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Subject: EE447-High Voltage Engineering
End Semester Examination

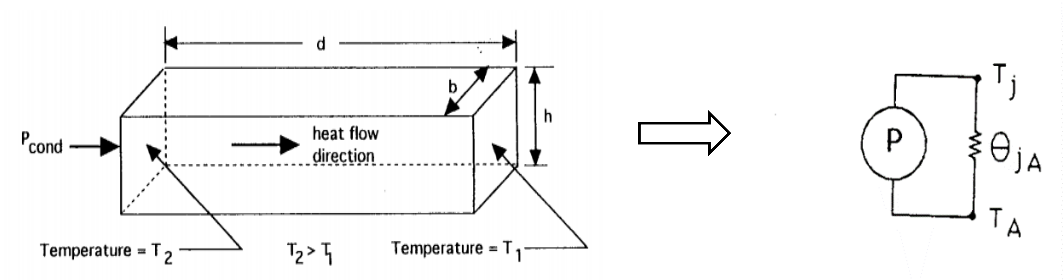
1.

<u>Considering dielectric material's thermal equivalent circuit using a DC Source:</u>

While using a DC Source, the thermal equivalent circuit of the dielectric under electrical stress will be in steady state always as the source always remains constant. Thus, the thermal equivalent circuit of the dielectric material stressesed electrically using a DC Source would only consist of the thermal resistance that is offered by the dielectric material. Also in the thermal equivalent circuit the quantity analogous to current in electical circuit will be the heat flow and the quantity analogous to potential difference in electical circuit will be the temperature difference. Thus, we can easily make the following equivalencies between thermal and electrical quantities:

Electrical	**Thermal**
Current	Heat Flow
Resistance	Thermal Resistance
Node Voltage	Temperature at a Spatial Location

Thus the thermal equivalent circuit under a DC source for a dielectric material will look something as shown below.



- Heat Flow $P_{cond} = \lambda(T_2\text{-}T_1)/d = (T_2\text{-}T_1)/R_{\theta cond}$
- Thermal Resistance $R_{\theta cond} = d/(\lambda A)$
- Here, Cross Section Area = hb.
- λ is the thermal conductivity of the dielectric material

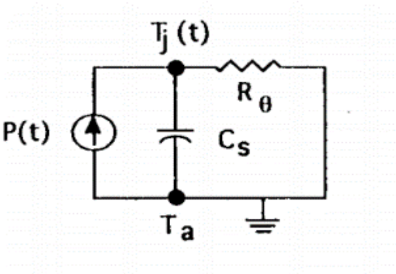
- A voltage source represented by ‘P’ Represents the temperature gradient
- θ_{jA} represents the thermal resistance
- T_A and T_J the temperatures of the nodes respectively in the equivalent circuit.

Fig: The above diagram shows generic heat flow via conduction in a dielectric material(left) and its corresponding thermal equivalent circuit(right).

<u>Considering dielectric material's thermal equivalent circuit under a Step Input Source:</u>

Since the input is now a function of time such that it provides fixed potential difference across the dielectric material after a fixed time, hence we see that the heat flow characteristics would follow a transient pattern for small period of time after the step input is applied. Thus, to consider these transient behaviours in the circuit the change that will be incurred in the thermal equivalent circuit is to use thermal impedance instead of the thermal resistance used in the previous case.

Since in a step input, voltage changes instantaneously, but due to this the temperature across the dielectric can't change instantaneously, hence to incorporate this feature in the thermal circuit we employ an electrical equivalent of the capacitor in the thermal equivalent circuit which doesn't allow instantaneous change of temperature across its sudden pulse of heat energy is applied. The modified equivalent thermal circuit that includes a heat capacity, which is analogous to the electrical capacitor is shown below:



- Heat capacity per unit volume (C_v=dQ/dt) of a material requires a finite amount of heat to be absorbed by a material before its temperature rises. With a step function of heat applied it results in a thermal time constant describing the temperature rise versus time.
- Also, here $C_s=C_v \times V$, where V is the volume of the dielectric material.
- Thus, the transient thermal impedance will be given as:
  - $Z_\theta(t) = (T_j(t) - T_a(t)) / P(t)$

2.

a.

