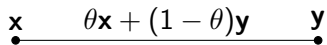


Convex Combination / Line Segment

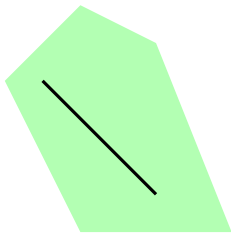


A horizontal line segment with two endpoints. The left endpoint is labeled \mathbf{x} and the right endpoint is labeled \mathbf{y} . Both labels are positioned above their respective dots. In the center of the line segment, the expression $\theta\mathbf{x} + (1 - \theta)\mathbf{y}$ is written.

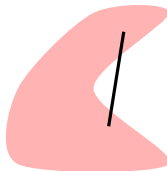
$$\mathbf{x} \quad \theta\mathbf{x} + (1 - \theta)\mathbf{y} \quad \mathbf{y}$$

Convex Sets

Convex Set



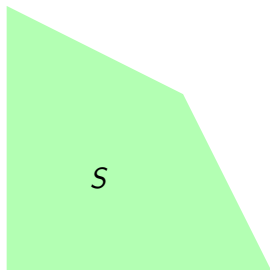
Non-Convex Set



Polytopes

Consider the set

$$S = \left\{ \mathbf{x} : \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x} \preceq \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$



Convex Functions

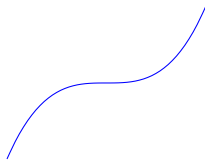
Convex Function



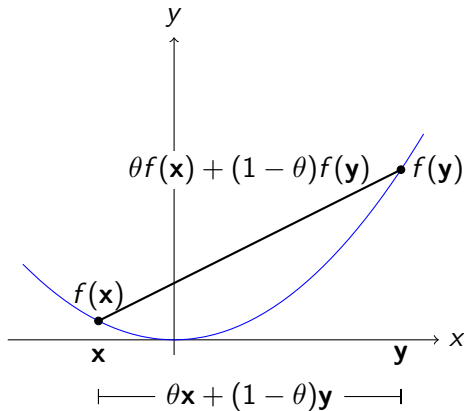
Concave Function



Neither



Convex Function

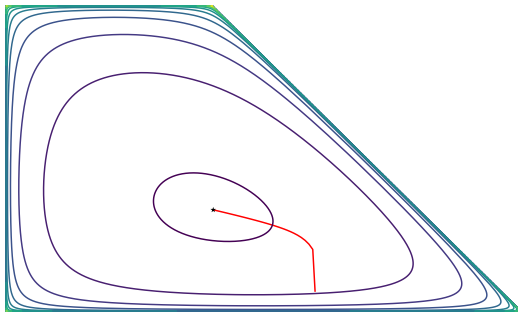


Example Problem - Analytic Centering

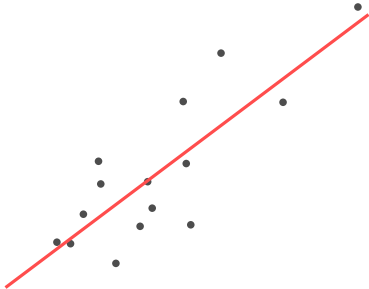
Want to find some sort of 'center' of a convex polygon (polytope)

$A\mathbf{x} \preceq \mathbf{b}$:

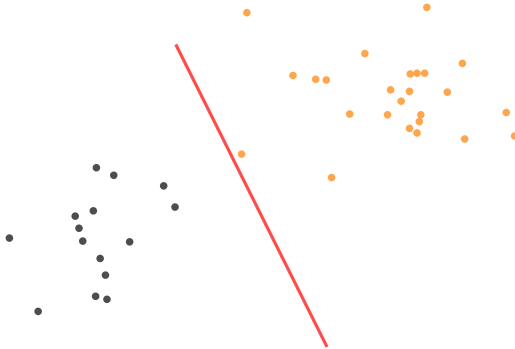
$$\text{minimize: } f_0(\mathbf{x}) = - \sum_i \log(\mathbf{b}_i - \mathbf{a}_i^\top \mathbf{x})$$



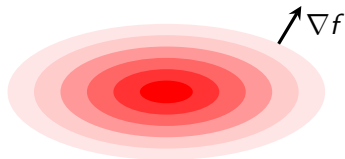
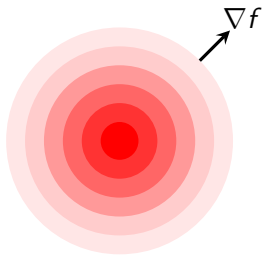
Example Problem - Least Squares Linear Regression



Example Problem - Logistic Regression



Gradient Descent



Gradient Descent

Gradient Descent

input : f , ∇f , $\eta(t)$, starting point \mathbf{x}_0 , tolerance ϵ

output: optimal point \mathbf{x}^*

$t \leftarrow 0$

while $\|\nabla f\|_2 \geq \epsilon$ **do**

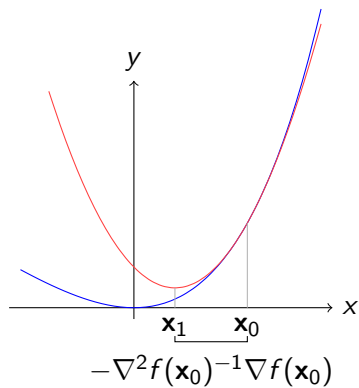
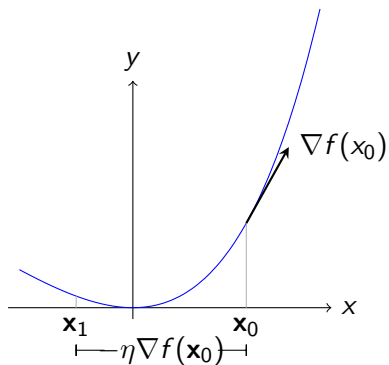
$\mathbf{x}_{t+1} \leftarrow \mathbf{x}_t - \eta(t)\nabla f(\mathbf{x}_t)$

$t \leftarrow t + 1$

end

return $\mathbf{x}^* = \mathbf{x}_{t+1}$

Newton's Method



Newton's Method

Newton's Method

input : $f, \nabla f, \nabla^2 f$, starting point \mathbf{x}_0 , tolerance ϵ

output: optimal point \mathbf{x}^*

$t \leftarrow 0$

while $\|\nabla f\|_2 \geq \epsilon$ **do**

 | solve $\nabla^2 f(\mathbf{x}_t)\mathbf{d}_t = -\nabla f(\mathbf{x}_t)$ for \mathbf{d}_t
 | $\mathbf{x}_{t+1} \leftarrow \mathbf{x}_t + \mathbf{d}_t$
 | $t \leftarrow t + 1$

end

return $\mathbf{x}^* = \mathbf{x}_{t+1}$