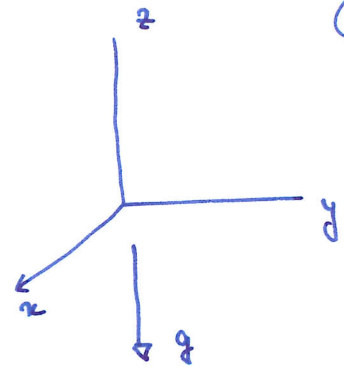
~~lectur~~

Lecture VI



6.1 Internal gravity waves:

waves in a stratified medium

stationary solution:

$$\frac{dp_0}{dz} = -\rho_0 g$$

$$v_0 = 0$$

$$p_0 = p_0(\rho)$$

$$c^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

is not a constant any more.

Linearized eqnr.

$$\rho = \rho_0 + \tilde{\rho}$$

$$v = v_0 + \tilde{v}$$

$$p = p_0 + \tilde{p},$$

$$\tilde{p} = \left.\frac{\partial p}{\partial \rho}\right|_s \tilde{\rho} = c^2(z)\,\tilde{\rho}$$

continuity eqn: $\partial_t \rho + \mathrm{div}(\rho v) = 0$

Linearized form: $\boxed{\partial_t \tilde{\rho} + \mathrm{div}(\rho_0 \tilde{v}) = 0}$

momentum eqn: $\partial_t(\rho v) + \mathrm{div}(\rho v_i v_j + p\,\delta_{ij}) = -\hat{z}\, g\rho$

linearized form $\partial_t(\rho_0 \tilde{v}) + \nabla \tilde{p} = -\hat{z}\, g\tilde{\rho}$

$$\Rightarrow \boxed{\partial_t(\rho_0 \tilde{v}) + \nabla(c^2 \tilde{\rho}) = -\hat{z}\, g\tilde{\rho}}$$

It is useful to consider the Lagrangian displacement. ξ

such that $\tilde{v} = D_t \xi = (\partial_t + \tilde{v}\cdot\nabla)\,\xi = \partial_t \xi$

Linearization

$$\Rightarrow \quad \partial_t \tilde{\rho} + \mathrm{div}(\rho_0 \partial_t \xi) = 0$$

with the condition $\tilde{\rho} = 0$, for $\xi = 0$

Integrating $\quad \partial_t \tilde{\rho} + \mathrm{div}(\rho_0 \xi) = 0$

substituting back in the linearized momentum eqn.

$$\rho_0 \partial_t^2 \xi - \nabla\left[c^2 \operatorname{div}(\rho_0 \xi)\right] = +\hat{z}\, g \operatorname{div}(\rho_0 \xi)$$

An eqn ~~quadratic~~ ~~in~~ containing ~~two~~ second order derivative of both space and time. The problem of understanding the solutions come from the inhomogeneity of the problem.

Expanding in three coordinate directions:

$$\rho_0 \partial_t^2 \xi_1 - \frac{\partial}{\partial x}\left(\rho_0 c^2 \operatorname{div}\xi\right) + \rho_0 g \frac{\partial \xi_3}{\partial x} = 0$$

$$\rho_0 \partial_t^2 \xi_2 - \frac{\partial}{\partial y}\left(\rho_0 c^2 \operatorname{div}\xi\right) + \rho_0 g \frac{\partial \xi_3}{\partial y} = 0$$

$$\rho_0 \partial_t^2 \xi_3 - \frac{\partial}{\partial z}\left(\rho_0 c^2 \operatorname{div}\xi\right) - \rho_0 g \left(\frac{\partial \xi_1}{\partial x} + \frac{\partial \xi_2}{\partial y}\right) = 0$$

where we have used $c^2 \frac{d\rho_0}{dz} = -\rho_0 g$

In general one can use

$$\xi = \tilde{\xi}(z) \exp i\left(k_1 x + k_2 y - \omega t\right)$$

substituting and simplifying one obtains an eqn. of the form:

$$\frac{d^2 \tilde{\xi}_3}{dz^2} + f(z) \frac{d\tilde{\xi}_3}{dz} + r(z)\tilde{\xi}_3 = 0$$

substitute $\tilde{\xi}_3 = h \exp\left[-\frac{1}{2}\int_0^z f(\sigma)\, d\sigma\right]$

Then one obtains:

$$\frac{d^2 h}{dz^2} + \gamma^2 h = 0 \qquad \gamma^2 = r - \frac{1}{4} f^2 - \frac{1}{2}\frac{df}{dz}$$

The solutions are oscillatory only if $\gamma^2 > 0$

Otherwise we have an ~~ust~~ unstable solution.

