Clustering of particles in inhomogeneous flows:

1. Introduction

The problem we want to address it is that of particle clustering for the case where the flow is NOT homogeneous.

Let us start with an analogy. If I release particles (inertial, heavy) in a homogeneous and isotropic turbulent flow, the ~~spee~~ speed of the particles follow a Maxwellian distribution:

$$\cancel{f(v) = \exp - v^2}$$

$$\boxed{f(v) = 4\pi\beta^{3/2} v^2 \exp\left[-\beta(St)\, v^2\right]} \quad -(1)$$

If we draw an analogy with ideal gas particles, then β plays the role of inverse (scaled) temperature

$$\beta = \frac{m}{2k_B T} \quad (2)$$

For the heavy inertial particles, β depends on the stokes number, $\beta(St)$.

Furthermore, β should also be a function of the fluid velocity; dimensionally speaking

$$\beta(St) = u_{rms}^2 \, g(St) \qquad (3) \quad (7)$$

If we ~~push th~~ push the analogy with temperature further we should observe the following:

If we bring into contact two bodies with different temperatures then heat flows from higher to lower temperature.

~~P~~ Continuing the analogy for heavy inertial particles: if we bring into contact two part of a simulation, with the same number density but different β there should be a flux of energy; carried by the particles, from lower β to higher β.

Interpreted this way turbophoresis should be determined by the flux carried by particles from lower u_{rms}^2 to higher u_{rms}^2.

There are two fluxes that one should consider:

$$J \equiv \text{flux of particles}$$

(4) $$\equiv n v$$ ~~(3)~~

$$J_q \equiv \text{flux of energy carried by the particles}$$

(5) $$= n v^2 v$$ ~~(4)~~

By the thermal analogy

(6) $$J_q = - \kappa \nabla \left(\frac{1}{\beta} \right)$$ ~~(5)~~

(7) $$\Rightarrow \quad J_q(st) = - g(st) \, \kappa \, \nabla \left(\frac{1}{u_{rms}^2} \right)$$ ~~(6)~~

Q1. Is this true? Can we measure this effect in a simulation? And also study the fluctuation of this flux?

The ~~main~~ main question is not really about this flux; but the flux of particles. The phenomenon of thermophoresis implies that due to the temperature gradient; there is not only a flux of energy but also a flux of particles. Phenomenologically the flux of particles may look like the following

$$\mathbf{J} = - \rho D \left(\nabla \ln \rho + K_T \nabla \ln T + K_p \nabla \ln p_h \right) \tag{8}$$

[Goldhirsch and Ronis, PRA 27 1616 1983]

where ρ is the density, and D, a diffusion coefficient.

By analogy let us write the flux of particles in a turbophoretic situation to be the following

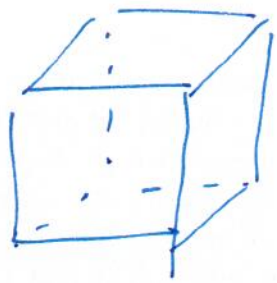
$$\mathbf{J} = - \varkappa_n \nabla n - \varkappa_u \nabla \left(\frac{1}{\beta} \right) \tag{9}$$

where both $\varkappa_n$ and $\varkappa_u$ are functions of Stokes number.

2. Model

Let us start with a very simple set up.

- periodic box.

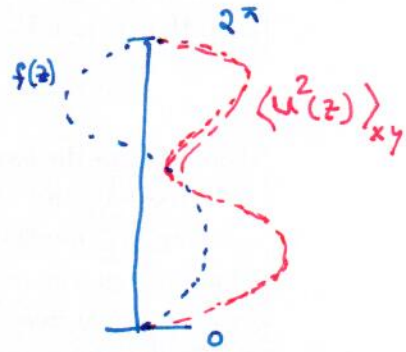


- forced turbulence (non-helical)

- The amplitude of the force varies as a periodic function of the z cordinate

$$f(x,y,z) = f_{NH}(x,y,k)\ \mathcal{F}(z)$$

usual forcing function in pencil-code



with $\quad \mathcal{F}(z) = \sin z$

Then we expect $\langle u^2\rangle_{xy}(z) \sim \sin^2 z$

The net flux of inertial particles will be:

$$J = -\varkappa_n \frac{\partial n}{\partial z} - \varkappa_u \frac{\partial}{\partial z}\left(\frac{1}{\beta}\right) \tag{10}$$

If the system reaches a stationary state then these two fluxes are going to cancel each other.

$$\Rightarrow \qquad \frac{\partial n}{\partial z} = - \frac{\varkappa_u}{\varkappa_n} \frac{\partial}{\partial z}\left(\frac{1}{\beta}\right)$$

$$(10) \qquad = - \frac{\varkappa_u(st)}{\varkappa_n} \frac{1}{g(st)} \frac{\partial}{\partial z}\left(\frac{1}{u_{rms}^2}\right)$$

$$= - \frac{\varkappa_u(st)}{\varkappa_n} \frac{1}{g(st)} u_{rms}^2 \frac{\partial}{\partial z}\left[\ln(u_{rms}^2)\right]$$

$$(11) \qquad \boxed{\frac{\partial n}{\partial z} = - \frac{\varkappa_u(st)}{g(st)} \left(\frac{u_{rms}^2}{\varkappa_n}\right) \frac{\partial}{\partial z}\left[\ln(u_{rms}^2)\right]}$$

$\Rightarrow$ Given the dependence of u_{rms}^2 and n as a function of z, and looking at the st dependence one should be able to figure out the ~~thermal~~ turbo-phoretic coefficient $\varkappa_u(st)$.