MDM 4U1 Name: Dhrumil Patel

## Unit #2 Test – Solving Problems with Counting Principles

K/U: ____ / 8 Application: ____ / 22 Communication: ____ / 9 TOTAL: ____ / 39

### Knowledge/Understanding: (8 marks)

**Multiple Choice: Circle the most appropriate answer from the following choices.** *<8 marks>*

1. $\frac{5!}{0!} =$
   a) 5
   b) 120
   c) 0
   d) Undefined

2. $8! - 5! =$
   a) 3!
   b) ~~6720~~
   c) 40 200
   d) 40 440

3. $\frac{_7P_3}{_7C_3} =$
   a) 6
   b) 1
   c) 1/6
   d) None of the above

4. $_nC_r =$
   a) $C(n,r)$
   b) $\binom{n}{r}$
   c) $\frac{_nP_r}{r!}$
   d) All of the above

5. How many ways can you arrange the letters of the word BIRTHDAY?
   a) 1
   b) 8
   c) 40 320
   d) 16 777 216

6. If at least one item is chosen, the total number of selections that can be made from *p* items of one type, *q* items of another, *r* items of another, and so on is:

   a) $\frac{n!}{(n-r)!}$ b) $(p+1)(q+1)(r+1)\ldots-1$ c) $\frac{n!}{(n-r)!r!}$ d) $\frac{n!}{p!q!r!}$

7. In general, if order matters, the number of arrangements of *n* items when *a* are alike, *b* are alike, *c* are alike and so on is:

   a) $\frac{n!}{(n-r)!r!}$ b) $\frac{n!}{a!b!c!}$ c) $(a+1)(b+1)(c+1)\ldots-1$ d) $\frac{n!}{(n-r)!}$

8. The number of permutations of 4 people chosen from a group of 15 is:
   a) Equal to the number of combinations of 4 people from a group of 15.
   b) Less than the number of combinations of 4 people from a group of 15.
   c) Greater than the number of combinations of 4 people from a group of 15.
   d) None of the above

$15P4 = 32760$

$15C4 = 1365$

MDM 4U1 Name: Dhrumil Patel

**Application: (22 marks)**

1. A varsity soccer team has ~~13~~ 16 senior and 8 junior players. The coach is allowed to carry only 15 players to their next tournament. How many ways can this group be selected: *<7 marks>*

a) With no restrictions *<1mark>*

$\binom{21}{15} = 54264$

$\therefore$ There are 54264 ways of selecting this group.

b) With exactly 10 seniors *<2marks>*

$\binom{13}{10}\binom{8}{5} = 16016$

$\therefore$ There are 16016 ways of selecting exactly 10 seniors.

c) Find the probability of at least 2 juniors attending the tournament *<4 marks>*

$$P(\geq 2 \text{ juniors}) = \frac{n(\text{all ways}) - n(\text{no juniors}) - n(1 \text{ junior})}{n(\text{all ways})}$$

(Using the indirect method)

$$= 1 - P(\text{no juniors}) - P(1 \text{ junior})$$

$$= 1 - \frac{n(\text{no juniors})}{n(\text{all ways})} - \frac{n(1 \text{ junior})}{n(\text{all ways})}$$

$$= 1 - \frac{16}{1{,}307{,}504} - \frac{960}{1{,}307{,}504}$$

$$= 0.99$$

$n(\text{all ways}) = \binom{24}{15}$

$= 1{,}307{,}504$

$n(\text{no juniors}) = \binom{16}{15}$

$= 16$

$n(1 \text{ junior})$

$= \binom{8}{1}\binom{16}{14}$

$= 960$

$\therefore$ There is a 0.99 probability of at least 2 juniors attending the tournament.

2. How many arrangements are there of the word MEMORIZE if the arrangement must begin with Z and end with M? *<2 marks>*

$Z\underbrace{\_ \_ \_ \_ \_ \_}_{\text{6 letters}}M$

↳ 2 E's

$$n(\text{arrangements}) = \frac{n!}{a!}$$

$$= \frac{6!}{2!}$$

$$= 360$$

$\therefore$ There are 360 arrangements.

3. An amusement park has a variety of rides. There are 4 roller coasters, 2 water rides and 1 Ferris Wheel. If Amar wants to try at least one ride, how many different combinations of rides could he choose? *<2 marks>*

$$n(\text{combinations}) = (p+1)(q+1) + \ldots + (k+1) - 1$$

$$= (4+1)(2+1)(1+1) - 1$$

$$= 5(3)(2) - 1$$

$$= 29$$

$\therefore$ There are 29 combinations of rides he could choose if he rides at least one.

4. How many different 7-card hands dealt from a standard deck of 52 would have at least 2 face cards? *<3 marks>*

Let $n(k\ f.c)$ be the number of ways to make a 7-card hand with k face cards.

