1. a)

x	$x<2$	$x=2$	$x>2$
$f'(x)$	$-$	0	$+$
$f(x)$	decreasing $-$	local min	increasing $+$

$\because f'(x) < 0$ on $x \in (-\infty, 2)$,

$f(x)$ is decreasing on $x \in (-\infty, 2)$

$\because f'(x) = 0$ at $x=2$, $f(x)$ has a

local max, local min, or

inflection point at $x=2$.

$\because f(x)$ is decreasing on $x \in (-\infty, 2)$

but increasing on $x \in (2, \infty)$,

$f(x)$ has a local min at $x=2$.

$\because f(1)=0$ and $f(3)=0$,

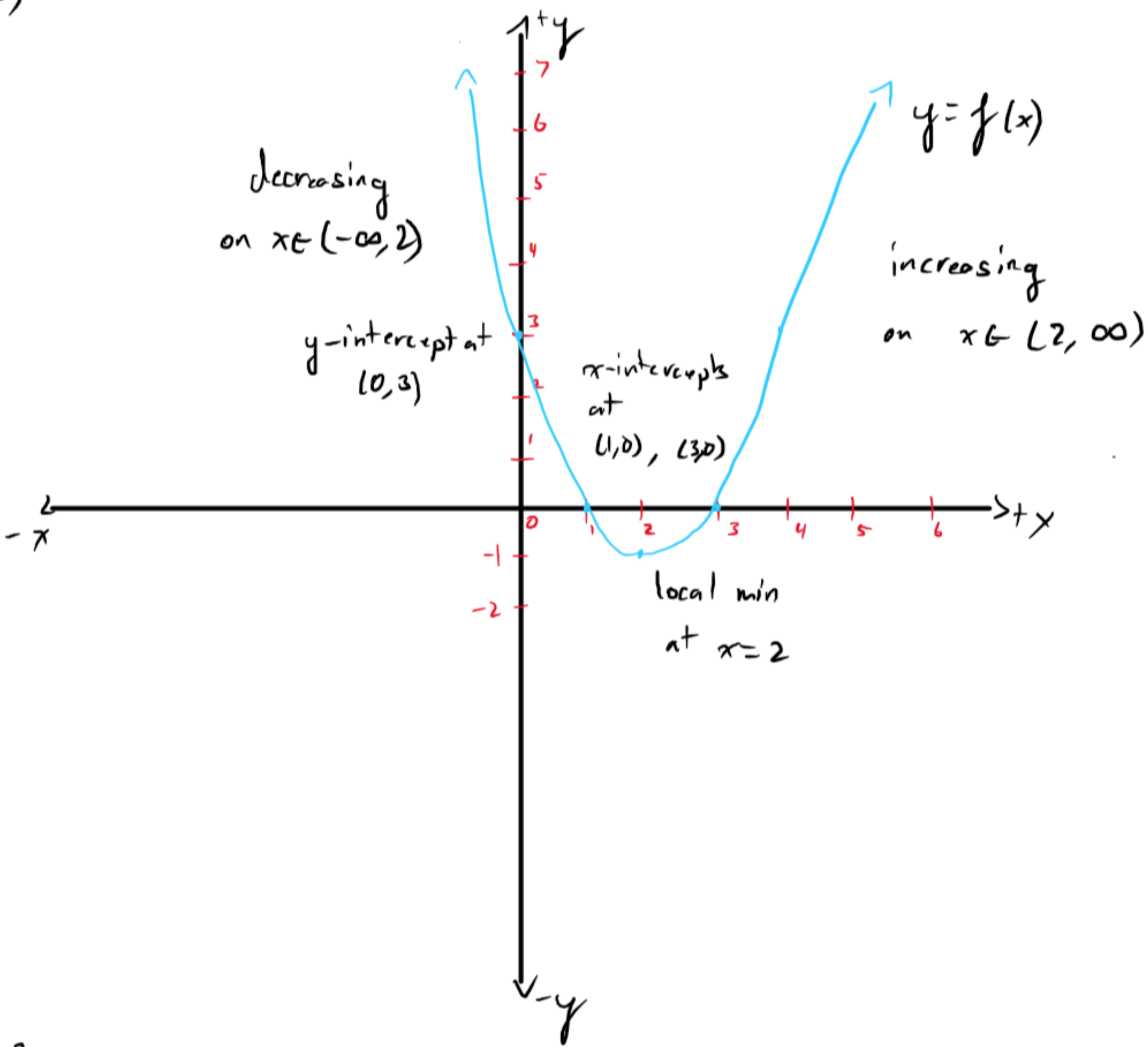
$(1,0)$ and $(3,0)$ are

x-intercepts of $f(x)$.

