(Q1) Roll. no. → 180103097 H→9 I = 7

$L = 1m$ $T_L = 0°C$ $T_\infty° = 197°C$ $h = 19.7$

general sol. → $\frac{d^2T}{dx^2} = -\frac{A}{k}(T - T_\infty)$

$T - T_\infty = C_1 \cos\sqrt{A/k} + C_2 \sin\sqrt{\frac{A}{k}}\, k$

B.C. → LHS ⇒ $-T_\infty = C_1$

RHS → $-k\frac{\partial T}{\partial x}\Big|_L = \bar{h}_{RHS}(T_L - T_\infty)$

$T_\infty \sin\sqrt{\frac{A}{k}}L - C_2\cos\sqrt{\frac{A}{k}}L = \underbrace{\bar{h}_{RHS}\sqrt{\frac{k}{A}}}_{Bi_{eff}}\left(T_\infty\cos\sqrt{\frac{A}{k}}L + C_2\sin\sqrt{\frac{A}{k}}\right)$

$\Rightarrow C_2\left(Bi_e \sin\sqrt{\frac{A}{k}}L + \cos\sqrt{\frac{A}{k}}L\right) = Bi_e\, T_\infty \cos\sqrt{\frac{A}{k}}L - T_\infty\sin\sqrt{\frac{A}{k}}L)$

$C_2 = T_\infty\left[\frac{Bi_e\cos\sqrt{\frac{A}{k}}L - \sin\sqrt{\frac{A}{k}}L}{Bi_e\sin\sqrt{\frac{A}{k}}L + \cos\sqrt{\frac{A}{k}}L}\right]$

$\therefore \frac{T_\infty - T}{T_\infty} = \cos\sqrt{\frac{A}{k}}x - \frac{Bi_e\cos\sqrt{\frac{A}{k}}L - \sin\sqrt{\frac{A}{k}}L}{Bi_e\sin\sqrt{\frac{A}{k}}L + \cos\sqrt{\frac{A}{k}}L}\sin\sqrt{\frac{A}{k}}x$

$h = 19.7$ $T_\infty° = 197°C$ $L = 1m.$ $A = 10$

$Bi_e = \frac{\bar{h}_{RHS}}{\sqrt{kA}} = \frac{19.7}{\sqrt{10}} = 6.2297$

Assuming $K = 1$

$1 - \frac{T}{T_\infty} = \cos(\sqrt{10}x) - \frac{6.2297\cos\sqrt{10} - \sin\sqrt{10}}{6.2297\sin\sqrt{10} + \cos\sqrt{10}}\sin(\sqrt{10}x)$

$\frac{T}{197} = 1 - \cos(\sqrt{10}x) + \cancel{4.59}\ 5.5 \sin\sqrt{10}x$

At $x = 0.5$ m.

$T\big|_{x=L/2} = \cancel{25.025°C\big|_{x=L/2}}$ $\underline{1282.479°C}\big|_{x=L/2}$

At $x = 1 \Rightarrow$ $\cancel{T = 50.181°C\big|_{x=L}}$ $T = \underline{371.547°C}\big|_{x=L}$

(Q2)

Roll no. → 180103097

Assuming fin to be cylindrical, $\frac{x}{L} = \xi$

General soln for this problem is →

$$T - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

$$= C_1 e^{ML\xi} + C_2 e^{-ML\xi}$$

BC →
$$q_1 = -\frac{k}{L}\frac{d}{d\xi}(T - T_\infty)\Big|_{\xi=0}$$

$$q_2 = \frac{k}{L}\frac{d}{d\xi}(T - T_\infty)\Big|_{\xi=1}$$

$$\frac{q_1}{kM} = -C_1 + C_2 \quad \& \quad \frac{q_2}{kM} = C_1 e^{ML} - C_2 e^{-ML}$$

So, $C_2 = \frac{q_1}{kM} + C_1$ & $\frac{q_2}{kM} = -\frac{q_1}{kM} e^{-ML} + C_1 2\sinh ML$

$$\therefore C_1 = \frac{\frac{q_1}{kM}\left(e^{-ML} + \frac{q_2}{q_1}\right)}{2\sinh ML}$$

$$\frac{T - T_\infty}{q_1/kM} = \frac{\left(e^{-ML} + \frac{q_2}{q_1}\right) e^{ML\xi} + 2\sinh ML\, e^{-ML\xi} + \left(e^{-ML} + \frac{q_2}{q_1}\right) e^{-ML\xi}}{2\sinh ML}$$

$$= \frac{(q_2/q_1)\, 2\cosh ML\xi + e^{-ML(1-\xi)} + e^{ML(1-\xi)} - \cancel{e^{-ML(1+\xi)}} + \cancel{e^{-ML(1+\xi)}}}{2\sinh ML}$$

$$\therefore \frac{T - T_\infty}{q_1/kM} = \frac{q_2}{q_1}\frac{\cosh ML\xi}{\sinh ML} + \frac{\cosh ML(1-\xi)}{\sinh ML}$$

for $q_2 = 0 \Rightarrow \frac{T_1 - T_\infty}{q_1/kM} = \frac{\cosh ML(1-\xi)}{\sinh ML}$

$$\Rightarrow \frac{T - T_\infty}{q_1/kM} = e^{-ML\xi}$$

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Roll no → 180103097   H = 9 I = 7

