

value(node n)

```
{ in cvalue; // cvalue is current value */
  if (n is terminal) return cvalue; // defined by depth bound */
  cvalue = (n is MIN node)?  $-\infty$ ;
  for (each successor  $n_i$  of n) {
    generate  $n_i$ ;
    cvalue = (n is MAX node)?  $\max(cvalue, value(n_i))$ ;
     $\min(cvalue, value(n_i))$ ;
  }
  return cvalue;
}
```

Remarks: 1) In the above algo, the scan is from left to right

2) It examines and evaluate all the terminal nodes

3) B^D is the no of terminal node at depth D
if B is the branching factor.

4) Memory req. is of the order B^D .

5) A move giving the root value must also be
outputted.

6) The game tree search algo. attempts to find the
value of the root by examining minimum no of terminal
nodes also intermediate nodes.

7) The no of terminal node examined serves as
performance
measure of the algorithm since evaluation of state
evaluation score of a terminal leaf is most time consuming
and costly.

Compute the maximum of two numbers.

$\text{max}(X, Y, X) :- X \geq Y, !$

$\text{max}(X, Y, Y).$

$\text{member}(X, [X|_]) :- !.$

$\text{member}(X, [_|L]) :- \text{member}(X, L).$

Add an element to a list without duplication.

write a relation $\text{efface}(X, Y, Z)$ to remove the first occurrence of element X from list Y giving a new list Z .

Max of a list.

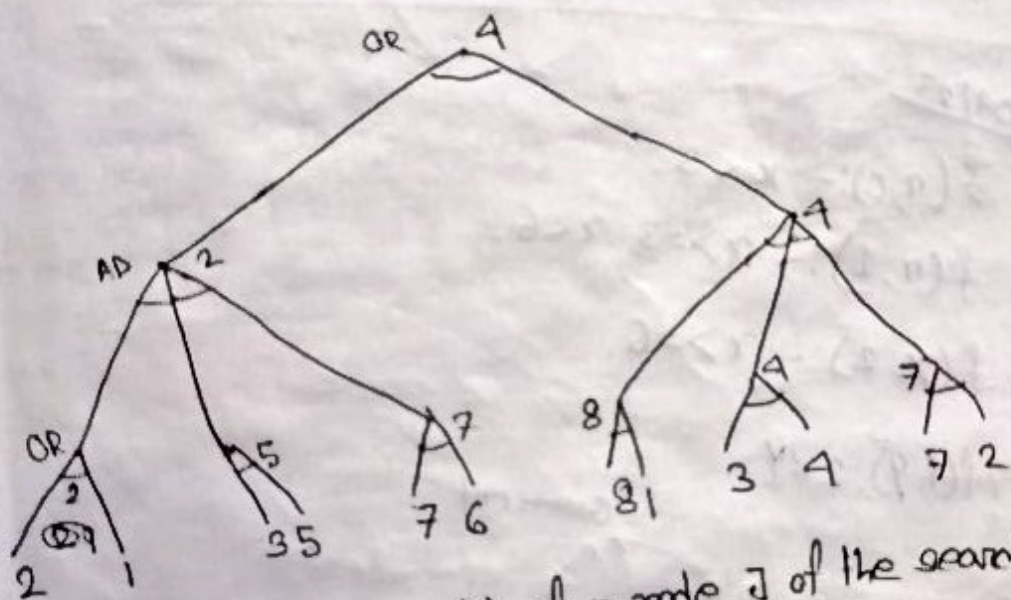
$\text{maxlist}([X], X).$

$\text{maxlist}([X|Y], \text{Rest})$

b) If C is to move the problem is solved with respect to C if a winning strategy can be found by any of its successors, hence OR-node.

c) If H is to move the problem is solved with respect to C if C can force a win from each of the H 's nodes hence AND-node.

Since easy to write programmes to win, as exhaustive evaluation of all plays will guide the computer to find its best move at any point of time.



Mini-Max Rule: (i) The value $V(i)$ of a node i of the search frontier is its static evaluation score otherwise

(ii) If i is a max node $V(i)$ is the maximum value of any of its successors.

(iii) If i is a min node $V(i)$ is the minimum value of any of its successors.

Game: MAX minimizes but wants
min to make the result worse

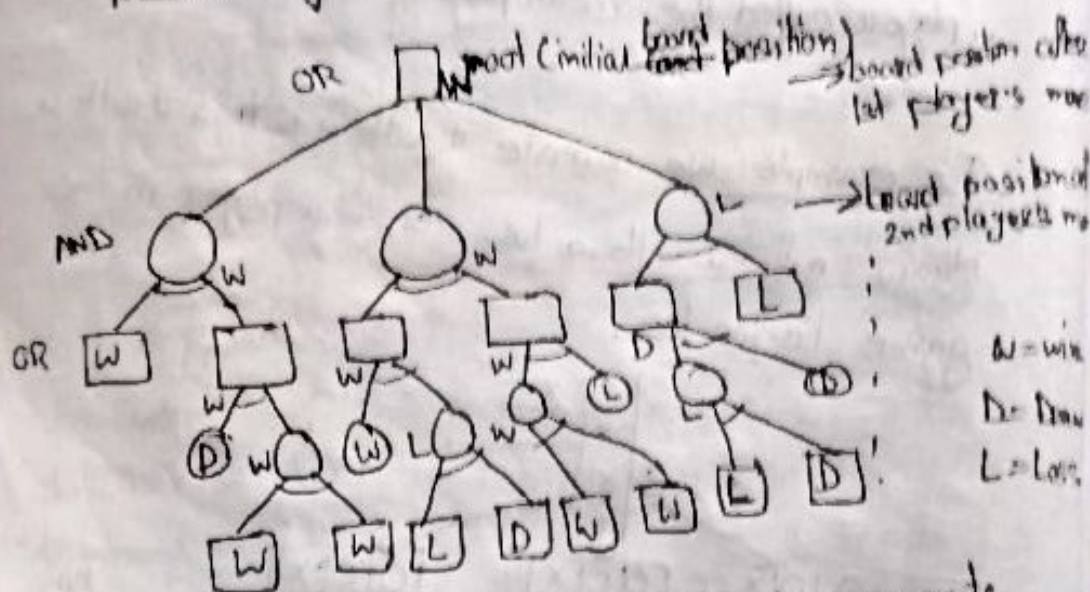
Component: Initial position function terminal state.

