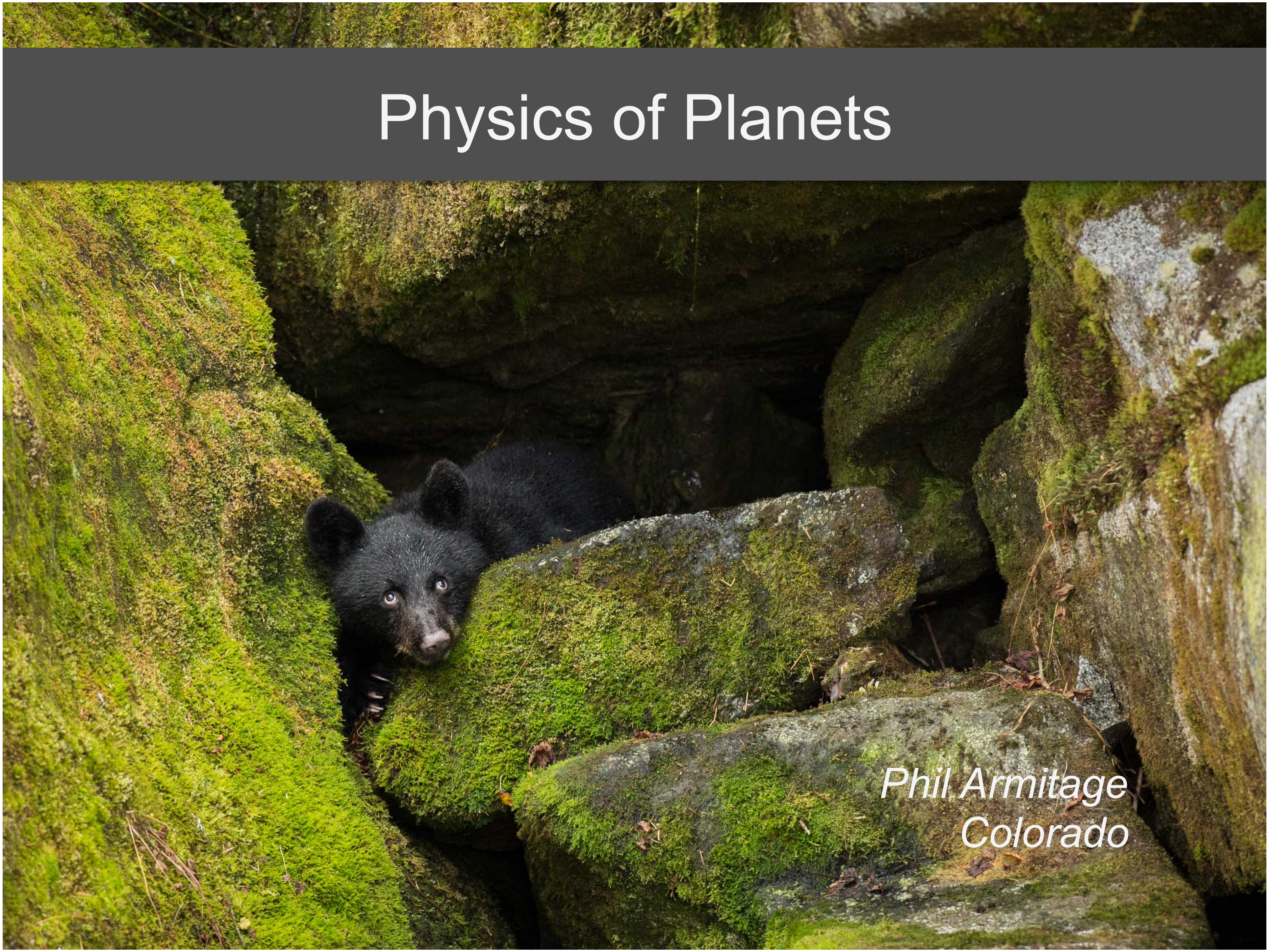
Physics of Planets



Phil Armitage
Colorado

α-model disks

Can make a predictive theory if we can write ν as a function of other disk parameters (T, r, ρ, x_e…)

$$\boxed{\nu = \alpha c_s h}$$

Shakura-Sunyaev '73 α-prescription

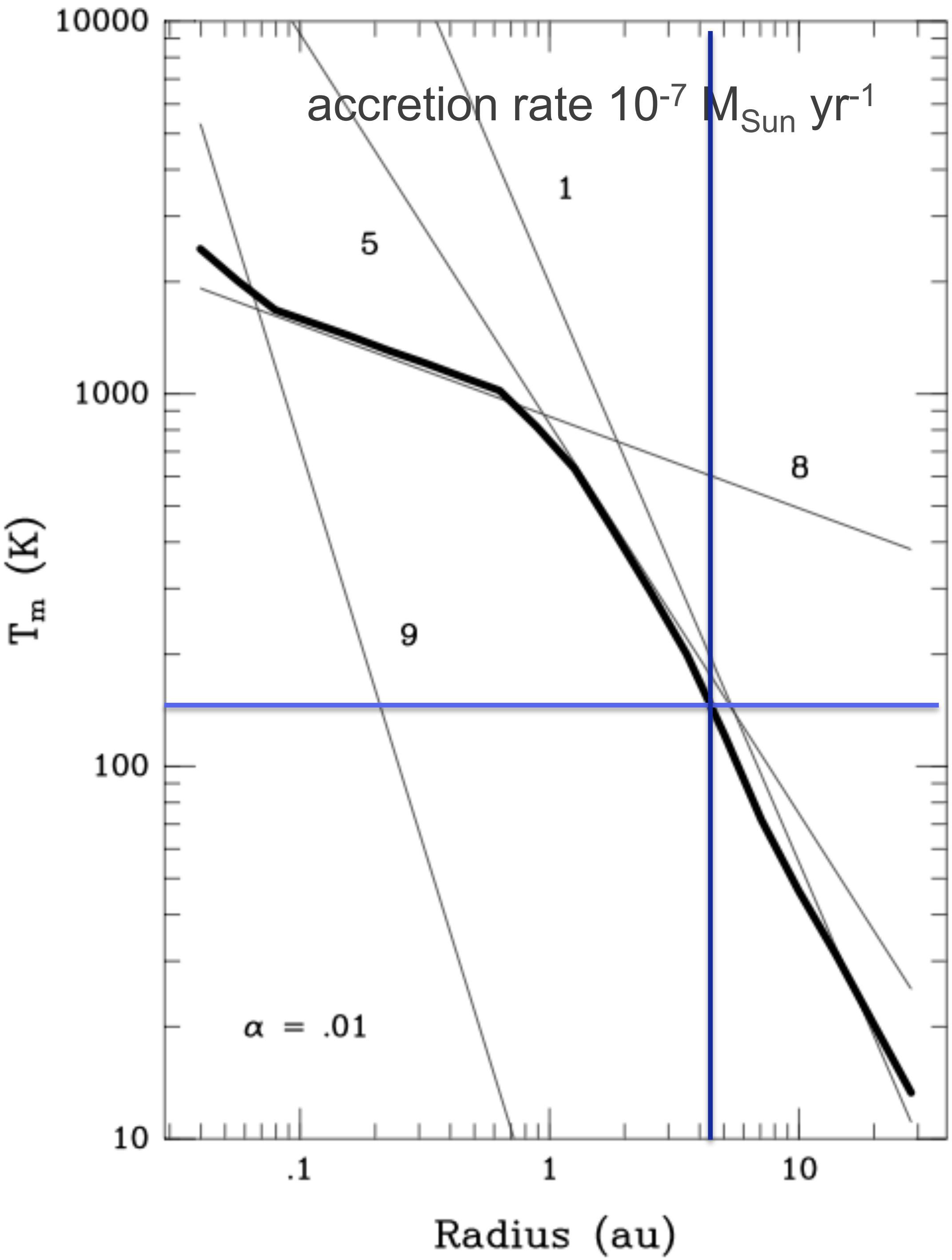
For α assumed constant, one parameter description of protoplanetary disk evolution

Identify disk lifetime with the viscous time at outer edge

$$t_\nu = \left(\frac{h}{r}\right)^{-2} \frac{1}{\alpha \Omega}$$

t_ν = 1 Myr at 30 AU, (h / r) = 0.05

$\Rightarrow$ $\boxed{\alpha = 0.01}$



If irradiation dominates, with fixed $T \sim r^{-1/2}$, then an α-disk is equivalent to $\nu \sim r$ (since $\nu = \alpha c_s^2 / \Omega$)

An α model predicts the time-varying radial (and vertical) structure for any accretion rate

e.g. snow line near 4 AU for this model

Bell et al. '97

$$\boxed{\nu = \alpha c_s h}$$

We can always choose to express the efficiency of angular momentum transport in terms of α

α-disk theory is useful **if** it encodes the "leading order" dependence of the stress on the local disk properties, i.e. so that α is a slowly varying function of Σ, r etc

Various caveats:

- α likely a strong function of T, Σ, *if* transport is due to MHD processes
- vertical structure also depends on how accretion energy is distributed vertically… even more uncertain
- for comparison against observations, reducing a possibly complex function to one number

