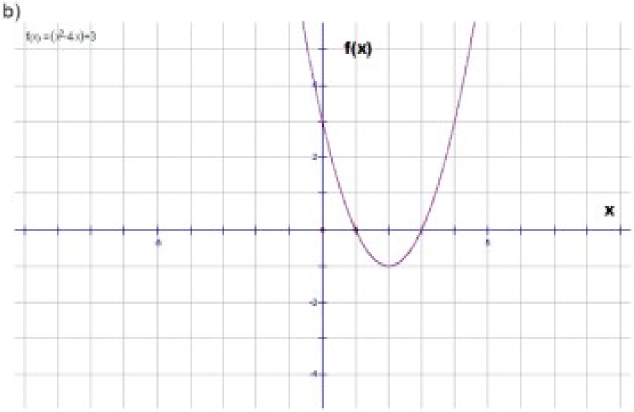
Unit 6 Assignment am Answer Key

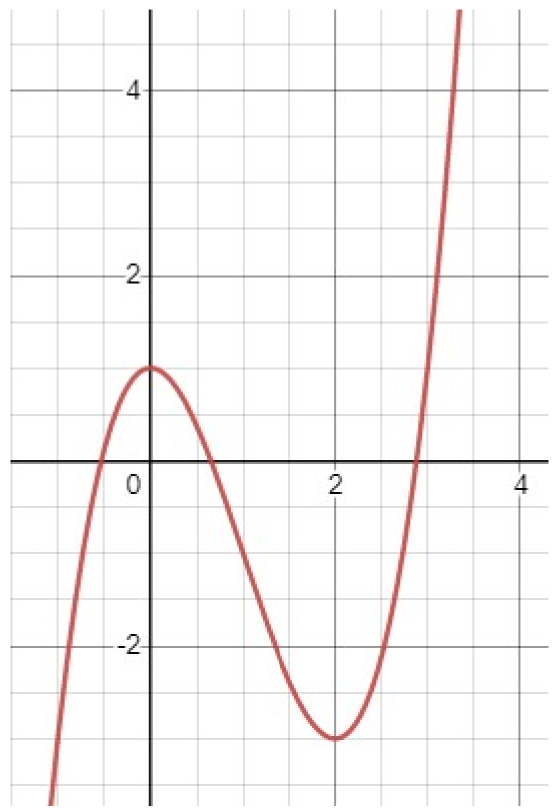
1. a) f(x) is decreasing when x < 2
   f(x) is increasing when x > 2.
   There is a local minimum at x=2.
   The y -intercept is 3 and the x intercepts are 1 and 3

b)



2. a) i) f(x) is increasing when x < 0 and x > 2 (since this is where f'(x) is positive)
   ii) f(x) is decreasing on 0 < x < 2 (since this is where f'(x) is negative)
   iii) f(x) has local extrema at x = 0 and x = 2 - there is a local maximum at x = 0 and a local minimum at x = 2
   iv) f(x) has a point of inflection at x = 1
   v) f(x) is concave down when x > 1 and concave up when x < 1

   b) $f(x) = ax^3+bx^2+cx+d$
   $f'(x) = 3ax^2 + 2b^x + c$
   f(0) = 1, so d =1
   f'(0) = 0 so c = 0
   f'(2) = 0 so 12a + 4b = 0 → 3a + b = 0 ...(1)
   f'(1) = -3 so 3a + 2b = -3 ... (2)
   (2) – (1) : b = -3
   Into (1) : a = 3/3 = 1
   So $f(x) = x^3 -3x^2 +1$
   So local max (0, 1), local min (2, -3), poi (1, -1)



3. a) $f(x) = x^3 - 3x^2$, $f'(x) = 3x^2-6x$, $f''(x) = 6x - 6$

Critical points: $f'(x) = 0$ so $x=0$ (point (0,0)) and $3x-6 = 0$ ➔ $x = 2$. (point (2,-4)

Factors of f' \ Intervals	X<0	0<x<2	x>2
x	-	+	+
x-2	-	-	+
f'(x) (f(x) inc/dec)	+ (inc)	- (dec)	+ (inc)

