University of Toronto
CSC343 Dhrumil Patel pate1385 — Younjee Kang kangyou8

Question 1.

a) State which of the given FDs violate BCNF.

FD	Closure	Violation?
$L \to NO$	$L^+ = LMNOPQRS$	Does Not Violate BCNF as L is a superkey
$M \to MP$	$M^+ = MP$	Violates BCNF as M is not a superkey
$N \to MQR$	$N^+ = MNPQR$	Violates BCNF as N is not a superkey
$O \to S$	$O^+ = OS$	Violates BCNF as O is not a superkey

$M \to MP$, $N \to MQR$, and $O \to S$ violate BCNF.

b) Employ the BCNF decomposition algorithm.From above, we see that $N \to MQR$ is one of the FDs that violate BCNF, and we will choose this FD to decompose the relation R. The two relations we get from the decomposition are R1=LNOS and R2=MNPQR. We will first see if R1 satisfies BCNF, and decompose it further if needed.

Project on R1=LNOS

L	N	O	S	FDs
✓			$L^+ = LNOMQRSP$	$L \to NOS$; L is a superkey of R1
	✓		$N^+ = NMQRP$	nothing
		✓	$O^+ = OS$	$O \to S$;violates BCNF;abort the projection

We must decompose R1 further. Decompose R1 using FD $O \to S$. This yields two relations: R3=LNO and R4=OS.

Project on R3=LNO

L	N	O	Closure	FDs	
✓			$L^+ = LNOMQRSP$	$L \to NO$;L is a superkey of R3	
	✓		$N^+ = NMQRP$	nothing	
		✓	$O^+ = OS$	nothing	
	✓	✓	$NO^+ = NOMQRPS$	nothing	

Don't need to check any supersets of L as it is a superkey and will only generate weaker FDs. The relation satisfies BCNF and has FD $\{L \to NO\}$.

Project on R4=OS

O	S	Closure	FDs
✓		$O^+ = OS$	$O \to S$;O is a superkey of R4
	✓	$S^+ = S$	nothing

The relation satisfies BCNF and has FD $\{O \to S.\}$

Project on R2=MNPQR

M	N	P	Q	R	Closure	FDs
✓					$M^+ = MP$	$M \to P$; violates BCNF; aborts the projection

We can stop checking further closures here, as we already have a FD which violates BCNF. Decompose R2 using FD $M \to P$. This yields two relations R5=MNQR and R6=MP.

Project on R5=MNQR

M	N	Q	R	Closure	FDs
✓				$M^+ = MP$	nothing
	✓			$N^+ = NMQRP$	$N \to MQR$; N is a superkey of R5
		✓		$Q^+ = Q$	nothing
			✓	$R^+ = R$	nothing
✓			✓	$MR^+ = MRP$	nothing
✓		✓		$MQ^+ = MQP$	nothing
		✓	✓	$QR^+ = QR$	nothing
✓		✓	✓	$MQR^+ = MQRP$	nothing

Don't need to check any supersets of N, as N is a superkey. This relation satisfies BCNF with FD $\{N \to MQR\}$.

Project on R6 = MP

M	P	Closure	FDs
✓		$M^+ = MP$	$M \to MP$; M is a superkey of R6
	✓	$P^+ = P$	nothing

This relation satisfies BCNF, with FD $\{M \to MP\}$.

Final Relations and their fds:
R3: LNO with FD $L \to NO$
R4: OS with FD $O \to S$
R5: MNQR with FD $N \to MQR$
R6: MP with FD $M \to P$

c) The schema preserves dependencies, since the final relations and their projected FDs, include ALL of the original FDs.

d) Chase Test
The chase test demostrates that it is a loseless-join decomposition. We start with:

Relation	L	M	N	O	P	Q	R	S
MP		m			p			
MQRN		m	n			q	r	
OS				o				s
OLN	l		n	o				

Because $M \to P$, we must have p in:

Relation	L	M	N	O	P	Q	R	S
MP		m			p			
MQRN		m	n		**p**	q	r	
OS				o				s
OLN	l		n	o				

Because $N \to MQR$, we must have m, q, r in:

Relation	L	M	N	O	P	Q	R	S
MP		m			p			
MQRN		m	n		p	q	r	
OS				o				s
OLN	l	**m**	n	o		**q**	**r**	

Because $O \to S$, we must s in:

Relation	L	M	N	O	P	Q	R	S
MP		m			p			
MQRN		m	n		p	q	r	
OS				o				s
OLN	l	m	n	o		q	r	**s**

Because $M \to P$, we must p in:

Relation	L	M	N	O	P	Q	R	S
MP		m			p			
MQRN		m	n		p	q	r	
OS				o				s
OLN	l	m	n	o	**p**	q	r	s

We can stop here, since we have found a full row of the relation. We observe that the tuple $< l, m, n, o, p, q, r, s >$ does occur. The Chase Test has succeeded, and this proves that its a lossless-join decomposition.

