Nordita Winter School 2017

PLANET GROWTH

UNIVERSITÉ CÔTE D'AZUR

Observatoire de la CÔTE d'AZUR

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PLANETESIMALS GROWTH

We assume that we have km sized bodies, so that the gravity is the dominant force. These bodies are called planetesimals.

ESCAPE VELOCITY

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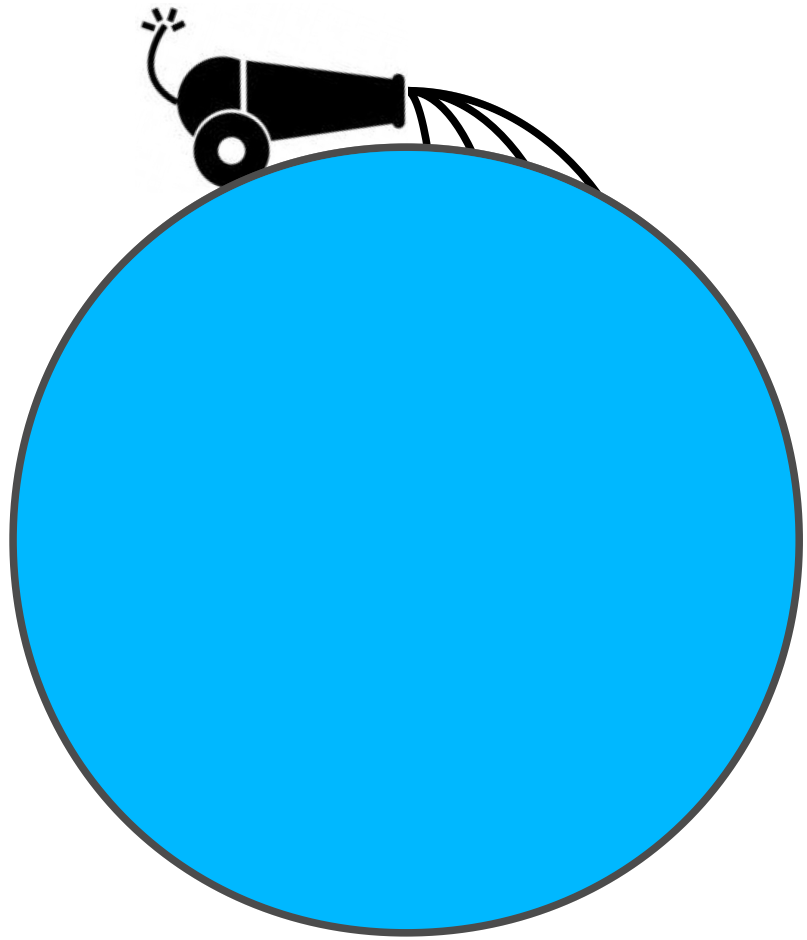
<u>Escape velocity :</u>

The stronger one throws an object, the further it goes.



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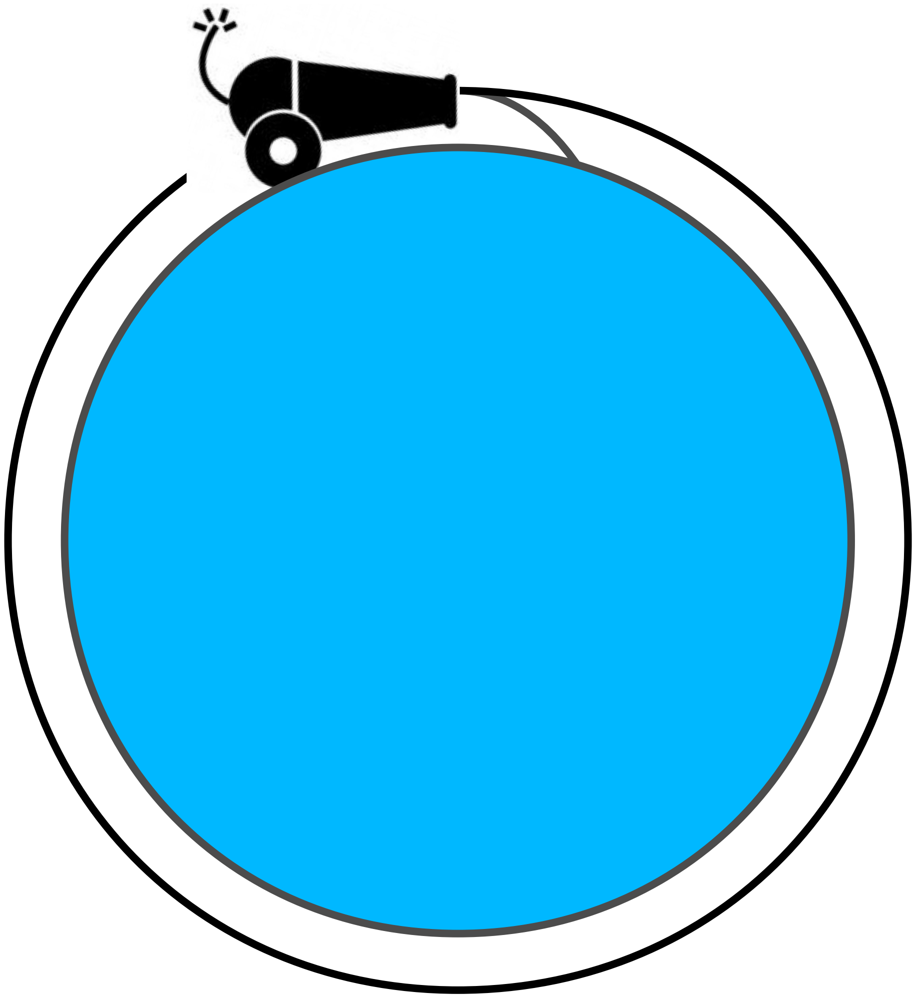
With a fast enough velocity, it orbits around the planet.



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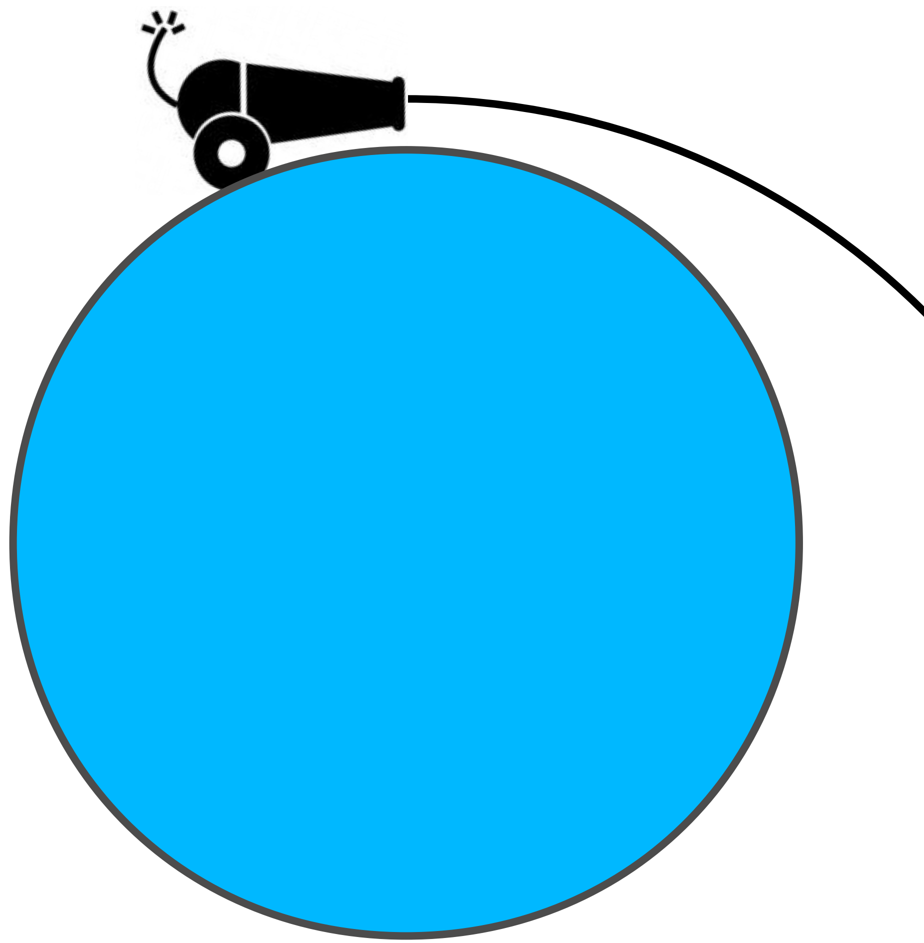
<u>Escape velocity :</u>

The stronger one throws an object, the further it goes.

With a fast enough velocity, it orbits around the planet.

If you throw fast enough, it never comes back...



ESCAPE VELOCITY

We assume that we have km sized bodies, so that the gravity is the dominant force. These bodies are called planetesimals.

<u>Escape velocity :</u>

velocity that a particle should have to escape from the surface of a planet of mass M_p and radius R_p.

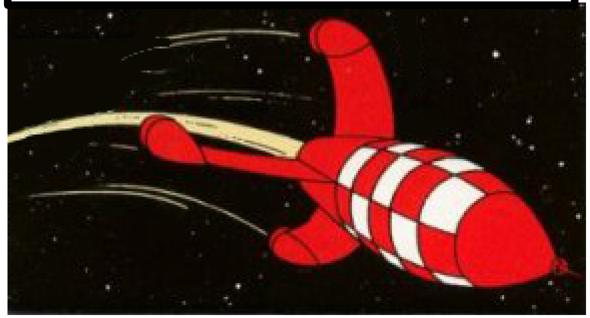
$E_{kin} + E_{pot} > 0$

$\frac{1}{2} m v^2 - GmM_p/R_p > 0$

$v > \sqrt{(2GM_p/R_p)} = v_{esc}$

Earth : $v_{esc} = 11$ km/s.

Allo, allo! Earth speaking! You just reached the escape speed, that is 13 km/s. You are therefore not subject to Earth's attraction anymore.

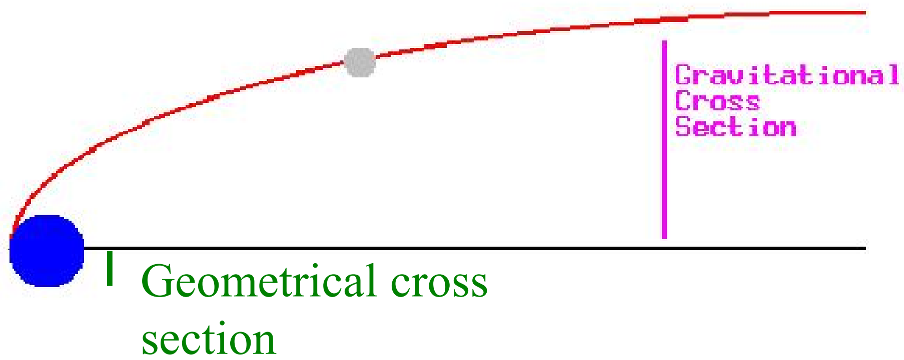


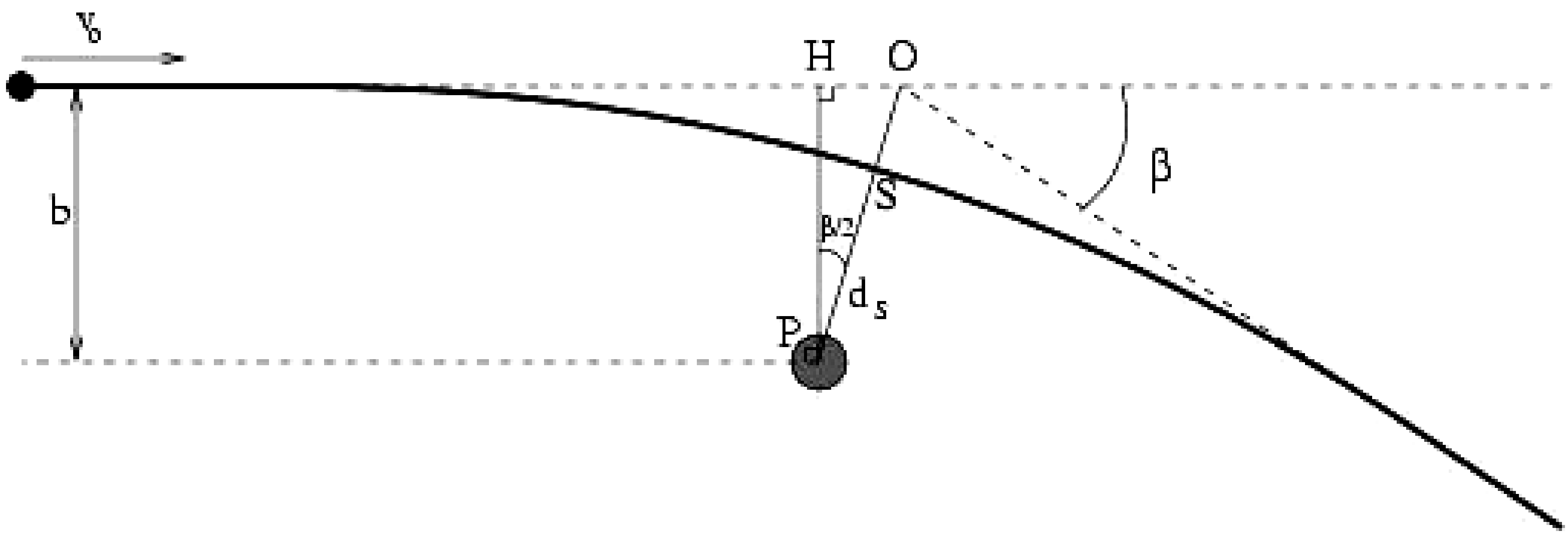
CROSS SECTIONS

<u>Geometric cross section :</u> $\sigma_{géom} = \pi R_p^2$

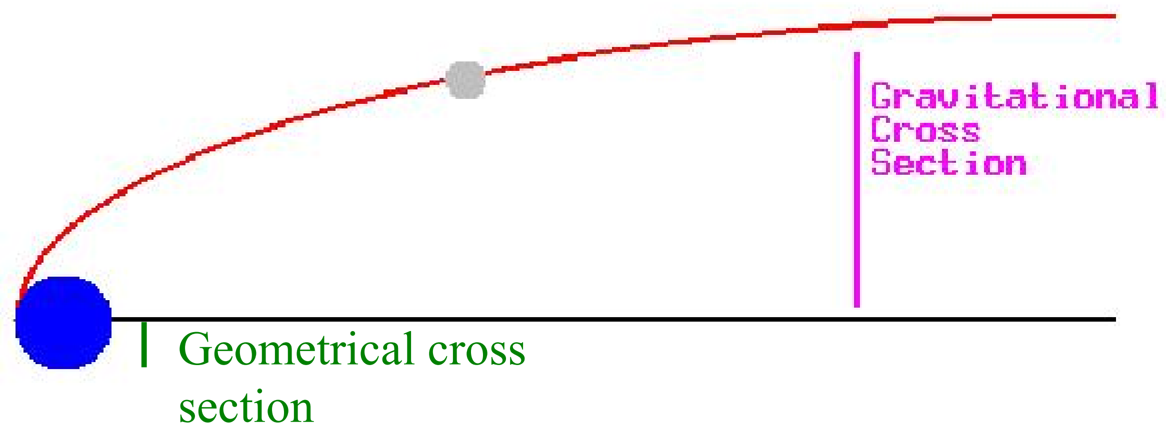
What is the gravitational cross-section ?

Which is the maximum impact parameter *b* , for a collision with the object of mass *M* and radius *R*, if the initial velocity is v_0 ?





CROSS SECTIONS



<u>Gravitational Focus :</u> $F_g = \sigma_{grav}/\sigma_{géom}$

$$= b_{max}^2/R_p^2$$

$$\mathbf{F_g = 1 + (v_{esc}/v_0)^2} \,,$$

$$\text{where} \quad v_{esc}^2 = 2GM_p/R_p.$$

<u>Growth :</u> $dM/dt \sim R_p^2\, F_g \sim M_p^{2/3}\, F_g$

RUNAWAY GROWTH

1) Case $v_0 << v_{esc}$ (dynamically cold disc).

Then, $F_g \sim v_{esc}^2 / v_0^2 \sim M^{2/3} / v_0^2$.

Take two objects of masses $M_1 > M_2$.

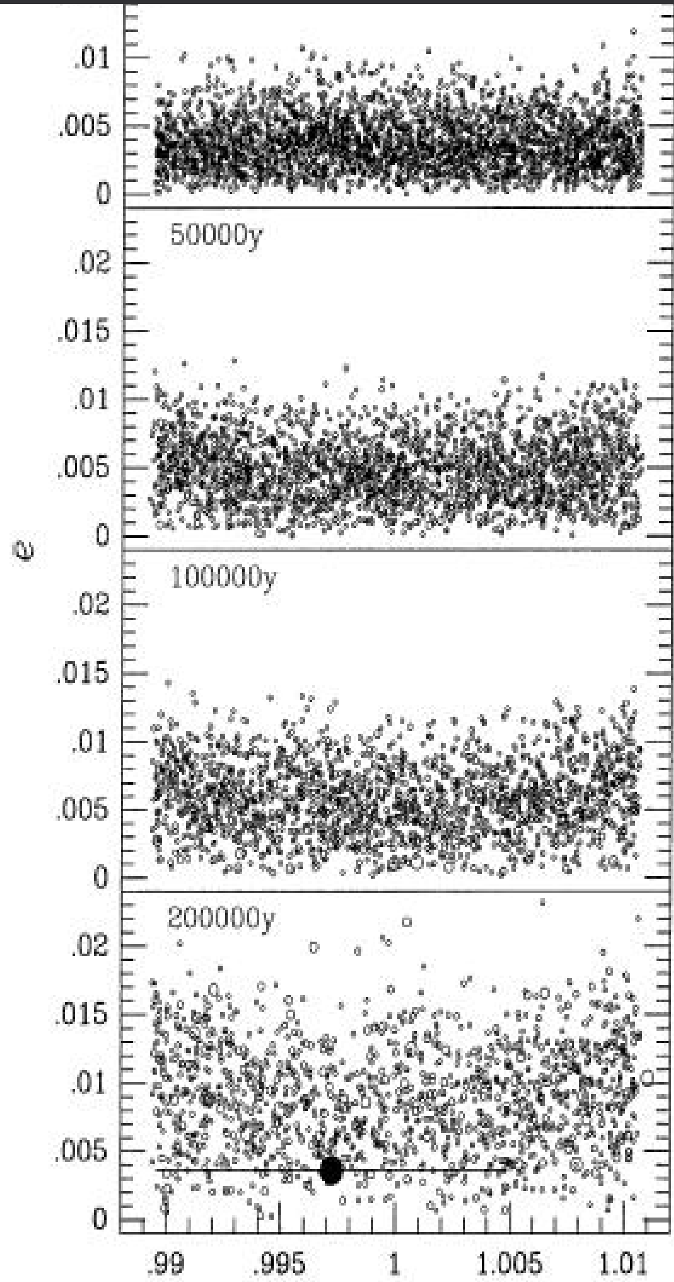
$dM_i /dt \sim M_i^{4/3} / v_0^2$;

$(1/M_i)\ (dM_i /dt) \sim M_i^{1/3} / v_0^2$.

$$d(M_1/M_2)/dt = M_1/M_2\ [\ (1/M_1)(dM_1/dt) - (1/M_2)(dM_2/dt)\ ] > 0.$$

The mass ratios increase : the largest objects grow faster, and become even larger : <u>Runaway growth</u>.

