A1

Notes Crypto Arithmetic Puzzle :→

Date | | |

Constraints :

1. There should be a unique digit to be replaced with a unique Alphabet
2. The result should satisfy the predefined arithmetic rules
3. Digits should be from (0 – 9)
4. There should be only one carry - forward while performing the addition operation of a Problem
5. The Problem can be solved from both sides (Left-hand-side) or (Right hand side)

Example :→

$$\begin{array}{r} SEND \\ +\ MORE \\ \hline MONEY \end{array}$$

Result → MONEY

⇒ Start from the L.H.S

$$\begin{array}{r} S \\ +\ M \\ \hline MO \end{array} \longrightarrow \begin{array}{r} 9 \\ 1 \\ \hline 10 \end{array}$$

Assign digits that can give a satisfactory Result.

∴ O = 0

⇒

$$\begin{array}{r} E \\ +\ O \\ \hline N \end{array} \longrightarrow \begin{array}{r} 4 \\ 0 \\ \hline 4 \end{array} \to \text{Constraint Violation}$$

$$\begin{array}{r} E \\ +\ O \\ \hline N \end{array} \longrightarrow \begin{array}{r} 1 \\ 4 \\ 0 \\ \hline 5 \end{array}$$

∴ S = 9, M = 1, O = 0, N = 5, E=4

$$\begin{array}{r} N \\ +R \\ \hline E \end{array} \rightarrow \begin{array}{r} 5 \\ 8 \\ \hline 13 \end{array} \times \rightarrow \text{Violation}$$

~~S = 9, R = 9~~ X

$$\Rightarrow \begin{array}{r} E \\ +O \\ \hline N \end{array} \rightarrow \begin{array}{r} 1 \\ 5 \\ 0 \\ \hline 6 \end{array}$$

1 → Carry

~~∴ S = 9, M = 1, O = 0, N = 6, E = 5~~

$$\begin{array}{r} N \\ +R \\ \hline E \end{array} \quad \begin{array}{r} N \\ +R \\ \hline E \end{array} \rightarrow \begin{array}{r} 5 \\ 8 \\ \hline 13 \end{array} \rightarrow \text{Violation} \quad E \neq 3$$

$$\begin{array}{r} 1 \\ 5 \\ 8 \\ \hline 14 \end{array}$$

1 → Carry.

∴ R = 8

$$\begin{array}{r} D \\ +E \\ \hline Y \end{array} \rightarrow \begin{array}{r} 7 \\ 4 \\ \hline 11 \end{array}$$

∴ S = 9, M = 1, O = 0, N = 5, E = 4, R = 8, D = 7, Y = 1

This assignments satisfies the constraints

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TWO
+ TWO
FOUR

F = 1   T = 5   F = 1, O = 0
T = 5
10
T = 6
T = 6
12

F = 1,
O = 2

3
3
6

{ T = 6, W = 3, U = 6
{ F = 1, O = 2, R = 4

O = 2
O = 2
4

TWO
+ TWO
FOUR

F = 1, T = 7, O = 4, R = 8,
W = 3, U = 6.

Example 2: TWO + TWO = FOUR

Solution 1 : T = 6, W = 3, U = 6, O = 2, F = 1, R = 4

Solution 2 : F = 1, T = 7, O = 4, R = 8, W = 3, U = 6

Types of constraints : Unary constraints
Binary constraints
Higher-order constraints

TWO
+ TWO
FOUR

Cryptoarithmetic Problem

Alldiff (F, T, U, W, R, O)