To see a simplified version ignore the variations along the y direction and consider the problem in 2-d, $x - z$ plane:

$$\rho_0 \ddot{\xi}_1 - \frac{\partial}{\partial x} \rho_0 c^2 \operatorname{div} \xi + \frac{\partial}{\partial x} \rho_0(z) g \xi_3 = 0$$

$$\operatorname{div} \xi = i k_1 \xi_1 + \frac{\partial \xi_3}{\partial z}$$

$$- \rho_0 \omega^2 \tilde{\xi}_1 - \rho_0(z) c^2(z) \, i k_1 \left(i k_1 \xi_1 + \frac{\partial \xi_3}{\partial z} \right) + \rho_0(z) g \, i k_1 \xi_3 = 0$$

$$\Rightarrow \left(- \rho_0 \omega^2 + \rho_0 c^2 k_1^2 \right) \xi_1 - i k_1 \rho_0 c^2 \frac{\partial \xi_3}{\partial z} + g \rho_0 i k_1 \xi_3 = 0$$

$$\Rightarrow \quad \xi_1 = \frac{i k_1}{\left(c^2 k_1^2 - \omega^2 \right)} \left(c^2 \frac{\partial \xi_3}{\partial z} + g \xi_3 \right)$$

From the z component:

$$\rho_0 \omega^2 \xi_3 - \frac{\partial}{\partial z} \left[\rho_0 c^2 \left(i k_1 \xi_1 + \frac{\partial \xi_3}{\partial z} \right) \right] - \rho_0 g \, i k_1 \xi_1 = 0$$

$$\Rightarrow \quad \frac{d^2 \xi_3}{d z^2} + f(z) \frac{d \xi_3}{d z} + r(z) \xi_3 = 0$$

with $$f(z) = \frac{d}{dz} \ln \left(\frac{\rho_0}{b^2} \right)$$

$$r(z) = b^2 - \frac{k_1^2}{\omega^2} g \frac{d}{dz} \ln \left(\frac{\rho_0}{b^2} \right) - \frac{k_1^2}{\omega^2} \frac{g^2}{c^2}$$

with $$b^2 = \frac{\omega^2}{c^2} - k_1^2$$

• ~~Now~~

• <u>Note 1</u> : at high frequencies, $\omega \to \infty$

$r(z) = b^2$ pure acoustic modes, effects of gravity is negligible.

• consider isothermal atmosphere :

$$\Rightarrow \quad \frac{dp_0}{dz} = -\rho_0 g \quad \Rightarrow \quad \frac{d\rho_0}{dz} = -\frac{\rho_0 g}{c^2} = -\frac{\rho_0}{H}$$

scale height

$$\Rightarrow \quad \rho_0(z) = \rho_{00} \exp(-Hz)$$

$$f(z) = \frac{d}{dz}(\ln \rho) \,, \qquad r = \frac{\omega^2}{c^2} - k_1^2 + k_1^2\left(\frac{N^2}{\omega^2}\right)$$

$$\text{where} \quad N^2 = -g\left[\frac{d}{dz}\ln \rho_0 + \frac{g}{c^2}\right]$$

– Brunt - Vaisalla frequency.

$$\gamma^2 = \frac{\omega^2}{c^2} - k_1^2 + \frac{k_1^2}{\omega^2}N^2 - \frac{1}{4}\left(\frac{d}{dz}\ln \rho\right)^2 - \frac{1}{2}\frac{d^2}{dz^2}\ln \rho$$

