1.

a)

$\vec{m} = (-2, 1, 3)$ is the direction vector of the line $\vec{r}$.

The symmetric equation can have the same direction vector.

$$\frac{x - P_x}{m_x} = \frac{y - P_y}{m_y} = \frac{z - P_z}{m_z}$$

Sub $P(5, 6, 10)$ and $\vec{m} = (-2, 1, 3)$:

$$\frac{x-5}{-2} = \frac{y-6}{1} = \frac{z-10}{3}$$

$\therefore \frac{5-x}{2} = y - 6 = \frac{z-10}{3}$ is a symmetric equation for the line.

b)

Let $x = 0$,

$$\frac{5-0}{2} = y - 6 = \frac{z-10}{3}$$

$y - 6 = \frac{5}{2}$

$y = \frac{5}{2} + 6$

$= \frac{17}{2}$

$\frac{z-10}{3} = \frac{5}{2}$

$z - 10 = \frac{15}{2}$

$z = \frac{35}{2}$

$\therefore \left(0, \frac{17}{2}, \frac{35}{2}\right)$ is a point on the line.

Let $x = 1$,

$$\frac{5-1}{2} = y - 6 = \frac{z-10}{3}$$

$y - 6 = 2$

$y = 8$,

$\frac{z-10}{3} = 2$

$z - 10 = 6$

$z = 16$

$\therefore (1, 8, 16)$ is a point on the line.

2.

$\vec{n_1} = (1, 1, -1) \qquad \vec{n_2} = (2, 4, -3)$

$\vec{n_1} \cdot \vec{n_2} = (1)(2) + (1)(4) + (-1)(-3)$

$= 2 + 4 + 3$

$= 9$

$\because \vec{n_1} \times \vec{n_2} \neq 0,$

$\therefore$ The two planes are not parallel and intersect at a line.

$x + y - z + 12 = 0 \quad ①$

$2x + 4y - 3z + 8 = 0 \quad ②$

$3① - ②$:

$x - y + 28 = 0$

$x = y - 28$

$4① - ②$:

$2x - z + 40 = 0$

$x = \dfrac{z - 40}{2}$

$\therefore x = y - 28 = \frac{z-40}{2}$ is a symmetric equation for the line.

$\vec{m} = (1, 1, 2)$, $\vec{p} = (0, 28, 40)$

Sub $\vec{m} = (1, 1, 2)$ and $\vec{p}$ $(0, 28, 40)$ into a line:

$\vec{r} = (0, 28, 40) + t(1, 1, 2)$

$\therefore \vec{r} = (0, 28, 40) + t(1, 1, 2)$ is the line of intersection between the two planes.

3.

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Let d be the distance between the point and the plane

$$= \frac{|3(1) + 0(2) + (-4)(3) - 1|}{\sqrt{3^2 + 0^2 + (-4)^2}}$$

$$= \frac{|3 - 12 - 1|}{5}$$

$\therefore d = 2.00$

$\therefore$ The distance between the point and the plane is 2.00 units.

4.

$x+3y+4z=10$ ①

$2x+4y-3z=23$ ②

$3x-y+6z=-4$ ③

② $-$ 2① :

$$
\begin{array}{r}
2x+4y-3z=23 \\
-(2x+6y+8z=20) \\
\hline
-2y-11z=3
\end{array}
$$

$2y+11z=-3$ ④

③ $-$ 3① :

$$3x - y + 6z = -4$$
$$-(3x + 9y + 12z = 30)$$

$$-10y - 6z = -34$$

$$5y + 3z = 17 \quad (5)$$

2 (5) − 5 (4):

$$10y + 6z = 34$$
$$-(10y + 55z = -15)$$

$$-49z = 49$$

$$\therefore z = -1$$

Sub $z = -1$ into (5):

$5y+3(-1)=17$

$\therefore y=4$

Sub $y=4$, $z=-1$ into ①:

$x+3(4)+4(-1)=10$

$x+12-4=10$

$\therefore x=2$

$\therefore (x,y,z)=(2,4,-1)$ is the solution to the system of equations and is the intersection of the three planes.