Utility function gives numerical value to all the terminal state.

High value, good for max & bad for min &
vice versa

A game begins from an initial state & ends in a position that using a simple criteria can be decided as best/worst

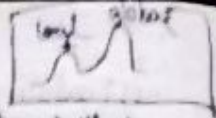
The game tree is an explicit representation of all possible ways of the game for a given position.



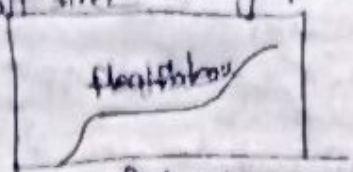
Each path from the root to a terminal node represents a different complete play of a game. Given a game tree it is easy to find a winning play for a player. Tree is nothing but a and-or tree since.

a) For a game position & a player playing the game means to find a winning strategy from it for computer C.

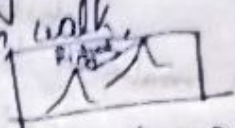
Limitations of hill climbing approach:



i) Local maximum: Once the top of a hill is reached, the algo will halt since every possible steps leads down.



ii) If the landscape is flat it means many states have the same goodness then the algo degenerates to a random walk.



iii) If a landscape contains ridges Ridges local improvements may follow a zigzag up the ridges slowing down the search.

Game Tree Search

We consider only two persons perfect info games i.e. chess, tic-tac-toe etc. Played by two players to move in turn. They respectively try to maximize or minimize a scoring function also called utility function. They each know completely what both the player done and can do. That is all informations about the board position & moves are available to the

There is no chance function like dice throwing game. Game playing is an intellectual activity. Our attempt is to increase the intellectual ability of the computers which in turn will increase the intellectual ability of humans.

i) Evaluate the initial state.

ii) Loop until a soln is found or there are no new operators left.

iii) Select and apply new operator.

iv) Evaluate new state:-

a) If it is a goal state then quite.

b) If it is better than the current state then make it the new

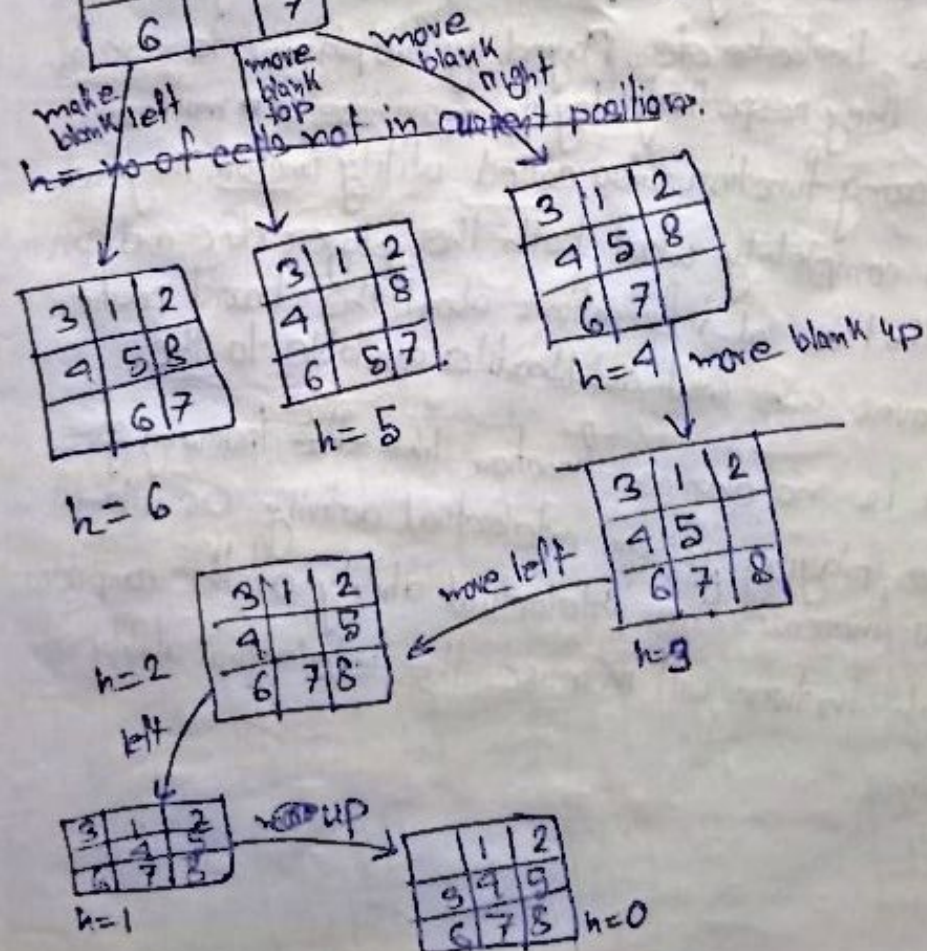
c) If it is not better than the current state then go to step (ii).

Initial state

3	1	2
4	5	8
6		7

Goal state

	1	2
3	4	5
6	7	8



CLOSED is the list of expanded nodes in the graph and is checking for duplicate nodes.

3) The idea of clean up is not applied here, because they are not considering the paths.

4) There are B^D nodes in a tree, having depth D and Branching factor B . In the worst case we have to store all the B^D NODES so the overhead is high compared to DFS.

5) DFS is recursive at the time of expanding it basically keeps the path in implicit ~~stack~~ stack but in case of BFS, it is relatively difficult to implement in recursive manner because it switches from one path to another.

6) Solution path is always optimal in terms of depth of a node but may not be optimal in terms of cost.

7) Duplicate nodes are not expanded but copies of the same node may appear in the OPEN.

8) Advantages are -

- i) Shallowest goal guaranteed. May not be optimal in terms of cost in case of graphs.
- ii) Faster solution in certain graphs.

9) Disadvantages are -

- i) Higher memory required.
- ii) More housekeeping work, more overhead and more difficult to implement.
- iii) Not suited to human problem solving approach.

Best First Search (1st page)

06/03/23

Q) Find the last member of a list.

~~member~~ ~~member~~ last(X, -

last(X, [X]).

last(X, [_:L]) :- last(X, L).

Q) Find the reverse of a list.

rev([X], [X]).

rev([_:X], [X]).

rev([X:L], L2) :- rev(L, L2), conc(L2, [X], L1).

Q) Check if a given list is palindrome.

~~Palindrome(X, L) :- rev(X, L)~~

checkpal(X) :- rev(X, X).