In general the dispersion relation is given by

$$\gamma = F(k_1, k_2, \omega)$$

• The waves are dispersive :

phase velocity $\frac{\omega}{k}$ is not a constant

$$v_g \equiv \text{group velocity} = \frac{\partial \omega}{\partial k}$$

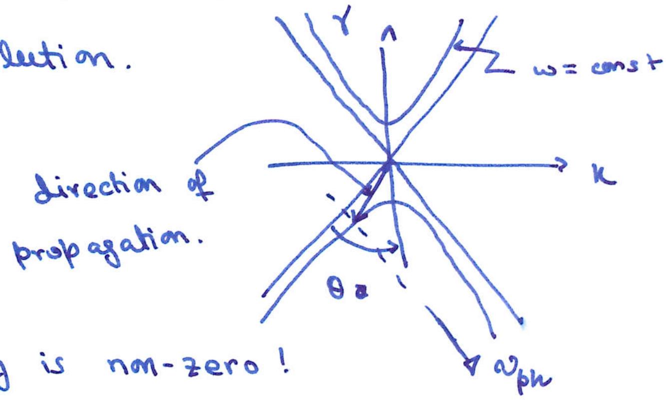
- The waves are anisotropic.

consider a surface in k_1, k_2, γ space over which ω is constant. These are called propagation surfaces.

e.g. $$\pm \frac{k^2}{a^2} \pm \frac{\gamma^2}{b^2} = 1 \qquad \text{with } k^2 = k_1^2 + k_2^2$$

These are either ellipsoids of revolution or hyperboloids of revolution.

$$\theta = \tan^{-1}\left(\frac{a}{b}\right)$$



- For this wave vorticity is non-zero!

- The ellipsoid case is similar to ~~wave~~ ~~for~~ propagation of electromagnetic waves in anisotropic crystals.

- The wave speed $\vec{v}_{ph} = \dfrac{\partial \omega}{\partial \vec{k}}$ is <u>not</u> along $\vec{k}$.

- comments on energy conservation in waves and a way of deducing the wave equation from a minimization principle.

- often a better intution about these waves can be obtained by assuming the $c(z)$ is a slow function z. slow compared to the wavelength

6.2 A flavour of Helioseismology :

Consider a spherical star. Let us ignore rotation and magnetic field. Consider a stationary state. with eqn. of state

$$p = p(\rho, T; X)$$

X — composition

We need not consider the exact eqn of state but assume that perturbations are adiabatic.

$$\gamma = \left.\frac{\partial \ln p}{\partial \ln \rho}\right|_s$$

and ideal gas

$$p = \frac{R \rho T}{\mu}$$

μ — mean molecular weight.

The ~~t~~ stationary state is provided by stellar evolution models. Those are not our concern. Assume that we know them.

The basic state satisfies (assuming spherical symmetry)

$$\frac{dp_0}{dr} + g_0 \rho_0 = 0 \quad , \quad g_0 = \frac{G m_0}{r^2}$$

$$m_0(r) = 4\pi \int_0^r \rho_0(s)\, s^2\, ds$$

These would be enough for us to start.

The question is : what are the waves that solves the linearized equations about this steady state ?

Linearized equations :

$$v = D_t \xi \simeq \partial_t \xi$$

$$\rho_0 \partial_t v = -\nabla p - g_0 \hat{r} \rho + \rho_0 \nabla \Phi$$

$$\nabla^2 \Phi = -4\pi G \rho$$

$$\rho + \mathrm{div}(\rho_0 \xi) = 0$$

$$\& \quad p = c_0^2 \rho$$

This is a new term compared to the case of internal gravity waves.

ρ, p, v, Φ, are perturbations.

$\rho_0, p_0, v_0 = 0, \Phi_0$ are the stationary state.

- consider only radial pulsations :

$$\xi = (\xi, 0, 0)\, r$$

merely convention.

