MCV4U Unit 1 Test

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Unit 1: VECTORS	Knowledge (18)	Communication (9)	Application (15)

Show ALL work for full marks.

1. Given u =(2, 4, –3), v =(3, –1, 7), and find [K(2,2,3,3)]

a) $3\vec{u} - 4\vec{v}$

$= 3(2,4,-3) - 4(3,-1,7)$

$= (6,12,-9) + (-12,4,-28)$

$= (6-12, 12+4, -9-28)$

$= (-6, 16, -37)$

b) $|\vec{u} - \vec{v}|$

$= |(2,4,-3) - (3,-1,7)|$

$= |(2-3, 4-(-1), -3-7)|$

$= |(-1, 5, -10)|$

$= \sqrt{(-1)^2 + 5^2 + (-10)^2} = \sqrt{126} = 3\sqrt{14}$

c) $\vec{u} \cdot \vec{v}$

$= (2)(3) + (4)(-1) + (-3)(7)$

$= 6 - 4 - 21$

$= -19$

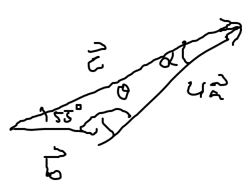
d) $\vec{u} \times \vec{v}$

$= (28-3, -9-14, -2-12)$

$= (25, -23, -14)$

$$\begin{matrix} 4 & -3 & 2 & 4 \\ -1 & 7 & 3 & -1 \end{matrix}$$

2. $|\vec{a}|$ = 5, and $|\vec{b}|$ = 8, and the angle formed by $\vec{a}$ and $\vec{b}$ is 55°. Determine $|4\vec{a} + \vec{b}|$. Round your answer to 2 d.p. [K4]



$\vec{c} = 4\vec{a} + \vec{b}$

$|\vec{c}| = |4\vec{a} + \vec{b}|$

$4|\vec{a}| = 4(5)$
$= 20$

$\frac{\sin 55°}{20} = \frac{\sin \alpha}{8}$

$\sin \alpha = \frac{8 \sin 55°}{20}$

$\alpha = 19.13°$

$\theta = 180° - 55° - \alpha = 105.87°$

$|4\vec{a} + \vec{b}|^2 = |4\vec{a}|^2 + |\vec{b}|^2 - 2|4\vec{a}||\vec{b}| \cos\theta$

$= 20^2 + 8^2 - 2(20)(8)\cos 105.87°$

$|4\vec{a} + \vec{b}| = \sqrt{400 + 64 - (40)(8)\cos 105.87°}$

$= 23.48$

$\therefore$ The magnitude of $4\vec{a} + \vec{b}$ is $|4\vec{a} + \vec{b}| = 23.48$

3. Determine the value(s) of k such that the angle between the vectors a = (1, 1, k) and b = (1, 0, 1) is 45°. [K4]

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a} \cdot \vec{b} = (1,1,k) \cdot (1,0,1)$
$= (1)(1) + (1)(0) + (1)k$
$= k + 1$

$|\vec{a}| = \sqrt{1^2 + 1^2 + k^2}$
$= \sqrt{k^2 + 2}$

$|\vec{b}| = \sqrt{1^2 + 0^2 + 1^2}$
$= \sqrt{2}$

$\cos 45° = \frac{k+1}{(\sqrt{k^2+2})(\sqrt{2})}$

$\frac{\sqrt{2}}{2} = \frac{k+1}{(\sqrt{k^2+2})\sqrt{2}}$

$\rightarrow 1 = \frac{k+1}{\sqrt{k^2+2}}$

$\sqrt{k^2+2} = k+1$

$k^2 + 2 = k^2 + 2k + 1$

$\rightarrow 2k + 1 = 2$

$2k = 1$

$k = \frac{1}{2}$

$\therefore$ The value of k that makes the angle between $\vec{a}$ and $\vec{b}$ $45°$ is $k = \frac{1}{2}$.

MCV4U Unit 1 Test

$a_y \; a_z \; a_x \; a_y$
$b_y \; b_z \; b_x \; b_y$
$b_y \; b_z \; b_x \; b_y$
$c_y \; c_z \; c_x \; c_y$

4. Prove whether $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$ is true or not for all vectors $\vec{a}, \vec{b}, and\ \vec{c}$ [C3]

Let $\vec{a} = (a_x, a_y, a_z)$, $\vec{b} = (b_x, b_y, b_z)$, $\vec{c} = (c_x, c_y, c_z)$

$\vec{a} \cdot (\vec{b} \times \vec{c})$

$= \vec{a} \cdot (b_y c_z - b_z c_y, b_z c_x - b_x c_z, b_x c_y - b_y c_x)$

$= a_x(b_y c_z - b_z c_y) + a_y(b_z c_x - b_x c_z) + a_z(b_x c_y - b_y c_x)$ ②

$(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$

$= (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x) \cdot (a_y c_z - a_z c_y, a_z c_x - a_x c_z, a_x c_y - a_y c_x)$

① $= (a_y b_z - a_z b_y)(a_y c_z - a_z c_y) + (a_z b_x - a_x b_z)(a_z c_x - a_x c_z) + (a_x b_y - a_y b_x)(a_x c_y - a_y c_x)$

