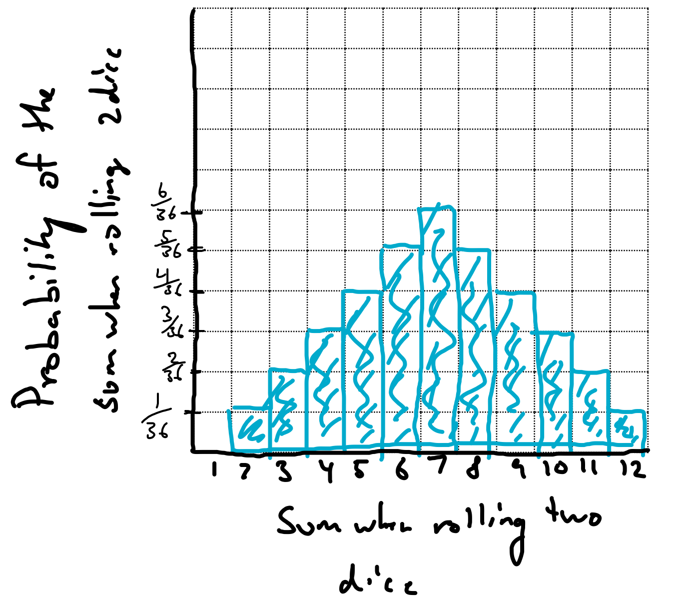
Date: ______________________ Name: Dhrumil Patel

MDM4U – Unit 5 Quiz: Pascal's Triangle & Probability Distributions /18K /1C (time)

1. Create a **probability distribution table** and **probability histogram** for the possible sums when rolling 2 dice:
{be sure to properly label the histogram} *<6 marks>*

x	P(X = x)
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

Distribution of Probabilities of Sums when Rolling 2 Dice



	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2. Use the probability distribution to find: *<6 marks>*
Show work for full marks

a) $P(X < 6)$ **b)** $P(X \neq 9)$ **c)** $P(X \text{ is prime})$

a)
$$P(X < 6) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$$
$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36}$$
$$= \frac{10}{36}$$
$$= \frac{5}{18}$$
$$\approx 0.278$$
$\therefore$ The probability that the sum is less than 6 is $\frac{5}{18}$ or 0.278.

b)
$$P(X \neq 9) = 1 - P(X=9)$$
$$= 1 - \frac{4}{36}$$
$$= \frac{32}{36}$$
$$= \frac{8}{9}$$
$\therefore$ The probability that the sum isn't 9 is $\frac{8}{9}$ or 0.889.

c)
prime = {2, 3, 5, 7, 11}
$$P(X \text{ is prime}) = P(X=2) + P(X=3) + P(X=5) + P(X=7) + P(X=11)$$
$$= \frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{2}{36}$$
$$= \frac{15}{36}$$
$$= \frac{5}{12}$$
$\therefore$ The probability that the sum is prime is $\frac{5}{12}$ or 0.4167.

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3. Use binomial expansion (combinations) to expand and simplify each of the following:

<4 marks>

a) $(a+b)^7$

$= \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^{i}$

$= \binom{7}{0} a^{7-0} b^{0} + \binom{7}{1} a^{7-1} b^{1} + \binom{7}{2} a^{7-2} b^{2} + \binom{7}{3} a^{7-3} b^{3} + \binom{7}{4} a^{7-4} b^{4} + \binom{7}{5} a^{7-5} b^{5} + \binom{7}{6} a^{7-6} b^{6}$
$+ \binom{7}{7} a^{7-7} b^{7}$

$= a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

$\therefore (a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7.$

b) $(x-3)^4$

$= \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^{i}$

$= \binom{4}{0} x^{4-0} (-3)^0 + \binom{4}{1} x^{4-1} (-3)^1 + \binom{4}{2} x^{4-2} (-3)^2 + \binom{4}{3} x^{4-3} (-3)^3 + \binom{4}{4} x^{4-4} (-3)^4$

$= x^4 + 4x^3(-3) + 6x^2(9) + 4x(-27) + 81$

$= x^4 - 12x^3 + 54x^2 - 108x + 81$

$\therefore (x-3)^4 = x^4 - 12x^3 + 54x^2 - 108x + 81$

4. Using **Pascal's Identity** write an expression that is equivalent to each of the following:

Determining an equivalent value just taken from Pascal's Triangle will result in ½ mark

<2 marks>

a) $\binom{26}{22} + \binom{26}{23}$

$\because \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

$\binom{26}{22} + \binom{26}{23} = \binom{26+1}{23}$

$\binom{26}{22} + \binom{26}{23} = \binom{27}{23}$

$\therefore \binom{26}{22} + \binom{26}{23} = \binom{27}{23}$

$\therefore$ An expression for $\binom{26}{22} + \binom{26}{23}$ is $\binom{27}{23}$.

b) $\binom{34}{16}$

$\because \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

$\binom{34}{16} + \binom{34}{17} = \binom{34+1}{17}$

$\binom{34}{16} + \binom{34}{17} = \binom{35}{17}$

$\therefore \binom{34}{16} = \binom{35}{17} - \binom{34}{17}$

$\therefore$ An expression for $\binom{34}{16}$ is $\binom{35}{17} - \binom{34}{17}$