1. The question's ambiguity in the interest rate of bank a) requires case work.

Compounded:

Should know which instead of giving both

Monthly:

$$P_1 = P_0(1+i)^n$$
$$= P_0(1+0.0317)^{2(12)}$$
$$\simeq P_0(2.115)$$

If the rate provided is the yearly rate:

$$P_{1_2} = P_0(1+i)^n$$
$$= P_0\left(1+\frac{0.0317}{12}\right)^{2(12)}$$
$$= P_0(1.06536)$$

Yearly:

$$P_2 = P_0(1+i)^n$$
$$= P_0(1+0.032)^2$$
$$P_2 = P_0(1.065024)$$

Continously:

$$P_3 = P_0 e^{rt}$$
$$= P_0 e^{(0.0316)(2)}$$
$$\simeq P_0(1.065240)$$

You should invest in the bank with the interest rate that results in the highest value for your investment after 2 years.

We compare this value by determining the componded interest by which your initial investment will increase. The bank with the largest factor will be the best investment.

Bank a) has a factor of about 2.115, which is larger than the other two banks' factors. With bank a), you would make $2.115P_0 - 1.065024P_0 \simeq 1.05P_0$ more than an investment at bank b) and $2.115P_0 - 1.065240P_0 \simeq 1.05P_0$ more than an investment at bank c.

If bank a)'s provided rate is yearly, you would make $1.06536P_0$, which is $1.06536P_0 - 1.065024P_0 \simeq 0.000336P_0$ more than an investment at bank b) and $1.06536P_0 - 1.065240P_0 \simeq 0.00012P_0$ more than an investment at bank c. You should invest in bank a).

2

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{4x+4h} - e^{4x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{4x}(e^{4h} - 1)}{h}$$

$$= \left(\lim_{h \to 0} e^{4x}\right)\left(\lim_{h \to 0} \frac{e^{4h}-1}{h}\right)\left(\frac{4}{4}\right)$$

$$= \left(\lim_{h \to 0} e^{4x}\right)\left(4 \lim_{h \to 0} \frac{e^{4h}-1}{4h}\right), \text{ Let } v = 4h, \lim_{h \to 0} \text{ becomes } \lim_{v \to 0}$$

$$= \left(\lim_{h \to 0} e^{4x}\right)\left(4 \lim_{v \to 0} \frac{e^{v}-1}{v}\right), \text{ Use } \lim_{v \to 0} \frac{e^{v}-1}{v} = \ln e$$

$$= \left(\lim_{h \to 0} e^{4x}\right)(4 \ln e)$$

$$= (e^{4x})(4)$$

$\therefore f'(x) = 4e^{4x}$

$\therefore$ The derivative of $f(x) = e^{4x}$ is $f'(x) = 4e^{4x}$.

3\.

a)

$P = P_0 e^{kt}$

$\frac{P}{P_0} = e^{kt}$

$\ln \frac{P}{P_0} = kt$

$k = \left(\frac{1}{t}\right)\left(\ln \frac{P}{P_0}\right)$

Sub $t = 38$, $\frac{P}{P_0} = 3$,

$k = \frac{1}{38} \ln(3)$

$k \simeq 0.0289$

$\therefore$ We define an expression as

$P(t) = P_0 e^{0.0289t}$

Sub $P_0 = 2000$

$\therefore P(t) = 2000 e^{0.0289t}$

$\therefore$ The amount of bacteria after t minutes is given by $P(t) = 2000e^{0.0289t}$.

b)

$t = 3.5$ hours

$t = 210$ minutes.

Sub $t = 210$ into expression from a)

$$P(210) = 2000e^{0.0289(210)}$$

$$\approx 866{,}468 \text{ bacteria} \quad \left(\text{Using } k = \frac{\ln(3)}{38}\right)$$

$\therefore$ There will be about 866,468 bacteria after 3.5 hours.

c) Sub $P(t) = 10800$ bacteria

$$10800 = 2000e^{0.028t} \quad \left(\text{Using } k = \frac{\ln 3}{38}\right)$$

$$\frac{108\cancel{00}}{20\cancel{00}} = e^{0.028t}$$

$$\frac{27}{5} = e^{0.028t}$$

$\therefore$ After about 58.33 minutes (or 58 min and 20 seconds), there will be 10,800 bacteria.

d)

$P'(t) = k(P(t))$

$= 0.028\left(2000e^{0.028t}\right)$ (Using $k = \frac{\ln 3}{38}$)

$= 57.82e^{0.028t}$

$t = 4$ hours $= 240$ minutes.

Sub $t = 240$ min:

$P'(240) = 57.82e^{0.028(240)}$

$= 59,633.22$ bacteria/min

$\therefore$ The instantaneous growth rate at $t = 4$ hours is about 59,633.22 bacteria/minute.

4.

a)

From the graph, we track the instantaneous slope at select points.
The function completes 2.5 cycles at π and thus 5 cycles at 2π.
We estimate the period of the function as $\frac{\pi}{5}$.

x	$f'(x)$
0	-10
$\frac{\pi}{10}$	0
$\frac{\pi}{5}$	10
$\frac{3\pi}{10}$	0
$\frac{2\pi}{5}$	-10
$\frac{\pi}{2}$	0
$\frac{3\pi}{5}$	10
$\frac{7\pi}{10}$	0
$\frac{4\pi}{5}$	-10

From the points and the amplitude of 2, we estimate $f(x) = -2\sin(5x)$.

$\therefore$ The derivative of $f(x)$ is $f'(x) = -10\cos(5x)$.

?

2 / 5

$\frac{9\pi}{10}$	0
π	10

b)

From the graph, we track the instantaneous slope at select points.
The function completes 1 cycle at π and thus 2 cycles at 2π.
We estimate the period of the function as π.

x	$f'(x)$
0	0
$\frac{\pi}{4}$	-1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	1
π	0
$\frac{5\pi}{4}$	-1
$\frac{3\pi}{\ }$	0

From the points and the amplitude of 1, we estimate $f(x) = \cos(x)$

$\therefore$ the derivative of $f(x)$ is $f'(x) = -\sin(x)$

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