Multiple motivations

Turbulence

- **angular momentum transport**
  - disk evolution, episodic accretion, planet-disk interactions

- **radial and vertical mixing / diffusion**
  - dust settling, chemistry, *Stardust* sample interpretation…

- **concentration of particles**
  - observations of transition disks, prelude to planetesimal formation, meteoritics

Turbulence

Fluid turbulence:

Define **Reynolds number** $\mathrm{Re} = \frac{UL}{\nu_m}$

Velocity U ~ km s^{-1}, L ~ AU, $\nu_m \sim 10^6$ cm^2 s^{-1}

➡ Re ~ 10^{12}

Any turbulence present will be fully developed – large inertial range

BUT linear stability of shear flow is given by Rayleigh criterion:

$$\frac{\mathrm{d}l}{\mathrm{d}r} < 0$$

…for *instability*, l ~ $r^{1/2}$ for Keplerian flow so disks are linearly stable to infinitesimal hydrodynamic perturbations

Linear instabilities

Consider a background disk model that is:

- described by some set of physics (isothermal hydrodynamics, MHD, hydro + self-gravity…)
- in equilibrium

Linearize equations, perturb with e.g. $\mathbf{s} \propto e^{i(\omega t - k\mathbf{r})}$

System is unstable if there are growing modes: $\omega^2 < 0$

Main disk instabilities are also **local**, apply in a "patch" of disk where shear is linearized, do not depend on boundary conditions

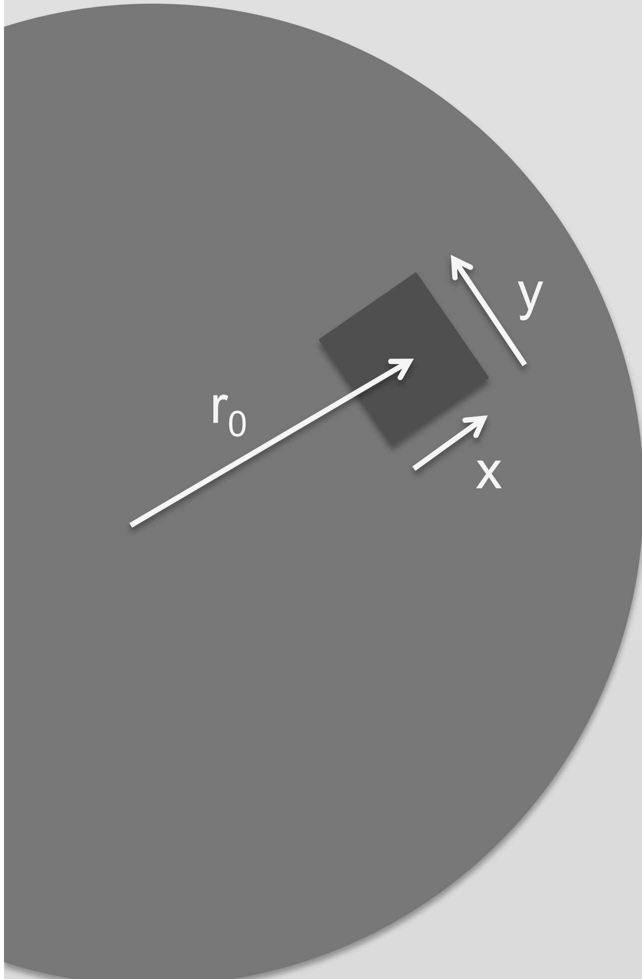
Interested in existence, growth rate, non-linear outcome

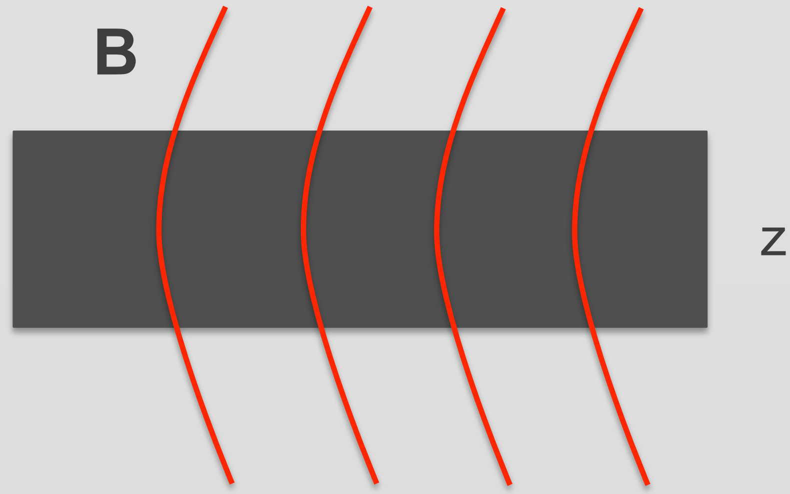
1. Magnetorotational instability

Balbus & Hawley ‘91, review Rev. Mod. Phys. ‘98

Physics: hydrodynamics ✔
magnetic fields ✔
self-gravity ✗
energy equation ✗

1. Magnetorotational instability





$$r = r_0 + x$$

$$y = \Omega t + \frac{y}{r_0}$$

Define cartesian co-ords for a small patch of disk (x,y,z)

Initial equilibrium state has a weak, uniform vertical magnetic field B_z, assume ideal MHD

Equations of motion

$$\ddot{x} - 2\Omega\dot{y} = -x\frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln r} + f_x$$
$$\ddot{y} + 2\Omega\dot{y} = f_y$$

coriolis force

magnetic tension force on the gas

Perturb: $\mathbf{s} \propto e^{i(\omega t - k\mathbf{r})}$

$$\delta\mathbf{B} = -ikB_z\mathbf{s}$$ perturbed magnetic field

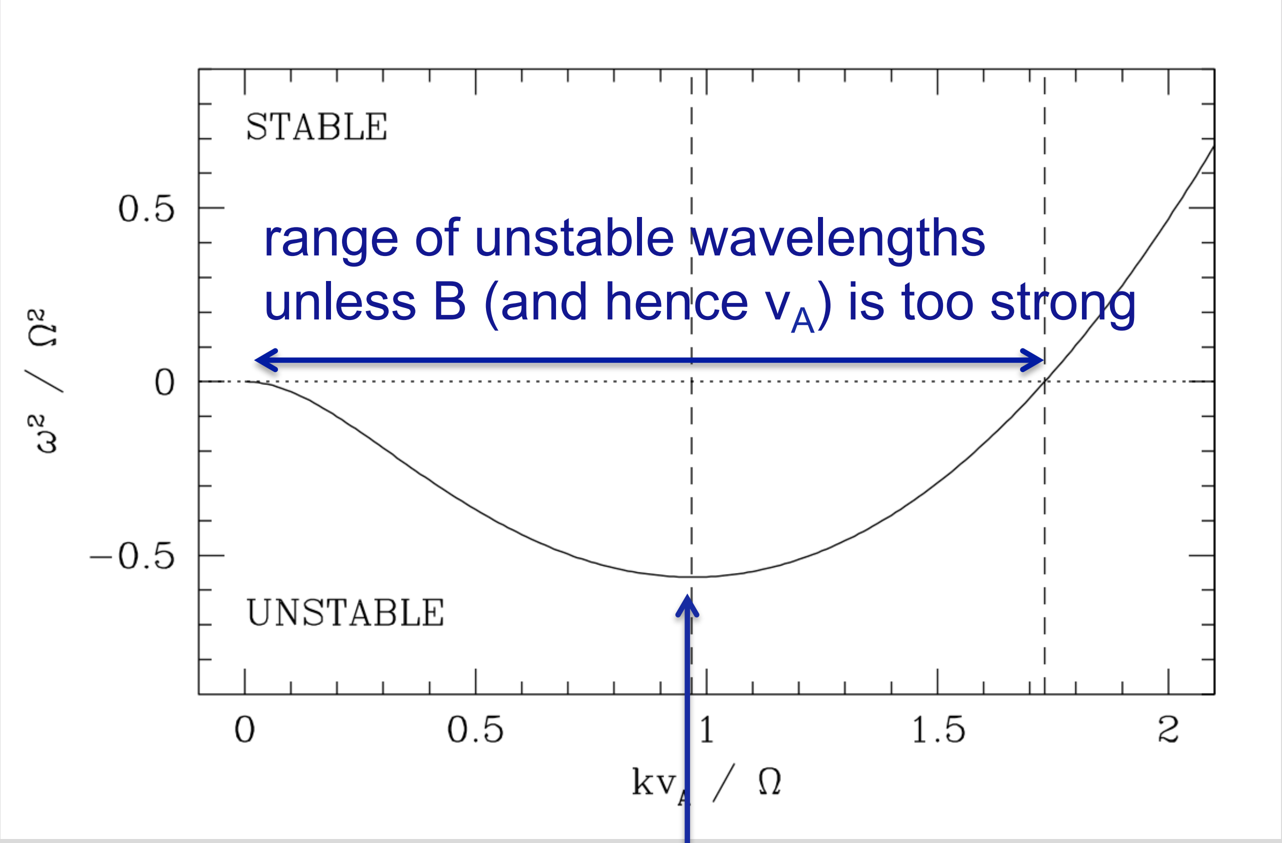
$$\mathbf{f} = -(kv_A)^2\mathbf{s}$$ magnetic tension force

$$v_A \equiv \sqrt{B_z^2/4\pi\rho}$$ Alfven speed in the gas

…substitute in the equations of motion

Dispersion relation:

$$\omega^4 - \omega^2 \left[\frac{d\Omega^2}{d\ln r} + 4\Omega^2 + 2\left(kv_A\right)^2 \right] + \left(kv_A\right)^2 \left[\left(kv_A\right)^2 + \frac{d\Omega^2}{d\ln r} \right] = 0$$



For a weak magnetic field, unstable if

$$\frac{d}{dr}\left(\Omega^2\right) < 0$$

In ideal MHD, always unstable

most unstable wavelength with growth rate $\omega \sim \Omega$ i.e. very fast!

