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Unit #5 Test– Probability Distributions

K/U: ______ / 13   Application: _____ / 19   Communication: _____ / 9   TOTAL: _____ / 41

Knowledge/Understanding: 14 marks

Multiple Choice: Circle the appropriate answer <9 marks>

1. **In a manufacturing process it is estimated that 15% of the products are defective. If a client places an order for 75 products and wants to determine the probability of getting no more than 3 defective items, what value will the client use as the probability of a failure?**

   a) 0.85
   ~~b) 25~~
   c) 0.15
   ~~d) 75~~

   $p = 0.15$
   $q = 0.85$

2. **Which of the following is NOT a property of the binomial expansion of $(a+b)^n$:**

   a) The degree of each term is n
   b) The exponents of a increase and the exponents of b decrease
   ~~c)~~ The coefficients of the terms are the entries in row n of Pascal's Triangle
   ~~d)~~ The number of terms in the expansion is n + 1

3. **A bag contains 13 red, 6 purple, 11 orange, and 9 yellow marbles. Approximately how many red marbles would you expect to get if 7 marbles are poured from the bag?**

   a) 1
   b) 2
   c) 7
   d) 39

   $\frac{ra}{n} = \frac{7(13)}{39} = 2.\overline{3}$

4. **Julia is trying to calculate the probability of having exactly 5 girls on a co-ed sports team if players are chosen randomly from a group of 27 people. If the following shows her calculations, then how many boys are there to choose from?**

$$P(X=5) = \frac{\binom{10}{5}\binom{17}{6}}{\binom{27}{11}}$$

$\frac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$

$x=5$
$a=10$
$n-a=17$

   a) 6
   b) 10
   c) 11
   d) 17

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5. **Given the probability function** $P(X=5)=\binom{8}{5}\left(\frac{3}{8}\right)^5\left(\frac{5}{8}\right)^3$**, identify the number of trials.**

$\binom{n}{x} p^x q^{n-x}$

a) 8
b) 5
c) 3
d) $\frac{3}{8}$

6. **Isabella rolls 4 dice. She is interested in the probability of getting three 2's. Which of the following statements best describes the random variable for this experiment?**

a) Let X represent the number of dice rolled
b) Let X represent the sum of the dice
c) Let X represent the number of 2's rolled
d) None of the above

7. **The quantity you can expect to obtain when an experiment is performed is known as:**

a) the Bernoulli value
b) the dependent value
c) the binomial value
d) the expected value

8. **How many terms are there in the simplified expansion of** $(8x + 9)^9$?

a) 8
b) 9
c) 10
d) 11

9. **Probability distribution for *Dependent trials* that have two possible outcomes, success or failure, are known as:**

a) Binomial Distribution
b) Probability Trials
c) Hypergeometric Distribution
d) Bernoulli Trials

10. Use **combinations** to expand and simplify $(4x - 3y)^4$. <4 marks>

$\binom{4}{0}(4x)^4(-3y)^0 + \binom{4}{1}(4x)^3(-3y)^1 + \binom{4}{2}(4x)^2(-3y)^2 + \binom{4}{3}(4x)^1(-3y)^3 + \binom{4}{4}(4x)^0(-3y)^4$

$= (1)(256x^4)(1) + (4)(64x^3)(-3y) + 6(16x^2)(9y^2) + 4(4x)(-27y^3) + (1)(1)(81y^4)$

$= 256x^4 - 768x^3y + 864x^2y^2 - 432xy^3 + 81y^4$

$\therefore (4x-3y)^4 = 256x^4 - 768x^3y + 864x^2y^2 - 432xy^3 + 81y^4$.

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Application: 19 marks

FOR THE FOLLOWING QUESTIONS, BE SURE TO SHOW ALL YOUR WORK TO RECEIVE FULL MARKS!

1. Rick and Dan are having a math party and decide to play some fun math games. They have 3 standard (six-sided) dice.

a) Create a probability distribution table for rolling the 3 dice. The random variable (X) represents the number of 5's.

