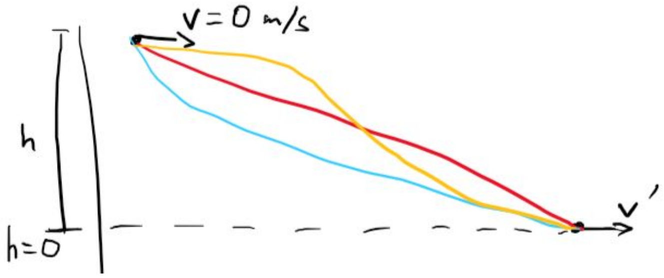
Dhrumil Patel

1.



Since the ball will start at the same height on all three ramps (A, B, and C), it has the same initial gravitational potential energy regardless of the ramp from which it starts. The ramps also end at the same height as each other (but not the height of the start of the ramp), which means that the displacement in the ball's height is equal for all three ramps. The acceleration due to gravity on the surface of the Earth does not change and we assume that the ball's mass has not changed once it reaches the bottom of the ramp. Without loss of generality, we assume that the ball is initially at rest and choose the height of the ball at the end of the ramp as h = 0. The ball's change in gravitational potential energy between the start and the end of the ramp must then be equal for all three ramps. The ramps are also frictionless, so all of the ball's gravitational potential energy is converted to kinetic energy without any loss to thermal energy (Law of Conservation of Energy). Equating the ball's gravitational potential energy at the start of the ramp and kinetic energy at the end of the ramp, we can solve for the velocity of the ball at the end of the ramp.

$$E_T = E_T'$$

$$E_k + E_g = E_k' + E_g'$$

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}m(v')^2 + mgh'$$

$$\frac{1}{2}m(0)^2 + mgh = \frac{1}{2}m(v')^2 + mg(0)$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}(v')^2$$

$$v' = \sqrt{2gh}$$

We assume that the ball's gravitational potential energy is converted completely into *translational* kinetic energy (this does not account for rotational kinetic energy, which would result in a lower, but still equal translational velocity for the ball). The velocity of the ball is clearly a function of the ball's height. Since the ball's displacement in height is the same for all three ramps, the velocity of the ball at the end of any of the three ramps will be the same.

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3.

3.

$m = 57.0\,kg$

$h = 15.0\,m$

$h' = 0\,m$

$h'' = -2.5\,m$

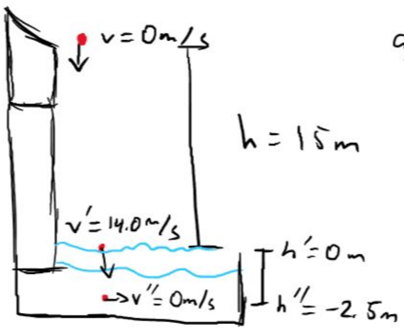
$v' = 14.0\,m/s$

$\vec{F}_b = 500\,N\,[\uparrow]$

$d_{water} = 2.5\,m$

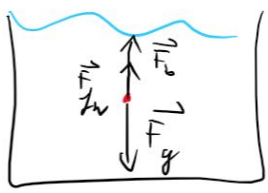
$F_{f_r} = ?$ (a)

$F_{f_w} = ?$ (b)



a)

b)



(a)

$E_T = E_T'$

$\cancel{m}gh = \frac{1}{2}\cancel{m}(v')^2$

$gh = \frac{1}{2}(v')^2$

$v' = \sqrt{2gh}$

$v' = \sqrt{2(9.8)(15.0)}$

$= 17.15\,m/s$

$\therefore$ The speed at which the diver would hit the water is 17.15 m/s without air resistance.

The loss in kinetic energy must be due to work performed by air resistance (friction).

$W_f = \Delta E_k$

$(F_{r_{avg}})(\Delta d)\cos\theta = E_k' - E_k$

$(F_{r_{avg}})(\Delta d)\cos(180°) = \frac{1}{2}m(v')^2 - \frac{1}{2}m(v)^2$

$(F_{r_{avg}})(0-15)(-1) = \frac{1}{2}(57.0)(17.15)^2 - \frac{1}{2}(57.0)(14.0)^2$

$F_{r_{avg}} = 186\,N$

$\therefore$ The average force due to air resistance is $186\,N\,[\uparrow]$

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(b)

$$W = \Delta E_k$$

$$W = E_k' - E_k$$

$$= \frac{1}{2}m(v')^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}(57.0)(14.0)^2 - \frac{1}{2}(57.0)(0)^2$$

$$= 5586 \text{ J}$$

$$W = (F_b)(\Delta d_{water})(\cos\theta) + (F_f)(\Delta d_{water})(\cos\theta)$$

$$5586 = (500)(-2.5-0)(\cos(180^\circ)) + (F_f)(-2.5-0)(\cos(180^\circ))$$

$$(2.5)F_f = 5586 - (500)(2.5)$$

$$F_f = 1734.4 \text{ N}$$

$\therefore$ The force of friction underwater is

1734.4 N [↑]

P.S. For q1, a solution that considered rotational kinetic energy also shows that the ball (assuming that it is a sphere) would have the same velocity at the end of any of the three ramps. I've shown the simple calculations below that yield the same conclusion as q1. It is interesting to note however, that some of the gravitational potential energy went into rotating the ball.

$$E_T = E_T'$$

$$E_g = E_k$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}m\cancel{r^2}\right)\left(\frac{v^2}{\cancel{r^2}}\right)$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{5}\cancel{m}v^2$$

$$gh = \frac{7}{10}v^2$$

$$v = \sqrt{\frac{10}{7}gh}$$