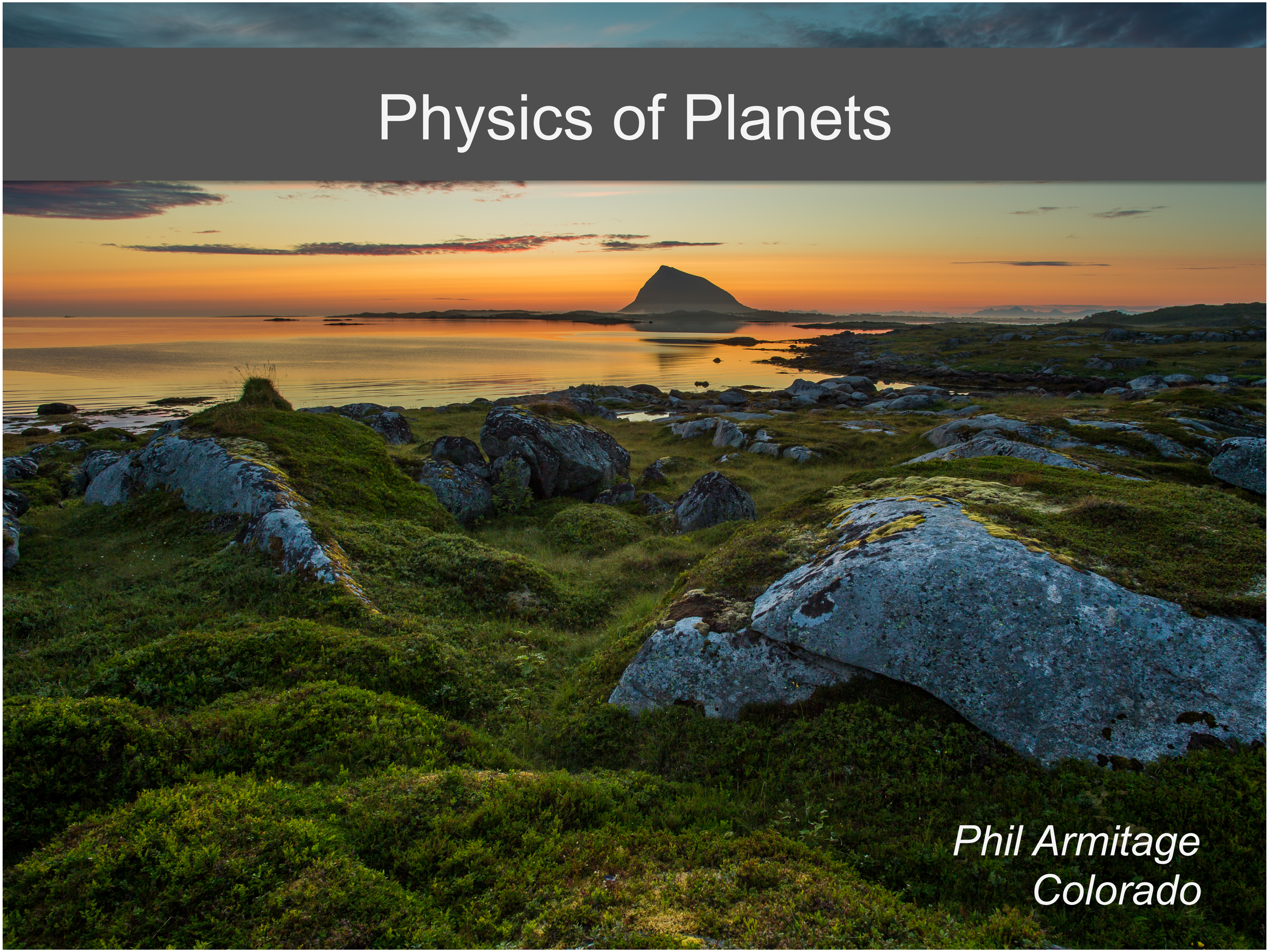
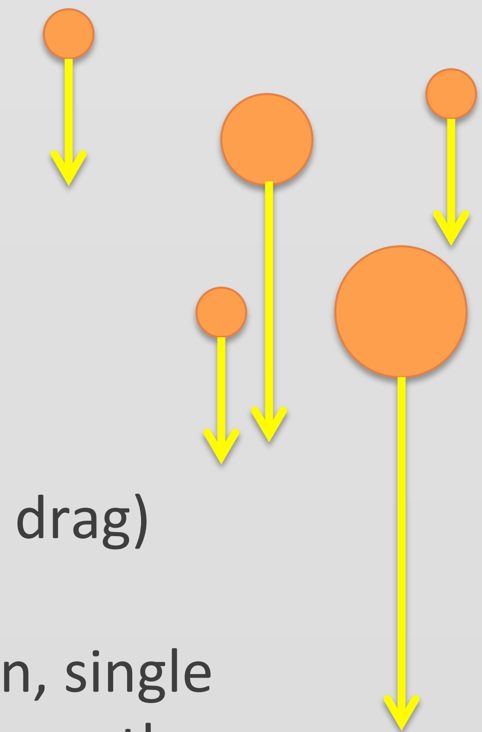


Physics of Planets

Phil Armitage
Colorado

Settling with perfect growth



$$v_{\mathrm{settle}} = \frac{\rho_m}{\rho}\frac{s}{v_{\mathrm{th}}}\Omega^2 z.$$

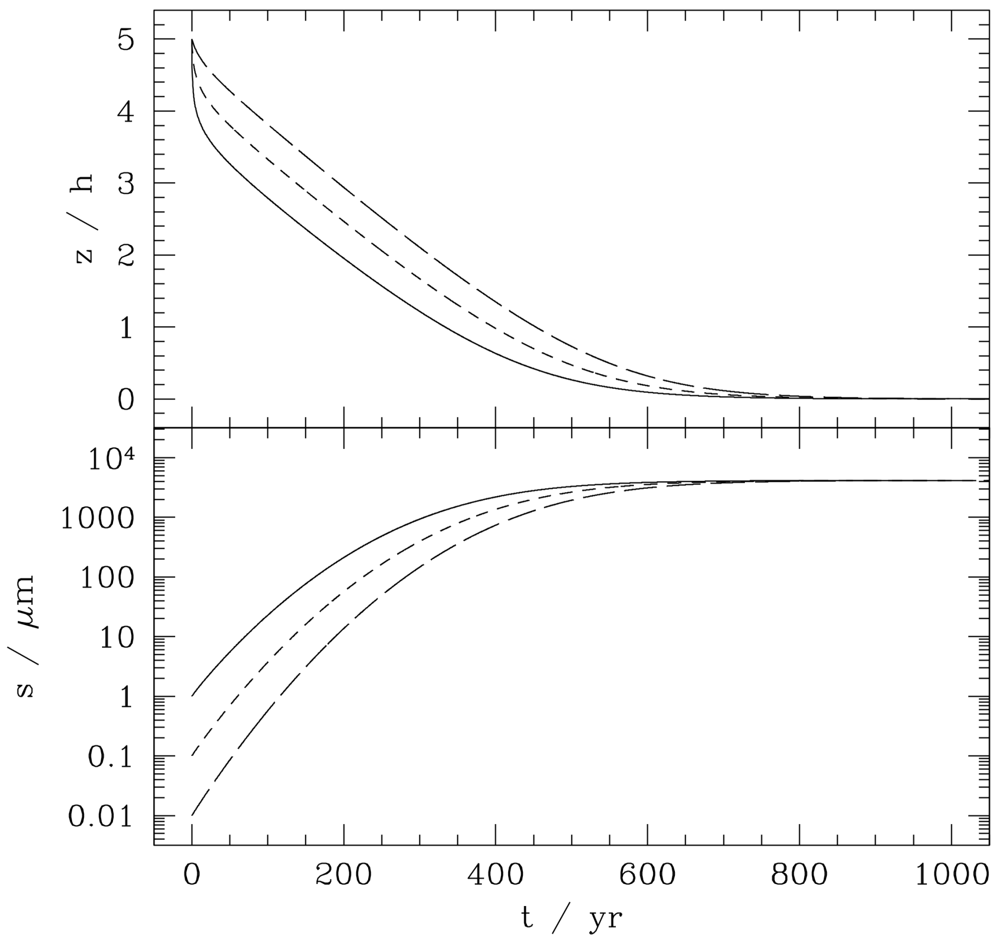
Settling speed proportional to size (Epstein drag)