$O + O = R + 10 X_1$

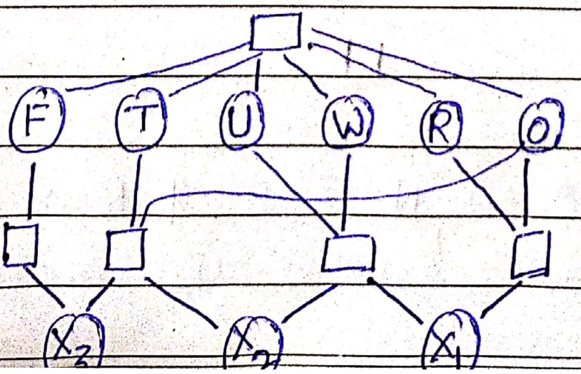
$X_1 + W + W = U + 10 . X_2$

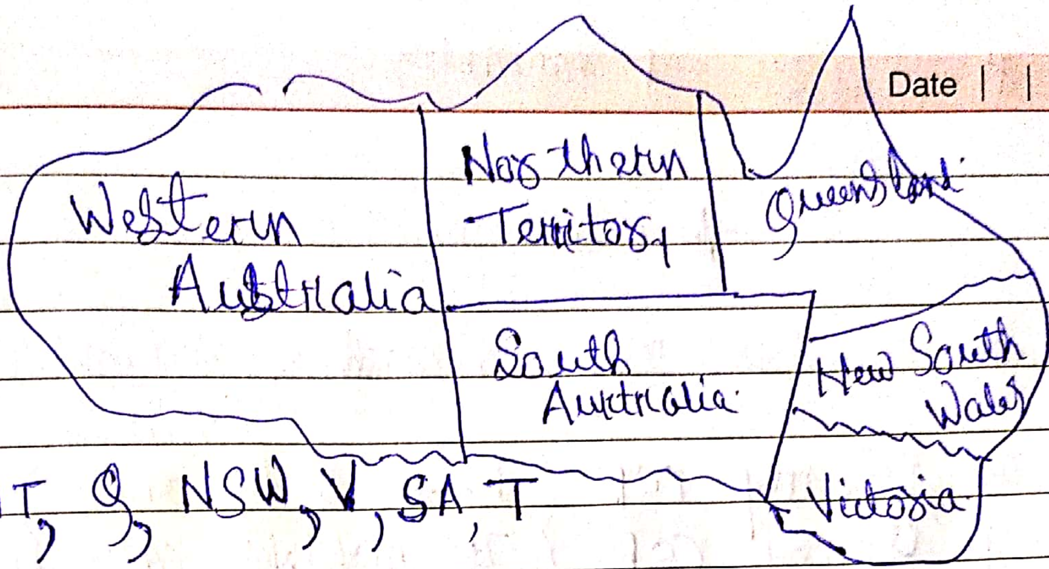
$X_2 + T + T = O + 10 . X_3$

$X_3 = F$

X_1, X_2, X_3
→ Auxiliary Variables (carried over into the next column)