$\because f(0)=3$, $(0,3)$ is

a y-intercept of $f(x)$.

b)



+y
y = f(x)
decreasing
on x∈(−∞, 2)
increasing
on x∈(2, ∞)
y-intercept at
(0,3)
x-intercepts
at
(1,0), (3,0)
local min
at x=2
−x
+x
−y

2.

a)

- $f(x)$ is increasing when $f'(x)$ is positive.

  $f'(x)$ is positive when $x \in (-\infty, 0)$ and $x \in (2, \infty)$ (From the graph)

- $f(x)$ is decreasing when $f'(x)$ is negative.

  $f'(x)$ is negative when $x \in (0,2)$ (From the graph)

- Local extrema of $f(x)$ occur when $f'(x)=0$.

  $f'(x)=0$ when $x=0$ and $x=2$. (From the graph). $f(0)$ is a local max since $f'(x)>0$ on $x \in (-\infty, 0)$ and $f'(x)<0$ on $x \in (0,1)$

  $f(2)$ is a local min since $f'(x)<0$ on $x \in (1,2)$ and $f'(x)>0$ on $x \in (2,\infty)$

- Points of inflection occur when $f''(x)=0$.

  $f''(x)$ corresponds to the instantaneous slope of $f'(x)$. $f''(x)=0$ at any maxima/minima of $f'(x)$.

  $\because$ $f'(x)$ has a local min at $x=1$, $f''(1)=0$

  $\therefore$ $f(x)$ has an inflection point at $x=1$.

- $f(x)$ is concave down when $f''(x)<0$ and is concave up when $f''(x)>0$.

$f''(x)$ corresponds to the instantaneous slope of $f'(x)$. $f''(x) < 0$ when $f'(x)$ is decreasing and $f''(x) > 0$ when $f'(x)$ is increasing.

From the graph, $f''(x) < 0$ on $x \in (-\infty, 1)$

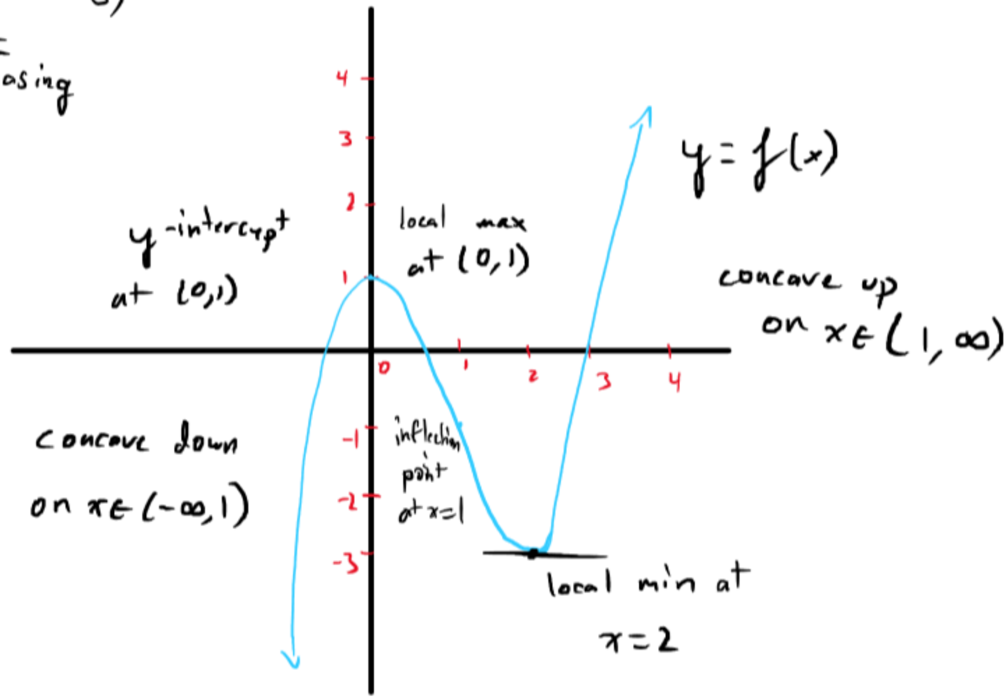
and

$f''(x) > 0$ on $x \in (1, \infty)$

$\therefore$ $f(x)$ is concave down on $x \in (-\infty, 1)$ and $f(x)$ is concave up on $x \in (1, \infty)$.

