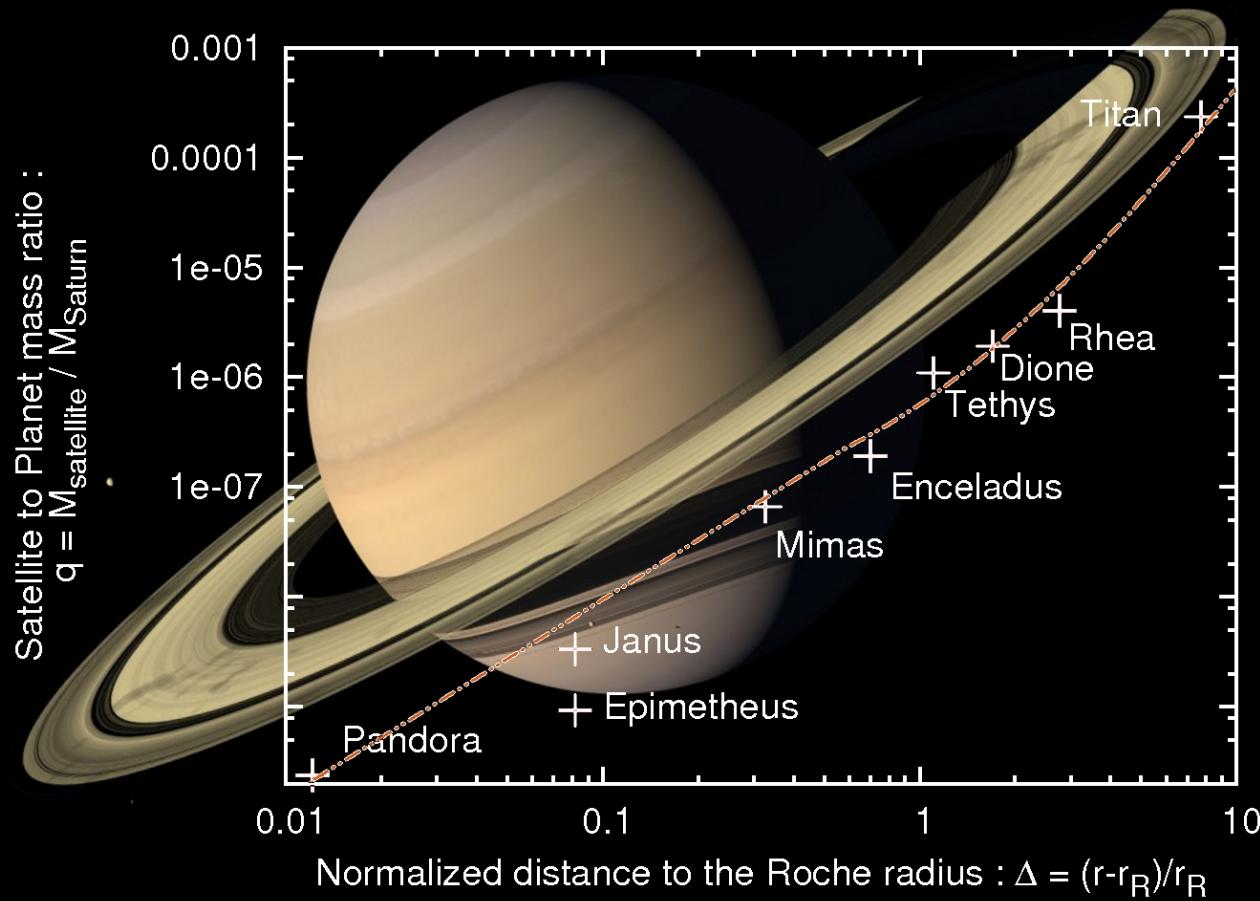


Formation of Satellites from Rings

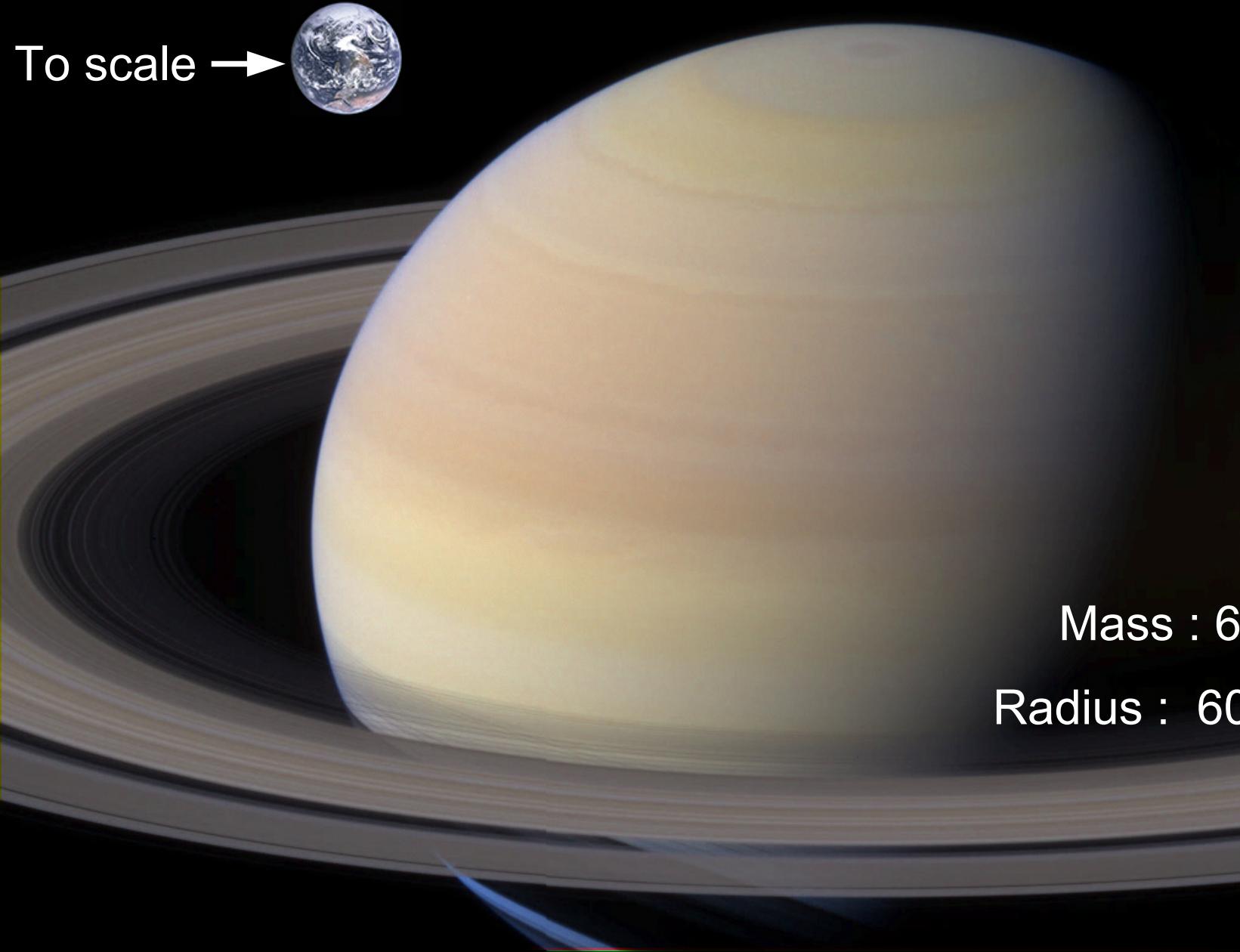


Aurélien CRIDA, with Sébastien CHARNOZ



SATURN

To scale →



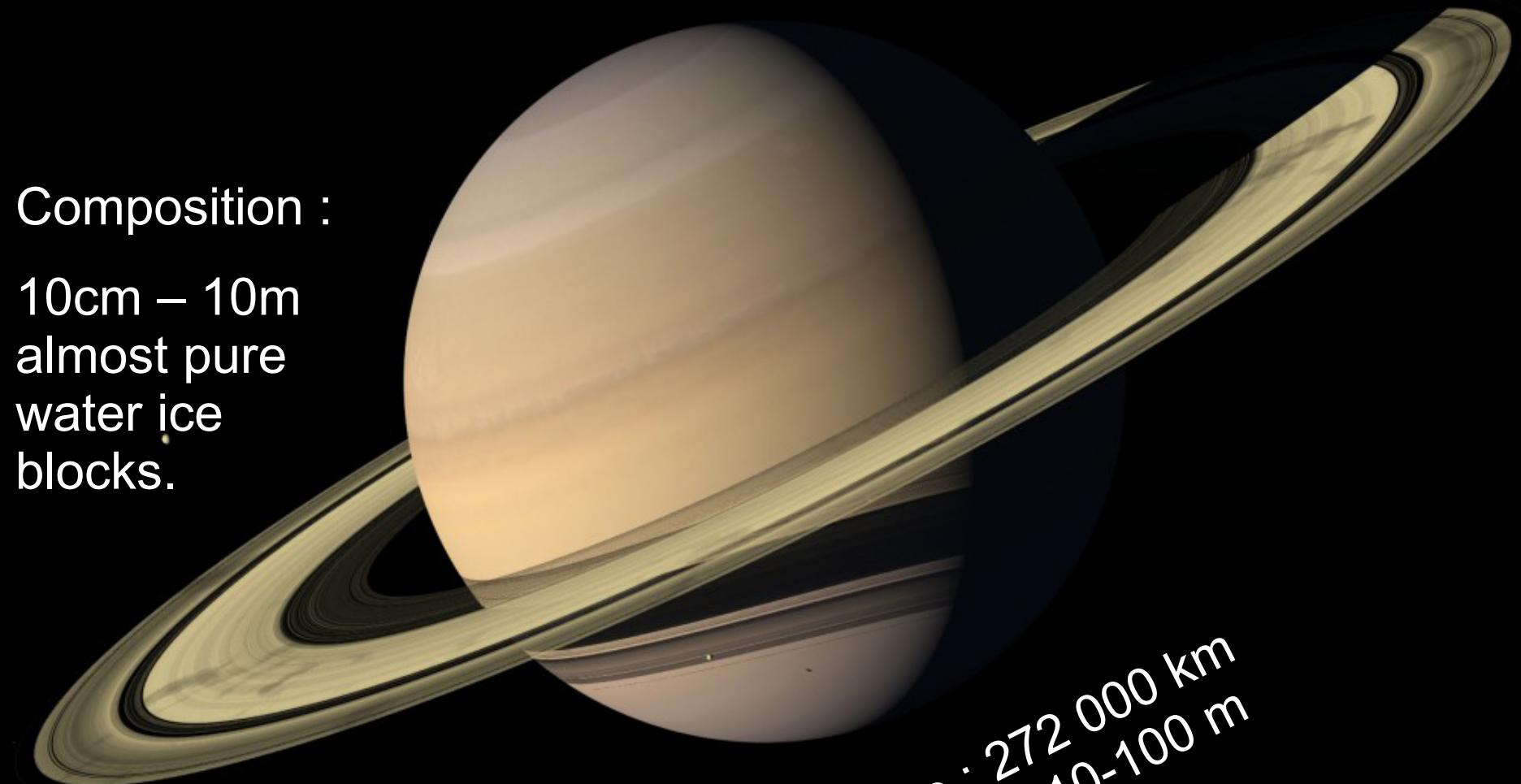
Mass : 6×10^{26} kg

Radius : 60 000 km.

RINGS

Composition :

10cm – 10m
almost pure
water ice
blocks.



Diameter : 272 000 km
Thickness : 10-100 m

PROMETHEUS & PANDORA



Sizes : ~80 km.

Masses : $\sim 10^{17}$ kg

Orbital radius : 140 000 km

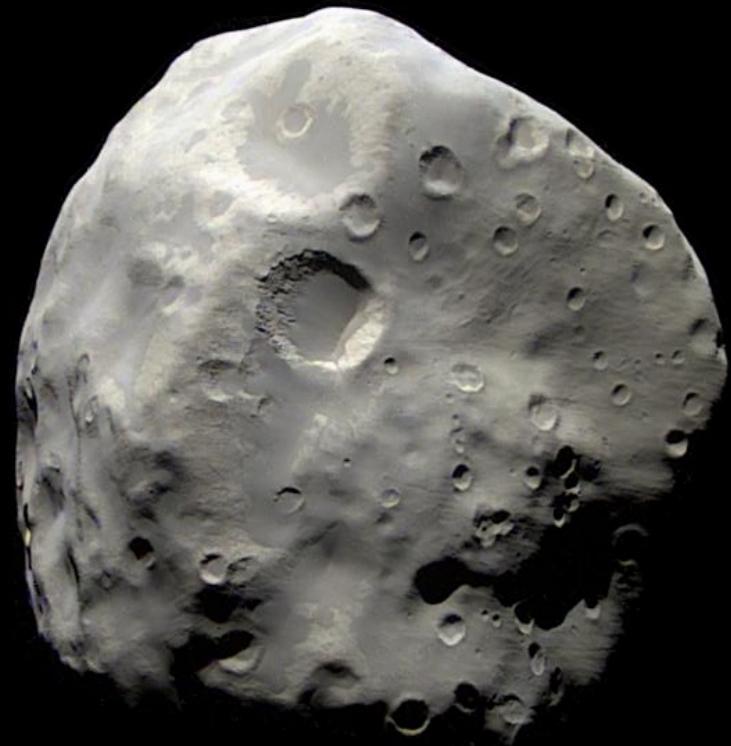
JANUS & EPIMETHEUS

Sizes : 180 & 110 km.

Masses : 2 & $0,5 \times 10^{18}$ kg

Orbital radius : 151 000 km

Distance to the rings : 11 000 km.



JANUS & EPIMETHEUS

Only known
system in mutual
horseshoe orbits

Rotating frame,
 $\omega = 21.6^\circ/\text{hour}$

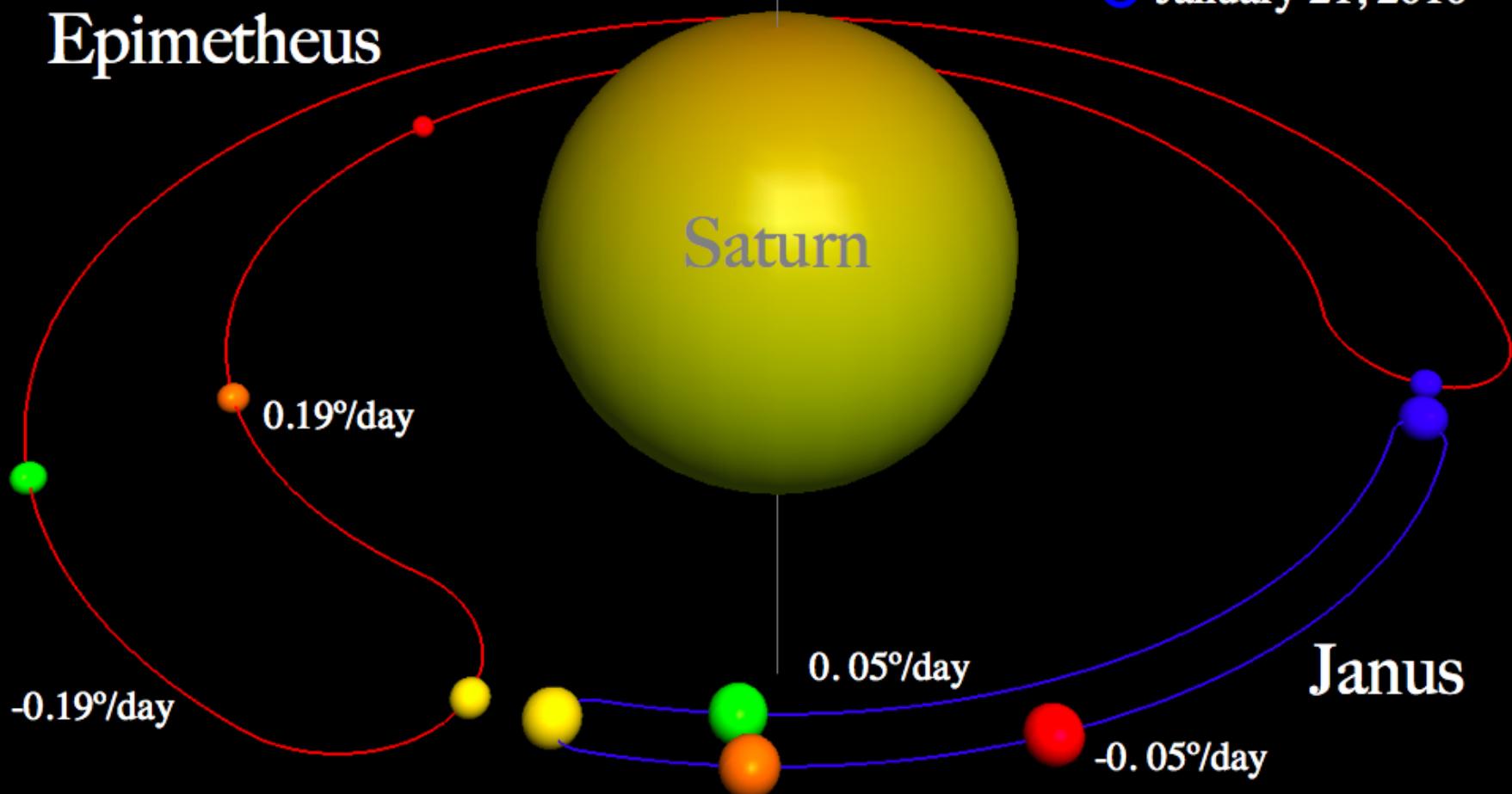


- July 1, 2004
- May 21, 2005
- January 21, 2006
- September 9, 2006
- January 21, 2010

