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Exercise ~~Set~~ 1.

To be returned on
28 Jan 2016

1. Show that if ρ and v satisfies the continuity eqn. then for any density variable ψ the following identity holds:

$$\rho D_t \psi \equiv \rho(\partial_t + v.\nabla)\psi \qquad (5 \text{ marks})$$

$$= \partial_t(\rho\psi) + \operatorname{div}(v\rho\psi)$$

2. Prove the vector identity

$$B \times (\nabla \times B) = \frac{1}{2}\nabla B^2 - (B\cdot\nabla)B \qquad (5 \text{ marks})$$

do you need to use $\operatorname{div} B = 0$. ?

3. Argue why ~~there are~~ only two independent numbers are necessary to describe an isotropic tensor of rank 4. You can look up the argument in a book if you do not remember it. (3 marks)

4. Show that the total energy

$$\mathcal{E} \equiv \int_V \left[\frac{1}{2}\rho v^2 + e + B^2\right] dV$$

e: internal energy per unit volume.

is a conserved quantity of the ideal MHD equations. Here the integral is over ~~all~~ ~~space~~. (7 marks)
a periodic domain.