1.

How many numbers $< 10{,}000$ have at least one "2" in them?

We consider n(natural numbers $< 10{,}000$) $= 9{,}999$. ~ The question asks for natural numbers that are strictly less than 10,000. (1-9999)

We can employ complementary counting to find n(number of natural numbers $< 10{,}000$ with no 2's).

$\therefore$ n(natural numbers $< 10{,}000$ that have at least one "2") $=$ n(natural numbers $< 10{,}000$) $-$ n(number of natural numbers $< 10{,}000$ with no 2's).

n(number of natural numbers $< 10{,}000$ with no 2's)

$=$ n(one-digit natural numbers with no 2's) $+$ n(two-digit natural numbers with no 2's) $+$ n(three-digit natural numbers with no 2's) $+$ n(four-digit natural numbers with no 2's)

n(one-digit natural numbers with no 2's) $= 8$ (1 - 9, excluding 2)

$\therefore$ There are 8 one-digit natural numbers with no 2's n (two-digit natural numbers with no 2's)

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8 9 ~ 8 options for first digit (1-9, excluding 2)

~ 9 options for second digit (0-9, excluding 2)

$8 \times 9 = 72$

∴ n (two-digit natural numbers with no 2's) = 72

∴ There are 72 two-digit natural numbers with no 2's.

n(three-digit natural numbers with no 2's)

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8 9 9 ~ 8 options for first digit (1-9, excluding 2)

~ 9 options for second digit (0-9, excluding 2)

~ 9 options for third digit (0-9, excluding 2)

$8 \times 9 \times 9 = 648$

∴ n(three-digit natural numbers with no 2's) = 648

$\therefore$ There are 648 three-digit natural numbers with no 2's

n(four-digit natural numbers with no 2's)

$\underline{8}\ \underline{9}\ \underline{9}\ \underline{9}$ ~ 8 options for first digit (1–9, excluding 2)

~ 9 options for second digit (0–9, excluding 2)

~ 9 options for third digit (0–9, excluding 2)

~ 9 options for fourth digit (0–9, excluding 2)

$8 \times 9 \times 9 \times 9 = 5832$

$\therefore$ n(three-digit natural numbers with no 2's) = 5832

$\therefore$ There are 5832 three-digit natural numbers with no 2's

n(number of natural numbers < 10,000 with no 2's)

= n(one-digit natural numbers with no 2's) + n(two-digit natural numbers with no 2's)

+ n(three-digit natural numbers with no 2's) + n(four-digit natural numbers with no 2's)

$= 8 + 72 + 648 + 5832$

$= 6560$

$\therefore$ n(number of natural numbers < 10,000 with no 2's) = 6560

n(natural numbers < 10,000 that have at least one "2") = n(natural numbers < 10,000) - n(number of natural numbers < 10,000 with no 2's)

$= 9999 - 6560$

$= 3439$

$\therefore$ There are 3439 natural numbers less than 10000 that have at least one "2".

2.

Let k be "DA".

"CANADIAN" becomes "kCNAIAN".

The number of arrangements of the new word will be equal to the number of arrangements of "CANADIAN" if the D immediately precedes the 'A'.

$$n(\text{arrangements}) = \frac{7!}{2!\,2!} \quad (2 \text{ A's}, 2 \text{ N's}$$

$$= 1260$$

$\therefore$ There are 1260 8-letter 'words' that can be formed from the "CANADIAN" so that the "D" always precedes the "A".