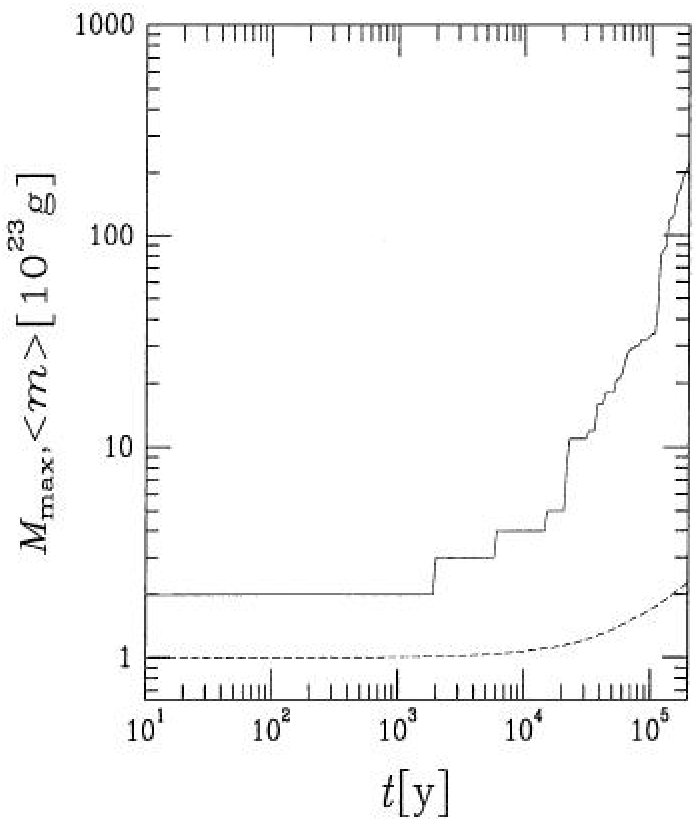
RUNAWAY GROWTH



Kokubo and Ida (2000)

Initially 3,000 10^{23}g planetesimals.

End: 1,322 planetesimals + 2×10^{25}g \`embryo'



RUNAWAY GROWTH

Runaway growth holds as long as $v_{rel} << v_{esc}$.

But v_{rel} grows in response to the presence of the largest bodies in the disk and it tends to become ~ v_{esc} .

It takes some time to get $v_{rel} \sim v_{esc}$. Runaway growth acts only during this time. This time is short if planetesimals are big, while it can be longer if planetesimals are numerous and small (collisional damping) and if there is gas in the system (gas drag). But the runaway growth time is short or null if there is a strong turbulent stirring of v_{rel}.

OLIGARCHIC GROWTH

2) Case $v_0 \sim v_{esc}$ (dynamically hot disc).

Then, $F_g \sim 1$.

Take two objects of masses $M_1 > M_2$.

$dM_i/dt \sim M_i^{2/3} / v_0^2$;

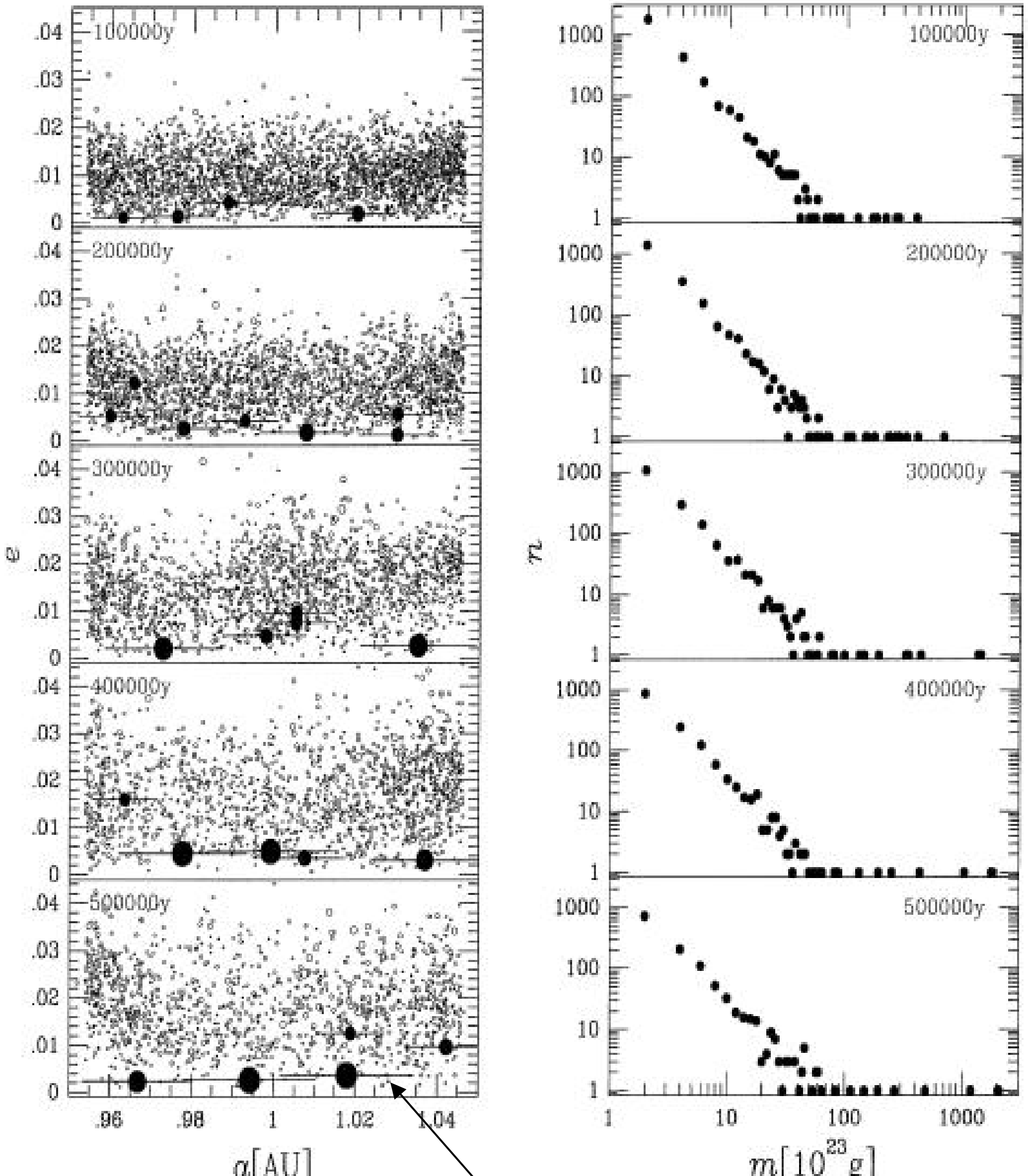
$(1/M_i)(dM_i/dt) \sim M_i^{-1/3} / v_0^2$.

$d(M_1/M_2)/dt = M_1/M_2 \, [\, (1/M_1)(dM_1/dt) - (1/M_2)(dM_2/dt) \,] < 0.$

The mass ratios decrease.

<u>Oligarchic growth :</u> the largest bodies (previously formed) dominate, and grow together by accreting the small stuff, until they have absorbed all the solids in their zone of influence (feeding zone).

OLIGARCHIC GROWTH



Lines: $5r_H$

Again from Kokubo and Ida (2000)

Filled dots: mass > 2×10^{25}g

OLIGARCHIC GROWTH

Masses of the oligarchs :

Zone of influence = Roche lobe, inside which the planet's gravity dominates over that of the star. It is close to a sphere (the Hill sphere), of radius: $R_H = a_p \, (M_p / 3M_*)^{1/3}$

R_H : Équilibre à $a = a_p - R_H$:
$-GM^*/a^2 + GM_p/R_H^2 + a\,\Omega_p^2 = 0$ $(/GM_*)$
$-a_p^{-2}(1+2R_H/a_p) + q/R_H^2 + (a_p-R_H)/a_p^3 = 0$
$- 3R_H / a_p^3 + q / R_H^2 = 0$ $(q=M_p/M_*)$

where the index p corresponds to the planet(esimal) and a_p is the semi-major axis, the distance to the star.

Final mass : $M_p = 2\pi a_p \, 2R_H \, \Sigma_{\text{planetesimals}}$

$$M_p = (4\pi/(3M_*)^{1/3} \, a_p^2 \, \Sigma_{pl})^{3/2}$$

which is ~ 0,1 $M_\oplus$ at 1 UA, 1 $M_\oplus$ at 5 UA, in 10^4 - 10^5 years.

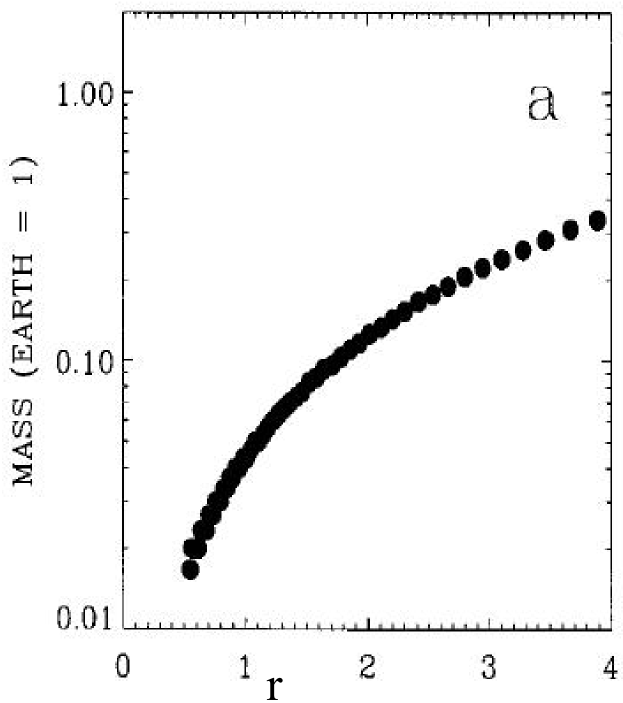
Which is good but not too good…

Every proto-planet has now absorbed everything in its feeding zone, and every feeding zone contains a proto-planet.

The system is densely packed.

OLIGARCHIC GROWTH

<u>Masses of oligarchs :</u> $M \sim [\, r^2\, \Sigma/(3M_{sun})^{1/3}]^{3/2}$ with $\Sigma \sim \Sigma_0/(r/r_0)^{3/2}$



<u>Snowline:</u> location in the disk beyond which the temperature is low enough that water is available in the form of ice. It was computed that Σ is enhanced by a factor 4-5 beyond the snowline.

This could make the oligarchs beyond the snowline as big as several Earth masses (comparable to the masses required to start to accrete gas from the disk). ???

However, studies suggest that Σ doesn't increase by more than a factor 2.

In addition, accretion can't be 100% efficient, and massive oligarchs scatter planetesimals instead of accreting them.

In the end, the oligarchs are not massive enough to be terrestrial planets or cores of giant planets.

FORMATION of EMBRYOS

<u>Conclusion :</u>

Embryos (or cores) form in two phases from planétésimals.

<u>Runaway growth</u> : the most massive grow the fastest.

<u>Oligarchic growth</u> : the mass differences between oligarchs get smaller.

Final mass = isolation mass = $\sim 15\pi r^2\Sigma \sim r^{1/2}$.

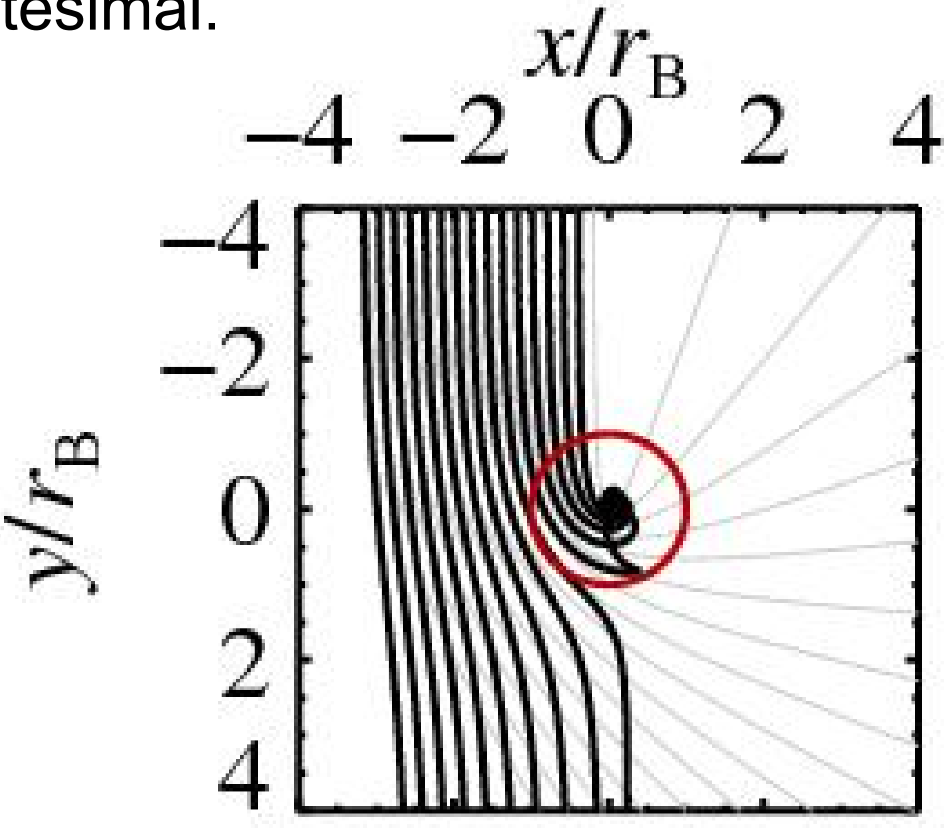
If Σ increases suddenly (snowline), easier formation of massive cores, but probably not massive enough.

PEBBLE ACCRETION

The key : pebble accretion (Lambrechts & Johansen 2012, see lecture by A. Johansen).

Pebbles are cm – m sized bodies, moderately coupled to the gas. Gas friction increases tremendously the cross section of a planetesimal.

Possible growth to several Earth masses for an oligarch beyond the snowline.



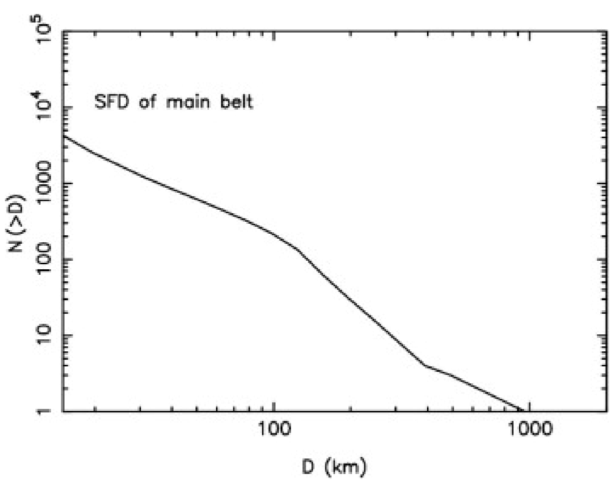
VORTICES

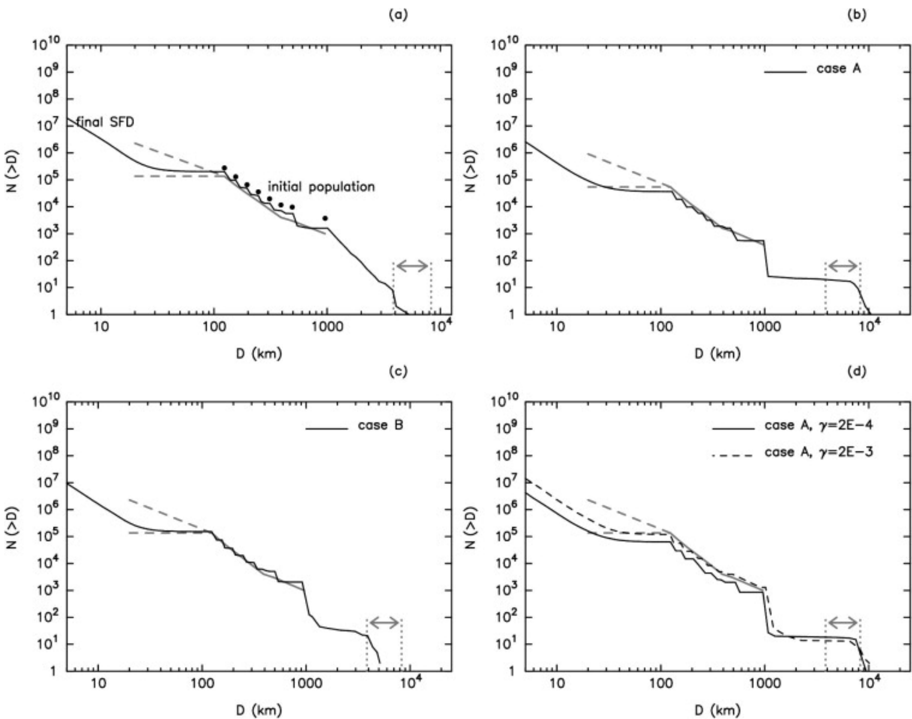
Alternative solution :

Direct formation through vortices, in a turbulent disk.

Real ?

Some people suggest that asteroids form big, and then are grained in smaller bodies by collisional evolution (Morbidelli et al. 2009).
But this is not yet accepted...