Problem Statement:

The given problem states that cubic fillers of side length(**d_a =50nm**) are incorporated into a cube of dimensions (**D×D×D=2000×2000×2000 nm**) which is the host material. The fillers are incorporated to improve the electrical properties of the host material. These fillers are incorporated into the material after a layer of coating is applied on them which further has variable length depending on a variable 'k' such that the coating thickness will be given by **t_a =k× d_a** where k=0.5,1,1.5 or 2. Also the number of fillers incorporated depends on the concentration of the fillers incorporated and can be denoted by **$N=V_f*(D/d_a)^3$.** Thus, **V_f**,**D** , **d_a**, **t_a** , **N** denote the filler concentrations, host size, and filler particle size, coating thickness and number of fillers respectively. Since the fillers can be randomly distributed inside the host material, it would be impossible for us to find a generalized equation which could calculate the overlapped coating. Hence, in such a situation it will not be feasible to find an equation which can represent coating volume fraction as a function of the filler concentrations. *So to solve the given problem we employ the concept of* ***Monte Carlo Simulation****. MC Simulation is used as it helps is to simulate an environment for number of sumilations, which helps us to find an answer to the problem taking into consideration the randomness.*

Proposed Approach:

Since the the fillers can get distributed randomly inside the host material(cube), so in every simulation, for a fixed concentration of fillers, we randomly distribute the fillers inside the Host cube and calculate the coating volume fraction for that random distribution of the fillers in that particular simulation. *To calculate the coating overlap, we choose the filler cubes pairwise and calculate their corresponding overlapped volume of coating, such that no pair is repeated and possibility of overlap between every pair of fillers are considered. Thus, we perform the same simulation for number of times such that we would have values of coating volume fraction for a particular concentration of* *fillers and then the final coating volume fraction is said to be the average of all the coating volume fractions which are calculated for that concentration of fillers in the host material.*

The main challenge in this approach is to keep a track of the coating overlap in the random distribution of the fillers which would be required to calculate the coating volume fraction later. *To calculate the overlap of coating we form 3 cases which can be seen in the diagram below. The calculation required are shown in the below section*. On calculating the total overlapped volume between the fillers we subtrat it with the total volume of coating used and on dividing this value with the total host material volume we get the coating volume fraction. *The more number of time we simulate the environment for a particular concentration of filler particles, more accurate is the answer of the coating volume fraction. This is solely because on simulating more number of time we would have considered more posiblilties of randomly distributed fillers inside the host cube.*

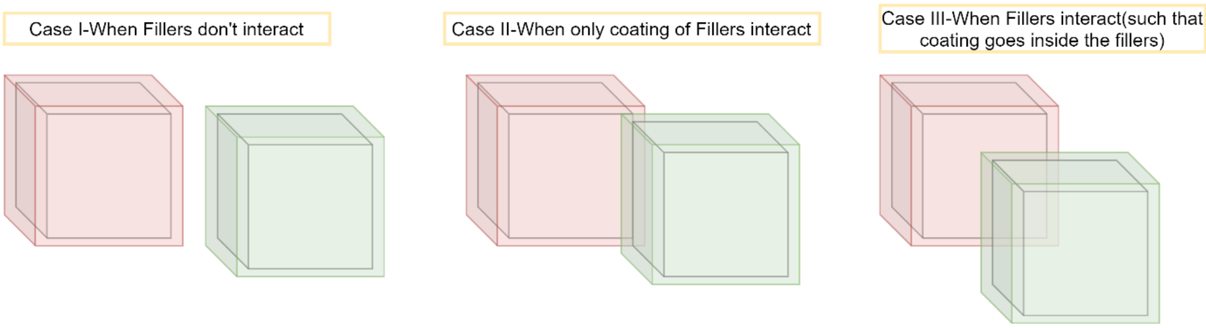


Fig: The above diagram shows all the possible interactions of fillers as we increase their concentration.

Calculations:

To randomly distribute the fillers inside the host cube we generate 3 arrays consisting random coordinates which lie in the closed interval [25,1975]. The first element of each of the 3 arrays defines the center of the first filler particle and so on for the rest. Now, since we have the coordinate of the centers of the filler particles we can calculate the distance between the centers in each of the 3 axis. This would help us in calculating the overlapped volume as shown below.

Consider two cubes in each of the three cases as shown in above figure. Also let,

dx= distance between x coordinates of the two cubes,
dy= distance between x coordinates of the two cubes,
dz= distance between x coordinates of the two cubes,
V=Overlapped volume between the two filler cubes interacting.