If assume collisions always lead to accretion, single particle settling time >> settling time with growth

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \pi s^2 |v_{\mathrm{settle}}| f \rho(z)$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\frac{\rho_m}{\rho}\frac{s}{v_{\mathrm{th}}}\Omega^2 z$$

Geometric "rain drop" model for growth (Dullemond & Dominik 2005; Safronov 1969)



With perfect growth, s -> mm at z = 0 on ~10^3 yr time scale at 1 AU

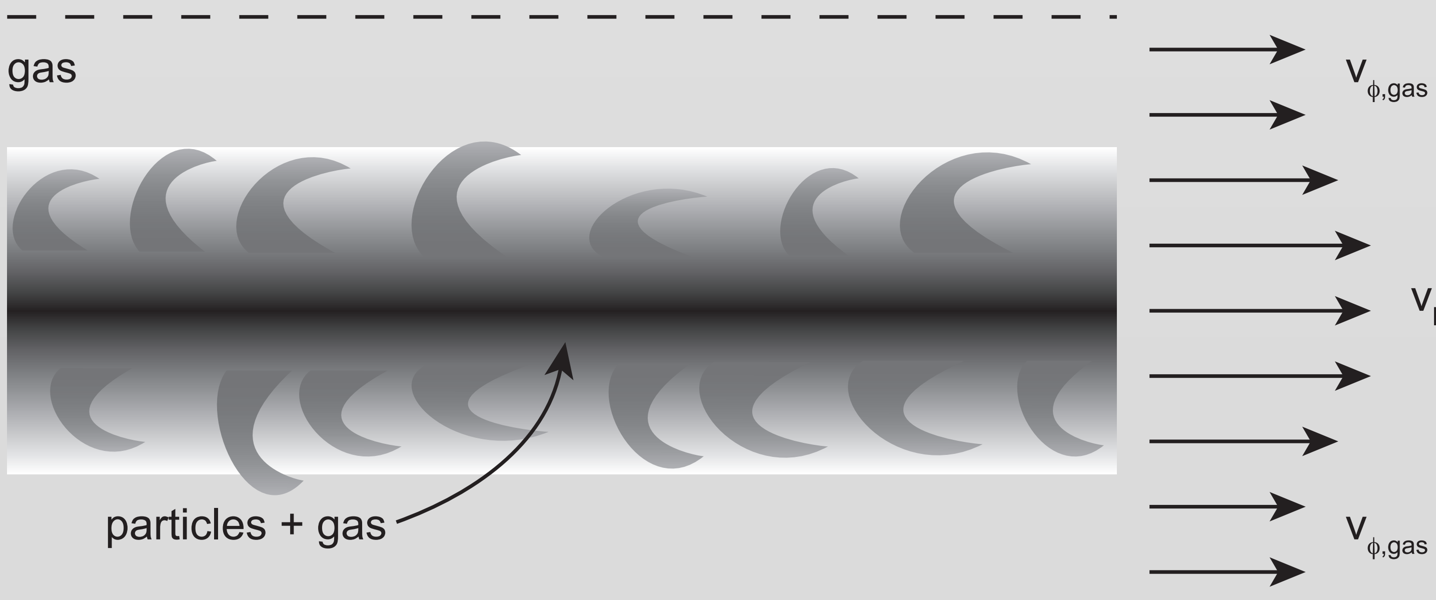
With perfect growth, s -> mm at z = 0 on ~10^3 yr time scale at 1 AU… obviously *too fast*

Observe small dust in atmospheres of protoplanetary disks

Argument that some fraction of collisions must lead to fragmentation to replenish small particles