SUMMARY

5 STEPS :

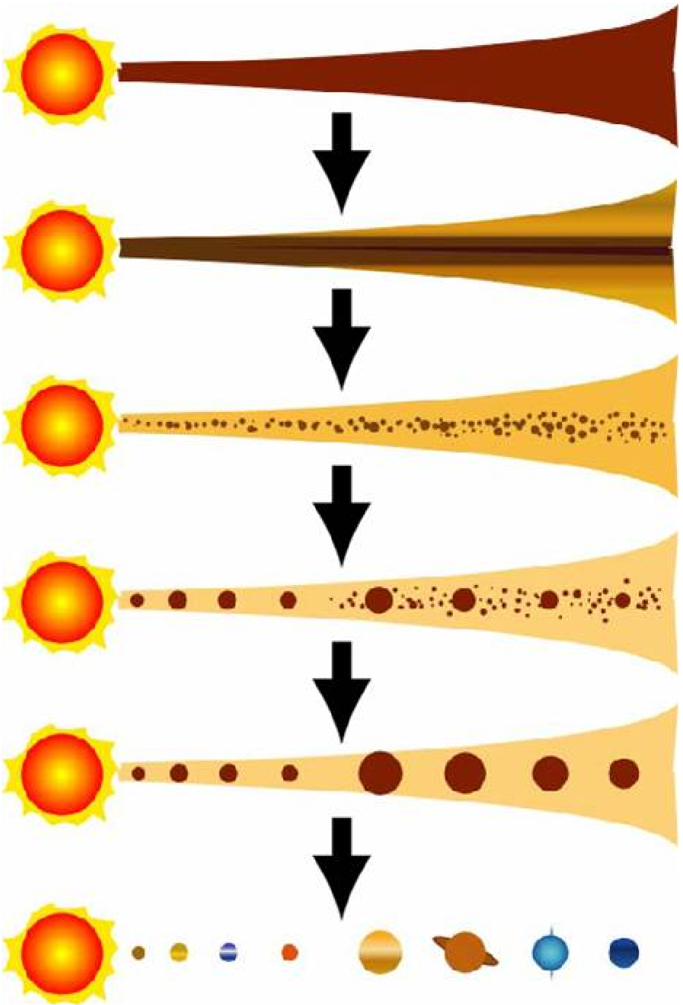
a/ Condensation.

b/ Sedimentation.

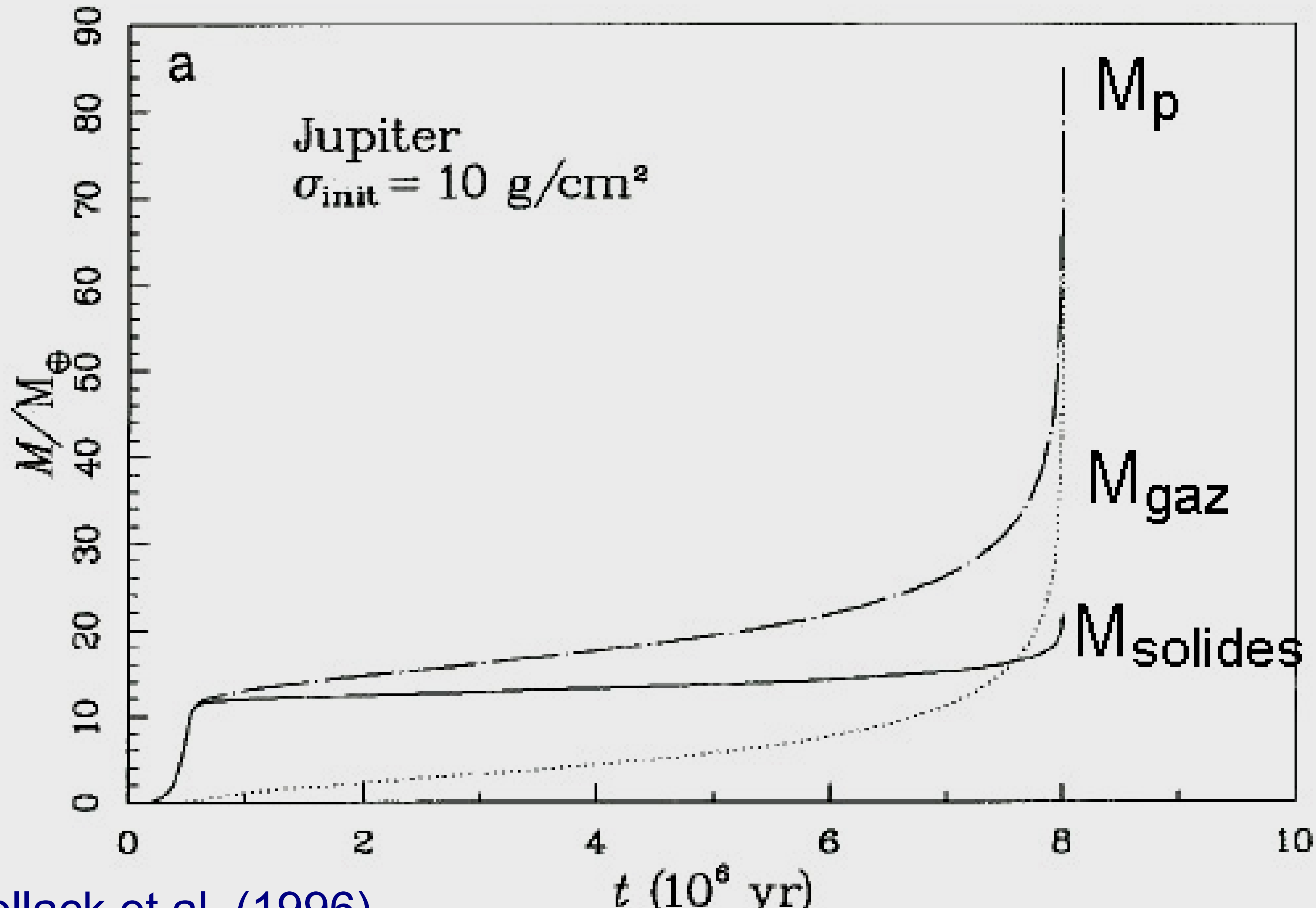
c/ Formation of planetesimals (~1km, 10^{-9} $M_{\oplus}$).

d/ Formation of embryos
runaway growth, oligarchic growth
$M \sim 10^{-1} - 10^{-2}$ $M_{\oplus}$, $M \sim \Sigma^{3/2} r^3$.

e/ Formation of terrestrial planets or cores

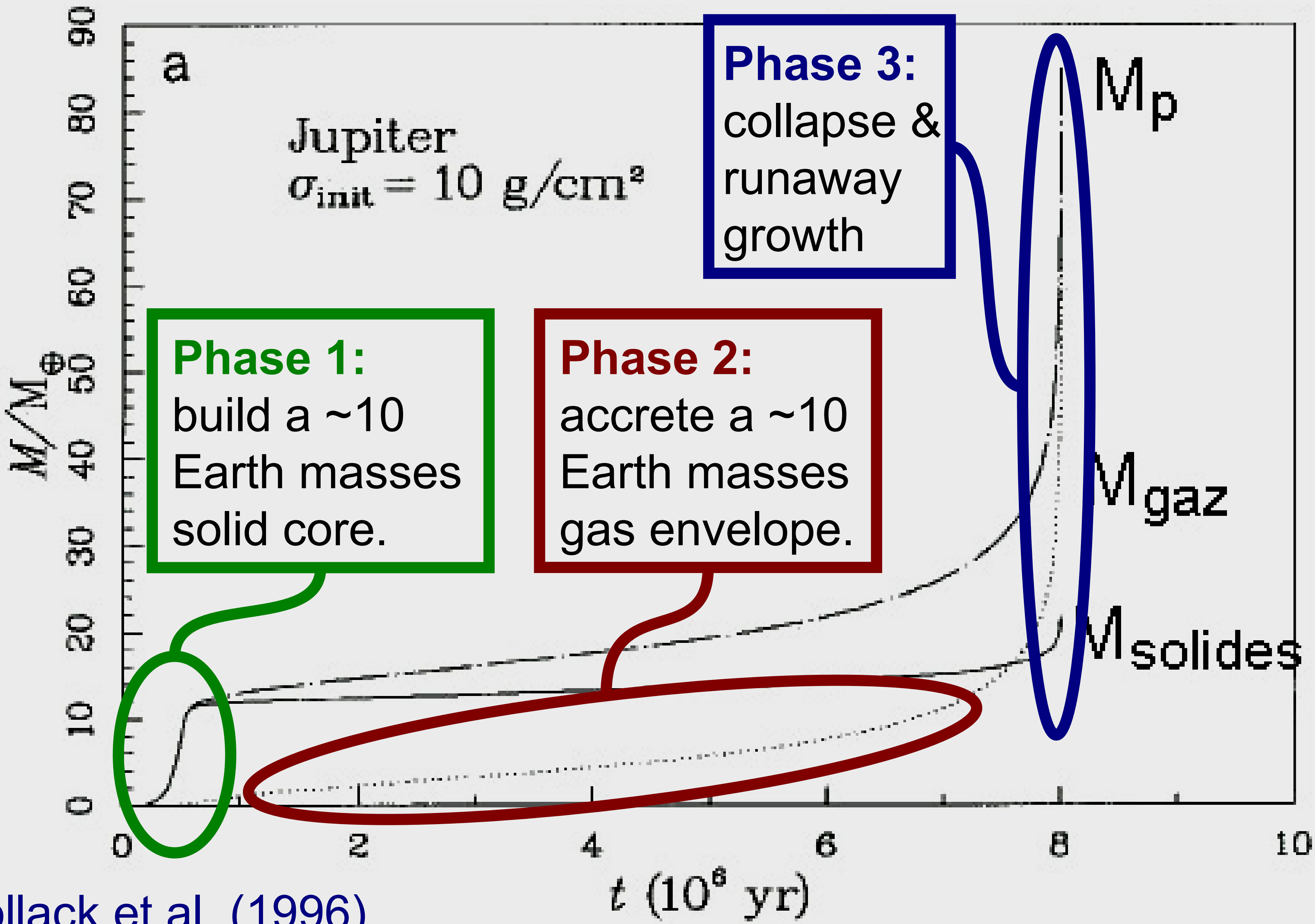


GAS ACCRETION



Pollack et al. (1996)

GAS ACCRETION



Pollack et al. (1996)

GAS ACCRETION

Reminder : The escape velocity at the surafce of a body of mass M_p and radius R_p is : $v_{esc} = \sqrt{(2GM_p/R_p)}$.

$\rightarrow$ For a given density, $v_{esc} \sim R_p \sim M_p^{\ 1/3}$

The average velocity of gas molecules is the celerity of sound $c_s \sim \sqrt{T(r)}$. On Earth : ~330 m/s.

<u>Bondi radius :</u>

r_B such that $\sqrt{(2GM_p/r_B)} = c_s \sqrt{2}$: $\boxed{r_B = GM/c_s^{\ 2}}$ ~1/T.

Capture of gas possible if $c_s < v_{esc}$ or $r_B > R_p$.

i.e. : large cores in a cold environment...

PHASE 2

Structure of the envelope (Papaloizou & Terquem 1999) :

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

Mass. $M_p = M(R_{Hill})$; $R_{Hill} = a_p (M_p/M_*)^{1/3}$

$$\frac{dP}{dr} = \rho(r) g(r)$$

Hydrostatic equilibrium. $P(R_{Hill}) = P_{out}$

$$g(r) = -\frac{GM(r)}{r^2}$$

Gravity.

$$\rho = f(P, T)$$

Equation of state (EOS)

Temperature.

$$\frac{dT}{dr} = \frac{-3\kappa\rho}{16\sigma T^3} \frac{L}{4\pi r^2}$$

L : luminosity through sphere of radius r.

$L = L_{core} + L_{convection}$

$$\kappa = f(\rho, T, \chi)$$

κ : opacity.

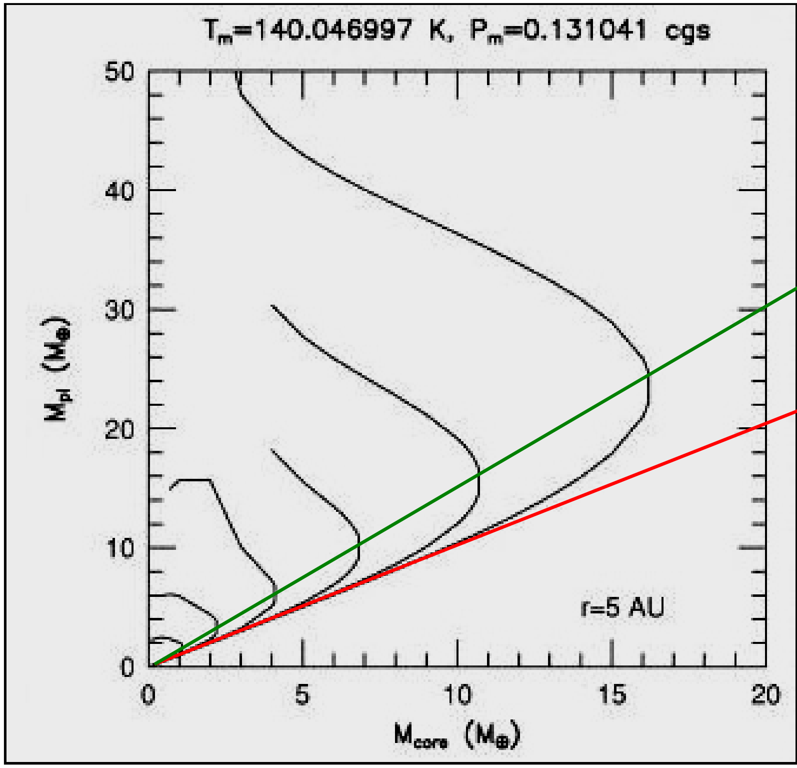
$$L_{core} = \frac{G M_{core}}{R_{core}} \frac{dM_{core}}{dt}$$

L_{core} = energy from the fall of solids.

PHASE 2

Papaloizou & Terquem, 1999 :
M_p as a function of M_{core}, at 5 UA,
for $dM_{core}/dt = 10^{-11, 10, 9, 8, 7, 6}$ $M_\oplus$/an.

For all M_{core}, there are 2 solutions.



Red line : $M_p = M_{core}$, no gas.

Green line : $M_p = 1.5\ M_{core}$.

Phase 2 :

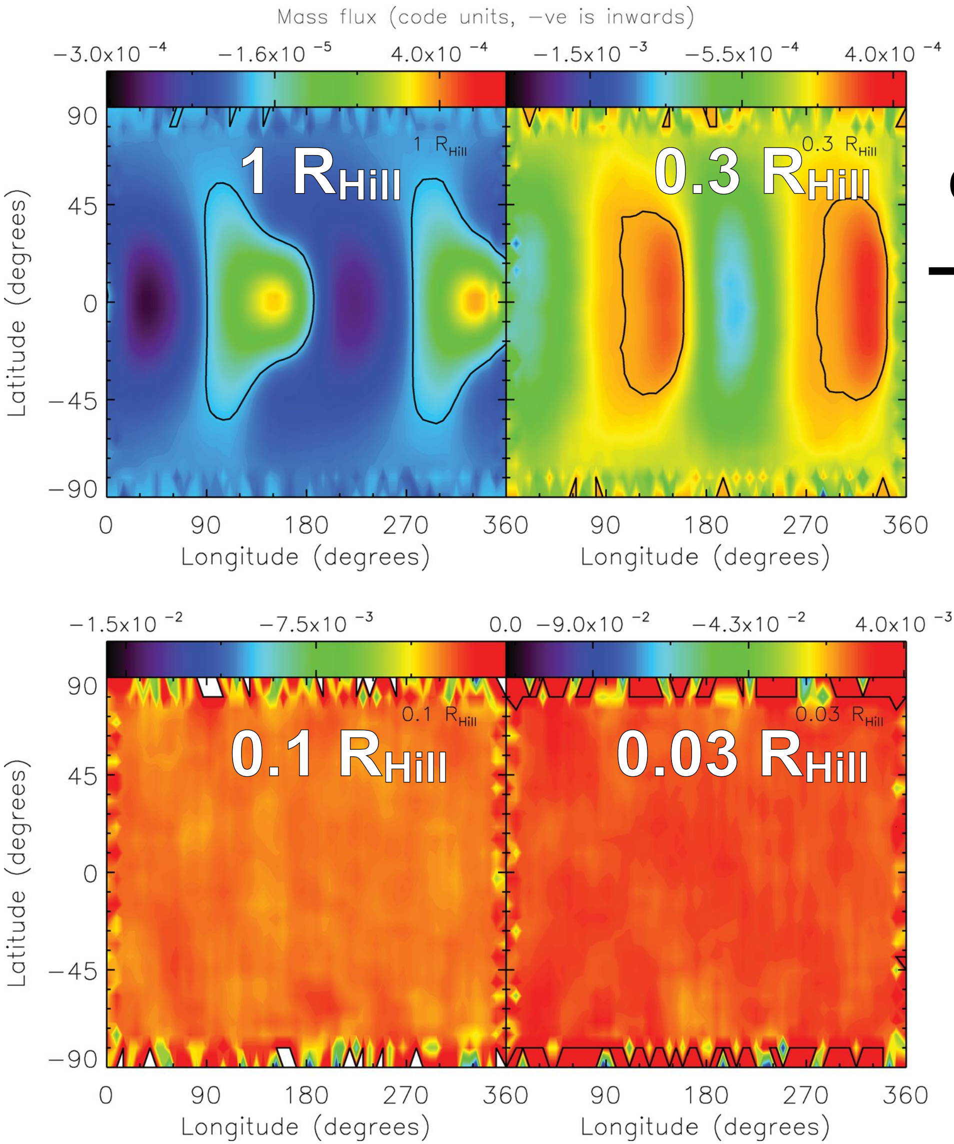
Core accretion slower than time to reach equilibrium.

Slow evolution following the line, starting from 0 hence on the lower branch. As the core increases, so does the envelope.

