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1.

$$y' = -12x^2 + 30x + 18$$
$$= -6(2x^2 - 5x + 3)$$
$$= -6(2x-3)(x-1)$$

Set y' to 0:

$$0 = -6(2x-3)(x-1)$$

if $2x-3=0$ if $x-1=0$

$x = \frac{3}{2}$ $x=1$

Factors of y'	$x<1$	$x=1$	$1<x<\frac{3}{2}$	$x=\frac{3}{2}$	$x>\frac{3}{2}$
$2x-3$	$-$		$-$	0	$+$
$x-1$	$-$	0	$+$		$+$
-6	$-$		$-$		$-$
y'	$-$		$+$		$-$
y inc/dec	dec		inc		dec

$\therefore$ y is increasing on $x \in (1, \frac{3}{2})$ and decreasing on $x \in (-\infty, 1)$ and on $x \in (\frac{3}{2}, \infty)$

2.

$$f'(x) = \frac{d}{dx}\left(3x^4 - 16x^3 + 24x^2 - 9\right)$$

$$= 12x^3 - 48x^2 + 48x$$

$$= 12x\left(x^2 - 4x + 4\right)$$

$$= 12x(x-2)^2$$

$$f''(x) = \frac{d}{dx}\left(12x^3 - 48x^2 + 48x\right)$$

$$= 36x^2 - 96x + 48$$

$$= 12\left(3x^2 - 8x + 4\right)$$

$$= 12(3x-2)(x-2)$$

Set $f''(x)$ to 0:

$$0 = 12(3x-2)(x-2)$$

if $3x-2=0$ $\quad$ if $x-2=0$

$\therefore x = \frac{2}{3}$ $\quad$ $\therefore x=2$

Sub $x=\frac{2}{3}$ and $x=2$ into $f(x)$:

$$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^4 - 16\left(\frac{2}{3}\right) + 24\left(\frac{2}{3}\right)^2 - 9$$

$$\simeq -2.481$$

$\therefore \left(\frac{2}{3}, -2.481\right)$ is a point of inflection

$f(2) = 3(2)^4 - 16(2) + 24(2)^2 - 9$
$= 7$

$\therefore (2, 7)$ is a point of inflection

factors of $f''(x)$	$x < \frac{2}{3}$	$x = \frac{2}{3}$	$\frac{2}{3} < x < 2$	$x = 2$	$x > 2$
$3x-2$	$-$	0	$+$		$+$
$x-2$	$-$		$-$	0	$+$
12	$+$		$+$		$+$
$f''(x)$	$+$		$-$		$+$
$f(x)$ concavity (up/down)	up		down		up

$\therefore f(x)$ is concave down on $x \in (\frac{2}{3}, 2)$ and concave up on $x \in (-\infty, \frac{2}{3})$ and $x \in (2, \infty)$

Set $g'(x) = 0$:

$$0 = 2(x+3)^2(2x+9)$$

if $(x+3)^2 = 0$ if $2x+9 = 0$

$(x+3)^2 = 0$ $2x = -9$

$\therefore x = -3$ $x = -\frac{9}{2}$

Sub $x = -3$ and $x = -\frac{9}{2}$ into $g(x)$:

$$g(-3) = (-3+3)^3(-3+5)$$

$$= 0$$

$\therefore (-3, 0)$ is a critical point

$$g\left(-\frac{9}{2}\right) = \left(-\frac{9}{2}+3\right)^3\left(-\frac{9}{2}+5\right)$$

$$\simeq -1.688$$

$\therefore \left(-\frac{9}{2}, -1.688\right)$ is a critical point

3.

$$g'(x) = 3(x+3)^2(x+5) + (1)(x+3)^3 \quad \text{(Using product rule)}$$

$$= (x+3)^2(3x+15+x+3)$$

$$= (x+3)^2(4x+18)$$

$$\therefore g'(x) = 2(x+3)^2(2x+9)$$

$$g''(x) = 2\left(2(x+3)(2x+9) + (2)(x+3)^2\right)$$

$$= 4(x+3)(2x+9+x+3)$$

$$= 4(x+3)(3x+12)$$

$$= 12(x+3)(x+4)$$

Set $g'(x) = 0$:

Sub $x=-3$ and $x=-\frac{9}{2}$ into $g''(x)$

$$g''(-3)=12(-3+3)(-3+4)$$
$$=0$$

$\therefore$ $(-3,0)$ is a point of inflection

$$g''(-\tfrac{9}{2})=12(-\tfrac{9}{2}+3)(-\tfrac{9}{2}+4)$$
$$=9$$

$\therefore$ $(-\frac{9}{2},-1.688)$ is a minimum

Set $g''(x)$ to 0:

$$0=12(x+3)(x+4)$$

if $x+3=0$ if $x+4=0$

$x=-3$ $x=-4$

Sub $x=-3$ and $x=-4$ into $g(x)$:

$$g(-3)=(-3+3)^3(-3+5)$$
$$=0$$

$\therefore$ $(-3,0)$ is a POI (previously calculated)