(Q3) $k_s = 19.7 \; W/mK$ $d_i = 3\,cm.$ $t = 2\,mm.$ $t_i = 6\,mm.$ $k_i = .797\,W/mK$

$h_1 = 1597\; W/m^2K$ $h_2 = 297\,W/m^2K$

$T_i = 597K$ $T_o = 379K$

Heat transfer $= q \Rightarrow q = 2\pi L\left(\frac{.03}{2}\right)(1597)(579 - \cancel{T_{a1}}T_{s1})$

$q = 2\pi l(1.5 + .2 + .6)\times10^{-2}(297)(T_{a2} - 379)$

$q = -2\pi l(1.5 + x)\times10^{-2} k \frac{dT}{dx}$

Let $\frac{q}{2\pi l\times10^{-2}} = y$

$\frac{y}{k}\frac{dx}{(1.5+x)} = -dT$

for steel, $\frac{y}{19.7}\ln\left(\frac{1.7}{1.5}\right) = T_{s1} - T_{s2}$ [for steel] - ①

$\frac{y}{.797}\ln\left(\frac{2.3}{1.7}\right) = T_{a1} - T_{a2}$ [for asbestos] ②

Also, $\frac{y}{1.5\times1597} = \cancel{1597}\; 579 - T_{s1}$ - ③

$\frac{y}{2.3\times297} = T_{a2} - 379$ - ④

Also $T_{a1} = T_{s2}$ [same surface]

Adding ①, ②, ③ & ④

$y\left[\frac{\ln\left(\frac{1.7}{1.5}\right)}{19.7} + \frac{\ln\left(\frac{2.3}{1.7}\right)}{.797} + \frac{1}{1.5\times1597} + \frac{1}{2.3\times297}\right] = \cancel{T_{s1}} - \cancel{T_{s2}} + \cancel{T_{a1}} - \cancel{T_{a2}} + 579 - \cancel{T_{s1}} + \cancel{T_{a2}} - 379$

$y\,[.0635 + 1.7108 + .0004 + .0015] = 200$

$y = 112.5999$

Substituting in eq (3) →

$$\cancel{\frac{112.5999}{19.7}\ln\left(\frac{1.7}{1.5}\right)} = \frac{112.5999}{1.5\times1597} = 579 - T_{s1}$$

$$T_{s1} = \underline{578.97\,K}$$

Sub. in eq (4) →

$$\frac{112.5999}{2.3\times297} = T_{a2} - 379$$

$$T_{a2} = \underline{379.165\,{}^{\circ}K}$$

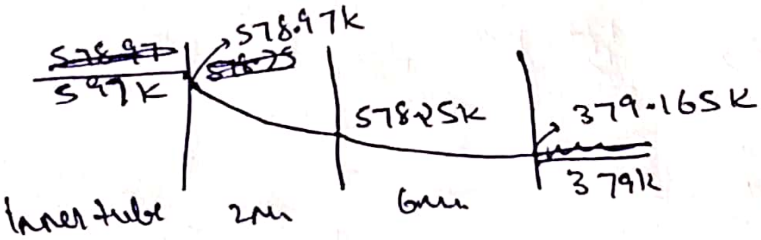
Sub. in eq (1) →

$$\frac{112.5999}{19.7}\ln\left(\frac{1.7}{1.5}\right) = 578.97 - T_{s2}$$

$$T_{s2} = T_{a1} = \underline{578.2546\,K}$$

$$\frac{q}{2\pi l\,4\times10^{-2}} = P$$

$$\left(\frac{q}{l}\right) = P\times2\pi\times10^{-2} = \underline{\underline{7.07}}\ W/m$$



