

FIND ALL PRIME NOS TILL  $N = 19$

Prime[]

|   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|   |   | T | T | T | T | T | T | T | T | T  | T  | T  | T  | T  | T  | T  | T  | T  | T  |

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|   |   | T | T | T | T | T | T | T | T | T  | T  | T  | T  | T  | T  | T  | T  | T  | T  |

$(N+1)$

Prime  $\Rightarrow 2$

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|   |   | T | T | T | T | T | T | T | T | T  | T  | T  | T  | T  | T  | T  | T  | T  | T  |
| ↑ | ↑ | ↑ |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |

$(N+1)$

Prime  $\Rightarrow 2,$

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| ↑ | ↑ | ↑ | ↑ | ↑ |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |

$(N+1)$

Prime  $\Rightarrow 2, 3$

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|   |   | T | T | T | T | T | T | T | T | T  | T  | T  | T  | T  | T  | T  | T  | T  | T  |
| ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  |

$(N+1)$

$$\left\lfloor \frac{N}{2} \right\rfloor$$

$19$

$$\frac{19}{2} = 9.5 \\ \Rightarrow \underline{9}$$

Prime  $\Rightarrow 2, 3, 5, 7, 11$

FIND ALL PRIME NOS TILL  $N = 19$

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|   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
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|   |   | T | T | T | T | T | T | T | T | T  | T  | T  | T  | T  | T  | T  | T  | T  | T  |
| ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  | ↑  |

$(N+1)$

$$\left\lfloor \frac{N}{2} \right\rfloor$$

$19$

$$\frac{19}{2} = 9.5 \\ \Rightarrow \underline{9}$$

Prime  $\Rightarrow 2, 3, 5, 7, 11, 13, 17, 19$

```

for (int i = 2; i ≤  $\frac{N}{2}$ ; i++)
{
    if (prime [i] == true)
    {
        // make all multiples false
        for (int j = 2*i; j ≤ N; j = j+i)
            prime [j] = false;
    }
}

```

2 optimizations :-

①

$$\begin{array}{ccc} \underline{X} & \cdot & \underline{Y} \implies \underline{N} \\ \uparrow & & \uparrow \\ \leq \sqrt{N} & & \geq \sqrt{N} \end{array}$$

② Elements needs to be marked from  $i^2$ .

2 3 4 5 6 7 8 9 10 11 12 13  
 14 15 16 17 18 19 20 21 22 23 24 25 26

```

for (int i = 2; i ≤  $\frac{N}{2}$ ; i++)
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    }
}

```

$\sim O(N \log N)$

$\frac{N}{2}$   
 $\leq \sqrt{N}$

2 optimizations :-

①  $X \cdot Y \Rightarrow N$   
 $\uparrow \quad \uparrow$   
 $\leq \sqrt{N} \quad \geq \sqrt{N}$

② Elements needs to be marked from  $i^2$ .

2 3 4 5 6 7 8 9 10 11 12 13  
 14 15 16 17 18 19 20 21 22 23 24 25 26

# Sieve of eratosthenes

```

for (int i = 2; i ≤  $\frac{N}{2}$ ; i++)
{
    if (prime[i] == true)
    {
        // make all multiples false
        for (int j = 2*i; j ≤ N; j = j+i)
            prime[j] = false;
    }
}

```

$\frac{N}{2}$  ←  $\sqrt{N}$   
 $\frac{N}{2}$   
 $\leq \sqrt{N}$   
*i* → prime or not  
 $2*i$  →  $\sim$   
 $3*i$   
 $4*i$

5:47 / 9:49

Scroll for details

```

for (int i = 2; i ≤  $\frac{N}{2}$ ; i++)
{
    if (prime[i] == true)
    {
        // make all multiples false
        for (int j = 2*i; j ≤ N; j = j+i)
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}

```

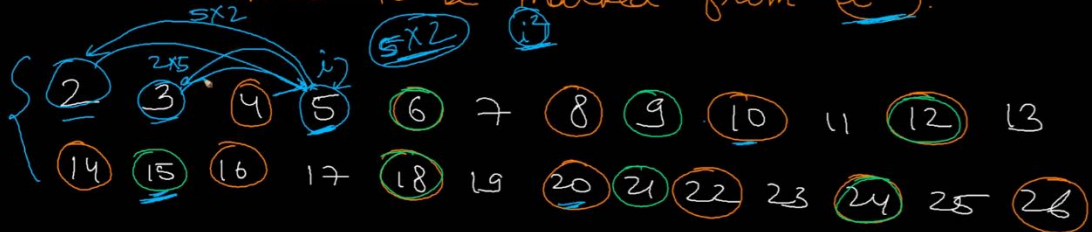
$i*i$   $i^2$   
*i* → prime or not  
 $2*i$  →  $\sim$   
 $3*i$   
 $4*i$

## 2 optimizations :-

①

$$\begin{array}{ccc} \underline{X} & \cdot & \underline{Y} \implies \underline{N} \\ \uparrow & & \uparrow \\ \leq \sqrt{N} & & \geq \sqrt{N} \end{array}$$

② Elements needs to be marked from  $i^2$ .



## Optimized Code

```
for (int i = 2; i <= sqrt(N); i++)  
{  
    if (prime[i] == true)  
    {  
        for (int j = i * i; j <= N; j = j + i)  
            prime[j] = false;  
    }  
}
```

# TIME COMPLEXITY

$P \Rightarrow$  Highest Prime No.  $\leq N$

$$\begin{aligned} & \left\{ \frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7} + \dots + \frac{N}{P} \right\} \\ &= N \left( \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots + \frac{1}{P} \right) \\ &= \underline{N \log \log N} \end{aligned}$$

$\swarrow$  HP of Sum of Primes