x	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<2$	$x=2$	$x>2$
$f'(x)$	decreasing $+$	0	decreasing $-$	min	increasing $-$	0	increasing $+$
$f''(x)$	$-$	$-$	$-$	0	$+$	$+$	$+$

b)



3.a) Domain: $x \in \mathbb{R}$, Range: $x \in \mathbb{R}$

x	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<2$	$x=2$	$2<x<3$	$x=3$	$x>3$
$f'(x)$	decreasing $+$	0	decreasing $-$	local min	increasing $-$	0	increasing $+$	increasing $+$	increasing $+$
$f''(x)$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$f(x)$	increasing $-$	max	decreasing $-$	inflection point	decreasing $-$	local min	increasing $-$	0	increasing $+$

$\because$ $f'(x) > 0$ on $x \in (-\infty, 0)$ and $f'(x) > 0$ on $x \in (2, \infty)$,

$\therefore$ $f(x)$ is increasing on $x \in (-\infty, 0)$ and on $x \in (2, \infty)$

$\because$ $f'(x) < 0$ on $x \in (0, 2)$

$\therefore$ $f(x)$ is decreasing on $x \in (0, 2)$

$\because$ $f'(0) = 0$ and $f'(x) > 0$ on $x \in (-\infty, 0)$ and $f'(x) < 0$ on $x \in (0, 2)$,

$\therefore$ $f(x)$ has a local max at $x = 0$.

$\because$ $f'(2) = 0$ and $f'(x) > 0$ on $x \in (0, \infty)$ and $f'(x) < 0$ on $x \in (0, 2)$,

$\therefore$ $f(x)$ has a local min at $x = 2$.

$f'(x) = 3x^2 - 6x$ (Power Rule)

Set $f'(x)$ to 0.

$0 = 3x^2 - 6x$

$= 3x(x-2)$

$\therefore f'(x) = 0$ at $x = 0$ and $x = 2$

Test $f'(-1)$:

$f'(-1) = 3(-1)^2 - 6(-1)$

$= 9$

$\because f'(-1) > 0$,

$\therefore f'(x) > 0$ on $x \in (-\infty, 0)$

Test $f'(1)$:

$f'(1) = 3(1)^2 - 6(1)$

$= -3$

$\because f'(1) < 0$,

$\therefore f'(x) < 0$ on $x \in (0, 2)$

Test $f'(3)$:

$f'(3) = 3(3)^2 - 6(3)$

$f''(x) = 6x - 6$ (Power rule)

Set $f''(y)$ to 0.

$0 = 6x - 6$

$x = 1$

$\therefore f''(x) = 0$ at $x = 1$

Test $f''(0)$:

$f''(0) = 6(0) - 6$

$= -6$

$\because f''(0) < 0$,

$\therefore f''(x) < 0$ on $x \in (-\infty, 0)$

Test $f''(2)$:

$f''(2) = 6(2) - 6$

$= 6$

$\because f''(2) > 0$,

$f''(x) > 0$ on $x \in (0, \infty)$

$\because f''(1)=0,$

$\therefore f(x)$ has an inflection point at $x=1.$

$\because f''(x)<0$ on $x \in (-\infty, 1)$

$\therefore f(x)$ is concave down on $x \in (-\infty, 1)$

$\because f''(x)>0$ on $x \in (1, \infty)$

$\therefore f(x)$ is concave up on $x \in (1, \infty)$

For intercepts:

For x-intercepts:

Set $f(x)$ to 0:

$$0 = x^3 - 3x^2$$
$$= x^2(x-3)$$

$f(x)=0$ if $x=0$ and if $x=3$

$\therefore (0,0)$ and $(3,0)$ are the x-intercepts.

