Electromagnetism (FK4005e)
Re-exam 16th June 2015

Each of the following problems carry 20 marks. While answering the questions, please write clearly all the assumptions you have made.

1. (a) Human nerve cells have a net negative charge and the material in the interior of the cell is a good conductor. If a cell has a net charge of -8.65 pico Coulomb what are the magnitude and direction (inward or outward) of the net flux (of the electric field) through the cell boundary ? (2p)

   (b) Some planetary scientists have suggested that the planet Mars has an electric field somewhat similar to that of the earth, producing a new electric flux of $3.63 \times 10^{16} \mathrm{Nm}^2/\mathrm{C}$ at the planet's surface, directed toward the center of the planet. Calculate: (i) The total electric charge of the planet; (2p) (ii) the electric field at the planet's surface; (2p) (iii) the charge density on Mars, (2p) assuming all the charge is uniformly distributed over the planet's surface. (Assume that Mars is a sphere with radius $R = 3000\mathrm{Km}$.)

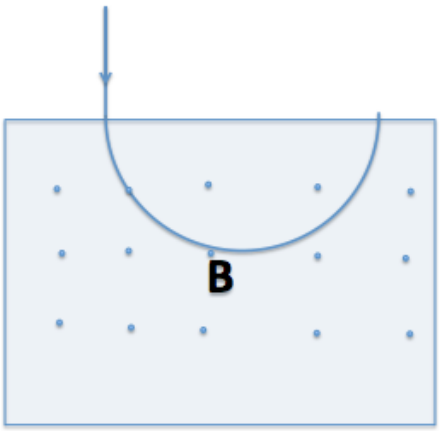
   (c) An insulating hollow sphere has inner radius a and outer radius b. Within the insulating material the volume charge density is given by

   $$\rho(r) = \frac{\alpha}{r} \tag{1}$$

   where α is a positive constant. What is the magnitude and direction of the electric field at a distance r from the center of the shell ? (12p)
   Hint: As ρ is a function of r only the charge inside a spherical shell between $r = R_1$ to $r = R_2$ is given by

   $$\int_{R_1}^{R_2} \rho(r) 4\pi r^2 dr \tag{2}$$

2. In an experiment with cosmic rays, a vertical beam of particles that have charge of magnitude $3e$ (where e is the electronic charge) and mass 12 times the proton mass enters a uniform horizontal magnetic field of 0.250tesla and is bent into semicircular path of diameter 95cm as shown in figure 2



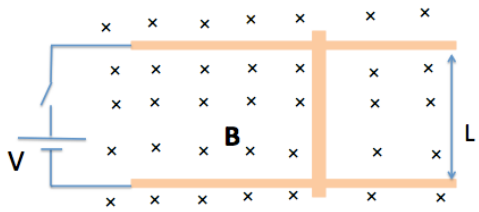
   (a) Find the speed of the particles and the sign of their charge. (10p)

   (b) Can you ignore gravity for these particles (gravity acts vertically downward in the figure) ? Give quantitative estimates. (3p)

   (c) How does the speed of the particles as they enter the field compare to their speed as they exit the field ? (2p)

(d) If an anti-proton (proton with a negative charge) entered the region of the magnetic field at the same position and with the same initial speed what would be their point of exit compared to these particles ? (5p)

3. A bar of length $L = 0.36$m is free to slide without friction on horizontal rails as shown in figure 3. There is a uniform magnetic field $B = 1.5$Tesla directed into the plane of the figure. At one end of the rails there is a battery with EMF $V = 12$volt and a switch. The bar has mass $m = 0.9$kg and resistance of 5Ohm. All other resistance in the circuit can be ignored. The switch is closed at time $t = 0$.



(a) Find the speed of the bar as a function of time and make a sketch. (10)

(b) Just after the switch is closed, what is the acceleration of the bar ? (3p)

(c) What is the acceleration of the bar when its speed is 2m/s ? (3p)

(d) What is the terminal speed (the speed when the acceleration of the bar is zero) of the bar. (4p)

4. Write down the the "source-free" or "empty-space" Maxwell's equations. Consider the following electric and magnetic field,

$$\boldsymbol{E} = E_0 \hat{z} \cos kx \cos ky \cos \omega t \quad (3)$$

$$\boldsymbol{B} = B_0 \left[\hat{x} \cos kx \sin ky - \hat{y} \sin kx \cos ky\right] \sin \omega t \quad (4)$$

What is the relationship between E_0 and B_0 and ω and k such that the electric and magnetic fields above satisfy the source-free Maxwell's equations ? (20p)

5. A capacitor with capacitance 6×10^{-5}F is charged by connecting it to a 12.0V battery. The capacitor is then disconnected and then connected across an inductor with inductance $L = 1.5$H. Both the capacitor and the inductor are ideal, i.e., the is no resistance.

(a) What are the angular frequency ω of the electrical oscillations and the period of these oscillations ? (2p)

(b) How much energy was initially stored in the capacitor ? (2p)

(c) What is the charge on the capacitor 0.023s after the connection to the inductor is made ? Interpret the sign of your answer. (3p)

(d) At the time given in part (d) what is the current in the inductor ? Interpret the sign of your answer. (3p)

(e) At the time given in part (d) how much electrical energy is stored in the capacitor and how much is stored in the inductor ? (2p)

(f) Now assume that the inductor is not ideal, but has a resistance of 1Ohm. Calculate the resonant frequency of this circuit and determine its impedance. (8p)