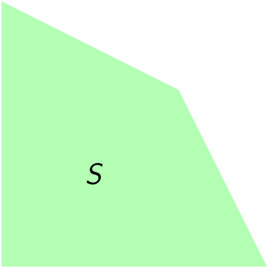
Non-Convex Set



Polytopes

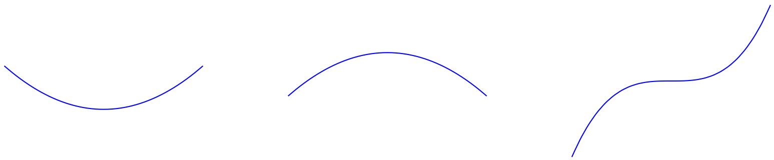
Consider the set

$$S = \left\{ \mathbf{x} : \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x} \preceq \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

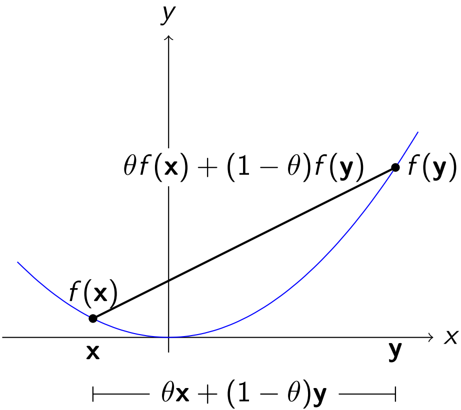


Convex Functions

Convex Function **Concave Function** **Neither**



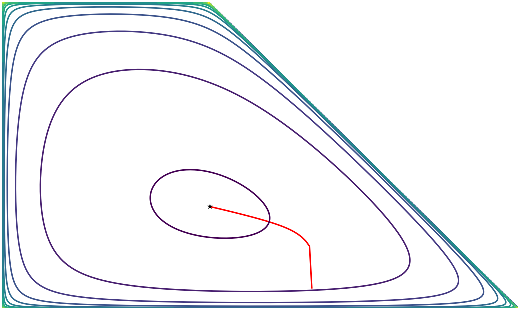
Convex Function



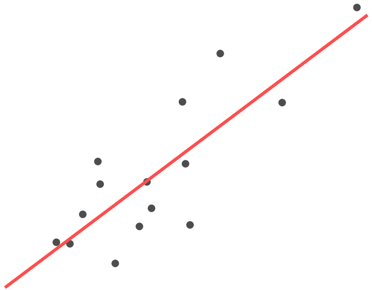
Example Problem - Analytic Centering

Want to find some sort of ‘center’ of a convex polygon (polytope) $A\mathbf{x} \preceq \mathbf{b}$:

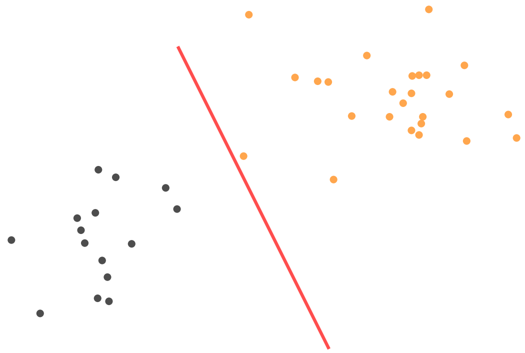
$$\text{minimize: } f_0(\mathbf{x}) = -\sum_i \log(\mathbf{b}_i - a_i^\top \mathbf{x})$$



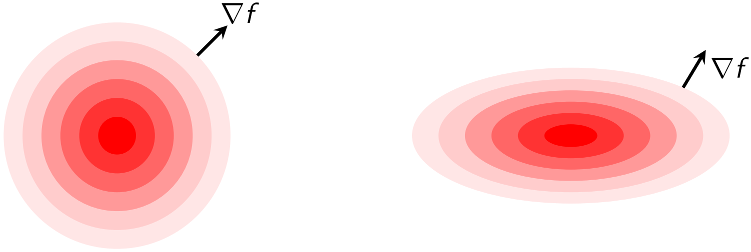
Example Problem - Least Squares Linear Regression



Example Problem - Logistic Regression



Gradient Descent



Gradient Descent

Gradient Descent

input : f, ∇f, $\eta(t)$, starting point $\mathbf{x}_0$, tolerance ϵ

output: optimal point $\mathbf{x}^\star$

$t \leftarrow 0$

while $||\nabla f||_2 \geq \epsilon$ **do**

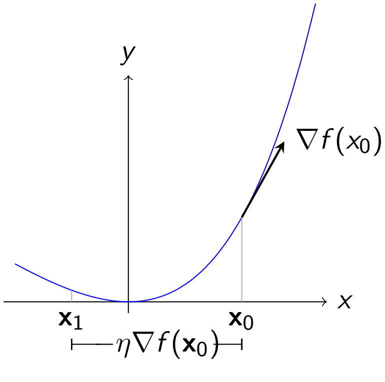
$\quad \mathbf{x}_{t+1} \leftarrow \mathbf{x}_t - \eta(t)\nabla f(\mathbf{x}_t)$

$\quad t \leftarrow t + 1$

end

return $\mathbf{x}^\star = \mathbf{x}_{t+1}$

Newton's Method



Newton's Method

Newton's Method

input : f, ∇f, $\nabla^2 f$, starting point $\mathbf{x}_0$, tolerance ϵ

output: optimal point $\mathbf{x}^\star$

$t \leftarrow 0$

while $||\nabla f||_2 \geq \epsilon$ **do**

  solve $\nabla^2 f(\mathbf{x}_t)\mathbf{d}_t = \nabla f(\mathbf{x}_t)$ for $\mathbf{d}_t$

  $\mathbf{x}_{t+1} \leftarrow \mathbf{x}_t - \mathbf{d}_t$

  $t \leftarrow t + 1$

end

return $\mathbf{x}^\star = \mathbf{x}_{t+1}$