Magnetohydrodynamics :

1.1 • Astrophysics :

(i) Cosmology [Gravity]

(ii) Astroparticle physics [Dark matter, Cosmic Rays, high- energy phenomenon]

(iii) Physics of plasma

+ Radiation

" Perhaps the fundamental equation that describes the swirling nebulae and the condensing, revolving and exploding stars and galaxies is just a simple equation for hydrodynamic behaviour of nearly pure hydrogen gas" Feynman, "Flow of wet water"

· plus magnetic field

Fundamental principle of astrophysics

" There are no new laws in astrophysics. It is an application of experimental laws foun terrestrially and applied astrophysically"

1.2 Plasma the 4th state of matter

It is a state of matter where there are no atoms but electrons and positive ions. Mixed up like a gas. By gas we mean that there are no order. What kind of equation will such a gas obey?

× Plasma is the most abundant state of ordinary matter.

× This is a topic of continuum mechanics, similar to fluid dynamics or elasticity.

× But plasma is more complex than ordinary gas because of it contains charges, hence can sustain magnetic field (why not electric field?)

× Fundamentally, the difficulty of dealing with plasma is the long-range nature of the Coulomb interaction, however 'shielding' provides some help. We shall come back to this topic later.

× Fusion plasma and the solution to all our problems.

1.3 Continuum mechanics:

- Traditionally derived as many-body formulation of Newton's laws. But with additional constitutive ~~equ~~. coefficients (elastic coefficients, viscosity, thermal conductivity)

- Theoretical physics and length and time scales. The concept of "effective theories."

~~quantum mechani~~cs → ~~many-body~~ ~~con~~

"fundamental theories"

↓

quantum mechanics

↓

classical mechanics

↓

few particles classical mech. ⟷ chaos theory

quantum mechanics → few particles q. mech. quantum chaos stat. mech. → many-body class. mech. → many body ~~stat~~ q mech. (superconductivity. ...)

↓

continuum mechanics.

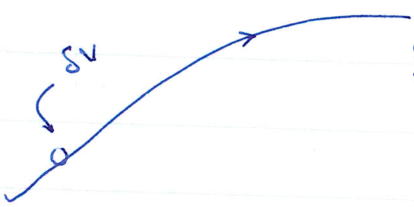
can also be formulated as non-equilibrium statistical mechanics.

× Each step is a change of scale, "coarse graining"

× ~~let us start by again~~

× Equations of a simple fluid :

× Apply Newton's laws to a fluid element :



$\rho\, \delta V$ (acceleration)

= pressure force

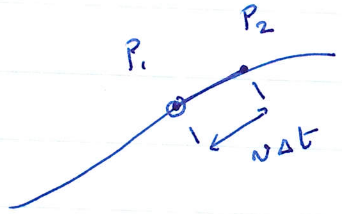
+ body forces (e.g. gravity)

+ viscous forces.

body forces : $-\rho \nabla \phi$

pressure forces : $-\nabla p$

<u>acceleration</u> : $\frac{d\vec{v}}{dt}$



$$\Delta v = v(x+\Delta x,\ y+\Delta y,\ z+\Delta z,\ t+\Delta t)$$

$$= v(x + v_x \Delta t,\ y + v_y \Delta t,\ z + v_z \Delta t,\ t+\Delta t)$$

$$= v(x, y, z) + \frac{\partial v_x}{\partial x} v_x \Delta t + \cdots + \frac{\partial v}{\partial t} \Delta t + h.o.t$$

The acceleration

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = (v \cdot \nabla) v + \frac{\partial v}{\partial t}$$

Putting togethere :

$$\rho \left[\frac{\partial v}{\partial t} + (v \cdot \nabla) v \right] = -\nabla p - \rho \nabla \phi + \text{viscous force.}$$

Beginning of hydrodynamics.

× A second way to derive hydrodynamics:

- look for conserved quantities:

mass, momentum, energy.