Now consider the cases:

Case- I:

Since the cubes are not overlaping hence overlapped volume is zero.
V=0

Case-II:

In this case only the coating on the two cubes overlap. Calculating the value of overlapped distances in x,y,z axis using simple geometry.

Overlap in X direction= Ox= $\mathbf{d_a + 2 \times t_a - dx}$
Overlap in Y direction= Oy= $\mathbf{d_a + 2 \times t_a - dy}$
Overlap in Z direction= Oz= $\mathbf{d_a + 2 \times t_a - dz}$

To calculate overlapped volume of coating we simply need to multiply above three overlapping lengths in each direction as only coating overlaps. Thuse, we get overlapped volume to be

$\mathbf{V = Ox \times Oy \times Oz}$

<u>Case-III:</u>

In this case a whole filler cube is inside another filler cube. Due to this there will be 4 parts of the overlapped volume i.e. the coating in the left and right sides of the intersection of the cubes as well as the top and bottom coating would have to be calculated. The calculations are shown as follows:

Overlap in X direction= Ox= $\mathbf{d_a + 2 \times t_a - dx}$
Overlap in Y direction= Oy= $\mathbf{d_a + 2 \times t_a - dy}$
Overlap in Z direction= Oz= $\mathbf{d_a + 2 \times t_a - dz}$

Calculating the overlapped volume of the left and the right surface of the intersection of the cubes portion:

$\mathbf{V_1 = t_a \times Ox \times Oz + t_a \times Ox \times Oz = 2 \times t_a \times Ox \times Oz}$

Similarly calculating the overlapped volume of the top and the botton surface of the intersection of the cubes portion:

$\mathbf{V_2 = t_a \times Oy \times Oz + t_a \times Oy \times Oz = 2 \times t_a \times Oy \times Oz}$

But in calculating the above we have added the overlapped volume of the coating at the corners also, so we need to subtract it. Thus, in this case the volume will look as:

$\mathbf{V = V_1 + V_2 - t_a \times t_a \times Oz}$

Now coating volume fraction is the ratio between the coating volume and the total volume of the host material. We know that the total coating volume of N particles will be given by

Total Coating Volume of all particles $\mathbf{V_{total} = N \times((d+2\times t_a)^3 - d^3)}$

Also Volume of Host material=$\mathbf{D^3}$
Now from the above calculations Final Coating Volume will reduce as:

$\mathbf{V_{final} = V_{total} - V}$ --------------------------------(here V_{total} and V are calculated above)

Thus, now coating volume fraction is written as:

Coating Volume Fraction = $\mathbf{V_{final} / D^3}$ **.**

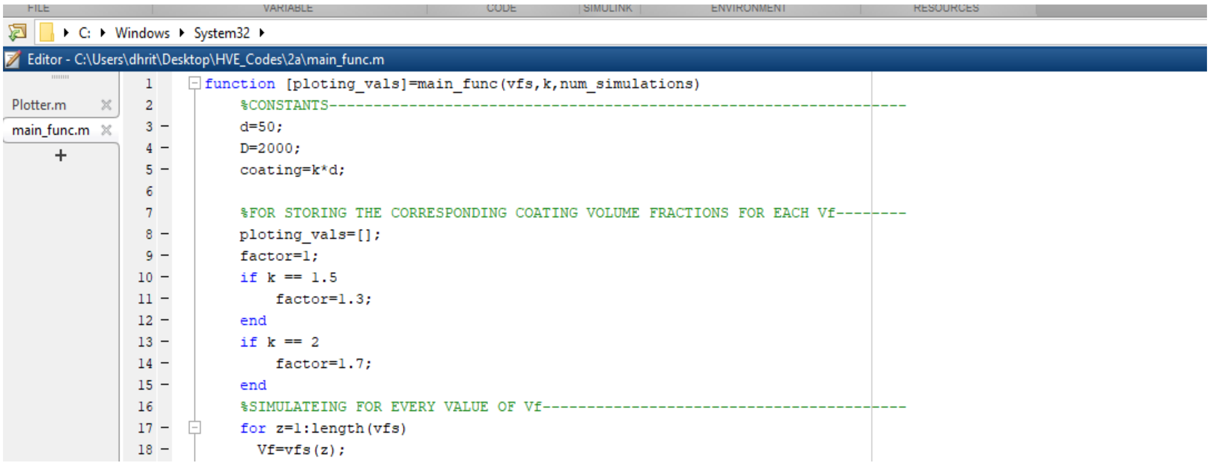
<u>Assumptions:</u>

- It is being assumed that the orientations of the filler cubes always remain along the 3-d coordinate axes. This assumption is made as if orientations of the cubes would be considered to be random then it would have led to complex calulations as overlapped shape between two cubles would have resulted a 3d shape finding whose area of overlap wasn't feasible.