5.

$\vec{n} = (1, 1, -1)$ is the normal vector of the plane.

$\vec{m} = (4, -3, -2)$ is the direction vector of the plane.

$\vec{n} \cdot \vec{m} = (1)(4) + (1)(-3) + (-1)(-2)$

$= 4 - 3 + 2$

$= 3$

$\because \vec{n} \cdot \vec{m} \neq 0,$

$\therefore$ The line and the plane are not parallel and intersect at a point.

Line (Parametric Eq's)

$x = 1 + 4t$

$y = -2 - 3t$

$z = 1 - 2t$

Sub x, y, z of the line into the plane to solve for t.

$x + y - z = 1$

$(1+4t) + (-2-3t) - (1-2t) = 1$

$\cancel{1} + 4t - 2 - 3t \cancel{-1} + 2t = 1$

$3t = 3$

$t = 1$

Sub $t=1$ into line:

$(x, y, z) = (1, -2, 1) + (1)(4, -3, -2)$

$= (5, -5, -1)$

Sub $(5, -5, -1)$ into plane:

$x+y-z=1$

LHS	RHS
$(5)+(-5)-(-1)$	1
$=1$	

$\therefore$ LHS = RHS,

$\therefore$ $(5,-5,-1)$ is the point of intersection of the line and the plane.

6.

$\vec{n} = (7, -3)$ is a normal vector for the line.

$7x - 3y + C = 0$

Sub $(x, y) = (-1, 2)$:

$7(-1) - 3(2) + C = 0$

$-7 - 6 + C = 0$

$C = 13$

$\therefore$ $7x - 3y + 13 = 0$ is the scalar equation for the line.

7. A direction vector for the line can be the cross prod. of $\vec{u}$ and $\vec{v}$.

$\vec{m} = \vec{u} \times \vec{v}$

$= (2, 0, 1) \times (0, 3, -1)$

$$\begin{matrix} 0 & 1 & 2 & 0 \\ 3 & -1 & 0 & 3 \end{matrix}$$

$= ((-1)(0) - (1)(3),\ 1(0) - (-1)(2),\ 6 - 0)$

$= (-3, 2, 6)$

$\vec{r} = \vec{P_0} + t(-3, 2, 6)$

Sub $\vec{P_0} = (5, 2, 1)$:

$\vec{r} = (5, 2, 1) + t(-3, 2, 6)$

$\therefore \vec{r} = (5, 2, 1) + t(-3, 2, 6)$ is a vector equation for the live.

8.

$\vec{n_1} = (2, 1, -1) \qquad \vec{n_2} = (1, 1, 2)$

$\vec{n_1} \cdot \vec{n_2} = \cancel{(2)(1)} + (1)(1) + \cancel{(-1)(2)}$

$= 1$

$\because \vec{n_1} \cdot \vec{n_2} \neq 0,$

$\therefore$ The two planes are not parallel and intersect at a line.

$\because$ The normal vectors of the planes are perpendicular to the line, they can be direction vectors for the new plane.

$\therefore \vec{r} = \vec{P_0} + s(2, 1, -1) + t(1, 1, 2)$

Sub $\vec{P_0} = (2, 0, -1)$:

$\therefore \vec{r} = (2,0,-1) + s(2,1,-1) + t(1,1,2)$

$\vec{n} = (2,1,-1) \times (1,1,2)$

$= (1(2)-(-1)(1), (-1)(1)-(2)(2), 2(1)-(1)(1))$

$= (3,-5,1)$

$$\begin{array}{cccc} 1 & -1 & 2 & 1 \\ & \times \quad \times & \times & \\ 1 & 2 & 1 & 1 \end{array}$$

$\therefore 3x-5y+z+D=0$ is a scalar equation for the plane.

Sub $(x,y,z) = (2,0,-1)$ to solve for D:

$3(2)-5(0)+(-1)+D=0$

$D=-5$

$\therefore 3x-5y+z-5=0$ is a scalar equation for the plane.