$p = \frac{n(5)}{n(all)} = \frac{1}{6}$

<3 marks>

X	P(X=x)	Answer
0	$\binom{3}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 = (1)(1)\left(\frac{125}{216}\right)$	$\frac{125}{216}$
1	$\binom{3}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 = (3)\left(\frac{1}{6}\right)\left(\frac{25}{36}\right)$	$\frac{75}{216}$
2	$\binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = (3)\left(\frac{1}{36}\right)\left(\frac{5}{6}\right)$	$\frac{15}{216}$
3	$\binom{3}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = (1)\left(\frac{1}{216}\right)(1)$	$\frac{1}{216}$

Use $P(X=x) = \binom{n}{x}p^x q^{n-x}$ ∵ X is a binomial random variable

$P(X=0) = \binom{n}{x}p^x q^{n-x}$
$= \binom{3}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 = \frac{125}{216}$

$P(X=1) = \binom{n}{x}p^x q^{n-x}$
$= \binom{3}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 = \frac{75}{216}$

$P(X=2) = \binom{n}{x}p^x q^{n-x}$
$= \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = \frac{15}{216}$

$P(X=3) = \binom{n}{x}p^x q^{n-x}$
$= \binom{3}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{1}{216}$

b) Calculate the number of 5's they can expect to get.

<2 marks>

$E(X) = np$
$= (3)(1/6)$
$= \frac{1}{2}$ ∴ They can expect to roll 0-1 5's.

c) What is the probability that they will roll a 5 at least once?

<1 marks>

$P(X \geq 1) = 1 - P(X=0)$
$= 1 - \frac{125}{216}$
$= \frac{91}{216} \simeq 0.421$

∴ The probability that they roll a 5 at least once is $\frac{91}{216}$ or 0.421.

2. Connor is playing a game where he draws cards from a standard 52-card deck. A card is drawn, then replaced and then another card is selected until 5 cards are drawn in total. Find the probability that **more than** three of his cards is a face card (J, Q, K).

<4 marks>

Let X be the number of face cards drawn, binomial random variable

$P(X>3) = P(X=4) + P(X=5)$

Use $P(X=x) = \binom{n}{x}p^x q^{n-x}$

$p = \frac{n(\text{face cards})}{n(\text{cards})} = \frac{3 \times 4}{52} = \frac{12}{52}$, $q = 1 - \frac{12}{52} = \frac{40}{52}$

$= \binom{n}{x_1}p^{x_1}q^{n-x_1} + \binom{n}{x_2}p^{x_2}q^{n-x_2}$
$= \binom{5}{4}\left(\frac{12}{52}\right)^4\left(\frac{40}{52}\right) + \binom{5}{5}\left(\frac{12}{52}\right)^5\left(\frac{40}{52}\right)^0$
$\simeq 0.0116$

∴ There is a probability of about 0.0116 that more than 3 of his cards are face cards.

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3. Twelve people need to be selected for a jury in a criminal court. There are 33 people to choose from, 19 of them are female.

a) What is the probability that there will be exactly six **men** on the jury?

<3 marks>

Let X be the number of men chosen, hypergeometric random variable.

$n(\text{men}) = n(S) - n(\text{women})$
$= 33 - 19$
$= 14$

Use $P(X=x) = \dfrac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$

$n = 33,\ a = 14,\ r = 12$

$$P(X=6) = \frac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}} = \frac{\binom{14}{6}\binom{19}{6}}{\binom{33}{12}} \simeq 0.230$$

∴ The probability that there will be exactly 6 men is 0.230.

b) Determine the probability that there will be at least 2 **women** on the jury?

<4 marks>

Let X be the number of women chosen, Hypergeometric random variable.

Use $P(X=x) = \dfrac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$
$$= 1 - \frac{\binom{a}{x_1}\binom{n-a}{r-x_1}}{\binom{n}{r}} - \frac{\binom{a}{x_2}\binom{n-a}{r-x_2}}{\binom{n}{r}}$$
$$= 1 - \frac{\binom{19}{0}\binom{14}{12}}{\binom{33}{12}} - \frac{\binom{19}{1}\binom{14}{11}}{\binom{33}{12}}$$
$$\simeq 0.999$$

∴ There is a 0.999 probability that there will be at least 2 women on the jury.

c) Determine the expected number of women.