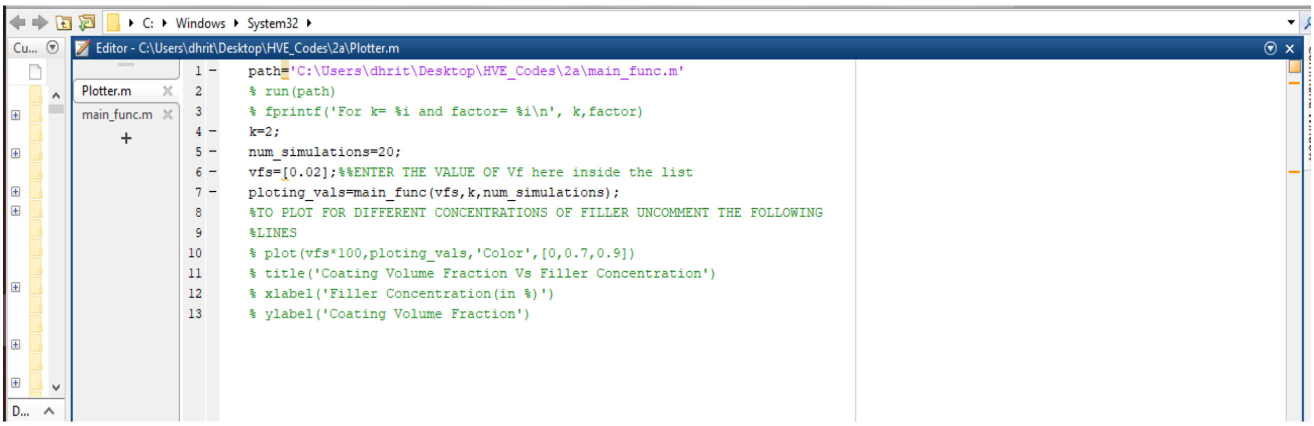
- Secondly, since in every simulation we consider a random distribution of the fillers, it might be possible that 3-4 or more cubes intersect majority of their volume at a common area. Due to this, the calculations considered in this approach would lead to excess overlapping volume deduction than actual overlapped volume to be deducted. Thus, we have considered a scaling factor for higher values of k only i.e., k=1.5 and 2 only, which downscales the overlapped volume so that it would be easier for us to visualize the plot or else we would have to plot negetive values of coating volume fraction.

Code:

The code is divided into two files. For proper error free working of the codes, *both the files should be kept in same directory* else the code will throw errors. The file named '**main_func.m**' contains a function which calculates the coating volume fraction for any given concentration. The input to this function is the filler concentration, the coating thickness scaling factor 'k' and the number of simulations that we want to run the code for as seen in the below image.



The file names 'plotter.m' is the file to be runned to get the desired value of coating fraction volume for given input concentration of fillers. As seen in the below image



To run the code,

- We first would require to write the path of the 'main_func.m' file in the first line. Then the input concentration of the filler concentration is to be given in the 6th line as written and then run the code.
- In the console we will see an output as shown in the last line of the below in the image.

```
Sim Number: 14  Actual Volume: 19840000000 Final Volume: 1.460900e+10  Overlapped Volume: 5.230999e+09 Coating Volume Fraction: 1.82
Sim Number: 15  Actual Volume: 19840000000 Final Volume: 1.464163e+10  Overlapped Volume: 5.198373e+09 Coating Volume Fraction: 1.83
Sim Number: 16  Actual Volume: 19840000000 Final Volume: 1.460650e+10  Overlapped Volume: 5.233503e+09 Coating Volume Fraction: 1.82
Sim Number: 17  Actual Volume: 19840000000 Final Volume: 1.442226e+10  Overlapped Volume: 5.417744e+09 Coating Volume Fraction: 1.80
Sim Number: 18  Actual Volume: 19840000000 Final Volume: 1.476789e+10  Overlapped Volume: 5.072112e+09 Coating Volume Fraction: 1.84
Sim Number: 19  Actual Volume: 19840000000 Final Volume: 1.458359e+10  Overlapped Volume: 5.256412e+09 Coating Volume Fraction: 1.82
Sim Number: 20  Actual Volume: 19840000000 Final Volume: 1.477510e+10  Overlapped Volume: 5.064903e+09 Coating Volume Fraction: 1.84
Final Coating Volume Fraction: 1.827978e+00 for Filler concentration: 2.000000e-02
>>
```

