

Knowledge Representation and Logic

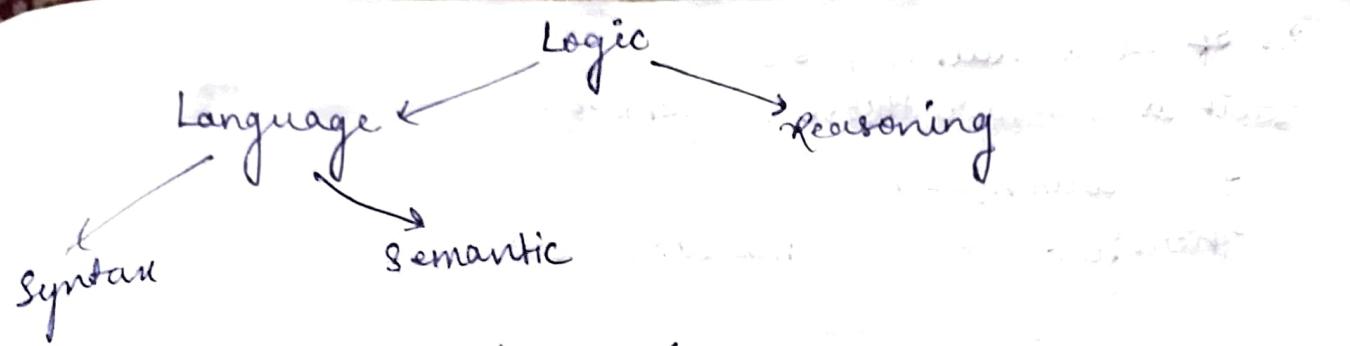
Intelligent agents
should have capacity of for

- ① Perceiving → acquiring info. from the environment
 - ② Knowledge representation → representing its understanding of the world
 - ③ Reasoning → inferring the implications of what it knows.
 - ④ Acting → choosing what it wants to do and carrying it out.
- advantages of using formal logic as a language of AI:-
- ① It is precise and definite
 - ② It allows programs to be written which are declarative - they describe what is true and not how to solve the problems.
 - ③ It allows for automated reasoning technique for general purpose inferencing.

→ Limitation of formal language:

- A large portion of reasoning carried out by humans depends on handling knowledge that is uncertain. Logic can not represent the uncertainty well.
- Natural language reasoning requires inferring hidden states namely the intention of the speaker

Logic



→ Different logic systems are :-

- There are a no. of logical system with different syntax and semantics

① First order predicate logic

② Propositional logic

③ Temporal

④ modal

⑤ high ordered logic

⑥ Non-monotonic etc.

Example of logical reasoning :-

→ Propositional logic

Ex:- When it rains it is humid.

It is raining now

Therefore, it is humid.

→ The first two statements are called premises or axioms. We take axioms to be true.

The third statement is called conclusion. The logical rule to come to a solution is called

Modus Ponens.

$\Delta \rightarrow$ represents

If $P \Delta Q$ it rains

$Q \Delta R$ it is humid

{ if P is true then Q is true.

$P \Rightarrow Q$

2. If it is raining or —
— it is summer then use umbrellas
It is summer now.

Therefore, we use umbrellas now.

3. If the sun rises in the west, then human beings can fly.
but the sun always rises in the east, hence, human beings can never fly.

→ Puzzle

Casket

• Perlia's Caskets →

There are three caskets made of gold, silver, lead.
A gold ring is in one of the three caskets. On each of the caskets, there is a statement. Out of which only one statement is true.

Gold → The ring is not in this casket {F
T}

Silver → "

Lead → The ring is not in the gold casket {F

The problem is, in which casket is the ring?

→ gold

• Logical Paradox → paradox has no solution.

e.g. → who is the follower of the Guru who says don't follow me?

~~definition~~

→ Can the almighty

→ In propositional logic, we are interested in declarative sentences that are either true or false, but not both.