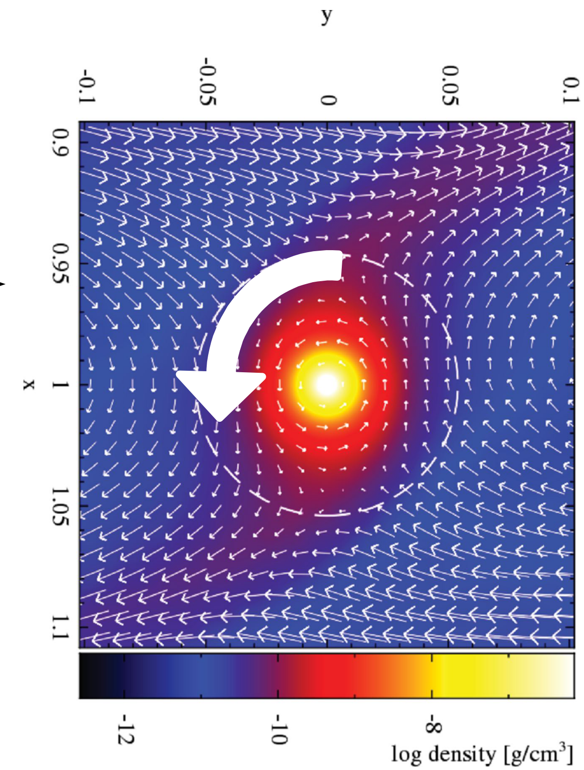
It can last ages...

PHASE 2

Ayliffe & Bate (2012) : mass flux through spherical shells



circulation



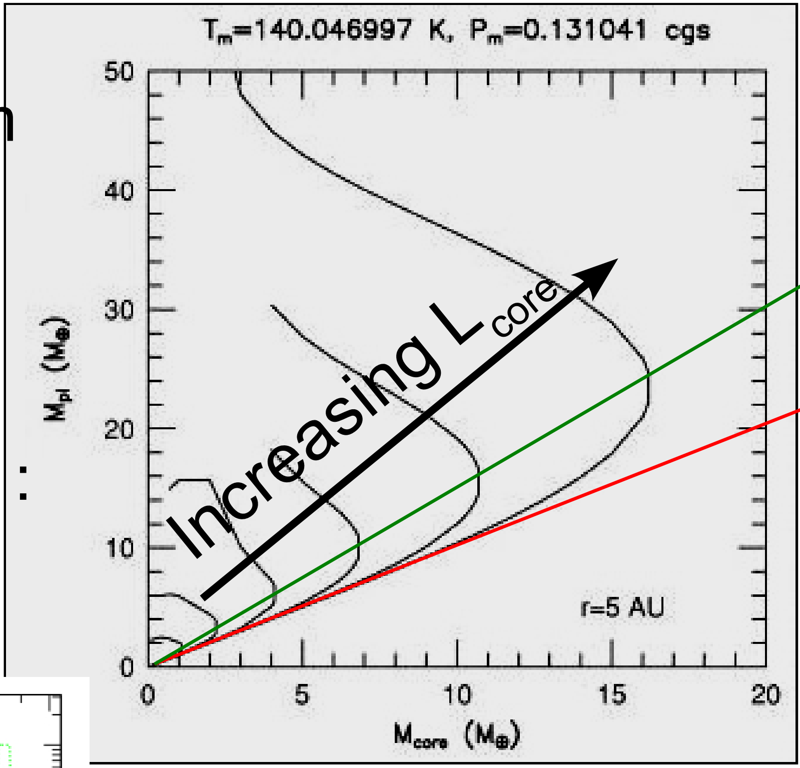
Contracting envelope,
no equilibrium
because $L_{core}=0$.

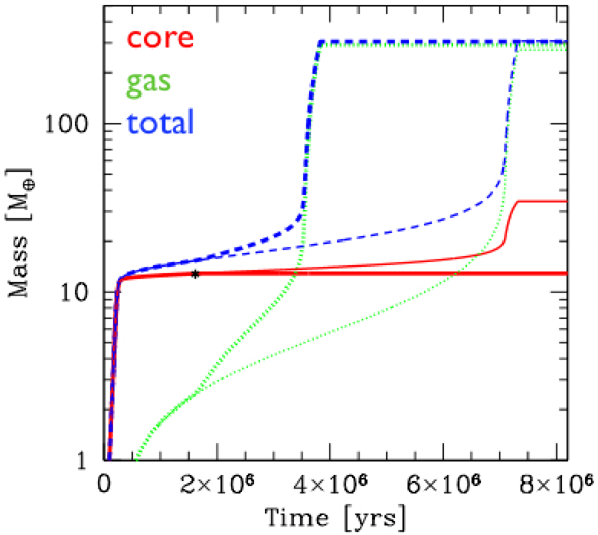
END OF PHASE 2

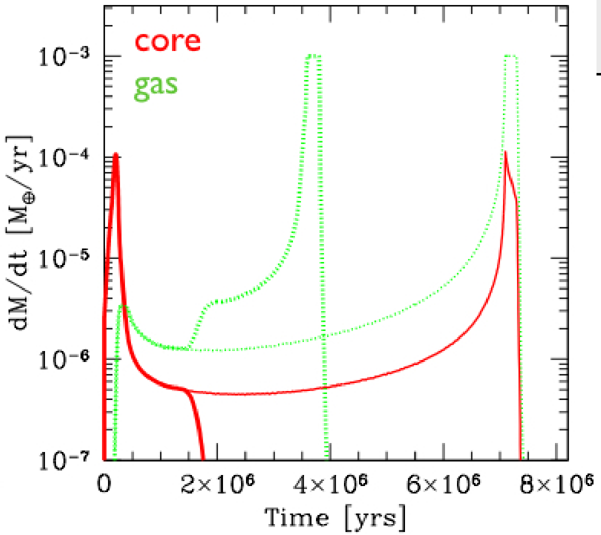
For all dM_{core}/dt, there is a maximum core mass beyond which no equilibrium is possible → collapse.

NB : This occurs for M_{gas} = 0.5 M_{core}.

The critical core (or planet) mass depends on the accretion rate of solids : L_{core} = 0 => no stability ever.





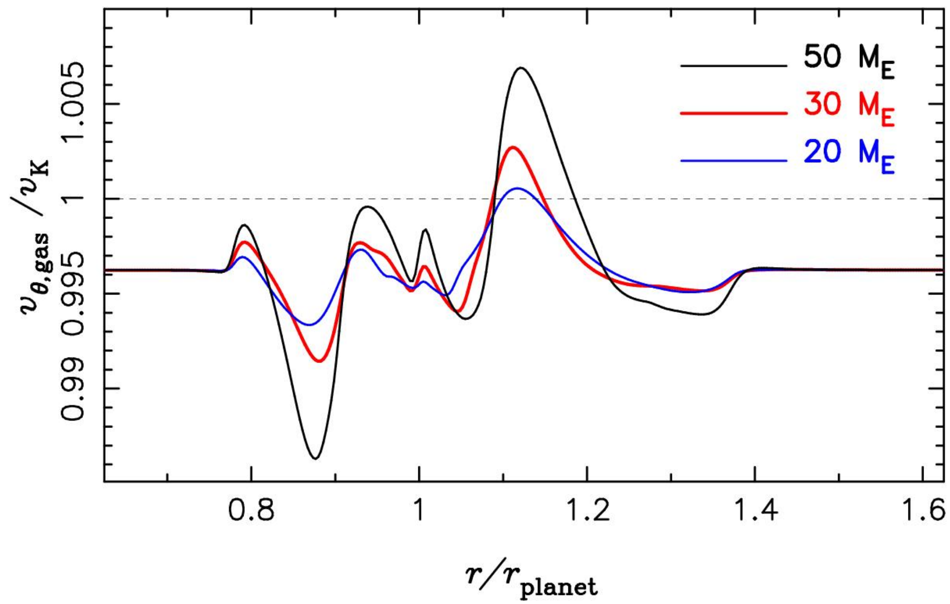


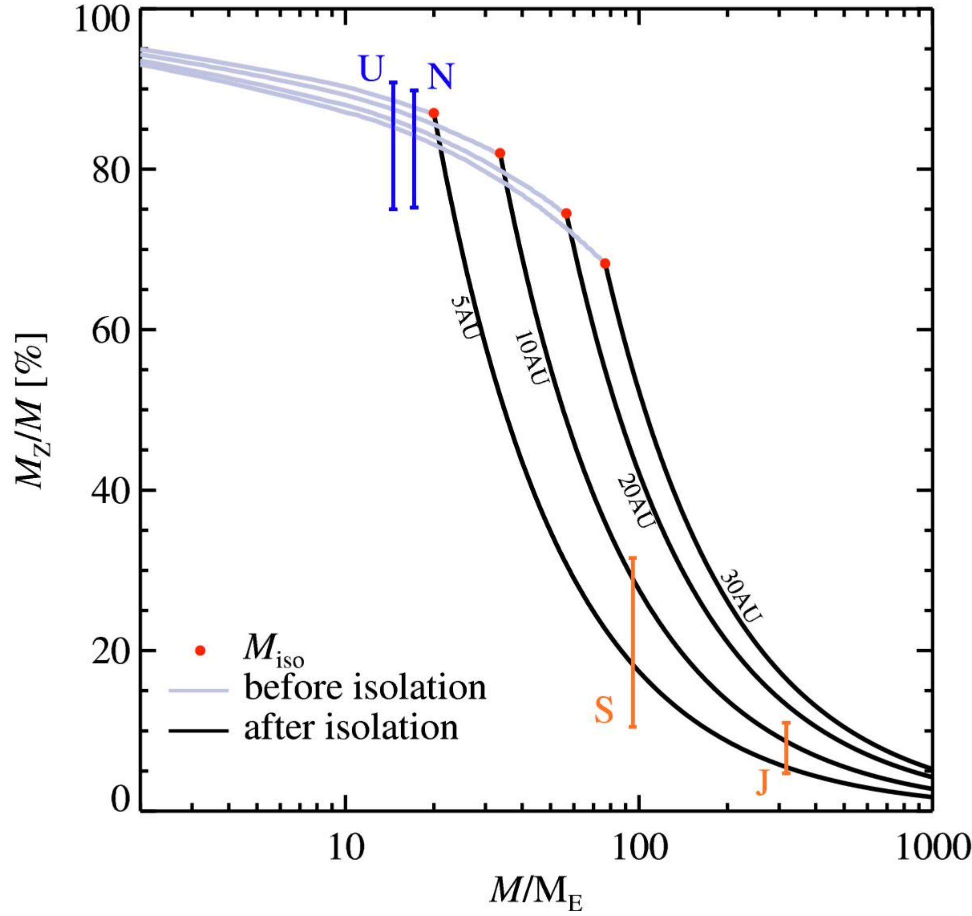
Mordasini et al. (2011) : cut the inflow of solids (black dot, thick curves) to trigger the collapse.

TRANSITION TO PHASE 3

Why would $L_{core} = 0$?

Lambrechts et al. (2014) : dM_p /dt (pebbles) => no collapse.
But $M_p > (20\ M_\oplus)\ (h/0.05)^3$ => pressure maximum outside of the planetary orbit, and no more pebbles flux.

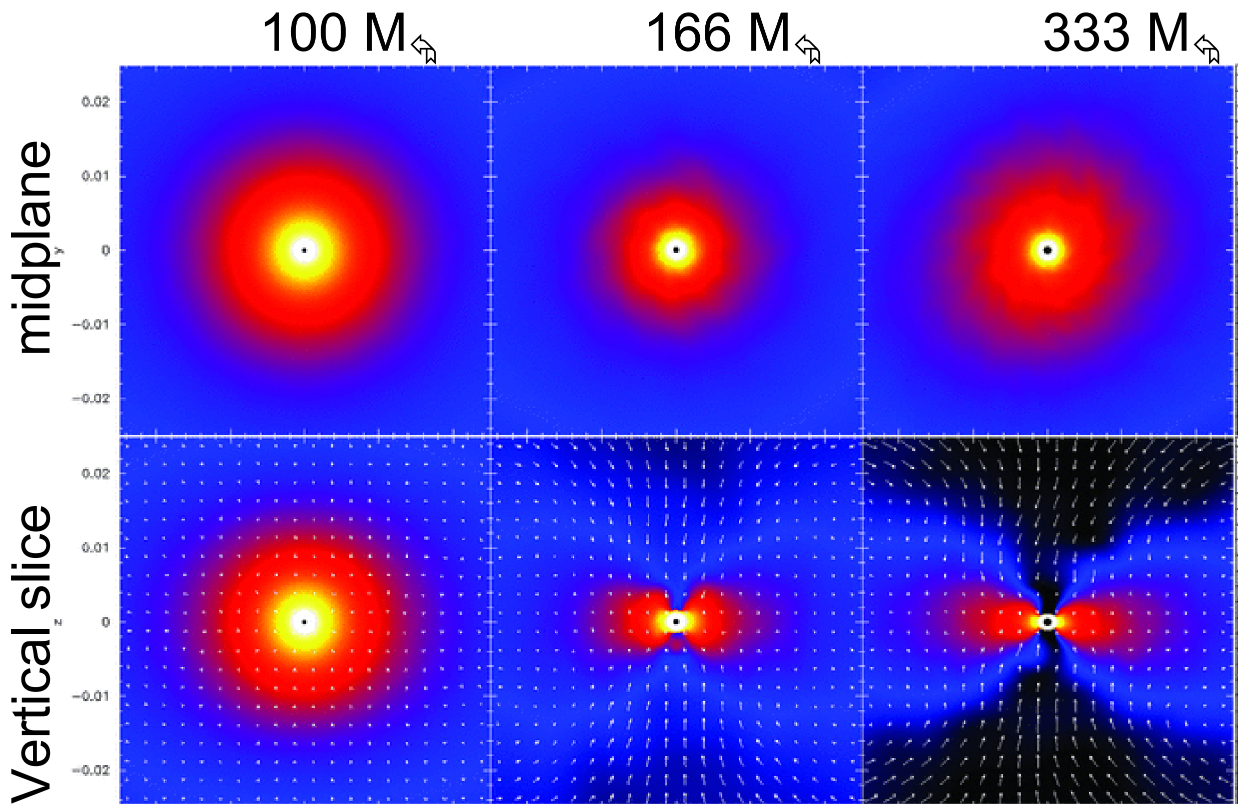




PHASE 3

<u>When does one get an enveloppe or a CircumPlanetary Disc ?</u>

Ayliffe & Bate (2009) :
3D radiative SPH simulations, inside boundary at 0.01 R_{hill} , low viscosity.



Spherical envelope for M_p < 100 M ,
Disc for M_p = M_{Jup} .

Szulagyi et al. (2016) : 3D, grid code with mesh refinement, viscous heating, no planet boundary, no dissociation nor ionization.

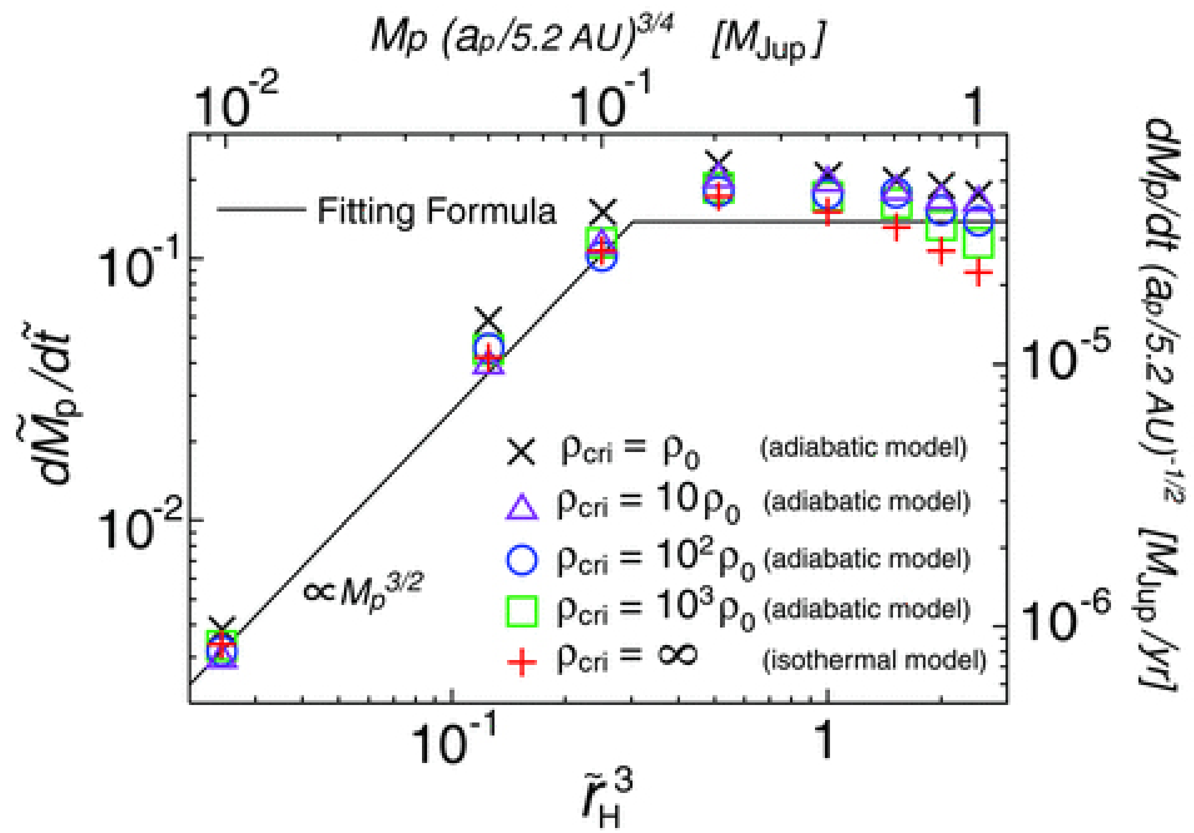
M_p=M_J : envelope or disc depending on cooling around the planet.

PHASE 3

ACCRETION RATE :

Machida et al. (2010) : 3D simulations, nested meshes, EOS isothermal for low ρ & adiabatic for large ρ.