For y-intercepts:

Set $x=0$:

$$f(x) = 0^3 - 3(0)^2$$
$$= 0$$

$\therefore (0,0)$ is the y-intercept

$$= 9$$

$\because f'(3)>0,$

$\therefore f'(x)>0$ on $x \in (2, \infty)$

For the min/max/inflection points:

$\because f(x)$ has a local max at $x=0,$

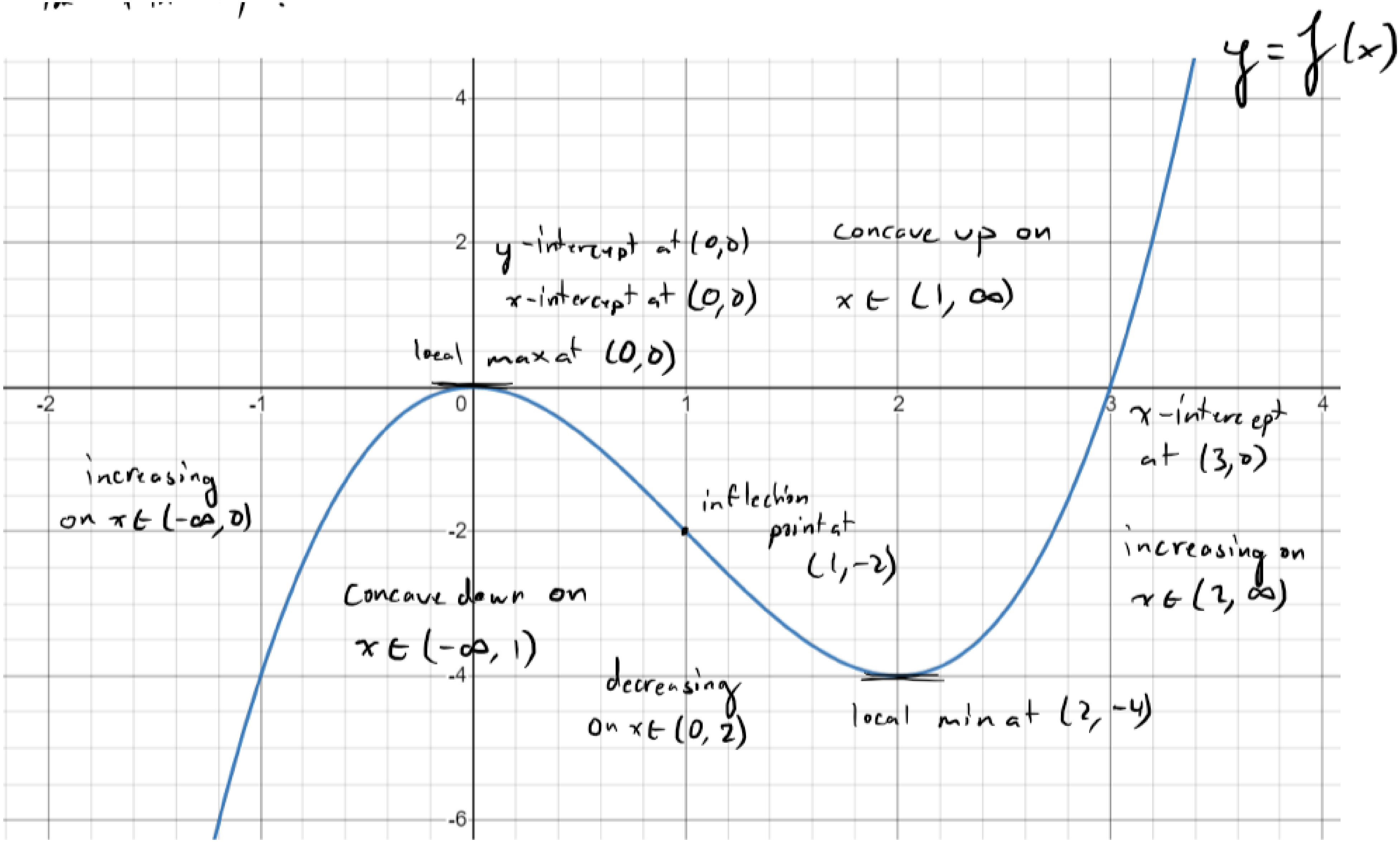
local max at $(0, f(0)) = (0, 0^3 - 3(0)^2) = (0,0)$

