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1.

$$y' = -12x^2 + 30x + 18$$

$$= -6(2x^2 - 5x + 3)$$

$$= -6(2x - 3)(x - 1)$$

Set $y'$ to 0:

$$0 = -6(2x - 3)(x - 1)$$

if $2x - 3 = 0$     if $x - 1 = 0$

    $x = \frac{3}{2}$         $x = 1$

| Factors of $y'$ | $x < 1$ | $x = 1$ | $1 < x < \frac{3}{2}$ | $x = \frac{3}{2}$ | $x > \frac{3}{2}$ |
|---|---|---|---|---|---|
| $2x - 3$ | $-$ | | $-$ | $0$ | $+$ |
| $x - 1$ | $-$ | $0$ | $+$ | | $+$ |
| $-6$ | $-$ | | $-$ | | $-$ |
| $y'$ | $-$ | | $+$ | | $-$ |
| $y$ inc/dec | dec | | inc | | dec |

$\therefore$ $y$ is increasing on $x \in (1, \frac{3}{2})$ and decreasing on $x \in (-\infty, 1)$ and on $x \in (\frac{3}{2}, \infty)$

2.

$$f'(x) = \frac{d}{dx}\left(3x^4 - 16x^3 + 24x^2 - 9\right)$$

$$= 12x^3 - 48x^2 + 48x$$

$$= 12x\left(x^2 - 4x + 4\right)$$

$$= 12x(x-2)^2$$

$$f''(x) = \frac{d}{dx}\left(12x^3 - 48x^2 + 48x\right)$$

$$= 36x^2 - 96x + 48$$

$$= 12\left(3x^2 - 8x + 4\right)$$

$$= 12(3x-2)(x-2)$$

Set $f''(x)$ to $0$:

$0 = 12(3x-2)(x-2)$

if $3x-2=0$     if $x-2=0$

$\therefore x = \dfrac{2}{3}$     $\therefore x=2$

Sub $x = \dfrac{2}{3}$ and $x=2$ into $f(x)$:

$f\left(\dfrac{2}{3}\right) = 3\left(\dfrac{2}{3}\right)^4 - 16\left(\dfrac{2}{3}\right) + 24\left(\dfrac{2}{3}\right)^2 - 9$

$\approx -2.481$

$\therefore \left(\dfrac{2}{3}, -2.481\right)$ is a point of inflection

$$f(2) = 3(2)^4 - 16(2) + 24(2)^2 - 9$$
$$= 7$$

$\therefore (2, 7)$ is a point of inflection

factors of $f''(x)$

| | $x < \frac{2}{3}$ | $x = \frac{2}{3}$ | $\frac{2}{3} < x < 2$ | $x = 2$ | $x > 2$ |
|---|---|---|---|---|---|
| $3x - 2$ | $-$ | $0$ | $+$ | | $+$ |
| $x - 2$ | $-$ | | $-$ | $0$ | $+$ |
| $12$ | $+$ | | $+$ | | $+$ |
| $f''(x)$ | $+$ | | $-$ | | $+$ |
| $f(x)$ concavity (up/down) | up | | down | | up |

$\therefore f(x)$ is concave down on $x \in \left(\frac{2}{3}, 2\right)$ and concave up on $x \in \left(-\infty, \frac{2}{3}\right)$ and $x \in (2, \infty)$

Set $g'(x) = 0$:

$$0 = 2(x+3)^2(2x+9)$$

if $(x-3)^2 = 0$      if $2x+1 = 0$

     $(x+3)^2 = 0$         $2x = -9$

     $\therefore x = -3$          $x = -\dfrac{9}{2}$

Sub $x = -3$ and $x = -\dfrac{9}{2}$ into $g(x)$:

$$g(-3) = (-3+3)^3(-3+5)$$

$$= 0$$

$\therefore (-3, 0)$ is a critical point

$$g\left(-\dfrac{9}{2}\right) = \left(-\dfrac{9}{2}+3\right)^3\left(-\dfrac{9}{2}+5\right)$$

$$\simeq -1.688$$

$\therefore \left(-\dfrac{9}{2}, -1.688\right)$ is a critical point

3.

$$q'(x) = 3(x+3)^2(x+5) + (1)(x+3)^3 \quad (\text{Using product rule})$$

$$= (x+3)^2(3x+15 + x+3)$$

$$= (x+3)^2(4x+18)$$

$$\therefore q'(x) = 2(x+3)^2(2x+9)$$

$$q''(x) = 2\left(2(x+3)(2x+9) + (2)(x+3)^2\right)$$

$$= 4(x+3)(2x+9 + x+3)$$

$$= 4(x+3)(3x+12)$$

$$= 12(x+3)(x+4)$$

Set $q'(x) = 0$:

Sub $x = -3$ and $x = -\frac{9}{2}$ into $g''(x)$

$g''(-3) = 12(-3+3)(-3+4)$

$= 0$

$\therefore (-3, 0)$ is a point of inflection

$g''(-\frac{9}{2}) = 12(-\frac{9}{2}+3)(-\frac{9}{2}+4)$

$= 9$

$\therefore (-\frac{9}{2}, -1.688)$ is a minimum

Set $g''(x)$ to $0$:

$0 = 12(x+3)(x+4)$

if $x + 3 = 0$      if $x + 4 = 0$

$x = -3$          $x = -4$

Sub $x = -3$ and $x = -4$ into $g(x)$:

$g(-3) = (-3+3)^3(-3+5)$

$= 0$

$\therefore (-3, 0)$ is a POI (previously calculated)

$g(-4) = (-4+3)^3(-4+5)$

$\qquad = -1$

$\therefore (-4, -1)$ is an inflection point

Set $x$ to $0$:

$g(0) = (0+3)^3(0+5)$

$\qquad = 135$

$\therefore (0, 135)$ is the y-intercept

Set $g(x)$ to $0$:

$0 = (x+3)^3(x+5)$

if $(x+3)^3 = 0$     if $x+5 = 0$

$(x+3)^3 = 0$        $\therefore x = -5$

$x+3 = 0$

$\therefore x = -3$

$\therefore (-3, 0)$ and $(-5, 0)$ are the x-intercepts.

$x$

**Factors of $g'$**

| Factors of $g'$ | $x < -\frac{9}{2}$ | $x = -\frac{9}{2}$ | $-\frac{9}{2} < x < -3$ | $x = -3$ | $x > -3$ |
|---|---|---|---|---|---|
| $(x+3)^2$ | $+$ | | $+$ | $0$ | $+$ |
| $2x+9$ | $-$ | $0$ | $+$ | | $+$ |
| $2$ | $+$ | | $+$ | | $+$ |
| $g'$ | $-$ | | $+$ | | $+$ |
| $g$ inc/dec | dec | | inc | | inc |

$\therefore$ $g$ is decreasing on $x \in (-\infty, -\frac{9}{2})$ and increasing on $x \in (-\frac{9}{2}, \infty)$

$x$

**Factors of $g''$**

| Factors of $g''$ | $x < -4$ | $x = -4$ | $-4 < x < -3$ | $x = -3$ | $x > -3$ |
|---|---|---|---|---|---|
| $x+3$ | $-$ | | $-$ | $0$ | $+$ |
| $x+4$ | $-$ | $0$ | $+$ | | $+$ |
| $12$ | $+$ | | $+$ | | $+$ |
| $g''$ | $+$ | | $-$ | | $+$ |
| $g$ concavity | up | | down | | up |

$\therefore$ $g$ is concave down on $x \in (-4, -3)$ and concave up on $x \in (-\infty, -4)$ and $x \in (-3, \infty)$

4.

For 0s to 3s:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 10}{3 - 0}$$

$$= -5$$

$\therefore v(t) = -5t$ from $t = 0s$ to $t = 3s$

$\therefore s(t) = -\frac{5}{2}t^2$ from $t = 0s$ to $t = 3s$.

At $t = 0$, $s(0) = 0$ m

At $t = 3$, $s(3) = -\frac{5}{2}(3)^2$

$$= -22.5 \text{ m}$$

$\therefore (0,0)$ and $(3, -22.5)$ are key points

For 3s to 4s

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - (-5)}{4 - 3}$$

$$= 0$$

$\therefore v(t) = 0t - 5$ from $t = 3s$ to $t = 4s$

$$= -5$$

$\therefore s(t) = -5t$ from $t = 3s$ to $t = 4s$

At $t = 3s$, $s(3) = -22.5$ m

At $t = 4s$, $s(4) = s(3) + (-5(1))$

$$= -27.5 \text{ m}$$

$\therefore (4, -27.5)$ is a key point

For $t = 4s$ to $t = 10s$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - (-5)}{10 - 4}$$

$$= \frac{5}{3}$$

$\therefore v(t) = \frac{5}{3} t$ from $t = 4s$ to $t = 10s$

$\therefore s(t) = \frac{5}{6} t^2$ from $t = 4s$ to $t = 10s$

At $t = 4s$, $s(4) = -27.5$ m

At $t = 10s$, $s(10) = s(4) + \frac{5}{6}(10)^2 - \frac{5}{6}(4)^2$

$$= 42.5 \text{ m}$$

$\therefore (10, 42.5)$ is a key point

$\therefore$ The displacement function has these key points:

$(0,0)$, $(3, -22.5)$, $(4, -27.5)$, $(10, 42.5)$

5.

$$f'(x) = 3ax^2 + 2bx + c$$

$$f''(x) = 6ax + 2b$$

At $x = 0$, $f''(x) = 0$

$$0 = 6a(0) + 2b$$

$$\therefore b = 0$$

At $x = 2$, $f'(2) = 0$

$$0 = 3a(2)^2 + 2(0)(2) + c$$

$$0 = 12a + c \quad \text{②}$$

$$6 = 8a + 2c + 2 \quad (f(2)) \quad \text{①}$$

$$\begin{cases} 4 = 8a + 2c \quad (①) \\ 0 = 24a + 2c \quad (② \times 2) \end{cases}$$

$-3 + c = 0$

$\therefore c = 3$

$-4 = 16a \quad (② \times 2 - ①)$

$\therefore a = -\frac{1}{4}$

Sub $(0, 2)$ into $f$; $b = 0, c = 0$

$2 = a(0)^3 + d$

$\therefore d = 2$

$\therefore f(x) = -\frac{1}{4}x^3 + 3x + 2$

Edit (Non-Change & For Ms. Kambo):
I use google docs to convert my screenshots from onenote into a pdf format to submit for the tests. Please let me know if there are concerns.