Examples of declarative sentence:-

$P \triangleq$ It is hot

$Q \triangleq$ It is humid

$C \triangleq$ one feels comfortable

$(P \wedge Q) \rightarrow C$

→ Logical connectives

① Conjunction $\rightarrow \wedge, \&$

② Disjunction $\rightarrow \vee$

③ Negation $\rightarrow \neg$

④ Implication $\rightarrow \rightarrow, \Rightarrow$

⑤ Equivalence $\rightarrow \leftrightarrow, \Leftrightarrow$

→ Syntax of the logic

① Propositional symbols or atoms, customarily denoted by upper case letters Q, P, R, etc.

② Truth constants $\rightarrow T \& F$

③ Logical connectives.

→ Well formed formula (WFF)

→ It is defined recursively as follows:

① T & F are WFFs

② An atom is a WFF

③ If G is a WFF ~~then~~ then $\neg G$ is a WFF.

④ If G and H are WFFs then $(G \wedge H), (G \vee H),$

⑤ If G and H are WFFs then $G \rightarrow H, G \leftrightarrow H$ are all WFFs

⑥ All WFFs are generated by finite no. of application

on the rules below:-

$$G \rightarrow H = \neg G \vee H$$

→ Conjunction

G	H	$G \wedge H$	$G \vee H$	$\neg G$	$\neg H$	$G \rightarrow H$	$G \leftarrow H$
T	T	T	F	F	F	T	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	T	F
F	F	F	T	T	T	T	T

→ Obtain the truth table for
 $(P \wedge Q) \rightarrow (R \leftrightarrow \neg S)$

Ans

$$\neg(P \wedge Q) \vee (R \leftrightarrow \neg S)$$

P	Q	R	S	$\neg S$	P	Q	R	S	$\neg S$
T	T	T	T	F	T	T	T	T	F
F	T	T	T	F	T	T	T	F	T
F	F	T	T	F	T	T	F	T	F
F	F	F	T	F	F	T	F	F	T
F	F	F	F	T	F	T	F	F	F
F	T	T	F	T	F	T	T	F	T
T	T	F	F	T	F	T	T	F	T
T	F	F	F	T	F	T	F	F	T
F	F	F	T	F	F	F	F	T	F
F	T	F	T	F	T	T	F	T	F
T	F	T	F	T	T	F	T	F	T
T	T	F	F	T	T	T	F	F	T
F	F	T	T	F	F	F	T	T	F
T	F	F	T	F	T	T	F	T	F
F	T	T	F	T	T	F	T	F	T

P	Q	R	S	$P \wedge Q$	$\neg S$	$R \leftrightarrow (\neg S)$	$\neg P \vee Q$
T	T	T	F	F	T	F	T
T	T	F	F	F	T	F	T
T	F	F	F	F	T	T	T
T	F	F	F	F	T	T	T
F	T	T	F	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	F	T	T
F	F	T	F	F	T	T	T
F	F	F	T	F	F	T	T
F	F	F	F	F	F	T	T

→ Model of WFF

An interpretation of WFF is said to be a model of WFF iff it evaluates to true under that interpretation

E.g. The interpretation $\{T, T, T, F\}$

is a model for $(P \wedge Q) \rightarrow (R \leftrightarrow (\neg S))$

counter model $\rightarrow \{T, T, T, T\}$

validity and consistency →

Consider the truth table for

$$q \equiv ((P \rightarrow Q) \wedge P) \rightarrow Q$$

P	Q	$\neg P$	$P \rightarrow Q$	$(P \rightarrow Q) \wedge P$	$((P \rightarrow Q) \wedge P) \rightarrow Q$
T	F	F	T	F	T
T	T	F	T	T	T
F	T	T	T	F	T
F	F	T	T	F	F

validity.

→ A WFF is said to be valid if it is true under all its interpretations. A valid formula is also called tautology.

E.g. → Ajay is at home OR NOT at home.

Invalid → A WFF is said to be invalid if it is not valid.

draw the truth table of $G \triangleq ((P \rightarrow Q) \wedge (P \wedge \neg Q))$

P	Q	$\neg P$	$(P \rightarrow Q) \wedge Q$	$(P \wedge \neg Q)$	$(P \rightarrow Q) \wedge (P \wedge \neg Q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	F	T	T	F	F

Inconsistency → A formula is said to be inconsistent or unsatisfiable or contradiction iff it is evaluated to false under all its interpretation.

E.g. → The sky is blue and The sky is not blue.

