1.

a) $\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$

$= \frac{s(5) - s(3)}{5-3}$

$= \frac{1}{2}\left((-4.9(5)^2 + 55(5)) - (-4.9(3)^2 + 55(3))\right)$

$= 15.8$

$\therefore$ the average rate of change in height over $t = 3\,s$ to $t = 5\,s$ is $15.8\ m/s$.

b)

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{s(5) - s(4)}{5 - 4}$$

$$= \left(-4.9(5)^2 + 55(5)\right) - \left(-4.9(4)^2 + 55(4)\right)$$

$$= 10.9$$

$\therefore$ The average rate of change in height over $t = 4s$ to $t = 5s$ is 10.9 m/s.

c)

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{s(5) - s(4.5)}{5 - 4.5}$$

$$= \frac{1}{0.5}\left(\left(-4.9(5)^2 + 55(5)\right) - \left(-4.9(4.5)^2 + 55(4.5)\right)\right)$$

$$= 8.45$$

$\therefore$ The average rate of change in height over $t = 4.5$s to $t = 5$s is 8.45 m/s.

d)

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{s(5) - s(4.9)}{5 - 4.9}$$

$$= \frac{1}{0.1}\left(\left(-4.9(5)^2 + 55(5)\right) - \left(-4.9(4.9)^2 + 55(4.9)\right)\right)$$

$$= 6.49$$

$\therefore$ the average rate of change in height over $t = 4.9$s to $t = 5$s is 6.49 m/s.

2.

As we approach $t=5$ from the left, the average rate of change decreases. This suggests that $t=5s$ is close to a local minimum/maximum. Since s is a quadratic function and has a negative coefficient on the t^2 term, $t=5s$ must be close to the global maximum (vertex). The instantaneous rate of change is 0 at this max. The decreasing average rate of change implies that $t=5s$ is nearing the function's vertex, which is also the peak height of the projectile.

3.

The peak height occurs when the rocket has no upward velocity. This implies that the instantaneous rate of change is 0. For a quadratic function like $s = -4.9t^2 + 55t$, this occurs only at the vertex.

Find the roots. The vertex is equidistant from both roots.

$s = -4.9t^2 + 55t$

Let $s = 0$:

$$0 = -4.9t^2 + 55t$$

$$= t(-4.9t + 55)$$

if $t = 0$: $\therefore t = 0$

if $-4.9t + 55 = 0$

$$55 = 4.9t$$

$$t = \frac{55}{4.9}$$

$\therefore$ The roots are $t = 0\,s$ and $t = \frac{55}{4.9}\,s$.

The value of t that is equidistant from these points is:

$$t_{eq} = \frac{t_0 + t_1}{2}$$

$$= \frac{0 + \frac{55}{4.9}}{2}$$

$$= \frac{55}{9.8}$$

$\therefore$ The rocket reaches its max height at $t = \frac{55}{9.8}$ s or $t = 5.61$ s.

4.

a)

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Sub $s = -4.9t^2 + 55t$ for f, $s' = v$

$$s' = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$v = \lim_{h \to 0} \frac{-4.9(t+h)^2 + 55(t+h) - (-4.9(t^2) + 55t)}{h}$$

$$v = \lim_{h \to 0} \frac{-4.9(t^2 + 2th + h^2) + \cancel{55t} + 55h + 4.9t^2 \cancel{- 55t}}{h}$$

$$v = \lim_{h \to 0} \frac{\cancel{-4.9t^2} - 9.8th - 4.9h^2 + 55h \cancel{+ 4.9t^2}}{h}$$

$$v = \lim_{h \to 0} \frac{-9.8th - 4.9h^2 + 55h}{h}$$

$$v = \lim_{h \to 0} \frac{\cancel{h}(-9.8t - 4.9h + 55)}{\cancel{h}}$$

$$v = \lim_{h \to 0} -9.8t - \cancel{4.9h}^{\to 0} + 55$$

$$v = -9.8t + 55$$

$\therefore$ The instantaneous rate of change (velocity) at any time t is given by $v = -9.8t + 55$.

b)

i)

$v = -9.8t + 55$

Sub $t = 3$:

$v = -9.8(3) + 55$

$= 25.6$

$\therefore$ The instantaneous rate of change (velocity) at time $a = 3s$ is $25.6\ m/s$.

ii)

$v = -9.8t + 55$

Sub $t = 5$:

$v = -9.8(5) + 55$

$= 6$

$\therefore$ The instantaneous rate of change (velocity) at time $a = 5s$ is $6\ m/s$.

c)

$s = -4.9t^2 + 55t$ describes the height of the projectile. The derivative of s would then be the instantaneous velocity in the upwards direction. The max height is achieved when this velocity is 0.