$\because f(x)$ has a local min at $x=2,$

local min at $(2, f(2)) = (2, 2^3 - 3(2)^2) = (2,-4)$

$\because f(x)$ has an inflection point at $x=1,$

inflection point at $(1, f(1)) = (1, 1^3 - 3(1)^2) = (0,-2)$



y = f(x)
y-intercept at (0,0)
x-intercept at (0,0)
local max at (0,0)
concave up on
x ∈ (1, ∞)
x-intercept
at (3,0)
increasing
on x ∈ (-∞, 0)
inflection
point at
(1,-2)
increasing on
x ∈ (2, ∞)
Concave down on
x ∈ (-∞, 1)
decreasing
on x ∈ (0, 2)
local min at (2, -4)

b) d = decreasing, i = increasing, POI = inflection point

x	$x<2$	$x=2$	$2<x<4$	$x=4$	$4<x<5$	$x=5$	$5<x<6$	$x=6$	$x>6$
$g'(x)$	i −	0 local max	d −	local min	i −	0	i +	i +	i +
$g''(x)$	+	0	−	0	−	+	+	+	+
$g(x)$	d +	POI	d −	POI	d −	min	i −	i 0	i +

$\because g'(x) < 0$ on $x \in (-\infty, 2)$ and $g'(x) > 0$ on $x \in (2, 4)$

$\therefore g(x)$ is decreasing on $x \in (-\infty, 2)$ and on $x \in (2, 4)$

$\because g'(x) > 0$ on $x \in (5, \infty)$

$\therefore g(x)$ is increasing on $x \in (5, \infty)$

$\because g'(5) = 0$ and $g'(x) < 0$ on $x \in (2, 5)$ and $g'(x) > 0$ on $x \in (5, \infty)$

$\therefore g(x)$ has a local min at $x = 5$.

$\because g'(2) = 0$ and $g'(x) < 0$ on $x \in (-\infty, 2)$,

Domain: $x \in \mathbb{R}$, Range: $x \in \mathbb{R}$

For $g'(x)$, Using product rule,

Let $u = (x-2)^3 \quad v = (x-6)$

$u' = 3(x-2)^2(1) \quad v' = 1$

$$\begin{aligned}\therefore g'(x) &= u'v + v'u \\ &= (3(x-2)^2)(x-6) + (1)(x-2)^3 \\ &= (x-2)^2(3(x-6) + 1(x-2)) \\ &= (x-2)^2(4x-20) \\ &= 4(x-2)^2(x-5)\end{aligned}$$

Set $g'(x)$ to 0.

$0 = 4(x-2)^2(x-5)$

$\therefore g'(x) = 0$ at $x = 2$ and $x = 5$

For $g''(x)$, Using product rule,

Let $u = 4(x-2)^2 \quad v = (x-5)$

$u' = 8(x-2)(1) \quad v' = 1$

$$\begin{aligned}\therefore g'(x) &= u'v + v'u \\ &= 8(x-2)(x-5) + (1)(4(x-2)^2) \\ &= 4(x-2)(2(x-5) + (x-2)) \\ &= 4(x-2)(3x-12) \\ &= 12(x-2)(x-4)\end{aligned}$$

Set $g''(x)$ to 0.

$0 = 12(x-2)(x-4)$

$\therefore g''(x) = 0$ at $x = 2$ and $x = 4$

Test $g''(0)$

$g'(x) < 0$ on $x \in (2,5)$, $g''(2) = 0$,

$\therefore g(x)$ has an inflection point at $x = 2$.

$\because g''(4) = 0$,

$\therefore g(x)$ has an inflection point at $x = 4$.