Accretion rate : $dM_p/dt = \Sigma H^2 \Omega$ x max{ 0.14 ; 0.83 $(R_{Hill}/H)^{9/2}$ }



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Accretion rate : $dM_p/dt = \Sigma H^2 \Omega$ x max{ 0.14 ; 0.83 $(R_{Hill}/H)^{9/2}$ }

$H/r=0.05$, $\Sigma=1270\ g/cm^2\ (r/1AU)^{-2}$ => $dM_p/dt = (10^{26}\ g)\ \Omega$

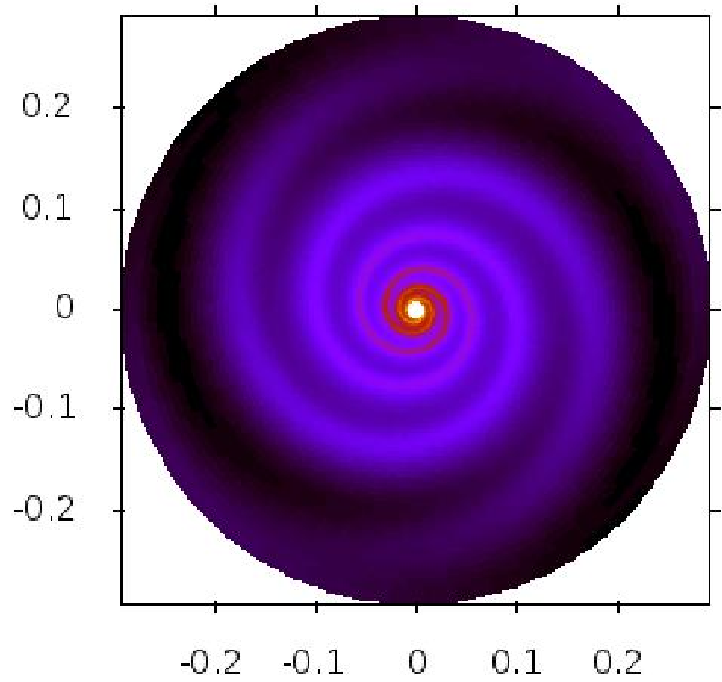
→ M_{Jup} in 2 x10^4 orbits. Too fast !

Rate confirmed in Szulagyi et al (2014) (isothermal) : $10^{-4}\ M_{jup}$/yr, but mainly due to numerical viscosity.

Problem : accretion of M_{Jup} in 200 000 years while the lifetime of the disk exceeds 2 million years => easy growth to several M_{Jup} , unless fine tuning. Not observed.

LIMITING GAS ACCRETION

1) The Circum-Planetary Disk, if not viscous (Fujii et al. 2011,2013 ; Turner et al. 2013) could act a bottleneck for the gas, that need to lose angular momentum to be accreted (Rivier et al. 2012, Szulagyi et al. 2014).



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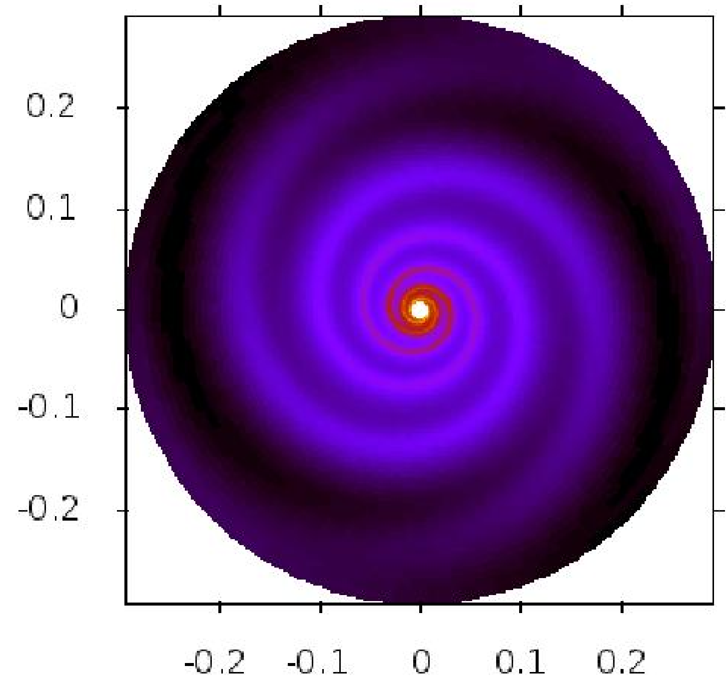
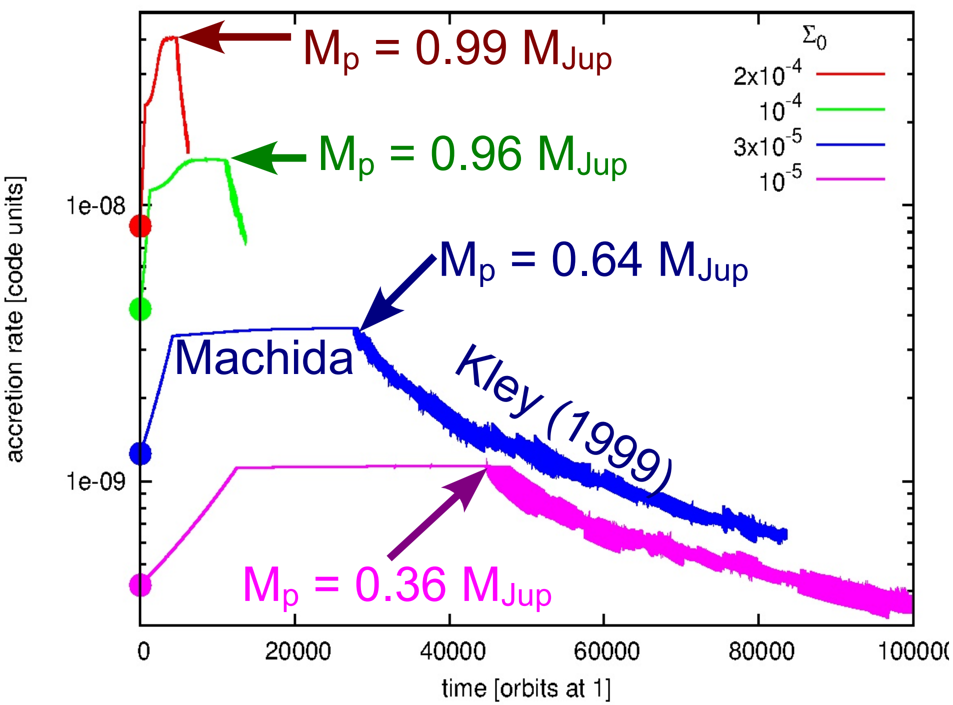




2) When the planet opens a gap, or has accreted all its horseshoe region, the accretion rate drops (Crida & Bitsch 2016).

GIANT PLANETS SUMMARY

Phase 1 :

→ Build a solid core via runaway / oligarchic growth
+ pebble accretion.

Phase 2 :

→ Equilibrium (complicated structure)

→ Needs $L_{core} > L_{threshold}(M_{core})$ or contraction, until collapse.

→ $M_p >$ $(20\ M_{\oplus})\ (h/0.05)^3$ => $L_{core}=0$ → trigger of phase 3.

Phase 3 :

→ CircumPlanetary Disc (inviscid ? Bottleneck?)

→ Possibly too fast accretion to explain the final masses.

TERRESTRIAL PLANETS

We left oligarchs / embryos densely packed in the inner solar system + a few planetesimals not accreted.
M_{embryo} = ~0.01 ~0.1 M_{Earth} .

A system of many planets around a star (or satellites around a planet) is stable if and only if

the distance between the orbits is larger than 5 mutual Hill radii,

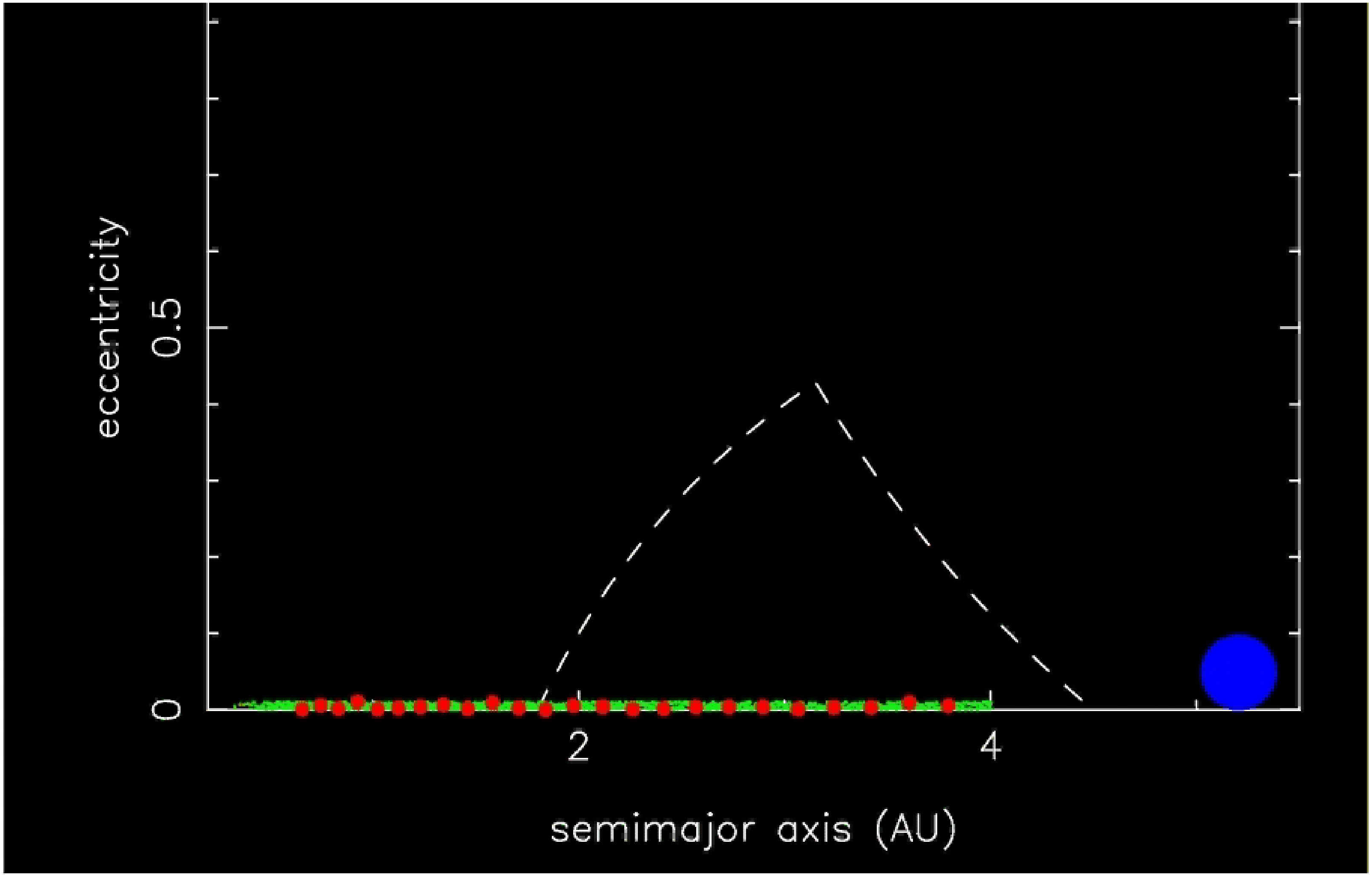
where $r_{H\,i,j} = [(M_i+M_j)/(3M_*)]^{1/3}\ (a_i+a_j) / 2$

Application: Is the Solar System stable now ?

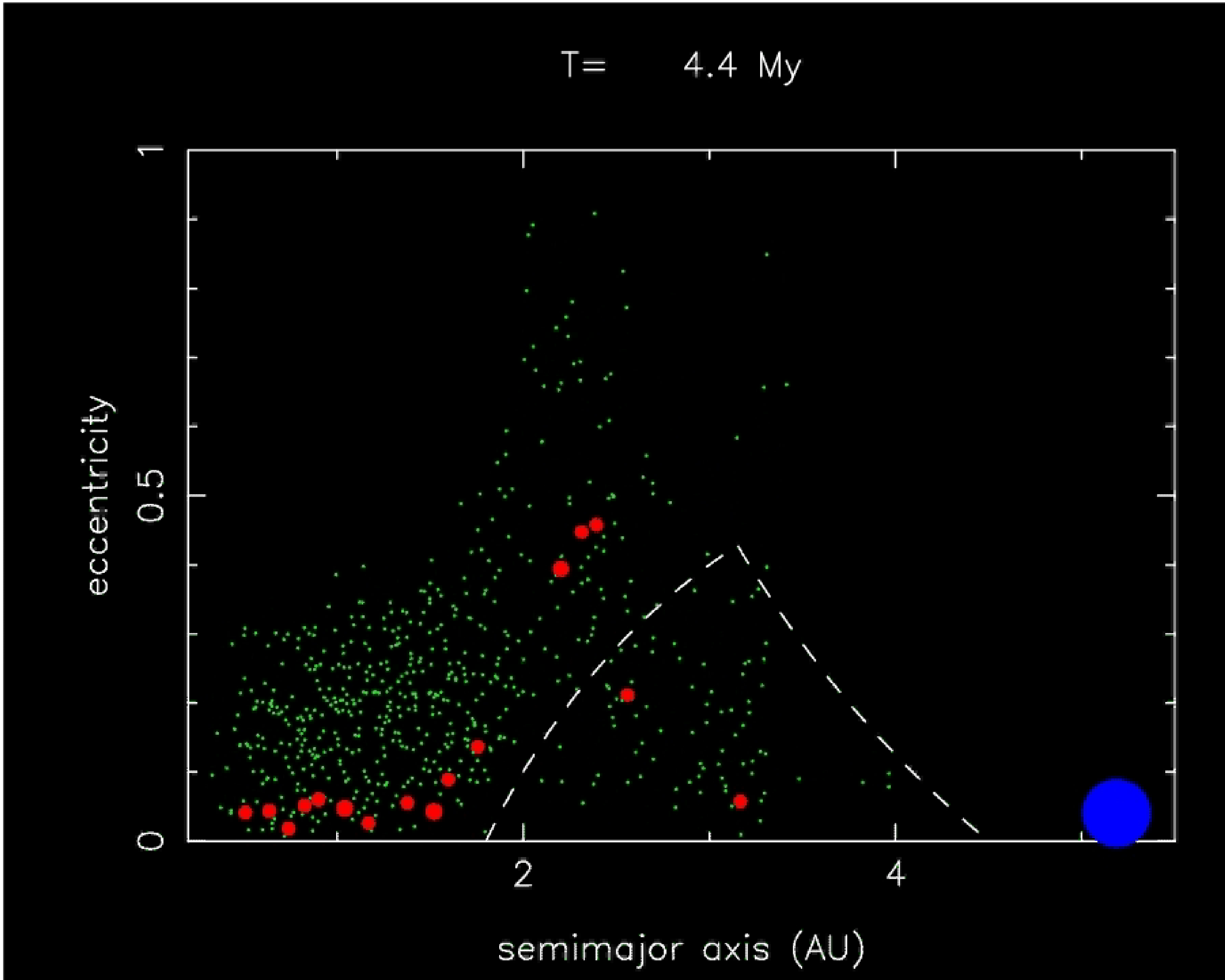
By construction, the system of embryos is NOT stable. While gas is present to circularise the orbits, nothing happens. But...

TERRESTRIAL PLANETS

Wetherhill's model of the dynamics of the formation of terrestrial planets (Chambers&Wetherhill 1998, Chambers 1999, O'Brien et al.2006).



TERRESTRIAL PLANETS



TERRESTRIAL PLANETS

Wetherhill's model of the dynamics of the formation of terrestrial planets (Chambers&Wetherhill 1998, Chambers 1999, O'Brien et al.2006).

Successes :

- Formation of a few terrestrial planets in the 0-3 AU zone.
- No formation of planet in the asteroid belt in most simulations.
- The most massive planets are ~ Earth mass.
- Good orbits (eccentricity and inclination excitation).
- Giant impacts are typical, several with geometries compatible with the Moon-forming event.
- Roughly correct accretion timescale for the Earth (tens of My).
- Delivery of water-rich bodies from the asteroid belt to the Earth

ANGULAR MOMENTUM DEFICIT

- Good orbits (eccentricity and inclination excitation) ?

Angular momentum deficit :

$$= \frac{\sum_j m_j \sqrt{a_j(1-e_j^2)} \cos i_j - \sum_j m_j \sqrt{a_j}}{\sum_j m_j \sqrt{a_j}}$$

« vertical » component of the a.m. normalized by circular coplanar orbits.

Angular mom. on a Keplerian orbit : $L = M_p [GM_* a(1-e^2)]^{1/2}$.

AMD Solar System : -0.0018

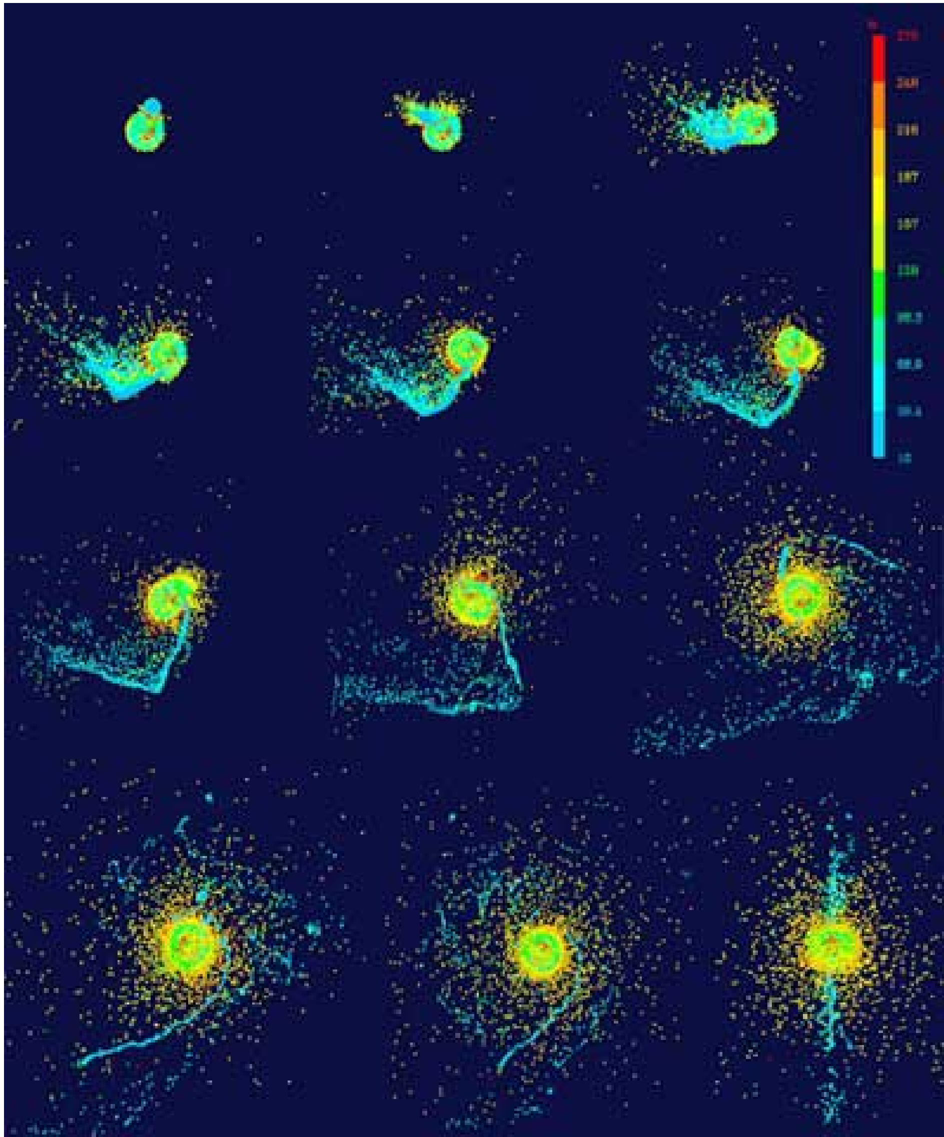
AMD Chambers (1999) : -0.007
Same simulation with more smaller planetesimals, hence more dynamical friction : AMD = -0.0010.