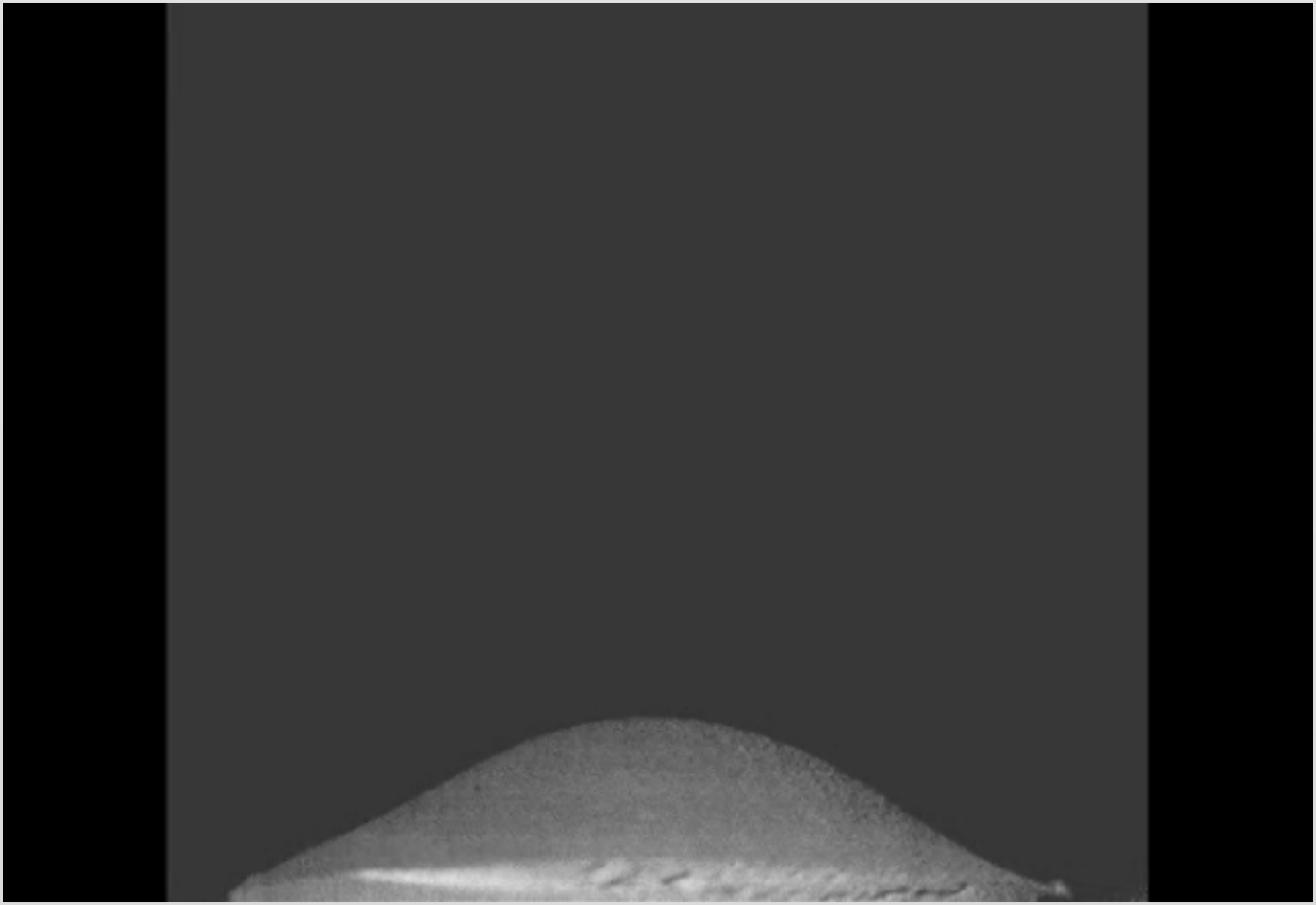
Turbulence suspends small particles

Even in *non-turbulent* disks, settling limited by Kelvin-Helmholtz instability between dust-rich / gas layer

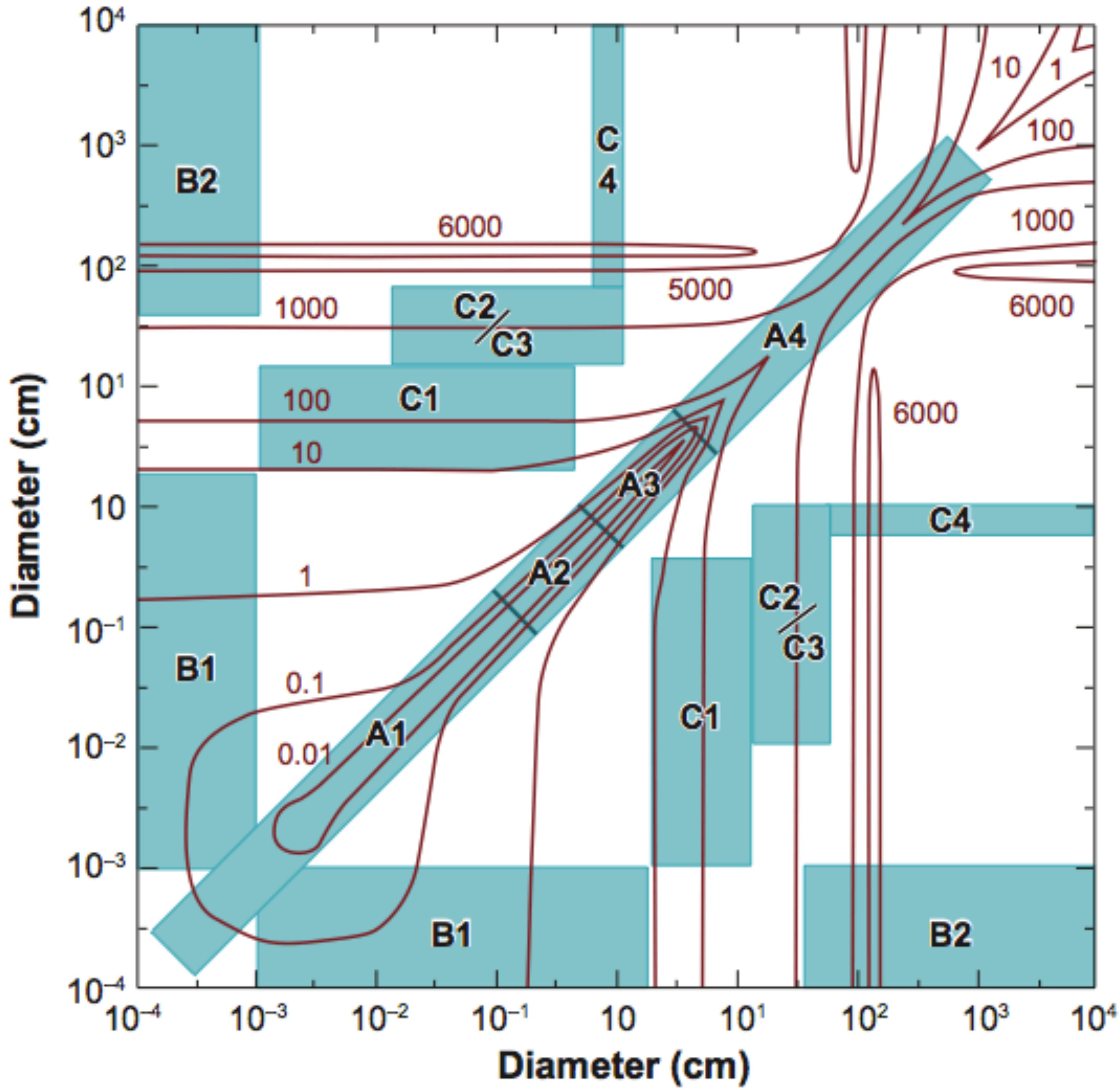


Collision outcomes

- studied experimentally in microgravity for silicates, less extensive work on ices
- often assumed that particles are **aggregates** of small (μm) monomers
- collision energy can be dissipated in re-arrangement of the aggregates, especially if very porous

