1 3 22.5 12 10

1.

a) Use product rule: $(fg)' = f'g + g'f$,

$f = 3x^5+2x, \quad g = 3\cos x$

$f' = 15x^4+2 \quad g' = -3\sin x$

$k'(x) = f'(x)g(x) + g'(x)f(x)$

$k'(x) = (15x^4+2)(3\cos x) + (-3\sin x)(3x^5+2x)$

$= 45x^4\cos x + 6\cos x - 9x^5\sin x - 6x\sin x$

$\therefore$ The derivative of $k(x)$ is

$k'(x) = 45x^4\cos x + 6\cos x - 9x^5\sin x - 6x\sin x$

b) Use product rule, $(fg)' = f'g + g'f$

$f = -e^{2x} \quad g = \cos 3x$

$f' = -2e^{2x} \quad g' = -3\sin 3x$

$m'(x) = (-2e^{2x})(\cos 3x) + (-e^{2x})(-3\sin 3x)$

$= -2e^{2x}\cos 3x + 3e^{2x}\sin 3x$

$\therefore$ The derivative of $m(x)$ is

$m'(x) = -2e^{2x}\cos 3x + 3e^{2x}\sin 3x$

c)

Let $u = \sqrt{x^5}$

$u = x^{\frac{5}{2}}$

$\frac{du}{dx} = \frac{5}{2}x^{\frac{3}{2}}$

h(x) is now a function of u

$h(u) = \sqrt{7+u}$

$h'(u) = \frac{d}{du}(u+7)^{\frac{1}{2}}$

$= \frac{1}{2}(u+7)^{-\frac{1}{2}}$

Apply chain rule:

$\frac{d}{dx}(h(x)) = \left(\frac{d(h(u))}{du}\right)\left(\frac{du}{dx}\right)$

$h'(x) = \frac{1}{2}(u+7)^{-\frac{1}{2}}\left(\frac{5}{2}x^{\frac{3}{2}}\right)$

Sub $u = \sqrt{x^5}$

$h'(x) = \frac{5}{4}\left(\sqrt{x^5}+7\right)^{-\frac{1}{2}}\left(x^{\frac{3}{2}}\right)$

$= \frac{5x^{3/2}}{4\sqrt{x^{\frac{5}{2}}+7}}$

d) Let $\frac{h}{g} = \frac{(2x-1)^2}{\sqrt{x-1}}$

Use quotient rule: $\left(\frac{h}{g}\right)' = \frac{h'g - g'h}{g^2}$

Use the chain rule and power rule,

Let $u = 2x-1$

$\frac{du}{dx} = 2$

$h' = 2(2x-1)^1(2)$

$= 8x-4$

$g = \sqrt{x-1}$

Use the power rule,

$g' = \frac{1}{2}(x-1)^{-\frac{1}{2}}$

$$f' = \frac{(8x-4)(\sqrt{x-1}) - \left(\frac{1}{2}(x-1)^{-\frac{1}{2}}\right)(2x-1)^2}{(\sqrt{x-1})^2}$$

$$= \frac{(8x-4)\sqrt{x-1} - \frac{(2x-1)^2}{2\sqrt{x-1}}}{x-1}$$

$$= \frac{4(2x-1)\sqrt{x-1}}{x-1} - \frac{(2x-1)^2}{2\sqrt{x-1}(x-1)}$$

$$= \frac{4(2x-1)\sqrt{x-1}\,(2\sqrt{x-1})}{(x-1)(2\sqrt{x-1})} - \frac{(2x-1)^2}{2\sqrt{x-1}(x-1)}$$

$\therefore$ The derivative of $h(x)$ is

$$h'(x) = \frac{5x^{\frac{3}{2}}}{4\sqrt{x^{\frac{5}{2}}+7}}$$

$$= \frac{(2x-1)\left(8(x-1) - (2x-1)\right)}{2(x-1)^{3/2}}$$

$$= \frac{(2x-1)(6x-7)}{2(x-1)^{\frac{3}{2}}}$$

$$= \frac{12x^2-20x+7}{2(x-1)^{\frac{3}{2}}}$$

$\therefore$ The derivative of $f(x)$ is

$$f'(x) = \frac{12x^2-20x+7}{2(x-1)^{\frac{3}{2}}}$$

2.

2.

The rate of change at time t is given by $h'(t)$.

$$h(t) = 5\sin\left(\frac{t}{2}+2\right)+6 \qquad \text{Let } v = \frac{t}{2}+2$$

$$h(v) = 5\sin(v)+6 \qquad \frac{dv}{dt} = \frac{1}{2}$$

$$\frac{d(h(t))}{dt} = \left(\frac{dh}{dv}\right)\left(\frac{dv}{dt}\right)$$

$$h'(v) = 5\cos(v)$$

$$h'(t) = 5\cos(v)\left(\frac{1}{2}\right)$$

$$= \frac{5}{2}\cos\left(\frac{t}{2}+2\right)$$

Sub $t=7$:

$$h'(t) = \frac{5}{2}\cos\left(\frac{7}{2}+2\right)$$

$$= \frac{5}{2}\cos\left(\frac{11}{2}\right)$$

$$= 1.7717 \text{ m/hour}$$

$\therefore$ The rate of change at 07:00 h is 1.7717 meters/hour

3.