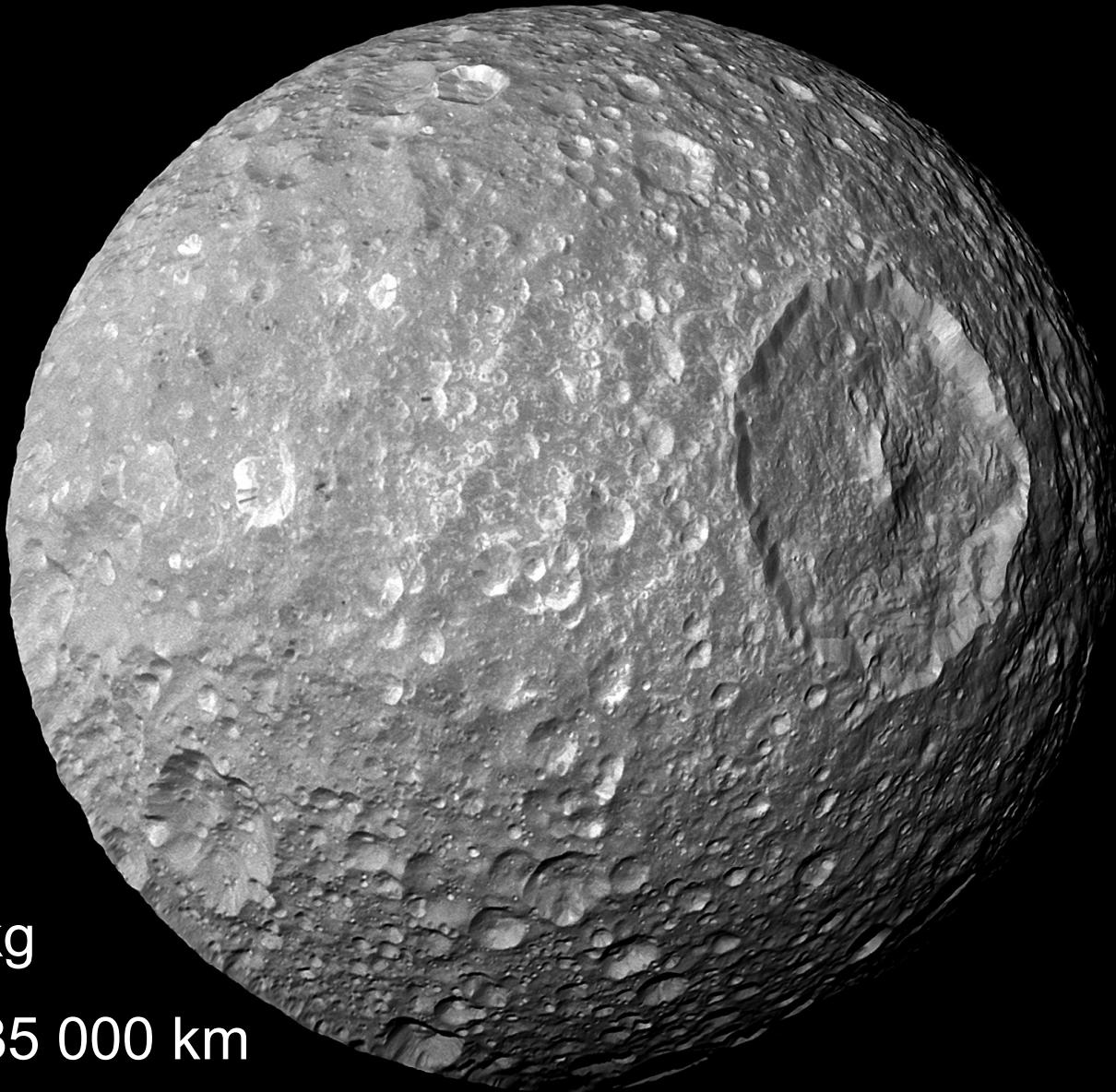
Epimetheus

Saturn

Janus



MIMAS



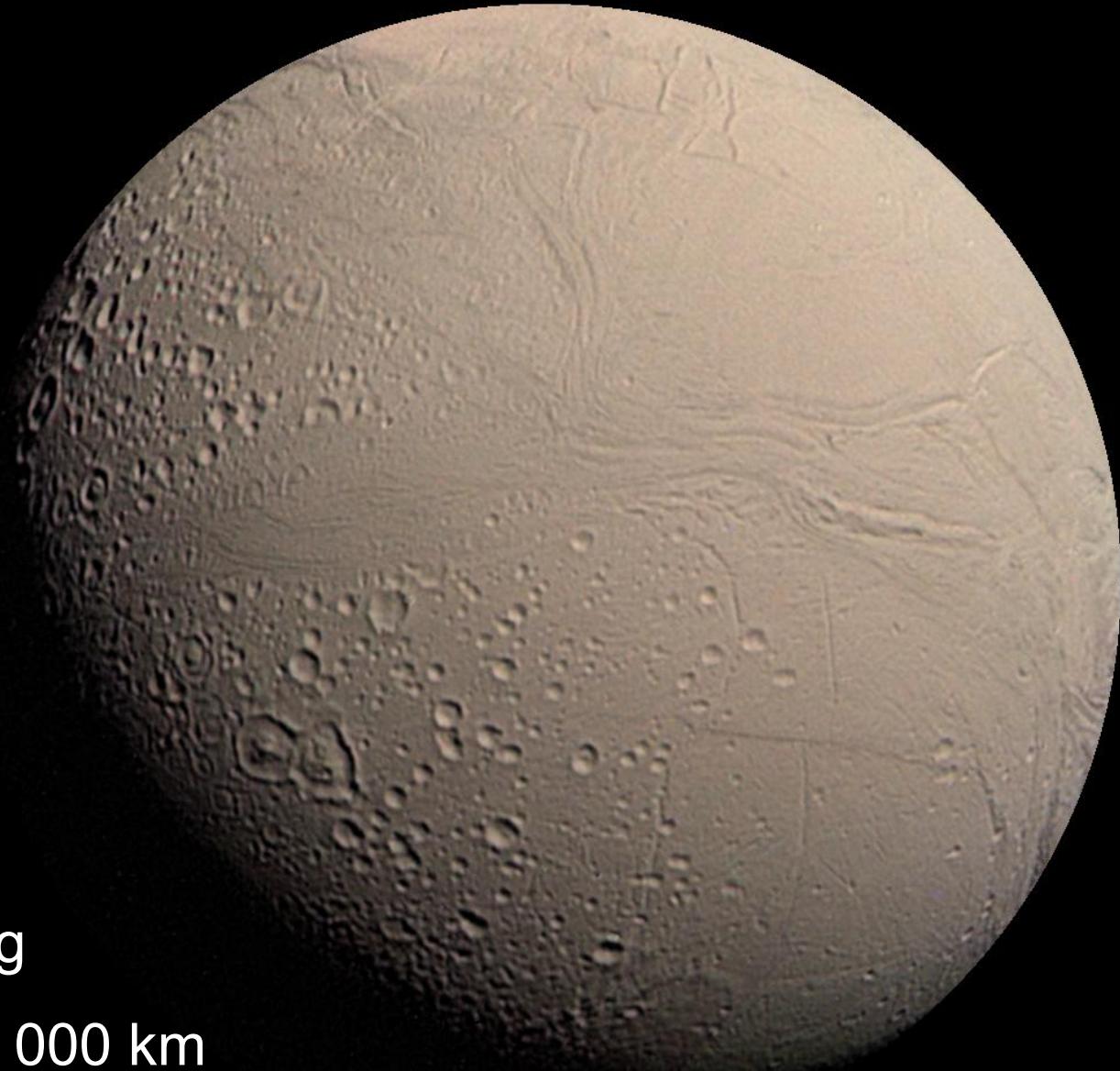
Size : 400 km.

Mass : 37×10^{18} kg

Orbital radius : 185 000 km

Distance to the rings : 45 000 km.

ENCELADUS



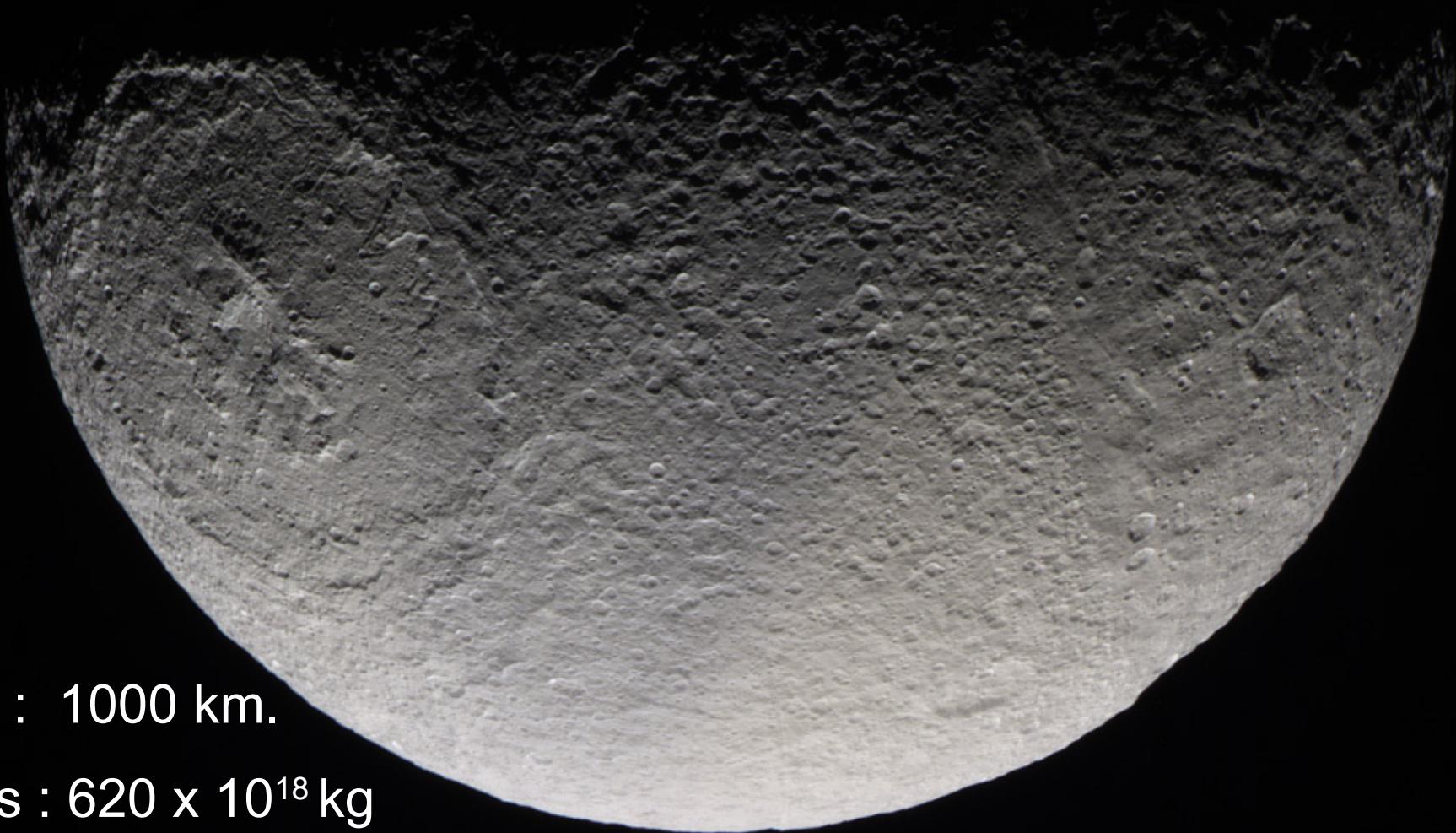
Size : 500 km.

Mass : 108×10^{18} kg

Orbital radius : 240 000 km

Distance to the rings : 100 000 km.

TETHYS



Size : 1000 km.

Mass : 620×10^{18} kg

Orbital radius : 300 000 km

Distance to the rings : 160 000 km.

DIONE



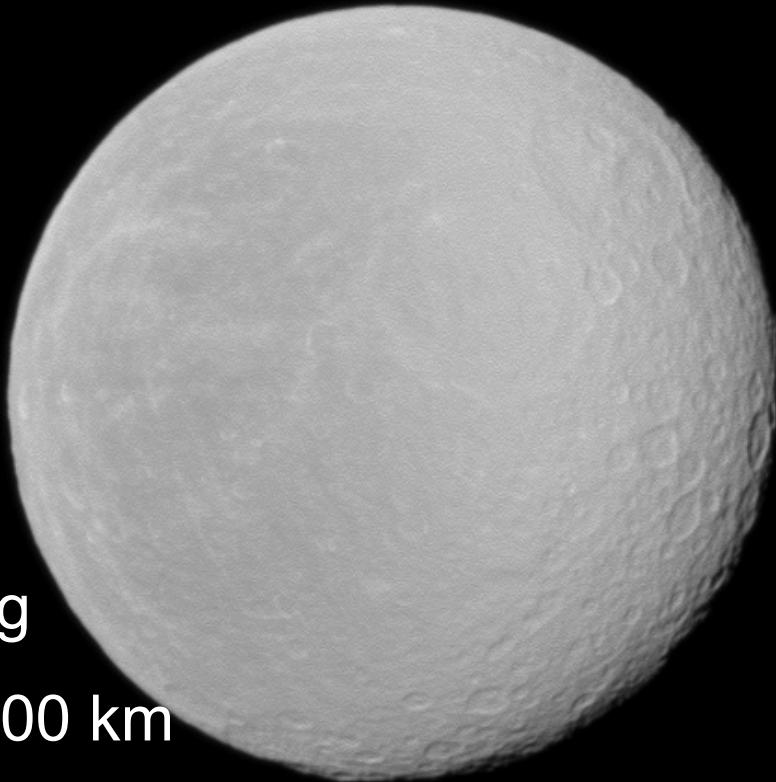
Size : 1120 km.

Mass : 1100×10^{18} kg

Orbital radius : 380 000 km

Distance to the rings : 240 000 km.

RHEA



Size : 1520 km.

Mass : 2300×10^{18} kg

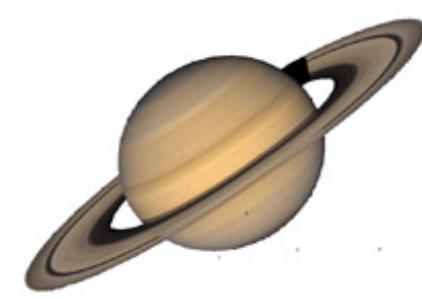
Orbital radius: 530 000 km

Distance to rings : 390 000 km.

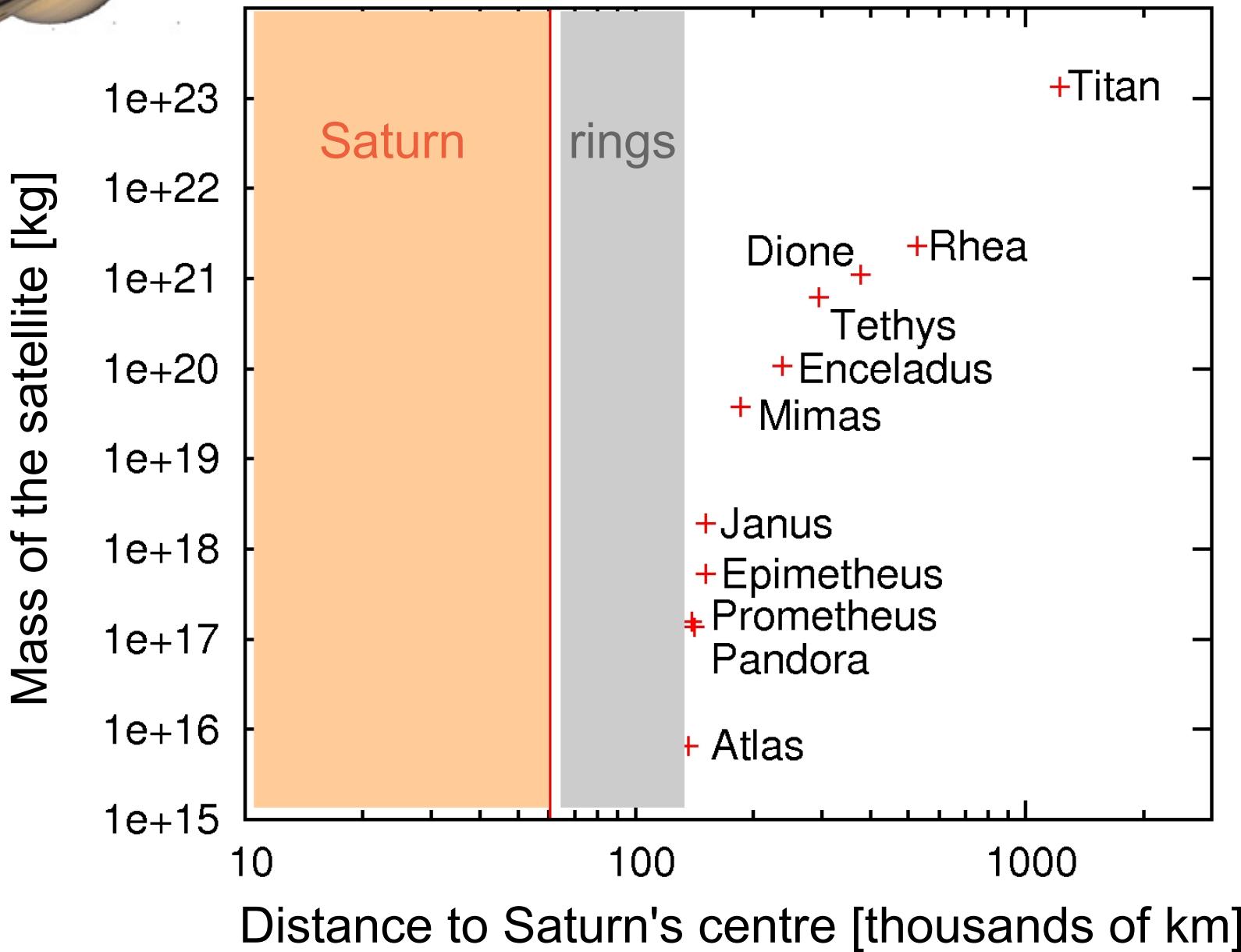
Dione Prometheus

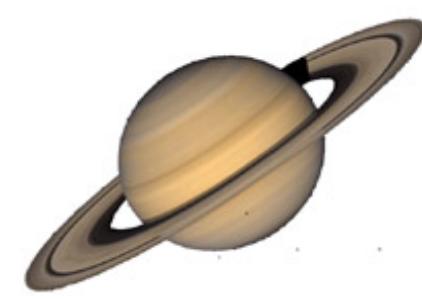
Epimetheus

Tethys

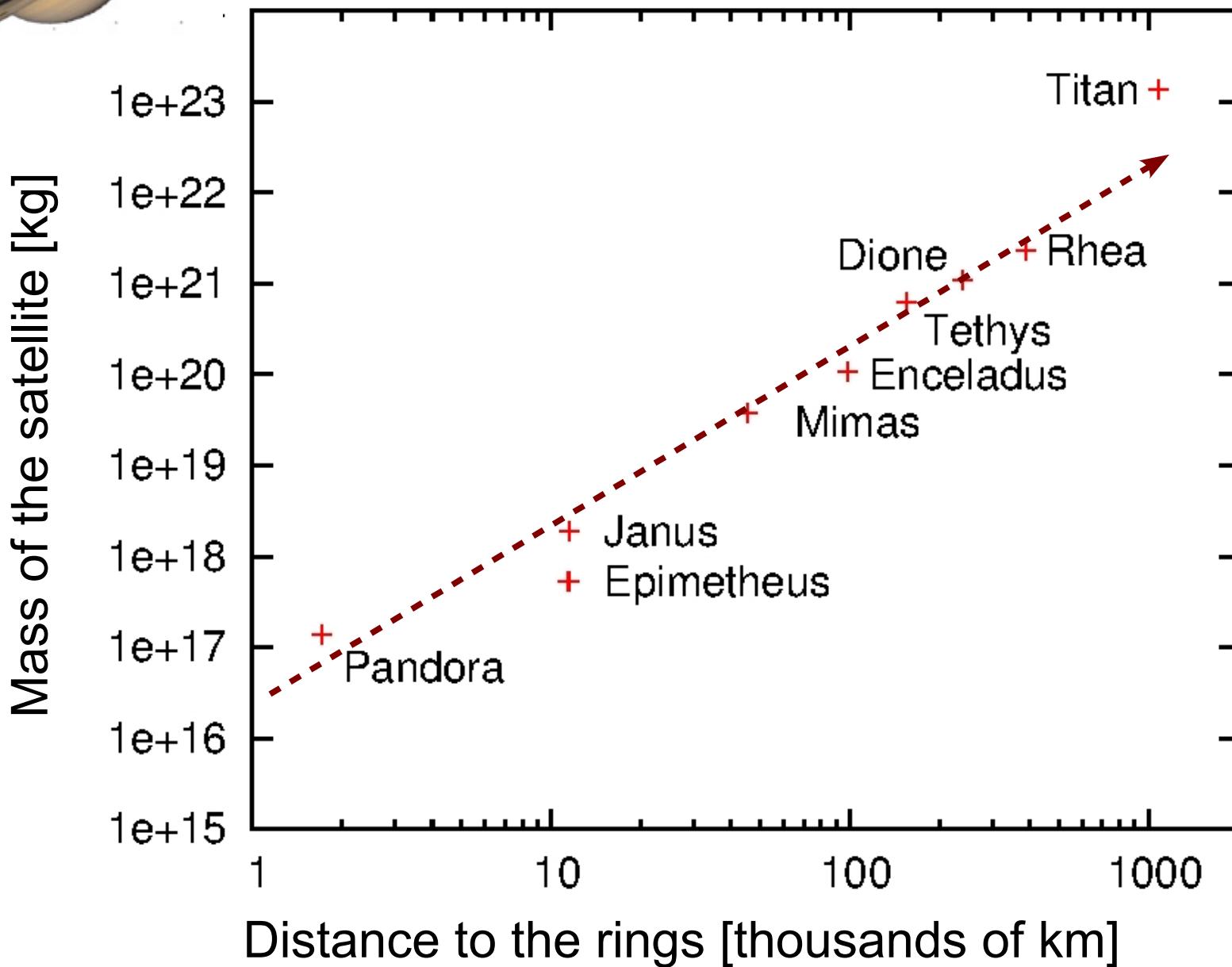


SATURN's SYSTEM





SATURN's SYSTEM



ORIGIN of the SATELLITES ?

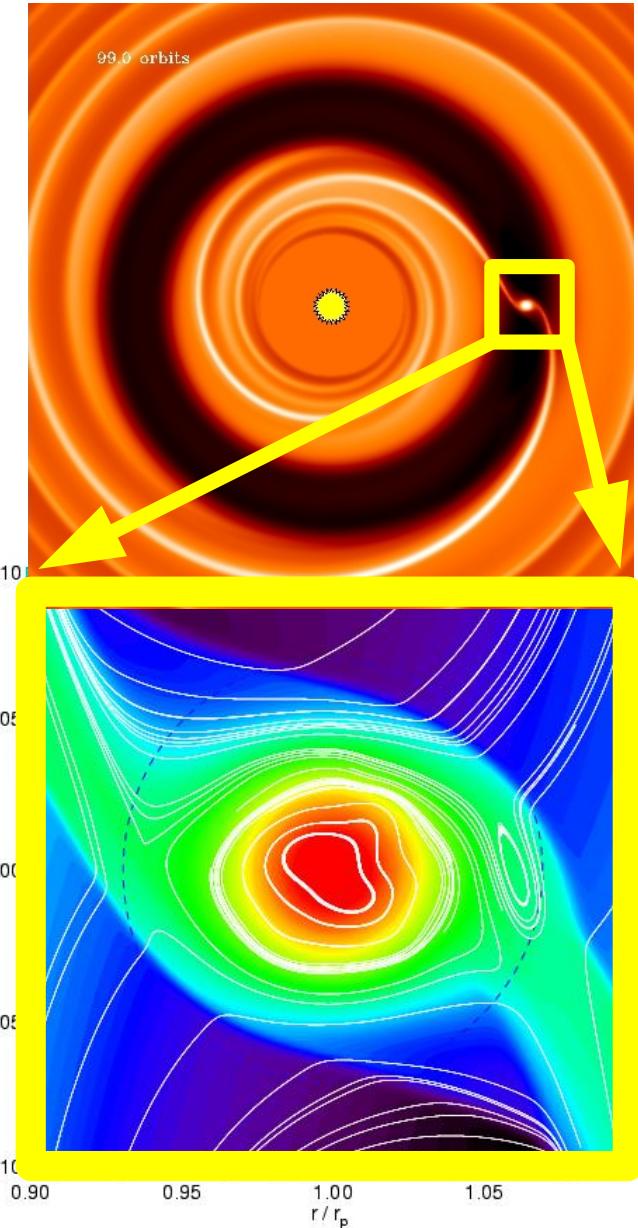
Planets form in a gas/dust disk around the Sun.