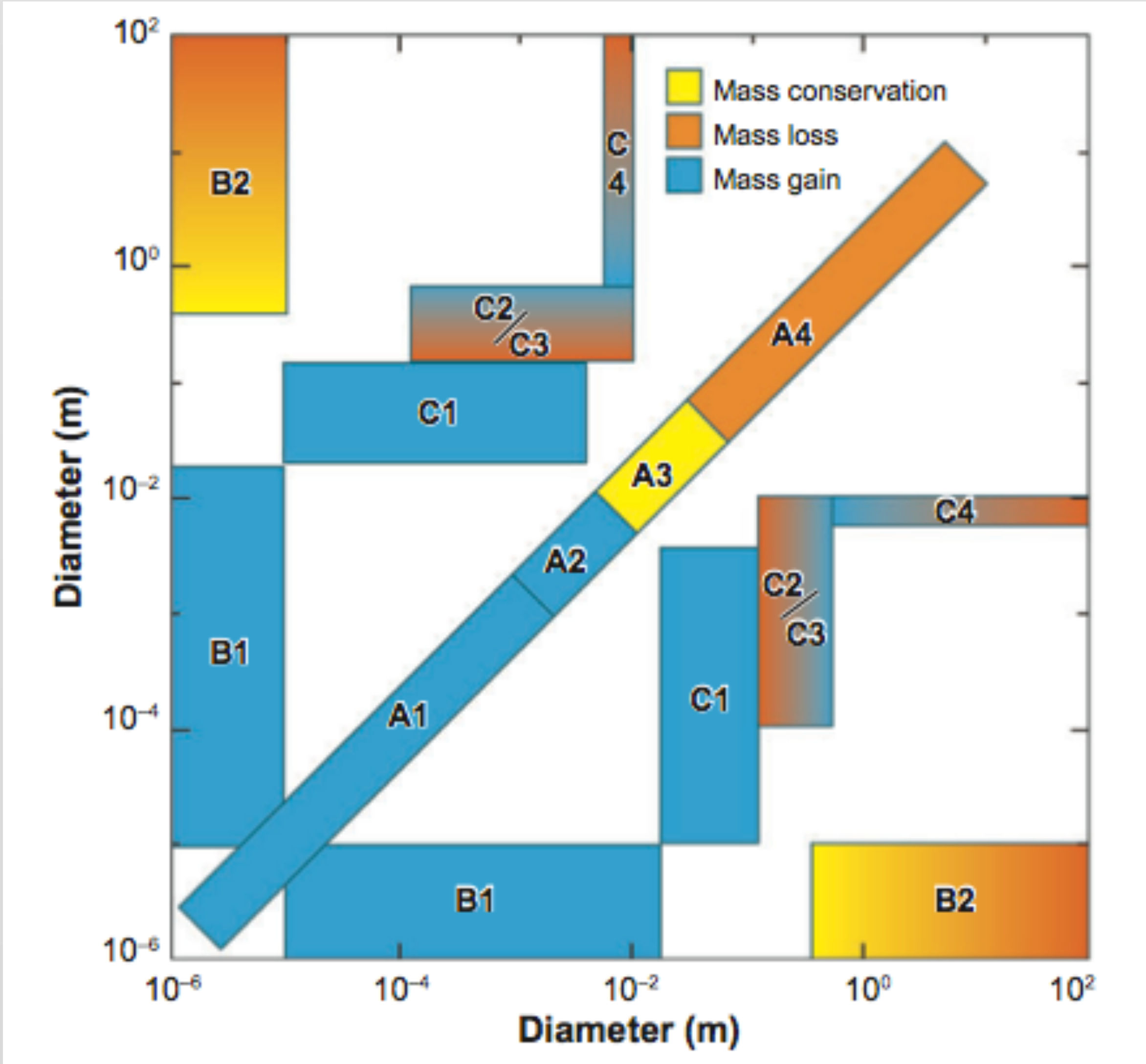


Blum & Wurm, ARA&A, 2008



B2
B1
C
4
6000
1000
C2
C3
C1
100
10
1
0.1
0.01
A1
A2
A3
A4
5000
10
1
100
1000
6000
6000
C4
B1
B2
Diameter (cm)
Diameter (cm)

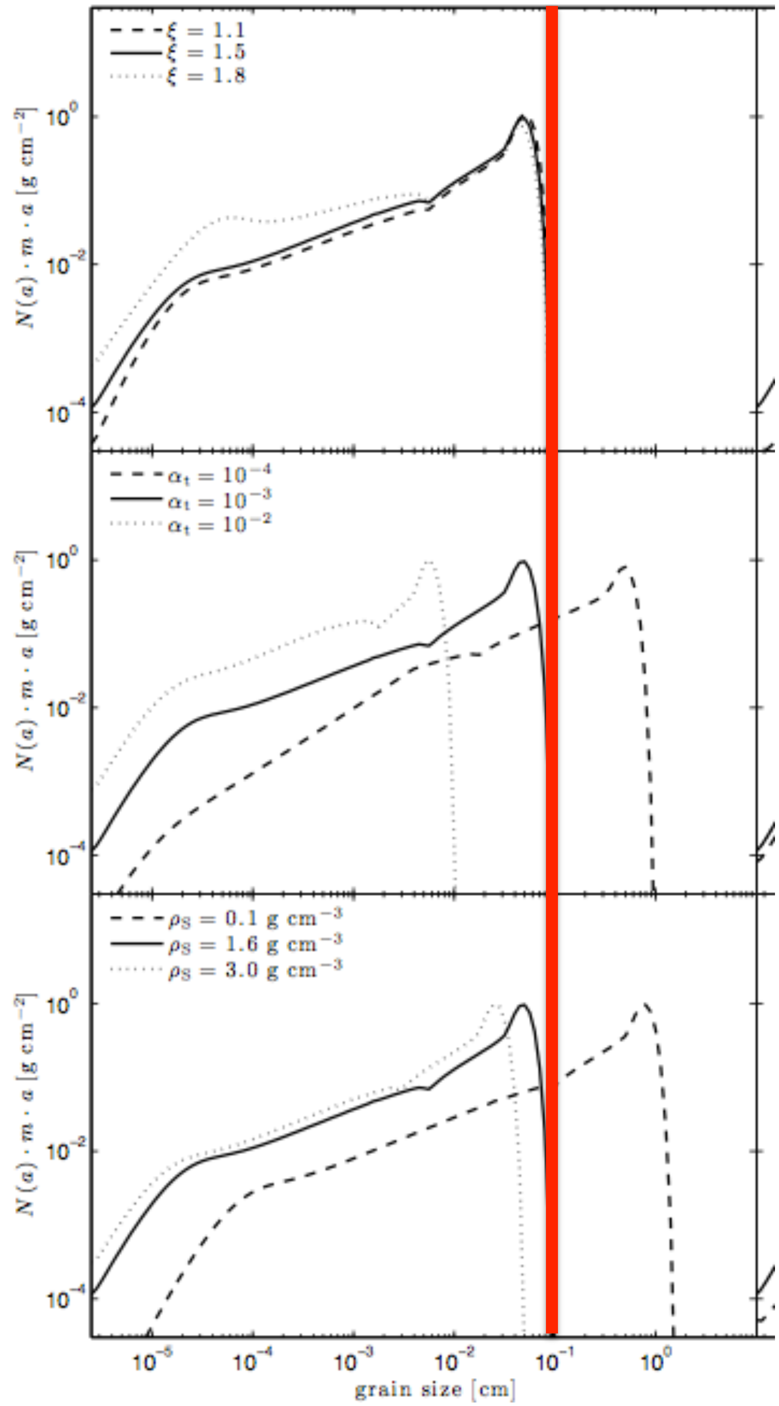
Material barriers to particle growth?



Blum & Wurm (2008)

Birnstiel, Ormel & Dullemond (2011) – with predicted collision velocities and experimentally measured collision outcomes:

- establish a coagulation / fragmentation equilibrium at small sizes
- in the inner disk, particles have difficulty growing beyond small macroscopic sizes
- outcomes for icy materials likely significantly different



Collision outcomes

- can also be modeled theoretically with N-body simulations
- examples: Wada et al. (2011), Paszun & Dominik (2009)
- input physics includes:
  - size and shape of the monomers
  - energy required to separate them
  - forces requires to induce rolling