Q) Insert a element at all possible position.

insert(~~[X]~~ X, [], [X]).

~~insert(X, L, [X:L]) :-~~

insert(X, [Y:T], [X,Y:T1]) :- insert(X, T, T1).

Note:

① Clean up ~~CLOSED~~ prunes from closed. All ~~closed~~ ancestors of the node that do not have sons in OPEN. The effects are -
i) ~~CLOSED~~ will only contain the expanded nodes on the current path
ii) Reduces the size of ~~closed~~ CLOSED, saves memory.

② The list CLOSED forms a single path from the start node to the currently expanded node. This restricts the maximum storage, which is depth \times branching factor, in case of a graph ~~OPEN~~ holds the node generated in the explicit graph not yet expanded.

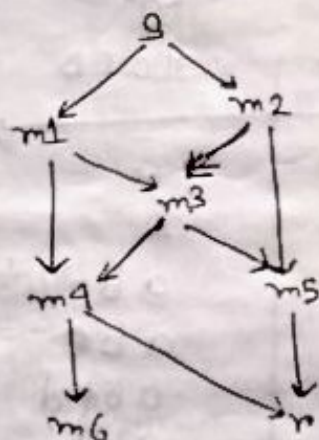
③ For a search graph (not a tree) the depth of the node, is the depth of its shallowest parents + 1. Maintaining depth first in true sense then leads to complication. In practice DFS takes depth of its current parents.

④ No duplications are not ~~checked~~ strictly checked in practice, only checked against the current path to avoid looping.

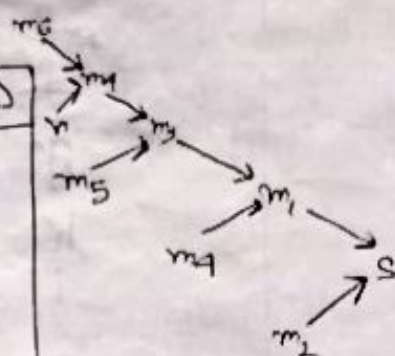
⑤ Effective recursive formulation are possible with lesser amount of storage requirements.

if OPEN is empty then Output "Solution path not found";
 else output • The solution path is obtained by
 tracking back through pointers;

end



OPEN	CLOSED
g	g
m1, m2	g
m1, m2	g, m1
m3, m4, m2	g, m1
m4, m2	g, m1, m3
m4, m5, m4, m2	g, m1, m3
m4, m5, m4, m2	g, m1, m3, m4
m4, m5, m4, m2	g, m1, m3, m4



open	closed
g	g
g	g
m1, m2	g, m1
m3, m5, m1	g, m1, m3
m3, m5, m1	g, m1, m3
m3, m5, m1	g, m1, m3
m6, m4, m5, m3, m1	g, m1, m3, m4

Time complexity: b^d
 Space complexity: bN

b: Branching factor.

d: Depth of the tree.

N: Maximum depth that can be used.

Depth First Search:

begin

put the start node in OPEN;

found = False;

while (OPEN is Empty and not found)

begin

remove the top most node n from

OPEN and put it in CLOSED;

if depth of n \geq depth-bound

then clean up CLOSED;

begin

Expand n generating all its successors

put these successors (in no particular order) on top of OPEN except the successors already appeared in CLOSED;

direct backward pointer to n for each successors n_i of n ;

if any of these successors is goal

then found = True;

else if any of the successor is a dead end then remove it from OPEN and clean up CLOSED;

end; end;

01/03/23 Effectiveness of search algo:-

state vs node

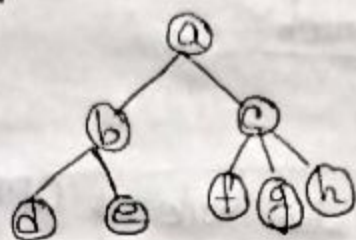
state < node

i) Completeness.

ii) Time complexity.

iii) Space complexity.

iv) Optimality.



bfs abcdefgh

dfs a b d e c f g h

Factors to evaluate

Problem solving performance of various search strat

① Completeness: The stratig guarantees to find a

② soln if one exists.

② Optimality: The soln has an optimal cost.

③ Time Complexity: Time taken (No of nodes expanded in worst case or avg case) to find a soln.

④ Space Complexity: Space used by the algorithm, measured in terms of maximum size of the fringe

[Fringe is a list of nodes, that have been generated but not yet explored. It represents the frontier of the search tree generated. Initially it contains a single node corresponding to the start state.]

Q) Adding an item X to the beginn-begining of the list L .

$\text{add}(X, L, [X|L])$.

using concatenation;

$\text{add}(X, L, L) :- \text{conc}([X], L, L)$.

Q) Delete the first ~~elem~~ occurrence of element X Assuming that the element exists.

$\text{del}(X, [X|L], L)$.

$\text{del}(X, [_Y|Rest], L) :- \text{del}(X, \text{Rest}, L)$.

$a, a, b, a ;$

$a \quad a \quad b \quad a ;$

$a \quad \quad b \quad a ;$

$a \quad a \quad b$

Q) Delete all occurance of X .

$\text{delall}(X, [], [])$.

$\text{delall}(X, [X|L], L1) :- \text{delall}(X, L, L1)$.

$\text{delall}(X, [_Y|L], L1) :- \text{delall}(X, L, L2), \text{conc}([Y], L2, L1)$.

22/02/23

List $[]$

$[X], [X], [-]$

$[X | \text{Rest}]$

$[X, Y | \text{Rest}]$

$[X | [Y | \text{Rest1}]]$

- Q) Given a list L we want to find whether an item X is member of that list or not.

$\text{member}(X, [X | -])$. $\text{member}(X, [Y | \text{Rest}])$:-
 $\text{member}(X, \text{Rest})$.

- Q) Concatenate list $L1$ & $L2$ to form list $L3$.

$\text{conc}([], L, L)$.

$\text{conc}([X | L1], L2, [X | L3])$:- $\text{conc}(L1, L2, L3)$.

- Q) Define Member function using concatenate.

$\text{member}(X, L)$:- $\text{conc}(L1, [X | L2], L)$.

$\text{conc}([], L, L)$.

$\text{conc}([X | L1], L2, [X | L3])$:- $\text{conc}(L1, L2, L3)$.

wolf goat cabbage problem

A farmer has a wolf, a goat, and a cabbage on the left bank of a river. He also has a boat that can carry almost one of the 3 and must transport this trio to the right bank. The problem is that he cannot keep this pairs at one side - (i) goat-cabbage.