6. Influence of the magnetic field on the stability of the rotating cloud

Some idea of the magnetic field's influence on the stability of the rotating cloud can be arrived at from certain results of Chandrasekhar (1961) for the motion of fluids between revolving cylinders (Couette flow) in the cases of a magnetic field H_z parallel to the axis of rotation and H_φ along the direction of rotation. For a field H_z of infinite conductivity, the stability condition is found to be

$$I_1 \frac{\mu H^2}{4\pi\rho} > -\int_{R_1}^{R_2} \frac{d\omega^2}{dR} R^2 \xi_R^2 \, dR. \qquad (27)$$

This result is somewhat unexpected, since the above does not reduce to Rayleigh's criterion when $H \to 0$. For a vanishingly small field when ω is a monotonic function of R, the necessary and sufficient condition for instability is that ω increase with R. In the protoplanetary cloud ω decreases with R

Safronov '72 – the MRI was *almost* discovered and understood by Chandrasekhar, Velikov & Safronov in the 1960s…

Conclusion that weakly magnetized disks are always violently unstable applies to *ideal MHD* (i.e. ionized disk), several complexities in protoplanetary disks

- currents decay due to collisions (**Ohmic** dissipation)
- magnetic field couples to charged particles, neutrals couple only via ion-neutral collisions (**ambipolar diffusion**)
- charge carriers moving in a magnetic field experience a Lorentz force, creating an additional electric field (**Hall effect**)

Very roughly, these "non-ideal" MHD effects damp the MRI in regions where the ionization fraction and / or density are very low

Ohmic Hall Ambipolar

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B} - \frac{\mathbf{J} \times \mathbf{B}}{e n_e} + \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{c \gamma \rho_i \rho} \right]$$

Ideal MHD: MRI grows on scale h on time scale:

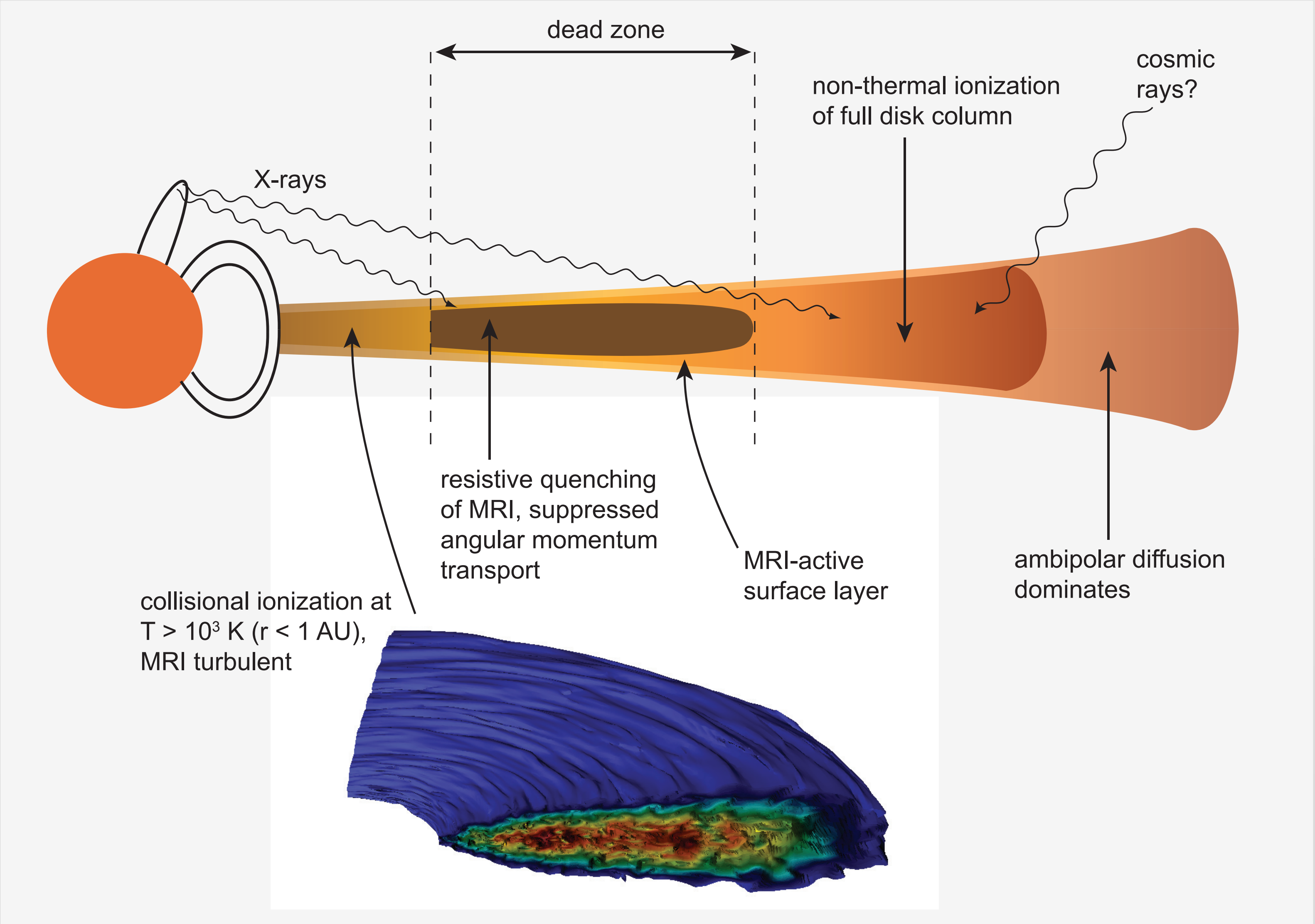
Diffusive term, leads to damping on time scale:

$$\tau \sim \frac{h}{v_A} = \tau \sim \frac{h^2}{\eta}$$

…with η the magnetic diffusivity ~ 1 / conductivity

MRI damped for $\eta > hv_A$
For inner disk $x_e > 10^{-12}$

Argument: *Gammie '96*



If MHD processes are dominant, expect suppressed turbulence near the mid-plane – a “dead zone”

2. Self-gravity

Physics: hydrodynamics ✔
magnetic fields ✗
self-gravity ✔
energy equation ✗

Linear instability if "Toomre Q" is below critical value:

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \lesssim Q_{\rm crit} \sim 1$$

Roughly this requires $M_{disk} / M_* > h / r$,
expect self-gravity to matter for massive disks

3. Vertical shear instability

Physics: hydrodynamics ✔
magnetic fields ✗
self-gravity ✗
energy equation ✔

$$\frac{\partial l^2}{\partial r} - \frac{k_r}{k_z}\frac{\partial l^2}{\partial z} < 0$$

and thermal diffusion due radiation, heating + cooling processes is "fast"