Proceeding in a way very similar to the internal gravity waves, we obtain :

$$r \rho \ddot{\xi} + 4 \frac{dp_0}{dr} \xi - \frac{\partial}{\partial r}\left[\gamma p_0 \left(r \frac{\partial \xi}{\partial r} + 3\xi\right)\right] = 0$$

- Lagrangian and Eulerian perturations:

For any quantity $f_0(\vec{x})$

The Eulerian variation $f(\vec{x})$

The Lagrangian variation $f(\vec{x}+\vec{\xi}) \equiv \delta f$

$$f(\vec{x}+\vec{\xi}) = f(\vec{x}) + \vec{\xi} \cdot \frac{\partial f}{\partial \vec{x}}$$

$$\Rightarrow \quad \boxed{\delta f = f + (\vec{\xi} \cdot \vec{\nabla}) f}$$

The momentum eqn. in Lagrangian frame:

$$r \rho \ddot{\xi} = -\frac{\partial \delta p}{\partial r} + 4 g \rho_0 \xi$$

The continuity eqn in Lagrangian frame:

$$\frac{\delta \rho}{\rho_0} + \frac{1}{r^2} \frac{\partial}{\partial r}(r^3 \xi) = 0$$

- ~~Boundary conditions~~:
- Self-Adjoint form:

$$\mathcal{L} \xi = 0$$

where
$$\mathcal{L} \xi := \frac{d}{dr}\left(\gamma p r^4 \frac{d\xi}{dr}\right) + \left\{ r^3 \frac{d}{dr}\left[(3\gamma - 4) p\right] + r^4 \rho \omega^2 \right\} \xi = 0$$

In principle the problem of radial pulsations is now solved. We merely have to solve for the eigenfunctions of this self adjoint operator with appropriate boundary conditions.

- Boundary conditions :

- Regularity at $r = 0$.

$$\xi = r^a \sum_{k=0}^{\infty} A_k r^k$$

$r=0$ is a singular point of the equation. we can do a power-series expansion around it and eventually :

$$\frac{d\xi}{dr} = 0 \quad \text{at} \quad r = 0.$$

- Outer boundary condition can have several choices.

One choice : the corona adjusts itself instantaniously to a hydrostatic equilibrium.

$$4\pi r^2 p = g\, m_c$$

mass of the corona.

Linearize, use def^n of ξ, and the continuity eqn. to obtain

$$\gamma R \frac{d\xi}{dr} + (3\gamma - 4)\xi = 0$$

- The condition at $r = 0$ is a reflecting condition.

  The condition at $r = R$ is also reflecting.

  So modes can be confined in a star.

- <u>nomenclature</u> :

  The problem

  $$\mathcal{L}\,\xi = 0$$

  can now be solved for eigenfunctions of $\mathcal{L}$ give a ω. There are a discrete set of eigenfunctions ξ_n with eigenfrequencies ω_n. The eigenfunctions can be shown to be orthogonal.

  When organized with frequency ω_n, the smallest frequency ω_1, for $n = 1$ is called the fundamental.

- It can be shown that the frequencies have a lower limit (

$$\omega^2 \geq (3\gamma - 4)\frac{GM_\odot}{R^3}$$
$$= (3\gamma - 4)\,\omega_0^2$$

- ~~The wave equa~~
- ~~Perturb~~
- Approximate solution of the wave equation:

Uses the JWKB method:

$$\xi = \mathrm{Re}\left[A \exp i\lambda \int \psi \, dr\right] \qquad \lambda = \frac{\omega}{\omega_0}$$

A similar but more mathematically involved method applies to pulsations that are not solely radial. These are (~~misnome~~ called non-radial pulsation, which is a misnomar because they always have a radial component.

They obey a dispersion relation:

$$K^2 = \frac{\omega^2 - \omega_c^2}{c^2} - \frac{l(l+1)}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)$$

with

$$\omega_c = \frac{c^2}{4\mathcal{H}^2}\left(1 - 2\mathcal{H}'\right) - \frac{g}{h}$$

$$N^2 = g\left(\frac{1}{\mathcal{H}} - \frac{g}{c^2} - \frac{2}{h}\right)$$

$$\frac{1}{\mathcal{H}} = \frac{1}{H} + \frac{1}{H_p} + \frac{1}{h} + \frac{1}{r}$$

density scale height

$$\frac{1}{h} = \frac{1}{H_g} + \frac{2}{r}$$

gravity scale height

- small ω modes are determined by N the Brunt-Vaissala frequency. These are the g modes.

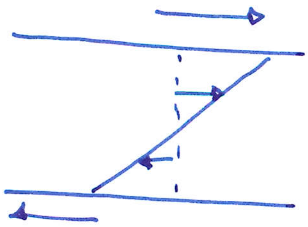
- Large ω modes are determined by sound waves dominated by pressure. These are the p modes.