(ii) wolf-goat

(l, [w, g, c], []) \rightarrow (r, [w, c], [g])

Initial state,

(r, [], [w, g, c])

Goal state.

Soln

Initial

(l, [w, g, c], [])

\downarrow
(r, [w, c], [g])

\downarrow
(l, [w, g], [c])

\downarrow
(r, [w], [g, c])

\downarrow
(l, [w, g], [c])

\downarrow
(r, [g], [w, c])

\downarrow
(l, [g], [w, c])

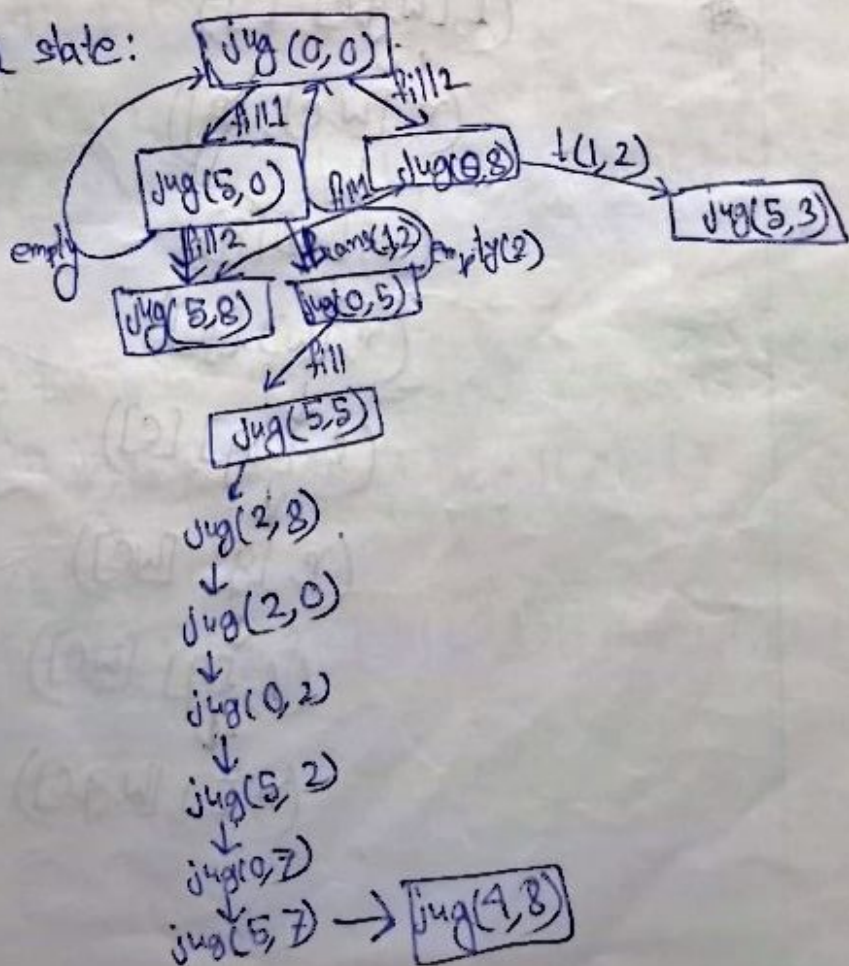
\downarrow
(r, [], [w, g, c])

Water jug problem

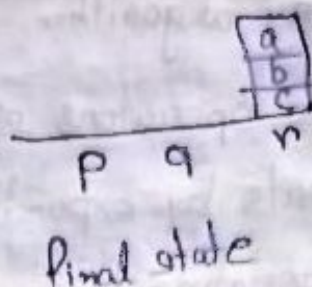
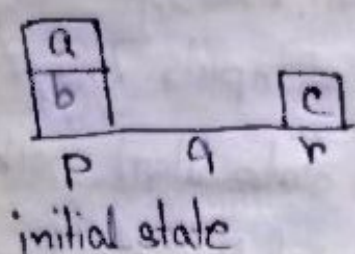
Two jugs of capacities 8 liters and five liters of no markings are given. The problem is to measure out exactly 4 liters from a vat containing enough water. The possible operations are-

- ① Filling up the jug from the vat.
- ② Emptying the jug into the vat.
- ③ Transferring the contents of one jug ^{into other} until the pouring jug is completely empty or the other jug is completely full to the capacity.

Initial state:



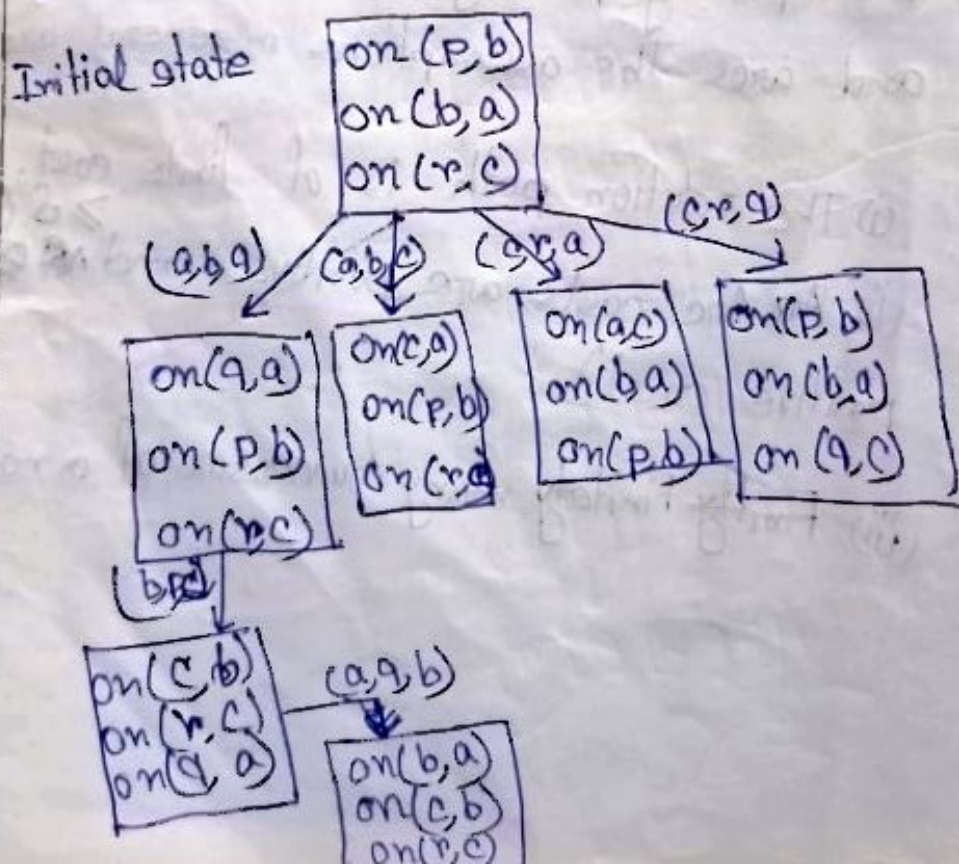
Example:



The types of moves are -

- i) The topmost block in one position to another position.
- ii) The topmost block in one position to an empty position.