radial gradient of specific angular momentum

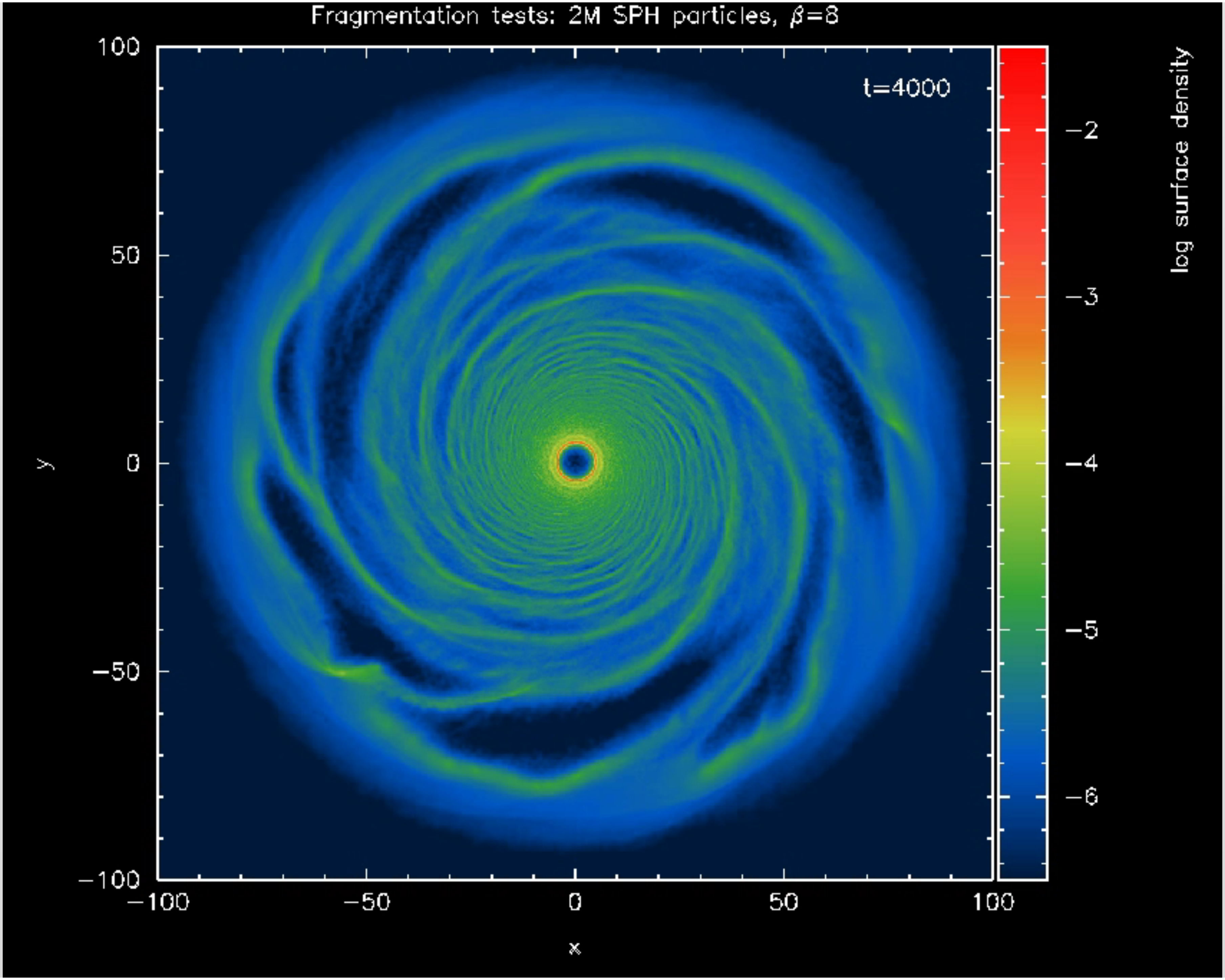
vertical gradient of specific angular momentum

Nelson, Gressel & Umurhan '13

Non-linear evolution

With few exceptions, need numerical simulations to assess:

- non-linear evolution of disk instabilities
- saturation level
- nature of turbulence (waves, vortices…)



Fragmentation tests: 2M SPH particles, β=8
t=4000
log surface density
x
y

Self-gravity: trailing spiral arms, "gravito-turbulence"

Assuming locality, condition that Q ~ Q_{crit} can be used to *analytically* estimate the efficiency of angular momentum transport *(Gammie '01)*

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \lesssim Q_{\rm crit} \sim 1$$

Heating rate $(9/4)\nu\Sigma\Omega^2$ must balance cooling σT^4

$$\alpha = \frac{4}{9\gamma(\gamma-1)} \frac{1}{t_{\rm cool}\Omega}$$

where t_{cool} is the thermal energy per unit area / cooling rate, and γ is the adiabatic index of the gas

"Clumpiness" of the disk increases as t_{cool} decreases, generally thought that fragmentation occurs for low $t_{cool}\Omega$

Vertical shear instability

Fluid stresses with $\alpha \sim 10^{-3}$

Nelson et al. '13

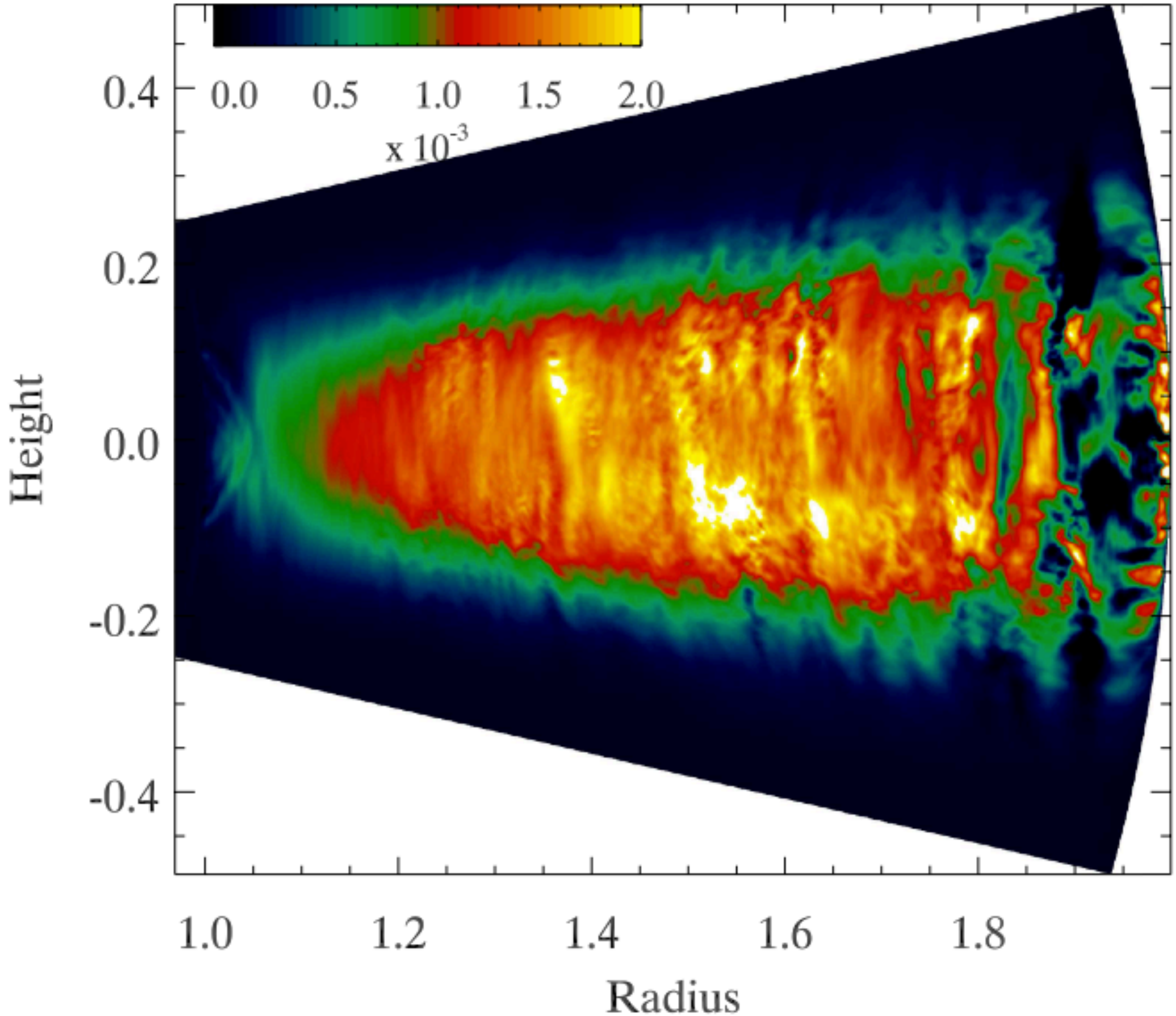
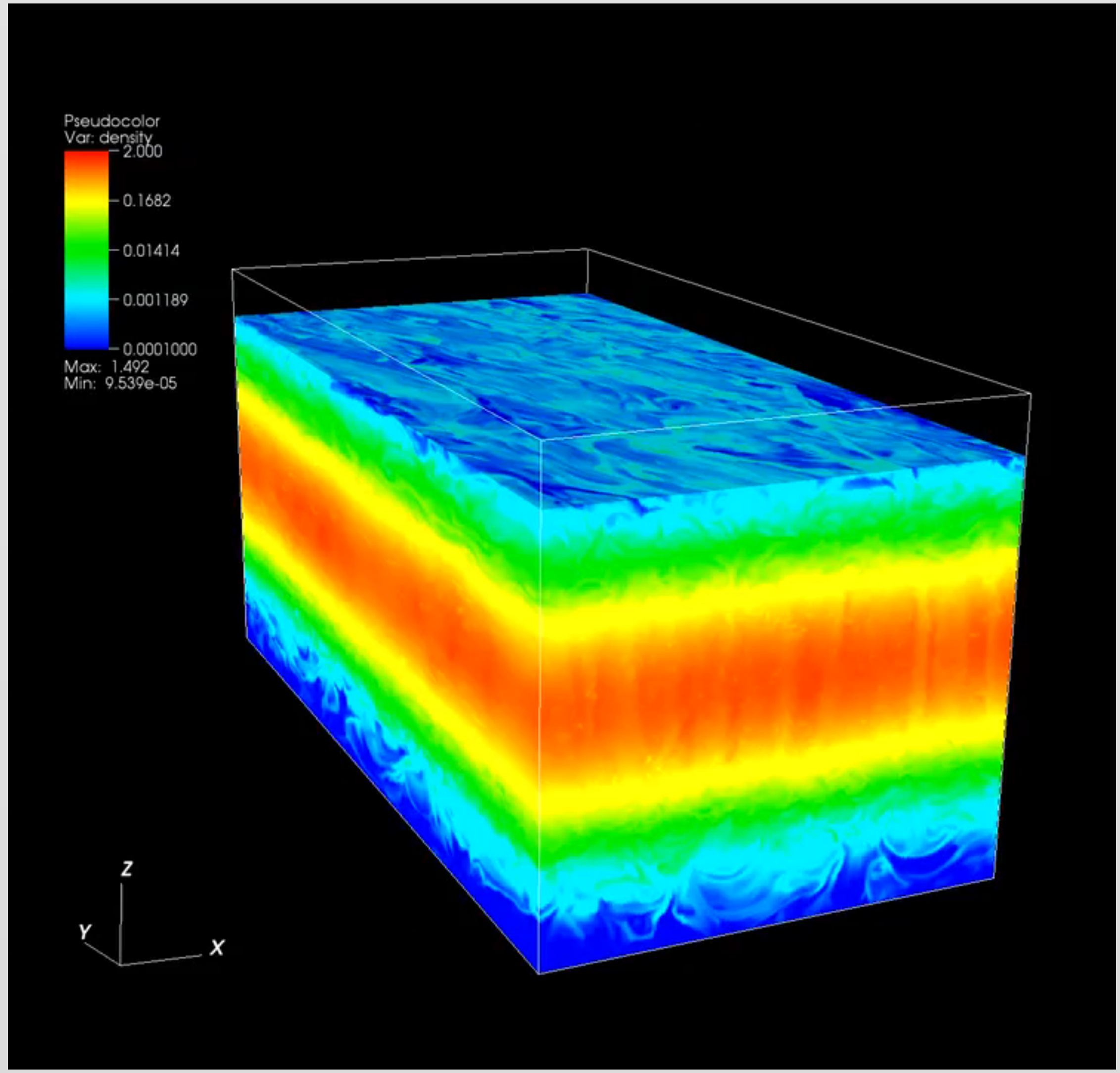