Also to plot the coating volume fraction for different values of concentrations we just need to uncomment the lines as mentioned in the code file and then run the code in plotter.m file to get desired plot.

Results and Inferences:

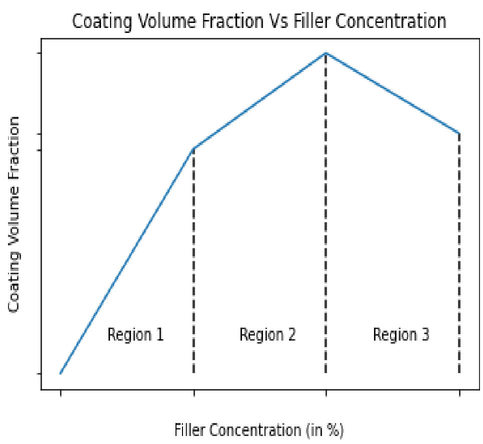


Fig 1: Above figure shows the ideal pattern of behaviour of Coating Volume Fraction with respect to filler concentration.

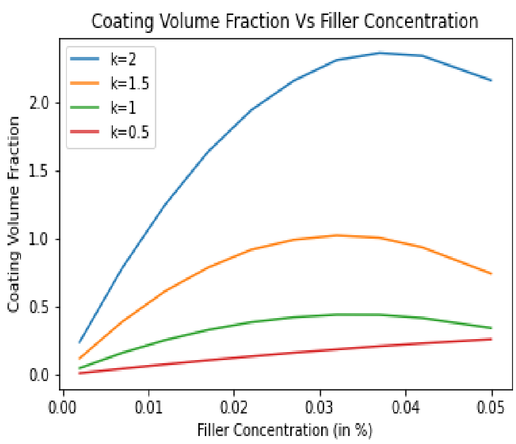


Fig 2: Above figure shows the plotted results of Coating Volume Fraction with respect to filler concentration for given filler particles as 'k' is varied.

The above **Fig: 2** shows the plotted values of Coating volume fraction for different values of k, with respect to the filler concentration. The graph can easily be explained. As we increase the concentration of the filler particles from very small value, due to their low concentration the overlapping of the coating volume is very small, hence the coating volume fraction increases gradually as in **region 1 of Fig: 1.** This can also be interpreted as Case I of the overlapping possibilities considered. Now as the filler concentration increases, their coating starts to overlap gradually leading to slowing the rate of increase of the coating volume fraction which could also be understood as **Case II** considered in the calculation section. This slowing in the rate of increase of coating volume fraction can be seen in **region 2 of Fig: 1**. Lastly, as we increase the concentration of the fillers further more the overlapped volume starts to increase due to the case considered as **Case III** in the calculation section. Hence, we see in **region 3 of Fig: 1** that the coating volume fraction decreases. The plotted graph in **Fig: 2** also depicts same behavior for higher values of k. Low value of k implies thinner coating hence, due to this the overlapping at such low concentrations of the fillers is not observed as interpreted from the graph. But as we increase the coating thikness by increasing the value of k, we see that even at such low concentrations all the three regions corresponding to the graph in **Fig: 1** can be observed. This is because on increasing the coating thickness we increase the chances of overlap also. Thus this explains the behavoir of the graph.

To plot the graph 20 simulations are considered for each concentration.

b.

Problem Statement:

The problem statement is based on the previous question and a **new filler of type 'B'** is introduced into the host material in order to improve the host materials thermal property. The size of the filler type 'B' depends on the size of filler type 'A' and is governed by the equation $\mathbf{d_b = f \times d_a}$,here f =0.5,1,1.5 or 2. Also the coating thickness of filler type 'B' was given by the equation $\mathbf{t_b = k \times d_b}$ where k=0.5,1,1.5 or 2. Lastly the concentration of filler type 'B' is given as a function of concentration of filler type 'A' as $\mathbf{V_{fb}=0.2 \times V_{fb}}$. Also, since concentration of both type of particles are different, let $\mathbf{N_a}$ be the number of fillers of type 'A' and $\mathbf{N_b}$ denote the number of fillers of type 'B'. We consider 0.2 as the scalling factor here, as it is the value by which the concentration of filler 'A' is scaled to get concnetration of filler type 'B'. The aim is to plot the coating fraction volume with respect to the concentration of filler type 'A'.

