MDM4U Quiz #2, Unit 2 – Permutations   /10K   /1C (time)

1. Evaluate each of the following without a calculator (expand factorials and show work):   *<2 marks>*
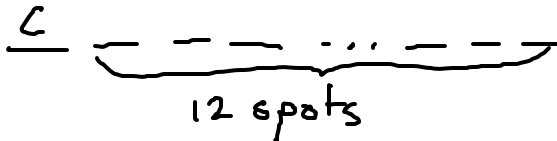
a) $_7P_4$

$$= \frac{7!}{(7-4)!}$$

$$= \frac{7!}{3!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

$$= 7 \times 6 \times 5 \times 4$$

$$= 840$$

b) $\frac{9!}{4!3!}$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1)(3 \times 2 \times 1)}$$

$$= 9 \times 8 \times 7 \times 5$$

$$= 2520$$

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  72
  56
  45
 280
2240
2520
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2. a) In how many ways can 13 students standing in a line be arranged if Chloe must be first?   *<2 marks>*

C ___ ___ ___ ... ___ ___ ___
         12 spots

$$n(\text{arrangements}) = 12P12$$

$$= \frac{12!}{(12-12)!}$$

$$= 12!$$

$$= 479,001,600$$

∴ There are 12! or 479,001,600 ways to arrange the students if Chloe must be first.

b) In how many ways can 13 students standing in a line be arranged if Wendy must be first and Benji last?

W ___ ___ ___ ... ___ ___ B
         11 spots

$$n(\text{arrangements}) = 11P11$$

$$= \frac{11!}{(11-11)!}$$

$$= \frac{11!}{0!}$$

$$= 39,916,800$$

∴ There are 11! or 39,916,800 ways to arrange the students if Wendy must be first and Benji must be last.

Date: _____   Name: Dhrumil Patel _____

1 2 3 4 5 6 7 8 9 10 11

3. In how many ways can the word FORECASTING be arranged if: <2 marks>

↳ 11 letters, all unique

a) All letters are used   b) Only 5 letters are used

$n(\text{arrangements}) = 11!$

$= 39,916,800$

∴ There are $11!$ or $39,916,800$

ways to arrange FORECASTING.

$n(\text{arrangements}, 5) = {}_{11}P_5$

$= \dfrac{11!}{(11-5)!}$

$= \dfrac{11!}{6!}$

$= 11 \times 10 \times 9 \times 8 \times 7$

$= 55440$

∴ There are 55,440 ways to arrange FORECASTING if only 5 letters are used.

4.   a) How many different ways can the word JAZZINESSES be arranged?   <1 mark>

↳ 11 letters, 2 Z's, 2E's, 3 S's

$n(\text{ways}) = \dfrac{11!}{2! \, 2! \, 3!}$

$= 1663200$

∴ There are $1,663,200$ different ways the word JAZZINESSES can be arranged

b) if it must begin and end with the letter S?   <1 mark>

$\underline{S} \ \underline{} \underline{} \underline{} \ \cdots \ \underline{} \underline{} \ \underline{S}$

$\underbrace{}$
9 letters

$n(\text{ways}) = \dfrac{9!}{2! \, 2! \, 1!}$

$= 90,720$

∴ There are 90,720 ways to arrange "JAZZINESSES" if it begins and ends with the letter "S".

c) if the Z's are kept together?   <2 marks>

Let K be the Z's together.
The word is now JAKINESSES. The # of arrangements of this word will be the same as if the original word's Z's were kept together.

$n(\text{arrangements}) = \dfrac{10!}{2! \, 3!}$ (10 letters, 2E's, 3 s')

$= 302,400$

∴ The number of arrangements of JAZZINESSES if the Z's are kept together is 302,400.