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Consider a grey image of $M \times N$ over gray intensity spectrum ~~P~~ $[p, P]$ p, P both non-negative integers with $p \leq P$. Convert it into an image of same size over gray spectrum $[q, Q]$, q, Q ~~Both~~ both non-negative integers with $q \leq Q$.

Further, $Q \leq P$ and $q \geq p$

Given that $\frac{Q}{P} = \frac{q}{p}$, determines the degree of compression.

[Degree of compression of a transformed image $g(x,y)$ over the original image $f(x,y)$ in $\frac{f(x,y) - g(x,y)}{f(x,y)}$, taken over the entire image]

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For a given image $M * N$

Initial Gray Itensity Spectrum $= [p, P]$

After Transformation Gray Itensity Spectrum $= [q, Q]$

p, P, q, Q are non-negative integer and $Q \leq P$, ~~and~~ $q \geq p$

We have to determine degree of compression when,

$$\frac{Q}{P} = \frac{q}{p} \Rightarrow Qp = Pq \quad \ldots\ldots (i)$$

Let intensity x, $x \in [p, P]$ and $F(x) \in [q, Q]$ to map $[p, P] \to [q, Q]$

To shift the left end to 0, we map $x \to x - p$

$$\therefore \text{interval} = [p - p, P - p] = [0, P - p] \ldots\ldots (ii)$$

Now to shift right end to 1, we map $x \to \frac{x}{P - p}$

$$\therefore \text{interval} = \left[\frac{0}{P - p}, \frac{P - p}{P - p}\right] = [0, 1] \ldots\ldots (iii)$$

Now to shift the right end to $Q-d$, we map $x \rightarrow (Q-d)x$

$$\therefore \text{interval} = [0 \times (Q-d),\ 1 \times (Q-d)] = [0,\ Q-d] \quad \text{......(iv)}$$

Now to shift the left end to d, we map $x \rightarrow x+d$

$$\therefore \text{interval} = [0+d,\ Q-d+d] = [d,\ Q] \quad \text{......(v)}$$

by comparing (ii), (iii), (iv), (v)

$$F(x) = \left(\frac{Q-d}{P-p}\right)(x-p) + d$$

Given, Degree of compression

$$= \sum\sum \frac{f(x,y) - g(x,y)}{f(x,y)}$$

Where $f(x,y) \rightarrow$ Initial Image

$g(x,y) \rightarrow$ Transformed Image

~~∴ Transformed Image~~ $\cancel{g(x,y) = F(f(x,y))}$

$$\cancel{= \left(\frac{Q-d}{P-p}\right)(f(x,y)-p) + d}$$

$$\cancel{= Qf(x,y) - Qp - df(x,y)}$$

∴ Transformed Image

$g(x,y)$

$$= F(f(x,y))$$

$$= \left(\frac{Q-d}{P-p}\right)(f(x,y) - p) + d$$

$$= \frac{Qf(x,y) - \cancel{Qp} - df(x,y) + \cancel{pd} + \cancel{Pd} - \cancel{pd}}{P-p} \quad \text{[using (i)]}$$

$$= \frac{Qf(x,y) - df(x,y)}{P-p}$$

$$= \left(\frac{Q-d}{P-p}\right) f(x,y)$$

$\therefore$ Degree of compression given $\frac{Q}{P} = \frac{q}{p}$

$$= \sum\sum \frac{f(x,y) - g(x,y)}{f(x,y)}$$

$$= \frac{f(x,y) - \left(\frac{Q-q}{P-p}\right) f(x,y)}{f(x,y)}$$

$$= \frac{\left\{1 - \left(\frac{Q-q}{P-p}\right)\right\} \cancel{f(x,y)}}{\cancel{f(x,y)}}$$

$$= \left\{1 - \left(\frac{Q-q}{P-p}\right)\right\}$$

For an image $M * N$
Degree of compression
$$= \left\{1 - \left(\frac{Q-q}{P-p}\right)\right\} * M * N$$

Answer

$\therefore$ For a given image $M * N$ with intensity spectrum of $[p, P]$ ~~[P, p]~~, when transformed into intensity spectrum ~~[Q, q]~~ $[q, Q]$, degree of compression is $\left\{1 - \left(\frac{Q-q}{P-p}\right)\right\} * M * N$, taken over the entire image.