MCV4U UNIT 4 Exponential and Trig

Name: _____ Date: _____

Knowledge (24)	Application (24)	Communication (13)

Show ALL work for full marks.

1. You open your newspaper and see 3 different ads from 3 different banks regarding investments and interest rates. Which of these should you invest in? How much more could you make after 2 years? Show supporting calculations. [9KA 3C]
   a. 3.17% interest rate, compounded every 3 months
   b. 3.2% yearly interest rate
   c. 3.16% interest rate, compounded continuously

a. $i = \frac{0.0317}{4}$

$n = 4 \times 2 = 8$

say $P = 1000$

$A = P(1+i)^n$

$= 1000\left(1+\frac{0.0317}{4}\right)^8$

$= \$1065.19$

b. $i = 0.032$

$n = 2$

$P = 1000$

$A = 1000(1.032)^2$

$= \$1065.02$

c. $r = 0.0316$

$t = 2$

$P = 1000$

$A = Pe^{rt}$

$= 1000e^{0.0632}$

$= 1065.24$

so you should invest with c as you make $0.05 ~ $0.22 more per $1000

v2. $1065.36, $1065.02, $1065.24

a is best, $0.34 & $0.12 per $1000

2. Find the derivative of the following function using the $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ Show all your steps. [3A 2C]

$f(x) = e^{3x}$

$f'(x) = \lim_{h\to 0}\frac{e^{3(x+h)} - e^{3x}}{h}$

$= \lim_{h\to 0}\frac{e^{3x}(e^{3h}-1)}{h}$

$= e^{3x}\lim_{h\to 0}\frac{e^{3h}-1}{h}$

use $s = 3h$

$\Rightarrow h = \frac{s}{3}$ and

$s \to 0$ as $h \to 0$

$\therefore f'(x) = e^{3x}\lim_{s\to 0}\frac{3e^s - 1}{s}$

$= 3e^{3x}\lim_{s\to 0}\frac{e^s - 1}{s}$

$= 3e^{3x}$

v2 $4e^{4x}$

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3. A strain of bacteria has a tripling time of 42 minutes. If we started off with 1000 bacteria.

a. Find an expression USING THE NATURAL EXPONENTIAL for the amount of bacteria we will have after t minutes. [3KA 2C] $b=3, d=42\text{min}, P_0=1000$

$b^{1/d}=e^k$

$\Rightarrow k=\frac{\ln b}{d}=\frac{\ln 3}{42\text{min}}=0.026\,1574/\text{min}$

so $P=P_0e^{kt}=1000e^{0.0261574t}$

1/2

$2000\,e^{0.02871087t}$

b. How many bacteria will we have after 3.5 hours? [3K 1A 2C]

$t=3.5\text{hrs}=210\text{min}$

so $P=1000e^{(0.0261574)(210)}=242798$ bacteria

≈ 86468

∴ there are 243000 bacteria after 3.5 hours.

c. At what time will there be 10800 bacteria? [3K 1A 1C]

$P=10800$

so $10800=1000e^{(0.0261574)(t)}$

$\Rightarrow t=\frac{\ln\left(\frac{10800}{1000}\right)}{0.0261574}=90.97\text{min}$

$\approx 5833\text{min}$

∴ you will have 10800 bacteria in about 90.97 min

d. What is the rate of growth at a time of 4 hours? You may use your knowledge of what the equation for the change in growth rate should be [3A 1C]

From "2" the derivative of dt is ke^{kt}

so $P'=1000\left[0.026574e^{0.0261574t}\right]$

$\text{so } P'(240)=1000\left[0.026574e^{(0.0261574)(240)}\right]$

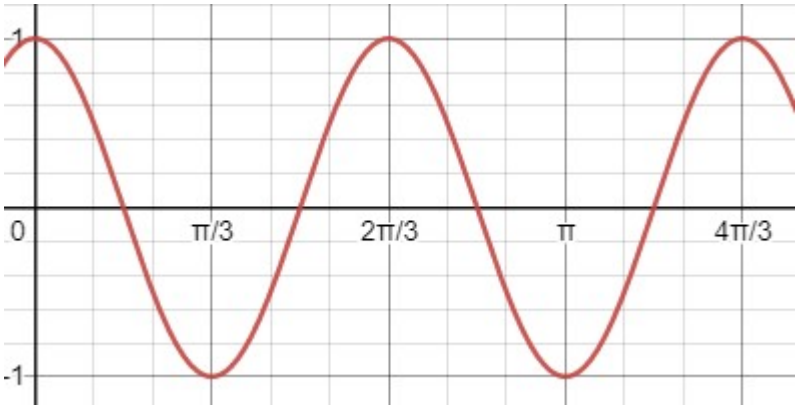
$=13931$ bacteria/min

$\approx 59637\ \frac{\text{bacteria}}{\text{min}}$

∴ the growth rate at 4hrs is $13931\ \frac{\text{bacteria}}{\text{min}}$

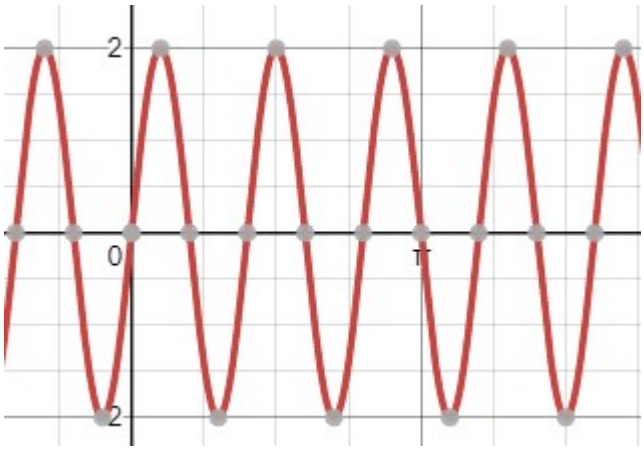
4. Below are two graphs of trigonometric functions. **Analyze** the function and, along with your knowledge of the derivatives of trigonometric functions, determine the derivative of each function. You DO NOT need to graph the function. [3K 2A 1C each]

a)



starts at 1
so $y = a\cos kx$
$a = 1, \frac{2\pi}{k} = \frac{2\pi}{3}$
so $y = \cos 3x$
$y' = -3\sin 3x$
(from assignment)

b)



starts at 0
so $y = a\sin kx$
$a = 2, \frac{2\pi}{k} = \frac{2\pi}{5}$
so $y = 2\sin 5x$
$y' = 10\cos 5x$

y_2 a) $-10\cos 5x$
b) $-2\sin 2x$