Figure 16. Spatial distribution of the time and horizontally averaged Reynolds stress (normalized by the mean pressure at each radius) for model T1R-0–3D.

Magnetorotational instability

In ideal MHD, leads to turbulence with $\alpha \sim 0.02$, with most of the stress coming from magnetic rather than fluid stresses

Relevant only to inner protoplanetary disks where $T > 10^3$ K



Simon, Beckwith & Armitage '12

Magnetorotational instability

Relationship of magnetic field instabilities
to turbulence in the cool part of the disk
where non-thermal ionization dominates…

Secure results:

- Ohmic diffusion damps the MRI near the mid-plane in the terrestrial planet-forming region
- ambipolar diffusion damps mid-plane turbulence in the outer disk, strongly if there is no net B_z

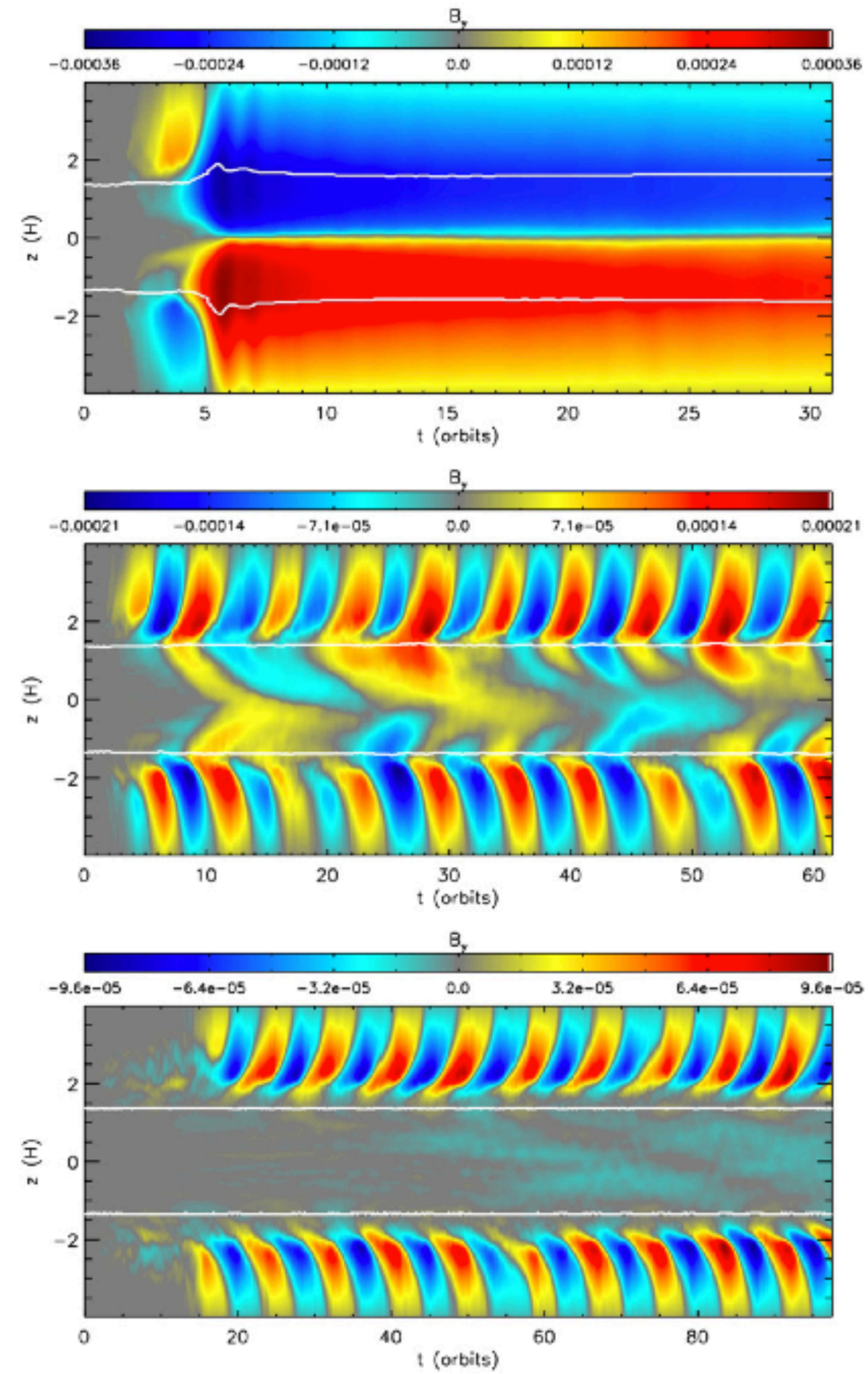
Provocative results:

- MRI + net field leads to disk winds
- where Hall effect dominates, get a **laminar** magnetic stress whose strength depends on **sign** of B_z

Simulations of the outer disk have turbulent surface layers, ambipolar damping near the mid-plane

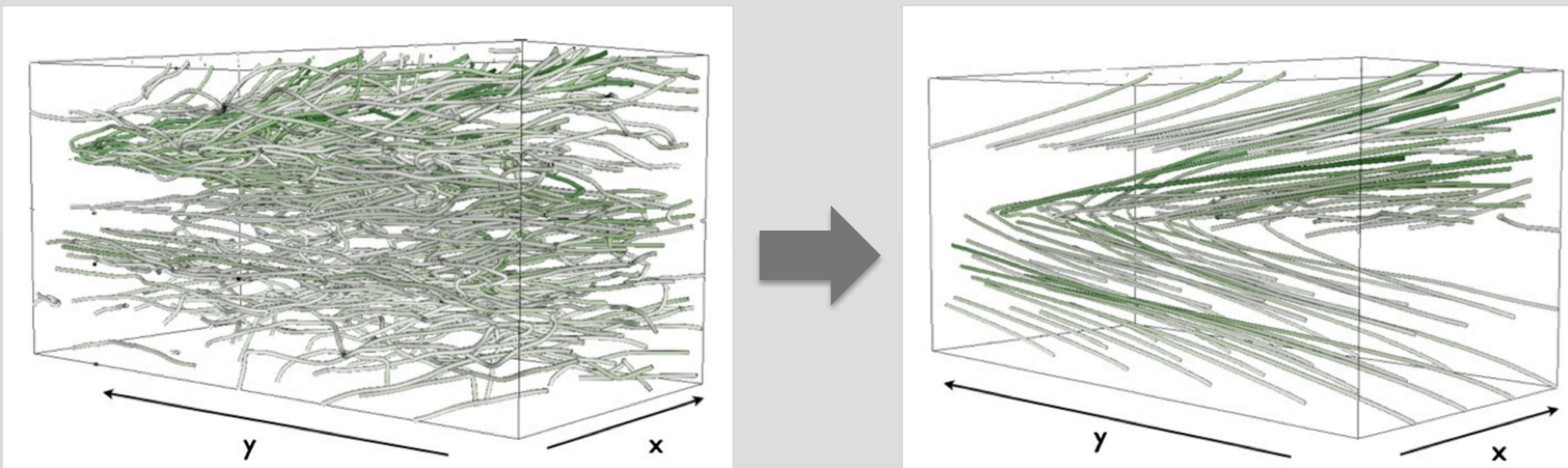
$$\beta_z = \frac{P_{\rm gas}}{B_z^2/8\pi} = 10^4$$