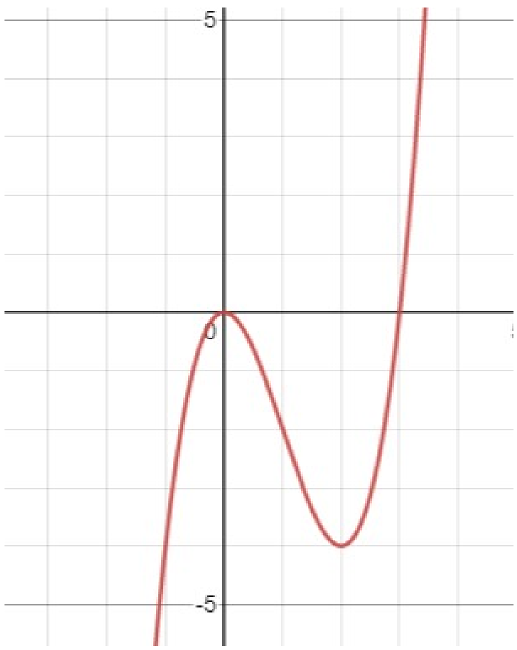
Check $f''(0) = -6$ so maximum. $f''(2) = 6$ so minimum

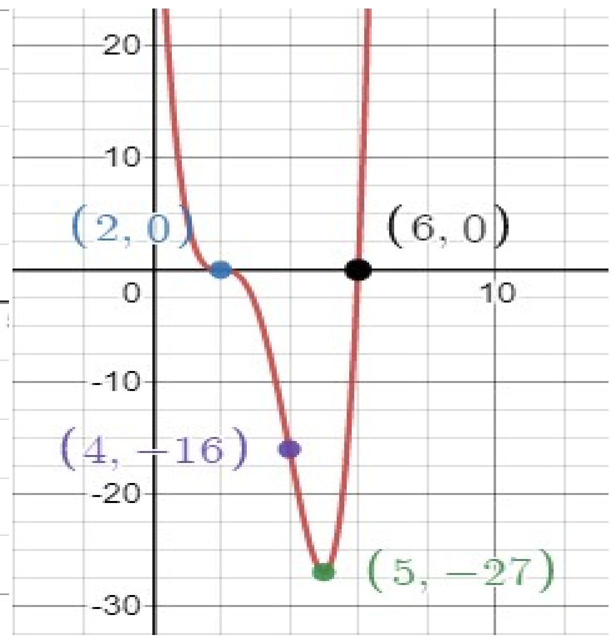
Poi: $6x-6 = 0$ ➔ $x = 1$ so (1, -2)

y – intercept is 0 ; x intercepts at 0 and 3

$x<1$, $f''(x) > 0$, concave up

$x>1$ $f''(x)<0$, concave down





b) $g(x) = (x-2)^3(x-6)$

$g'(x) = 3(x-2)^2(x-6)+(x-2)^3$

$= (x-2)^2(4x-20)=4(x-2)^2(x-5)$

$g''(x) = 4[2(x-2)(x-5)+(x-2)^2]$

$= 4(x-2)(3x-12)=12(x-2)(x-4)$

Factors of g' \ Intervals	x<2	2<x<5	x>5
$(x-2)^2$	+	+	+
x-5	-	-	+
g'(x) (g(x) inc/dec)	- (dec)	- (dec)	+ (inc)

Critical points: $g'(x) = 0$ ➔ $x=2$ (2,0), and $x=5$ (5,-27)

Check $g''(2) = 0$ so poi, $g''(5) = 36$ so minimum

Poi: $g''(x) = 0$ → $x=2$ (2,0) and $x=4$ (4,-16)