A giant planet carves a gap in the disk, and has its own circum-planetary disk.

A mini-planetary system would then form around the planet.

(Canup & Ward 2002, 2006 ;
Sasaki et al. 2010 ;
Mosqueira & Estrada 2003a,b...)

This model can't explain the mass-distance feature.

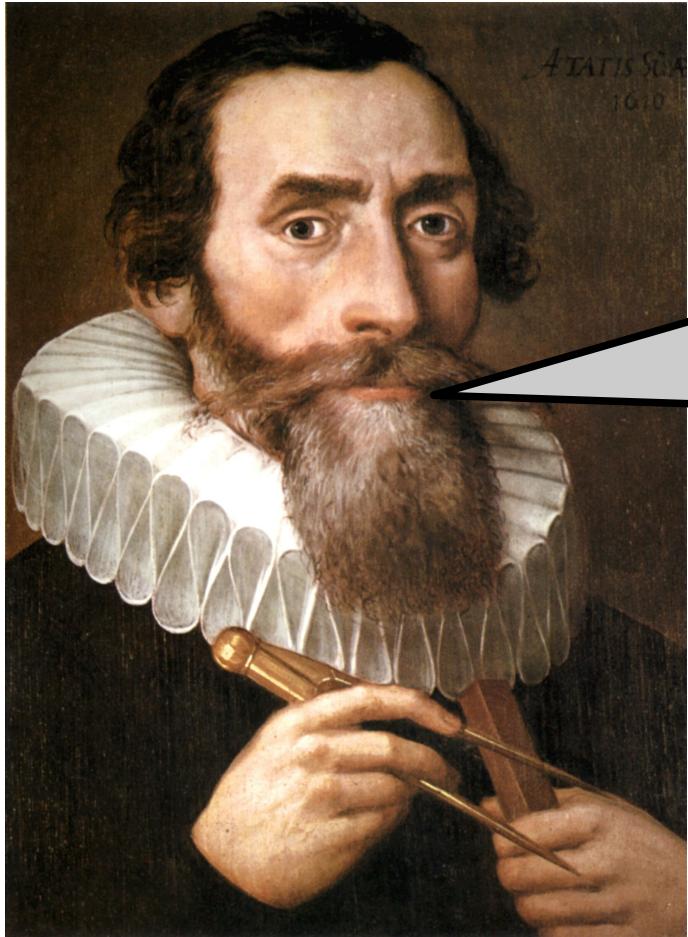


Reminder:

Kepler's law &
rings spreading



Kepler's law



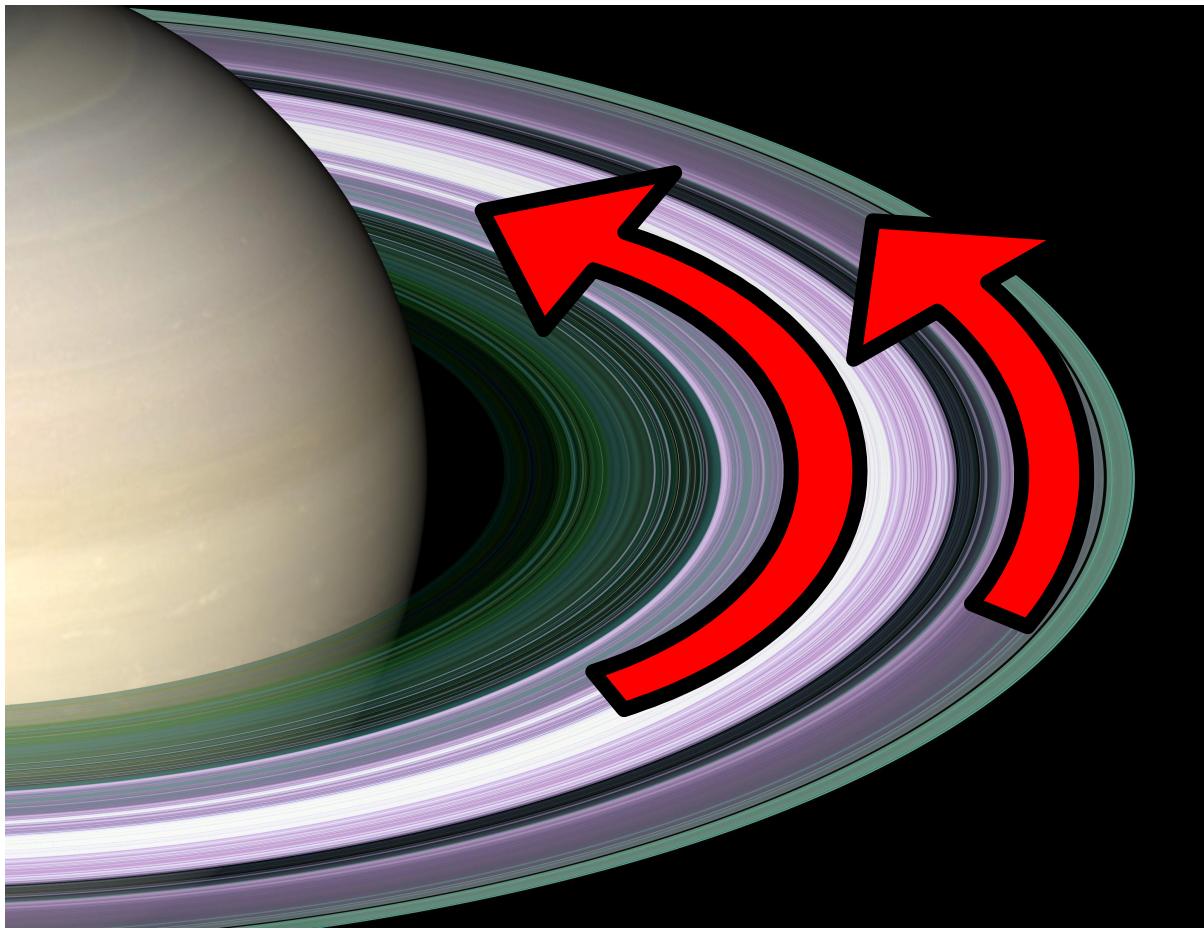
*The cube of the radius of an orbit
is proportionnal to
the square of the period.*

$$P^2 = \left(4\pi^2/GM_* \right) r^3$$

angular velocity : $\Omega = (GM_*/r^3)^{1/2}$
decreases with r .

specific orbital
angular momentum : $j = (GM_*r)^{1/2}$
increases with r !

Rings spreading



The inside rotates faster than the outside, so friction accelerates the outside (positive torque, increase of j thus r), and slows down the inside (negative torque, r decrease). Total: spreading.

Any astrophysical disk in Keplerian rotation spreads by viscous friction (eg. [Lynden-Bell & Pringle 1974](#)).

Reminder:

Roche radius



Roche Radius

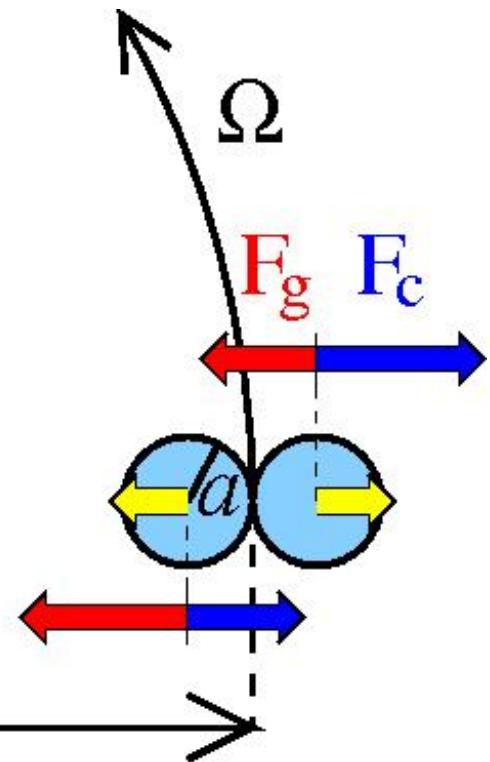
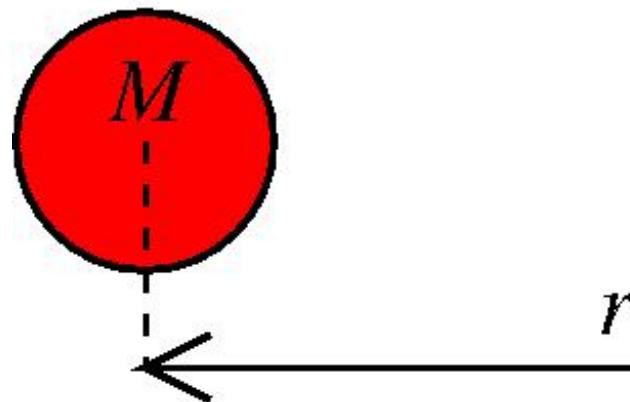
Reminder : Tidal forces (per mass unit) :

$$\Omega = (GM/r^3)^{1/2}$$

$$F_g = GM / (r+/-a)^2$$

$$F_c = \Omega^2(r+/-a)$$

$$F_{\text{tide}} = 3\Omega^2 a$$



Roche Radius

Self-gravity force of the two bodies (per mass unit) :

$$F_{sg} = G^*(4/3)\pi\rho a^3 / (2a)^2$$

Condition for stability of the aggregate : $F_{sg} > F_{tide}$,

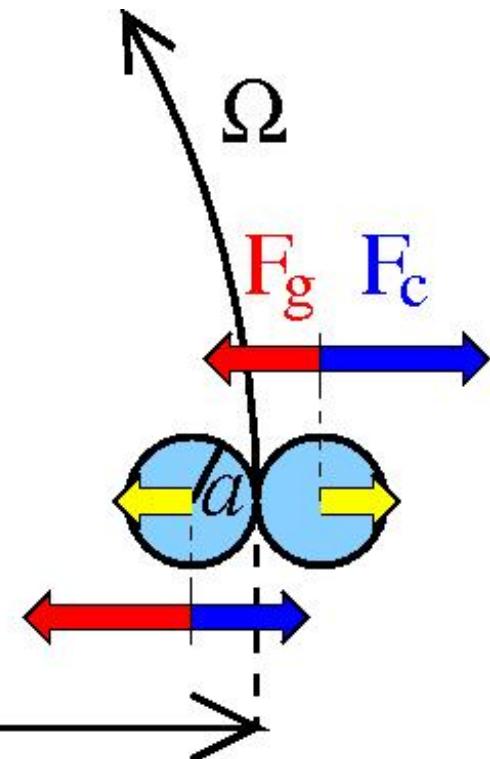
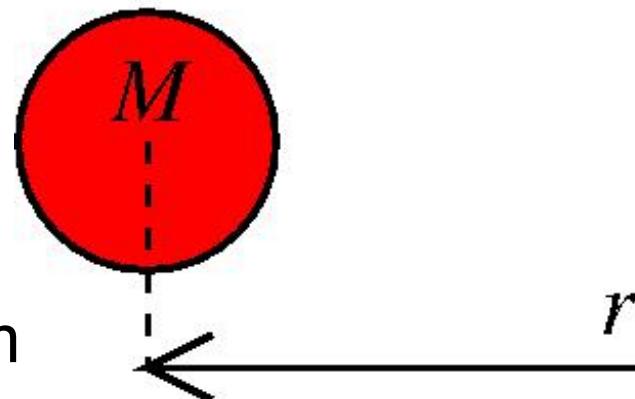
or : $r > (9M/\pi\rho)^{1/3} = r_{\text{Roche}}$

Application:

$$M = M_{\text{Saturn}},$$

$$\rho = 600 \text{ kg.m}^{-3}$$

$$r_{\text{Roche}} = 1,4 \cdot 10^8 \text{ m}$$



Roche Radius

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Application:

$$M = M_{\text{Saturn}},$$

$$\rho = 600 \text{ kg.m}^{-3}$$

$$r_{\text{Roche}} = 1,4 \cdot 10^8 \text{ m} > r_{\text{rings}}$$

This is why Saturn's rings remain rings.

(see movie)

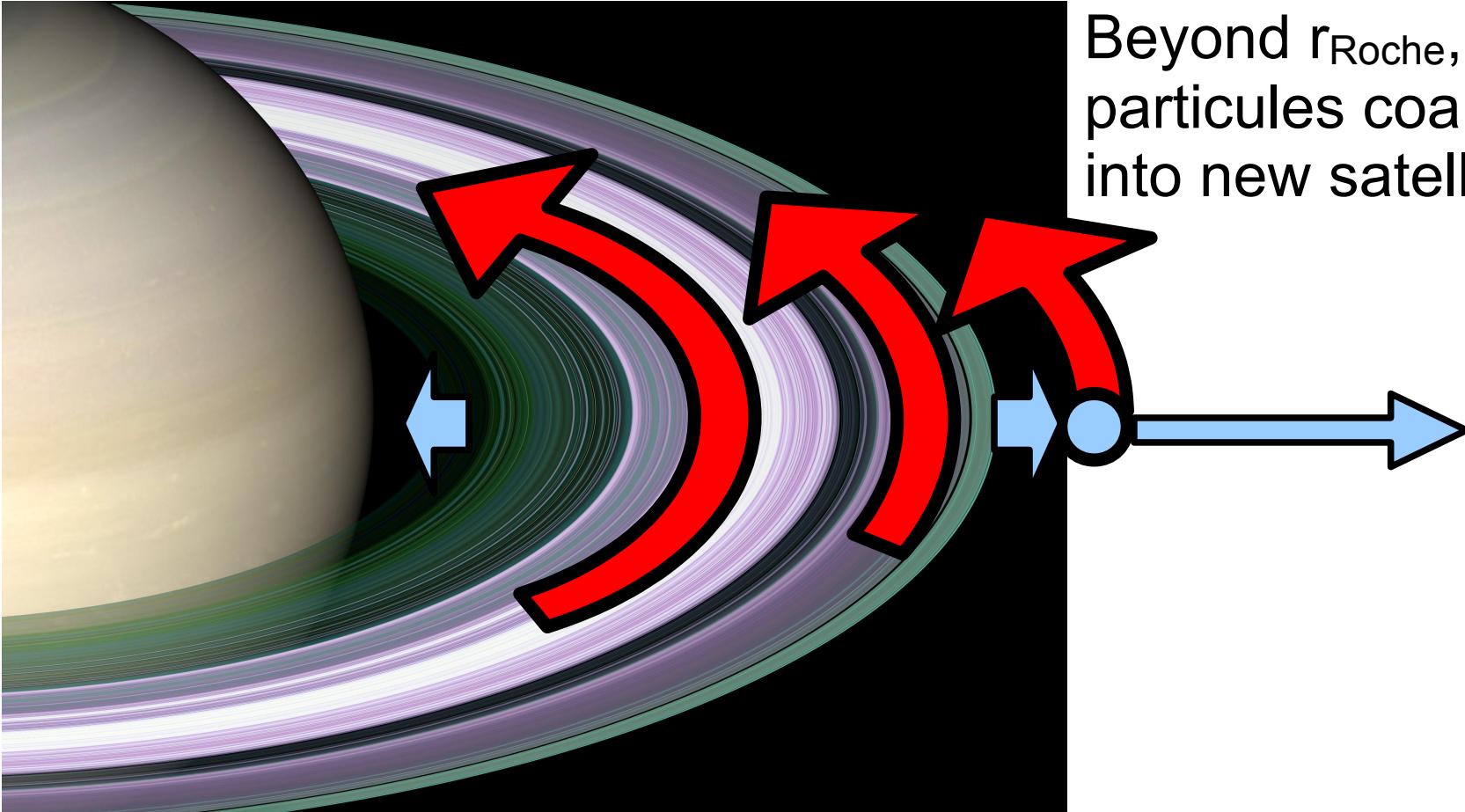


Put things together



- 1) the rings spread.
- 2) Beyond the Roche radius,
self-gravity wins.

Satellites children of the rings



Beyond r_{Roche} , ring particles coalesce into new satellites !

The new satellites have a smaller angular velocity than the rings particles. Therefore, they are accelerated and repeled outwards...

Satellites children of the rings

Total torque : $\Gamma = \frac{8}{27} \left(\frac{M_{satellite}}{M_{Saturne}} \right)^2 \sum r^4 \Omega^2 \Delta^{-3}$ Eq.(1)

proportionnal to $M_{satellite}^2$ and to Δ^{-3} , where $\Delta = (r - r_R) / r_R$.
(Lin & Papaloizou 1979)

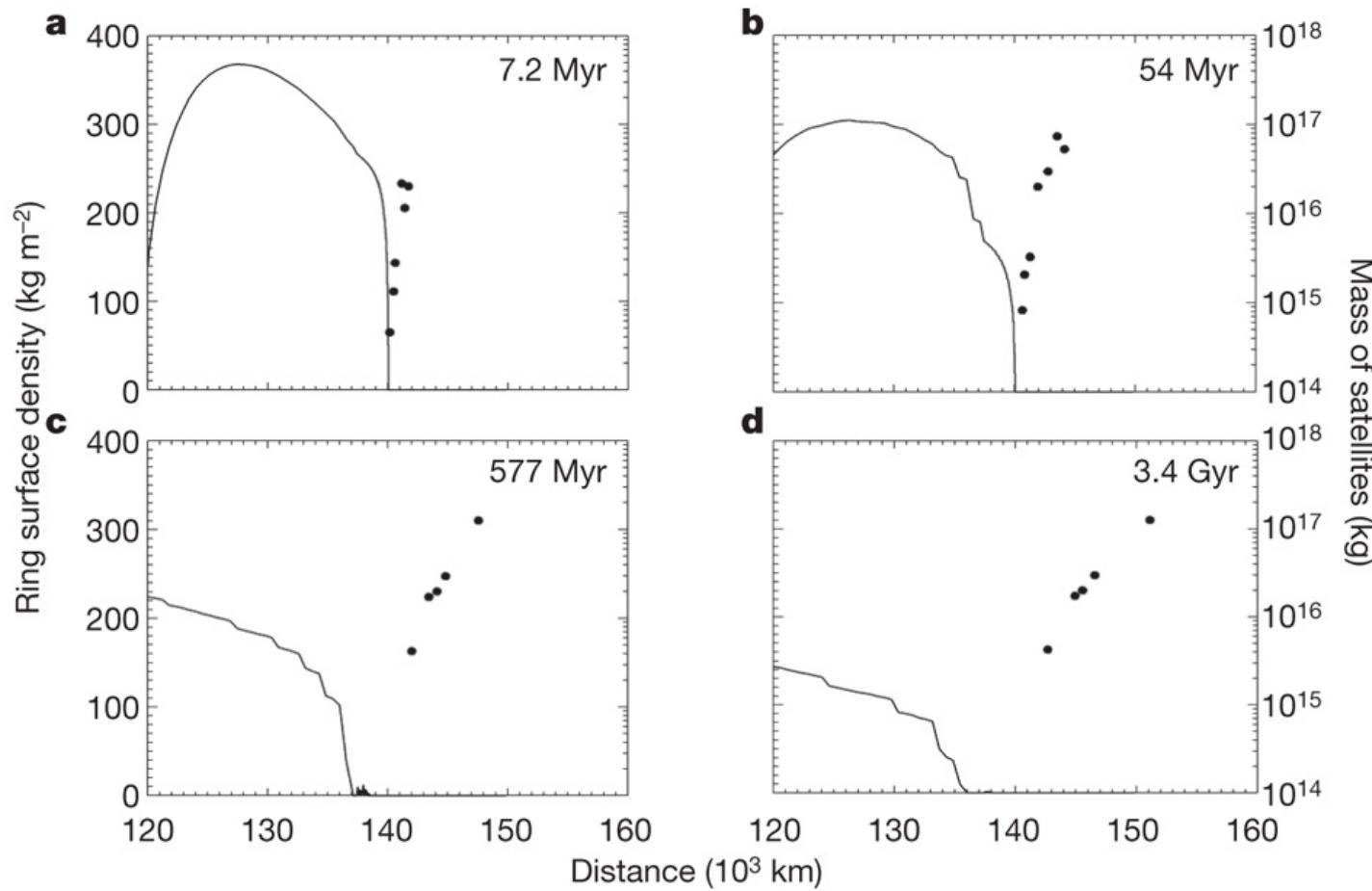
The bigger satellites migrate outwards faster,
the further you are, the more slowly you move.

Numerical application :

~ 10^8 years ago, Janus was in the rings !

Satellites children of the rings

Numerical simulation of present day Saturn's rings,
with satellite formation beyond r_{Roche} :
Formation of Prometheus, Pandora, Epimetheus, Janus.