- In reaching this conclusion the perturbation of the gravitational potential has been ignored. — cowling's approximation.

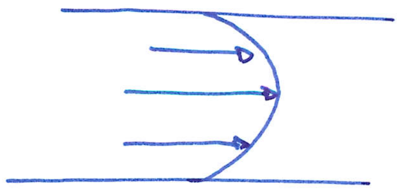
6.3 Instabilities in shear flows.

So far we have dealt with instabilities of flows for which the unperturbed state had zero velocity. The problem can become quite a bit more interesting if the ~~of~~ unperturbed state has shear : one component of velocity is a function of a different coordinate direction ~~to~~ $u_x(y)$. Such flows are very relevant.

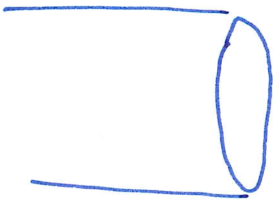
Some examples are :



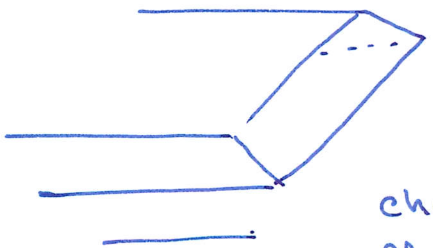
Plane Couette



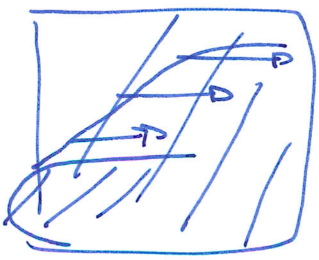
Plane Poiseuille



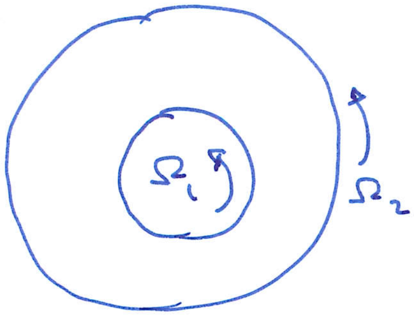
Pipe flow



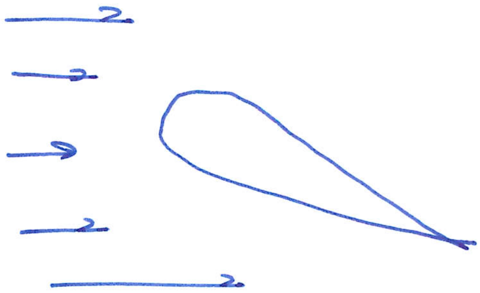
channel flow



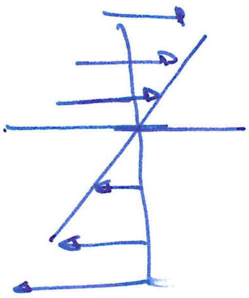
They can be considered in both viscous and inviscid formulation. Although the inviscid formulation can have fundamental problem with boundary conditions.



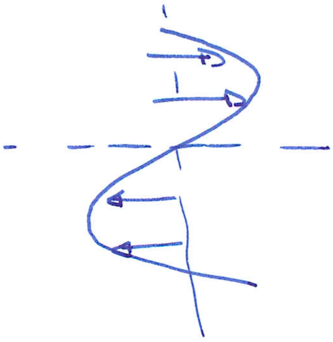
Taylor-couette flow



unbounded shear flows.



homogeneous shear flow



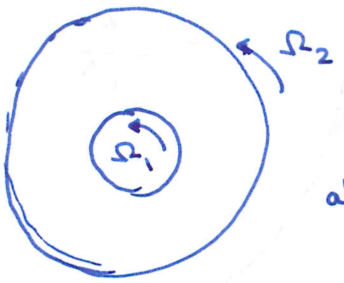
Kolmogorov flow.

And so on and so forth.

