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MDM1D3
Story of Maths - Episodes 1 to 4

Episode 1 - The Language of the Universe

- Mathematics outlines the rules and patterns of our surroundings. It determines the qualities and the relationships they have between us and each other. Mathematics may have began as a method to count and understand the natural patterns of the world.
- Egyptians recorded floodings of the Nile by counting the number of days between two floodings, giving birth to a counting system. Egyptians used a palm as the width of a hand, a cubit as the length of an elbow to the fingertips, and a land cubit as one cubit by a hundred. Egyptians recorded results with a decimal system; a stroke was 1, a heel bone was 10, a coil of rope was 100, and a lotus plant was 1000. However, it did not have a place value system.
- The Rhind mathematical papyrus outlined solutions to everyday problems and operations with Egyptian numbers. Their method of multiplying two numbers together introduced the use of binary. Hints of a geometric series were included as pieces of Horus' torn up eye. Egyptians found the area of a circle by approximating the area from known shapes, approximating pi.
- Egyptians used an instance of Pythagoras' theorem; 3 knots, 4 knots, and 5 knots with rope gave perfect right angled triangles. Egyptians calculated the volume of a truncated pyramid, which would be the first hint of calculus.
- Babylonians too used mathematics for practical problems and were written like recipes; a scribe would follow the instructions to get a result. One could add and subtract a certain arrangement of objects to isolate values.
- Babylonians used a base 60 number system and set up place value to chart the night sky. They also made a system of angular measurement; 360 degrees in a circle, 60 minutes in a degree, and 60 seconds in a minute. Babylonian engineers and surveyors used ingenious methods of transporting water to crop fields using mathematics.
- Babylonian mathematicians used quadratic equations for the first time when calculating areas of fields, but did not use symbols.
- Babylonians played a version of backgammon for over 5000 years, using arithmetic to outwit their opponent.2
- Plimpton 322 is a Babylonian tablet that shows the Babylonians knew that the square of a diagonal equals the sum of the squares of the other two sides. The Babylonians approximated the value of the square root of two, and etched it into the Yale Tablet.
- Pythagoras turned mathematics into an analytical subject, and opened a school in Samos. Pythagoras also shared multiple proofs of the Pythagorean Theorem. Pythagoras found that intervals between harmonious musical notes were always represented in integer ratios.
- Pythagoras assumed that the square root of 2 was a fraction, but was disproved by Hippasus, who noticed the importance of the square root of 2; its irrationality.
- Other schools opened up across Greece like Plato's Academy in Athens. Plato supported the Pythagorean world view, and believed mathematics was the bedrock of knowledge. Plato said that geometry is the key to unlocking the secrets of the universe in his book, *Timaeus*. Plato crystallized the universe into 5 symmetrical shapes, known as the Platonic solids.
- Alexandria became a hub of academic excellence under the rule of Ptolemy, but its library was destroyed when the Muslims conquered Egypt.
- Euclid's mathematics exists in his most famous textbook, *The Elements*, which contains the culminations of mathematical revelations in Greece. *The Elements* also proves that there can only be 5 Platonic Solids.
- Archimedes designed weapons of mass destruction that were used as protection. He produced formulas for regular shapes, such as circles and spheres. Archimedes was working on a theorem in the sand when he was murdered by a Roman soldier.
- Hypatia was a very prestigious and influential mathematician at her time; this combination made her a figure of hatred for Christian mobs. Hypatia was dragged off her chariot by Christian mobs and tortured and brutally murdered in a church.

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Episode 2 - The Genius of the East