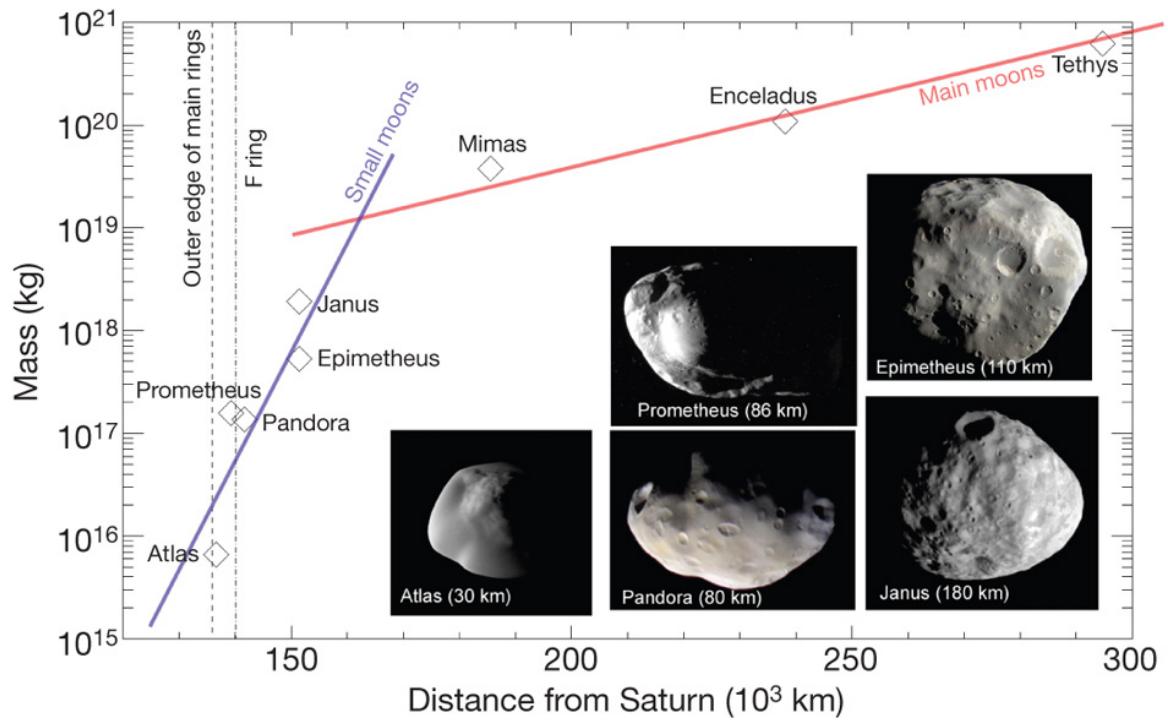


Satellites children of the rings

This explains surprising properties of the small moons :

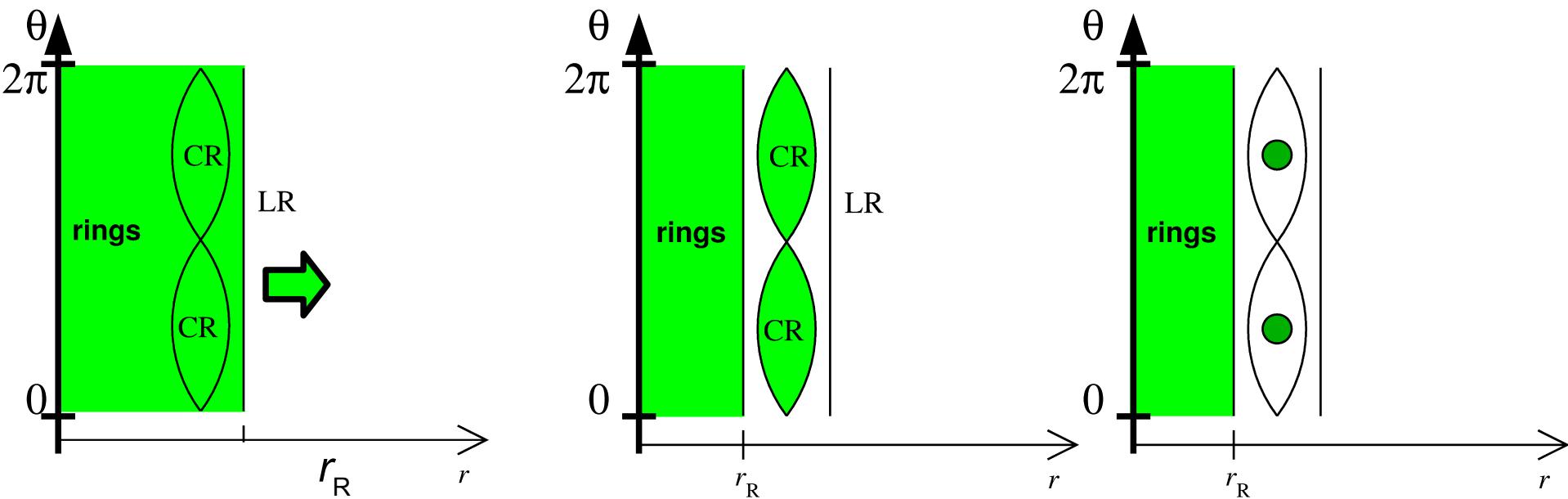
- underdense ($\sim 600 \text{ kg.m}^{-3}$)
- same spectrum as the rings
- dynamically young
- young surfaces

(Charnoz, Salmon,
& Crida, 2010)



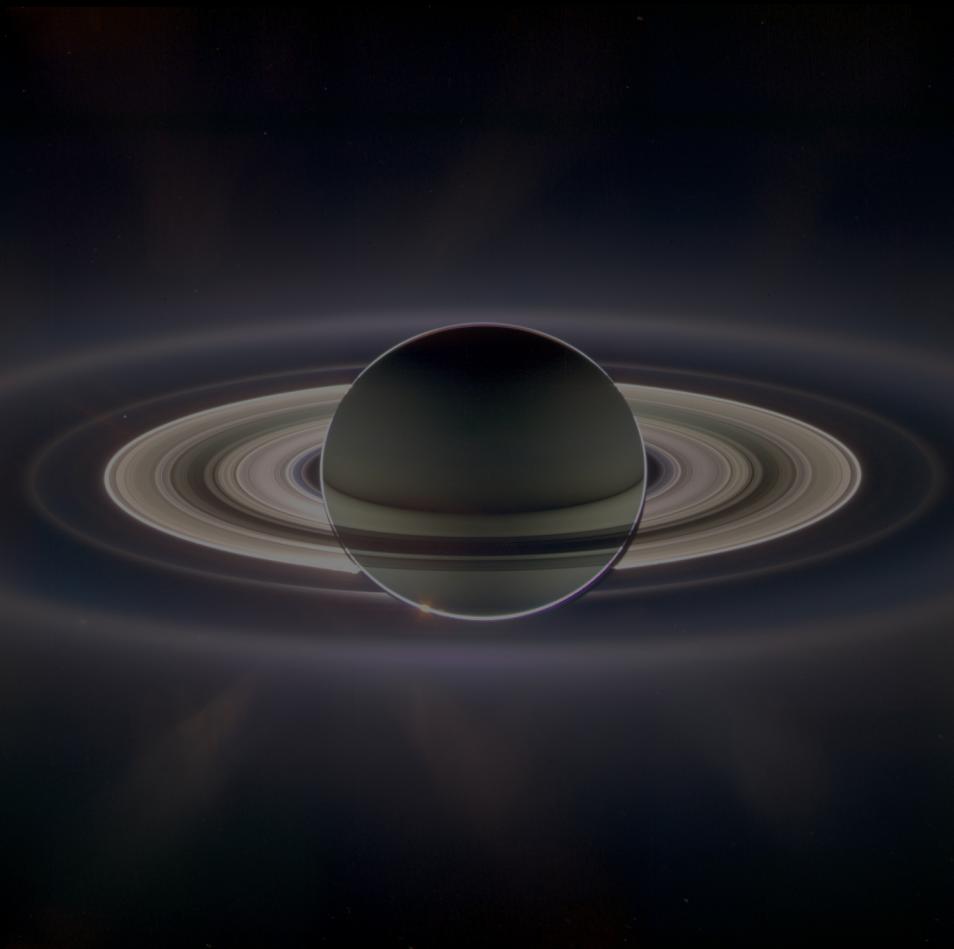
Janus & Epimetheus' horseshoe

Mimas used to confine the rings with its 2:3 Lindblad resonance. When Mimas receded, its 2:3 corotation resonance could have captured ring material and brought it beyond $r_{\text{Roche}} \rightarrow$ 2 bodies on the same orbit = seeds of Janus & Epimetheus ?



Work in progress. (Crida & El Moutamid, DPS 2016)

Do the maths

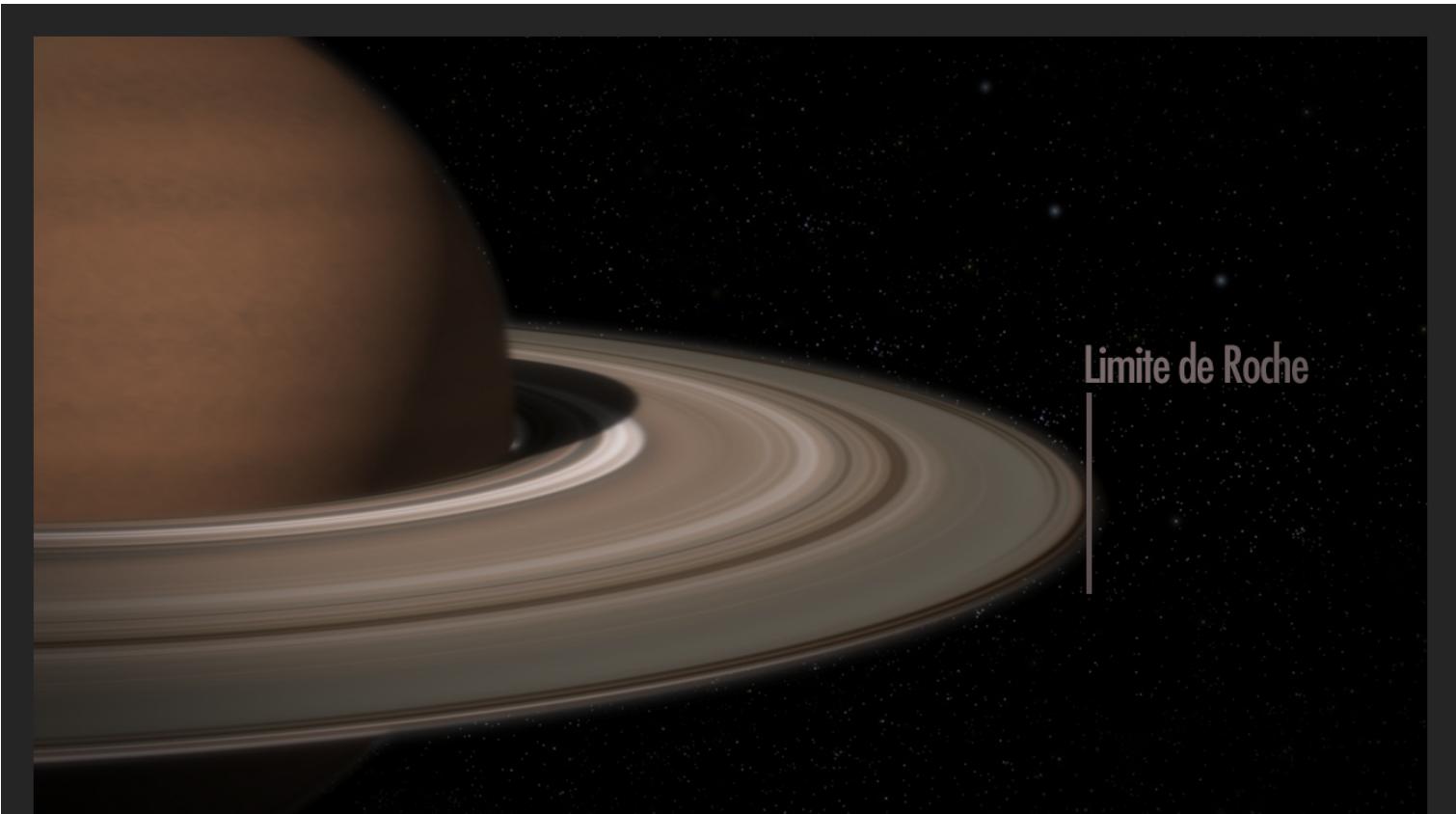


Notations

Be T_R the orbital period at the Roche limit r_R ,

F the flow through r_R , and

$\tau_{disk} = M_{disk} / FT_R$, the normalized life-time of the disk.



Notations

Be T_R the orbital period at the Roche limit r_R ,
 F the flow through r_R , and

$\tau_{disk} = M_{disk} / FT_R$, the normalized life-time of the disk.

The disk spreads with a viscous time $t_v = r_R^2/v$.

Using Daisaka et al. (2001)'s prescription for v ,
we find $\tau_{disk} = t_v / T_R = 0.0425 D^{-2}$ where $D = M_{disk}/M_p$,
and $F = 23 D^3 M_p / T_R$.

Continuous regime

Say 1 satellite forms. Its mass is : $M = F t$ (3)

It feels a torque from the disk : $\Gamma = \frac{8}{27} \left(\frac{M}{M_p} \right)^2 \Sigma r^4 \Omega^2 \Delta^{-3}$ (1)

where $\Delta = (r - r_R) / r_R$.

→ Migration rate :

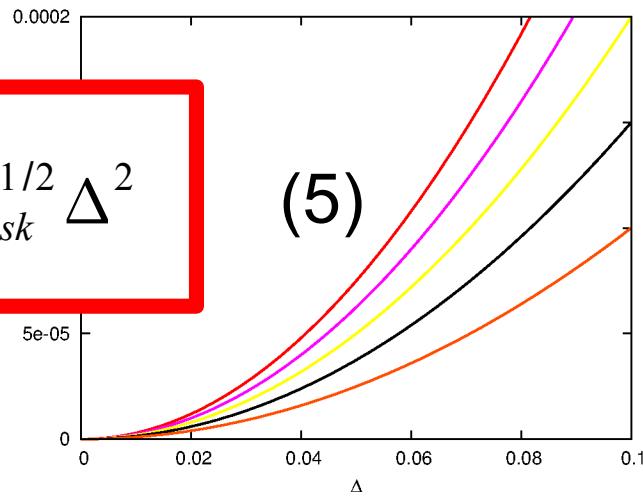
$$\frac{d\Delta}{dt} = \frac{32}{27} q D T_R^{-1} \Delta^{-3} \quad (4)$$

where $q = M / M_p$.

Solution of (3) & (4) :

$$q = \left(\frac{\sqrt{3}}{2} \right)^3 \tau_{disk}^{-1/2} \Delta^2 \quad (5)$$

We call this the *continuous regime*.



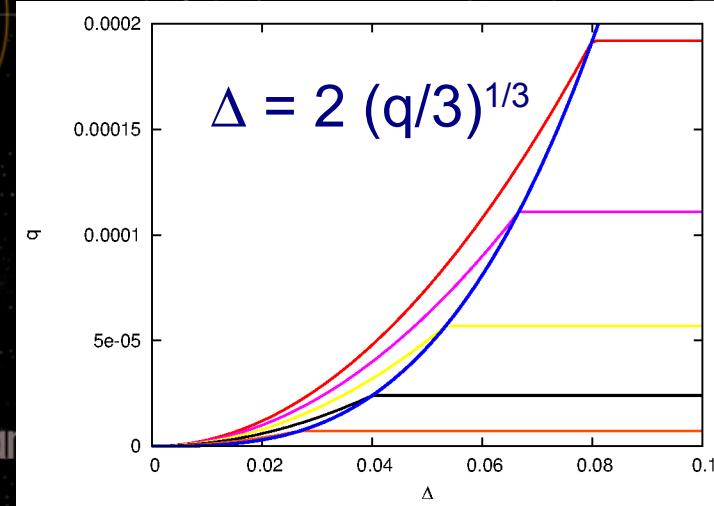
Continuous regime

This holds as long as the satellite captures immediately what comes through r_R .

That is, as long as $(r - r_R) < 2 r_{\text{Hill}}$, or $\Delta < 2 (q/3)^{1/3}$.



Régime continu:
formation d'une lune par accrétion de la matière des anneaux qui fr



Continuous regime

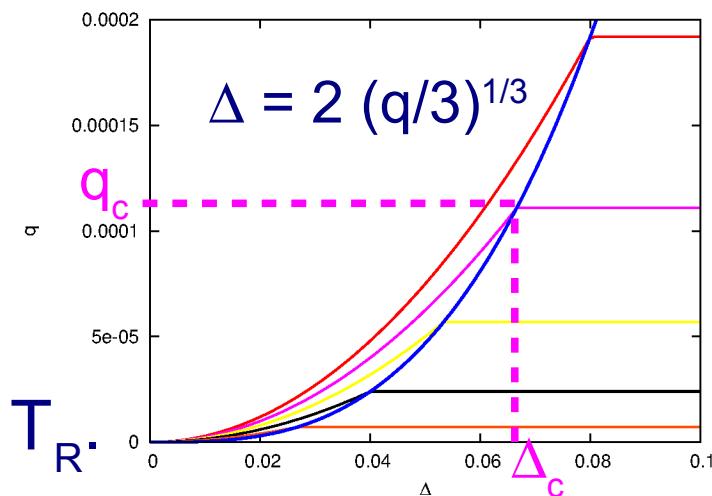
This holds as long as the satellite captures immediately what comes through r_R .

That is, as long as $(r-r_R) < 2 r_{\text{Hill}}$, or $\Delta < 2 (q/3)^{1/3}$.

Input into Eq.(5), this gives a condition of validity for the continuous regime :

$$\Delta < \Delta_c = \sqrt{\frac{3}{\tau_{disk}}} = \sim 8.4 D$$

$$q < q_c = \frac{3^{5/2}}{2^3} \tau_{disk}^{-3/2} = \sim 222 D^3$$



Duration of the continuous regime: $10 T_R$.

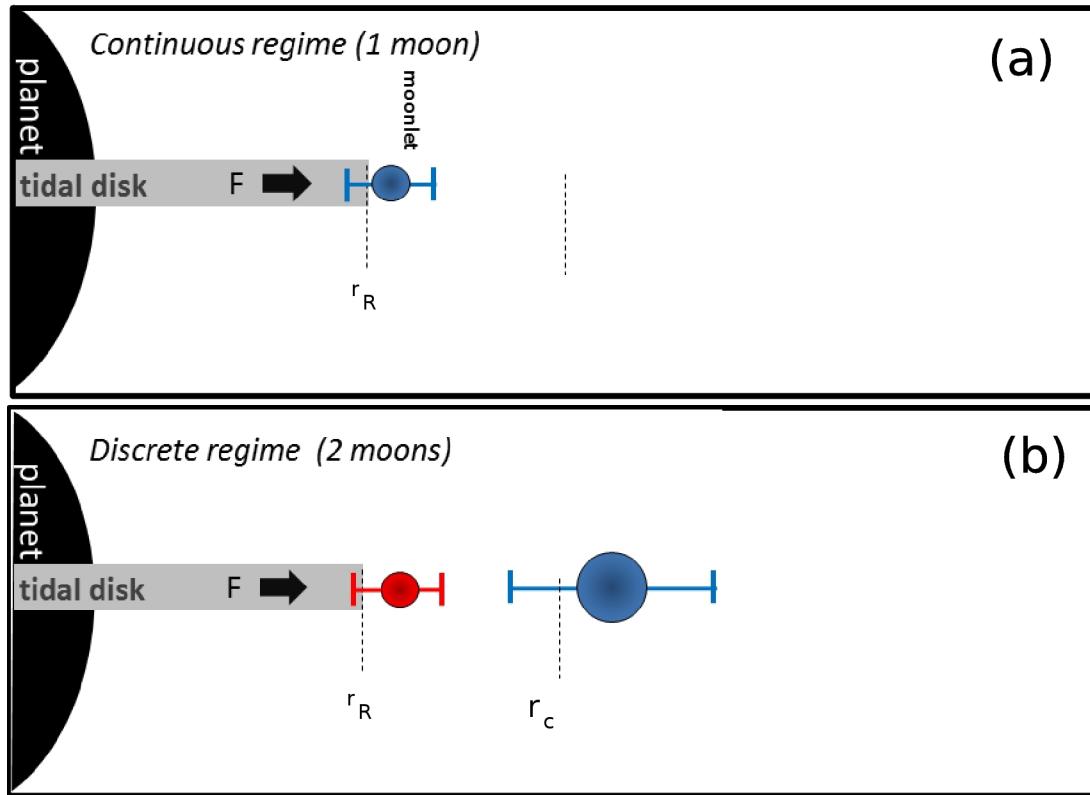
Discrete regime

When the satellite is beyond Δ_c (or q_c), the material flowing through r_R forms a new satellite at r_R .

This new satellite is immediately accreted by the first one.

And so on...