$\because g''(x) > 0$ on $x \in (-\infty, 2)$

$\therefore g(x)$ is concave up on $x \in (-\infty, 2)$

$\because g''(x) < 0$ on $x \in (2,4)$

$\therefore g(x)$ is concave down on $x \in (2,4)$

$\because g''(x) > 0$ on $x \in (4, \infty)$

$\therefore g(x)$ is concave up on $x \in (4, \infty)$

Test $g'(0)$

$$g'(0) = 4(0-2)^2(0-5) = -80$$

$\because g'(0) < 0$,

$\therefore g'(x) < 0$ on $x \in (-\infty, 2)$

Test $g'(3)$:

$$g'(3) = 4(3-2)^2(3-5) = -8$$

$\because g'(3) < 0$

$\therefore g'(x) < 0$ on $x \in (2,5)$

Test $g'(6)$:

$$g'(6) = 4(6-2)^2(6-5) = 16$$

$\because g'(6) > 0$

$\therefore g'(x) > 0$ on $x \in (5, \infty)$

$$g''(0) = 12(0-2)(0-4) = 96$$

$\because g''(0) > 0$,

$\therefore g''(x) > 0$ on $x \in (-\infty, 2)$

Test $g''(3)$:

$$g''(3) = 12(3-2)(3-4) = -12$$

$\because g''(3) < 0$

$\therefore g''(3) < 0$ on $x \in (2,4)$

Test $g''(5)$

$$g''(5) = 12(5-2)(5-4) = 24$$

$\because g''(5) > 0$

$\therefore g''(x) > 0$ on $x \in (4, \infty)$

For intercepts:

For x-intercepts:

Set $g(x)$ to 0:

$$0 = (x-2)^3(x-6)$$

$g(x) = 0$ if $x = 2$ and if $x = 6$

$\therefore$ $(2,0)$ and $(6,0)$ are the x-intercepts.

For y-intercepts:

Set $x = 0$:

$$g(x) = (0-2)^3(0-6)$$
$$= 48$$

$\therefore$ $(0, 48)$ is the y-intercept

For the min/max/inflection points:

$\because$ $g(x)$ has a local min at $x = 5$,

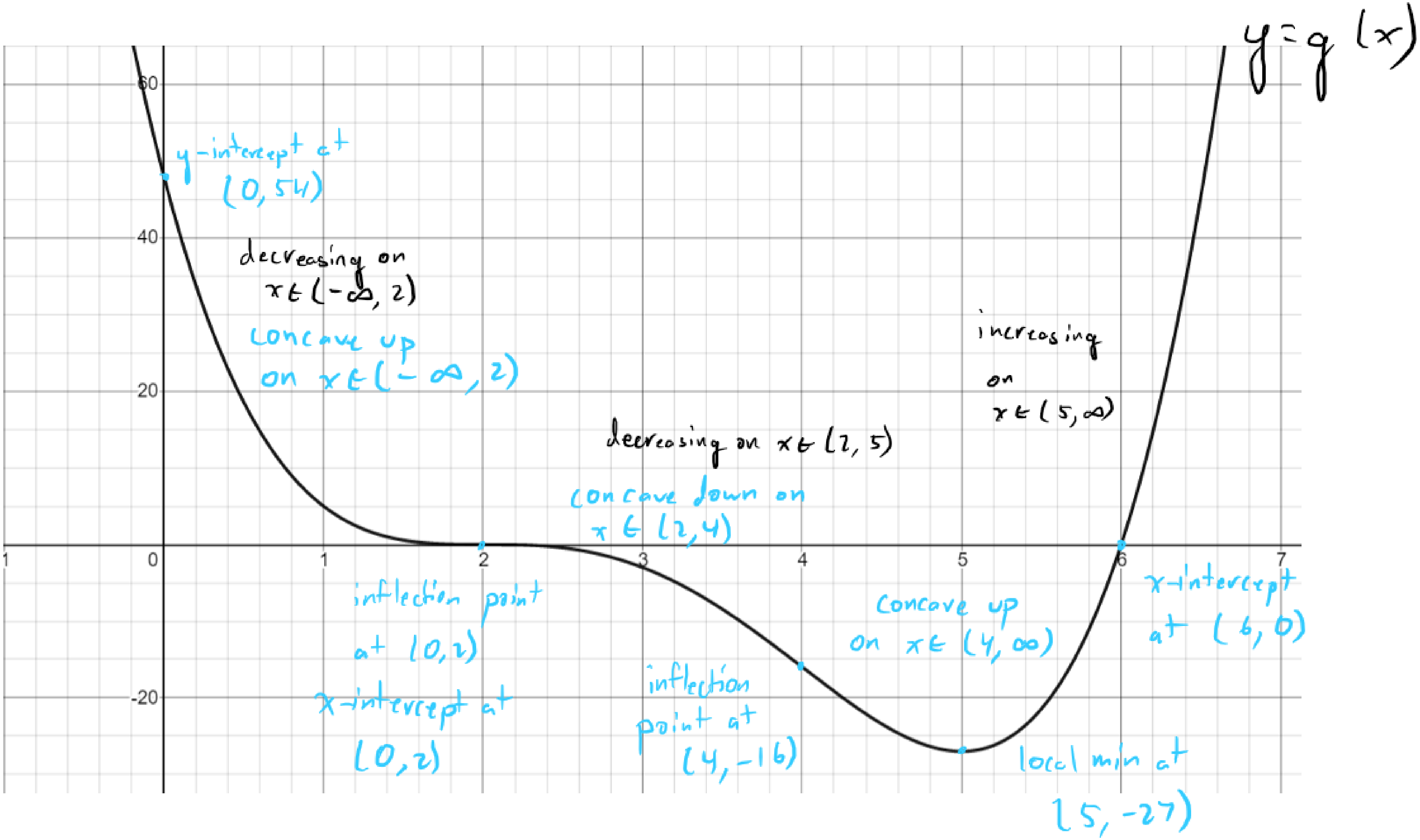
local min at $(5, g(5)) = (5, (5-2)^3(5-6)) = (5, -27)$

