

RSA (Rivest-Shamir-Adleman) is a widely used public-key cryptography algorithm, which is based on the mathematical properties of large prime numbers.

The algorithm involves the following steps:

#### 1. Key Generation:

- Select two large prime numbers,  $p$  and  $q$  (for example,  $p = 61$  and  $q = 53$ )
- Compute  $n = pq$  ( $n = 61 * 53 = 3233$ )
- Compute  $\phi(n) = (p-1) * (q-1) = (61-1) * (53-1) = 3120$
- Select an integer  $e$  such that  $1 < e < \phi(n)$  and  $e$  is coprime to  $\phi(n)$  (for example,  $e = 17$ )
- Compute  $d$  such that  $d \equiv e^{-1} \pmod{\phi(n)}$  (i.e.,  $d$  is the modular inverse of  $e$  with respect to  $\phi(n)$ ).
- The public key is  $(n, e)$  and the private key is  $(n, d)$

#### 2. Encryption:

To encrypt a message,  $m$ , the sender first represents the message as a number between 0 and  $n-1$ . Let's say the message is 42.

- The sender then computes the ciphertext,  $c$ , as  $c \equiv m^e \pmod{n}$  (for example,  $c = 42^{17} \% 3233 = 811$ )

#### 3. Decryption:

To decrypt the message, the recipient computes the original message,  $m$ , as  $m \equiv c^d \pmod{n}$  (for example,  $m = 811^{2753} \% 3233 = 42$ )

This example demonstrates the basic idea behind RSA encryption and decryption. In practice, much larger prime numbers and more secure methods for key generation are used.



**DS** Show that RSA encryption is inverse process of decryption



In RSA, encryption and decryption are inverse operations because they use the same modulus,  $n$ , and the encryption exponent,  $e$ , is the modular inverse of the decryption exponent,  $d$ .

Consider a message,  $m$ , that we want to encrypt and decrypt. The encryption process computes the ciphertext,  $c$ , as:

$$c \equiv m^e \pmod{n}$$

And the decryption process computes the original message,  $m'$ , as:

$$m' \equiv c^d \pmod{n}$$

Since  $e$  and  $d$  are the modular inverse of each other with respect to  $\phi(n)$ , we have:

$$ed \equiv 1 \pmod{\phi(n)}$$

Therefore, we can rewrite the decryption equation as:

$$\begin{aligned} m' &\equiv c^d \pmod{n} \\ &\equiv (m^e)^d \pmod{n} \text{ (substituting } c = m^e) \\ &\equiv m^{ed} \pmod{n} \\ &\equiv m^1 \pmod{n} \\ &\equiv m \pmod{n} \end{aligned}$$

Thus, we have  $m' = m$ , which shows that decryption is the inverse of encryption.



