MCV4U UNIT 5 Rates of Change

Name: ______________________________ Date: ______________________________

Knowledge (13)	Application (26)	Communication (15)	Thinking (12)

<u>Show ALL work for full marks.</u>

1. Determine the derivatives of the following functions [3K each]

   a. $f(x) = x^3\cos x$

   b. $h(x) = (x^2-1)^3(x^2+1)^2$

   c. $g(x) = \frac{x^2}{e^x}$

2. Determine the equation of the tangent to the curve $f(x) = (2x^2-1)^{-5}$ at $x = 1$. [4K 2A 2C]

3. Find the value(s) of k for which $y = e^{kx}$ satisfies the equation $y + y' = y''$. (y'' is the 2nd derivative) [2A 2C]

5. The temperature of a warm object can be found by $T = A + (T_0 - A)e^{kt}$, where A is the temperature of the surroundings, T_0 is the initial temperature of the object, t is the time in minutes since the initial temperature was measured, and k is a characteristic of the object. At what rate is a cup of tea cooling 4 minutes after its initial temperature of *88°C* was taken, with a room at a temperature of *27°C*, and has a k value of *0.033*? [3A 1C]

6. An electronics store can sell 30 ipods per week when the ipods are priced at $290. For each ten dollar increase in price, there will be a loss of one sale per week. The cost of the ipod to the store is $120 each. What is the price that will give the maximum profit? [5A 3C]

6. Consider the circle $x^2 + y^2 = 25$. What is the area of the largest rectangle that can be inscribed in the circle. [3AT 1C]

7. After you eat something containing sugar, the pH or acid level in your mouth changes. This can be modeled by the function below, where L is the pH level and n is the number of minutes that have elapsed since eating. $L(n) = 6 - \frac{20.4n}{n^2+49}$

Calculate the instantaneous rate of change at 4 minutes. What does this value represents in this situation. [4A 2T 2C]

8. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2? [3AT 2C]

9. A spherical balloon is being inflated. Given that the volume of a sphere in terms of its radius is $V(r) = \frac{4}{3}\pi r^3$ and the surface area of a sphere in terms of its radius is $S(r) = 4\pi r^2$, calculate the rate at which the volume of the balloon is changing with respect to its surface area when the surface area measures 75 cm^2. [4AT 2C]