Proposed Approach:

The approach of this problem will be same as the approach i.e., to use **Monte Carlo Simulations** as done in the previous problem and that we will divide the overlapping between the coatings into 3 cases as done in the previous question, the only difference being that this time we would need to keep a track of the filler type which are interacting as while calculating the overlaps we will have to keep track of coating thickness as well as their dimensions. The pattern of calculation will remain same as we will again consider 3 cases as done above but this time keeping a track of the filler type which overlap. The calculations to be done are shown below. All notations are considered as done in above calculations.

Calculations:

We as done earlier randomly distribute both type of the fillers inside the host cube, so again we generate 3 arrays consisting random coordinates which lie in the closed interval [25,1975]. The first element of each of the 3 arrays defines the center of the first filler particle and so on for the rest. Now, since we have the coordinate of the centers of the filler particles we can calculate the distance between the centers in each of the 3 axis. This would help us in calculating the overlapped volume as shown below.

Consider two cubes in each of the three cases as shown in Fig. Again let,

dx= distance between x coordinates of the two cubes,
dy= distance between x coordinates of the two cubes,
dz= distance between x coordinates of the two cubes,
V=Overlapped volume between the two filler cubes interacting.
Also lets consider that one of the cube is of type X and the other of type Y. Here , type X or Y both can be any of type A or B which we have to keep a track while coding the solution. For the sake of

caalculations let the thickness and side length of filler type X be denoted as $\mathbf{t_x}$ and $\mathbf{d_x}$ respectively. Similarly for filler type 'Y' let it be $\mathbf{t_y}$ and $\mathbf{d_y}$ respectively. It should be kept in mind that $\mathbf{t_x}$ and $\mathbf{t_y}$ can take any of values of $\mathbf{t_a}$ or $\mathbf{t_b}$. And similarly $\mathbf{d_x}$ and $\mathbf{d_y}$ can take any of values of $\mathbf{d_a}$ or $\mathbf{d_b}$.

Now consider the cases:

<u>Case- I:</u>

Since the cubes are not overlaping hence overlapped volume is zero.
V=0

<u>Case-II:</u>

In this case only the coating on the two cubes overlap. Calculating the value of overlapped distances in x,y,z axis using simple geometry.

Overlap in X direction= $\mathbf{Ox= (d_a + d_b)/2 + t_a + t_b - dx}$
Overlap in Y direction= $\mathbf{Oy= (d_a + d_b)/2 + t_a + t_b - dy}$
Overlap in Z direction= $\mathbf{Oz= (d_a + d_b)/2 + t_a + t_b - dz}$

Again to calculate overlapped volume of coating we simply will need to multiply above three overlapping lengths in each direction as only coating overlaps. Thuse, we get overlapped volume to be

$\mathbf{V=Ox \times Oy \times Oz}$

Case-III:

In this case a whole filler cube is inside another filler cube. Due to this there will be 4 parts of the overlapped volume i.e. the coating in the left and right sides of the intersection of the cubes as well as the top and bottom coating would have to be calculated. The calculations

Overlap in X direction= $\mathbf{Ox= (d_a + d_b)/2 + t_a + t_b - dx}$
Overlap in Y direction= $\mathbf{Oy= (d_a + d_b)/2 + t_a + t_b - dy}$
Overlap in Z direction= $\mathbf{Oz= (d_a + d_b)/2 + t_a + t_b - dz}$

Calculating the overlapped volume of the left and the right surface of the intersection of the cubes portion:

$\mathbf{V_1 = t_b \times Ox \times Oz + t_b \times Ox \times Oz = 2 \times t_b \times Ox \times Oz}$

Similarly calculating the overlapped volume of the top and the botton surface of the intersection of the cubes portion:

$\mathbf{V_2 = t_a \times Oy \times Oz + t_a \times Oy \times Oz = 2 \times t_a \times Oy \times Oz}$