Same simulation with Jupiter & Saturn on circular orbits : -0.003.

→ Role of dynamical friction and J & S's orbits.

MOON FORMATION

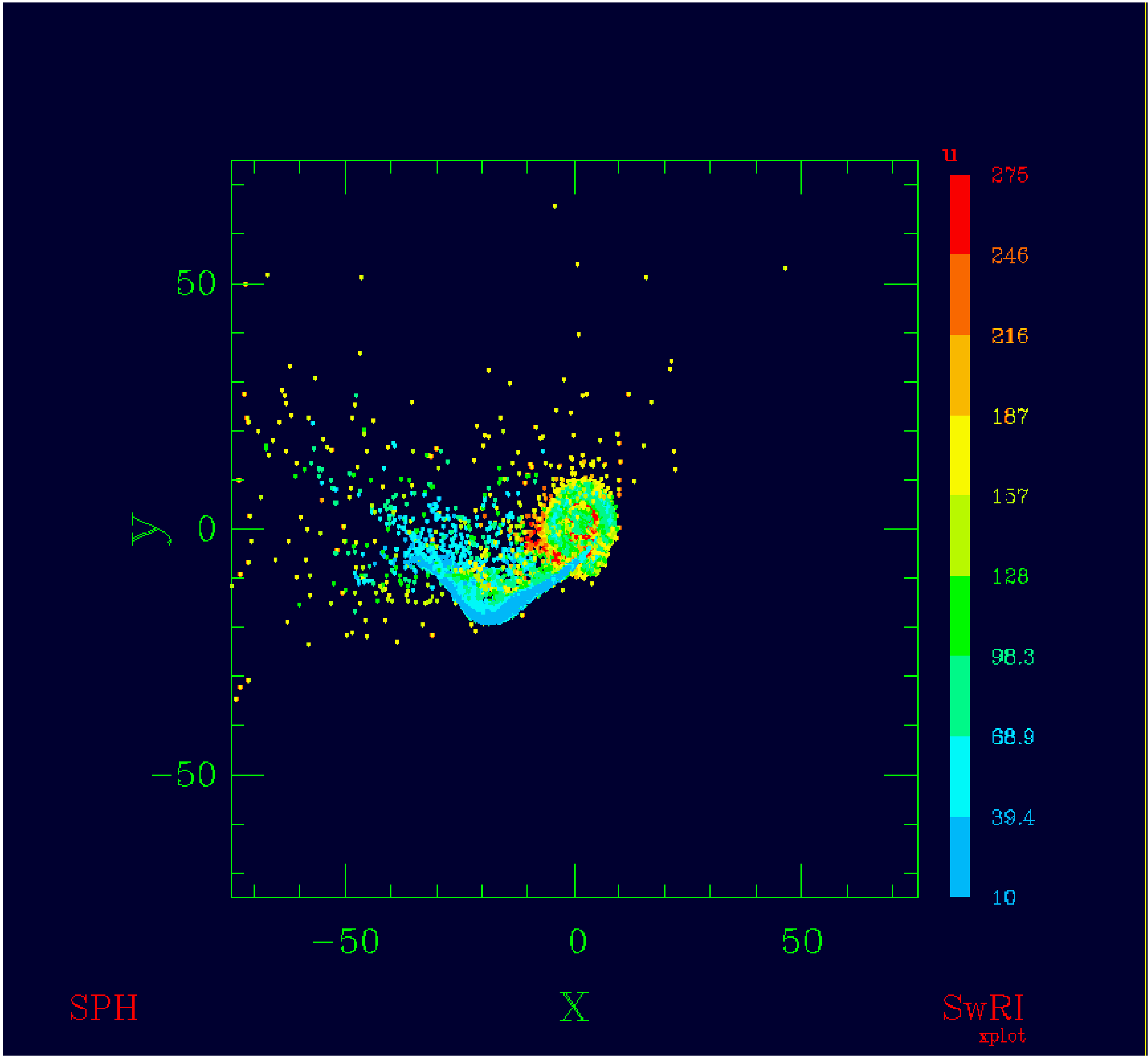
- Giant impacts are typical, several with geometries compatible with the Moon-forming event.



The Moon forming impact was NOT an exceptional, lucky event (Agnor et al. 1999).

MOON FORMATION

Canup & Asphaug (2001)

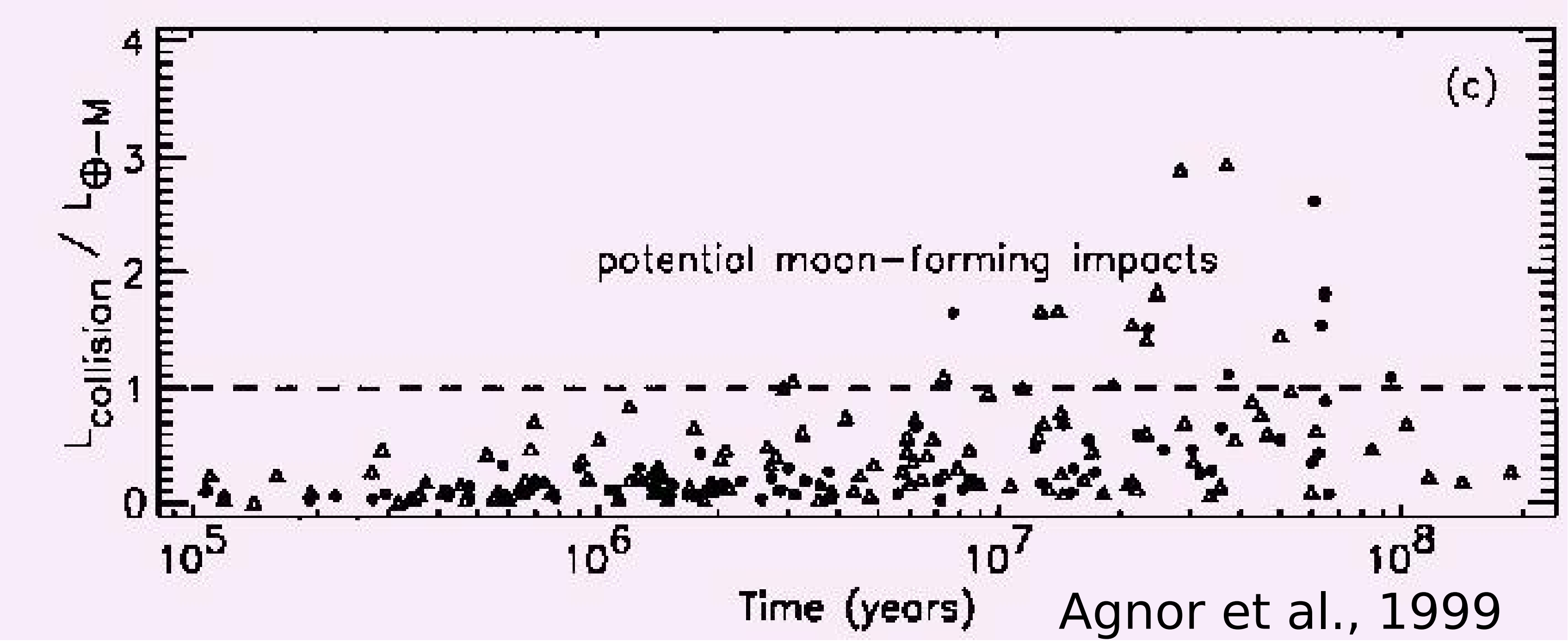


MOON FORMATION

Potential Moon-forming impacts are those that carry an angular momentum larger than that of the Earth-Moon system

They are not rare: They occur for 25% of the planets that have a final mass > 0.5 M_{earth}

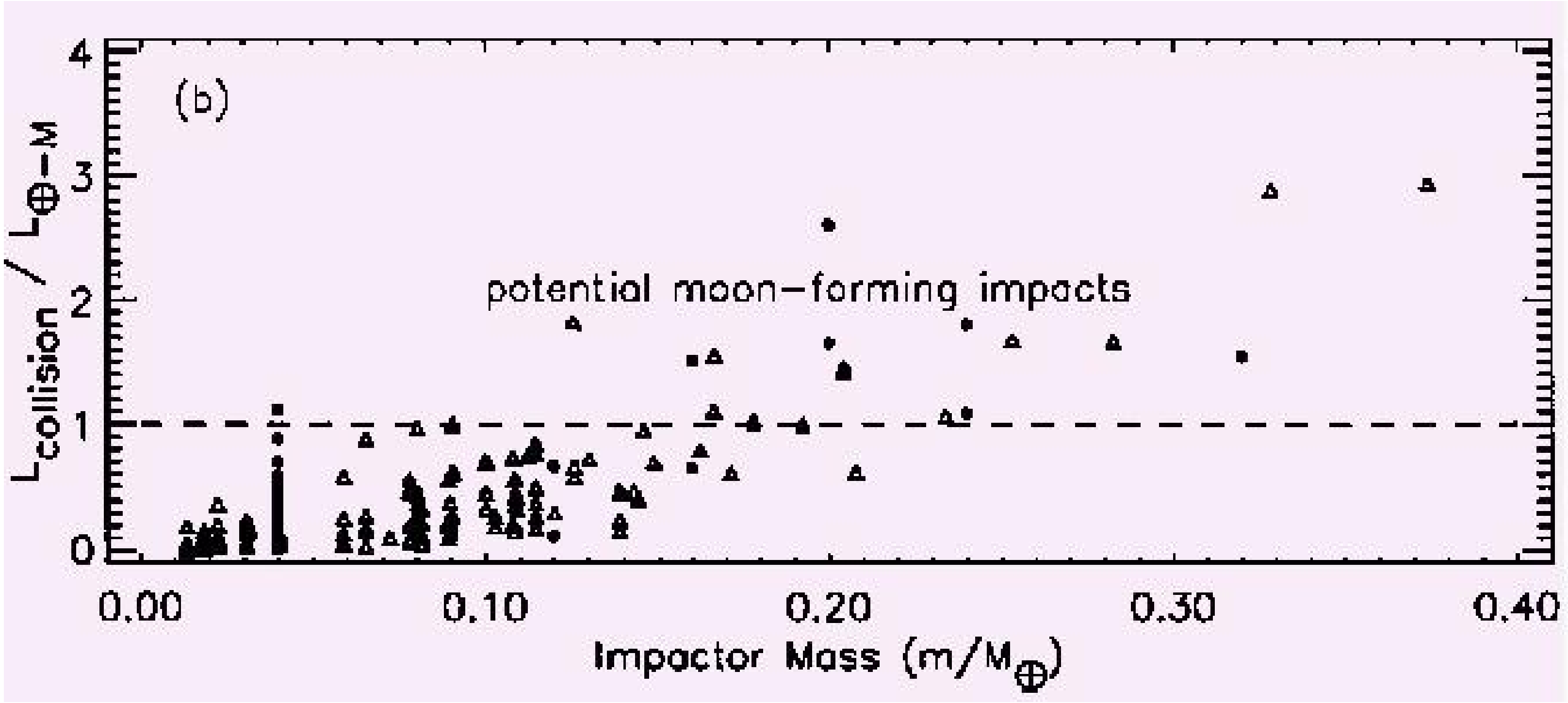
They tend to occur towards the end of the accretion.



Agnor et al., 1999

MOON FORMATION

They involve impactors which are at least as massive as Mars.



Agnor et al., 1999

MOON FORMATION

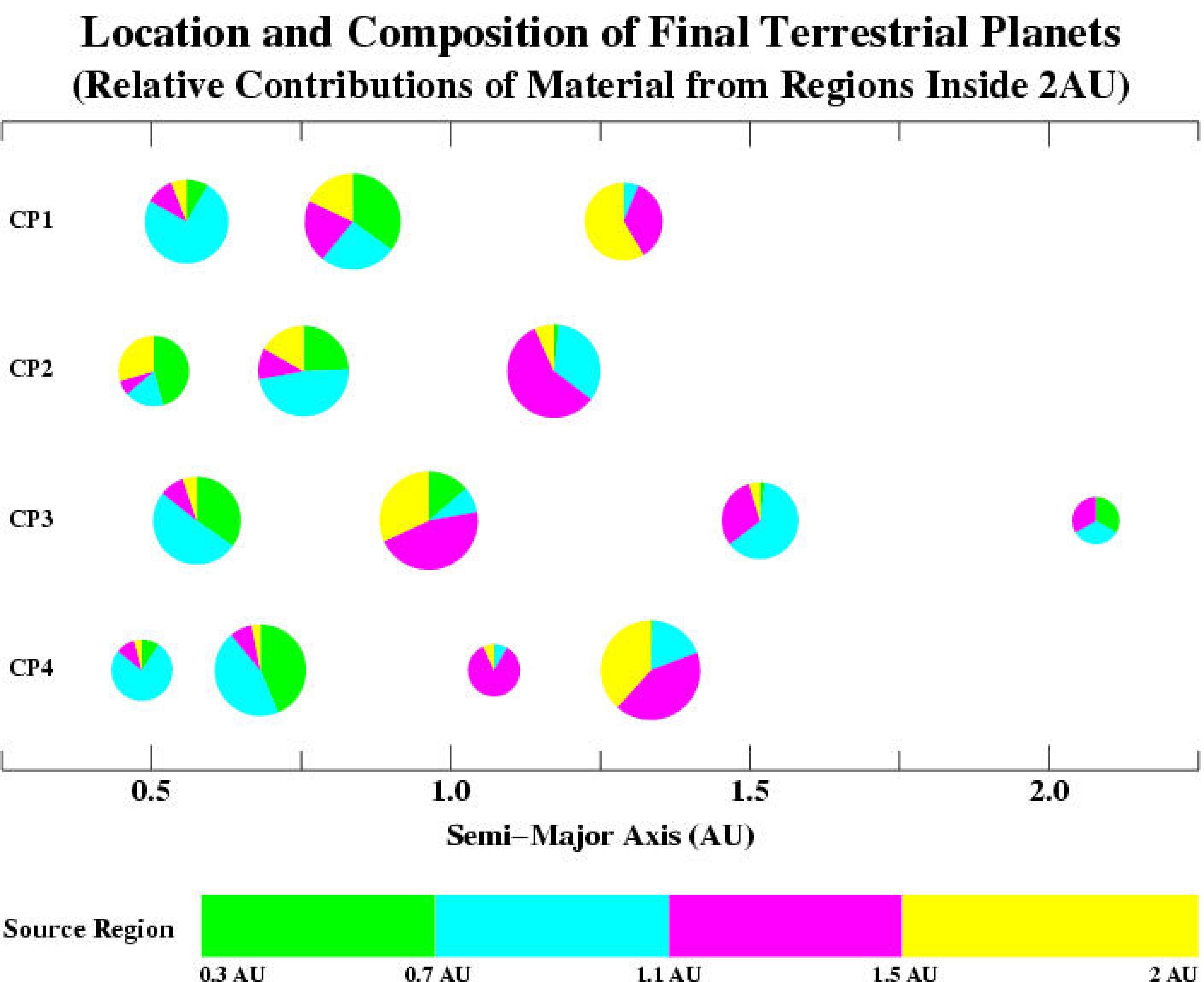
<u>Successive simulations are those that:</u>

Carry an angular momentum larger (but not much: up to ~30%) of the angular momentum in the Earth-Moon system

Produce a proto-lunar disk that is more massive than the Moon (but not much: up to ~80%)

The mass of iron in the proto-lunar disk is <3% of the mass of the disk

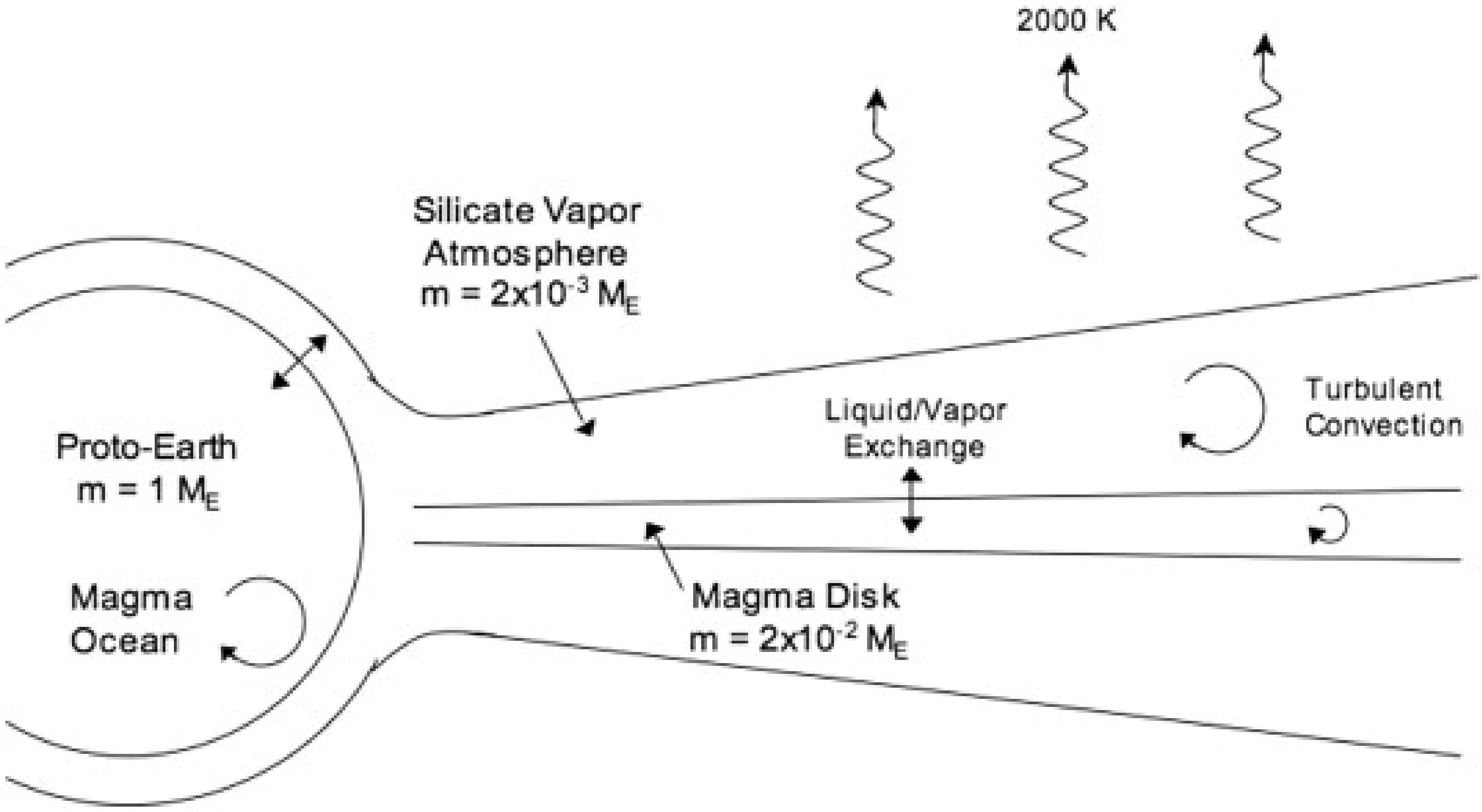
MOON FORMATION



Even if planets had accreted NO material from the asteroid belt, it is unlikely that the proto-Earth and the proto-Lunar impactor could have the same composition even if they formed very close to each-other.

