Knowledge: _____/5	Application: _____/9

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Student Exploration: Estimating Population Size

Directions: Follow the instructions to go through the simulation. Respond to the questions and prompts in the orange boxes.

Vocabulary: percent error, population, sample

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. A jar contains 100 marbles total. Some are black. If you pull 4 marbles out and 1 is black, how many out of the 100 marbles would you guess are black? Explain.

   25 marbles; if 1 out of the 4 marbles of the sample is black, an estimate for the ratio of black to non-black marbles in the jar is 0.25. The estimated number of black marbles is 0.25 * 100 = 25 marbles.

2. A different jar has 100 black marbles. (The total number of marbles is unknown.) If you pull 4 marbles out and 1 is black, what is your guess for the total number of marbles? Explain.

   The ratio of black to total marbles in the sample is 1:4, so 100 black marbles in the jar would imply 400 marbles in total.

Gizmo Warm-up

In the *Estimating Population Size* Gizmo, you will estimate the total number of fish in a pond. You will do this by tagging and releasing a certain number of fish, and then "recapturing" some.

To change the number of fish to tag or catch, drag the slider, or click on the number in the text field next to a slider, type a new value, and hit **Enter**.

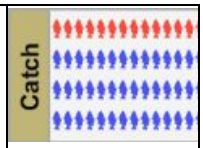
1. Suppose a scientist tags 100 fish, and releases them. (Set the **Tagged fish in pond** to 100 to show this.) Later, a fisherman catches 50 fish from the same pond. (Set **Fish to catch** to 50 to show this.) The fisherman's catch is a **sample** of all fish in the pond.

A. Click **Catch and check**. Look at the **Results** table. How many tagged fish did the fisherman catch?	24 fish
B. What percent of the fish in his sample were tagged?	48%

2. At the bottom of the Gizmo, select **Show total fish in pond**. How many fish are in this pond?

218 fish

This number is the **population** of fish in this pond.

Activity A: **Capture/recapture method**	Get the Gizmo ready: • Be sure **Show total fish in pond** is selected on the **POND** tab.	

The “capture/recapture” method lets you estimate the size of a population. To apply this to a fish pond, first catch and tag some fish. (This is the “capture” part.) Then release the tagged fish into the pond and wait. Later, catch a second group of fish. (This is the “recapture” part.) Count the tagged fish in the second group and use a proportion to estimate the pond’s fish population.

1. Set the **Tagged fish in pond** to 50. Click **New pond**.

   A. How many fish are in this pond?

   336 fish

   B. If you catch one fish out of this pond, what is the probability that your fish is tagged?

   Fraction: 50 / 336

   Decimal (to nearest hundredth): 0.15

   C. Suppose you catch 100 fish out of this pond. About how many of these fish would you expect to be tagged? Explain.

   Since N = Mn/m, which can be rearranged to m = Mn/N, I would expect m = (100)(50)/336 = 15 fish to be tagged since M = 100 fish, n = 50 fish, N = 336 fish.

   D. Set the **Fish to catch** to 100. Click **Catch and check**. How does the number of tagged fish shown in the Gizmo table compare to your estimate above?

   The simulation shows that 19 fish were tagged in the sample. This value is higher than the prediction of 15 fish by 19 fish - 15 fish = 4 fish. This may be due to randomness, which would be averaged out after averaging multiple data points.

   E. If you don’t know how many fish are in the pond, you can use the results from the catch to write a proportion to estimate the fish population. Fill in the blanks below with words to write this proportion.

# of tagged fish in sample	=	# of tagged fish in pond (population)
# of fish in sample		# of fish in pond (population)

F. Now, in the space to the right, use the catch above to write the proportion to estimate the fish population with numbers. Then, solve the proportion. Select the **CALCULATE** tab to check your work.

N = Mn/m, N = (100 fish)(50 fish)/(19 fish) = 263 fish, since M = 100 fish, n = 50 fish, m = 19 fish

G. How close is your estimate to the actual fish population?

Not very close. I calculated N = 263 fish, but N = 336 fish.

2. On the **POND** tab, turn off **Show total fish in pond**. Click **New pond**. Set the **Tagged fish in pond** and the **Fish to catch** to the same number of your choice under 75.

A. How many fish were tagged and released? 50 fish

B. Now click **Catch and check**. Fill in the blanks below to describe the number of tagged fish (and total fish) in your sample.

21 out of 50 fish in the sample are tagged.

C. In the space to the right, write a proportion that you can use to estimate the fish population. Solve the proportion. Check your answer on the **CALCULATE** tab.

N = Mn/m
N = (50)(50)/21
N = 119 fish

(Since M = 50 fish, n = 50 fish, m = 21 fish)

D. Turn on **Show total fish in pond**. What is the actual fish population?

167 fish

E. How far off was your estimated population from the actual value?

Far off by 167 - 119 = 48 fish

F. Do you think your estimate would be more accurate if you increased the number of fish to catch? **[A: 2]**

Explain. Yes. A larger sample size includes more data, making the prediction more accurate. The calculated proportion is closer to the true average with more data.