2) Assuming Arc costs are unity then draw search graph



In general, the search is supplied implicitly to any algorithm. The start node, goal nodes and the operators are given as inputs. Any search algorithm starts by expanding the state start node i.e by generating the successor of the start node with the help of operators and adding its successors to ~~the~~ the implicit graph. —

At any state in general a node in explicit graph is chosen for expansion and the algorithm terminates when the goal node is chosen. This rule will be followed in every algorithm.

The implicit graph may have infinitely many nodes and arcs. The assumptions in general are

- (i) The solution path is of finite cost.
- (ii) ~~Are~~ Arc costs are positive and ~~are~~ $\geq \delta$ (some positive const).
- (iii) ~~Finite~~ Finitely many successor of a node.

The problem may not come naturally indicate a state space representation. The problem may have to be reformulated in order to be suited for solving by search methods. This itself is an interesting exercise for various problems. ~~An~~

The state space is basically a directed graph known as space graph with a special node called the start node called s and a set of goal nodes called g . If an arc is directed from n_i to n_j , then n_j is said to be the successor of node n_i . Given a node (state), the problem defines the operators which can be applied to the nodes to generate a successor node. There is a cost associated with the generation of each successor node and each arc is labeled with the operator and the cost of applying the operator. Nodes of the graphs are labeled by description. Solving the problem reduces to finding the sequence of states or steps or move required from the start node to a goal node.

Artificial Intelligence

state search space

Input: Starting S

set of operators/moves

set of goal states



To find a solution path (optimal) from S to any of the goal nodes.

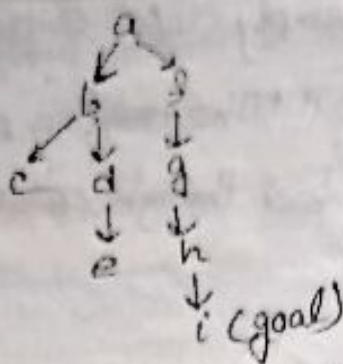
A state is defined by the specification of the values of all attributes of interest in the world.

An operator changes one state into another.

The initial state is where we start. The goal state is where we finish; it is partial description of solution. A plan is a sequence of actions.

search space graph / search graphs

Search methods are used to solve problems by searching ~~the~~ a state space to find a path from a given initial state to a final state.



OPEN

CLOSED

~~a~~
~~a~~
 b
~~cd~~
~~d~~
~~e~~
~~f~~
~~g~~
~~h~~
 i

a
 a
 ab (clean up)
 ab
 abd
 abd
 abd (clean up)
 abdef
 abdefg
 abdefgh

OPEN

CLOSED

~~a~~
~~a~~
 b
~~cd~~
~~ed~~
~~d~~
~~e~~
~~f~~
~~g~~
~~h~~
 i

a
 a
 ab
 ab (clean up)
 ab
 abd
 abd (clean up)
 a
 af
 afg
 afgh

⑥ Advantages are -

- (i) low memory and overhead are easy to implement
- (ii) Quick solution in certain graphs.
- (iii) Human problem solving methods.

⑦ Disadvantages are -

- (i) Sometimes exploring non promising path deeper.
- (ii) Unnecessary node expansion.
- (iii) solution paths not optimal in terms of depth.
- (iv) A node may be expanded more than once along various paths. In fact it may be exponential in certain cases.

⑧ The algorithm takes exponential time in the worst case the algo will take $O(b^d)$ time, where b is the branching factor & d is the depth where soln can be found.

⑨ The space taken is linear in the depth of the search tree is $O(B * N)$

where n is the length of the longest path or maximum depth.

⑩ If the search tree has infinite depth the algo may not terminate. This can happen if the search space is infinite or search contains cycles.

Thus DFS is not complete.

15/03/23

BFS

Begin

put start node s in OPEN;

found = false;

while OPEN is not empty and not found do

begin

remove the topmost node n from OPEN and put it in CLOSED;expand n generating all its successors. put these successors (in particular order) at the end of OPEN except the successors already appearing in CLOSED;Direct backward pointers to n from each successor n_i ;

if any of these successors is a goal node then

found = true

else if one of these successors is a dead end then remove it from OPEN.

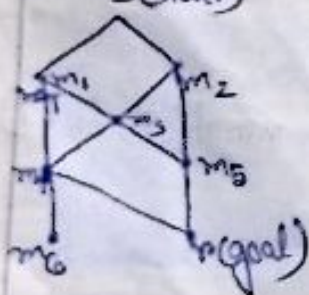
end

if OPEN is empty then

Output failure message.

else

end Output solution by tracking back through pointers

Ex: s (start)

OPEN	CLOSED
s	s
m_1	m_1
m_2	m_2
m_3	m_3
m_4	m_4
m_5	m_5
m_6	m_6
$goal$	$goal$

Note: 1) OPEN is the list of nodes generated in the explicit graph but not yet expanded (just like DFS)

Algorithm: Uniform Cost

Input: Root/Start node, goal node

Output: Optimal solution path.