$v = -9.8t + 55$

Set $v = 0$:

$$0 = -9.8t + 55$$

$$t = \frac{55}{9.8}\text{ s.}$$

$\therefore$ The maximum height occurs at $t = \frac{55}{9.8}$ s or $t = 5.61$s. This is also the same value of t found in q 3).

5.

$$\frac{f(a+h) - f(a)}{h}$$

$$= \frac{2(a+h)^3 - 8(a+h)^2 + 3 - (2a^3 - 8a^2 + 3)}{h}$$

$$= \frac{2(a^3 + 3a^2h + 3ah^2 + h^3) - 8(a^2 + 2ah + h^2) \cancel{+3} - 2a^3 - 8a^2 \cancel{-3}}{h}$$

$$= \frac{\cancel{2a^3} + 6a^2h + 6ah^2 + 2h^3 \cancel{-8a^2} - 16ah - 8h^2 \cancel{-2a^3} \cancel{-8a^2}}{h}$$

$$= \frac{\cancel{h}(6a^2+6ah+2h^2-16a-8h)}{\cancel{h}}$$

$$= 6a^2+6ah+2h^2-16a-8h$$

a.

Sub $a=5$, $h=2$:

$6a^2+6ah+2h^2-16a-8h$

$= 6(5)^2+6(5)(2)+2(2^2)-16(5)-8(2)$

$= 122$

$\therefore$ The average rate of change of f at $a=5$ and $h=2$ is 122.

b.

Sub $a=5$, $h=0.5$

$6a^2+6ah+2h^2-16a-8h$

$= 6(5)^2+6(5)(0.5)+2(0.5)^2-16(5)-8(0.5)$

$= 81.5$

$\therefore$ The average rate of change of f at $a=5$ and $h=0.5$ is 81.5.

6.

i)

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{(a+h)^3 - 2(a+h) + 1 - (a^3 - 2a + 1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{a^3} + 3a^2h + 3ah^2 + h^3 \cancel{-2a} - 2h \cancel{+1} \cancel{-a^3} \cancel{+2a} \cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{3a^2h + 3ah^2 + h^3 - 2h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3a^2 + 3ah + h^2 - 2)}{\cancel{h}}$$

$$= 3a^2 - 2$$

$$\therefore f'(x) = 3x^2 - 2$$

Min and max occur at $f'(x)=0$:

Set $f'(x)=0$:

$0=3x^2-2$

$x^2=\frac{2}{3}$

$x=\pm\sqrt{\frac{2}{3}}$

$x\simeq\pm 0.816$

$\therefore$ Approximate x-values for max/min points are $x=-0.816$ and $x=0.816$.

$\because$ $f(x)$ is a cubic with a positive coefficient, the function is increasing, then decreasing, then increasing on $x\in(-\infty,\infty)$, $x\in\mathbb{R}$. Thus, $x=-0.816$ is the local max and $x=0.816$ is the local min.

ii)

	$-\infty$	$x=-0.816$		$x=0.816$	$+\infty$
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	increasing	(local max)	decreasing	(local min)	increasing

$, x \in \mathbb{R}$

Test $x < -\sqrt{\frac{2}{3}}$, choose $x=-1$.

$f'(-1) = 3(-1)^2 - 2$

$= 1$

$\because f'(-1)$ is positive,

$\therefore f(x)$ is increasing

on $x \in \left(-\infty, -\sqrt{\frac{2}{3}}\right)$

Test $-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$,

Choose $x = 0$.

$f'(0) = 3(0)^2 - 2$

$= -2$

$\because f'(0)$ is negative,

$\therefore f(x)$ is decreasing

on $x \in \left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$.

Test $x > \sqrt{\frac{2}{3}}$,

Choose $x=1$.