The first satellite still grows as $M=Ft$, but by steps : *discrete regime*.



Discrete regime

This holds as long as $\Delta < \Delta_c + 2(q/3)^{1/3}$.

It gives the condition :

$$\Delta < \Delta_d = 3.14 \Delta_c = \sim 26 \text{ D}$$

$$q < q_d = 9.9 q_c = \sim 2200 \text{ D}^3$$

The duration of the discrete regime is $\sim 100 T_R$.

Application :

Saturn's rings : $q_d = \sim 10^{-18}$.

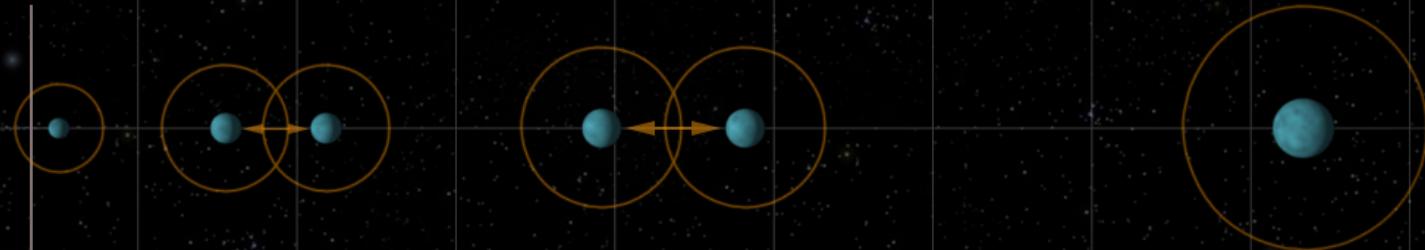
→ Only tiny things form. Then what ?

Pyramidal regime

Satellites of mass q_d are produced at Δ_d every q_d / F .

Then, many satellites of constant mass migrate outwards, at decreasing rates. They approach each other, and merge.

Limite de Roche



Régime pyramidal:

Formation de lunes par fusion gravitationnelle

Pyramidal regime

Satellites of mass q_d are produced at Δ_d every q_d / F .

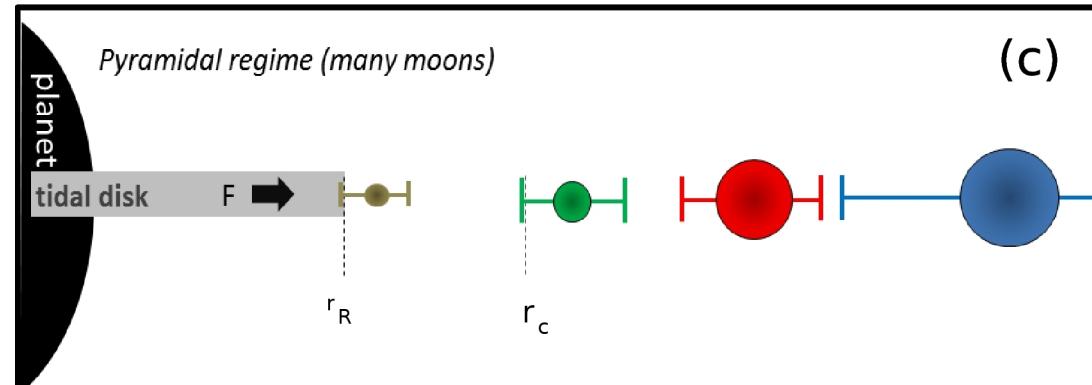
Then, many satellites of constant mass migrate outwards, at decreasing rates. They approach each other.

If their distance decreases below 2 mutual Hill radii, they merge.

=> Formation of satellites of masses $2q_d$, every $2 q_d / F$, which migrate away and merge further...

And so on, hierachically...

We call this *the pyramidal regime*.



Pyramidal regime

- Using Eq.(4), we show that in the pyramidal regime, while the mass is doubled, Δ is multiplied by $2^{5/9}$.

Thus, $q \propto \Delta^{9/5}$.

In addition, the number density of satellites should be proportionnal to $1/\Delta$, explaining the pile-up.

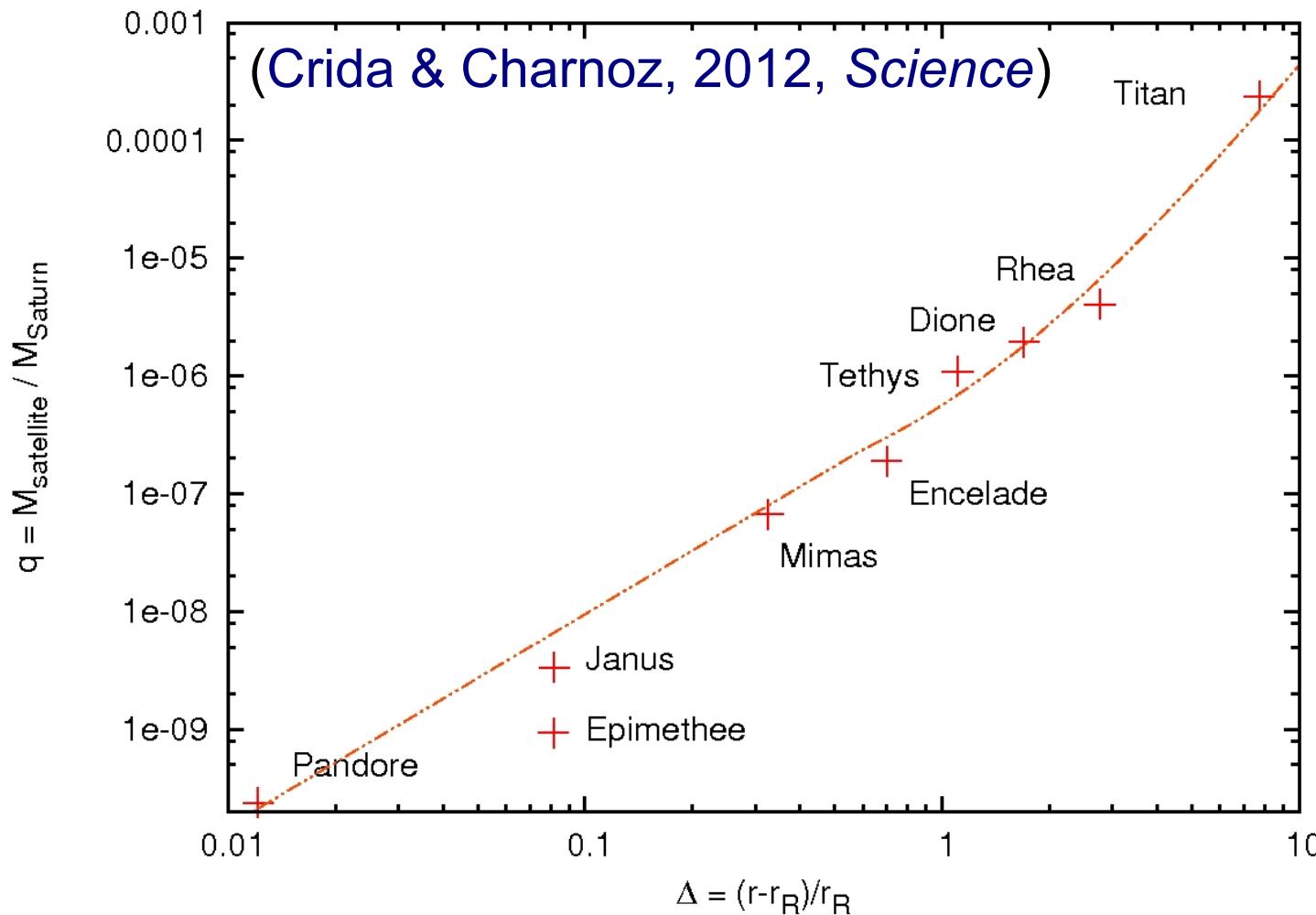
- Beyond the 2:1 Lindblad resonance with r_R ($\Delta=0.58$), Eq.(4) doesn't apply. Migration is driven by planetary tides:

$$\frac{dr}{dt} = \frac{3 k_{2p} M \sqrt{G} R_p^5}{Q_p \sqrt{M_p} r^{11/2}} \quad (2)$$

Using Eq.(2), we find $q \propto r^{3.9}$.

Pyramidal regime

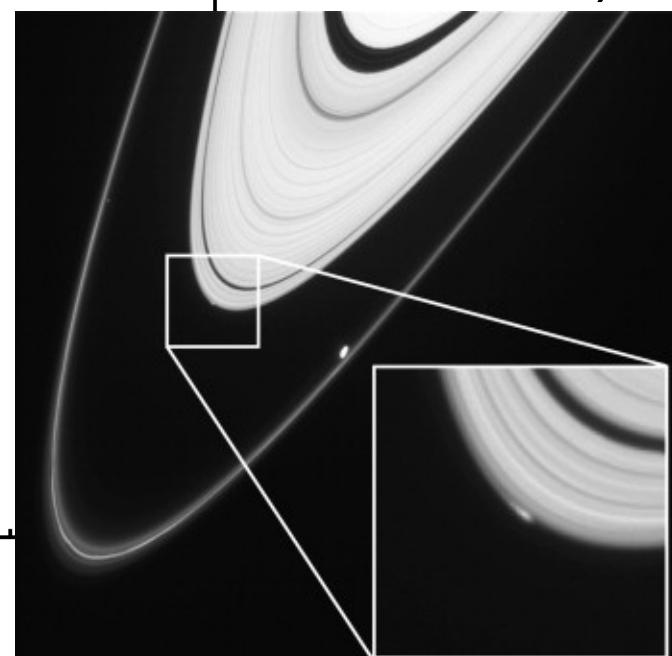
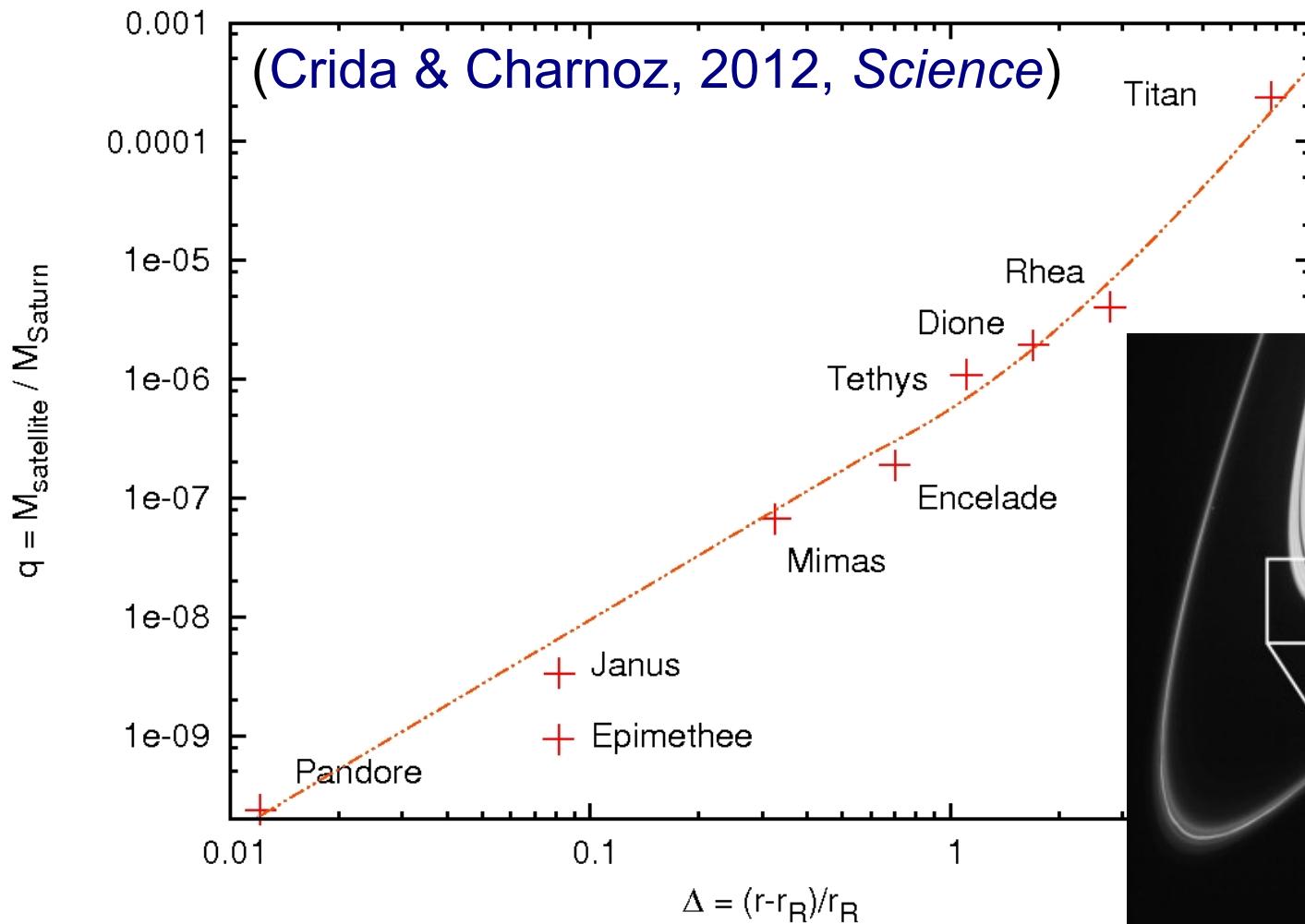
The result spectacularly matches the distribution of the Saturnian system !



Note : the + symbols
are **not** errorbars.
Masses and positions
are precisely known.

Pyramidal regime

The result spectacularly matches the distribution of the Saturnian system !



Can this be true ?



- 1) Was there enough mass in the rings ?**

- 2) Was there enough time ?**

Rings evolution: a first clue

Be T_R the orbital period at the Roche limit r_R , $\bar{t} = t / T_R$,
 τ_{disk} the normalized life-time of the disk = $M_{disk} / (dM_{disk}/d\bar{t})$.

Using Daisaka et al. (2001)'s prescription for v ,
we find $\tau_{disk} = t_v / T_R = 0.0425 D^{-2}$ where $D = M_{disk}/M_p$.

Thus $(dD / d\bar{t}) = \sim -30 D^3$

$$D(\bar{t}) = 1 / \sqrt{60 \bar{t} + D_0^{-2}}$$

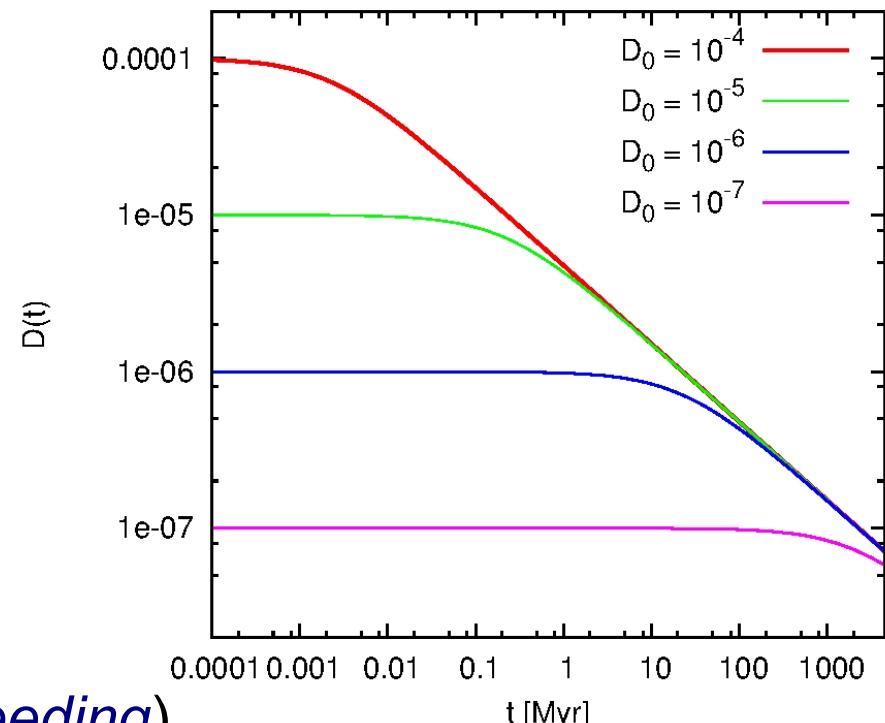
If $\bar{t} \gg 1/60D_0^2$,

$D(\bar{t}) = (1/60\bar{t})^{1/2}$, **indep of D_0 !**

$$\bar{t} = 4.5 \text{ Gyr} \rightarrow D < \sim 10^{-7}$$

$$\text{Now, } D = 8 \times 10^{-8} \dots$$

(Crida & Charnoz 2014, IAU proceeding)



Rings evolution: simulations

Implicit 1D code solving :
(mass + momentum conservation)

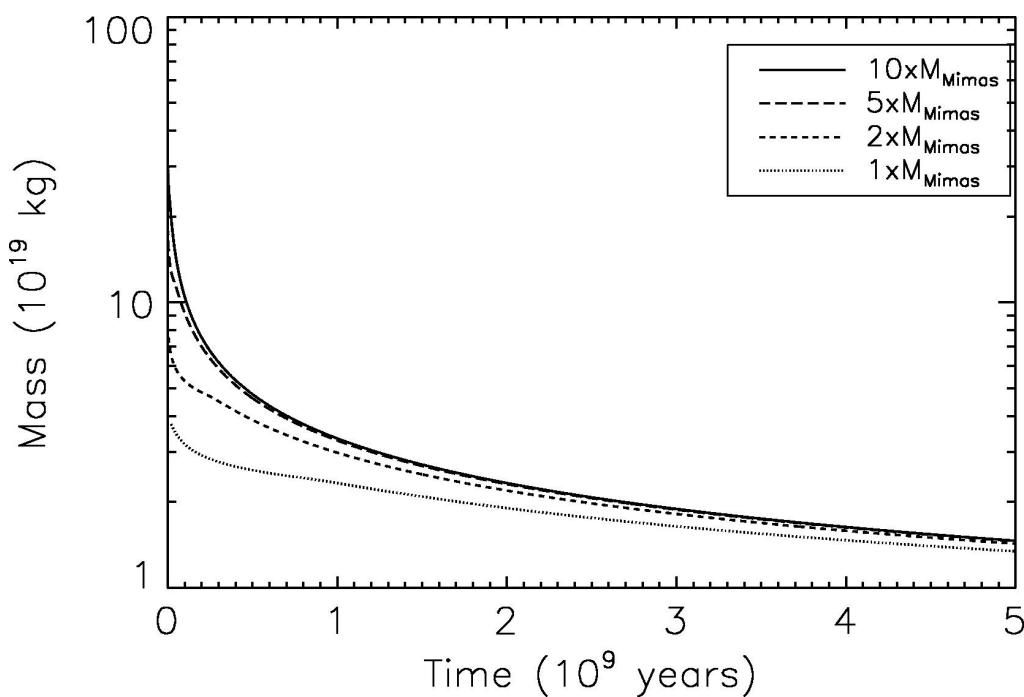
$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[\sqrt{r} \frac{\partial}{\partial r} \left(\nu \Sigma \sqrt{r} \right) \right]$$

With Daisaka et al (2001)'s viscosity $\nu(\Sigma)$,
the less massive the rings are, the slower they spread

→ with their present mass, they can survive for Gyrs.

→ they could have been
more massive in the past:
whatever their initial mass,
they should have ~present
mass after ~4.5 Gyrs.

(Salmon, Charnoz, Crida,
& Brahic, 2010, *Icarus*)

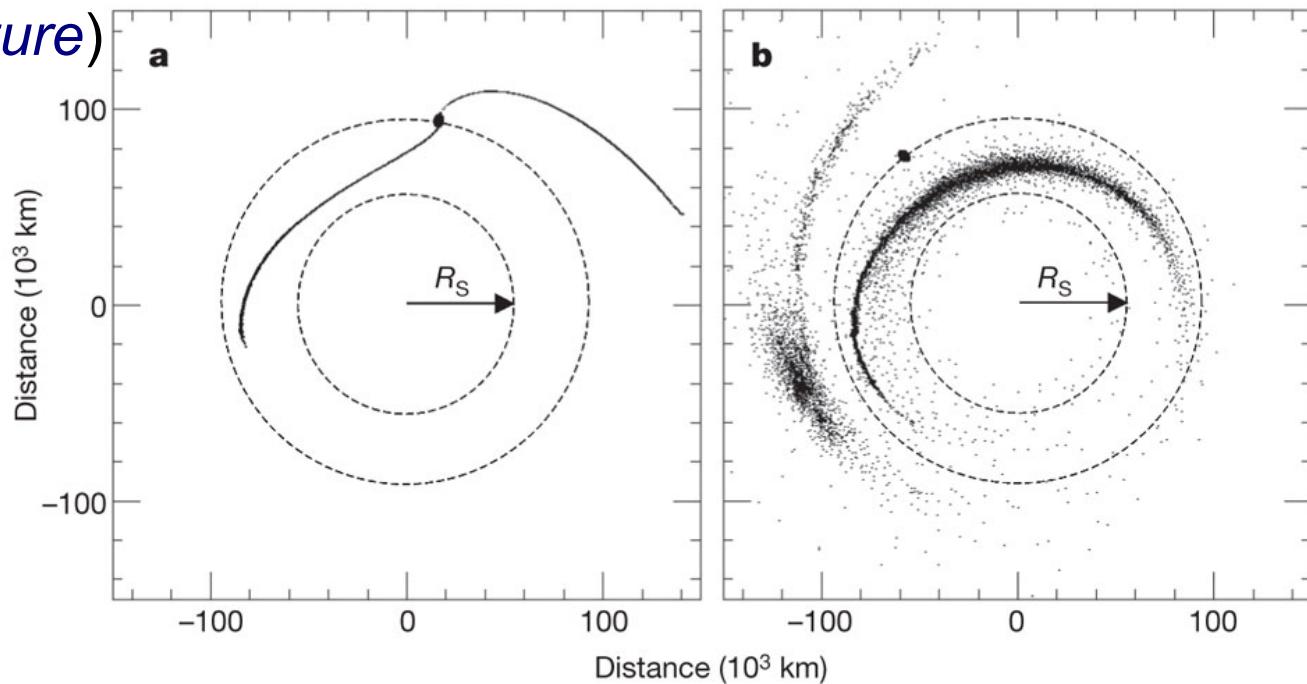


Origin of Saturn's rings

If a satellite forms in the circum planetary disk around Saturn, it migrates inwards, and can be lost
(Canup & Ward 2002, 2006).