<2 marks>

Let X be the number of women chosen, Hypergeometric random variable

$$E(X) = \frac{ra}{n} = \frac{(12)(19)}{33} \simeq 6.91$$

∴ There will be 6 – 7 women on the jury.

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Communication: 9 marks

1. You are flipping a coin five times, and counting the number of heads. Explain TWO different ways of calculating the probability of getting AT LEAST 2 heads.

<2 marks>

Since the probability of landing heads vs. tails is constant across trials, this has a binomial probability distribution. We can calculate the probability directly or indirectly.

Let X be the number of times the coin landed heads, binomial random variable

Directly

$P(X\geq 2)=P(X=2)+P(X=3)+P(X=4)+P(X=5)$

Each probability is calculated using $P(X=x)=\binom{n}{x}p^x q^{n-x}$, $p=\frac{1}{2}$, $q=1-p=\frac{1}{2}$, $n=5$.

$P(X\geq 2)=\binom{5}{2}(\frac{1}{2})^2(\frac{1}{2})^3+\binom{5}{3}(\frac{1}{2})^3(\frac{1}{2})^2+\binom{5}{4}(\frac{1}{2})^4(\frac{1}{2})^1+\binom{5}{5}(\frac{1}{2})^5(\frac{1}{2})^0$

$=0.8125$

∴ The probability of landing at least 2 heads is 0.8125 (Directly).

Indirectly

$P(X\geq 2)=1-P(X=0)-P(X=1)$

Each probability is calculated using $P(X=x)=\binom{n}{x}p^x q^{n-x}$, $p=\frac{1}{2}$, $q=1-p=\frac{1}{2}$, $n=5$

$P(X\geq 2)=1-\binom{5}{0}(\frac{1}{2})^0(\frac{1}{2})^5-\binom{5}{1}(\frac{1}{2})^1(\frac{1}{2})^4$

$=0.8125$

∴ The probability of landing at least 2 heads is 0.8125 (Indirectly).

2. Describe in your own words when you would use a binomial distribution and when you would use a hypergeometric distribution. Give an example of each distribution.

<4 marks>

Suppose:

1. What is the probability of rolling 3 4's when rolling a 6-sided dice 5 times? (Binomial)
2. What is the probability of drawing 4 Aces from 5 draws without replacement? (Hypergeometric)

1. The probability of rolling a 4 doesn't change across dice rolls - on roll 1, the probability of rolling a 4 is $\frac{1}{6}$, which is the same as the probability of rolling a 4 on roll 5. This means that the probability distribution is binomial since there are a fixed number of trials (5) and a constant probability across trials ($\frac{1}{6}$). Let X be the number of 4's rolled. Use $P(X=x)=\binom{n}{x}p^x q^{n-x}$, $P(X=3)=\binom{5}{3}(\frac{1}{6})^3(\frac{5}{6})^2\simeq 0.0322$.

2. The probability of drawing an ace on draw 1 is DIFFERENT than the probability of drawing an ace on draw 3 because there are 2 fewer cards in the deck on draw 3. Since the probability of drawing an ace (success) depends on previous trials ($P(\text{Ace}\mid\text{roll }3)\neq P(\text{Ace}\mid\text{roll }1)$), the probability distribution is hypergeometric. Let X be the number of aces drawn. Use $P(X=x)=\frac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$, $P(X=4)=\frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}}\simeq 0.000018$.

Mathematical Form	Level R	Level 1	Level 2	Level 3	Level 4
Proper Use of formulas and notation	Never uses formulas and/or notation properly	Rarely uses formulas and/or notation properly	Sometimes uses formulas and/or notation properly	Often uses the formulas and/or notation properly	Always uses formulas and/or notation properly

Completing the test within the given time: 1C

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Equations:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} x^r a^{n-r}$$

$$t_{r+1} = \binom{n}{r} a^{(n-r)} b^r$$

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$E(X) = \sum_{i=1}^{n} x_i P(X = x_i)$$

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

$$E(X) = np$$

$$P(X = x) = \frac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$$

$$E(X) = \frac{ra}{n}$$