Stress and nature of the solution strong function of how much magnetic field threads the disk *(Simon et al. 2013)*



Vertical fields lead to a transition from turbulence to disk winds, which *may* carry away significant mass and angular momentum

$\beta_z = 10^4$ $\beta_z = 10^3$

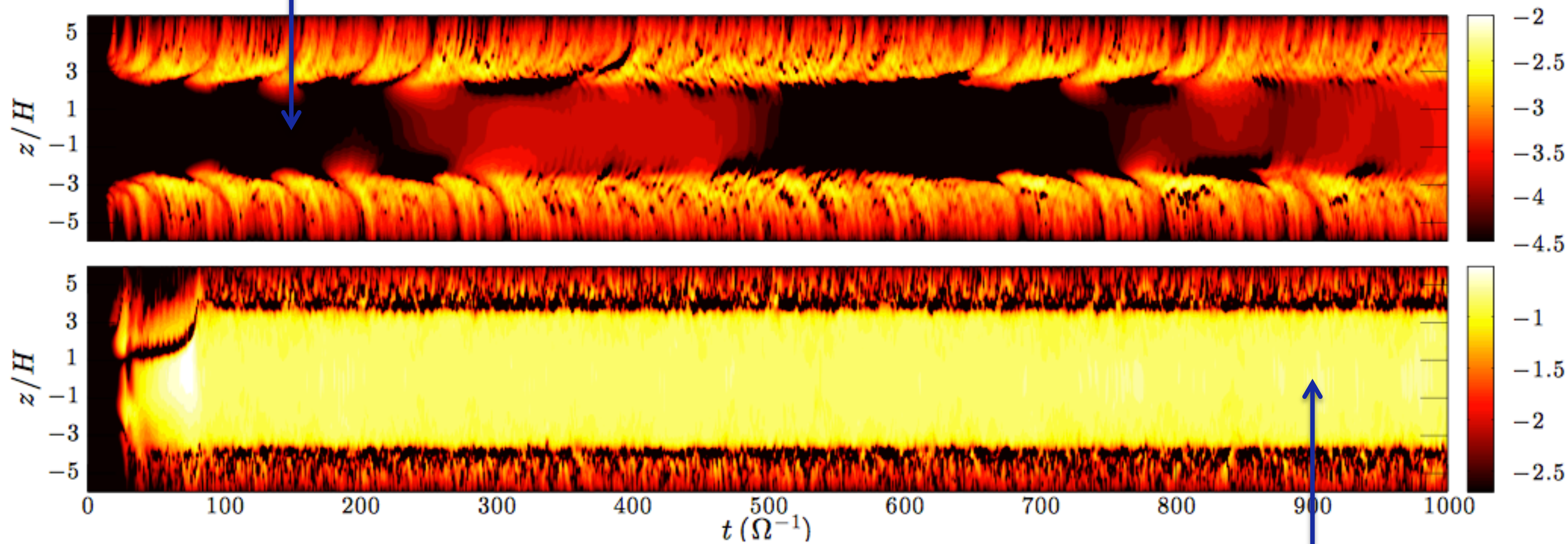


Simon et al. 2013

Generic result (Suzuki & Inutsuka ’09; Fromang et al. ‘13; Bai & Stone ‘13; Lesur et al. ‘13)

At r ~ AU, expect Hall effect to be dominant term

Ohmic diffusion only, get a dead zone with very weak mid-plane stress / turbulence

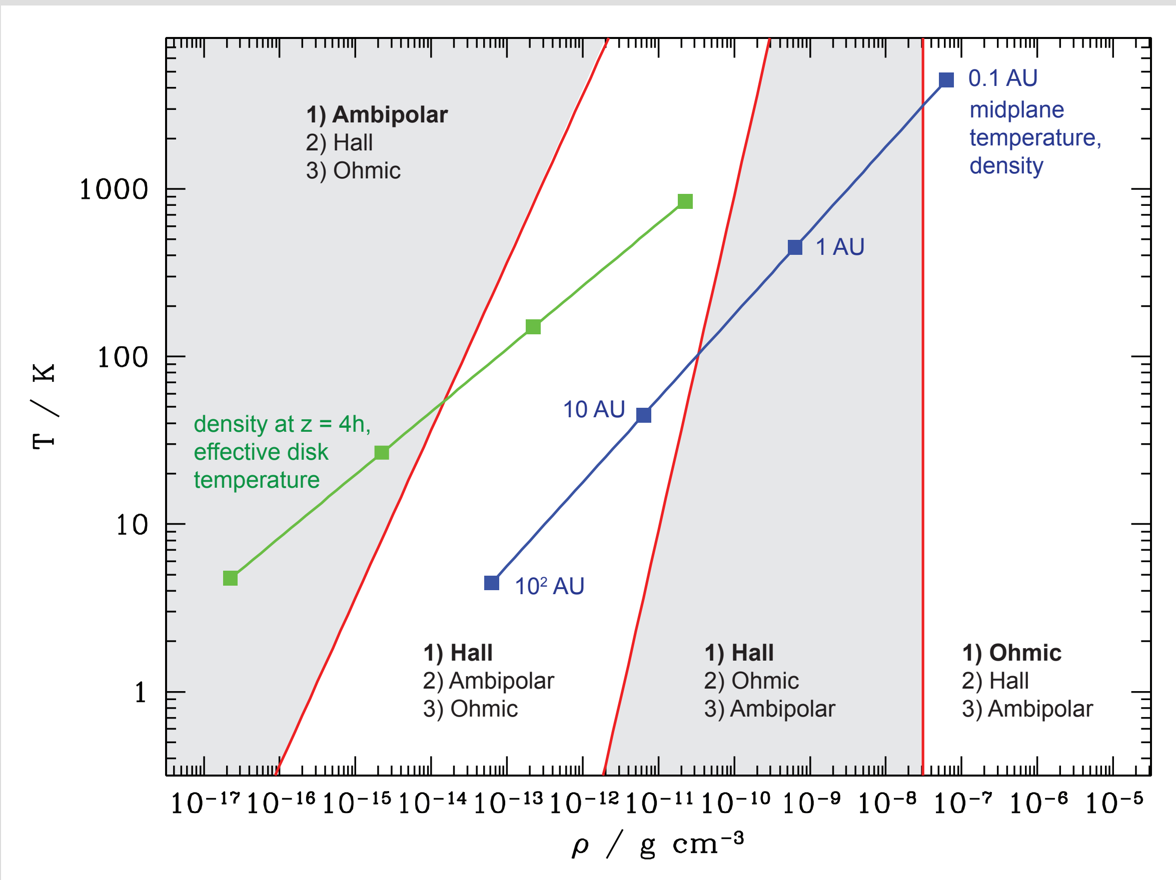


with the Hall effect, strong laminar transport of angular momentum due to Maxwell stress B_rB_ϕ

Lesur, Kunz & Fromang '14

Hall effect depends on the **sign** of (**Ω.B**)

Predict different levels of stress and turbulence in disks where the net field is aligned / anti-aligned with rotation



Is disk structure bimodal on ~AU scales?