$f'(1) = 3(1)^2 - 2$

$= 1$

$\because f'(1)$ is positive,

$\therefore f(x)$ is increasing

on $x \in \left(\sqrt{\frac{2}{3}}, \infty\right)$

iii)

For:

on $x \in (-\sqrt{3}, \sqrt{\frac{2}{3}})$. on $x \in (\sqrt{3}, \infty)$

iii)

x	$f'(x) = 3x^2 - 2$
-3	25
-2	10
-1	1
0	-2
1	1
2	10
3	25

For:

$$f'(-3) = 3(-3)^2 - 2 = 25$$

$$f'(-2) = 3(-2)^2 - 2 = 10$$

$$f'(-1) = 3(-1)^2 - 2 = 1$$

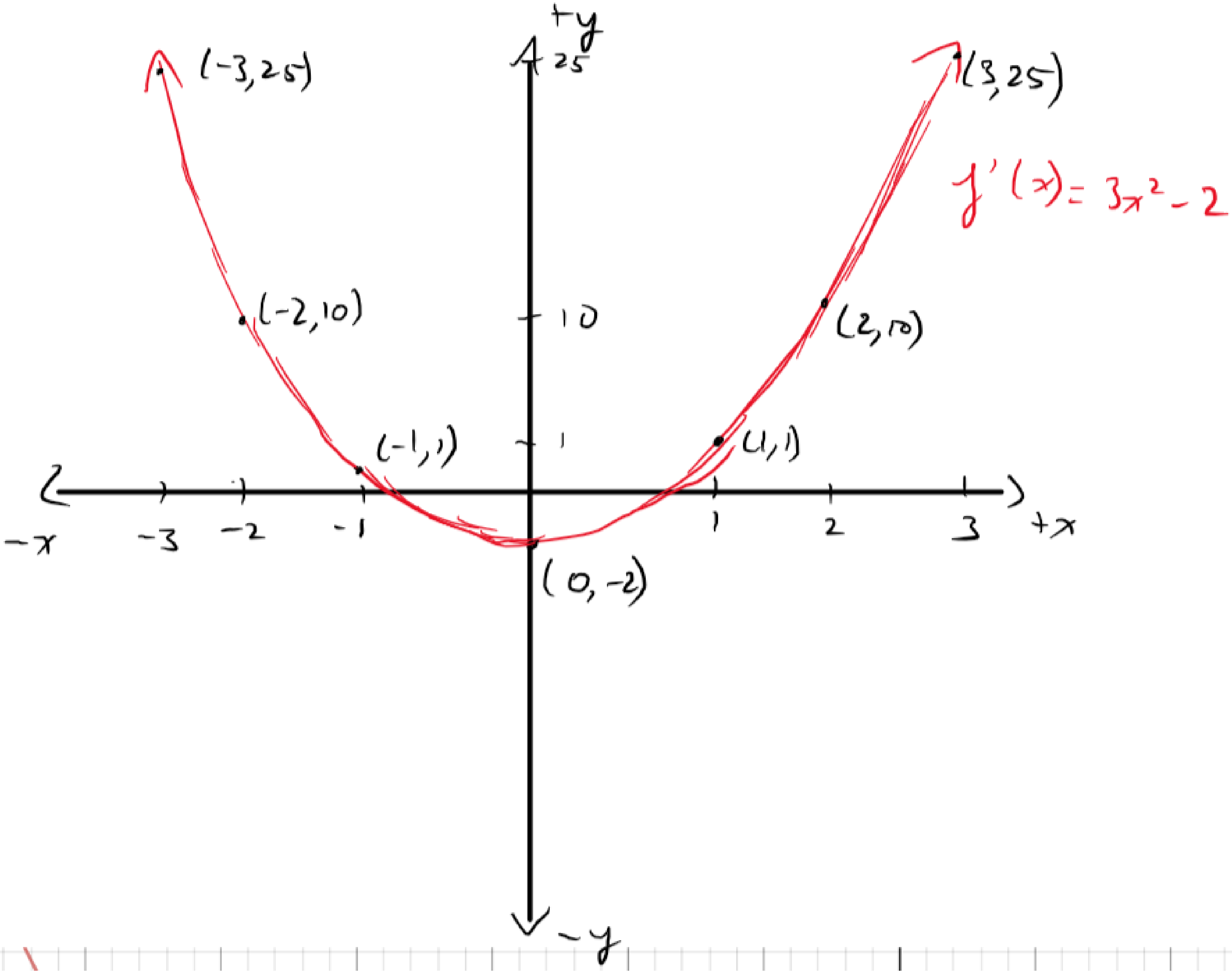
$$f'(0) = 3(0)^2 - 2 = -2$$

$$f'(1) = 3(1)^2 - 2 = 1$$

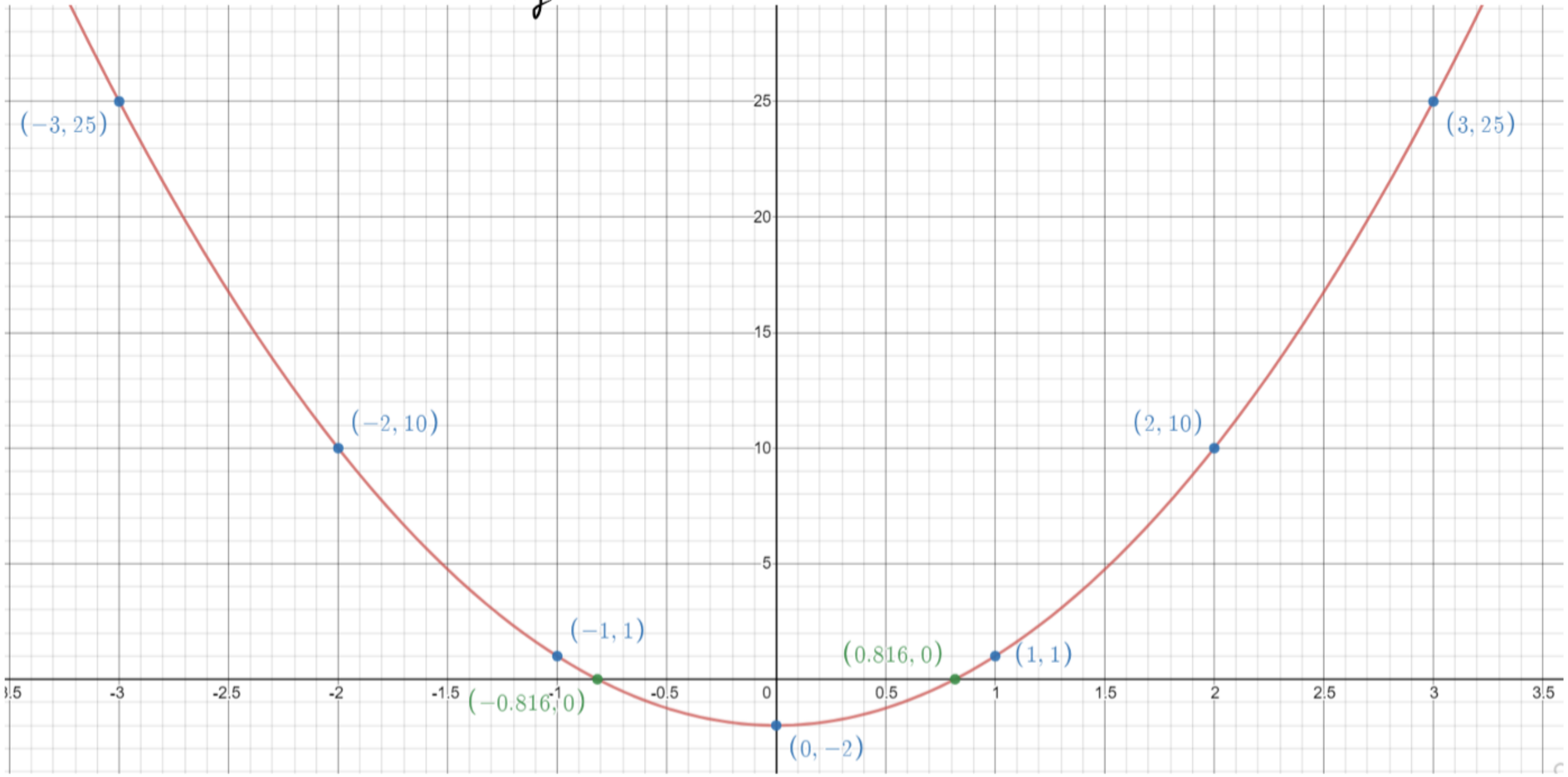
$$f'(2) = 3(2)^2 - 2 = 10$$

$$f'(3) = 3(3)^2 - 2 = 25$$

iv)



$\downarrow -y$



(−3, 25)
(3, 25)
(−2, 10)
(2, 10)
(−1, 1)
(1, 1)
(0.816, 0)
(−0.816, 0)
(0, −2)
25
20
15
10
5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5