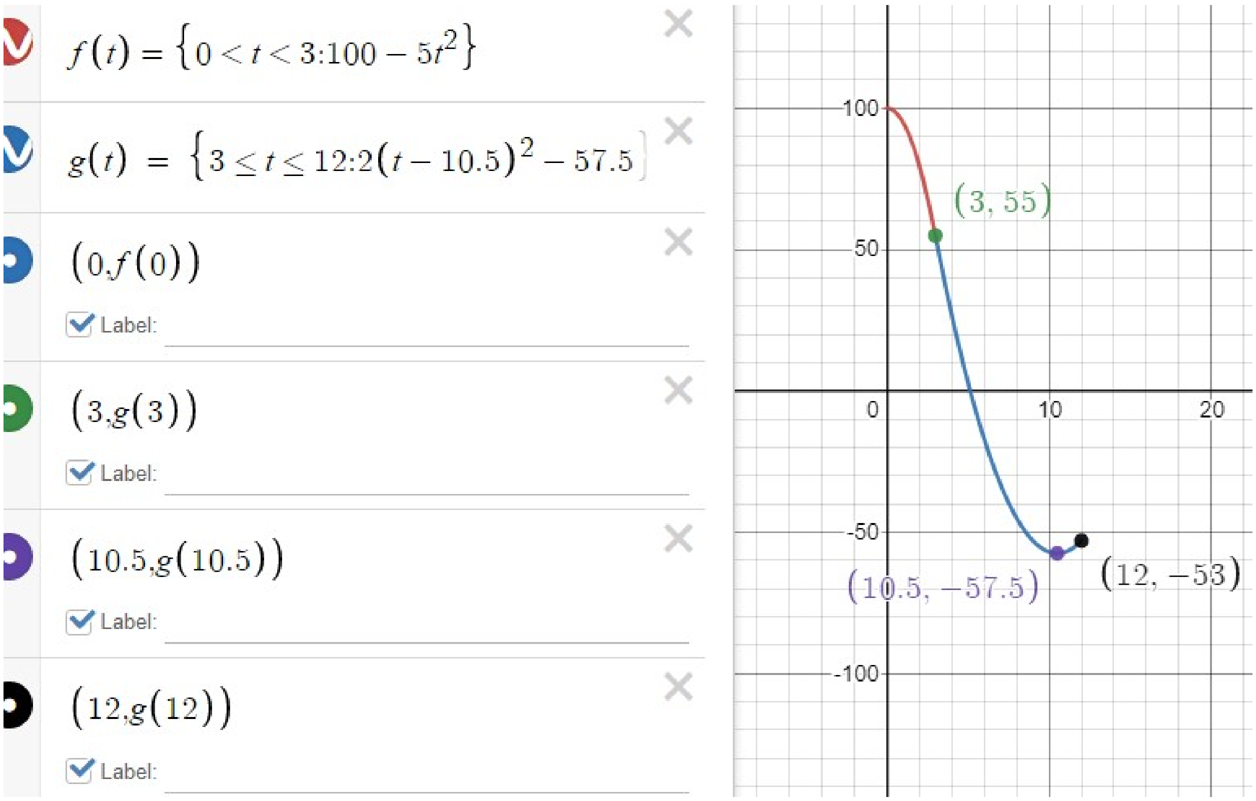
$x<2$, $g'(x) < 0$ so decreasing, $2<x<5$ decreasing, $x>5$ increasing

y-intercept is 48; x intercepts at 2 and 6

$x<2$, $g''(x) > 0$ so concave up, $2<x<4$ concave down, $x>4$ concave up

Factors of g'' \ Intervals	x<2	2<x<4	x>4
x-2	-	+	+
x-4	-	-	+
g''(x) (g(x) up/dn)	+ (up)	- (dn)	+ (up)

4. Note: the jumper is falling so a “positive” increase in velocity will cause a decrease in the height of the jumper
0s – 3s, slope = (30-0)/3 = 10 so v(t) = 10t and change in height will be $5t^2$ (from 0 to 45, so height goes from 100 to 55)
3s – 10.5s, slope = (0-30)/7.5 = -4 so v(t) = -4t and change in height will be $-2t^2$ (from “-7.5 to 0” as its on the positive slope side so goes from -112.5 to 0 so height goes from 55 to -57.5)
10.5s-12s , slope = -4 so v(t) -4t, and change in height will be -2t (from “0 to 1.5” as its on the negative slope side so goes from so goes from 0 to -4.5 so height goes from -57.5 to -53)
Vertex of parabola (10.5,-57.5)



5. (2,-4) ➔ 8+4b+d = -4 … (1)
Critical point → h’(2) = 0 or 12 + 4b =0 ➔ b=-3
Into (1), d = -12+ 12 =0
So h(x) = x^3 $-3x^2$