But in calculating the above we have added the overlapped volume of the coating at the corners also, so we need to subtract it. Thus, in this case the volume will look as:

$\mathbf{V = V_1 + V_2 - t_a \times t_a \times Oz - t_b \times t_b \times Oz}$

Now again since the coating volume fraction is the ratio between the coating volume and the total volume of the host material and we know that the total coating volume of N particles will be given by
Total Coating Volume of all particles $\mathbf{V_{total} = N_a \times((d_a+2\times t_a)^3 - d_a^3) + N_b \times((d_b+2\times t_b)^3 - d_b^3)}$

Also Volume of Host material=**D^3**

Now from the above calculations Final Coating Volume will reduce as:

$V_{final} = V_{total} – V$ -------------------------------(here V_{total} and V are calculated above)

Thus, now coating volume fraction is written as:

Coating Volume Fraction = **V_{final} / D^3** .

Assumptions:

Same assumptions are made as in above question.

Code:

The code is divided *into 3 files* namely '**calc_overlap.m**', '**main_func.m**' and '**plotter.m**'. **To get the plot of Coating fraction Volume Vs the concentrations of filler type 'A', we need to only run the 'plotter.m' file** whose snapshot is shown below.

```
Editor - C:\Users\dhrit\Desktop\HVE_Codes\2b\Plotter.m
calc_overlap.m | Plotter.m | main_func.m

path='C:\Users\dhrit\Desktop\HVE_Codes\2b\main_func.m'
% run(path)
% fprintf('For k= %i and factor= %i\n', k,factor)
k=2;
f=2;
num_simulations=30;
vfs=[0.002,0.007,0.015,0.02,0.025,0.03,0.035,0.04,0.045,0.05];
scalling_factor_of_concentration=[0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,2];
final=[];
for i=1:length(scalling_factor_of_concentration)
    fprintf('For SCALLING FACTOR = %i\n',scalling_factor_of_concentration(i) );
    ploting_vals=main_func(vfs,k,f,scalling_factor_of_concentration(i),num_simulations);
    plot(vfs*100,ploting_vals,'Color',[rand(1),rand(1),rand(1)]);
    final=[final; ploting_vals];
    hold on;
end
legend(num2str(0.2),num2str(0.4),num2str(0.6),num2str(0.8),num2str(1.0),num2str(1.2),num2str(1.4),num2str(1.6),num2str(1
title('Coating Volume Fraction Vs Filler Concentration')
```

It should be noted that all *these **3 files** should be kept in the same directory else it will throw error*.

- The **'main_func.m'** file contains a function which returns the coating volume fraction for a set of values of filler type 'A' concentrations.
- The '**calc_overlap.m**' file contains a function which calculates the overlap between any two cubes given their side lengths as well as their coating thicknesses.
- Thus, the '**plotter.m**' file is to be runned to get the desired plot.

Also, through the console the data of overlapped volume, total volume etc., can be kept track of as they are printed after every simulation is completed as shown in the folllowing image

```
hold o
end
legend(num
title('Coa
xlabel('Fi
ylabel('Co

Sim Number: 15  Actual Volume: 133920000000 Final Volume: 8.551665e+10  Overlapped Volume: 4.840335e+10 Coating Vc
Sim Number: 16  Actual Volume: 133920000000 Final Volume: 8.525646e+10  Overlapped Volume: 4.866354e+10 Coating Vc
Sim Number: 17  Actual Volume: 133920000000 Final Volume: 8.461974e+10  Overlapped Volume: 4.930026e+10 Coating Vc
Sim Number: 18  Actual Volume: 133920000000 Final Volume: 8.487634e+10  Overlapped Volume: 4.904366e+10 Coating Vc
Sim Number: 19  Actual Volume: 133920000000 Final Volume: 8.562295e+10  Overlapped Volume: 4.829705e+10 Coating Vc
Sim Number: 20  Actual Volume: 133920000000 Final Volume: 8.571769e+10  Overlapped Volume: 4.820231e+10 Coating Vc
Final Coating Volume Fraction: 1.068391e+01 for Filler concentration: 4.500000e-02
For Vf= 5.000000e-02 and and Vfb =1.000000e-01
Sim Number: 1  Actual Volume: 148800000000 Final Volume: 8.935957e+10  Overlapped Volume: 5.944043e+10 Coating Vol
Sim Number: 2  Actual Volume: 148800000000 Final Volume: 8.848169e+10  Overlapped Volume: 6.031831e+10 Coating Vol
Sim Number: 3  Actual Volume: 148800000000 Final Volume: 8.919254e+10  Overlapped Volume: 5.960746e+10 Coating Vol
Sim Number: 4  Actual Volume: 148800000000 Final Volume: 8.845940e+10  Overlapped Volume: 6.034060e+10 Coating Vol
```

Also the **hold on function** is used as can be seen from the image of code snippet of the 'plotter.m' file show above to plot all the plots together in the same plot for all the given scaling factors given in the question.