Satisfiable → A WFF is satisfiable if it is not unsatisfiable.

Observations → ~~A WFF is~~

- ① A WFF is valid if the negation of it is inconsistent or contradiction.
- ② A WFF is inconsistent iff the negation of it is valid.

- (iii) A formula is said to be invalid if there exists at least one counter model
- (iv) A formula is said to be consistent or satisfiable if there exists at least one model
- (v) If a WFF is valid then it is consistent or satisfiable but not vice-versa.
- (vi) If a WFF is inconsistent, then it is invalid but not the vice-versa.

H.W.

→ For the following formula, determine whether they are valid, invalid, consistent, inconsistent or some combinations of this.

$$(I) \neg(\neg P \rightarrow P)$$

$$(II) P \rightarrow (\neg P)$$

$$(III) (\neg P) \rightarrow P$$

$$(IV) (P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$$

$$(V) (P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$$

$$(VI) (P \vee Q) \wedge (\neg P \vee Q)$$

Equivalence forms of WFF

→ A formula P of WFF is said to be equivalent to a formula Q of WFF if $P \equiv Q$ iff the truth values of P and Q are the same under all interpretations.

$$G_1 = P \rightarrow Q$$

$$G_2 = \neg P \vee Q$$

$$\text{Then } G_1 \equiv G_2$$

→ List of Some equivalence formulas →

1. $P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$

2. $(P \rightarrow Q) = \neg P \vee Q$.

3(a) $P \wedge Q = Q \wedge P$

3(b) $P \vee Q = Q \vee P$ } commutative law

4.(a) $(P \vee Q) \vee R = P \vee (Q \vee R)$

(b) $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$ } associative Law.

5.(a) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

(b) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$ } distributive Law

6. ~~$P \vee \square = P$~~

True \square

$P \wedge \blacksquare = P$

False \blacksquare

7. (a) $P \vee \blacksquare = \blacksquare$

(b) $P \wedge \square = \square$

8. (a) $P \vee \neg P = \blacksquare$

(b) $P \wedge \neg P = \square$

9. $\neg(\neg P) = P$

10 (a) $\neg(P \vee Q) = \neg P \wedge \neg Q$

(b) $\neg(P \wedge Q) = \neg P \vee \neg Q$ } de Morgan's Law

Literal \rightarrow

A literal is an atom or negation of an atom.

E.g. $\rightarrow P, \neg Q$ etc.

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Normal forms \rightarrow

① Conjunctive \rightarrow a WFF is in conjunctive normal form (CNF) iff the formula is of the form

$$q = G_1 \wedge G_2 \wedge \dots \wedge G_n, n \geq 1$$

where each of G_1, G_2, \dots, G_n are conjunction of literals.

E.g. $\rightarrow P \wedge Q, (P \wedge Q) \wedge (P \vee \neg S)$

② Disjunctive Normal form \rightarrow a WFF is in disjunctive normal form (DNF) iff the formula is of the form

$$q = G_1 \vee G_2 \vee \dots \vee G_n; n \geq 1$$

where each of G_1, G_2, \dots, G_n are conjunction of literals.

E.g. $\rightarrow P \vee Q, (P \wedge Q) \vee (P \wedge \neg S)$

Example \rightarrow Obtain the DNF of $(P \vee \neg Q) \rightarrow R$

$$\Rightarrow \neg(P \vee \neg Q) \vee R$$

$$\Rightarrow \neg P \neg Q \vee \neg \neg Q \vee R \quad (\neg P \wedge Q) \vee R \rightarrow \text{DNF}$$

$$\Rightarrow (\neg P \vee R) \wedge (Q \vee R) \rightarrow \text{CNF}$$

\rightarrow Obtain the CNF of $(P \wedge (Q \rightarrow R)) \rightarrow S$

$$\Rightarrow \neg P \wedge \neg(Q \rightarrow R) \quad (P \wedge (\neg Q \vee R)) \rightarrow S$$

$$P \wedge \neg Q \vee P \wedge R \rightarrow S$$

$$\Rightarrow (\neg P \wedge Q \vee \neg P \wedge \neg R) \vee S$$

$$\Rightarrow \neg P$$

$$\Rightarrow \neg P \vee Q \vee \neg R \vee S$$

→ Logical consequence →
 Given a set of statements or facts and rules
 (or axioms or premises) and a conclusion from if
 the conclusion follows from the set of statements
 then we can say that the conclusion is
 the logical consequence of the statement.