OPEN and CLOSED are initially empty.

begin

u1: $g(s) = 0$; put s in OPEN; found = false;

while OPEN not empty and not(found) do

begin

u2: select a node n from OPEN with minimum g -value;

* resolve ties arbitrarily but in favour of a goal node *

remove n from OPEN and put it in CLOSED;

if n is a goal node then found = true

else

begin

expand n generating all its immediate successors

for each immediate successor n_i :

begin

$g = g(n) + c(n, n_i)$

if n_i is not already in OPEN or CLOSED then

begin

2	8	3
1	6	4
	7	5

h_1 = no. of tiles not in proper position.

$$= 7$$

h_2 = Manhattan-distance

$$= 1+2+0+1+1+2+2+1$$

$$= 10$$

1	2	3
4	5	6
7	8	

Heuristic estimates

i) Non negative heuristic estimates $h(n)$ associated with each node in a graph G , here $h(n)$ is an estimate of the cost of a minimum cost path from n to a goal node; $h(n)$ is estimated from the problem domain.

ii) $f(n) = g(n) + h(n)$, $f(n)$ gives an estimate of the cost of the minimal path from a start node S to a goal node which is a constraint to pass through n .

iii) $h^*(n)$ is the actual cost of a minimal cost path from n to a node. (infinite no path exists)
 $g^*(n)$ is the actual cost of the minimal cost path from S to n ; $f^*(n) = g^*(n) + h^*(n)$ is the actual cost of a minimal cost path from S to a goal node which is constraint to pass through n .

Find all prefixes of a list.

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

Find the permutation of a list.

22/03/23

Heuristic means rule of thumb

Heuristic are criteria methods or principle for deciding which among several alternative courses of promises to be more efficient in order to achieve some goal. A heuristic function at node n denoted by $h(n)$ is an estimate of the optimum cost from the current node to a goal node.

example 1: The heuristic for the path from Bolpur to Kolkata may be the straight line distance from between Bolpur & Kolkata

$$h(\text{Bolpur}) = \text{Euclidean-distance}(\text{Bolpur}, \text{Shantimiketan})$$

Note 1: - Uniform cost search lacks problem specific knowledge. Such methods are probably inefficient in many cases. Using problem specific knowledge can dramatically improve the search speed.

Note 2: Uniform cost works with $g(n)$ values of nodes. It does not use any info about the cost of the remaining path from a node to a goal node. So in many problems this method tends to expand to many nodes.

Note 3: It is possible to cut down the number of node expansion by using the estimate of the cost of the remaining path from a node to a goal node. Heuristic search approach attempts to direct the search by using heuristic estimates of the cost of the path from a node to a goal node.

Note 4: Several approaches to use heuristic search are there such as A* approach, marking approach, propagation of values, Recursive approach.

20/03/23

Check palindrome using concatenate

palindrome(L).

palindrome(L-).

palindrome(L|T):-conc(L, H, T), palindrome(L).

Check whether even or odd in length.

even(L).

odd(L-).

odd(L, X, Y|T):-odd(T).

even(L, X, Y|T):-even(T).

direct backward pointer from v_i to v_j
and
else if $g(v_i) > g$ then

begin

$$g(n) = g; \quad f(n) = g + h(n)$$

redirect backward pointer from n_i to n_j

if v_j is in CLOSED then remove

n_1 : from CLOSED and put in GEN.

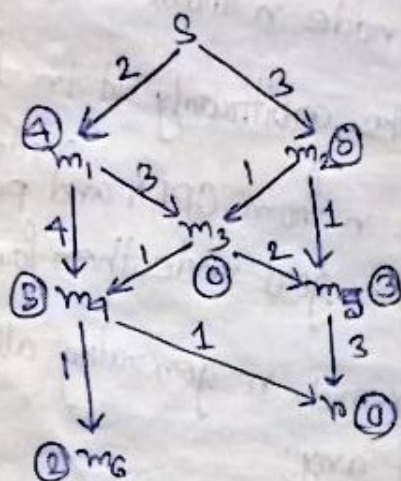
end

end

end

and

if found then output len and solution path by tracing back through pointers else output failure message.

[illegible]

(iv) Assumption about $h(n)$: a) $h(n) \geq 0$ for every node n .

b) $h(s) = 0$ and $h(n) \geq 0$.

c) If n lies on the soln path then $h(n)$ is finite otherwise $h(n)$ may be infinite.

Algorithm A*:

begin

A1: $g(s) = 0$; $f(s) = 0$; found = false;

put s in OPEN;

while OPEN is not empty and not (found) do

begin

A2: select a node n from OPEN with minimum f -value;

/ Resolve ties arbitrarily but in favour of a goal node */*

Remove n from OPEN and put it in CLOSED;

if n is a goal node then found = true else

begin expand n generating all its successors if any;

for each intermediate successors ~~if~~ n_i of n do

begin

$g = g(n) + c(n, n_i)$

if n_i is not in OPEN or CLOSED

then

begin

$g(n_i) = g$; $f(n_i) = g + h(n_i)$

put n_i in OPEN;

Optimal algorithm

However the number of nodes searched is still exponential in the worst case.

iv) A^* must keep all nodes it's considering in memory.

v) A^* is much more efficient than ~~any~~ uninformed methods.

Limitations of A^* :

- i) worst case performance is poor.
- ii) Size of memory is appreciable is ~~noted~~ to 'DFA'.
- iii) Repeated node expansion.
- iv) In case of ~~admissible~~ inadmissible heuristic we have no idea about the quality of soln.
- v) Overhead of open is just as BFS.
- vi) OPEN must be implemented as priority queue.

Advantages of A^* :