Results and Inferences:

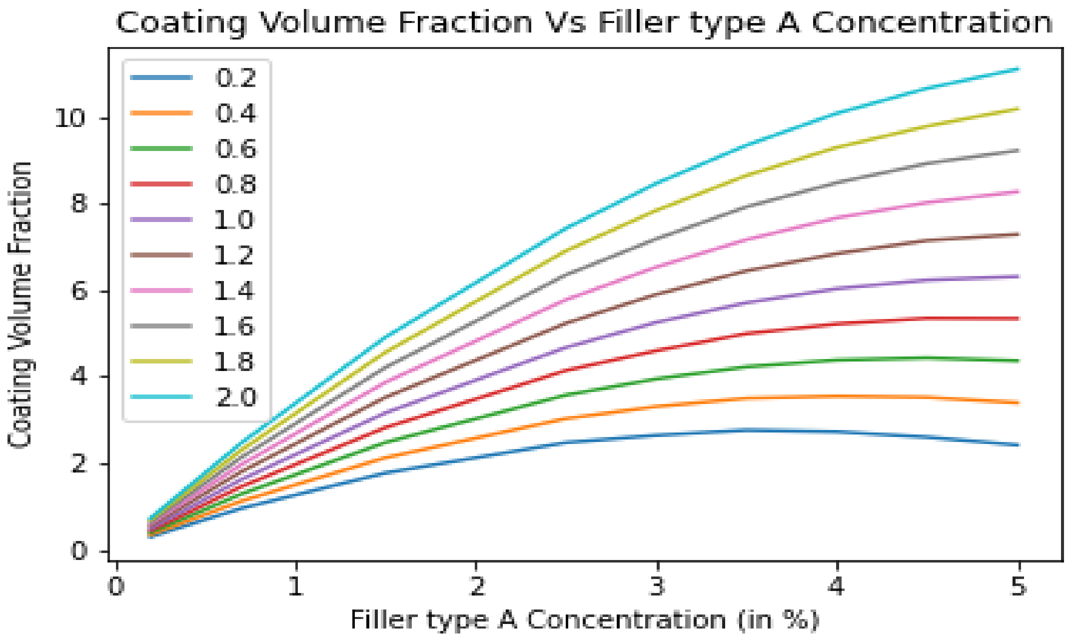


Fig 3: The above plot shows the variation of Coating Fraction Volume with respect to Filler type 'A' concentrations, along with the variations in scaling factors used to calculate the concentration of filler type 'B' as per the question. While plotting the above graph k=2 and f=2 is considered.

As from the above plot the behaviour of the coating volume fraction with respect to the concentration of filler type 'A' can be observed. Also, variation in scaling factor by which concentration of filler type 'B' depends can also be seen in the plot. We see that as we increase the concentration of filler type 'B' the coating volume fraction increases and also, we see that the decrease in coating volume fraction also vanishes. For lower scaling factor used to find concentration of filler type 'B' we see that the for such low concentrations between (0.2% to 5%) the coating volume fraction first increases and the decreases explaining the phenomenon described earlier in the previous question. Also, as the scaling factor is increased the region 3 which is visible for lower scaling factors starts to vanish and probably shifts towards right side of the curve i.e., for higher scaling factor values the coating volume fraction will decrease but for higher values of filler type 'A' concentrations which are more than 5%.

The above graph is plotted using value k=2, f=2 and for every simulation 30 simulations are performed to find the value of coating fraction volume.

References:

1. *Lecture 30: Simple Heat Flow Modelling and Sample Temperature Calculations. Link: https://www.engr.colostate.edu/ECE562/98lectures/l30.pdf*
2. *Thermo-electro-mechanical behaviour of dielectric elastomer actuators: experimental investigations, modelling and simulation, Smart Materials and Structure.*