- Mathematics has become ever present in today's world, and mathematics of the East gave birth to many principles of the modern world.
- The Great Wall of China was constructed to protect China's growing empire, and was built over China's rough and high countryside. The wall required calculations about distances, angles of elevation, and amount of material, inspiring mathematics in China.
- When writing numbers down, the Chinese didn't use a decimal place value system, they used symbols to denote powers of ten. However, the Chinese used a decimal place value system when calculating. Numbers held a different significance in China, representing real life concepts. The Chinese invented an early version of Sudoku called the Magic Square, in which numbers in a straight line add up to the same amount. Calculations of the calendar and lunar cycles dictated the emperor's plans, decisions, and sleep schedule, which was described through a geometric progression.
- A mathematical textbook, *The Nine Chapters*, was aimed at providing civil servants a competent understanding of mathematics. The book is a compilation of 246 practical problems and how to solve equations.
- The Chinese Remainder Theorem is where the remainder and divisor is given, and the dividend can be ascertained. This theorem has become an integral part of cryptography.
- The Golden Age of Mathematics started when 30 mathematics schools popped up across China, and mathematics was a long-established part of the curriculum. Qin Jiushao was the most important mathematician at the time. He was interested in cubic equations, and was able to approximate them.
- Indians had discovered the value of the decimal place value system and created precursors to the currently used nine numerals and the number zero. Zero was first recorded on the wall of a small temple in the fort of Gwalior in Central India. The Indians transformed zero from a placeholder to a number in its own right. The Indian mathematician Brahmagupta proved some essential properties of zero, and his principles of zero are taught all over the world today. Bhaskara II stated that dividing by zero gives infinitely many pieces, infinity.
- The Indians also invented negative numbers as a way to represent numbers less than zero, and referred to them as "debts".
- Brahmagupta also discovered that quadratic equations always have two solutions, and one could be negative. He also went on to solve quadratic equations with 2 unknown variables, which wouldn't be considered in the west until much later.
- The Indians also began further developing trigonometry, and used it to survey the land, navigate the seas, and chart the night sky. Furthermore, the Indians began to predict the sine of any angle.
- Madhava was an Indian mathematician who enjoyed working with infinite series and was exploring the connections between infinite series and trigonometry. He discovered that he could use infinity to get an exact value for pi.
- A great library called the "House of Wisdom" was constructed in the center of Baghdad, and had subjects such as astronomy, medicine, chemistry, zoology, and mathematics. Its director was a persian scholar named Muhammad Al-Khwarizmi, showed the Indian numerals were very efficient for calculations, and made algebra.
- A persian mathematician and poet by the name of Omar Khayyam tried to find the general method to solve all cubic equations, but wasn't able to find it because he was unable to separate the geometry from the numbers. Leonardo da Piza (Fibonacci) promoted the Hindu-Arabic numeral system in place of the Roman numeral system, and discovered the Fibonacci sequence, which is widely found in nature.
- Tartaglia found the general method to solve all types of cubic equations. He reluctantly shared his method with Cardano, who shared it to his student Ferrari, who found the solution to quartic equations.

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Episode 3 - The Frontiers of Space

- Piero della Francesca was a Renaissance artist and mathematician who used perspective perfectly in his painting, *The Flagellation of Christ*
- Descartes was a renowned French mathematician and philosopher who published a dictionary containing his philosophical views and his work with geometry. He developed the Cartesian plane, and linked geometry to algebra. He also laid out the potential for geometry in higher dimensions, but did not explore it.
- Marin Mersenne was a Parisian mathematician who did a lot of work with prime numbers and shared mathematical ideas. He urged people to read Descartes' and Fermat's work.
- Fermat had many conjectures and theorems that, among other things, laid out the foundations of modern number theory, on which modern technology is built on.
- Newton produced a new theory of light, discovered gravitation, and made calculus, which has wide reaching modern applications. He shared his thoughts with his friends instead of publishing.
- Leibniz also came up with his calculus shortly after Newton, developing integral and differential calculus in 2 months. He also worked for the Royal Family of Hanover, and had plenty of time to think about philosophy and logic. He also constructed one of the first practical calculating machines. He worked out all the details of his calculus in 5 years and published, much to the dismay of Newton. Leibniz was accused of plagiarism and was greatly discredited. Upon his death, he was buried in a small church, while Newton was buried in the grandeur of Westminster Abbey. In the end, Leibniz's mathematics eventually triumphed Newton's.
- The Bernoullis spread Leibniz' calculus through the scientific community, and applied mathematics to the world. The Bernoullis used calculus to calculate the quickest route a ball should take to get to the end of a ramp and created Calculus of the Variation. This form of calculus has many wide applications in today's world.
- Euler found his intellectual home in the Peter's Academy in Russia, and worked on topology and analysis as well as past mathematics. He also created a very famous modern equation, and applied his skill to prime numbers, optics, astronomy, music, etc. He also devised a new system of weights and measures and wrote a textbook on mechanics. Euler also solved a famous question with infinite sums.
- Joseph Fourier's work on sound waves lives on to today's world
- Carl Gauss was a revolutionary mathematician, coming up with theorems and his diary showed intimations of functions that were very advanced such as elliptic functions and the Riemann Zeta function. The square root of negative 1 was given the symbol i for imaginary number in 16th and 17th century Europe. Little of its importance was thought at the time, but imaginary numbers are key to understand pieces of today's world. While working on surveying the land of Hanover, Gauss thought of the shape of space and realized that Euclidean geometry only worked if the universe was flat. Gauss had made interesting discoveries concerning non-Euclidean geometry, but didn't want to cause controversy and consequently didn't publish.
- Bolyai Jonas made a type of geometry he called imaginary geometry, and has become hyperbolic geometry. In it, a triangle's angles can add up to less than 180 degrees.
- Bernhard Riemann was a brilliant mathematician who showed mathematical prowess at a very early age. Later in life, Riemann described what geometry actually was and sketched out what it could be. Although it wasn't completely understood at the time, Riemann's mathematics fundamentally changed how we see the world; introducing higher-dimensional geometry. Evidence of Riemann's work like hyperspace have been recognized in today's world.