Interestingly Taylor-couette although may look quite complicated is in many ~~ma~~ ways one of the best to study through linear analysis. There is a kaleidoscope of possible behaviour. The simplest case is the following:

~~Let us consider the incompress~~

6.3.1 ~~6.4~~ Inviscid Taylor - Couette flow :



$$\Omega(r) = A + \frac{B}{r^2}$$

at $R = R_1$, $\Omega = \Omega_1$

$R = R_2$ $\Omega = \Omega_2$

$$\Rightarrow \quad A = -\Omega_1 \eta^2 \frac{1 - \mu/\eta^2}{1-\eta^2} \qquad \mu = \frac{\Omega_2}{\Omega_1}$$

$$B = \Omega_1 \frac{R_1^2(1-\mu)}{1-\eta^2} \qquad \eta = \frac{R_1}{R_2}$$

In the absence of viscosity; there ~~is~~ $\Omega(r)$ can be in general any function of r. What are the necessary and sufficient conditions for linear stability of the flow?

Answer : $\frac{d}{dr}(r^2\Omega)^2 > 0 \iff$ stability

If $(r^2\Omega)^2$ decreases with r anywhere in the domain $\Rightarrow$ instability.

$l \equiv$ angular momentum per unit mass $= r^2\Omega$

stratification of angular momentum is stable iff it increases monotonically outward.

$$\boxed{\text{Rayleigh Criterion}}$$

Argument :

consider only axisymmetric perturbations.

$$\partial_t u_\theta + u_r \frac{\partial u_\theta}{\partial r} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = 0$$

$$\Rightarrow \quad \cancel{\frac{d}{dt}} \quad D_t (r u_\theta) = 0 \qquad u_\theta = \Omega r$$

$$\Rightarrow \quad D_t (r^2 \Omega) = 0$$

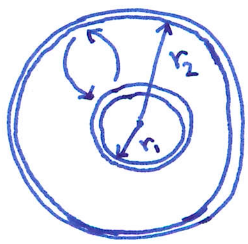
$\longrightarrow$ angular momentum is conserved.

The radial eqn.

$$\partial_t u_r + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} = \underbrace{\frac{u_\theta^2}{r}}_{(\ell^2/r^3)} - \frac{\partial}{\partial r}(p) \qquad \rho = 1$$

$$= - \frac{\partial}{\partial r}\left(\frac{\ell^2}{2r^2}\right)$$

an effective potential.



$$2\pi r_1 \, dr_1 = 2\pi r_2 \, dr_2$$ mass conservation.

The change in energy after the ~~ta~~ "exchange"

$$\left(\frac{\ell_2^2}{r_1^2} + \frac{\ell_1^2}{r_2^2}\right) - \left(\frac{\ell_1^2}{r_1^2} + \frac{\ell_2^2}{r_2^2}\right)$$

$$= \left(\ell_2^2 - \ell_1^2\right)\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \equiv \Delta E$$

For stability this change in energy must be negative

$r_2 > r_1$, $\Rightarrow$ $\Delta E < 0$ when $\ell_2^2 < \ell_1^2$

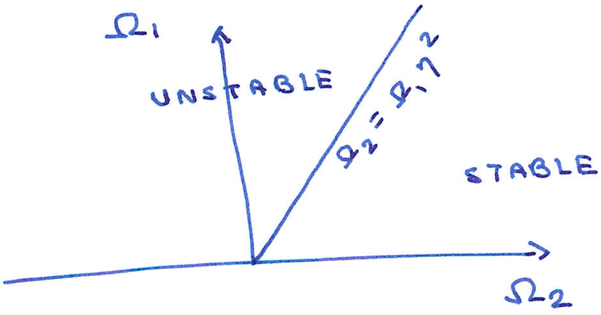
$\Rightarrow$ angular momentum should decrease outward everywhere in the domain.