We find the derivative for y.

$y = e^{-x}$

$y' = -e^{-x}$

Sub $x = -1$,

$y' = -e^{-(-1)}$

$= -e$

Find a point on y using $x = -1$,

$y = e^{-(-1)}$

$= e$

$\therefore$ a point on the line is $(-1, e)$

Sub $(-1, e)$ and $m = -e$:

$y = mx + b$

$e = (-e)(-1) + b$

$b = 0$

$\therefore y = -ex$

$\therefore$ The equation of the tangent line to $y = e^{-x}$ at $x = -1$

is $y = -ex$.

$\checkmark$

4.

We model the ticket prices with an objective function of the revenue.

$R(d) = (12-d)(11000+1000d)$, d is the number of dollars by which the ticket price is reduced, $0 \le d \le 4$

why?

$-12 \cdot 1a$

For $R'(d)$, use product rule.

$(fg)' = f'g + g'f$

$f = 12-d \qquad g = 11000+1000d$

$f' = -1 \qquad g' = 1000$

$$\therefore R'(d) = (-1)(11000+1000d) + (1000)(12-d)$$
$$= -11000 - 1000d + 12000 - 1000d$$
$$= 1000 - 2000d$$

Maximum Revenue occurs when $R'(d) = 0$.

Set $R'(d) = 0$,

$$0 = 1000 - 2000d$$

$$2000d = 1000$$
$$d = \frac{1}{2}$$

$\therefore$ The maximum revenue occurs when the ticket price is reduced by $\frac{1}{2}$ dollars, or 50 cents. The ticket price is $\$12 - \$0.50 = \$11.50$

$R(\frac{1}{2}) = (12-\frac{1}{2})(11\,000 + 1000(\frac{1}{2}))$

$= (\frac{23}{2})(11500)$

$= 132\,250$

$\therefore R(\frac{1}{2}) = \$132,250$

$\therefore$ The maximum revenue is \$132, 250.

t is not 5730 @ 79%

5.

$N = N_0 e^{-\lambda t}$

Sub $N_0 = 1$, $\lambda = 1.21 \times 10^{-4}$

$N = (1) e^{-(1.21 \times 10^{-4}) t}$

$N = e^{-(1.21 \times 10^{-4}) t}$ Let $u = -(1.21 \times 10^{-4}) t$

$\frac{du}{dt} = -(1.21 \times 10^{-4})$

$N = e^{u}$

$N' = e^{u}$

Use chain rule,

$N' = e^{u}(-1.21 \times 10^{-4})$

$= -(1.21 \times 10^{-4}) e^{-(1.21 \times 10^{-4}) t}$

-1.5 A

$\therefore$ The rate of change of an initial amount of 1 gm and the decay constant is

$N' = -(1.21 \times 10^{-4}) e^{-(1.21 \times 10^{-4}) t}$

t = 0

b) $N = N_0 e^{-\lambda t}$

$\frac{N}{N_0} = e^{-\lambda t}$

$\ln \frac{N}{N_0} = -\lambda t$

$\lambda = \frac{-\ln \frac{N}{N_0}}{t}$

$= \frac{-\ln(0.79)}{5730}$

$= 0.0000411$

$\therefore$ The decay rate is 4.11×10^{-5} gm/year.

6.

Find slope of the line:

$3x-y+6=0$

$y=3x+6$

$y'=3$

$\therefore$ The slope of the line is 3.

The point of the curve that is tangent and parallel to $3x-y+6=0$ is the point when the derivative of the curve is 3.

$y=x\sqrt{x}$

$y=x^{\frac{3}{2}}$

$y'=\frac{3}{2}x^{\frac{1}{2}}$

Sub $y'=3$,

$3=\frac{3}{2}x^{\frac{1}{2}}$

$2=x^{\frac{1}{2}}$

$x=4$

Sub $x=4$ into $y=x\sqrt{x}$:

$$y = 4\sqrt{4}$$
$$= 8.$$

$\therefore$ $(4,8)$ is the point on the curve that is tangent and parallel to $y=3x+6$.

7.