MOON FORMATION

Pahlevan & Stevenson (2007) suggest that the proto-lunar disk was molten, with a gaseous silicate atmosphere. The Earth was also a magma ocean, with a gaseous silicate atmosphere.
In the atmosphere, the isotopic compositions equilibrate.



ACCRETION TIMESCALE

- Roughly correct accretion timescale for the Earth (tens of My).

Relative datation using the Hafnium – Tungtene chronometer :

Hafnium 182 decays into Tungsten 182. Half life $t_{1/2}$~ 9 Myr :

$$^{182}W = {}^{182}W_{init} + {}^{182}Hf_{init}$$

$$\underbrace{\left(\frac{^{182}W}{^{184}W}\right)}_{y} = \underbrace{\left(\frac{^{182}W}{^{184}W}\right)_{init}}_{b} + \underbrace{\left(\frac{^{182}Hf}{^{180}Hf}\right)_{init}}_{a} * \underbrace{\left(\frac{^{180}Hf}{^{184}W}\right)}_{x}$$

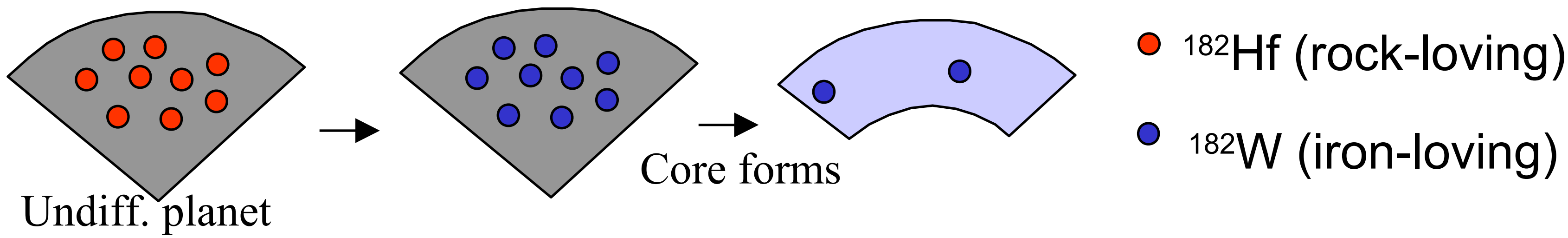
The higher the rate ^{182}Hf/^{180}Hf was (a),
the more the proportion of ^{182}W inside the tungsten (y)
increases with the general Hf/W ratio (x).

ACCRETION TIMESCALE

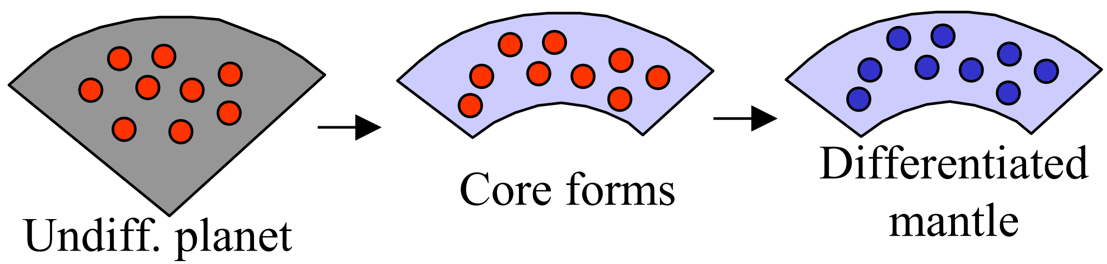
<u>Age of the Moon, through</u> ^{182}Hf / ^{182}W <u>chronometer :</u>

Hf is lithophile → mantle. W is siderophile → core.

Late core formation : no excess ^{182}W in the mantle.



Early core formation : excess ^{182}W in the mantle.



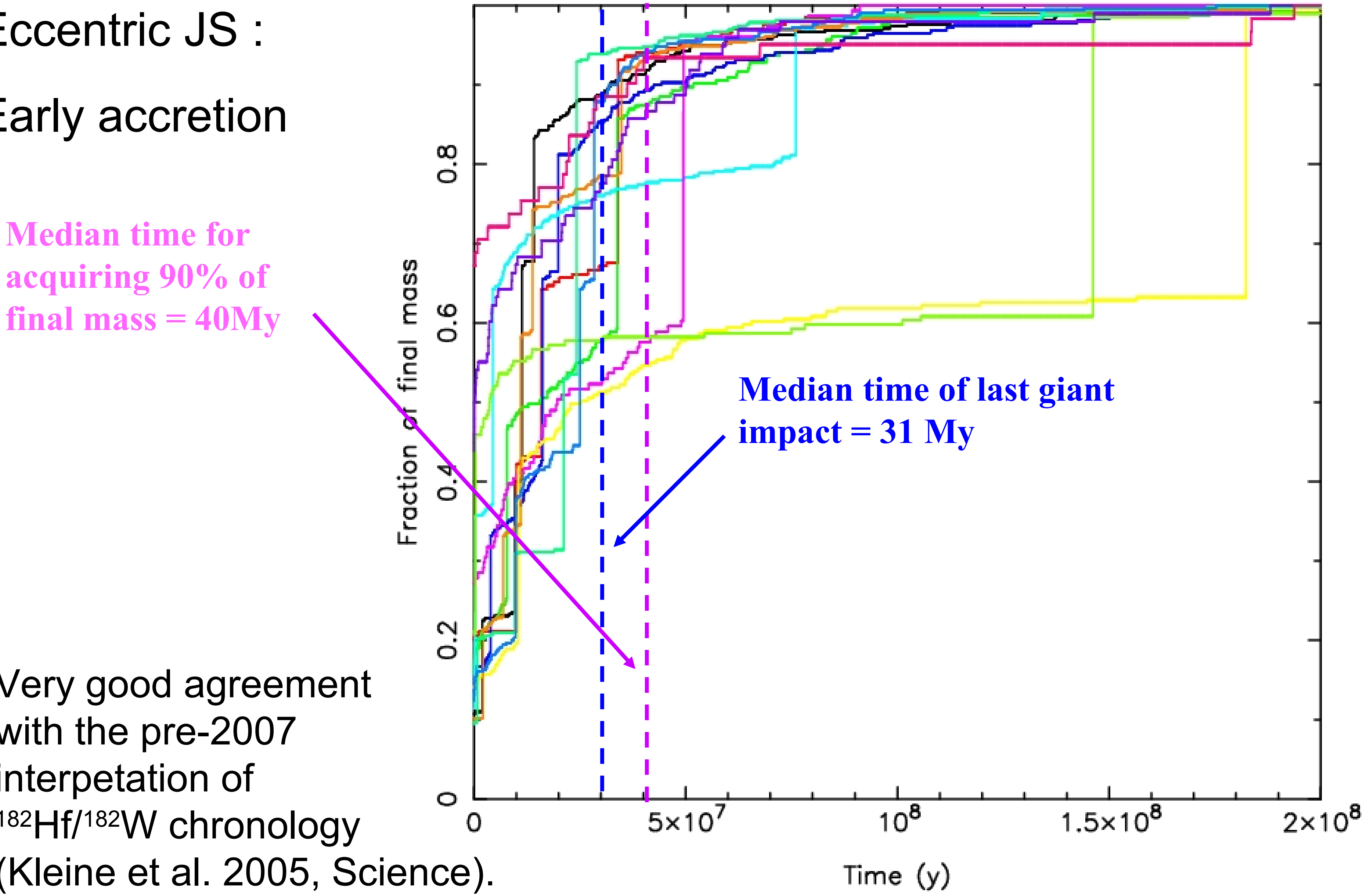
(sketch courtesy: F. Nimmo)

Touboul et al. (2007) : "*The dominant ^{182}W component in most lunar rocks reflects cosmogenic production*"
"*lunar and terrestrial mantles have identical ^{182}W / ^{184}W. This* [...] *constrains the age of the Moon and Earth to 62 Myrs (+90/-10)*".

ACCRETION TIMESCALE

Eccentric JS :

Early accretion



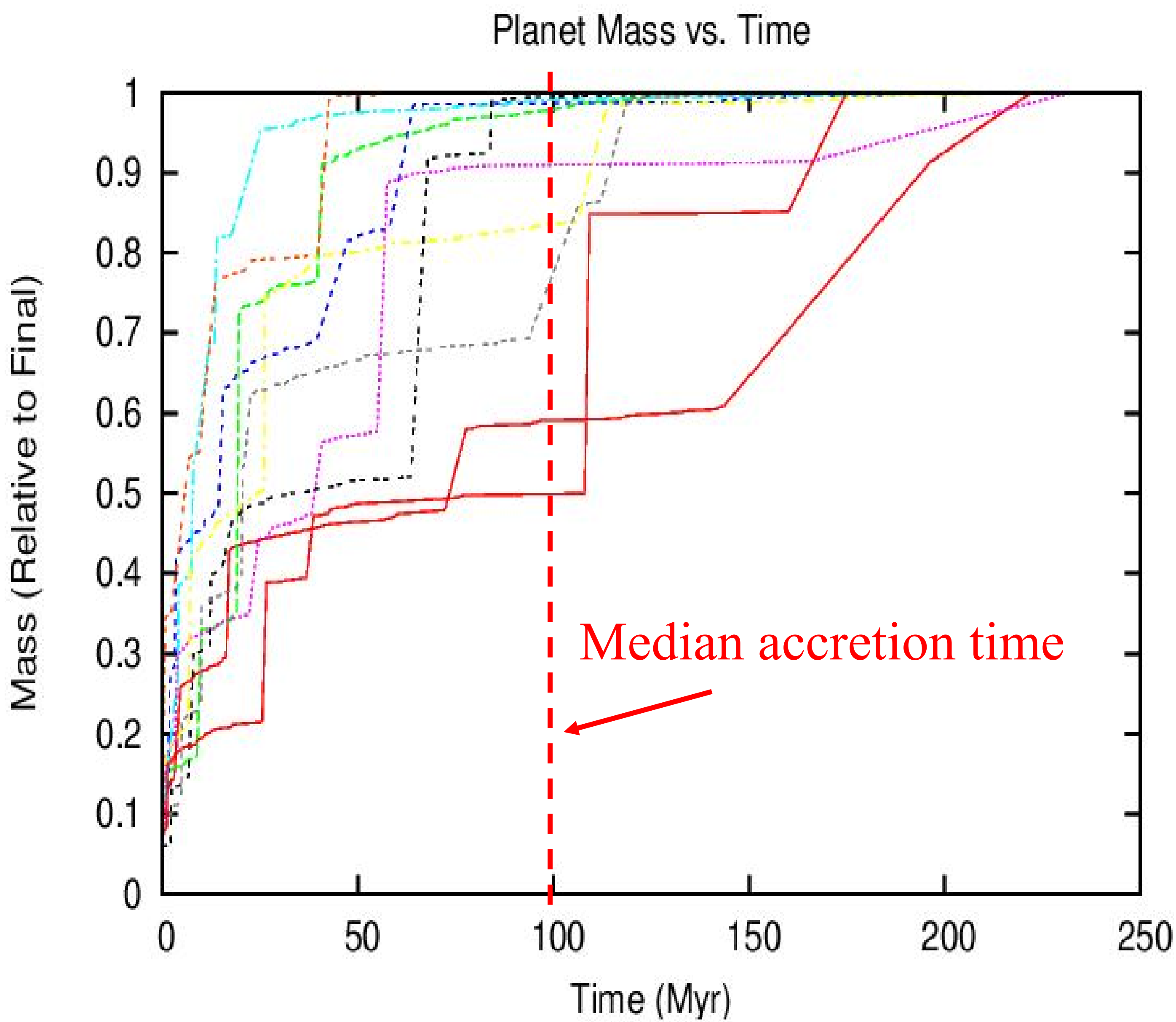
Very good agreement with the pre-2007 interpetation of $^{182}Hf/^{182}W$ chronology (Kleine et al. 2005, Science).

ACCRETION TIMESCALE

Circular JS :

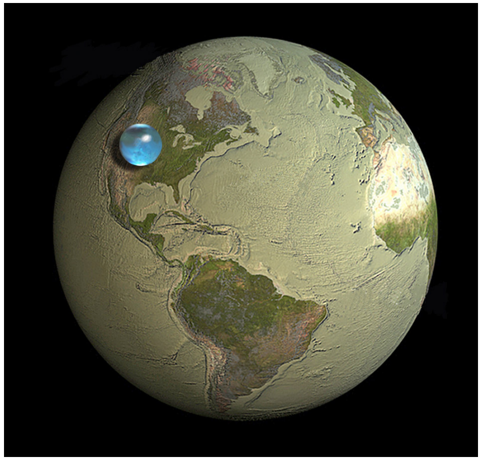
the terrestrial planets accretion timescale also becomes longer (~100 My),

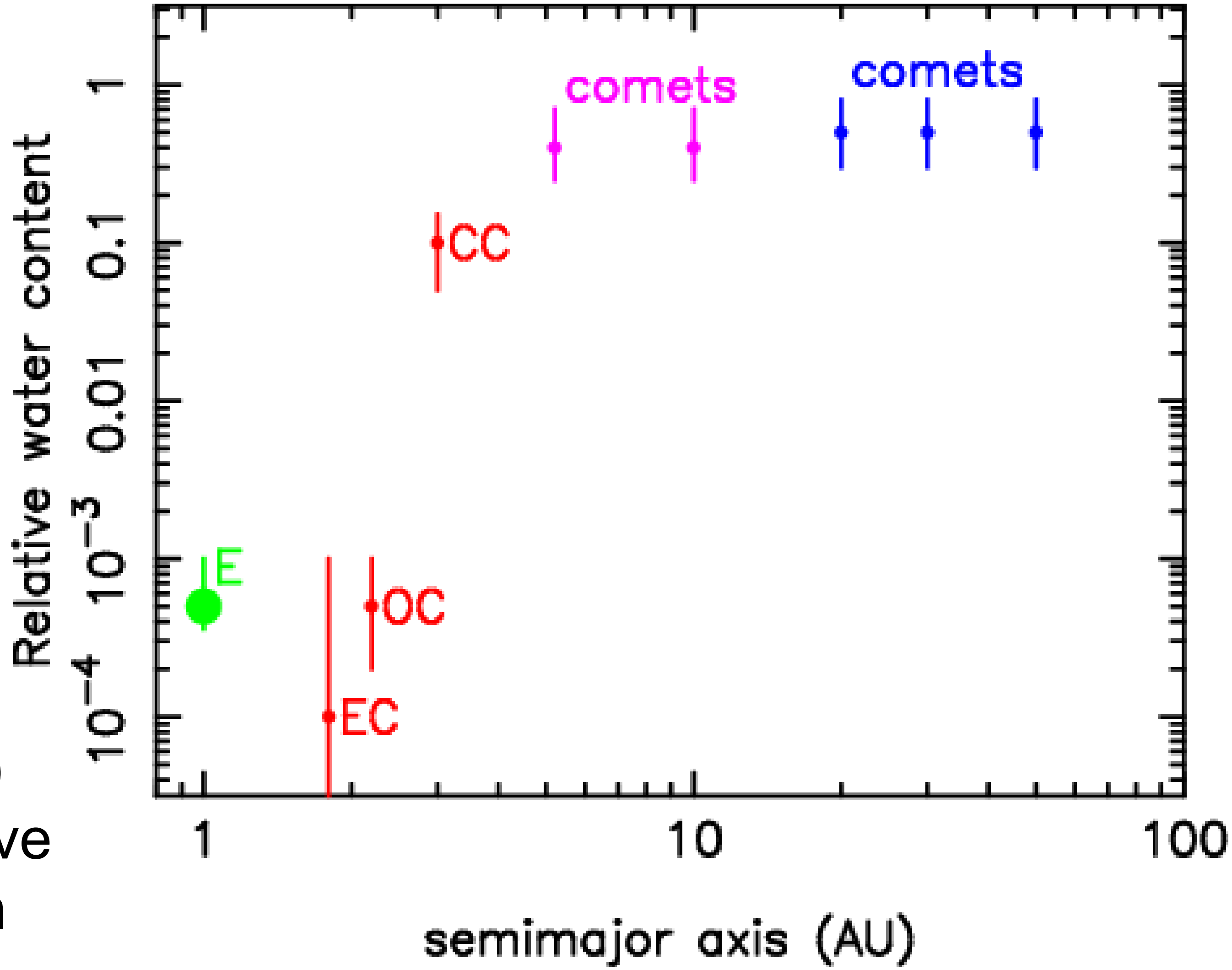
in agreement with the post-2007 datation of the Moon (Touboul et al. 2007)



