1. The water flow rate is given by the derivative of V.

$$V' = \frac{d}{dt}\left(1000\left(1-\frac{t}{30}\right)^2\right)$$

$$= 1000\left(2\left(1-\frac{t}{30}\right)^1\right)\left(\frac{-1}{30}\right)$$

$$= \frac{-200}{3}\left(1-\frac{t}{30}\right)$$

Sub $t = 10$:

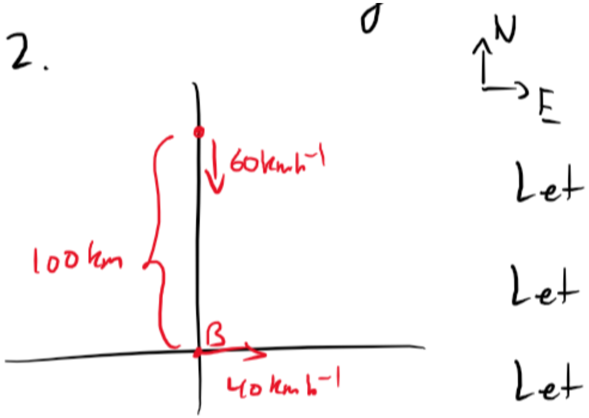
$$V(t) = \frac{-200}{3}\left(1-\frac{10}{3}\right)$$

$$= \frac{-200}{3}\left(\frac{2}{3}\right)$$

$$= \frac{-400}{9} \simeq -44.44 \text{ L/min}$$

$\therefore$ Water is flowing out of the tank at 44.44 L/min after 10 min.

2.



Let A be the distance from the station to the train travelling South.

Let B be the distance from the station to the train travelling East.

Let t be the elapsed time in hours.

$$A(t) = 100 - 60t$$

$$B(t) = 40t$$

The distance between the trains is given by:

$$d = \sqrt{A^2+B^2}$$

$$= \left((100-60t)^2+(40t)^2\right)^{\frac{1}{2}}$$

$$= \left(100^2 - 2(60)(100)t + 60^2t^2 + 40^2t^2\right)^{\frac{1}{2}}$$

$$= \left(10000 - 12000t + 5200t^2\right)^{\frac{1}{2}}$$

Find d':

$$d' = \frac{1}{2}\left(10000 - 12000t + 5200t^2\right)^{-\frac{1}{2}}\left(-12000 + 10400t\right)$$

$$d' = \frac{10400t - 12000}{2\sqrt{5200t^2 - 12000t + 10000}}$$

$$d' = \frac{520t - 600}{\sqrt{52t^2 - 120t + 100}}$$

Set d' to 0:

$$0 = \frac{520t - 600}{\sqrt{52t^2 - 120t + 100}}$$

$$0 = 520t - 600$$

$$t = \frac{600}{520}$$

$$= \frac{15}{13} \simeq 1{,}15 \text{ hours}$$

Create a sign chart around $t = \frac{15}{13}$ for d':

t		$t = \frac{15}{13}$	
d'	$-$		$+$
d	decreasing		increasing

Choose $t = 1$:

$$d'(1) = \frac{520(1) - 600}{\sqrt{52(1)^2 - 120(1) + 100}}$$

$$= -14.14$$

$\therefore d'(1)$ is negative

Choose $t = 2$:

$$d'(1) = \frac{520(2) - 600}{\sqrt{52(2)^2 - 120(2) + 100}}$$

$$= 77.78$$

$\therefore d'(2)$ is positive

$\because d$ is decreasing on $x \in (-\infty, \frac{15}{13})$ and increasing on $x \in (\frac{15}{13}, \infty)$,

$t = \frac{15}{13}$ is a minimum.

$\therefore$ At $t = \frac{15}{13} \approx 1.15$ hours, the distance between the trains is minimized.

Sub $t = \frac{15}{13}$ into d:

$$d\left(\frac{15}{13}\right) = \left(10000 - 12000\left(\frac{15}{13}\right) + 5200\left(\frac{15}{13}\right)^2\right)^{\frac{1}{2}}$$

$\simeq (3076.9209)^{\frac{1}{2}}$

$\simeq 55.47$ km

$\therefore$ The trains are 55.47 km apart at their closest distance, achieved at $t = \frac{15}{13} \simeq 1.15$ hours.

3.

Let A be the area of the field in m^2.

$A = 5000$

Let l, w be the length and width of the field. w is the shorter side.