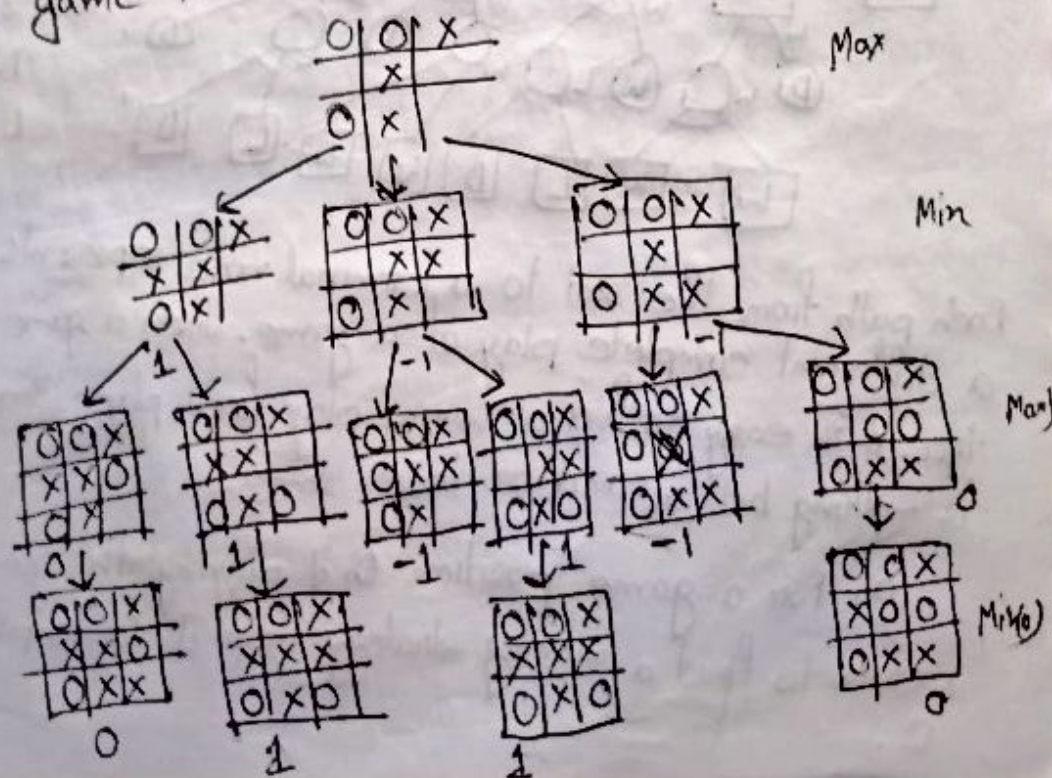
- i) A^* is quite fast on the average.
- ii) A^* never ~~use~~ stores the entire explicit graphs and ~~never~~ ^{hence} makes repeated expansion.

This is a local search algorithm with greedy approach and with no backtracking.

Game tree: Adversarial real search can be represented as a tree where the nodes represent the current state of and the arcs represent the moves. The game tree consists of all possible moves for the current player starting at the root and all possible moves for the next player as the children of these nodes and so forth as far as to the future of the goal as desired. Each individual move by game player called is called ply.

The leaves of a game tree represent terminal position where the outcome of a game is clear (win, loss, draw). Each terminal position has a score. If are good for one of the player called the max player. The other player called the min player tries to minimize the score.

For example: we associate 1 with a win & 0 with a draw & -1 with a loss for max player in the game tic-tac-toe.



Min-Max Algo: To determine $V(\alpha)$ do the following.

(i) If α is a terminal value return $V(\alpha) = \alpha$ otherwise,

(ii) Generate α 's successors say $\alpha_1, \dots, \alpha_b$ where b is the branching factor.

(iii) Evaluate $V(\alpha_1), V(\alpha_2), \dots, V(\alpha_b)$ from left to right

(iv) If α is a max node return $V(\alpha) = \max_{1 \leq i \leq b} [V(\alpha_i)]$

(v) If α is a min node return $V(\alpha) = \min_{1 \leq i \leq b} [V(\alpha_i)]$

03/04/23

$f(\alpha, 0) :- \alpha < 3.$

$f(\alpha, 1) :- \alpha \geq 3, \alpha < 6.$

$f(\alpha, 2) :- \alpha \geq 6.$

? $f(\alpha, Y), 2 < Y$

$f(\alpha, 0) :- \alpha < 3, !$

$f(\alpha, 1) :- \alpha \geq 3, \alpha < 6, !$

$f(\alpha, 2) :- \alpha \geq 6$

$f(\alpha, 0) :- \alpha < 3, !$

$f(\alpha, 1) :- \alpha < 6, !$

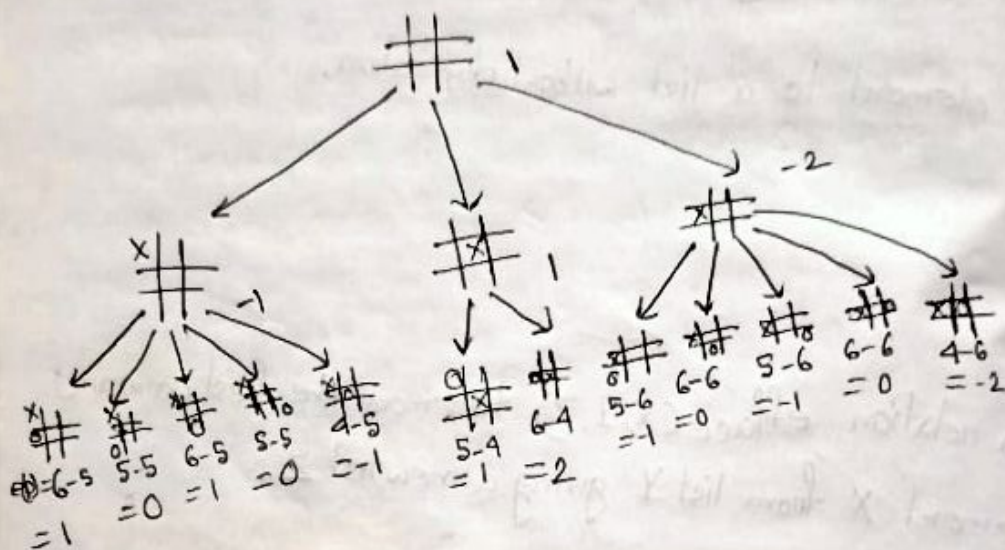
$f(\alpha, 2).$

Green cut

Red cut.

05/04/23

Let's assume that max (the computer) marks process(x) and MIN marks (o) and MAX is to play first in the game of Tic-Tac-Toe.
 Let the static evaluation score $e(p) = (\text{no of complete rows, columns or diagonals that are still OPEN for MAX}) - (\text{no of com. OPEN for MIN})$

