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Unit #6 Test – Continuous Probability Distributions

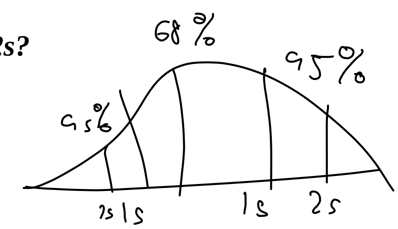
K/U: ____	Application: ____	Communication: ____	TOTAL: ____
9	21	9	39

Knowledge/Understanding: 9 marks

Multiple Choice: Circle the appropriate answer <7 marks>

$\frac{95-68}{2} = 13.5$

1. **What percent of data lies between x + *1s* and *x* +*2s*?**
   a) 13.5%
   b) 34%
   c) 68%
   d) 81.5%



2. **The Normal Distribution is written in the form:**
   a) $X \sim N\left(s^2, \bar{x}\right)$ b) $X \sim N\left(\bar{x}, s\right)$ c) $X \sim N\left(x, s^2\right)$ d) $X \sim N\left(\bar{x}, s^2\right)$

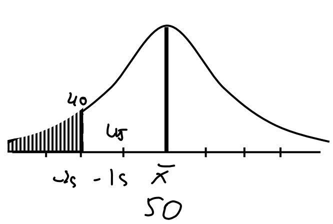
3. **Student scores on a math test are normally distributed. If the probability that Sarah passes this test is 0.8451, then the probability that she gets less than a 50 is:**
   a) 0.8451
   b) 0.5000
   c) 0.1549
   d) 1.8451

$1 - 0.8451 = 0.1549$

4. **A positive z-score indicates**
   a) that the value is located to the right of the mean
   b) that the value is located to the left of the mean
   c) that there has been a calculation error
   d) The probability represents a failure

5. **To use the Normal Approximation of the Binomial Distribution which conditions must be met?**
   a) $np \geq 5$ and $nq \leq 5$
   b) $np \leq 5$ and $nq \leq 5$
   c) $\bar{x}s^2 \leq 5$ and $\bar{x}s^2 \leq 5$
   d) $np \geq 5$ and $nq \geq 5$

6. **If X~N(50, 5^2), which of the following intervals is represented by the diagram:**
   a) x ≤ 50
   b) x > 40
   c) x ≤ 40
   d) x > 60



Name:____

7. **If Andy's final exam mark is in the 77th percentile that would imply**

a) He got 77% on the exam
b) 77 students wrote the exam
c) His mark was higher than 77% of the students writing the exam
d) 77% of students writing the exam did better than him

Problems

8. Determine the number of standard deviations the piece of data below lies above or below the mean:

<1 mark>

$X \sim N(6,3^2),\ x = 0.8$

$$z = \frac{x-\bar{x}}{s} = \frac{0.8-6}{3} = -1.73$$

$\therefore$ 0.8 is 1.73 standard deviations below the mean.

9. Given a normally distributed data set whose mean is 50 and standard deviation is 10, what value of x would the following z-score have:

<1 mark>

z = -0.74

$$z = \frac{x-\bar{x}}{s}$$

$$x = sz + \bar{x} = (10)(-0.74) + 50 = 42.6$$

$\therefore$ A value of x of 42.6 would yield a z-score of -0.74.

Application: 21 marks

FOR THE FOLLOWING QUESTIONS, BE SURE TO SHOW ALL YOUR WORK TO RECEIVE FULL MARKS!

1. The amount of coffee an automatic machine dispenses (in ounces) can be represented by the normal distribution X~N(10.2, 0.4²).

a) What range does 68% of the amount of coffee dispensed lie between (answer in ounces)?

<1 mark>

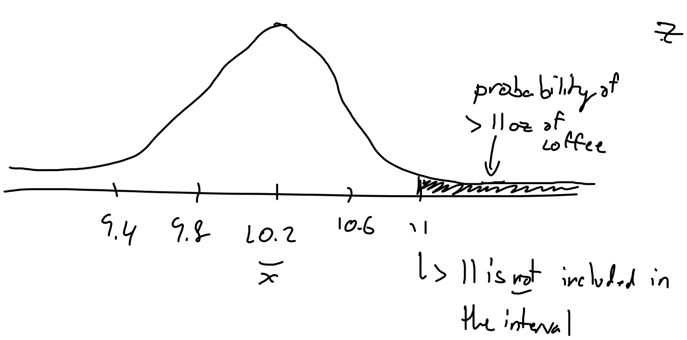
$\because$ $-1s$ to $+1s$ encompasses 68% of the data:

$10.2 - 0.4 = 9.8$ $(\bar{x} - 1s)$
$10.2 + 0.4 = 10.6$ $(\bar{x} + 1s)$

$\therefore$ 68% of the amount of coffee dispensed lies between 9.8 ounces and 10.6 ounces (0.8oz range)

b) Draw a diagram that represents the percent of cups dispensed that contain more than 11 oz. of coffee.

<1 mark>



$$z = \frac{x-\bar{x}}{s} = \frac{11-10.2}{0.4} = 2$$

$\therefore$ 11 is 2 standard deviations above the mean (10.2)

Name:____

2. Ella has a bag full of marbles. She has 12 red, 13 yellow and 20 blue marbles. Ella takes a marble from the bag, puts it back, and repeats this process until she has chosen 20 marbles in total. Determine the probability that she chooses:

a) exactly 5 blue marbles.