E.g. → suppose stock price goes down if
 the prime interest goes up.
 Also suppose that most people are
 unhappy when stock price goes down.
 Assume that the prime interest goes up.
 Show that you can conclude most people
 are unhappy.

Let us denote,

$$\begin{aligned} P &\triangleq \text{prime interest goes up} \\ S &\triangleq \text{stock price goes down} \\ U &\triangleq \text{most people are unhappy.} \end{aligned}$$

→ Then the given statement of the problem
 can be symbolized as

$$A_1 \triangleq \text{if the prime interest goes up then stock price goes down} \\ = P \rightarrow S$$

$$A_2 \triangleq \text{if the stock price goes down then most people are unhappy.} \\ = S \rightarrow U$$

$$A_3 \triangleq \text{prime interest goes up} \\ = P$$

We have to show

$$A_4 = \text{most people are unhappy.} \\ = U$$

$$(A_1 \wedge A_2 \wedge A_3) \rightarrow U$$

$$(P \rightarrow S) \wedge (S \rightarrow U) \wedge P \rightarrow U$$

$$\rightarrow (\neg P \vee S) \wedge (\neg S \vee U) \wedge P \rightarrow U$$

$$(a+b)(b+c) \\ ab + ac + b^2 + bc$$

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$$\neg(\neg P \vee S) \wedge \neg(\neg S \vee U) \wedge P \vee V : U \\ ((P \vee \neg S) \wedge (\neg S \vee U) \wedge \neg P) \vee U$$

$$P \wedge S \vee P \wedge U \vee \neg S \wedge S \vee \neg S \wedge U \wedge \neg P \vee U$$

~~PQQR~~

O

→ Given a set of WFFs, F_1, F_2, \dots, F_n and WFF G ,
 G is said to be a logical consequence of F_1, F_2, \dots, F_n
if there exists an interpretation I such that
 ~~$F_1 \wedge F_2 \wedge \dots \wedge F_n$~~ is true in I , G
is also true in I

Theorem 1 → Given a set of WFFs (F_1, F_2, \dots, F_n) and another WFF G , G is said to be a logical consequence of F_1, F_2, \dots, F_n iff
 $(F_1 \wedge F_2 \wedge \dots \wedge F_n) \rightarrow G$ is valid.

Theorem 2 → Given a set of WFFs (F_1, F_2, \dots, F_n) and another WFF G , G is said to be a logical consequence of F_1, F_2, \dots, F_n iff
 $(F_1 \wedge F_2 \wedge \dots \wedge F_n) \wedge \neg G$ is inconsistent or a contradiction.

Q: Consider the formula $F_1 \triangleq P \rightarrow Q$

$$F_2 = \neg Q$$

$$G = \neg P$$

Show that G is the logical consequence of $F_1 \wedge F_2$

→ Method 1 $(F_1 \wedge F_2) \rightarrow G$

$$(P \rightarrow Q) \wedge (\neg Q) \rightarrow G$$

$$(\neg P \vee Q) \wedge (\neg Q) \rightarrow Q \wedge \neg P$$

$$\neg P \wedge \neg Q \vee Q \wedge \neg Q \rightarrow \neg P \quad (Q \wedge \neg Q = \boxed{0})$$

$$(\neg P \wedge \neg Q) \rightarrow \neg P$$

$$\neg(\neg P \wedge \neg Q) \vee \neg P$$

$$P \wedge Q \vee \neg P$$

$$= \boxed{1}$$

$$\text{Method II} \rightarrow F_1 \wedge F_2 \wedge \neg G = (\neg P \wedge \neg Q) \wedge P \\ = \square$$

Thus we can conclude Method III \rightarrow

P	Q	$\neg Q$	$P \rightarrow Q$	$F_1 \wedge F_2$	$\neg P$	$(F_1 \wedge F_2) \rightarrow G$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Method 4

$$F_1 \wedge F_2 \wedge \neg G$$

$$\begin{matrix} F \\ F \\ F \\ F \end{matrix}$$

By theorem 2, we can say that the conclusion is a logical consequence.