- What does this imply if $\Omega(r)$ satisfies the viscous solution $\Omega(r) = A + \frac{B}{r}$?

$$\Phi(r) = \frac{1}{r^3}\frac{d}{dr}\left(r^2\Omega\right)^2$$

$$\Phi(r) > 0$$

$$\Rightarrow \quad \mu = \frac{\Omega_2}{\Omega_1} > \eta^2 \qquad \left(\eta = R_1/R_2\right)$$



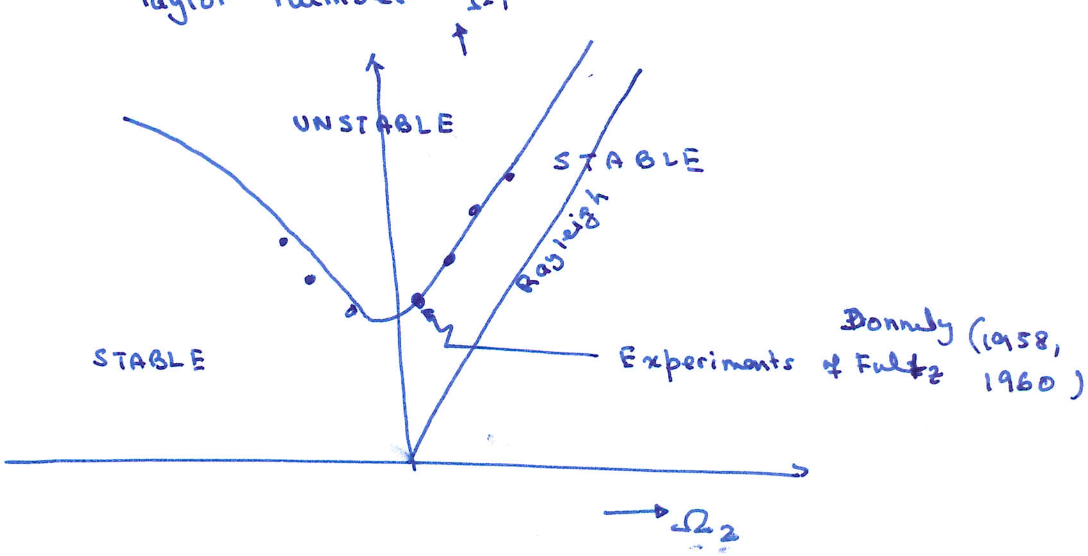
- Actual viscous calculation (starting from G I Taylor)

viscosity may postpone the onset of instability upto a critical value (always ?)

$$T = \frac{4\Omega_1^2 R_1^4}{\nu^2}\frac{(1-\mu)(1-\mu/\eta^2)}{(1-\eta^2)^2}$$

$\uparrow$

Taylor number

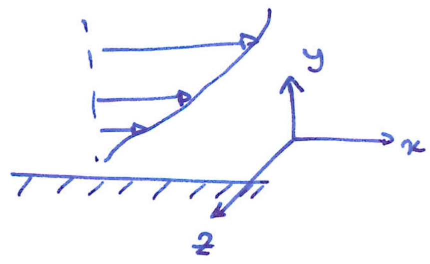


6.3.2

Orr - Sommerfeld equations

$$\mathbf{v} = \mathbf{U} + \mathbf{u}$$

$U = \hat{x}\, U(y)$



$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \frac{1}{Re}\nabla^2 \mathbf{u} - \nabla p$$

$$(\mathbf{U}\cdot\nabla)\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{U}$$

$$U(y)\partial_x \mathbf{u} \qquad \hat{x}\, u_y \partial_y U$$

$$\eta = \omega_y, \qquad v = u_y$$

$$\left(\partial_t + U\partial_x - \frac{1}{Re}\nabla^2\right)\eta = -\partial_z v \frac{dU}{dy} \qquad \text{Squire Eqn.}$$

$$\left(\partial_t + U\partial_x - \frac{1}{Re}\nabla^2\right)\nabla^2 v = \partial_x v \frac{d^2U}{dy^2}$$

A set of closed equations. Orr- Sommerfeld equations.

+ Boundary conditions.

no slip $\Rightarrow$ $\left.\begin{matrix} v = 0 \\ \omega = 0 \end{matrix}\right\}$ at the walls