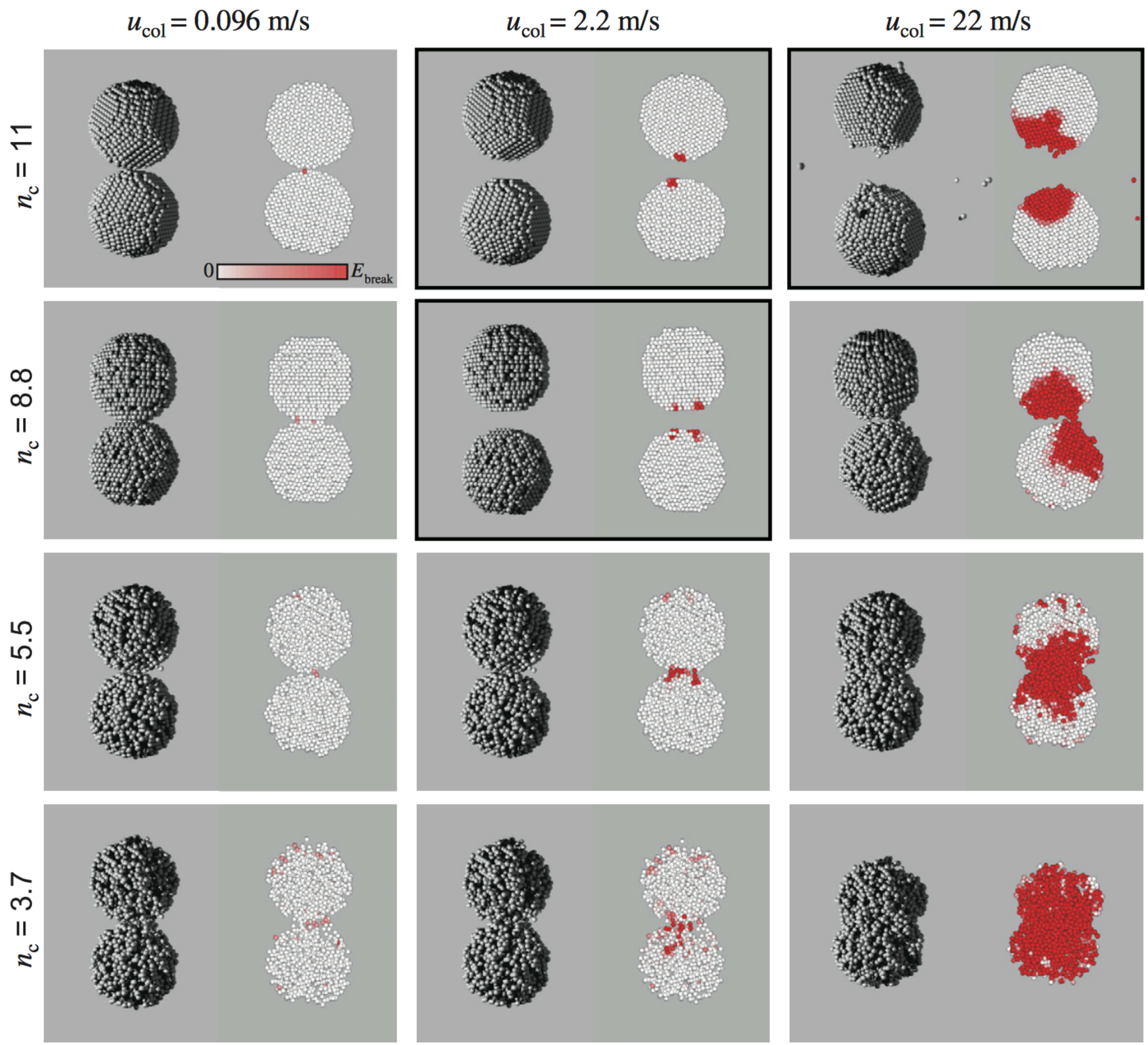


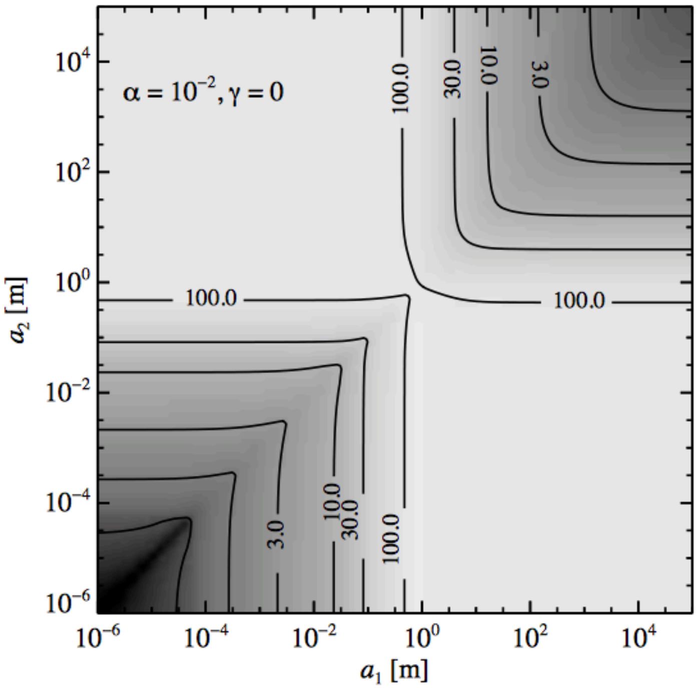
Figure 5. Examples of collisional outcomes of icy CPE aggregates for various n_c and u_{col}. Panels are arranged by n_c (row) and u_{col} (column). In each panel, the left is the appearance of the outcome and the right is its cross-sectional view. Particles in the cross-sectional views are painted, depending on the amount of dissipation energy in them (0 to E_{break}) as indicated in the scale bar. Panels of rebound cases are framed by lines.

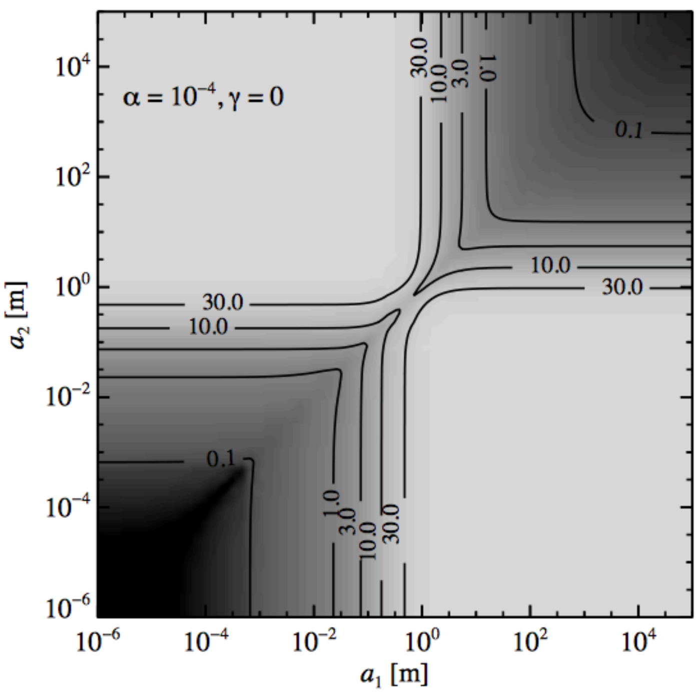
Critical velocity for fragmentation may be:

- order of m/s for silicate particles
- several 10s of m/s for icy particles

Particles may grow as extremely porous bodies

Time scales for growth to large macroscopic size typically still exceed radial drift time





Collision velocities from Johansen et al. PP6 review (2014)

Diffusive evolution

Start by considering evolution of a trace species of *gas* within the disk (e.g. gas-phase CO)

Define **concentration** $C = \frac{\Sigma_d}{\Sigma}$

Continuity implies: $$\frac{\partial C}{\partial t} + \nabla \cdot \mathbf{F}_d = 0$$