Armitage, ARA&A '11

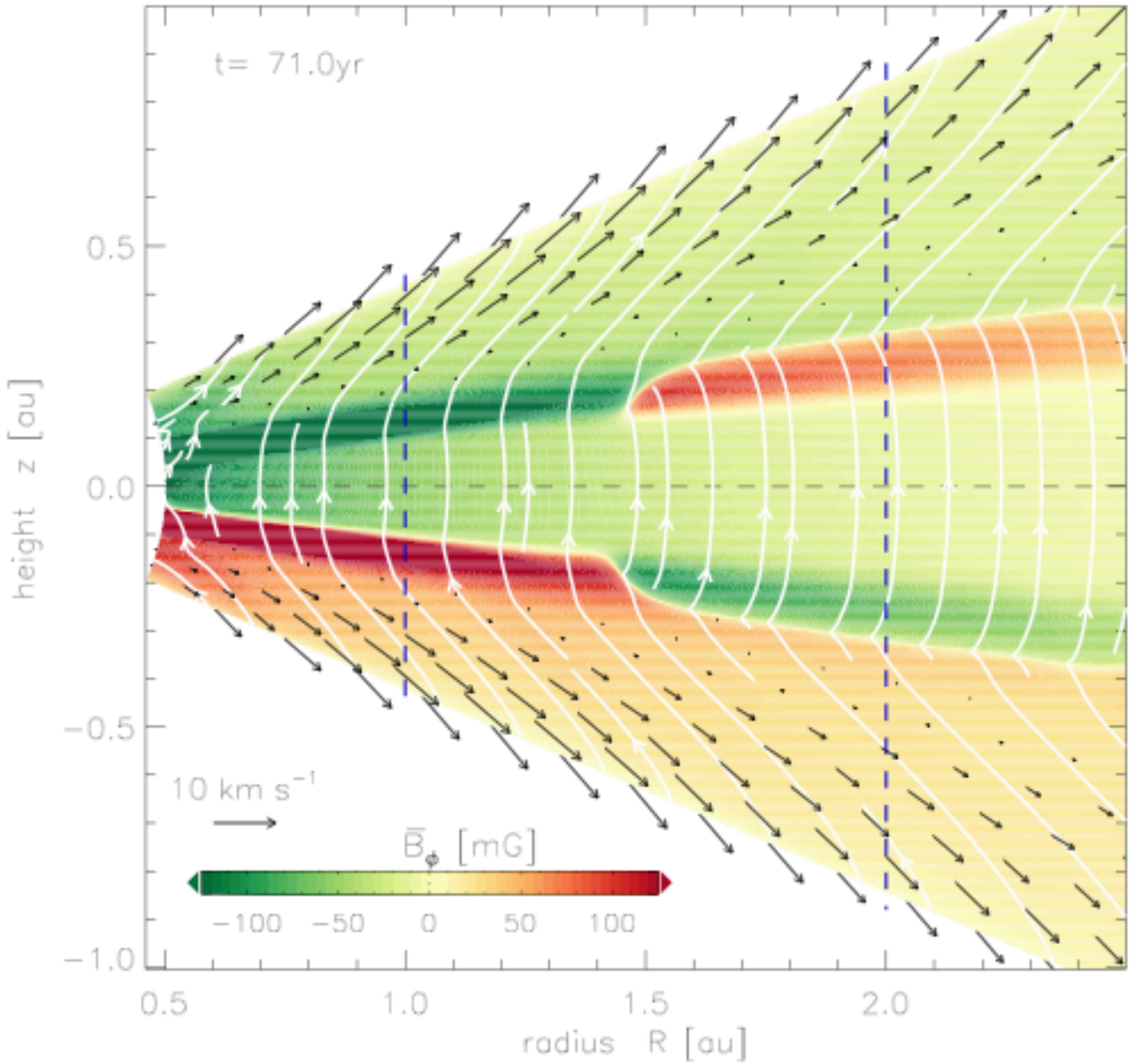


Figure 5. Field topology of our fiducial simulation at different evolution times. The azimuthal magnetic field (color) has been restricted to values $|B_\phi| < 125\,\mathrm{mG}$ for clarity; peak values are a few hundred mG. We also show projected magnetic field lines (white) and velocity vectors (black). Additional lines indicate the position, z_b, of the wind base (dot-dash), and the radial location of the profiles plotted in Figs. 6 and 7 (dashed lines).

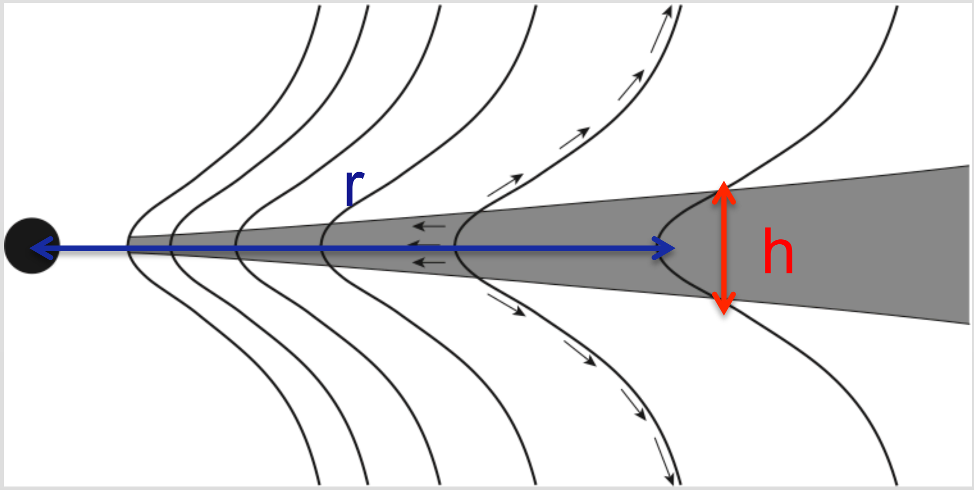
Gressel et al. '15: disk models with net field defined via poloidal pressure with respect to gas pressure:

$$\beta_z \equiv \frac{B_z^2/8\pi}{\rho c_s^2}$$

Here $\beta_z \sim 10^5$

Observational constraints on this scenario?

If net field is important, what determines how B_z evolves?



Standard answer: assume turbulence provides both an effective viscosity and an effective magnetic resistivity

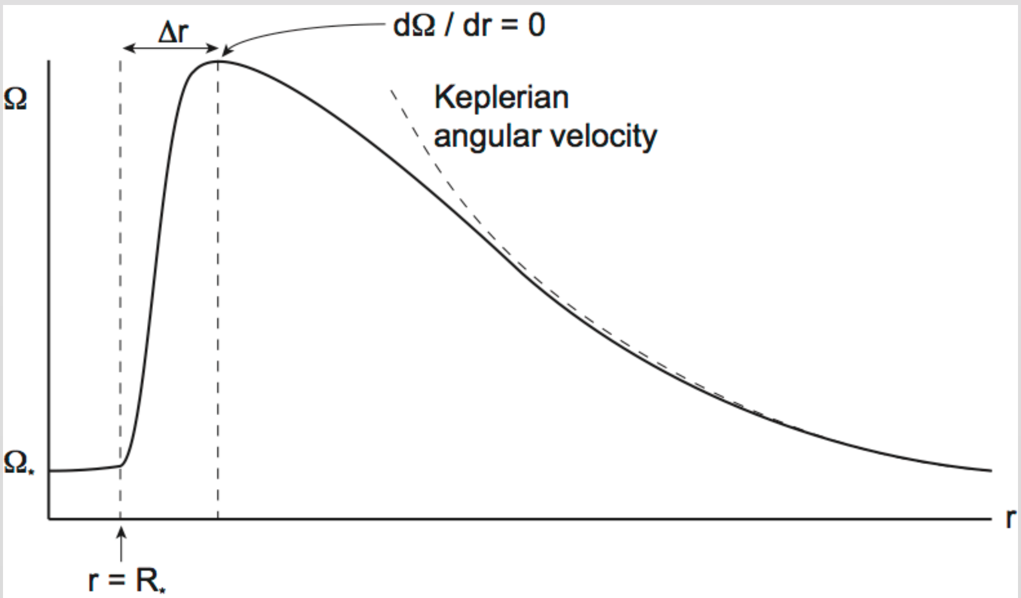
Radial scale for accretion r >> h vertical scale for magnetic reconnection… expect poloidal flux ψ to diffuse radially faster than it is "dragged" inward *(Lubow et al. 1994)*

$$\frac{\partial \psi}{\partial t} + \underbrace{r v_{\mathrm{adv}}}_{-\nu/r} B_z + \underbrace{r v_{\mathrm{diff}}}_{+\eta/h} B_{rs} = 0$$

c.f. Guilet & Ogilvie '14

Star-disk interactions

For a weakly-magnetized star: **boundary layer**



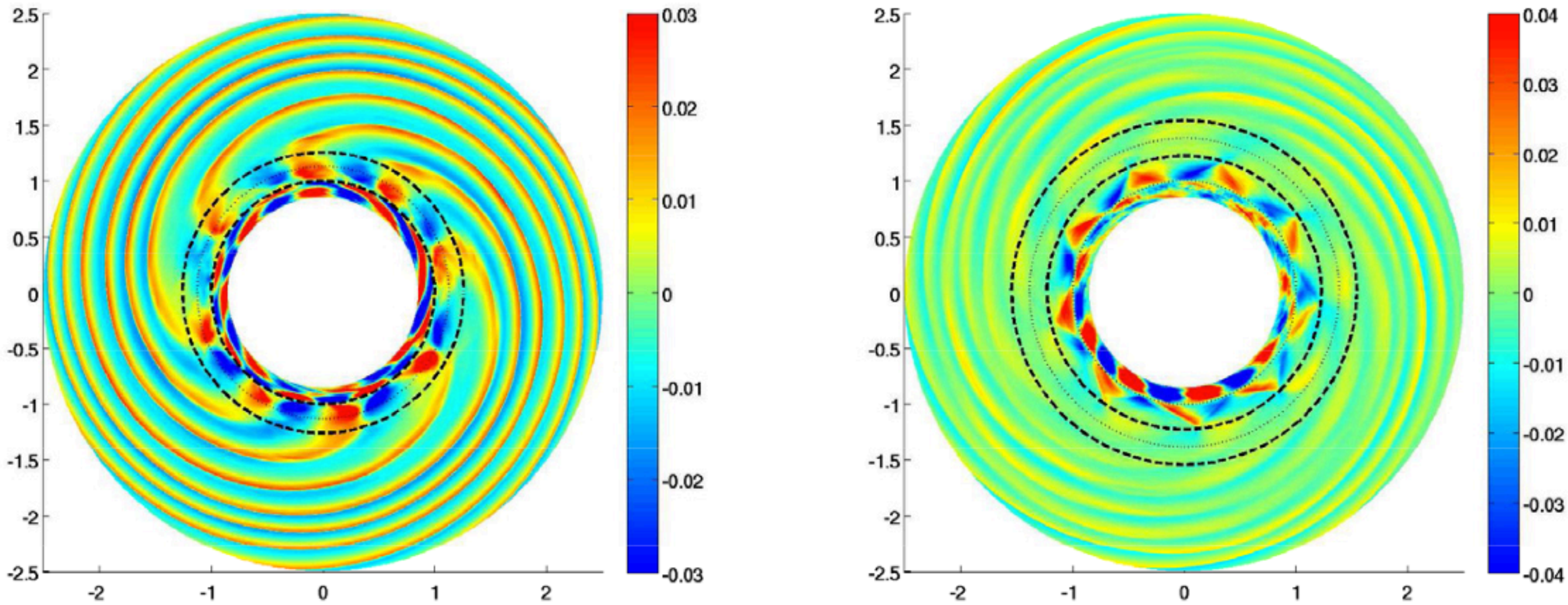
Classical theory:

- point in flow at $r \simeq R_*$ where dΩ/dr = 0
- viscous stress vanishes

$$G = 2\pi r \cdot \nu \Sigma r \frac{\mathrm{d}\Omega}{\mathrm{d}r} \cdot r$$

- disk has a boundary condition of zero torque

- star accretes gas with high angular momentum
- kinetic energy of disk is dissipated in narrow boundary layer, expected to be hot and luminous



Belyaev et al. '13

Boundary layer models are sensitive to the nature of disk angular momentum transport: dΩ / dr has opposite **sign**

Boundary layer flow is not unstable to the magnetorotational instability, rather evidence for transport by acoustic **waves** (non-local, not a "viscosity" at all!)

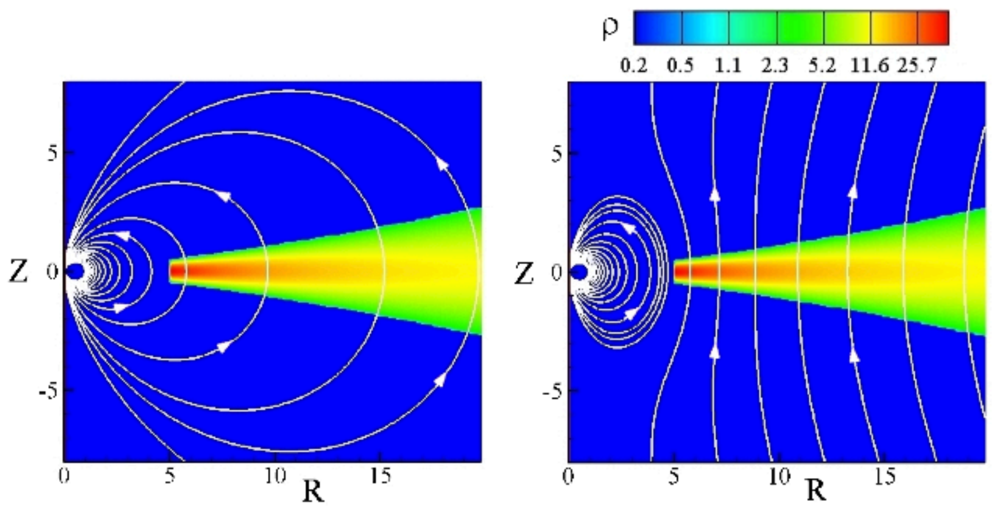


Figure 2. Plot of density ρ and poloidal field lines for a stellar dipole (left), a stellar dip
dipole and an anti-parallel disc-field.

At low accretion rates, expect **magnetospheric accretion**

Simulation: Dyda et al. '15

Suppose vertical field at disk surface is a dipole, toroidal component similar

$$B_z = B_* \left(\frac{r}{R_*} \right)^{-3} \qquad B_\phi \sim B_z$$

Then magnetic torque on surface of disk

$$T_m = \frac{B_z^s B_\phi^s}{2\pi} r$$

Time scale for stellar torque to drive inflow is shorter than viscous time inside some *magnetospheric radius* r_m

$$r_m \simeq \left(\frac{B_*^2 R_*^6}{\dot{M}\sqrt{GM_*}} \right)^{2/7}$$

$$r_m \simeq \left(\frac{B_*^2 R_*^6}{\dot{M}\sqrt{G M_*}} \right)^{2/7}$$

Very rough, but weak function due to rapid dipole fall off

For kG fields, 10^{-8} M_{Sun} yr^{-1}, typically r_m = 10-20 R_{Sun}

Consequences:

- gas accretes along magnetic field lines (free-fall, accretion shock on surface)
- magnetic field allows star to exert a non-zero torque on disk inner edge (in principle, star may spin down)
- innermost disk is missing

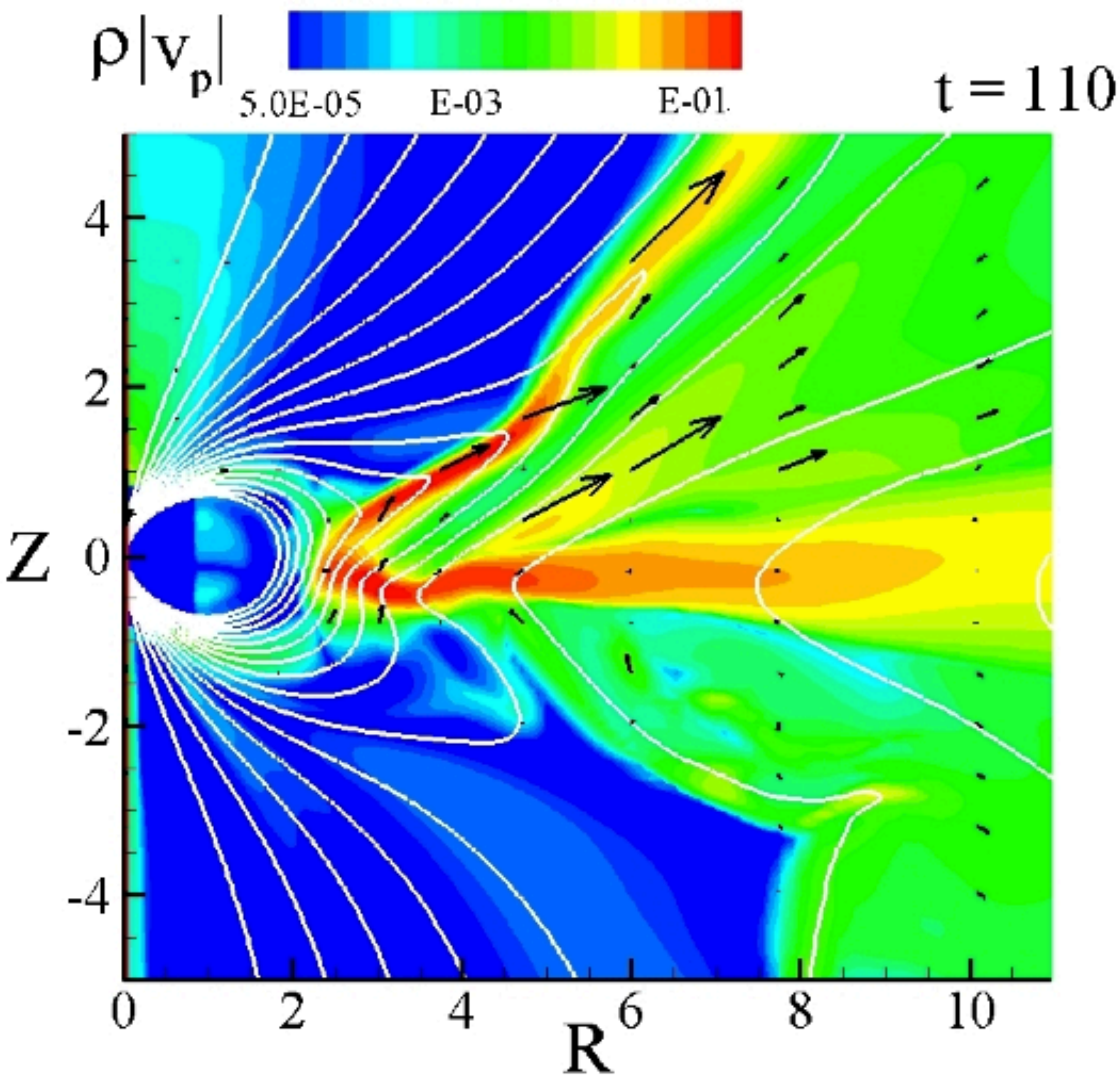


Figure 11. *Pure Stellar Dipole:* Poloidal mass flux $\rho|\mathbf{v}|$ (color), magnetic field lines (white lines) and coronal poloidal mass flux vectors $\rho\mathbf{v}_p$ at $t = 110$ for the case of a pure stellar dipole field. Note the presence of two outflows, a magnetospheric wind and a disc wind.

Interaction between disk and stellar field close to r_m favorable location for launching **jets**

Dyda et al.'15

Most basic questions are open!

Disk evolution may be driven primarily by:

- turbulence (innermost, ionized disk?)
- magnetic winds (scales ~AU?)
- thermal mass loss (~10^2 AU?)

How do disk magnetic fields evolve from disk formation through to disk dispersal?

How much turbulence is present if disk winds dominate angular momentum evolution?

Are there observational tests of turbulence and winds?