If differentiated, its icy mantle is peeled off by tidal forces inside the Roche limit, while the denser silicate core keeps migrating and falls inside Saturn.

(Canup, 2010, *Nature*)



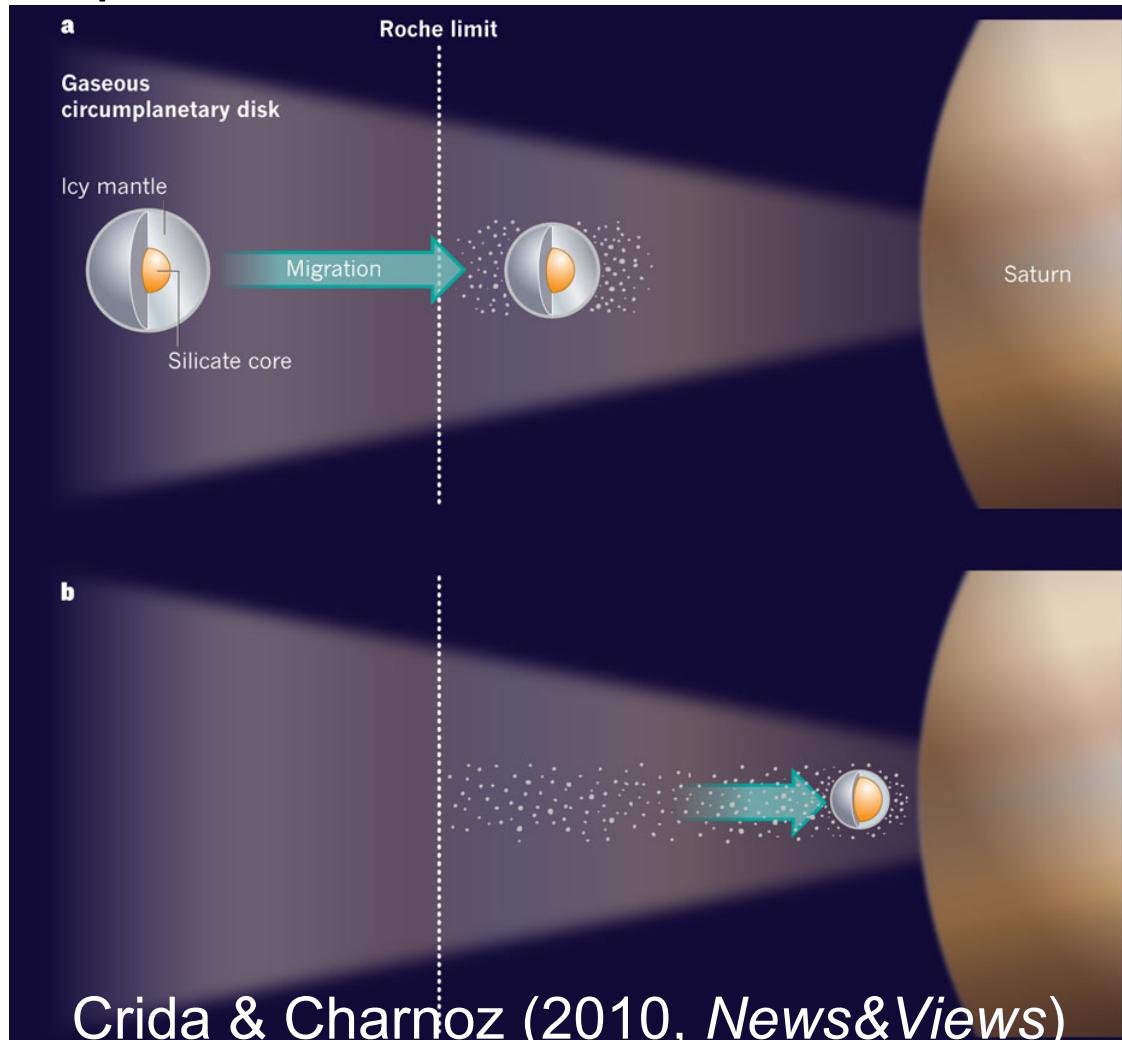
Origin of Saturn's rings

Key points of Canup (2010)'s model :

- **differentiated** satellite => pure water ice
- **migration** =>
progressive pealing off
+ get rid of the core
- **last** big satellite lost =>
the debris stay there.

In the end :

- a very massive ring
(~ 10^{13} kg)
- from ~100 years ago.
- no gap between
the rings & Titan.



Crida & Charnoz (2010, News&Views)

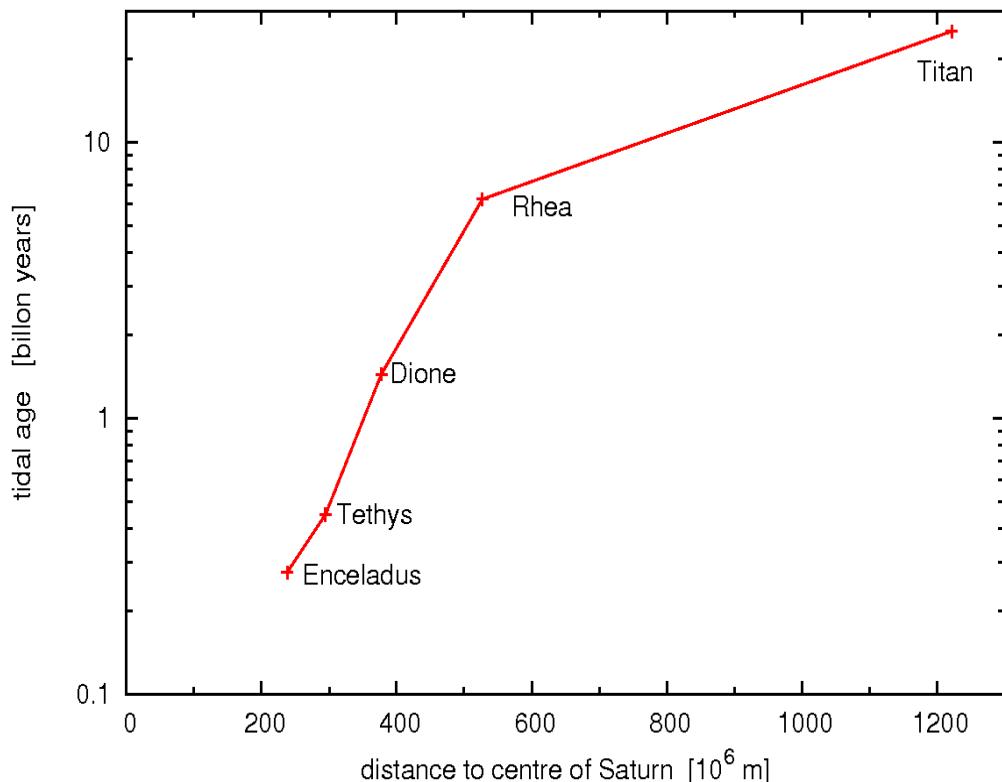
Timing: are tides efficient ?

Using this equation : $\frac{dr}{dt} = \frac{3 k_{2p} M_{satellite} \sqrt{G} R_{Saturne}^5}{Q_{Saturne} \sqrt{M_{Saturne}} r^{11/2}}$ Eq.(2)

one finds the « **tidal age** » : how long it takes to bring the satellites at their present position from 222 000 km, where interactions with the rings vanish.

Standard $Q_{Saturn} = 18\,000$,
→ negligible migration.

With $Q_{Saturn} = 1700$
(Lainey *et al.* 2012, 2016,
based on observations)
even Rhea is younger
than the Solar System !

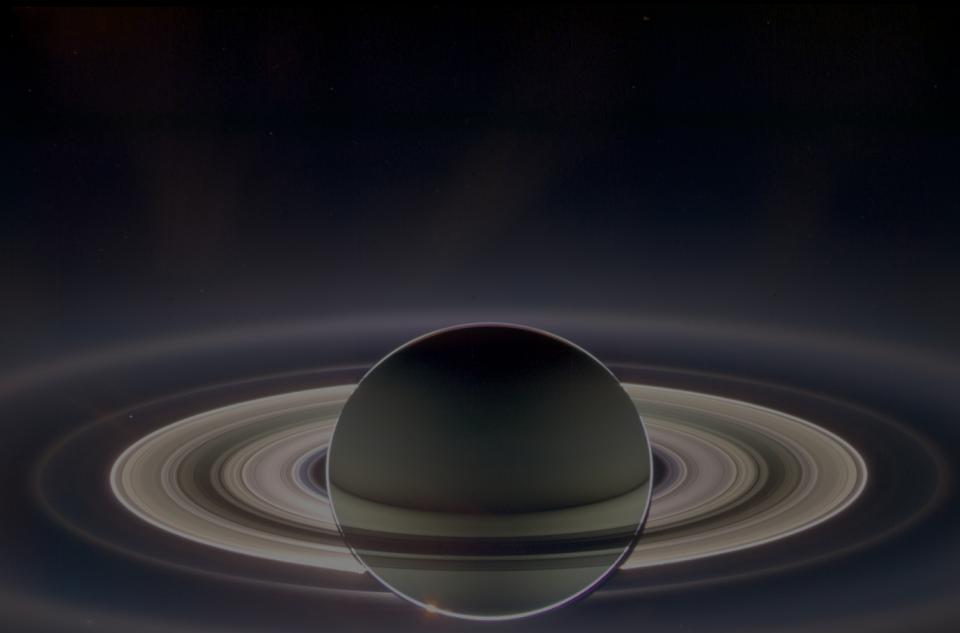


Other good aspects

- ✓ Ages are ranked by distance : the further, the older.
→ formation of one after the other, in logical order.
- ✓ The mid-sized moons have irregular cores, and irregular global composition : stochastic formation of silicate aggregates in the rings, coated with ice.
→ at least for Dione & Rhea's core.
- ✓ The mid sized moons have young cratering ages, and couldn't survive the Late Heavy Bombardment, hence must have formed less than 4 Gyrs ago.
→ to be checked carefully + study of debris impacts

(Charnoz, Crida, Castillo-Rogez, et al. 2011, *Icarus*)

It's not over yet !



What about the other planets ?

Discrete regime: our Moon

This holds as long as $\Delta < \Delta_c + 2(q/3)^{1/3}$.

It gives the condition :

$$\Delta < \Delta_d = 3.14 \Delta_c = \sim 26 \text{ D}$$

$$q < q_d = 9.9 q_c = \sim 2200 \text{ D}^3$$

Application :

Earth's Moon forming disk :

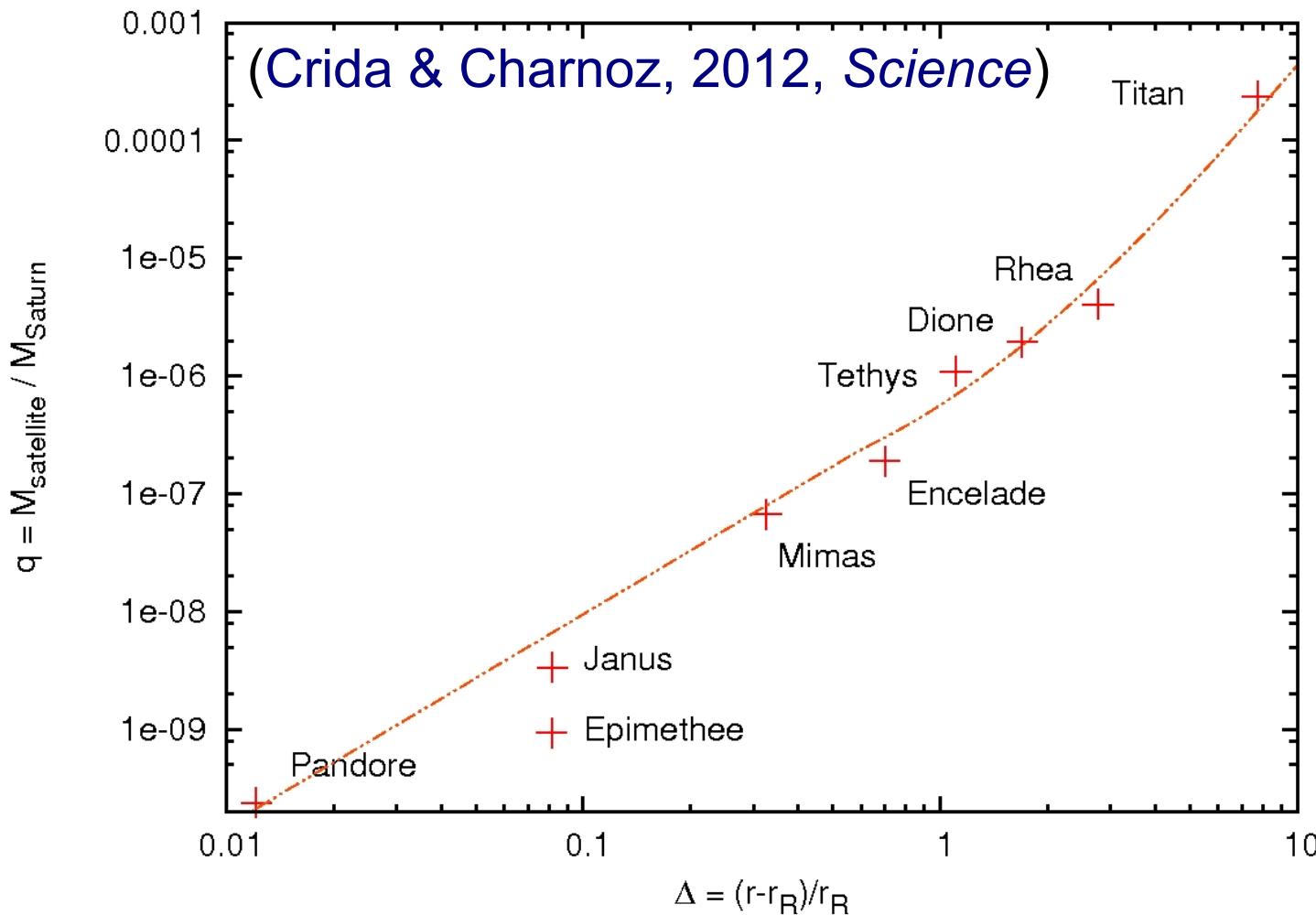
q_d = mass of the Moon !

→ Only 1 satellite formed, in ~ 1 month.



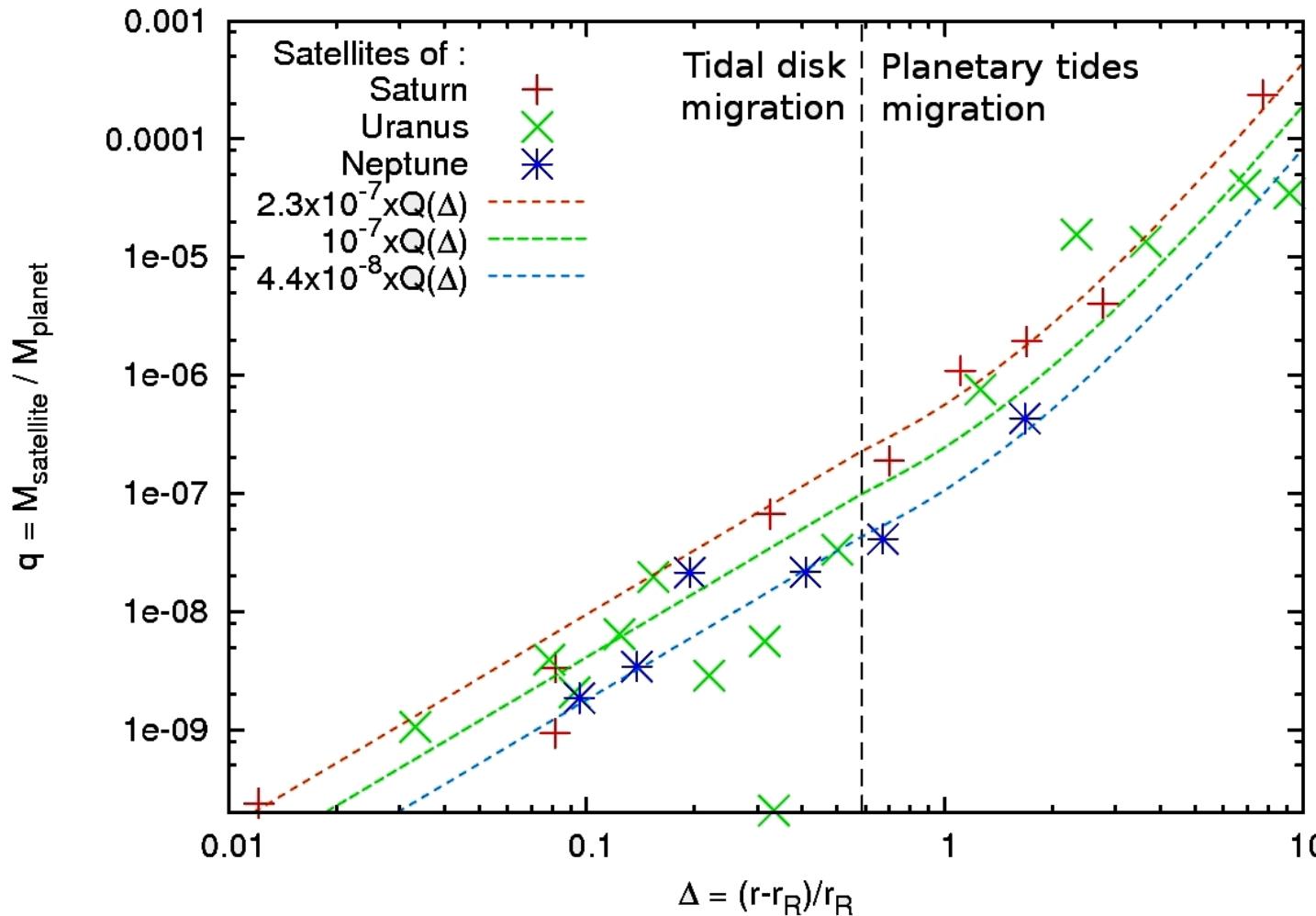
Pyramidal regime

The result spectacularly matches the distribution of the Saturnian system !



