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Exam in Fluid mechanics (MO5001)

Write the solution of each problem on a separate paper, and write your identification number on every paper.

Allowed aids: calculator, sheet with vector analysis relations.

Grading: A 90-100%, B 80-89%, C 65-79%, D 55-64%, E 50-54%, Fx 45-49%, F 0-44%

1. Answer the following short questions. You just need to write the final answer. Each question is worth 3 points.

   (a) A vector field, $\boldsymbol{u}$, with components u_x, u_y and u_z, as a function of space (described by x, y, and z coordinates) is give by the following expression

   $$\begin{aligned} u_x &= \alpha[2x + \cos(y) + 5z^3] \\ u_y &= \alpha[e^{-x} - y + \sin(y)] \\ u_z &= \alpha[\sin(x) + \cos(y) - z] \end{aligned} \tag{1}$$

   Let $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{u}$. Calculate $\boldsymbol{\nabla} \cdot \boldsymbol{\omega}$.

   (b) A velocity field $\boldsymbol{u}$ in two-dimensions (x, y) is given by the following expression

   $$u_x = x \tag{2a}$$
   $$u_y = Sx - y. \tag{2b}$$

   Calculate the gradient matrix $G_{\alpha\beta} \equiv \partial_\beta u_\alpha$, where ∂_β denote spatial derivative, as a function of x and y. Is this velocity field incompressible ?

   (c) From Eq. 2 calculate vorticity and rate-of-strain as a function of space coordinates, x, y.

   (d) Which of the following are true ? (More than one may be true.) Viscosity of a Newtonian fluid is

   i. a scalar.
   ii. can be described by two scalar quantities.
   iii. is a fourth rank tensor.

   (e) Organize the images in figure in sequence of increasing Reynolds number.

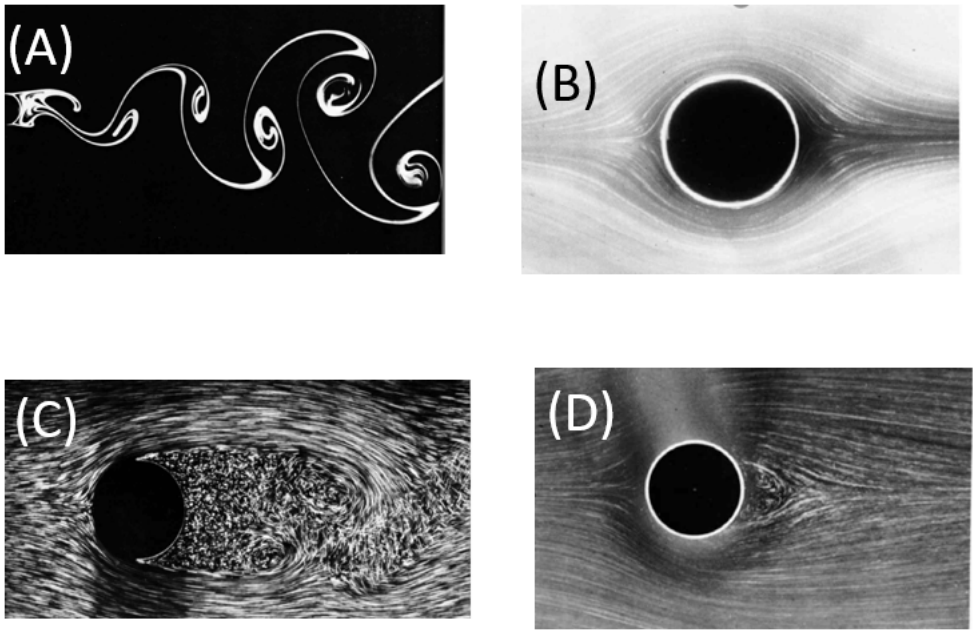

Figure 1: Organize these images in sequence of increasing Reynolds number

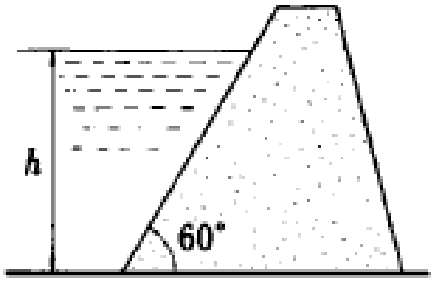

Figure 2: Sketch of dam

2. (**5 pt**) The figure 2 sketches a dam wall. The height of the water in the reservoir is h. What is the net *horizontal* force acting on the wall of the dam ?

3. (**5 pt**) A thin horizontal disc of radius R is located in a cylindrical cavity filled with oil with dynamic viscosity μ. The clearence between the disc and the horizontal planes of the cavity is equal to h. Using lubrication approximation calculate the power necessary to rotate the disc with a constant angular velocity ω. Ignore the end effects.