The average rate of change from $t=1$ to $t=4$ is given by

$$\Delta C_{avg} = \frac{C(4)-C(1)}{4-1}$$

$$= \frac{2\sqrt{4^3}+25 - (2\sqrt{1^3}+25)}{4-1}$$

$$= \frac{2(8)\cancel{+25} - 2 \cancel{-25}}{4-1}$$

$$= \frac{14}{3}$$

$\therefore$ The average rate of change from $t=1$ to $t=4$ is $\frac{14}{3}$.

$$C = 2\sqrt{t^3}+25$$

$$C = 2\left(t^{\frac{3}{2}}\right)+25$$

$$C' = 3\left(t^{\frac{1}{2}}\right)$$

Set $C' = \frac{14}{3}$,

$$\frac{14}{3} = 3\left(t^{\frac{1}{2}}\right)$$

$$t^{\frac{1}{2}} = \frac{14}{9}$$

$$t = \frac{196}{81}$$

$\therefore$ The time t at which the instantaneous rate of change of C is equal to the average rate of change from $t=1$ to $t=4$ is $t = \frac{196}{81}$ hours or $t \simeq 2.42$ hours.

8.

$\because$ $f(x)$ has a horizontal tangent at $(-1, 8)$, $f'(-1) = 0$.

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

$f'(-1) = 2a(-1) + b$

$0 = -2a + b$

$b = 2a$

Sub $b = 2a$,

$f(x) = ax^2 + 2ax + c$

Sub $(-1, 8)$ and $(2, 19)$,

$f(-1) = a(-1)^2 + 2a(-1) + c$

$8 = a - 2a + c$

$8 = c - a$ ①

$f(2) = a(2^2) + 2a(2) + c$

$19 = 4a + 4a + c$

$19 = 8a + c$ ②

② − ①:

$$8a + c = 19$$
$$-(-a + c = 8)$$
$$9a = 11$$
$$a = \frac{11}{9}$$

Sub $a = \frac{11}{9}$ into ②:

$$c - \left(\frac{11}{9}\right) = 8$$
$$c = 8 + \frac{11}{9}$$
$$c = \frac{83}{9}$$

Sub $a = \frac{11}{9}$ into $b = 2a$,

$$b = 2a$$
$$= 2\left(\frac{11}{9}\right)$$
$$= \frac{22}{9}$$

$\therefore f(x) = \frac{11}{9}x^2 + \frac{22}{9}x + \frac{83}{9}$ is a quadratic function that has a horizontal tangent at $(-1, 8)$ and passes through $(2, 19)$.

9.

We attempt to apply power rule in reverse.

Let $y = x^n$.

From the power rule,

$$y' = nx^{n-1}.$$

The "reverse" power rule for y' will transform y' into y.

We observe:

→ The exponent n for y is 1 more than the exponent for y';

→ The value of the exponent of y becomes a coefficient for y'. Reversing this would require dividing this coefficient out.

From our observations, we suggest that the reverse power rule for x^n is:

$$\text{RevPower}(x^n) = \frac{x^{n+1}}{n+1}, \quad n \neq -1.$$

If this is correct, its derivative should be x^n.

$$\frac{d}{dx}\left(\text{RevPower}(x^n)\right) = \frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right), n \neq -1$$

Constants come out of the derivative,

$$= \left(\frac{1}{n+1}\right) \frac{d}{dx}\left(x^{n+1}\right)$$

Using the power rule,

$$= \left(\frac{1}{n+1}\right)(n+1)x^{n+1-1}$$

$$= x^n$$

$\therefore$ The derivative of $\frac{x^{n+1}}{n+1}$, $n \neq -1$, is x^n, we can transform y' into y using this "reverse" power rule.

$$\text{RevPower}(y) = \text{RevPower}(n x^{n-1})$$

$$= \frac{n x^{n-1+1}}{n-1+1}$$

$$= \frac{\cancel{n} x^n}{\cancel{n}}$$

$$= x^n$$

$\therefore$ The reverse power rule for x^n is $\frac{x^{n+1}}{n+1}$. However, differentiating a constant will yield a value of 0, which would be lost in the derivative but kept in the original function. Thus, we add a C to represent an arbitrary constant in the reversed function.

$\therefore \text{RevPower}(x^n) = \dfrac{x^{n+1}}{n+1} + C, n \neq -1.$

Applying this rule to $f'(x) = 12x^2 + 4x - 10$, we have

$$\text{RevPower}(f'(x)) = \text{RevPower}(12x^2 + 4x - 10)$$

$$f(x) = \text{RevPower}(12x^2) + \text{RevPower}(4x) + \text{RevPower}(-10)$$

$\because$ Addition doesn't affect the reversed power of individual parts.

$$= \frac{12x^{2+1}}{2+1} + C + \frac{4x^{1+1}}{1+1} + C + \frac{(-10)x^{0+1}}{0+1} + C$$

$$= \frac{12x^3}{3} + \frac{4x^2}{2} + \frac{-10x}{1} + C$$

← We represent all constants as C

$$= 4x^3 + 2x^2 - 10x + C.$$

For a specific function, let $C = 0$.

$$\therefore f(x) = 4x^3 + 2x^2 - 10x + 0$$

$\therefore$ A function with a derivative of $f'(x) = 12x^2 + 4x - 10$ could be

$$f(x) = 4x^3 + 2x^2 - 10x.$$