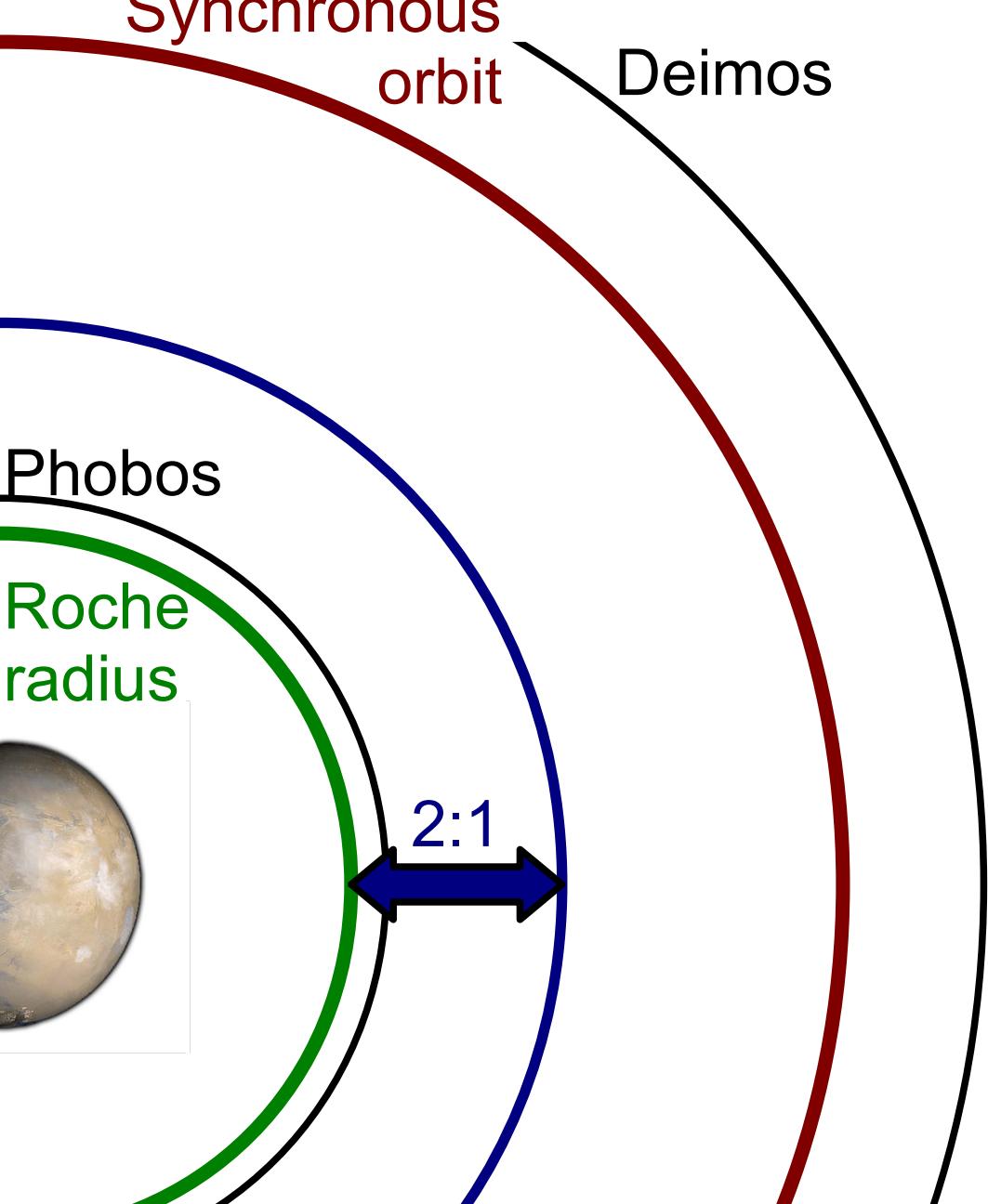
Pyramidal regime

The result spectacularly matches the distribution of the Saturnian, Uranian, and Neptunian systems !



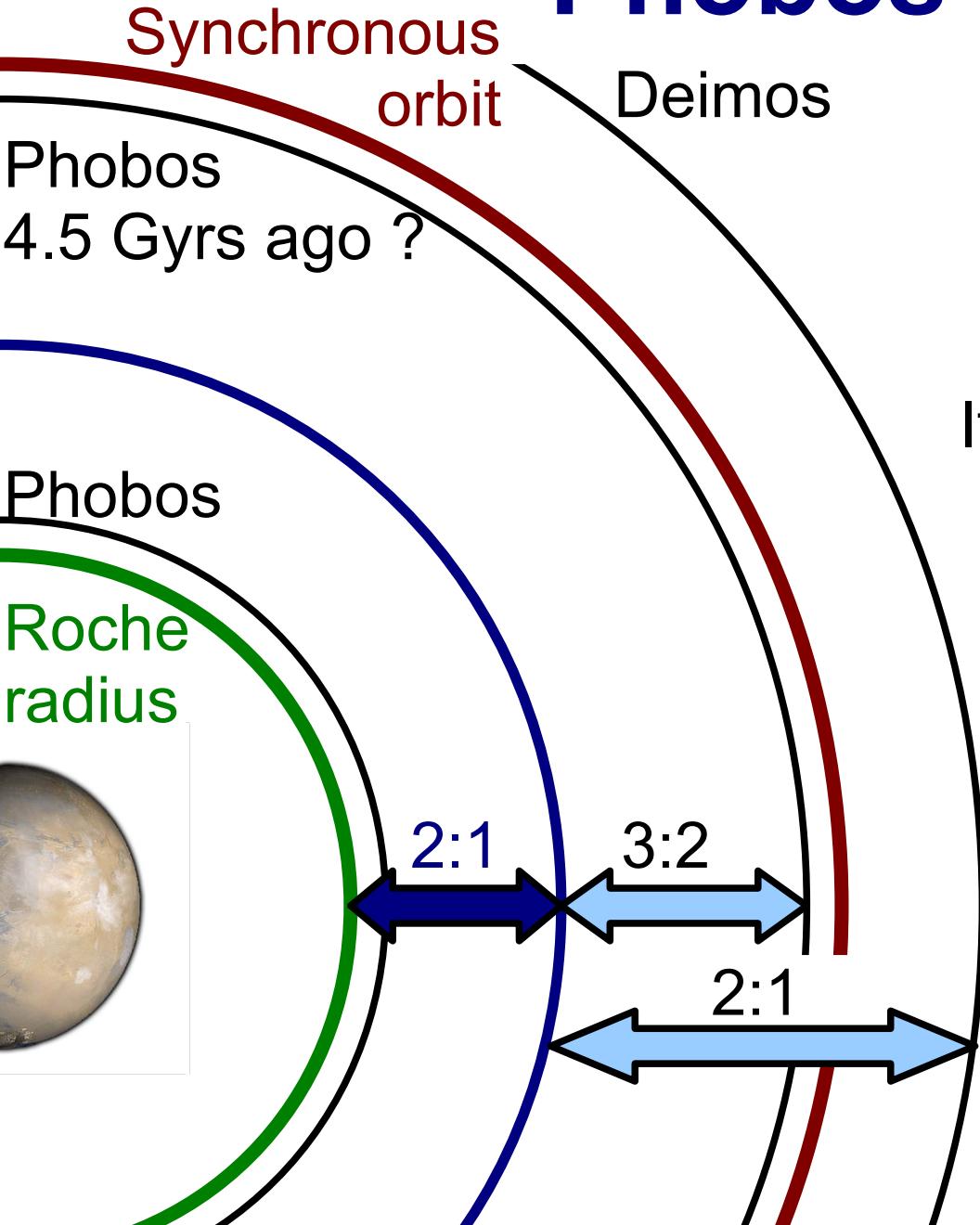
I claim that
Uranus and
Neptune
had massive
rings,
from which
their regular
satellites
were born.

Phobos and Deimos



Impossible to produce Phobos, and mostly Deimos by direct pyramidal regime.

Phobos and Deimos



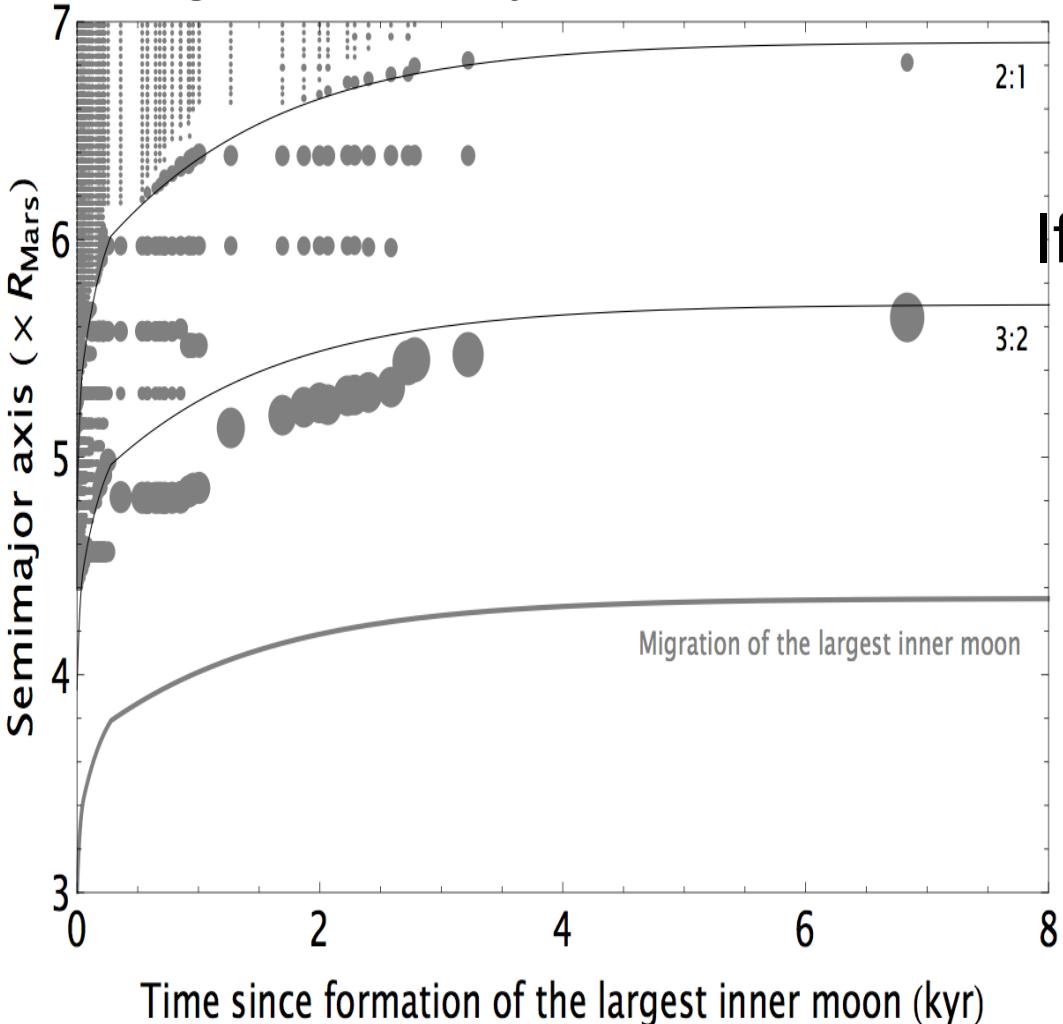
Impossible to produce Phobos, and mostly Deimos by direct pyramidal regime.

If a giant impact created a massive ring within r_{Roche} + debris beyond r_{Roche} , the debris are parked at resonances with satellites formed in the pyramidal regime, that later fall back onto Mars.

(Rosenblatt et al. 2016,
Nature Geoscience)

Phobos and Deimos

Figure courtesy : S. Charnoz



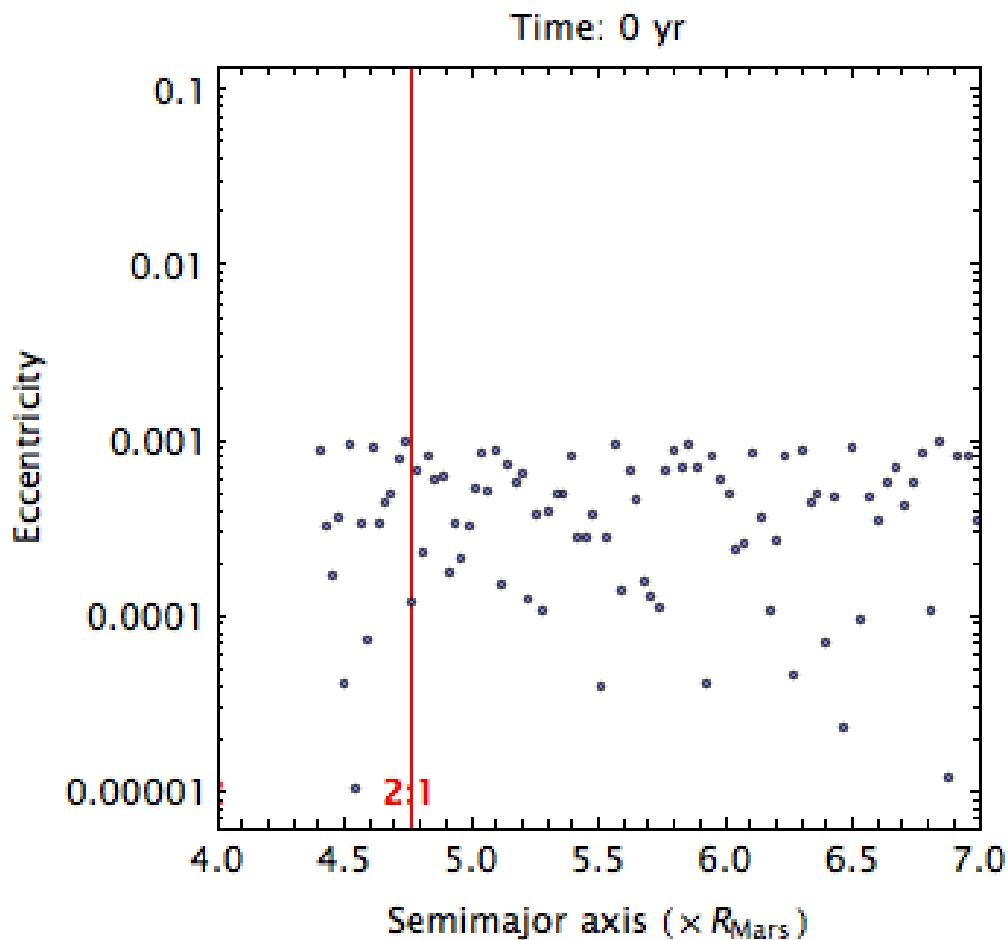
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Phobos and Deimos

Movie courtesy : S. Charnoz



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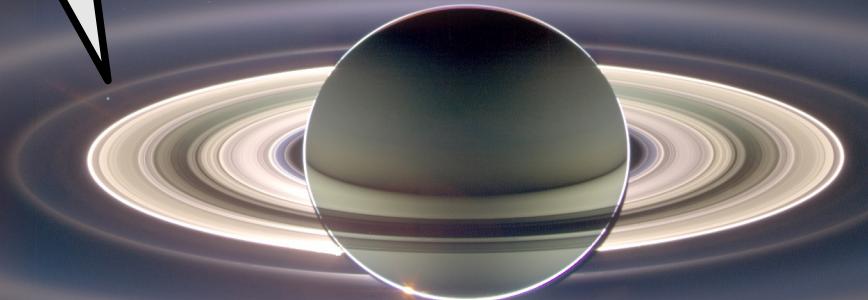
(Rosenblatt et al. 2016,
Nature Geoscience)

Summary

The spreading of a tidal disk beyond the Roche radius

- ✓ explains the mass-distance distribution of the regular satellites of the giant planets
(observational signature of this process)
 - ✓ unifies terrestrial and giant planets in the same paradigm.
 - ✓ most Solar System regular satellites formed this way.
-
- ✗ Jupiter doesn't fit in this picture : probably formed in a circum-planetary disk (e.g. Canup & Ward 2002, 2006 ; Sasaki et al 2010)
 - Titan fits very well in this picture, though its « tidal age » is too large... Coincidence ?

Merci !
Thanks !



References : Crida & Charnoz (2014, IAU#310)

Crida & Charnoz (2012, *Science*)

Charnoz, Salmon, Crida (2010, *Nature*)

Charnoz, Crida, *et al.* (2011, *Icarus*)

Crida & El Moutamid (in prep.)

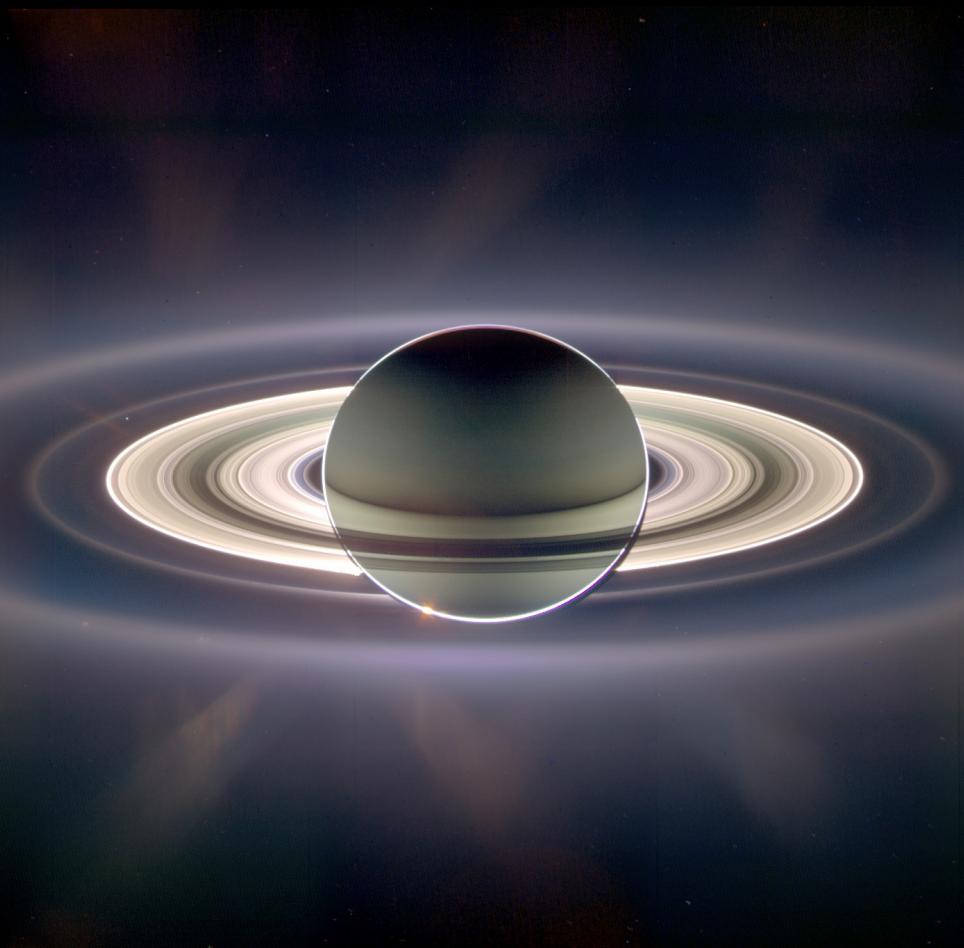
Canup (2010, *Nature*)

Lainey *et al.* (2012, *ApJ*)

Salmon *et al.* (2010, *Icarus*)

Rosenblatt *et al.* (2016, *Nature Geo*)

EXTRA SLIDES



Summary

1) Continuous regime:

1 moon grows

$$q \propto \Delta^2$$

until Δ_c or q_c .

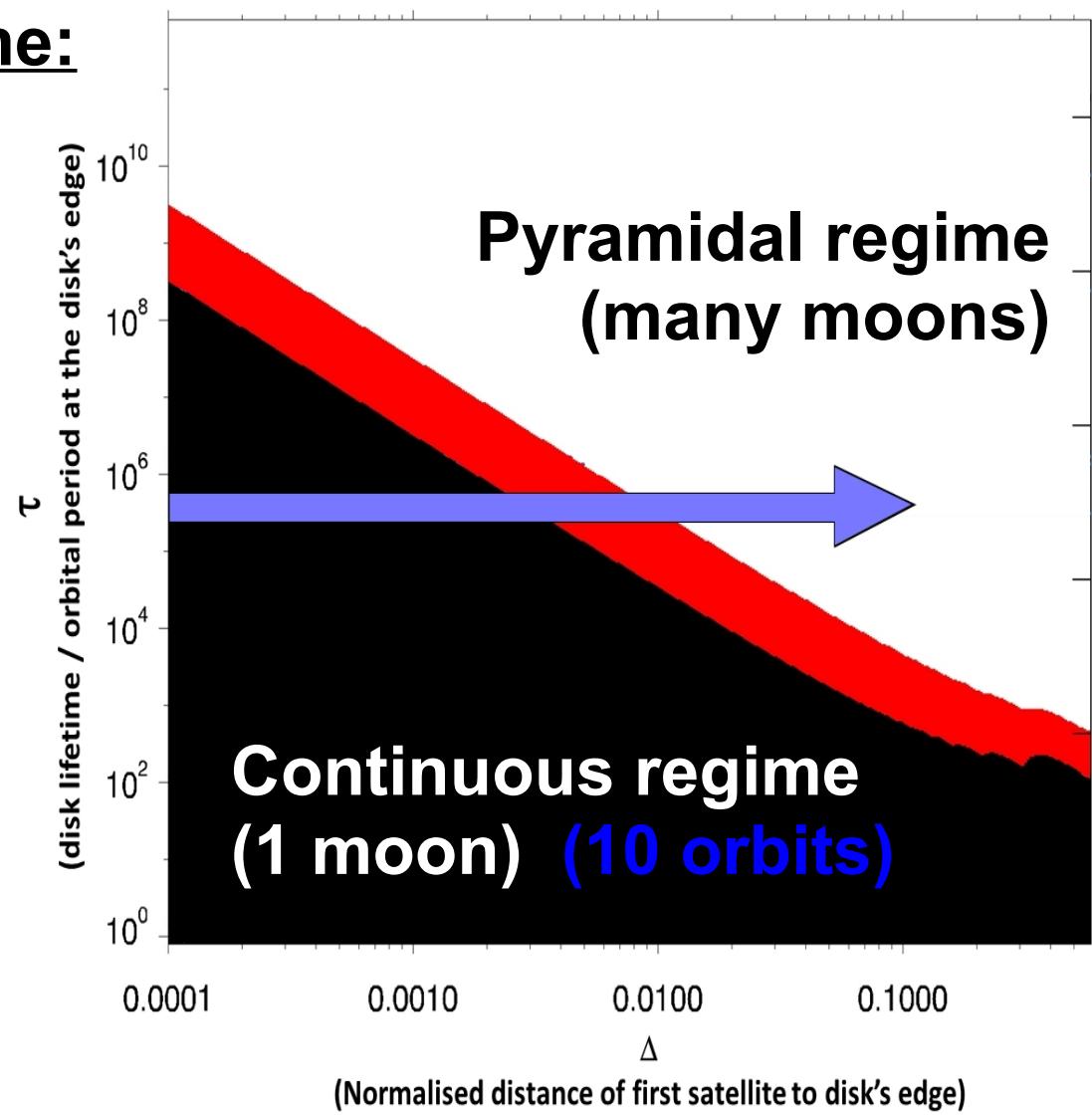
2) Discrete regime:

2 moons,
growth by steps
until Δ_d or q_d .