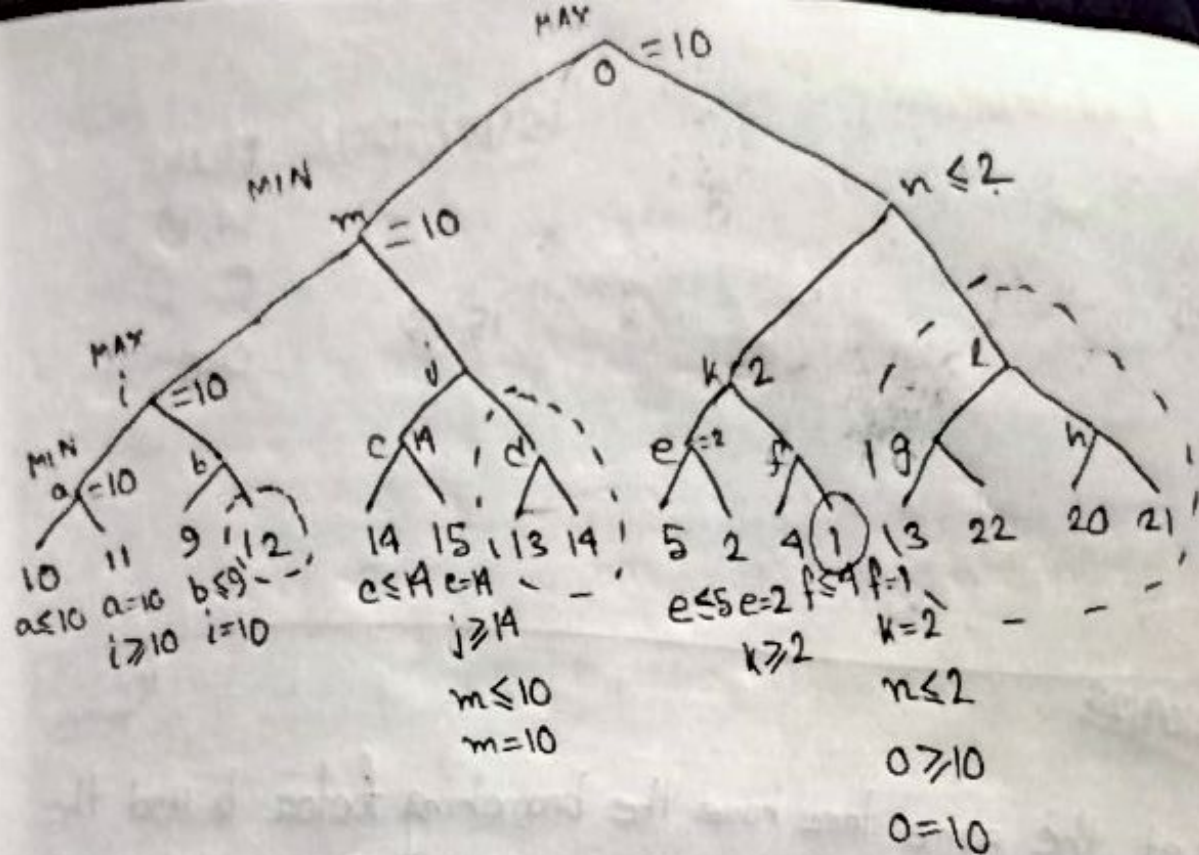


Disadvantage: To use this method we have to -

- i) Generate the full game tree first.
- ii) Evaluate all the terminal nodes.
- iii) Examine the full game tree.

Recursive version of MIN-MAX algorithm:

```
main()
{
    v = value(root);
    output v;
}
```

$L = [a, b]$

$- ([- | [- | Rest]], Rest)$

10/04/23

~~double tone~~
Whether a list is ^{not} double tone ~~and not~~

~~double tone~~ $([])$ ~~[]~~

~~[]~~ ~~[]~~

$([])$

$([x | [y | [z | Rest]]])$

Whether two list are same length or not.

even-odd length.

X is leaf

$$(\leq (15), (20)) = (15, 20)$$

	14, 19
	0,
15, 15	0, 0

13/09/23

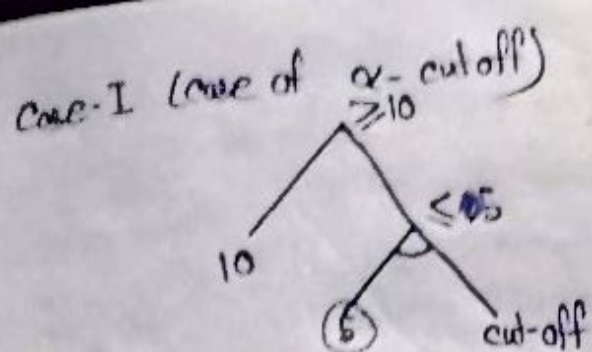
Let the game tree have the branching factor b and the depth d , then the minimum no. of static evaluation required to solve the game tree is -

$$E = b^{\lfloor d/2 \rfloor} + b^{\lceil d/2 \rceil} - 1$$

$$= \begin{cases} 2b^{\lceil d/2 \rceil} - 1 & \text{if } d \text{ is even} \\ b^{\frac{d-1}{2}} + b^{\frac{d+1}{2}} - 1 & \text{if } d \text{ is odd} \end{cases}$$

We can say the bound on a node as a current value of a node. Actual value of a node is set to its current value -

- (i) If all the sons are yet to be examined
- (ii) If some of the sons are yet to be examined but their examinations can in no way affect the value of the root.



The max ancestor of a MIN node cuts-off the sons of the MIN node if the current value of MIN node is \leq the current value of MAX ancestor (Reason since current value of a MIN node in such ^{case} can never increase the value of a MAX ancestor).

The MIN ancestor of a MAX node cuts-off the sons of the MAX node if the current value of a MAX node is \geq the current value of MIN ancestor (Reason since current value of a MAX node in such a case can never ~~decrease~~ ^{increase} the value of a MIN ancestor).

The current value of a MAX node is known as α -value or α bound and the current value of the MIN node is known as β -value or β bound.

Modification of MIN-MAX algorithm.

```

100
main()
{
  va

```

α -bound is the cut off bound and is equal to the current ~~low~~ value of all MAX ancestors of J i.e. highest current lower bound of all " " " J When the current value of the ~~low~~ MIN node $J \leq \alpha$ -bound for J , the sons of

β -bound

α - β pruning algorithm

main()

$\{ v = \alpha\beta\text{value}(\text{root}, -\infty, \infty) \quad /* \alpha = -\infty, \beta = \infty */$
 output v ;
 $\}$

$\alpha\beta\text{value}(\text{node } n, \text{int } \alpha, \text{int } \beta)$

$\{ \text{int } \text{evalue};$

if (n is a terminal) ~~no~~
 return $e(n)$;

if (n is a MAX node)

$\{ \text{evalue} = \alpha;$

for (each successor n_i of n)

$\{ \text{generate } n_i;$

$\text{evalue} = \max(\text{evalue}, \alpha\beta\text{value}(n_i, \text{evalue}, \beta));$

if ($\text{evalue} \geq \beta$)

return $\text{evalue};$

$\}$

return $\text{evalue};$

$\}$