WATER

- Delivery of water-rich bodies from the asteroid belt to the Earth



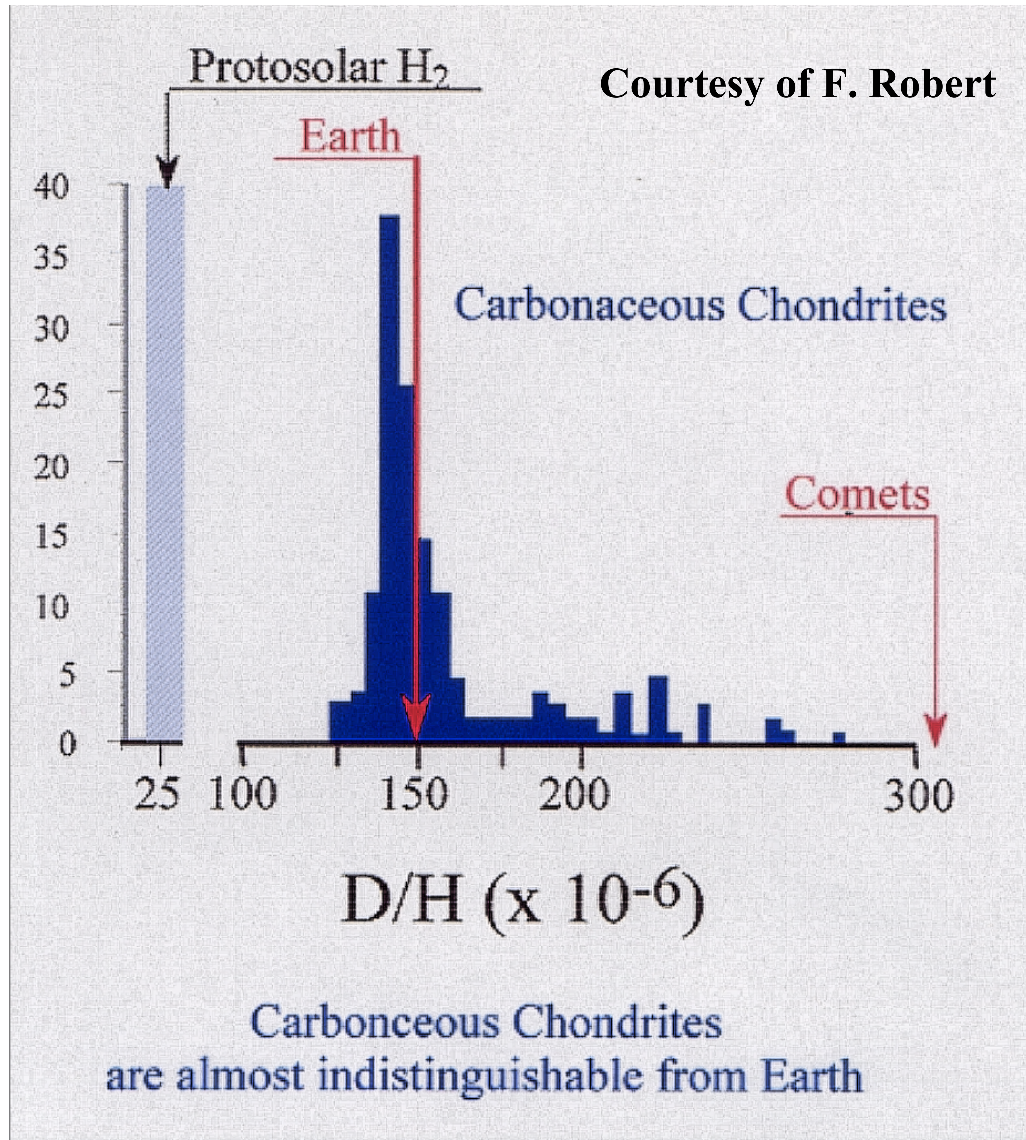


Gradient of relative water content with heliocentric distance (snowline) → water should have been accreted from distant material.

WATER

- Delivery of water-rich bodies from the asteroid belt to the Earth

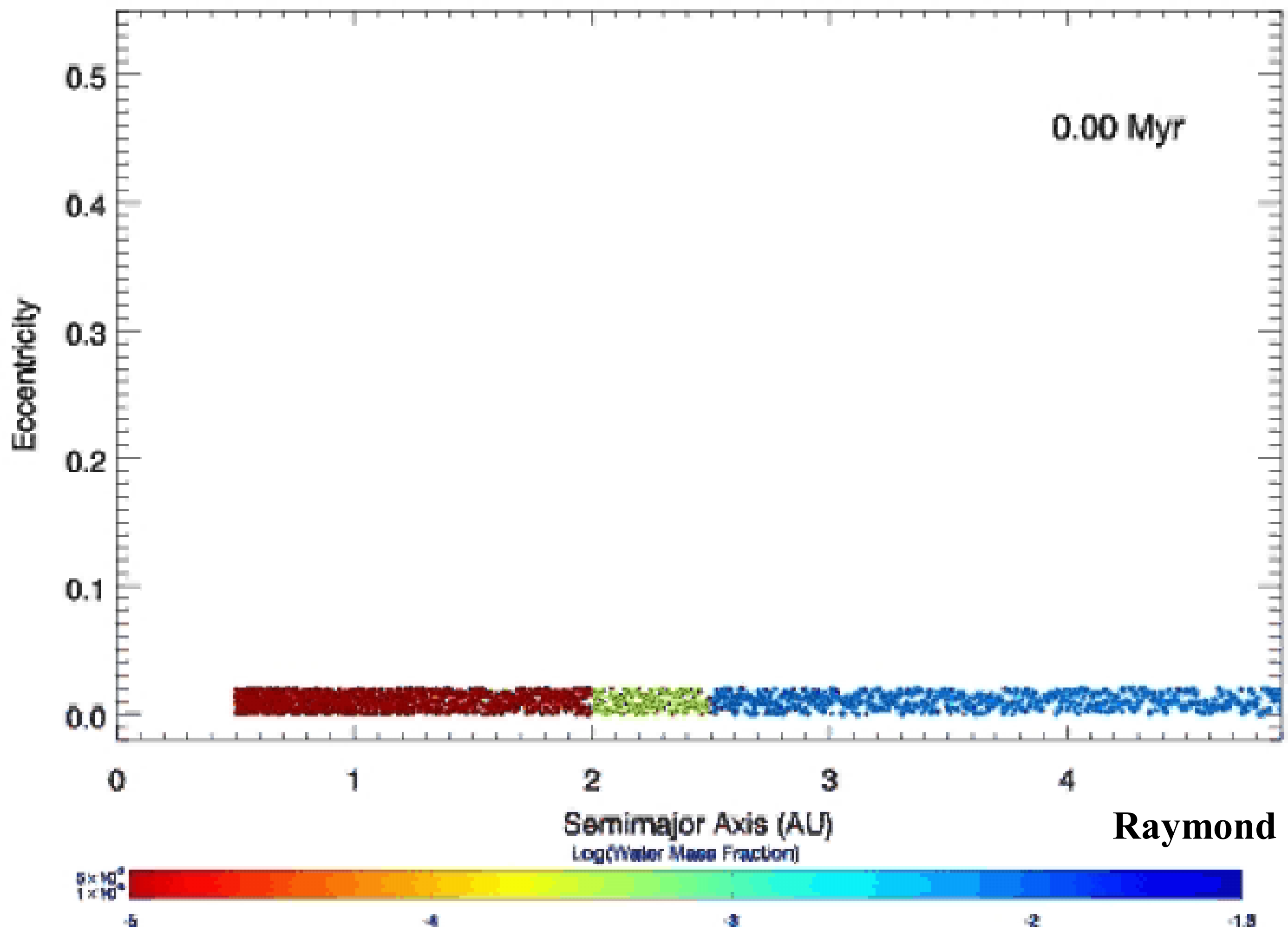
The D/H ratio constraint:



Numerical integrations also show that comets could have contributed at most 10% of the current water on Earth Levison et al. (2000); Morbidelli et al. (2000)

WATER

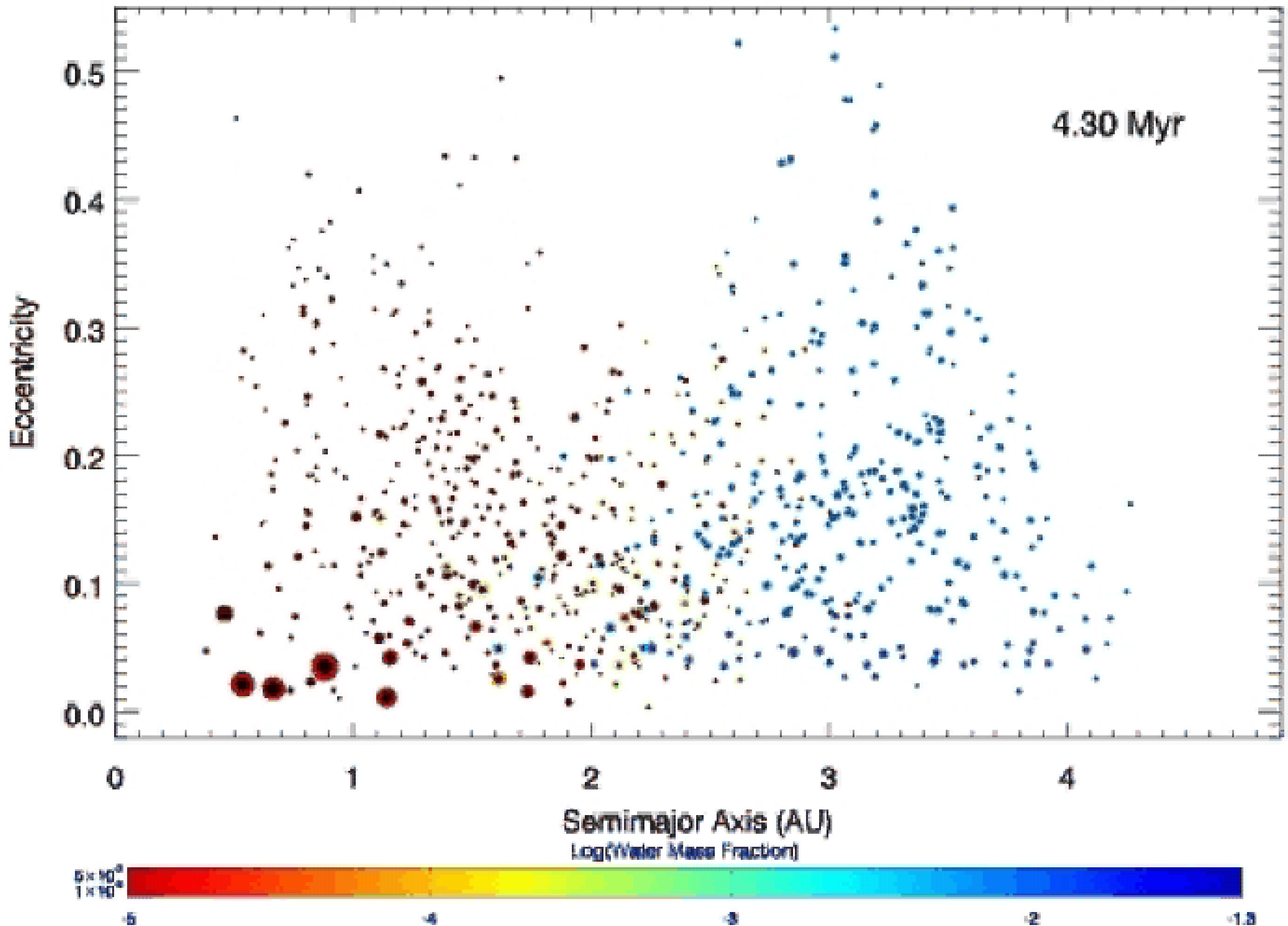
The Earth acquires the water relatively late, but nevertheless during its own accretion process.



Raymond et al., 2006

WATER

The Earth acquires the water relatively late, but nevertheless during its own accretion process.



al., 2006

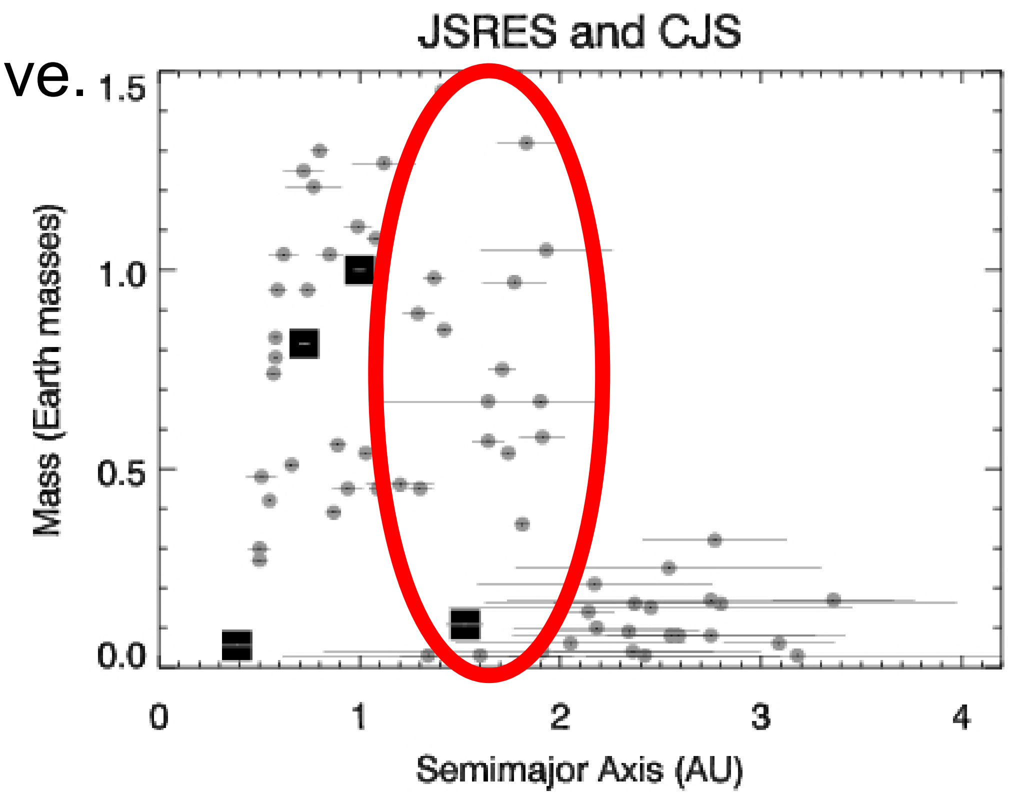
PROBLEM(S)

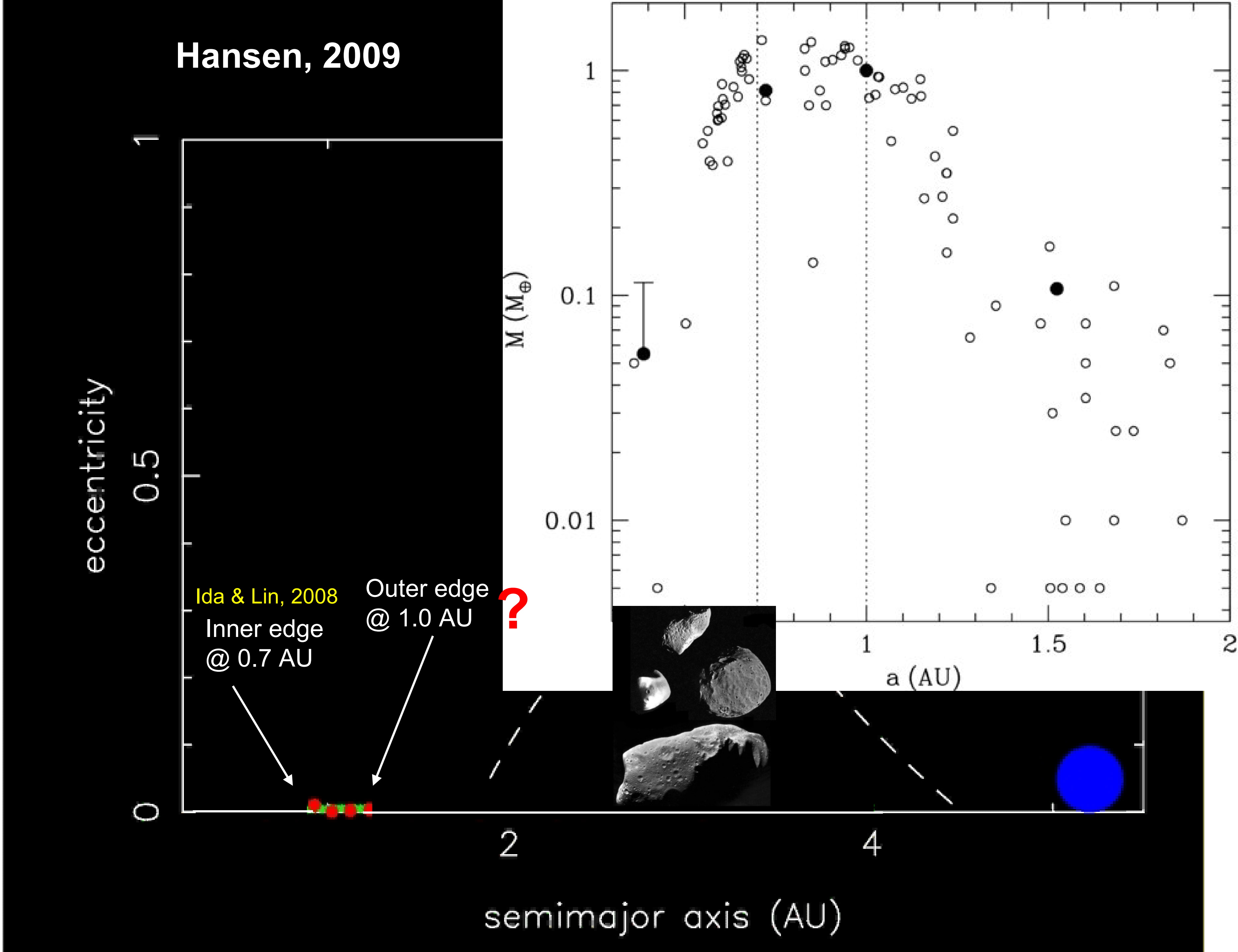
- Things work a little bit better if Jupiter and Saturn are not on their present orbits, but on cirular orbits.

Note: They should form on circular orbits in the gas disk...

- Mars is always too massive.

Only possible solution:
start from a disk of embryos and planete--simals truncated between 0.7 and 1 AU (Hansen 2009).




Hansen, 2009
eccentricity
1
0.5
0
semimajor axis (AU)
2
4
Ida & Lin, 2008
Inner edge
@ 0.7 AU
Outer edge
@ 1.0 AU
?
M ($M_\oplus$)
1
0.1
0.01
a (AU)
1
1.5
2