3) Pyramidal regime:

Many moons in the system.

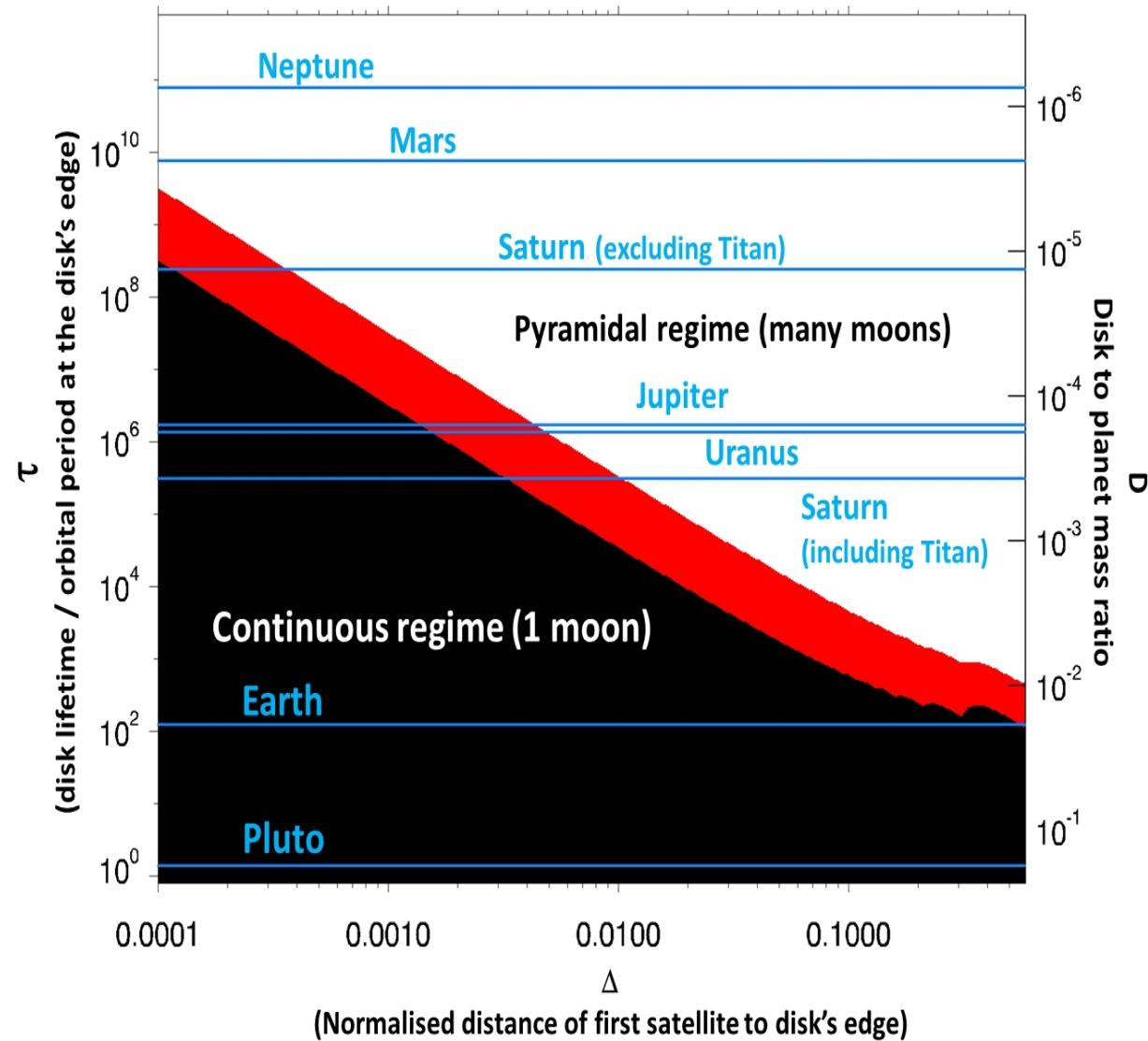
$$q \propto \Delta^{9/5} \text{ or } r^{3.8}$$



Summary

Take $M_{\text{disk}} = 1.5 \times$
the mass of the
present satellite
system.

Giant planets must
be dominated by the
pyramidal regime,
while we expect the
Earth and Pluto to
have 1 large satellite.



Evolution of Saturn's rings

Mass conservation :

$$r \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r} (r \Sigma v_r) = 0$$

Angular momentum conservation :

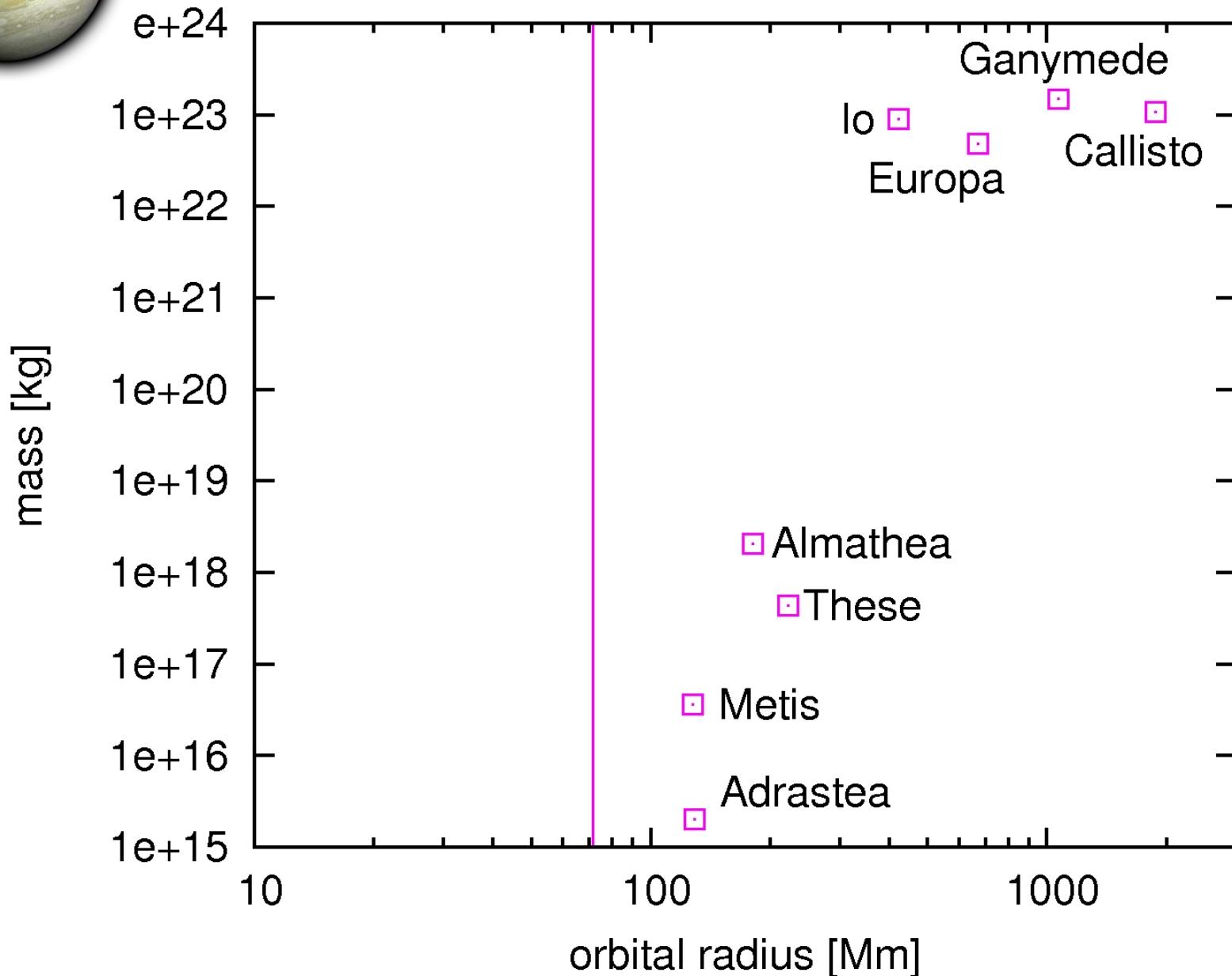
$$\frac{\partial}{\partial t} (\Sigma r^2 \Omega) + \frac{1}{r} \frac{\partial}{\partial r} \left(\nu \Sigma r^3 \frac{\partial \Omega}{\partial r} \right) = 0$$

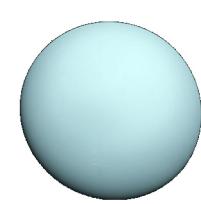
Thus density evolution :

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[\sqrt{r} \frac{\partial}{\partial r} (\nu \Sigma \sqrt{r}) \right]$$

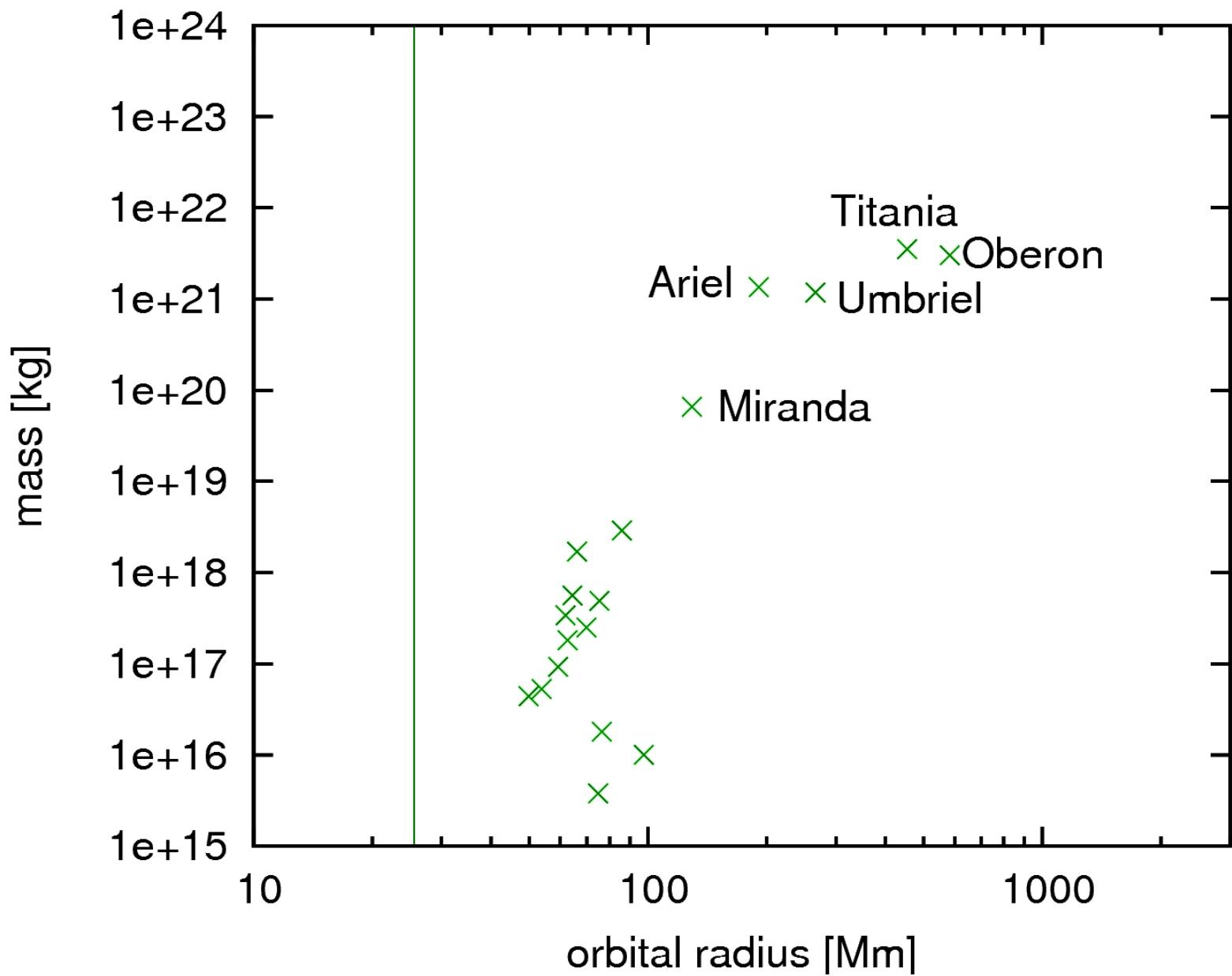


JUPITER



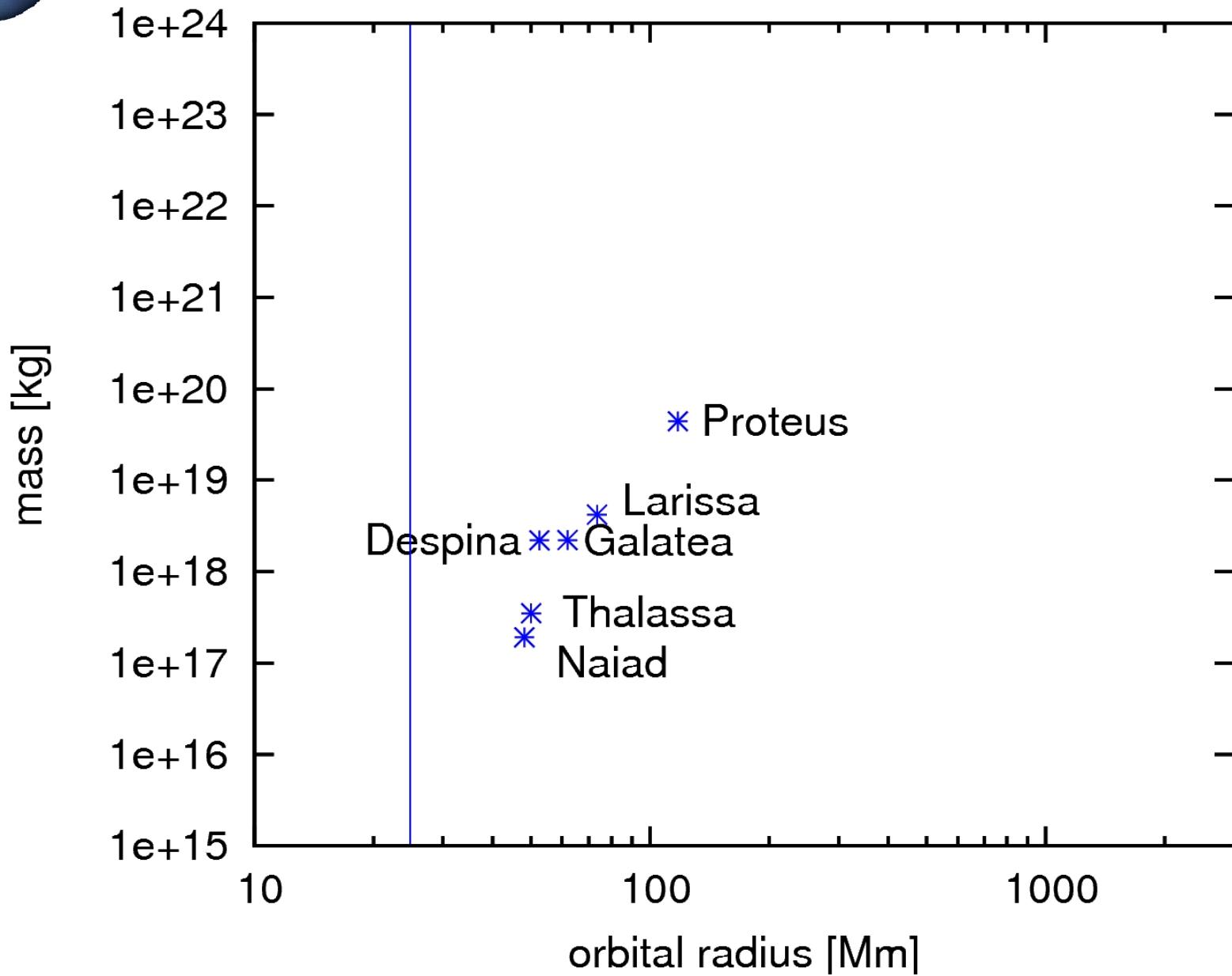


URANUS

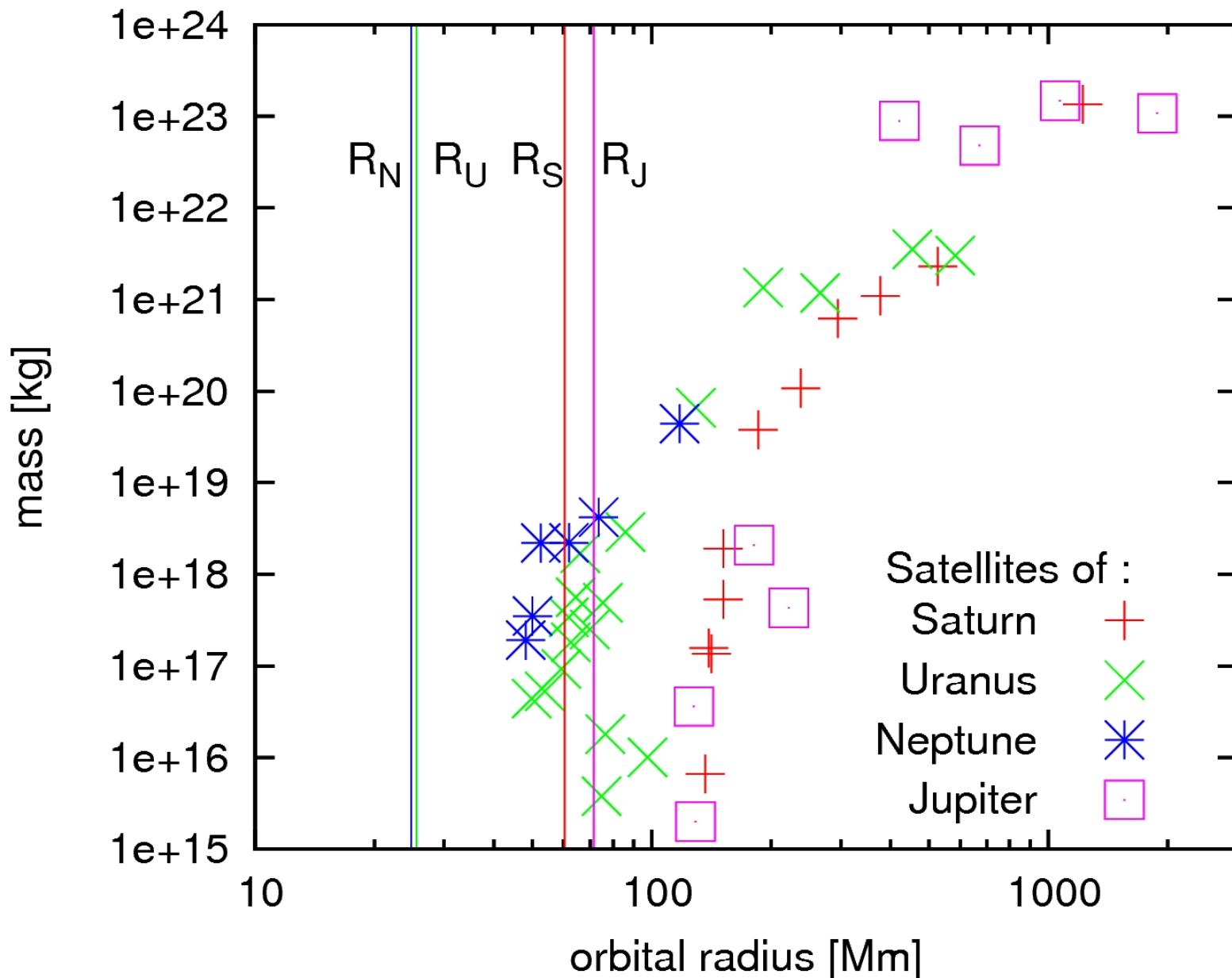




NEPTUNE



ALL GIANT PLANETS



ALL GIANT PLANETS

Distributions of giant planets' regular satellites :

- don't reach the planet
 - ranked by mass
 - pile-up at a few planetary radii
(small bodies)

Why ?