$A = lw$

Let C be the cost of the fences in \$.

$$C = 10(2(l+w)) + 4(w)$$

$$= 20l + 20w + 4w$$

$$= 24w + 20l$$

Using $A = lw$,

$$5000 = lw$$

$$w = \frac{5000}{l}$$

Sub $w = \frac{5000}{l}$ into C:

$$C = 24\left(\frac{5000}{l}\right) + 20l$$

$$= \frac{120{,}000}{l} + 20l$$

Find C':

$$C' = \frac{-120000}{l^2} + 20$$

Set C' to 0:

$$0 = \frac{-120000}{l^2} + 20$$

$$20 = \frac{120000}{l^2}$$

$$l^2 = \frac{120000}{20}$$

$l = \sqrt{6000}$ $\quad\therefore\ l = -\sqrt{6000}$ is extraneous (no negative lengths allowed)

$= 20\sqrt{15} \simeq 77.46\text{ m}$

Find C'' for concavity:

$$C' = \frac{-120000}{l^2} + 20$$

$$C'' = \frac{240000}{l^3}$$

$\because l > 0$, C'' is always positive

$\therefore l = 20\sqrt{15}$ m is a minimum

Sub l into w:

$$w = \frac{5000}{l}$$

$$= \frac{\cancel{5000}^{250}}{\cancel{20}\sqrt{15}}$$

$$= \frac{250}{\sqrt{15}} \simeq 64.55\text{m}$$

Sub l, w into C:

$$C = 24w + 20l$$

$$C = 24\left(\frac{250}{\sqrt{15}}\right) + 20\left(20\sqrt{15}\right)$$

$\simeq \$3098.39$

$\therefore$ A length of $20\sqrt{15}$m or about 77.46m and a width of $\frac{250}{\sqrt{15}} \simeq 64.55$m yields a minimum cost of \$3098.39.

4.

Let p be the ticket price.

Let s be the number of spectators.

Let R be the ticket revenue as a function of the change in ticket price.

Let d be the change in ticket price.

Let $p = 60 - d$ describe the ticket price.

Let $s = 18000 + \frac{2000}{5} d$ describe the number of spectators.

$= 18000 + 400d$

Then,

$$R = ps$$
$$= (60-d)(18000 + 400d)$$

$$R' = (-1)(18000 + 400d) + (400)(60-d)$$
$$= -18000 - 400d + 400(60) - 400d$$
$$= -800d + 6000$$

$d = \frac{15}{2} = 7.5$

Check R'' for concavity:

$$R'' = \frac{d}{d(d)}(-800d + 6000)$$

$$= -800$$

$\because$ R'' is negative, R is concave down and $d = 7.5$ is a max.

Sub $d = 7.5$ into p:

$$p = 60 - d$$

$$p = 60 - 7.5$$

$$= \$52.5$$

Sub $d = 7.5$ into s:

$$s = 18000 + 400(7.5)$$

$$= 21000 \text{ spectators}$$

$\because$ $21000 < 24000$, $d = 7.5$ is valid.

Sub $d = 7.5$ into R:

$$R = (60 - d)(18000 + 400d)$$

$$= (60 - 7.5)(18000 + 400(7.5))$$

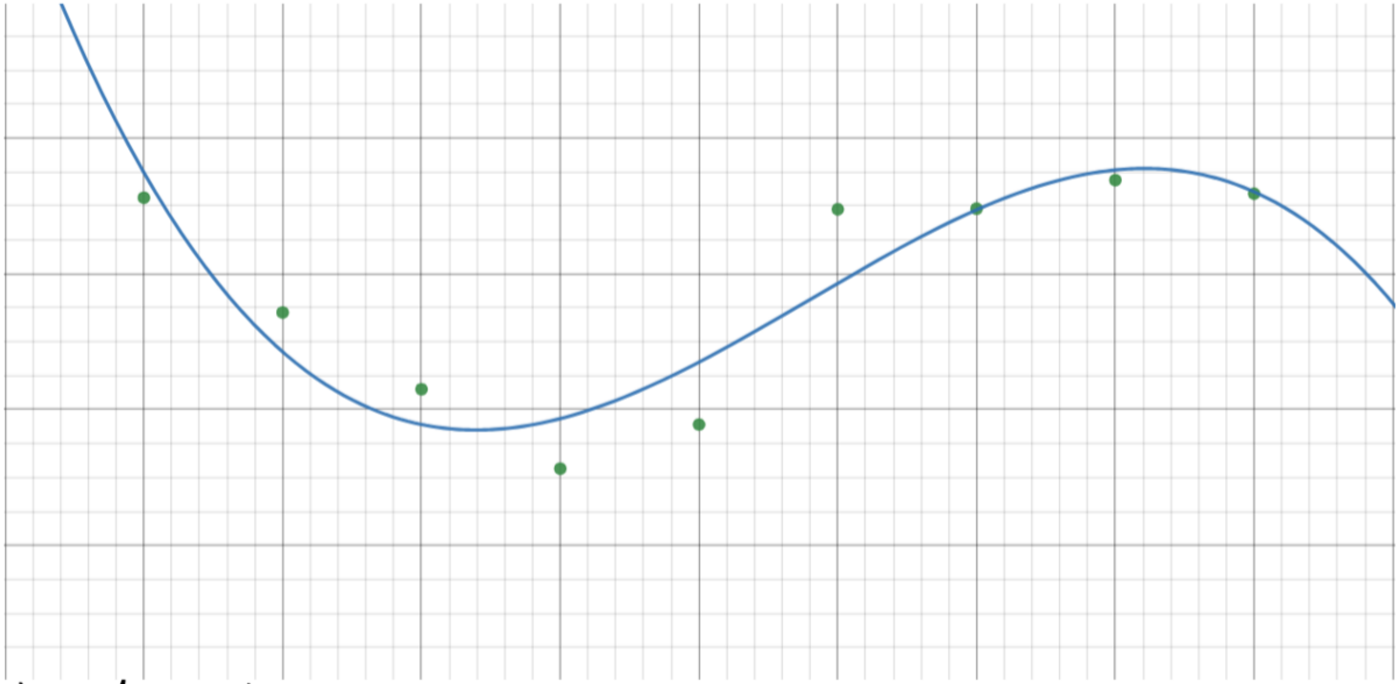
$$= \$1102\,5000$$

$\therefore$ A ticket price of \$52.5 maximizes revenue at \$1,102,500.

5.

a)

Year	Diagnoses	1st difference	2nd difference	3rd difference
1997	2512			
		−169		
1998	2343		56	
		−113		−52
1999	2230		4	
		−117		178
2000	2113		182	
		65		70
2001	2178		252	
		317		−568
2002	2495		−316	
		1		357
2003	2496		41	
		42		−103
2004	2538		−62	
		−20		
2005	2518			
Avg. diff		0.75	22.43	−19.667



We use a cubic function to model the data since it has the highest coefficient of determination (r^2 value) of 0.851.

Let f describe the number of HIV diagnoses per year.

$$f(t) = -6.883t^3 + 41335.2t - (8.2745 \times 10^7)t + (5.5213 \times 10^{10})$$

b)

b)

Sub $t = 1995$ in $f(t)$:

$$f(t) = -6.883t^3 + 41335.2t^2 - (8.2745\times10^7)t + (5.5213\times10^{10})$$

$$f(1995) = -6.883(1995)^3 + 41335.2(1995)^2 - (8.2745\times10^7)(1995) + (5.5213\times10^{10})$$

$$= 3713.254 \simeq 3713 \text{ diagnoses}$$

$\therefore$ There were an estimated 3713 diagnoses in 1995.

Sub $t = 2008$ in $f(t)$:

$$f(t) = -6.883t^3 + 41335.2t - (8.2745\times10^7)t + (5.5213\times10^{10})$$

$$f(2008) = -6.883(2008)^3 + 41335.2(2008) - (8.2745\times10^7)(2008) + (5.5213\times10^{10})$$

$$= 1466.796 \simeq 1467 \text{ diagnoses}$$

$\therefore$ There were an estimated 1467 diagnoses in 1995.

c)

Find $f'(t)$:

$$f'(t) = \frac{d}{dt}\left(-6.883t^3 + 41335.2t^2 - (8.2745\times10^7)t + (5.5213\times10^{10})\right)$$

$$= -20.649t^2 + 82670.4t - (8.2745\times10^7)$$

Sub $t = 2010$:

$$f'(2010) = -20.649(2010)^2 + 82670.4(2010) - (8.2745\times10^7)$$

$$= -1520.9 \text{ diagnoses/year}$$

$\therefore$ The number of diagnoses is decreasing by 1520.9 diagnoses/year.

d)

$\therefore$ In 2006, there are $\left(\frac{1232}{0.5}\right) = 2464$ diagnoses of HIV.

The curve of best fit is still the cubic function, but $f(t)$ becomes:

$$f(t) = -6.0068t^3 + 36076.5t^2 - (7.2224 \times 10^7)t + (4.8197 \times 10^{10})$$

Find $f'(t)$:

$$f'(t) = -18.0204t^2 + 72153t - (7.2224 \times 10^7)$$

Sub $t = 2010$:

$$f'(2010) = -18.0204(2010)^2 + 72153(2010) - (7.2224 \times 10^7)$$

$$= -688.04 \simeq -688 \text{ diagnoses/year}$$

$\therefore$ The number of diagnoses is decreasing by 688 diagnoses/year.