1.

a)

This is a binomial probability distribution.

b)

Let X be the number of times you roll a '1'.

$$P\begin{pmatrix}\text{winning LARGE} \\ \text{prize}\end{pmatrix} = P(X=5)$$

$$P(X=5) = \binom{n}{x} p^x q^{n-x}$$

Sub $n=5$, $x=5$, $p=\frac{1}{12}$, $q=1-p=\frac{11}{12}$

$$P(X=5) = \binom{5}{5}\left(\frac{1}{12}\right)^5\left(\frac{11}{12}\right)^0$$

$$= (1)\left(\frac{1}{12}\right)^5(1)$$

$$= \frac{1}{248832}$$

$\therefore$ The probability of winning the LARGE prize is

$$\frac{1}{248832}.$$

c)

Let $X$ be the number of times you roll a '1'.

$$P\begin{pmatrix}\text{Winning A}\\ \text{prize}\end{pmatrix} = P(X=3) + P(X=4) + P(X=5) = P(X \geq 3)$$

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$= \binom{n}{x} p^x q^{n-x} + \binom{n}{x} p^x q^{n-x} + \binom{n}{x} p^x q^{n-x}$$

Sub $n=5$, $x = 3, 4, 5$ respectively, $p = \frac{1}{12}$, $q = 1-p = \frac{11}{12}$

$$= \binom{5}{3}\left(\frac{1}{12}\right)^3\left(\frac{11}{12}\right)^2 + \binom{5}{4}\left(\frac{1}{12}\right)^4\left(\frac{11}{12}\right)^1 + \binom{5}{5}\left(\frac{1}{12}\right)^5\left(\frac{11}{12}\right)^0$$

$$= (10)\left(\frac{1}{1728}\right)\left(\frac{121}{144}\right) + (5)\left(\frac{1}{20736}\right)\left(\frac{11}{12}\right) + (1)\left(\frac{1}{248832}\right)(1)$$

$$= \frac{1210}{248832} + \frac{55}{248832} + \frac{1}{228832}$$

$$= \frac{1266}{248832}$$

$$= \frac{211}{41472} \simeq 5.09 \times 10^{-3}$$

$\therefore$ The probability of winning a prize is $\frac{211}{41472}$ or $5.09 \times 10^{-3}$.

2.

The amount you can expect to win is given by the expected value of the spinner.

Let X be the number spun.

$$E(x) = \sum_{i=1}^{n} x_i \, P(X = x_i)$$

$$= (0)\left(\tfrac{1}{10}\right) + \tfrac{1}{10}(1) + (0)\left(\tfrac{1}{10}\right) + \tfrac{1}{10}(2) + (0)\left(\tfrac{1}{10}\right) +$$

1 2 3 4 5

$$\tfrac{1}{10}(5) + (0)\left(\tfrac{1}{10}\right) + \tfrac{1}{10}(10) + (0)\left(\tfrac{1}{10}\right) + \tfrac{1}{10}(100)$$

6 7 8 9 10

$$= 11.8$$

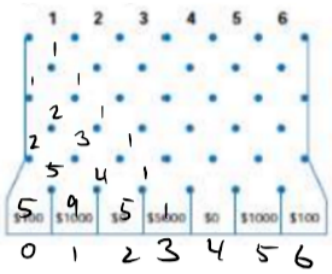
You paid \$1 to play the game, so you can expect to win

\$ Win = \$11.8 - \$1 = \$10.8

$\therefore$ You can expect to win \$10.8 or about \$11.

3.

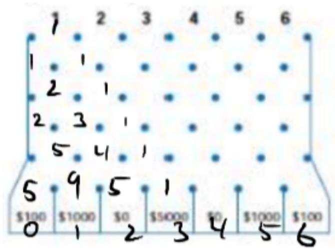
We number the 7 slots from 0 to 6.




We find the probability of landing in each slot using Pascal's Triangle. Each trial has a $\frac{1}{2}$ probability of going left or right, which stays constant throughout the trials.

i) From position 1.

Let X be the slot that the ball lands in.




→ If released from position 1, we track the number of ways for the ball to reach the $5000 slot using Pascal's Triangle. The number of ways to reach each slot would represent a binomial distribution if the walls did not exist. The probabilities are thus shifted.

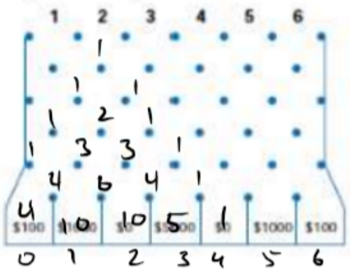
There is 1 way that the ball reaches the $5000 slot.

$$\therefore P(X = \$5000 \text{ slot} \mid \text{position } 1) = \frac{n(\text{ways to get } \$5000)}{n(\text{all ways})}$$

$$= \frac{1}{5+9+5+1} = \frac{1}{20}$$

∴ The probability that the ball lands in the \$5000 slot if it's released from position 1 is $\frac{1}{20}$.

ii) Similar reasoning in part i) applies. Let X be the slot that the ball lands in.




→ If released from position 2, we track the number of ways for the ball to reach the \$5000 slot using Pascal's Triangle. The number of ways to reach each slot would represent a binomial distribution if the walls did not exist. The probabilities are thus shifted.

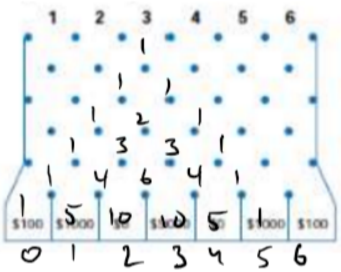
There are 5 ways that the ball reaches the \$5000 slot.

$$\therefore P(X = \$5000 \text{ slot} \mid \text{position } 2) = \frac{n(\text{ways to get } \$5000)}{n(\text{all ways})}$$

$$= \frac{5}{4+10+10+5+1} = \frac{5}{30} = \frac{1}{6}$$

∴ The probability that the ball lands in the \$5000 slot if it's released from position 2 is $\frac{1}{6}$.

iii) Similar reasoning in part i) applies. Let X be the slot that the ball lands in.




→ If released from position 3, we track the number of ways for the ball to reach the \$5000 slot using Pascal's Triangle. The number of ways to reach each slot represents a binomial distribution.

There are 10 ways that the ball reaches the \$5000 slot.

$$\therefore P(X = \$5000 \text{ slot} \mid \text{position } 1) = \frac{n(\text{ways to get } \$5000)}{n(\text{all ways})}$$

$$= \frac{10}{1+5+10+10+5+1} = \frac{10}{32} = \frac{5}{16}.$$

∴ The probability that the ball lands in the \$5000 slot if it's released from position 3 is $\frac{5}{16}$.