If the diffusion depends only on the gas properties, then the flux is:

$$\mathbf{F}_d = \Sigma_d \mathbf{v} - D \Sigma \nabla C$$

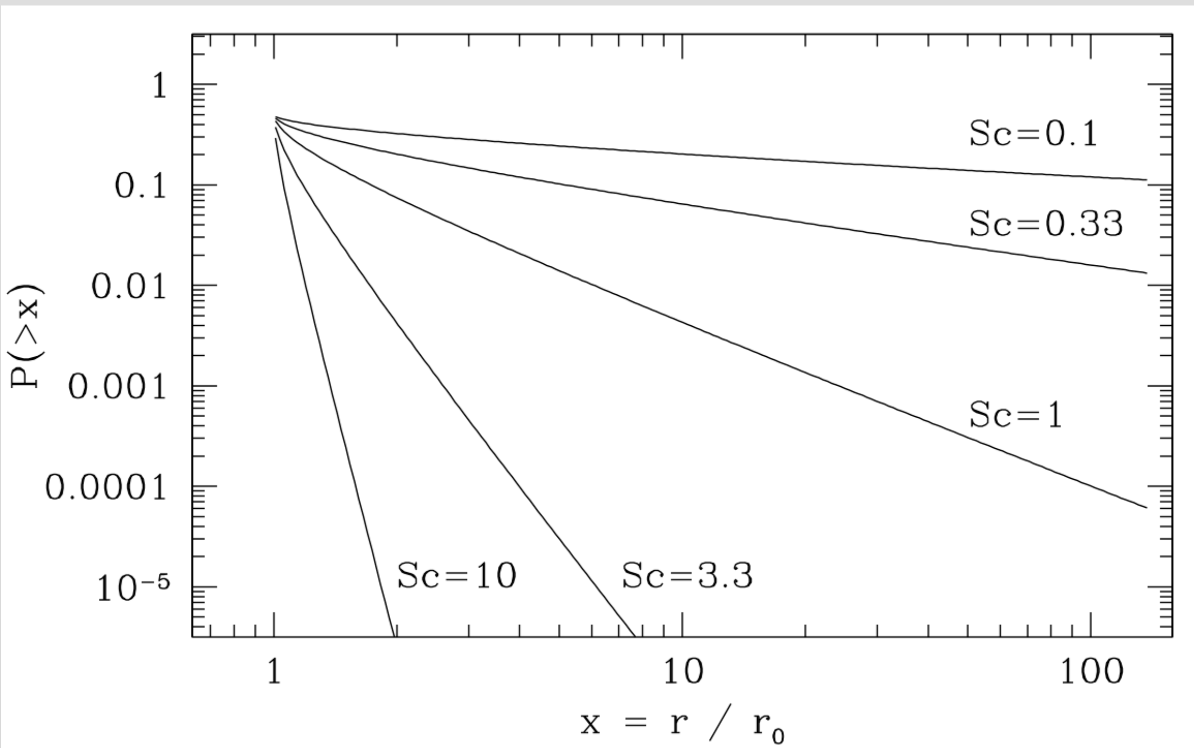
advection with mean gas flow **v**

diffusion where there is a gradient in concentration

For an axisymmetric disk, obtain:

$$\frac{\partial C}{\partial t} = \frac{1}{r\Sigma}\frac{\partial}{\partial r}\left(Dr\Sigma\frac{\partial C}{\partial r}\right) - v_r\frac{\partial C}{\partial r}$$

In a steady disk, $v_r = -3\nu / 2r$. Solutions are very strongly dependent on relative strength of viscosity and diffusivity: $Sc = \nu / D$ (the Schmidt number)



For $\Sigma \sim r^{-2}$, maximum fraction of contaminant, released at x = 1, that is ever at radius x or larger

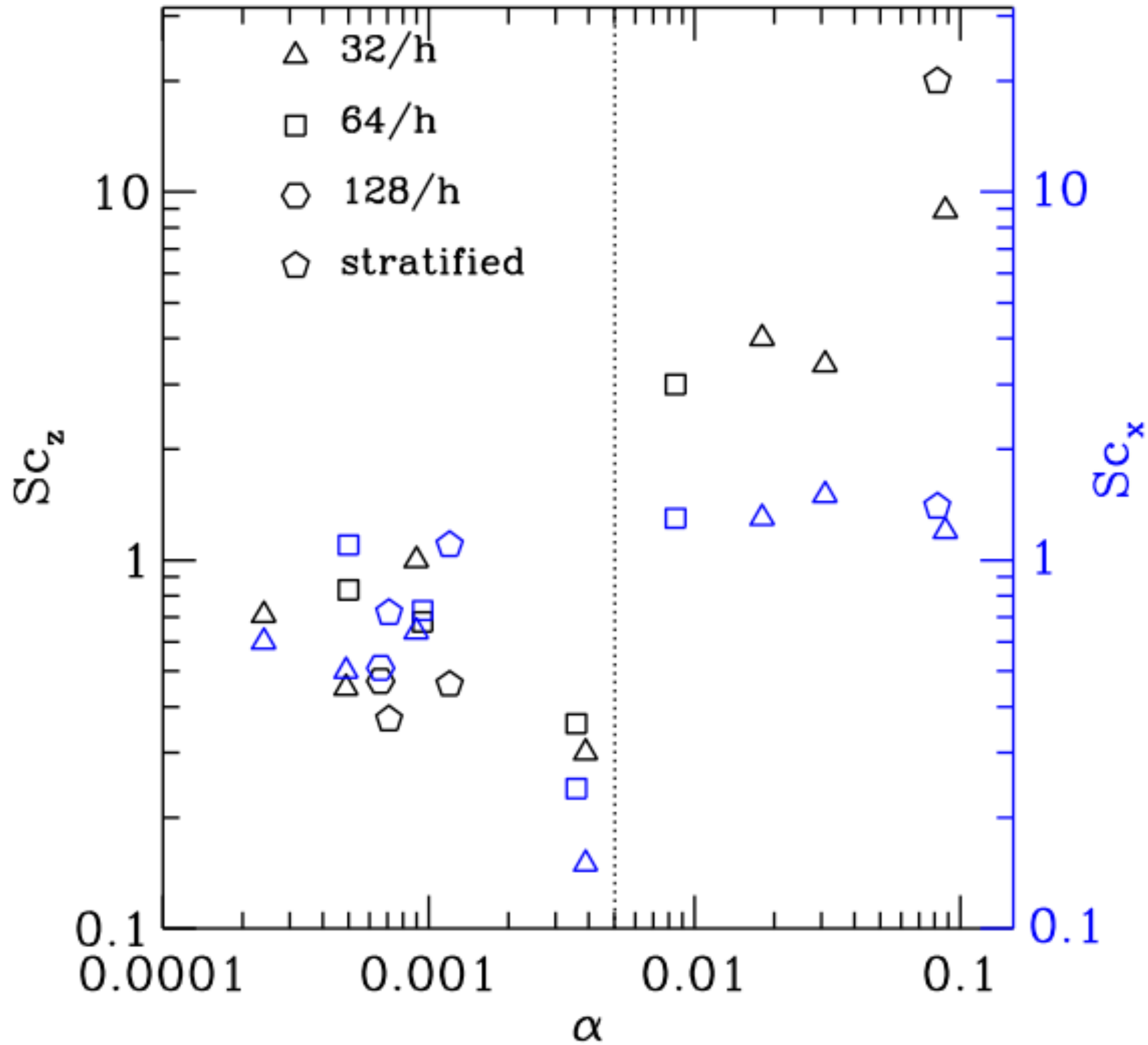


Figure 16. Sc_x (blue points) and Sc_z (black points) for all our shearing box simulations. The dotted line separates ideal MHD simulations (on the right) from MHD simulations with AD (on the left). Clearly, Sc_z is smaller than 1 for AD runs, while $Sc_z \gtrsim 3$ for ideal MHD runs.