(Q4) $H = 9$ $I = 7$

$L = .1m$

$k = 45\ W/m^2K$ $\rho = 800\ kg/m^3$ $c_p = 500\ J/kgK$

$T_i = 397°C$ $T_\infty = 779°C$

$h = 597\ W/m^2K$

Temp. min.:- Centerline temp. $T_o = 559°C$

$$\alpha = \frac{k}{\rho c_p} = \frac{45}{800 \times 500} = 1.125 \times 10^{-4}$$

$$Bi = \frac{hL}{k} = \frac{597 \times .05}{45} = .663$$

From Table, using linear interpolations

$$\frac{.663 - .6}{.7 - .6} = \frac{\lambda_1 - .7051}{.7506 - .7051} \Rightarrow \lambda_1 = 1.13$$

$$\frac{.663 - .6}{.7 - .6} = \frac{c_1 - 1.0814}{1.0919 - 1.0814}$$

$c_1 = 1.088$

$$Fo = \frac{\alpha t}{L^2}$$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = c_1 e^{-\lambda_1^2 Fo}$$

$$\frac{559 - 779}{397 - 779} = c_1 e^{-\lambda_1^2 Fo}$$

$Fo = .498 > .2$ (OK for app. solution)

$$t = \frac{Fo\, L^2}{\alpha} = \frac{.498 \times .05 \times .05}{1.125 \times 10^{-4}} = \underline{\underline{11.066\ sec.}}$$

(Q5) Roll no. → 180103097 H= 9 I=7

T_i = 4149.7 K ⟶ T_f = 0K $\alpha_{rock} = 1.197 \times 10^{-6}$

$\frac{\partial T}{\partial x} = 27°C/m$

$$q = k \left.\frac{\partial T}{\partial x}\right|_{x=0} = k(T_i - T_\infty)\,erf(\pi \alpha t)^{-1/2}$$

$$t = \frac{(T_i - T_\infty)^2}{(27)^2} \frac{1}{\pi \times 1.197 \times 10^{-6}} = \cancel{4.58152 \times 10^{12} \text{ sec}}$$

$= 6284.66 \times 10^{6}$ sec

$=$ 199 years

His estimates were wrong because $\frac{\partial T}{\partial x} = 27°C/m$ which is ~~very~~ ~~very~~ ~~greater~~ a huge number.

He also didn't consider radioactive internal heat generation due to radiation of material on surface.

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(Q6) Roll no → 180103097 $\quad n = 9$

$T|_{x=0} = 190°C \quad T|_{x=5} = 0°$

5m.

Steady state ⇒ $q_x = q_{x+dx}$

$$\frac{d}{dx}(q)\,dx = 0$$

$$\Rightarrow \frac{d}{dx}\left(-kA\frac{dT}{dx}\right) = 0$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} = 0$$

$T = C_1 x + C_2$

$x = 0 \Rightarrow T = C_2 \Rightarrow 190°C \; \cancel{463.15}$

$x = L \Rightarrow T = C_1 L + C_2 = 0 \Rightarrow C_1 = \cancel{-\frac{190}{L}} \; - \frac{\cancel{463.15}\;190}{5}$

$= -\cancel{92.63}\;38$

$T = -\cancel{92.63}\;38\,x + 463.15 \quad —(1)$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

Given ⇒ $\Delta x = 1m.$ $\quad \alpha = .25$

$\Delta t = 1s.$

$$\cancel{\frac{\partial T}{\partial x}} \quad \frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha\left(\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}\right)$$

$$\Rightarrow T_i^{n+1} - T_i^n = \alpha\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right)$$

$T_1^0 \quad T_2^0 \quad T_3^0 \quad T_4^0 \quad T_5^0 \quad T_6^0 \quad t = 0s$

$T_1^1 \quad T_2^1 \quad T_3^1 \quad T_4^1 \quad T_5^1 \quad T_6^1 \quad t = 1s$

$T_1^2 \quad T_2^2 \quad T_3^2 \quad T_4^2 \quad T_5^2 \quad T_6^2 \quad t = 2s$

$T_1^3 \quad T_2^3 \quad T_3^3 \quad T_4^3 \quad T_5^3 \quad T_6^3 \quad t = 3s$

Now, $T_1^n = 0°C = \cancel{273.15 K}$ (BC 1 node)

$T_6^n = 190°C = \cancel{463.15 K}$ (BC last node)

$T = 463.15 - 92.63 m$ $\quad T_2^0 = \cancel{372.52 K}$ $152°C$

$T_3^0 = \cancel{279.89 K}$ $114°C$ $\quad T_4^0 = \cancel{187.26}$ $76°C$ $\quad \cancel{T_2^0} = \quad T_5^0 = \cancel{94.63 K}$ $38°C$

Now, $T_i^{n+1} = \dfrac{T_{i+1}^n + 2T_i^n + T_{i-1}^n}{4}$

$\cancel{T_2^n \; T_2^1 = \dfrac{279.89 + 2 \times 372.52 + 273.15}{4}}$

$i =$		1	2	3	4	5	6
$t =$	0	0	152	114	76	38	190
	1	0	104.5	114	76	85.5	190
	2	0	80.75	102.175	87.875	109.25	190
	3	0	65.90	93.228	96.781	124.93	190