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Episode 4 - To Infinity and Beyond

- David Hilbert set 23 defining mathematics problems in the 1900 International Congress of Mathematics. He set an agenda for 20th century mathematics and succeeded. He believed that all 23 of his problems would eventually be solved. He brought everything that was known about number theory together, revolutionized the theory of the integral equation, and proved that there are ways to divide all equations so that they are built up of a finite set. His work is still present in today's advanced mathematics. During Nazi Germany, he arranged for some of his brightest students to flee.
- Georg Cantor gave mathematical precision to infinity, and showed what infinity really was. Cantor compared types of infinity and proved that some infinities are the same as other infinities, and showed ways to count infinities. Cantor suffered from manic depression, and used the clinic as a way to recharge and tackle infinity. Cantor's study of the infinite was not well taken by others except for Henri Poincare.
- Henri Poincare was an exceptional mathematician with expertise in algebra, geometry, analysis, etc. He had an arsenal of mathematical techniques he would use to attempt problems and theorems. One of such techniques leading to the realization of chaos theory.
- A famous problem of the time, known as the Seven Bridges of Konigsberg, was proven that it was impossible by Leonhard Euler, but led to the creation of topology, which has become ever present in today's modern society.
- Kurt Godel studied math at the Vienna University, and came up with a revolutionary mathematical idea; the Incompleteness Theorem, which states that some statements are true but unprovable. Kurt Godel's pessimism grew as he attended the institute that opened up in Princeton. His pessimism turned into paranoia as time went by and he slowly stepped more and more into reclusion.
- Paul Cohen was a young mathematician who found that both answers to Cantor's continuum hypothesis were true, opening two branches of mathematics, each dependent on whether the continuum hypothesis is true and false. Cohen worked on the Riemann Hypothesis for the remainder of his life.
- Julia Robinson made history when she was the first female president of the American Mathematical Society. She set out to solve Hilbert's tenth problem. Julia developed the Robinson hypothesis, which argued that to show that no solution exist, one had to get an equation with very specific solutions. Yuri Matiyasevich built on the Robinson hypothesis and solved Hilbert's tenth problem.
- Evariste Galois believed that mathematics should be the study of structure. He used new techniques to tell if certain equations had solutions or not. His use of geometry to analyze equations proved very useful and was picked up by Andre Weil, who built on Galois' ideas and built algebraic geometry. Weil was able to connect number theory, algebra, geometry, and topology.
- Nicolas Bourbaki had written over 30 to 40 books about geometry, algebra, and other subjects. Bourbaki was actually the name for a French group of mathematicians led by Andre Weil, who decided to write a coherent account of the mathematics of the 20th century. The Bourbaki was handed to the next generation of French mathematicians, and the most exceptional of them was Alexandre Grothendieck. Grothendieck created a new mathematical language that has seen use in various parts of modern mathematics.