Q: Given that if the congress refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the president of the firm resigns. Will the strike not be over if the congress refuses to act and the strike just starts.

$P \triangleq$ the congress refuses to enact new laws

$Q \triangleq$ the strike is over

$R \triangleq$ the president of the firm resigns

$S \triangleq$ the strike lasts more than one year.

$F_1 \triangleq$ if the congress refuses to enact new laws then the strike will not be over unless it lasts & more than one year and the president of the firm resigns.

$$= P \rightarrow (\neg Q \vee (S \wedge R))$$

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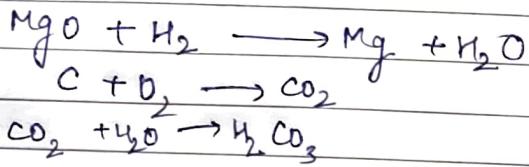
$F_2 \triangleq$ the congress refuses to act
 $= P$

$F_3 \triangleq$ the strike just stands
 $= ns$

$G \triangleq$ the conclusion that its the strike will not be over.
 $= \neg q$

→ Complete it ...

Q Suppose we can perform the following chemical reactions



Suppose we have some quantities of MgO , H_2 , C & O_2 .
Show that we can make H_2CO_3 .

$A_1 \triangleq (MgO \wedge H_2) \rightarrow (Mg \wedge H_2O)$

$A_2 \triangleq (C \wedge O_2) \rightarrow CO_2$

$A_3 \triangleq (CO_2 \wedge H_2O) \rightarrow H_2CO_3$

$A_4 \triangleq MgO$

$A_5 = A_2$

$A_6 = O_2$

$A_7 = C$

$G \triangleq H_2CO_3$

$(A_1 \wedge A_2 \wedge A_3 \wedge A_4 \wedge A_5 \wedge A_6 \wedge A_7) \longrightarrow G = T$
 $) \wedge \neg q = F$

There are 9 rules of inferences in propositional logic. The rules and their mathematical logics are given below:-

	Rule	Mathematically
1. Modus ponens	if $P \rightarrow Q$ is true and P is true then Q is true	$P \rightarrow Q$ P $\therefore Q$
2. Modus Tollens	if $P \rightarrow Q$ is true and $\neg Q$ is true then $\neg P$ is true	$P \rightarrow Q$ $\neg Q$ $\therefore \neg P$
3. Hypothetical Syllogism	if $P \rightarrow Q$ is true and $Q \rightarrow R$ is true then $\neg P$ is true	$P \rightarrow Q$ $Q \rightarrow R$ $\therefore P \rightarrow R$
4. Disjunctive Syllogism	if $P \vee Q$ is true and $\neg P$ is true then Q is true	$P \vee Q$ $\neg P$ $\therefore Q$
5. Permutation	if $(P \rightarrow Q) \wedge (R \rightarrow S)$ is true and $\neg Q \vee \neg R$ is true $\neg Q$ is true then $\neg P \vee \neg R$ is true	$(P \rightarrow Q) \wedge (R \rightarrow S)$ $\neg Q \vee \neg R$ $\therefore \neg P \vee \neg R$
6. Simplification	if $P \wedge Q$ is true then P is true	$P \wedge Q$ $\therefore P$
7. Conjunction	if P is true and Q is true then $P \wedge Q$ is true	P Q $\therefore P \wedge Q$

8. Addition

if P is true

then $P \vee Q$ is true.

P

$\therefore P \vee Q$

9. Conjunctionive
dilemma.

if $(P \rightarrow Q) \wedge (R \rightarrow S)$ is
true then $Q \vee S$ is
true

$(P \rightarrow Q) \wedge (R \rightarrow S)$

$P \vee R$

$\therefore Q \vee S$

Q. Symbolize the following statements that is ~~is~~ right.
WFFs

① a relation is an equivalence relation iff it is
reflexive, symmetric and transitive.

$E \triangleq$ equivalence reln.

