Ex 1

You have got most of the answers correct except the last one about energy conservation in MHD.
It is essentially a question of manipulating vector analysis.
Let me show you how it works in the simplest case.
We have:

$$\partial_t (\rho v) + \mathrm{div}(\rho v_i v_j + p\delta_{ij}) = J \times B$$

$$\partial_t B = \nabla \times (v \times B)$$

Multiply the first eqn. by v, the second by B and then add them:

$$\partial_t (\rho v^2 + B^2) + v\, \mathrm{div}(\rho v_i v_j + p\delta_{ij}) = v \cdot J \times B + B \cdot (\nabla \times v \times B)$$

The second term on the left was already shown to be a div( ) in class. So we only consider the two terms of the right:

$$v \cdot J \times B = v \cdot \left[(\nabla \times B) \times B \right]$$

$$= v \cdot \left[- \nabla \frac{B^2}{2} + (B \cdot \nabla) B \right]$$

put $A = B$ in (A.6) page 389 of PFP.

$$B \cdot \nabla \times (v \times B) = B \cdot \left[v(\underbrace{\nabla \cdot B}_{0}) - B(\underbrace{\nabla \cdot v}_{0}) - (v \cdot \nabla) B + (B \cdot \nabla) v \right]$$

(assume incompresibility)

Then

$$v\cdot(J\times B) = -\frac{1}{2}\, v_j \partial_j B_k B_k + v_j B_k \partial_k B_j$$

$$= -B_k v_j \partial_j B_k + v_j B_k \partial_k B_j$$

$$B\cdot \nabla\times(v\times B) = -B_j v_k \partial_k B_j + B_j B_k \partial_k v_j$$

$$\mathrm{SUM} = -\frac{1}{2}\, v_j \partial_j (B_k B_k) + B_k \partial_k (v_j B_j)$$

$$\cancel{\mathrm{SUM}} = -\frac{1}{2}\partial_j (v_j B_k B_k) + \frac{1}{2} B_k B_k \partial_j v_j \quad (=0,\ \text{by incompressibility})$$

$$+ \partial_k (B_k v_j B_j) - v_j B_j \partial_k B_k \quad (=0,\ \text{because } \nabla\cdot B = 0)$$

$$= \partial_j\left(-\frac{1}{2} v_j B_k B_k\right) + \partial_k (v_j B_j B_k) = \mathrm{div}[\quad]$$

Both div[ ] term

In the calculations above we have assumed incompressibility but it would have worked without it too.