Each conserved quantity will have a density and a current.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot g = 0$$

$$\frac{\partial g_i}{\partial t} + \nabla \cdot \Pi_{ij} = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot j_\varepsilon = 0$$

- clearly current of mass is the momentum.

$$g = \rho v$$

~~This~~ ~~is~~ ~~the~~ Current of a conserved quantity is also conserved.

To proceed let us be a little bit more careful.

- We consider a fluid that has local thermodynamic equilibrium. In other words one can define a local temperature.
- So we are dealing with thermodynamics of moving systems.
- Remind ourselves some thermodynamics:

- An interacting classical system is described by a Hamiltonian $\mathcal{H}$

All of thermodynamics is in the partition function

$$Z = \mathrm{Tr}\, e^{-\beta \mathcal{H}}$$

Tr ← integral over all degrees of freedoms.

$$\mathcal{F} = -T \ln Z \qquad \text{Helmholtz potential.}$$

$$= E - TS.$$

~~But now we are in a moving system.~~

$$d\mathcal{F} = dE - TdS - SdT \qquad TdS = dE + p\,dV$$

$$= -SdT - p\,dV$$

Now consider ~~a~~ thermodynamics with motion.

$$Z_N(T, V, v) = e^{\beta N m v^2/2} \, Z_N(T, V, 0)$$

Because in a classical system velocity is independent of position.

$$\Rightarrow \quad \mathcal{F}(T, V, N, v) = \mathcal{F}(T, V, N, 0) - \frac{1}{2} N m v^2$$

The momentum operator

$$P_j = - \left. \frac{\partial \mathcal{F}}{\partial v_j} \right|_{T, V, N}$$

$$= N m v_j$$

$$\Rightarrow \quad \cancel{d\mathcal{F}} =$$

$$\Rightarrow \quad d\mathcal{F} = - S\, dT - p\, dV - P \cdot dv$$

Now introduce the grand potential:

$$\mathcal{A} = \mathcal{F} - \mu N$$

chemical potential.

Clearly

~~μ~~ ~~μ_0~~

$$\mu = \frac{\partial F}{\partial N} = \mu_0 - \frac{1}{2} m v^2$$

The grand potential is related to the pressure by

$$\Omega = - V p(\mu, T\, v)$$

$$\Rightarrow \quad p = - \frac{\Omega}{V}$$

$$= -\frac{1}{V} \left(F - \mu N \right)$$

$$= -\frac{1}{V} \left[E - TS - \mu N - \frac{1}{2} N m v^2 \right]$$

$$= - \varepsilon - Ts - \alpha \rho - \vec{g} \cdot \vec{v}$$

$$\rho = \frac{Nm}{V}, \quad \alpha = \frac{\mu}{m},$$

Then the entropy eqn.

$$T ds = d\varepsilon - \alpha d\rho - \vec{v} \cdot d\vec{g}$$

Now write an equation of entropy transport:

$$T\left[\frac{\partial s}{\partial t} + \nabla\cdot\left(v s + \frac{\Phi}{T}\right)\right]$$

$$= -\,\varphi\cdot\frac{\nabla T}{T} - (g - \rho v)\cdot\nabla\alpha$$

$$-\left(\Pi_{ij} - p\,\delta_{ij} - v_i g_j\right)\nabla\cdot v_j$$

Demand zero dissipation

$$g = \rho\vec{v}$$

$$\Pi_{ij} = p\,\delta_{ij} + v_j g_i$$

$$j_\varepsilon = (\varepsilon + p)\,v = \left(\varepsilon_0 + p + \frac{1}{2}\rho v^2\right)v$$

$$\boxed{\begin{aligned} &\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0 \\ &\partial_t(\rho v) + \nabla\cdot(\rho v v) = -\nabla p \\ &\partial_t s + \nabla\cdot(v s) = 0. \end{aligned}}$$

Dissipationless hydrodynamics.