Constraint Hypergraph



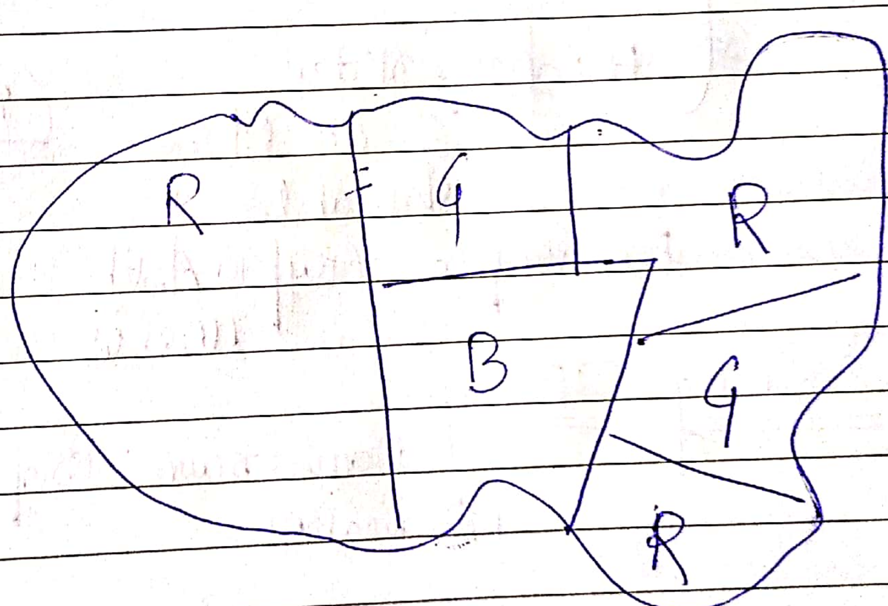


WA, NT, Q, NSW, V, SA, T

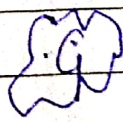
D_i = {Red, Green, Blue}.
R, G, B


Tasmania

(WA, NT)

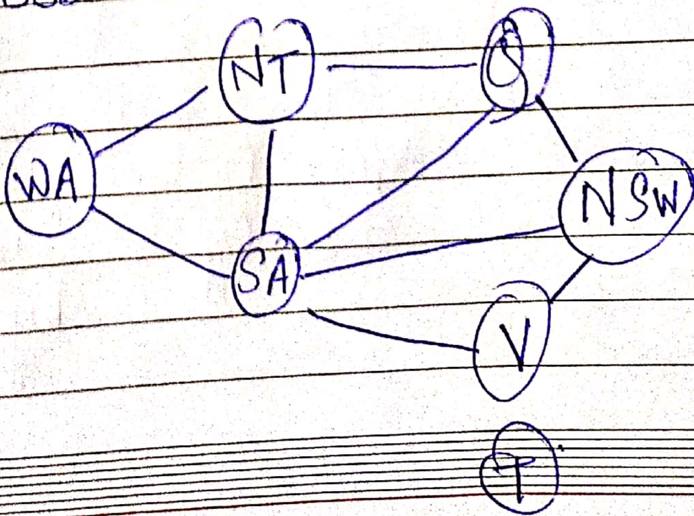


WA = Red.
NT = green
Q = Red.
NSW = green
V = Red,
SA = blue
T = green.



* State Space Search.
* Constraint Satisfaction Problem.

Constraint graph:



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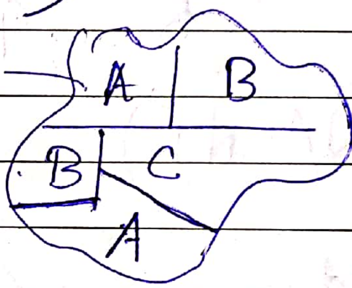
Notes* Models $\longrightarrow$ Variables.
arcs $\longrightarrow$ Constraints

* Standard representation : Variables
Values

SA ≠ WA. * Binary CSP (Relate 2 Variables)
Unary CSP (It involves only a single Variable)
B ≠ Green.



* Higher-order
3 - or More Variables

Example Crypto Arith metic

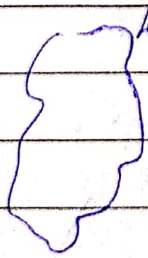
A ← Green

Solving CSP:

1. Constraint Propagation
2. Search

$V_i \longrightarrow V_j$, (V_i, V_j)