Question 2.

a) Finding Minimal Basis

Step 1. Split RHS
Simplifying FDs to a singleton right-hand sides, we get these resulting FDs:

ACD $\to E$
$B \to C$
$B \to D$
$BE \to A$
$BE \to C$
$BE \to F$
$D \to A$
$D \to B$
$E \to A$
$E \to C$

Step 2. Reduce LHS

FD	Closures	Reduced LHS-FDs(if possible)
$ACD \to E$	$\{A^+ = A, C^+ = C, D^+DABCE, AC^+ = AC\}$	$\{D \to E\}$
$BE \to A$	$\{B^+ = BCDAEF, E^+ = EAC\}$	$\{B \to A, E \to A\}$
$BE \to C$	$\{B^+ = BCDAEF, E^+ = EAC\}$	$\{B \to C, E \to C\}$
$BE \to F$	$\{B^+ = BCDAEF, E^+ = EAC\}$	$\{B \to F\}$

Note all the FDs with only one attribute in their LHS do not need to be checked, as they can't become any smaller.

So now we have the following FDs. We will number those resulting FDs for easy reference :

1. B $\rightarrow A$
2.$B \rightarrow C$
3.$B \rightarrow D$
4.$B \rightarrow F$
5.$D \rightarrow A$
6.$D \rightarrow B$
7.$D \rightarrow E$
8.$E \rightarrow A$
9.$E \rightarrow C$

Let us call the set of FDs that we have after reducing LHS S2.

Step 3.
(1: B $\rightarrow A, 2 : B \rightarrow C, 3 : B \rightarrow D, 4 : B \rightarrow F, 5 : D \rightarrow A, 6 : D \rightarrow B, 7 : D \rightarrow E, 8 : E \rightarrow A,$)

FD	Exclude these from $S2$ when computing closure	Closure	Decision
1	1	B$^+ = BCDFAE$	discard
2	1,2	$B^+ = BDFAEC$	discard
3	1, 2, 3	$B^+ = BF$	keep
4	1, 2, 4	$B^+ = BDAEC$	keep
5	1, 2, 5	$D^+ = DBEFAC$	discard
6	1, 2, 5, 6	$D^+ = DEAC$	keep
7	1, 2, 5, 7	$D^+ = DBF$	keep
8	1, 2, 5, 8	$E^+ = EC$	keep
9	1, 2, 5, 9	$E^+ = EA$	keep

The minimal basis after discarding certain fds is:
(B $\rightarrow D, B \rightarrow F, D \rightarrow B, D \rightarrow E, E \rightarrow A, E \rightarrow C$) OR (B $\rightarrow DF, D \rightarrow BE, E \rightarrow AC$)

b) FDs : (B $\rightarrow DF, D \rightarrow BE, E \rightarrow AC$)

Attribute	Appears on LHS	Appears on RHS	Conclusion
G, H	–	–	must be in every key
	✓	–	must be in every key
A, C, F	–	✓	is not in any key
B, D, E	✓	✓	must check

B	D	E	closure	key
✓			$BGH^+ = BGHDFBEAC$	BGH
	✓		$DGH^+ = DGHBEACDF$	DGH
		✓	$EGH^+ = EGHAC$	none

We don't need to check any closures with B and D, as they will only create superkeys.
Therefore, the keys in this relation are BGH and DGH.

c) Now we will check if we can find any superkeys from the fds.
FDs : $\{B \rightarrow DF, D \rightarrow BE, E \rightarrow AC\}$

Relation	Closure	Superkey?
BDF	BDF$^+ = BDFEAC$	not a superkey
DBE	DBE$^+ = DBEFAC$	not a superkey
EAC	EAC$^+ = EAC$	not a superkey

Therefore, we need to add the key BGH, in order to have 3NF. Now we have 4 final relations:
R1: (A, C, E), R2: (B, D, E), R3: (B, D, F), R4: (B, G, H)

d) There may be other FDs that violate BCNF so allow redundancy. The only way to check is to project the FDs onto each relation and do a BCNF property check for each.

Project on R1=ACE

A	C	E	Closure	FDs
✓			$A^+ = A$	nothing
	✓		$C^+ = C$	nothing
		✓	$E^+ = EAC$	$E \rightarrow AC$; E is a superkey in R1
✓	✓		$AC^+ = AC$	nothing

Don't need to check any supersets of E, as E is a superkey. So none of FD(s) projected onto R1 violates the BCNF property.

Project on R2=BDE

B	D	E	Closure	FDs
✓			$B^+ = BDFEAC$	$B \rightarrow DE$; B is a superkey in R2
	✓		$D^+ = DBEFAC$	$D \rightarrow BE$; D is a superkey in R2
		✓	$E^+ = EAC$	nothing

Don't need to check any supersets of B or D, as each of B and D is a superkey. So none of FD(s) projected onto R2 violates the BCNF property.

Project on R3=BDF

B	D	F	Closure	FDs
✓			$B^+ = BDFEAC$	$B \to DF$; B is a superkey in R3
	✓		$D^+ = DBFEAC$	$D \to BF$; D is a superkey in R3
		✓	$F^+ = F$	nothing

Don't need to check any supersets of B or D, as each of B and D is a superkey. So none of FD(s) projected onto R3 violates the BCNF property.

Project on R4=BGH

B	G	H	Closure	FDs
✓			$B^+ = BDFEAC$	nothing
	✓		$G^+ = G$	nothing
		✓	$H^+ = H$	nothing
✓	✓		$BG^+ = BGDFEAC$	nothing
✓		✓	$BH^+ = BHDFEAC$	nothing
	✓	✓	$GH^+ = GH$	nothing

(Note there can be no FD as G and H is neither in the RHS or LHS of FDs.)So none of FD(s) projected onto R4 satisfies the BCNF property(as there's none in the first place).

We have looked at all the FDs projected for every relation, and did not find any violation. Therefore, we can conclude that this schema doesn't allow redundancy.