Use the Gizmo to check your prediction.

3. Suppose you use the "capture/recapture" method to come up with an estimate of 200 fish living in a pond. Assume that this result is accurate to within 20% of the estimate.

   A. If *p* is the actual number of fish, write an inequality to describe the minimum number of fish in the pond. **[A: 1]**

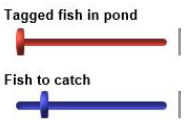
   p ≥ 160 fish

   B. Write an inequality to describe the maximum number of fish in the pond. **[A: 1]**

   p ≤ 240 fish

   C. Combine your answers above into a compound inequality. **[A: 1]**

   160 fish ≤ p ≤ 240 fish

Activity B: **Sample size**	Get the Gizmo ready: • Select the **POND** tab. • Turn off **Show total fish in pond**.	

The "capture/recapture" method lets you estimate the size of a population. To apply this to a fish pond, first catch and tag some fish. (This is the "capture" part.) Then release the tagged fish into the pond and wait. Later, catch a second group of fish. (This is the "recapture" part.) Count the tagged fish in the second group and use a proportion to estimate the pond's fish population.

1. Set **Tagged fish in pond** to 75 and **Fish to catch** to 10. Click **New pond**.

   A. Click **Catch and check.** In the space to the right, write and solve a proportion to estimate the total fish in the pond. Select the **CALCULATE** tab to check.

   N = Mn/m
   N = (75)(10)/(6)
   N = 125 fish

   (Since M = 75 fish, n = 10 fish, m = 6 fish)

   B. Turn on **Show total fish in pond**. What is the actual fish population?

   173 fish

   C. In the space to the right, find the difference between your estimate and the actual fish population. Then express this as a percent of the actual population.

   %diff = |(estimate - actual)/actual| * 100%
   %diff = |(125 - 173)/173| * 100%
   %diff = 27%

   This is known as the **percent error** and is given by the formula below:

$$\text{percent error} = \frac{\text{estimated value} - \text{actual value}}{\text{actual value}} \cdot 100$$

Note: If your estimate is less than the actual value, then the percent error is negative.

D. Fill in the first row of the table for the catch above. Then, change the **Number of fish to catch** to the numbers shown in the first column and fill in the rest of the table.

Number of fish caught	Number of tagged fish in catch(fish)	Estimate of fish population(fish)	Actual fish population(fish)	Percent error
10	3	250	173	30.80%
30	18	125	173	27.75%
80	24	250	173	30.80%

2. On the **POND** tab, click **New pond**. Be sure **Tagged fish in pond** is still set to 75.

A. Fill in the following table for the new pond.

Number of fish caught	Number of tagged fish in catch (fish)	Estimate of fish population(fish)	Actual fish population(fish)	Percent error
10	5	150	273	45.05%
30	14	160.71	273	41.13%
80	20	300	273	9.89%

B. On the **POND** tab, click **New pond** again. Fill in the following table for this pond.

Number of fish caught	Number of tagged fish in catch(fish)	Estimate of fish population(fish)	Actual fish population(fish)	Percent error
10	3	250	263	4.94%
30	11	204.55	263	22.23%
80	23	260.87	263	0.81%

C. Compare the last three tables you filled in. What tends to happen to the percent error as the number of fish caught (the sample size) increases? **[A: 1]**

> The percent error tends to decrease as sample size increases.

D. Why do you think larger sample sizes usually lead to better estimates? **[A: 1]**

> Larger sample sizes capture more of the population data, leading to more accurate sample statistics of population parameters. In this case, the estimated fish population is closer to true value of fish with a larger sample size.

3. On the **POND** tab, click **New pond**. Be sure the **Tagged fish in pond** is still 75.

A. Select the **CALCULATE** tab. For each of the following sample sizes, click **Catch and check** 5 times. Record all 5 estimates below, and then fill in the rest of the table.

Number of fish caught	Five estimates of fish populations(fish)					Mean of estimates(fish)	Actual fish populations(fish)	Percent error
10	5	7	6	7	7	6.4	118	0.69%
30	13	19	22	19	22	19	118	0.36%
80	42	53	50	58	50	50.6	118	0.49%

B. Compare the percent errors for this 5-sample method to the percent errors for the 1-sample method above. Does the 5-sample method seem more reliable? Explain. **[A: 2]**

The 5-sample method has very low percent errors in comparison to the percent errors for 1-sample methods. (5-sample method percent errors: 0.69%, 0.36%, 0.49% vs. 1-sample method percent errors: 4.94%, 22.23%, 0.81%). The 5-sample method seems more reliable since using multiple samples averages out randomness/noise that may exist in a single sample.