$\because$ $g(x)$ has an inflection point at $x = 2$,

inflection point at $(2, g(2)) = (2, (2-2)^3(2-6)) = (2, 0)$

$\because$ $g(x)$ has an inflection point at $x = 4$

inflection point at $(4, g(4)) = (4, (4-2)^3(4-6)) = (4, -16)$


y = g(x)
y-intercept at (0, 54)
decreasing on x ∈ (-∞, 2)
concave up on x ∈ (-∞, 2)
decreasing on x ∈ (2, 5)
concave down on x ∈ (2, 4)
increasing on x ∈ (5, ∞)
inflection point at (0, 2)
x-intercept at (0, 2)
inflection point at (4, -16)
concave up on x ∈ (4, ∞)
x-intercept at (6, 0)
local min at (5, -27)
60
40
20
0
-20
1
2
3
4
5
6
7

4. i = increasing, d = decreasing, PoI = point of inflection

t	$t=0$	$0<t<3$	$t=3$	$3<t<10.5$	$t=10.5$	$10.5<t<12$
$v(t)$	0	i +	local max	d +	0	d −
$a(t)$	+	+	undef	−	−	−
$s(t)$	0	i +	PoI	i +	local max	d +

Let $s(t)$ be the displacement function, let $a(t)$ be the acceleration function, $s'(t) = v(t)$, $v'(t) = a(t)$

$\because$ $v(t) > 0$ on $t \in (0, 10.5)$, $s(t)$ is increasing on $t \in (0, 10.5)$

$\because$ $v(t) < 0$ on $t \in (10.5, 12)$, $s(t)$ is decreasing on $t \in (10.5, 12)$