In ideal and ambipolar MHD, Sc for radial diffusion (Sc_x) is generally within factor of ~2 of unity

Larger variations for vertical diffusion

Zhu et al. '15

Particle transport

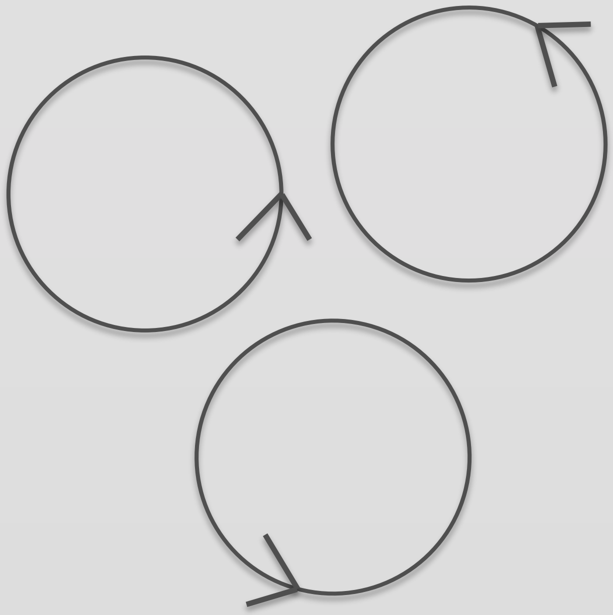
How will this change if the trace species is a solid particle? *Very* small particles will be so well-coupled as to behave like gas. For larger particles:

$$\frac{\partial C}{\partial t} = \frac{1}{r\Sigma}\frac{\partial}{\partial r}\left(Dr\Sigma\frac{\partial C}{\partial r}\right) - v_r\frac{\partial C}{\partial r}$$

diffusion is now not only different (in principle) from gas viscosity, but also size-dependent… large particles' inertia means they are less affected by turbulence

radial velocity needs to include the aerodynamic drift term

Particle diffusion in turbulence



Describe turbulence as eddies, time scale t_{eddy}, velocity δv_g

Make dimensionless $\tau_{\rm eddy} = \Omega t_{\rm eddy}$

Consider particle, stopping time τ, in limit $\tau \gg 1$ and $\tau_{eddy} \ll 1$

In time Ω^{-1}, particle receives $N \sim \tau^{-1}_{eddy}$ kicks, each of

$$\delta v_p \sim \frac{\tau_{\rm eddy}}{\tau} \delta v_g$$

Add up as a random walk

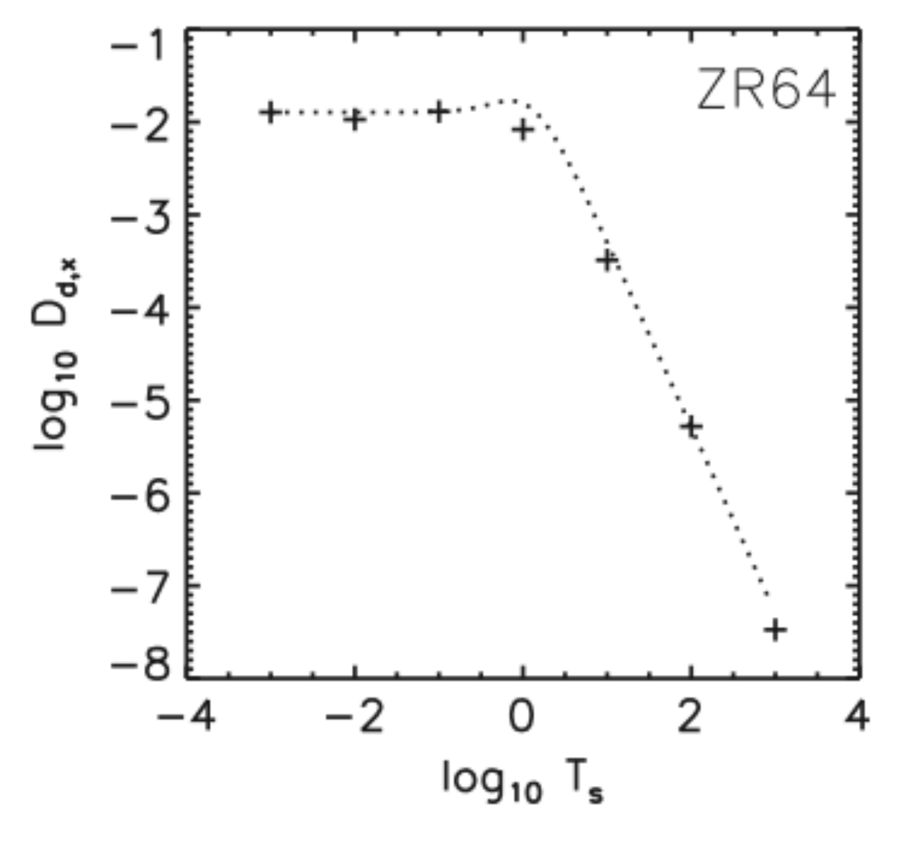
$$\delta v_p \sim \frac{\tau_{\rm eddy}}{\tau} \delta v_g \sqrt{N} \sim \frac{\sqrt{\tau_{\rm eddy}}}{\tau} \delta v_g$$

Distance travelled $\delta l \sim \delta v_p \Omega^{-1}$

This implies an effective diffusion coefficient $D_p \sim D/\tau^2$

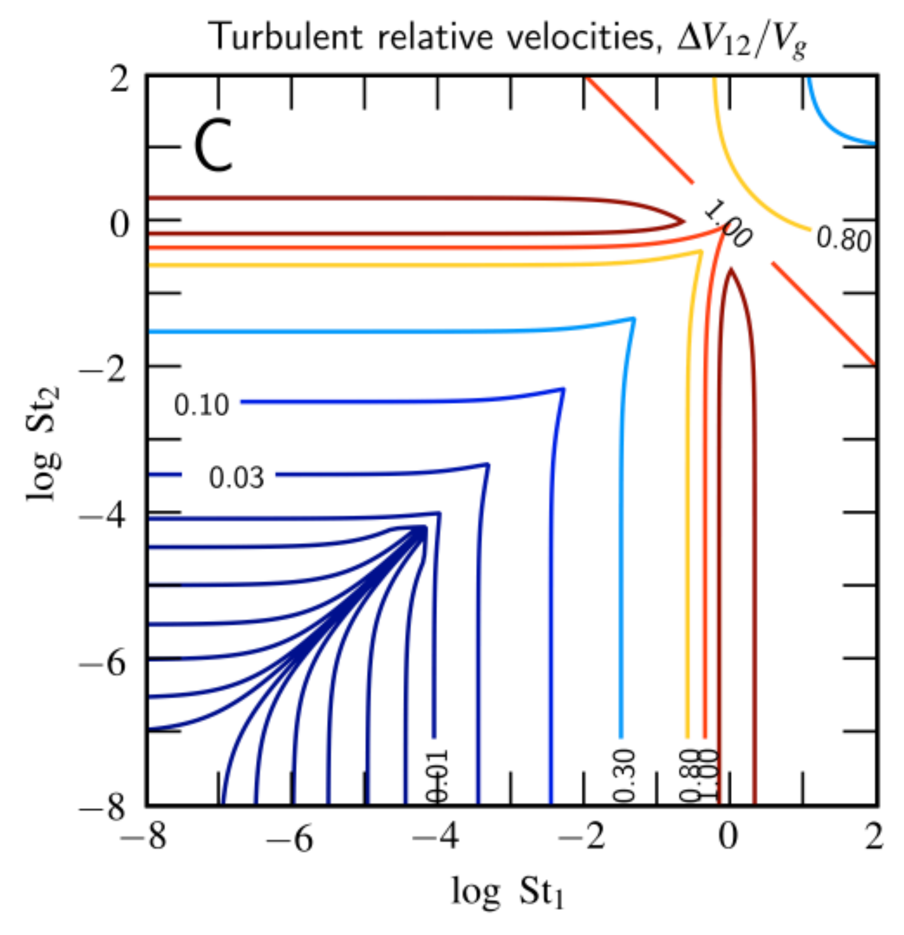
Since for small particles, D_p = D, generally, $\frac{D_p}{D} \sim \frac{1}{1+\tau^2}$