$R \triangleq$ Reflexive

$S \triangleq$ Symmetric

$T \triangleq$ Transitive

~~$R \wedge S \wedge T$~~

$E \longleftrightarrow (R \wedge S \wedge T)$

② If the humidity is ~~so~~ high, it will rain this rain ~~tomorrow~~
afternoon or in the evening.

$H \triangleq$ high humidity

$A \triangleq$ rains in afternoon

$E \triangleq$ rains in evening.

$H \rightarrow (A \vee E)$

③ Cancer will not be cured unless its cause is ~~not~~
or determined and a new drug for cancer
is found.

$C \triangleq$ cancer is ^{not} cured

$D \triangleq$ cause determined

$N \triangleq$ new drug

$$\neg C \rightarrow (D \wedge N)$$

$$C \vee (D \wedge N)$$

④ If ~~M~~ is a man who can campaign so hard
the he probably he will be elected.

$M \triangleq$ a man

$C \triangleq$ campaigns hard

$E \triangleq$ elected

$$M \wedge C \rightarrow E$$

⑤ I attend a concert or I wake up early.

$C \triangleq$ attend a concert

$E \triangleq$ wake up early

$$(C \wedge \neg E) \vee (\neg C \wedge E)$$

⑥ I will attend the concert unless I have
exam tomorrow.

$C \triangleq$ attend the concert

$E \triangleq$ I have exam tomorrow,

$$\neg E \rightarrow C / E \vee C$$

⑦ I will attend the concert inspite of my exam.

$C \triangleq$ attend the concert

$E \triangleq$ exam

$A \triangleq$ Attend

$$A \rightarrow C$$

⑧ In Belpur it's either hot or rainy except in winter when it is cool.

$H \triangleq$ hot

$R \triangleq$ rainy

$C \triangleq$ cool

$$(\neg C \wedge (H \vee R)) \vee (W \wedge \neg (H \vee R))$$

⑨ Show that the following statement F_2 is a logical consequence of F_1 .

$$\hookrightarrow F_2 \circledcirc F_1 \quad F_1 \rightarrow F_2 = \text{true}.$$

$F_1 \rightarrow$ Tom cannot be a good student unless he is smart or his father supports him.

$F_2 \rightarrow$ Tom is a good student if his father supports him.

Let $P \triangleq$ Tom is good student

$Q \triangleq$ he is smart

$R \triangleq$ his father supports him

$$F_1 \circledcirc F_2 = (Q \vee R) \rightarrow P$$

$$F_2 = \cancel{P \rightarrow Q \wedge R} \rightarrow P$$

$$(\neg Q \vee R) \rightarrow P$$

$$\neg(\neg Q \vee R) \vee P$$

$$(\neg \neg Q \vee \neg R) \vee P$$

$$\neg \neg P \vee P \vee \neg \neg R \vee P$$

$$\neg \neg P \vee (\neg \neg R \vee P)$$

T

$$R \rightarrow P$$

$$\neg R \vee P$$

First order predicate logic

→ In propositional logic, the basic elements are ~~useless~~ atoms. A proposition is a declarative sentence which is either true or false but not both.

But if a sentence has some non-declarative part then it can not be expressed in the framework of propositional logic.

Examples:-

① If Every man is mortal

Socrates is a man

Therefore Socrates is a mortal.

② Five is an odd number

and it is prime.

Therefore there exists some odd prime number.

Suppose, x is greater than 3

father of x

x loves y

GREATER ($x, 3$)

FATHER (x)

LOVES (x, y)

John's father loves John.

→ LOVES (FATHER (John), John).

If there are n predicate symbols
 $\rightarrow n$ -ary predicate symbols.

A term can be recursively defined as follows

- (I) A constant is a term, ~~or variable is~~.
- (II) a variable is a term.
- (III) If f is n -place function symbol and $T_1, T_2 \dots T_n$ are terms, then $f(T_1, T_2, \dots, T_n)$ is a term.
- (IV) All terms can be derived by finite applications of the above rules.