<2 marks>

Let x be the # of blue marbles

$n = 12 + 13 + 20 = 45$ marbles

$p = \frac{20}{45} \quad q = 1 - p = 1 - \frac{20}{45} = \frac{25}{45}$

$P(X=5) = \binom{n}{x} p^x q^{n-x}$

$= \binom{20}{5}\left(\frac{20}{45}\right)^5\left(\frac{25}{45}\right)^{15} = 0.0399 \approx 0.040$

∴ There is a 0.040 probability that she chooses exactly 5 blue marbles.

b) at least 11 blue marbles, using the **normal approximation of a binomial distribution.**

<6 marks>

Let X be the number of blue marbles

Check

$np = (20)\left(\frac{20}{45}\right) = 8.88 > 5$ ✓

$\bar{x} = np$

$nq = (20)\left(\frac{25}{45}\right) = 11.11 > 5$ ✓

$s = \sqrt{npq}$

$= \sqrt{(20)\left(\frac{20}{45}\right)\left(\frac{25}{45}\right)}$

$= 2.22$

$\therefore X \sim N(8.88, 2.22^2)$

10.5 11 11.5

∴ We calculate $P(X \leq 10.5)$ to include 11 in the $P(X \geq 11)$ calculation

$z_{10.5} = \frac{x - \bar{x}}{s}$

$= \frac{10.5 - 8.88}{2.22}$

$= 0.73$ ∴ 10.5 is 0.73 standard deviations above the mean

$\therefore P(X \leq 10.5) = 0.7673$ (From the table)

$P(X \geq 11) = 1 - P(X \leq 10.5)$

$= 1 - 0.7673$

$= 0.2327$

∴ There is a 0.2327 probability that she chooses at least 11 blue marbles.

Name:____

3. Salmon in a lake have a mean length of 30 cm and a standard deviation of 6 cm. Tyson and Becky are in a fishing derby. Tyson catches a Salmon that is 29 cm long.

a) What percentile is his catch in?

<3 marks>

$z = \frac{x-\bar{x}}{s}$ Let x be the length of his catch

$= \frac{29-30}{6}$

$\doteq -0.17$

$P(\text{catch} \le -0.17) = 0.4325 = 43.25\%$

$\therefore$ His catch is in the 43^{rd} percentile

b) If Becky caught a fish with a length greater than 40% of the others, how long would her catch be?

Let X be the length of her catch

<4 marks>

$z = \frac{x-\bar{x}}{s}$

$-0.25 = \frac{x-30}{6}$ $\because$ The z-score closest to $40\% = 0.4$ is -0.25

$x - 30 = 6(-0.25)$

$\therefore x = 28.5$ cm

$\therefore$ Her catch is 28.5 cm long.

4. IQs are normally distributed with a mean of 100 and a standard deviation of 15. Cassandra has an IQ of 125. If there are 850 students at her school, how many of them have an IQ greater than Cassandra's?

<4 marks>

Let X be the IQ of a student at her school.

$n = 850$

$\bar{x} = 100$

$s = 15$

$\therefore X \sim N(100, 15^2)$

$125/z$

124.5 125 125.5

$\therefore$ We use 125.5 to exclude 125 from the $P(X > 125)$ calculation

$z = \frac{x-\bar{x}}{s}$

$= \frac{125.5-100}{15}$

$= 1.70$

$P(X \le 125.5) = 0.9554$

$\therefore P(X > 125) = 1 - P(X \le 125.5)$

$= 1 - 0.9554$

$= 0.0446$

$n\begin{pmatrix}\text{students with a} \\ \text{higher IQ}\end{pmatrix} = (850)(0.0446)$

$= 37.91 \doteq 38$

$\therefore$ There are about 38 students with an IQ greater than Cassandra's.

Name:____

Communication: 9 marks

1. When can we use a normal distribution to approximate data that is discrete? *<2 marks>*

We can use a normal distribution to approximate discrete data when that data can be described by a roughly symmetrical probability distribution. For data that has a binomial probability distribution, we use the conditions $np \geq 5$ and $nq \geq 5$ to ensure that there is no left or right skew that would otherwise invalidate the normal distribution approximation. Suppose you roll a die 6 times and want the probability of rolling at least 3 2's, the probability distribution has too much skew (imagine rolling a 5 twice and a 3 4 times) to be approximated as a normal distribution. Here, $np = 6\left(\frac{1}{6}\right) = 1$. Since $1 \not\geq 5$, the data is too skewed to be symmetric

2. List and explain three characteristics of a normal distribution. *<4 marks>*

Symmetrical

– The normal distribution is symmetrical around the mean. This provides a baseline value to calculate probabilities and ensures that high values (outliers) in either the positive or negative directions are treated similarly. i.e. a data point that is −2.8 standard deviations from the mean is just as much an "outlier" as a point that is 2.8 standard deviations from the mean. This helps calculate extremes for determining if a data point is "notable" or not.

Continuous and Positive

– The normal distribution is continuous, so it is defined on all possible values ($-\infty$ to $+\infty$). This ensures that the probability of any interval of data points can be calculated (even if it is 0.0000...001 or 0.999...994). The normal distribution is also always positive, so probabilities calculated will always be valid.

Area under the curve is 1

– This characteristic is integral to using the normal distribution to find probabilities. Since the area of the entire distribution is 1, the probability of a value is calculated as the area under the curve from $-\infty$ to that value (Use integrals from calculus). This ensures that the calculated probability of a data point is a valid probability and, when combined with the other characteristics of the normal distribution, enables the normal distribution's widespread use.

Communication:	Level 1	Level 2	Level 3	Level 4	Mark
Use of conventions, vocabulary, and terminology of the discipline in visual and written forms. ie. Using equal signs appropriately	Uses conventions, vocabulary, and terminology of the discipline with limited effectiveness.	Uses conventions, vocabulary, and terminology of the discipline with some effectiveness.	Uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness.	Uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness.	____ 2

Complete the test within allotted time: /1C