Agreement with formal analysis by *Youdin & Lithwick '07*, which in turn agrees with measurements of particle diffusivity in (ideal) MHD turbulence by *Zhu et al. '15*



Same type of argument gives the typical collision velocities between particles in turbulence *(Ormel & Cuzzi '07; see also Pan et al. '14)*

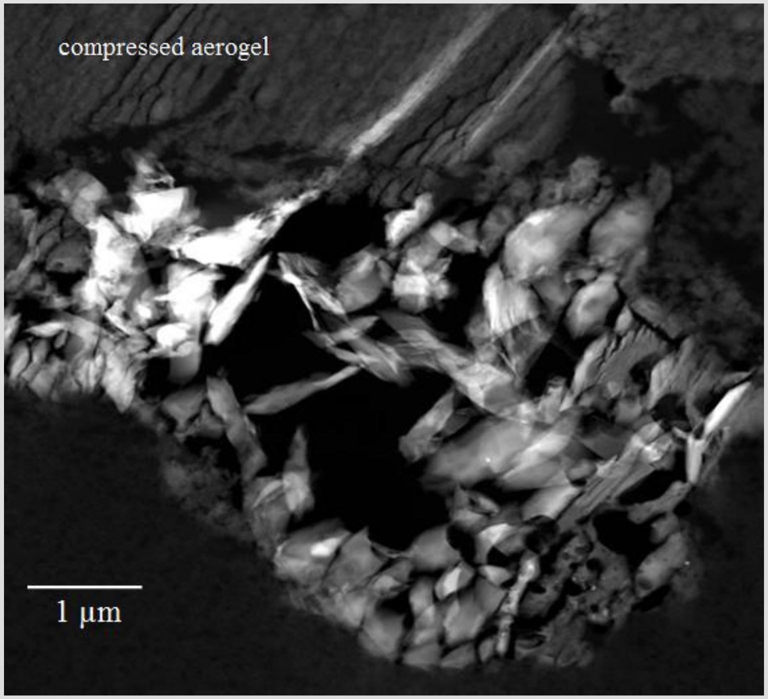
Note: large-scale nature of turbulence is not so critical here, expect fluid turbulence to be good limit



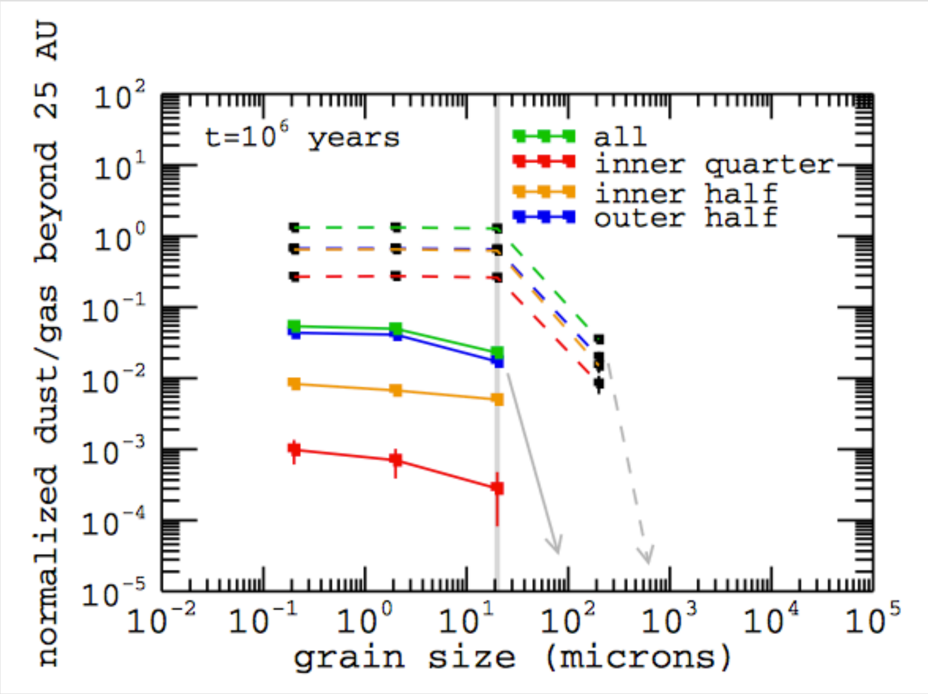
General results:

- turbulent diffusion rapidly negligible for $\tau > 1$
- for smaller particles, either turbulent component to collision velocities, or differential radial drift, can dominate depending on disk model

The *Stardust* problem



Brownlee et al. ‘06



Hughes & Armitage ‘10

Stardust mission recovered crystalline silicate particles (processed at T > 10^3 K) and CAIs from a Jupiter-family comet… quite hard for “upstream” radial diffusion to move such particles from inner disk to comet-forming region

Particle feedback

"Rule of thumb" – in many planetesimal formation models, pre-requisite is *local* dust to gas ratio $\rho_d / \rho \sim 1$

→ solids are no longer trace contaminant, feedback of particles on gas should not be neglected

What is the equilibrium radial drift solution in this limit?
Is it stable?

Equations for particle fluid interacting with an incompressible gas disk via aerodynamic forces only:

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p \boldsymbol{V}_p) = 0,$$

$$\nabla \cdot \boldsymbol{V}_g = 0,$$

$$\left.\begin{aligned} \frac{\partial \boldsymbol{V}_p}{\partial t} + \boldsymbol{V}_p \cdot \nabla \boldsymbol{V}_p &= -\Omega_{\mathrm{K}}^2 \boldsymbol{r} - \frac{\boldsymbol{V}_p - \boldsymbol{V}_g}{t_{\mathrm{stop}}}, \\ \frac{\partial \boldsymbol{V}_g}{\partial t} + \boldsymbol{V}_g \cdot \nabla \boldsymbol{V}_g &= -\Omega_{\mathrm{K}}^2 \boldsymbol{r} + \frac{\rho_p}{\rho_g} \frac{\boldsymbol{V}_p - \boldsymbol{V}_g}{t_{\mathrm{stop}}} - \frac{\nabla P}{\rho_g} \end{aligned}\right\} \text{symmetric momentum exchange}$$

e.g. Youdin & Goodman ‘05

Well defined equilibrium solution by *Nakagawa et al. 86*

$$v_{r,\mathrm{sum}} = v_{r,p} + v_{r,\mathrm{gas}} = -2\rho_g \frac{\rho_g - \rho_p}{\rho^2} \frac{\eta v_K \tau}{1 + (\tau \rho_g / \rho)^2}$$

Depends explicitly on relative densities of gas, solids… gas has non-zero v_r in *absence* of angular momentum transport