\rightarrow Suppose $x + y$ $\text{plus}(x, y)$
 $2x + 1$ $(\text{plus}(\text{times}(x, 2), x))$
 $2x + 1$ greater than x .
GREATER($(\text{plus}(\text{times}(x, 2), x), x)$)

Quantifiers \rightarrow once atoms are defined, we can use five logical connectives
(and, or, not, \rightarrow , \leftrightarrow)

Further more in F.O.P.L., we can use quantifiers to quantify the variables.

- (1) Universal quantifier \forall read as for all.
- (2) Existential quantifier \exists read as there exists.

~~$\forall x (\text{MORTAL}(x))$~~

E.g. 1 $\rightarrow \forall x (\text{MAN}(x) \rightarrow \text{MORTAL}(x))$

$\text{MAN}(\text{Socrates})$

$\forall x (\text{MAN}(x) \rightarrow \text{MORTAL}(x)) \wedge \text{MAN}(\text{Socrates})$
 $\rightarrow \text{MORTAL}(\text{Socrates})$

b.g. 3 ②

$$\text{odd}(x) \wedge \text{prime}(x)$$

$$\exists x (\text{odd}(x) \wedge \text{prime}(x))$$

$$(\text{odd}(5) \wedge \text{prime}(5)) \rightarrow (\exists x (\text{odd}(x) \wedge \text{prime}(x)))$$

g. Let the domain be a set of people invited for a meeting. Use $M(x)$ to denote that the person has attended the meeting and $S(x)$ denote that person x spoke at the meeting. Now using F.O.P.i. describe the following statements.

① Ram and Shyam attended the meeting.

$$M(\text{Ram}) \wedge M(\text{Shyam})$$

② Everyone is present at the meeting,

$$\forall x [M(x)]$$

③ Some people could not come for the meeting

$$\exists x [\neg M(x) / \neg (\forall x M(x))]$$

④ Some people who came to the meeting did not speak

~~$\exists x Q(x)$~~

$$\exists x [(M(x) \wedge \neg S(x))]$$

⑤ Everyone who came to the meeting spoke on some topic

~~$\forall x [M(x) \Rightarrow S(x)]$~~

$$\forall x [M(x) \rightarrow S(x)]$$

(6) Only one person spoke in the meeting
Equal ($x, 1$)

$$\exists x (s(x) \wedge \text{Equal}(x, 1))$$

(7) Ram and Shyam attended the meeting, but only one spoke in the meeting.

$$(M(\text{Ram}) \wedge M(\text{Shyam})) \rightarrow \cancel{(s(\text{Ram}) \vee s(\text{Shyam}))} \wedge \underline{s(\text{Ram}) \vee \neg s(\text{Shyam})}$$

$$(M(\text{Ram}) \wedge M(\text{Shyam})) \wedge ((s(\text{Ram}) \wedge \neg s(\text{Shyam})) \vee (\neg s(\text{Ram}) \wedge s(\text{Shyam})))$$

(8) Exactly 3 person spoke in the meeting.

$$\exists y_1 \exists y_2 \exists y_3 (s(y_1) \wedge s(y_2) \wedge s(y_3) \wedge \\ (\forall y (s(y) \rightarrow \text{Equal}(y, y_1) \vee \text{Equal}(y, y_2) \vee \\ \text{Equal}(y, y_3)) \wedge (\forall y (\neg \text{Equal}(y, y_1) \wedge \\ \neg \text{Equal}(y, y_2) \wedge \neg \text{Equal}(y, y_3))))$$

(9) Everyone didn't come to all the meetings.

$$N(i, x)$$

$$\forall i \forall x \neg N(i, x)$$

(10) Someone did not turn up for any meeting.

$$\exists x \forall i \neg N(i, x)$$

E.g. \rightarrow

(11) Some patients like all doctors

No patients like any doctor
Hence, no doctor is a quack.

let $P(x) \triangleq x \text{ is a patient}$

$D(x) \triangleq x \text{ is a doctors}$

$Q(x) \triangleq x \text{ is a queen.}$

$\exists x P(x)$

$A \triangleq \exists x P(x) \wedge \forall y (D(y) \rightarrow \text{Likes}(x, y))$

$B \triangleq \forall x P(x) \rightarrow \forall y (Q(y) \rightarrow \neg \text{Likes}(x, y))$

$C \triangleq \neg \forall x (D(x) \rightarrow \neg Q(x))$