$\therefore$ ① and ② are not the same. $\therefore \vec{a} \cdot (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$

5. Determine the value(s) of x such that the area of the parallelogram formed by the vectors a = (x + 1, 1, −2) and b = (x, 3, 0) is $\sqrt{41}$. [A4]

$1 \; -2 \; x+1 \; 1$
$3 \; 0 \; x \; 3$

$A = |\vec{a} \times \vec{b}|$

$\sqrt{41} = |(0 - (-2)(3), -2x - 0, 3(x+1) - x)|$

$\sqrt{41} = |(6, -2x, 2x+3)|$

$\sqrt{41} = \sqrt{6^2 + (-2x)^2 + (2x+3)^2}$

$41 = 36 + 4x^2 + 4x^2 + 12x + 9$

$8x^2 + 12x + 4 = 0$

$2x^2 + 3x + 1 = 0$

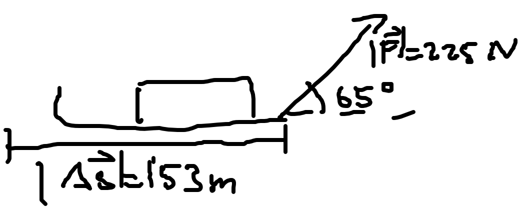
$(2x+1)(x+1) = 0$

if $2x+1=0$: $2x+1=0$, $x = -\frac{1}{2}$

if $x+1=0$: $x+1=0$, $x=-1$

$\therefore$ $x = \frac{1}{2}$ and $x=-1$ are the values of x that make the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ $\sqrt{41}$.

6. Chris pulls a sled 153m by exerting a constant force of 225N at a constant angle of 65° to the level ground. Find the work done in pulling the sled, correct to 2 d.p. [3A 2C]



$W = \vec{F} \cdot \Delta\vec{s}$

$= |\vec{F}||\Delta\vec{s}|\cos\theta$

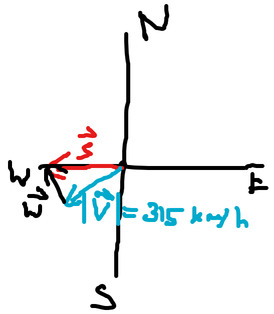
$= (225)(153)\cos 65°$

$= 14\,548.63\ J$

$\therefore$ The work done in pulling the sled is 14548.63 J

MCV4U Unit 1 Test

7. The nose of a plane is pointing west with an airspeed of 350 km/h. The plane's resultant ground velocity is 315 km/h [S75°W]. Determine the speed and direction of the wind, correct to 2 d.p. Include a labeled diagram with your solution. [4A 2C]



$\vec{s} = \vec{v} + \vec{w}$

$\vec{w} = \vec{s} - \vec{v}$

	x	y
$\vec{v}$	$-(315\cos 15^\circ)$ $= -304.27$	$-(315\sin 15^\circ)$ $= -81.53$
$\vec{s}$	-350	0

$\vec{s} - \vec{v} = (-350 - (-304.27), 0 - (-81.53))$

$\vec{w} = (-45.73, 81.53)$

$|\vec{w}| = \sqrt{(-45.73)^2 + (81.53)^2}$

$= 93.45$ km/h

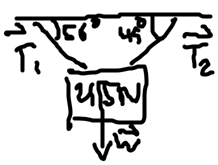
$\theta_w = \tan^{-1}\left(\frac{w_y}{w_x}\right)$

$= \tan^{-1}\left(\frac{81.53}{-45.73}\right)$

$= 60.71^\circ$

$\therefore$ The wind is blowing at 93.45 km/h at W60.71°N.

8. A box weighing 415 N is hanging from two chains attached to an overhead beam at angles of 56° and 49°. Find the magnitude of the tension in each chain algebraically, correct to 2 d.p. [4A 2C]



$\vec{T_1} + \vec{T_2} + w = 0$

$|\vec{T_1}|\cos 56^\circ = |\vec{T_2}|\cos 45^\circ$

$|\vec{T_1}| = \frac{|\vec{T_2}|\left(\frac{\sqrt{2}}{2}\right)}{\cos 56^\circ}$

$|\vec{T_1}| = \frac{(236.41)\frac{\sqrt{2}}{2}}{\cos 56^\circ}$

$= 298.94$N

$|\vec{T_1}|\sin 56^\circ + |\vec{T_2}|\sin 45^\circ = 415$

$\left(\frac{|\vec{T_2}|\cos 45^\circ}{\cos 56^\circ}\right)\sin 56^\circ + |\vec{T_2}|\sin 45^\circ = 415$

$|\vec{T_2}|\left(\left(\frac{\cos 45^\circ}{\cos 56^\circ}\right)\sin 56^\circ + \sin 45^\circ\right) = 415$

$|\vec{T_2}| = \frac{415}{\left(\frac{\cos 45^\circ}{\cos 56^\circ}\right)\sin 56^\circ + \sin 45^\circ}$

$= 236.41$ N

$\therefore$ The magnitude of the tension in each chain is 298.94N and 236.41 N for the 56° chain and 45° chain respectively.