In Fourier space : $v = \hat{v}(y)\, e^{\lambda t} e^{i(\alpha x + \gamma z)}$

$\eta = \hat{\eta}(y)\, e^{\lambda t} e^{i(\alpha x + \gamma z)}$

$$\left[\lambda + i\alpha U - \frac{1}{Re}(D^2 - k^2)\right]\hat{\eta} = -i\gamma\hat{v}\, U'$$

$$\left[\lambda + i\alpha U - \frac{1}{Re}(D^2 - k^2)\right](D^2 - k^2)\,\hat{v} = U'' i\alpha \hat{v}$$

$$D = \frac{d}{dy}, \qquad k^2 = \alpha^2 + \gamma^2$$

$$\left[\tilde{\lambda} + ikU - \frac{1}{Re'}(D^2-k^2)\right](D^2-k^2)\hat{v} - U''ik\hat{v} = 0$$

$$\tilde{\lambda} = \frac{\lambda k}{\alpha}, \quad Re' = \frac{Re\,\alpha}{k}$$

- Squire's theorem:

  A three dimensional perturbation (α, γ) at a fixed Re with growth rate λ is equivalent to a two-dimensional perturbation with wavenumber $(k, 0)$ but with $Re' = \frac{Re\,\alpha}{k} < Re$.

  with growth rate $\tilde{\lambda} = \frac{\lambda k}{\alpha} > \lambda$ [only real part matters]

  so for any 3D unstable mode, there exists a 2D unstable mode with a growth rate greater than the 3D one at a critical Re smaller than the 3D one. So, to find the first critical Re we need to look at only 2D perturbations.

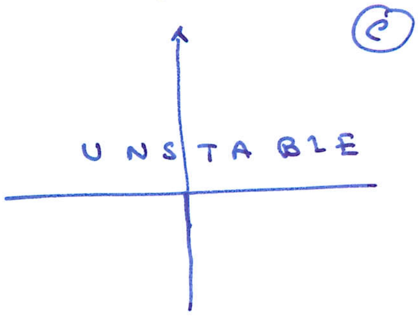
- The linear stability problem has now been reduced to finding eigenfunctions and eigenvalues of the Orr-Sommerfeld equation. <u>But this is not a self-adjoint operator</u>.

• Inviscid case: $\frac{1}{Re} = 0$

$$(U - c)(D^2 - \alpha^2) v - U'' v = 0$$ Rayleigh eqn.

with $\lambda = -i\alpha c$, $c = -\frac{\lambda}{i\alpha} = \frac{i\lambda}{\alpha}$

Instability can appear when $\Im m(c) > 0$



with Boundary condition $v = 0$
(only one boundary condition)

$$\int_{y_1}^{y_2} \left[v'' - \alpha^2 v - \frac{U'' v}{U - c_1 - i c_2} = 0 \right] v^* \, dy$$

$$\Rightarrow \int_{y_1}^{y_2} \left(|v'|^2 + k^2 |v|^2 \right) dy + \int_{y_1}^{y_2} \frac{(U - c_1 + i c_2) U'' |v|^2}{(U - c_1)^2 + c_2^2} dy = 0$$

If $c_2 \neq 0$

The imaginary part of this eqn. becomes

$$i \int_{y_1}^{y_2} \frac{c_2 U'' |v|^2}{(U - c_1)^2 + c_2^2} dy = 0$$

$\Rightarrow$ $U''(y)$ must change sign in the domain.

$\Rightarrow$ If $U''(y)$ is non-zero in the whole domain

we ~~cannot~~ ~~have~~ must have $c_2 = 0$,

~~complete~~. the imaginary part of c must be zero.

This may look like a convincing proof of stability of inviscid flows without <u>inflection points</u>. But the actual matter is somewhat more delicate.

For $\Im m(c) = 0$ we should get a neutral wave solution. It can be shown that for a plane-parallel flow the phase velocity of the ~~neutr~~ neutral wave c must be $U_{min} \leq c \leq U_{max}$

$\Rightarrow$ There will be at least one point in the domain with $U(y_0) = c$ where the Rayleigh eqn. will ~~becom~~ become singular.

What one should really do is to go back to the Orr-Sommerfeld equation and study its solutions for the limit $Re \to \infty$.

Summary of results

- Couette flow is lineary stable for all Re.
- Plane Poiseuille flow is lineary stable in the inviscid case. BUT is lineary unstable for the viscous case with an ~~ist~~ instability at $Re \sim 5772$.

  This counter intutive result was first shown by Heisenberg. (His PhD thesis)
- Pipe flow is belived to be lineary stable for all Re but not proven yet.
- In general linear stablity is a poor predictor of critical numbers where a flow becomes unstable.