$n(\geq 2\ f.c) = n(7\text{-card}) - n(0\ f.c) - n(1\ f.c)$

$n(7\text{-card}) = \binom{52}{7}$
$= 133{,}784{,}560$

$n(0\ f.c) = \binom{12}{0}\binom{40}{7}$
$= (1)(18\,643\,560)$
$= 18{,}643{,}560$

$n(1\ f.c) = \binom{12}{1}\binom{40}{6}$
$= (12)(3\,838\,380)$
$= 46{,}060{,}560$

$\therefore n(\geq 2\ f.c) = 133{,}784{,}560 - 18{,}643{,}560 - 46{,}060{,}060$
$= 69{,}080{,}440$

$\therefore$ There are 69,080,440 different 7-card hands with these conditions.

5. A bag contains five red, three green, and four yellow marbles. Three are drawn, one at a time and are not replaced. What is the probability that the order they are drawn is: red, green, green? *<4 marks>*

$$P(r,g,g) = \frac{n(\text{ways to get } r,g,g)}{n(\text{total draws})}$$

$n(\text{total draws}) = {}_{12}P_3$
$= 1320$

$n(\text{ways to get } r,g,g) = 5 \times 3 \times 2$
$= 30$

$P(r,g,g) = \frac{30}{1320}$
$= \frac{1}{44}$
$= 0.023$

$\therefore$ There is a $\frac{1}{44}$ or 0.023 probability that the order drawn is red, green, green

6. There are seven speakers scheduled for a seminar on careers. How many different orders of speaking are possible if <4 marks>

a) there are no special conditions?

$n(\text{orders}) = {}_7P_7 = \frac{7!}{0!} = 7! = 5040$

$\therefore$ There are 5040 different orders of speaking.

b) the marine biologist must speak first so that she can get back to her lab?

$B\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{6}$

$n(\text{orders}) = {}_6P_6 = \frac{6!}{0!} = 6! = 720$

$\therefore$ There are 720 different orders of speaking if the marine biologist must speak first

c) the electrical engineer and machinist are to speak one after the other?

Let k be the electrical engineer and machinist.

$\underbrace{k\ \_\ \_\ \_\ \_\ \_}_{6}$

2! accounts for switching the order of electrical engineer and machinist.

$n(\text{orders}) = {}_6P_6 \times 2!$
$= 6! \times 2! = 1440$. $\therefore$ There are 1440 orders of speaking under these conditions.

**Communication: (9 marks)**

| Mathematical Form | Level R | Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|---|---|
| **Proper Use of formulas and notation.** | Never uses formulas and/or notation properly | Rarely uses formulas and/or notation properly | Sometimes uses formulas and/or notation properly | Often uses the formulas and/or notation properly | Always uses formulas and/or notation properly |

1C – Completing the test in the allotted time

1. A committee of 5 people is randomly chosen from a group of 17 people. Should you use combinations or permutations? Explain. *<2 marks>*

Combinations. Since there is no explicit order in the committee members, the order of the group chosen from the 17 people does not matter. Thus, we only consider the distinct combinations of 5 people from the 17. The number of possible committees is $n(\text{committees}) = \binom{17}{5} = 6188$ committees

2. When should a person use the indirect method to count the amount of a given situation? Provide an example. *<2 marks>*

Suppose: Count the number of possibilities of landing at least one tails on 1000 coin flips.

Direct counting here would require counting the possibilities for one tail, adding that to the number of possibilities for 2 tails, 3 tails, ..., to 1000 tails. This is a long and inefficient process. Indirect counting makes calculations like these much easier and works well for problems like this one.

Indirect counting would just be $n(\text{total possibilities}) - n(0 \text{ tails})$. $n(\text{total possibilities})$ is $2^{1000}$ and there is only 1 way to land no tails. Therefore, $n(\geq 1 \text{ tail}) = n(\text{total}) - n(0 \text{ tails}) = 2^{1000} - 1$.

3. You are taking a chemistry test and are asked to list the first 10 elements of the periodic table in order from 1st to 10th as they appear in the table. You know the first 10 elements but not the order. Explain why the probability of guessing the correct answer is $\frac{1}{3628800}$. *<2 marks>*

There is only 1 correct order of the first 10 elements ($n(\text{correct order}) = 1$).

However there are $_{10}P_{10} = 10! = 3{,}628{,}800$ possible orders of the first 10 elements.

$n(\text{possible orders}) = {}_{10}P_{10} = \frac{10!}{0!} = 10! = 3{,}628{,}800$. Therefore, the probability of guessing the correct order is $P(\text{correct order}) = \frac{n(\text{correct order})}{n(\text{possible orders})} = \frac{1}{3{,}628{,}800}$.