$g(-4) = (-4+3)^3(-4+5)$

$= -1$

$\therefore (-4, -1)$ is an inflection point

Set x to 0:

$g(0) = (0+3)^3(0+5)$

$= 135$

$\therefore (0, 135)$ is the y-intercept

Set g(x) to 0:

$0 = (x+3)^3(x+5)$

if $(x+3)^3 = 0$ if $x+5=0$

$(x+3)^3 = 0$ $\therefore x = -5$

$x+3=0$

$\therefore x = -3$

$\therefore (-3, 0)$ and $(-5, 0)$ are the x-intercepts.

Factors of g'	$x<-\frac{9}{2}$	$x=-\frac{9}{2}$	$-\frac{9}{2}<x<-3$	$x=-3$	$x>-3$
$(x+3)^2$	$+$		$+$	0	$+$
$2x+9$	$-$	0	$+$		$+$
2	$+$		$+$		$+$
g'	$-$		$+$		$+$
g inc/dec	dec		inc		inc

$\therefore$ g is decreasing on $x\in(-\infty,-\frac{9}{2})$ and increasing on $x\in(-\frac{9}{2},\infty)$

Factors of g''	$x<-4$	$x=-4$	$-4<x<-3$	$x=-3$	$x>-3$
$x+3$	$-$		$-$	0	$+$
$x+4$	$-$	0	$+$		$+$
12	$+$		$+$		$+$
g''	$+$		$-$		$+$
g concavity	up		down		up

$\therefore$ g is concave down on $x\in(-4,-3)$ and concave up on $x\in(-\infty,-4)$ and $x\in(-3,\infty)$

4.

For 0s to 3s:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 10}{3 - 0}$$

$$= -5$$

$\therefore v(t) = -5t$ from $t = 0s$ to $t = 3s$

$\therefore s(t) = -\frac{5}{2}t^2$ from $t = 0s$ to $t = 3s$.

At $t = 0$, $s(0) = 0$ m

At $t = 3$, $s(3) = -\frac{5}{2}(3)^2$

$$= -22.5 \text{ m}$$

$\therefore (0,0)$ and $(3, -22.5)$ are key points

For 3s to 4s

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - (-5)}{4 - 3}$$

$$= 0$$

$\therefore v(t) = 0t - 5$ from $t = 3s$ to $t = 4s$

$$= -5$$

$\therefore s(t) = -5t$ from $t = 3s$ to $t = 4s$

At $t = 3s$, $s(3) = -22.5$ m

At $t = 4s$, $s(4) = s(3) + (-5(1))$

$$= -27.5 \text{ m}$$

$\therefore (4, -27.5)$ is a key point

For $t = 4s$ to $t = 10s$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - (-5)}{10 - 4}$$
$$= \frac{5}{3}$$

$\therefore v(t) = \frac{5}{3}t$ from $t = 4s$ to $t = 10s$

$\therefore s(t) = \frac{5}{6}t^2$ from $t = 4s$ to $t = 10s$

At $t = 4s$, $s(4) = -27.5$ m

At $t = 10s$, $s(10) = s(4) + \frac{5}{6}(10)^2 - \frac{5}{6}(4)^2$
$= 42.5$ m

$\therefore (10, 42.5)$ is a key point

$\therefore$ The displacement function has three key points:

$(0,0)$, $(3, -22.5)$, $(4, -27.5)$, $(10, 42.5)$

5.

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

At $x = 0$, $f''(x) = 0$

$0 = 6a(0) + 2b$

$\therefore b = 0$

At $x = 2$, $f'(2) = 0$

$$\begin{cases} 0 = 3a(2)^2 + 2(0)(2) + c \\ 0 = 12a + c \end{cases}$$

$6 = 8a + 2c + 2$

$$\begin{cases} 4 = 8a + 2c \\ 0 = 24a + 2c \end{cases}$$

$-3 + c = 0$

$\therefore c = 3$

$-4 = 16a$

$\therefore a = -\frac{1}{4}$

Sub $(0,2)$ into f; $b = 0, c = 0$

$2 = a(0)^3 + d$

$\therefore d = 2$

Sub $(2,6)$ into f:

$6 = a(2)^3 + 2$

$4 = 8a$

$a = \frac{1}{2}$

$\therefore f(x) = -\frac{1}{4}x^3 + 3x + 2$