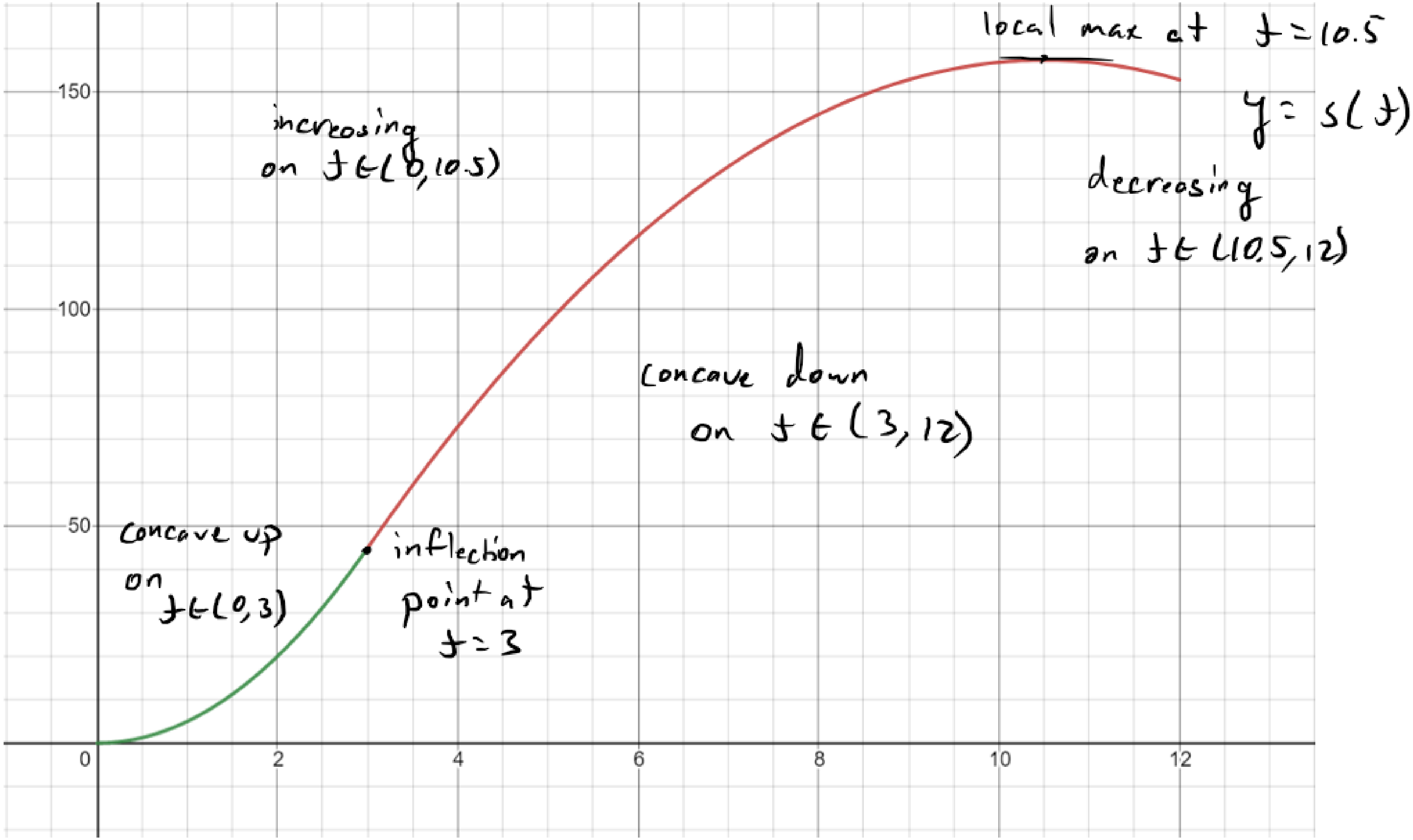
$\because$ $v(t) = 0$ at $t = 10.5$ and $v(t) > 0$ on $t \in (0, 10.5)$, $v(t) < 0$ on $t \in (10.5, 12)$, there is a local max at $t = 10.5$.

$\because$ $v(t)$ is linear on $t \in (0, 3)$ and $t \in (3, 12)$, $s(t)$ is parabolic on $t \in (0, 3)$ and $t \in (3, 12)$ (Power Rule).

$\because$ $v(3)$ is not differentiable, $s(t)$ has an inflection point at $t = 3$.

$\therefore$ $a(t) > 0$ on $t \in (0, 3)$, $s(t)$ is concave up on $t \in (0, 3)$

$\because a(t) < 0$ on $t \in (3, 12)$, $s(t)$ is concave down on $t \in (3, 12)$



5.

Critical points are points where the derivative is 0 or the function is non-differentiable.

$\because$ $h(x)$ is a polynomial, $h(x)$ is continuous on $x \in (-\infty, \infty)$. $\therefore$ all points on the derivative are differentiable.

$h'(x) = 3x^2 + 2bx$

Sub $x = 2$, $h'(x) = 0$

$0 = 3(2)^2 + 2b(2)$

$0 = 12 + 4b$

$\therefore b = -3$

Sub $b = -3$, $(2, -4)$ into h:

$-4 = 2^3 + (-3)(2)^2 + d$

$\therefore d = 0$.

$\therefore$ The equation of $h(x)$ is $h(x) = x^3 - 3x^2$.