Di



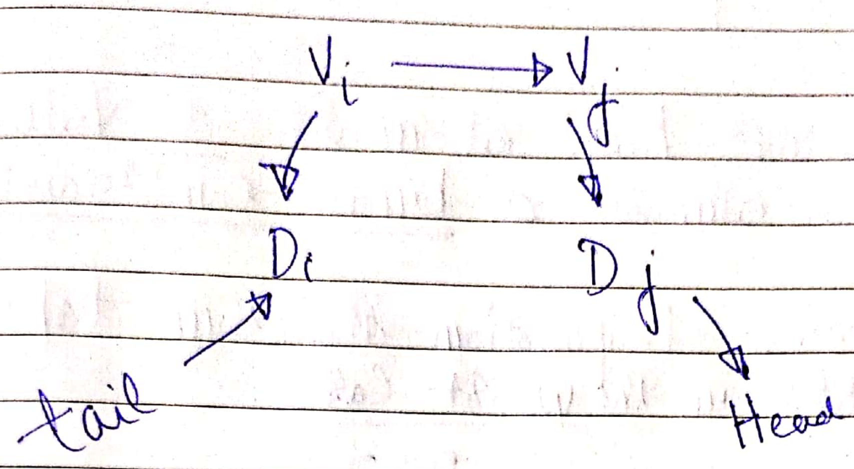
Dj

Arc Consistent

if

$\forall x \in D_i,$
$\exists y \in D_j$ } such that (x, y) is allowed by the constraint on the arc.

Notes Date



* Approach to solve the CSP — Constraint Propagation + Search.

↳ Constraint Propagation :→ Eliminate values that could not be part of any solution.

↳ Search : Explore the valid Assignments.

Points.

* Suppose there are some values in the domain at the tail of the constraint Arc (V_i) and do not have any consistent partner in the domain at the head of the arc (V_j). We achieve the arc consistency by dropping those values from D_i. Note that if we change D_i, we now have to check to make sure that any other constraint Arcs that have D_i at their head are still consistent.

↳ Constraint Propagation.

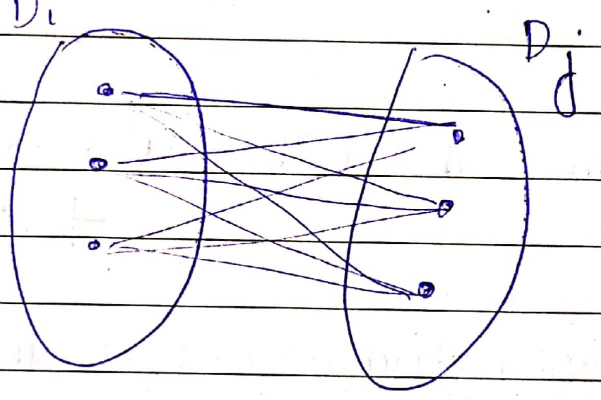
Notes

Date

Analysis :→

Domains have at-most 'd' values (Elements) and there are atmost 'e' binary constraints (arcs)

Constraint Propagation algorithm takes $O(ed^3)$ arc tests in the worst case.



$3 \times 3 = 3^2$

$O(d^2)$

If 'e' such arcs present, then once to check for all —

$O(ed^2)$

We have to go through and look at the arcs more than once as the deletions to a node's domain propagates.

$O(ed^3)$

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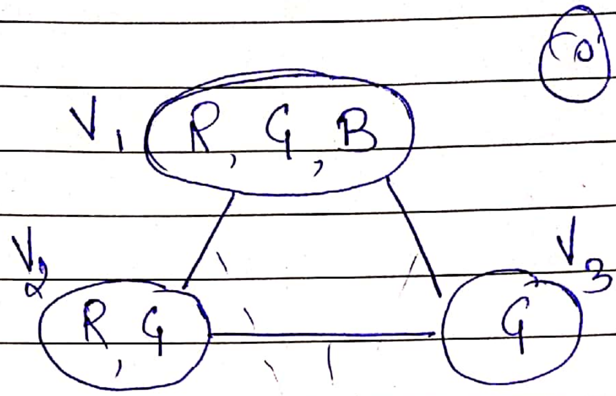
Example of Constraint Propagation

Graph coloring:

Initial domains are indicated. →



Different color constraint

Arc Examined	Value Deleted
$V_1 - V_2$	None
$V_1 - V_3$	V_1 (G)
$V_2 - V_3$	V_2 (G)
$V_1 - V_2$	V_1 (R)
$V_1 - V_3$	None
$V_2 - V_3$	Home.