MCV4U - Unit 6 TEST CURVE SKETCHING 13K 16A 12C 5T

Give full/appropriate answers and justification for best marks.

1. Find the intervals of increase and decrease algebraically for: $y = -4x^3 + 15x^2 + 18x + 3$ [3K 2A 2C]

$y' = -12x^2 + 30x + 18$

$= -6(2x^2 - 5x - 3)$

$= -6(2x+1)(x-3)$

$y' = 0 \quad x = -\frac{1}{2} \quad x = 3$

x	$x < -\frac{1}{2}$	$-\frac{1}{2}$	$< x < 3$	3	$< x$
y'	$-$	0	$+$	0	$-$
y	$\searrow$	L Min	$\nearrow$	L Max	$\searrow$

y increasing for $x \in (-\frac{1}{2}, 3)$

decreasing $x \in (-\infty, -\frac{1}{2}) \cup (3, +\infty)$

2. Find the point(s) of inflection and intervals of concavity for: $f(x) = 3x^4 - 16x^3 + 24x^2 - 9$ [4K 2A 2C]

$f'(x) = 12x^3 - 48x^2 + 48x$

$f''(x) = 36x^2 - 96x + 48$

$= 12(3x^2 - 8x + 4)$

$= 12(3x-2)(x+2)$

$f''(x) = 0 \quad x = \frac{2}{3}, x = 2$

		$\frac{2}{3}$		2	
$f''(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\cup$ up	PoI	$\cap$ down	PoI	$\cup$ up

$f(\frac{2}{3}) = -\frac{67}{27} \quad f(2) = 7 \quad$ PoI $(\frac{2}{3}, -\frac{67}{27}), (2, 7)$

concave up $x \in (-\infty, \frac{2}{3}) \cup (2, +\infty)$

" down $x \in (\frac{2}{3}, 2)$

3. Use calculus and algebraic methods to do a complete analysis (i.e., intervals of increase and decrease, intercepts, critical points, points of inflection, and intervals of concavity) for $g(x) = (x+3)^3(x+5)$. You DO NOT need to graph.[6K 3A 3C]

$g'(x) = 3(x+3)^2(x+5) + (x+3)^3(1)$

$= (x+3)^2(4x+18) \quad g'(x) = 0$

$= 2(x+3)^2(2x+9) \quad x = -3 \quad x = -\frac{9}{2}$

CP $(-3, 0) \quad (-\frac{9}{2}, \frac{27}{16})$

$g''(x) = 4(x+3)(2x+9) + 2(x+3)^2(2)$

$= 4(x+3)(3x+12) \quad g''(x) = 0$

$= 12(x+3)(x+4) \quad x = -3, x = -4$

	$x < -\frac{9}{2}$	$-\frac{9}{2}$	$< x < -3$	-3	$< x$
$g'(x)$	$-$	0	$+$	0	$+$
$g(x)$	$\searrow$ dec.	L Min	$\nearrow$ inc		$\nearrow$ inc

$g(x)$ increasing $x \in (-\frac{9}{2}, -3) \cup (-3, +\infty)$

$g(x)$ decreasing $x \in (-\infty, -\frac{9}{2})$

	$x < -4$	-4	$< x < -3$	-3	$< x$
$g''(x)$	$+$	0	$-$	0	$+$
	$\cup$ up	PoI	$\cap$ down	PoI	$\cup$ up

$g(x)$ concave up $x \in (-\infty, -4) \cup (-3, +\infty)$

$g(x)$ concave down $x \in (-4, -3)$

$g(-4) = -1 \quad g(-3) = 0$

PoI $(-4, -1), (-3, 0)$

x int $(x+3)^3(x+5) = 0$

$x = -3 \quad x = -5$

x int $(-3, 0)$ and $(-5, 0)$

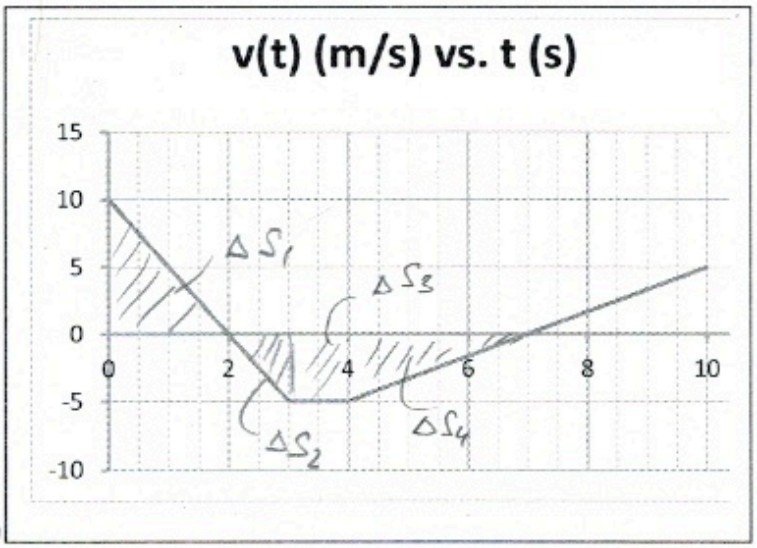
y int $y = (0+3)^3(0+5)$

$= 135$

y int $(0, 135)$

4. The graph shows the velocity-time graph of an object. Determine the displacement of the object AT KEY POINTS IN TIME from 0s to 10s. You can use the methods discussed in class and assume that s(0) = 0m. You DO NOT need to graph the displacement-time graph

[5A 2T 3C] $\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 \ldots$



Key points in time
$t = 0, 2, 3, 4, 7, 10.$

Area under v-t graph gives the displacement (change in position)

$\Delta S_1 = \frac{10 \times 2}{2} = 10\,m$

$\Delta S_2 = \frac{(-5)(1)}{2} = -2.5\,m$

$\Delta S_3 = (-5)(1) = -5$

$\Delta S_4 = \frac{(3)(-5)}{2} = -7.5$

$\Delta S_5 = \frac{(3)(5)}{2} = 7.5$

at $t = 3s$ $\Delta S_T = 10 - 2.5 = 7.5\,m$

$t = 4s$ $\Delta S_T = 7.5 - 5 = 2.5\,m$

$t = 7s$ $\Delta S_T = 2.5 - 7.5 = -5\,m$

$t = 10s$ $\Delta S_T = -5 + 7.5 = 2.5\,m$

5. Let $f(x) = ax^3+bx^2+cx+d$. Determine the values of a, b, c, and d if f(x) has a point of inflection at (0, 2) and a critical point at (2, 6). [4A 3T 2C]

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

(0, 2) P.O.I.

$f''(0) = 0$

$6a(0) + 2b = 0$

$\boxed{b = 0}$

$f(0) = 2$

$a(0)^3 + b(0)^2 + c(0) + d = 2$

$\boxed{d = 2}$

(2, 6) C.P

$f'(2) = 0$

$3a(2)^2 + c = 0$

$12a + c = 0$

$f(2) = 6$

$a(2)^3 + c(2) + 2 = 6$

$8a + 2c = 4$

$4a + c = 2$

$12a + c = 0$

$-4a + c = -2$

$8a = -2$

$\boxed{a = -\frac{1}{4}}$

$\boxed{c = 3}$

$\therefore a = -\frac{1}{4},\ b = 0,\ c = 3,\ d = 2$