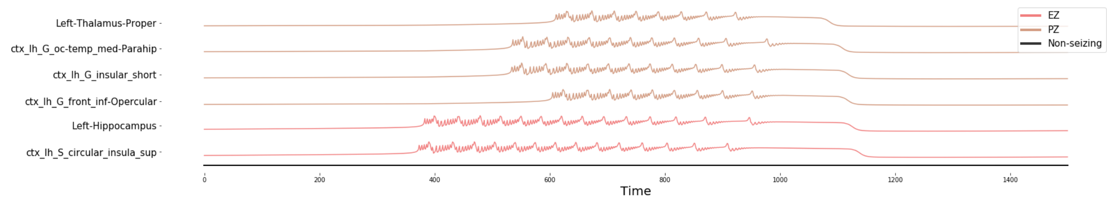
Report for diagnosing problems with HMC

January 8, 2020

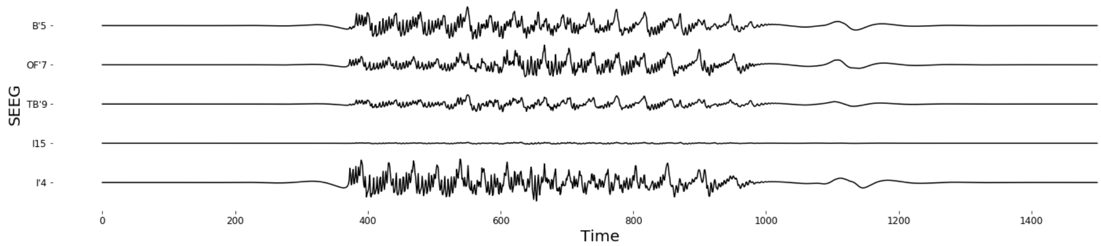
The objective of this work is to identify spatio-temporal seizure propagation patterns i.e to find the regions of brain where seizure starts and propagates towards. The network of regions which initiate the seizure is called as Epileptogenic Zone(EZ) and the network of regions where seizure propagates is called as Propagation Zone(PZ)

Dataset

In order test the model, a synthetic dataset is generated using 5D Epileptor



(a) Source activity



(b) SEEG activity

Figure 1: Simulated seizure propagation pattern

Modeled data features

Given appropriate parameter values 2D Epileptor can capture some of the important characteristics of a seizure namely seizure onset and seizure length, which are sufficient for the purposes of identifying spatio-temporal seizure propagation patterns. Log. SEEG power encompasses both these features and hence forms a good candidate data feature that can be modeled using 2D Epileptor. Figure 2 shows the features extracted from synthetic dataset shown in figure 1

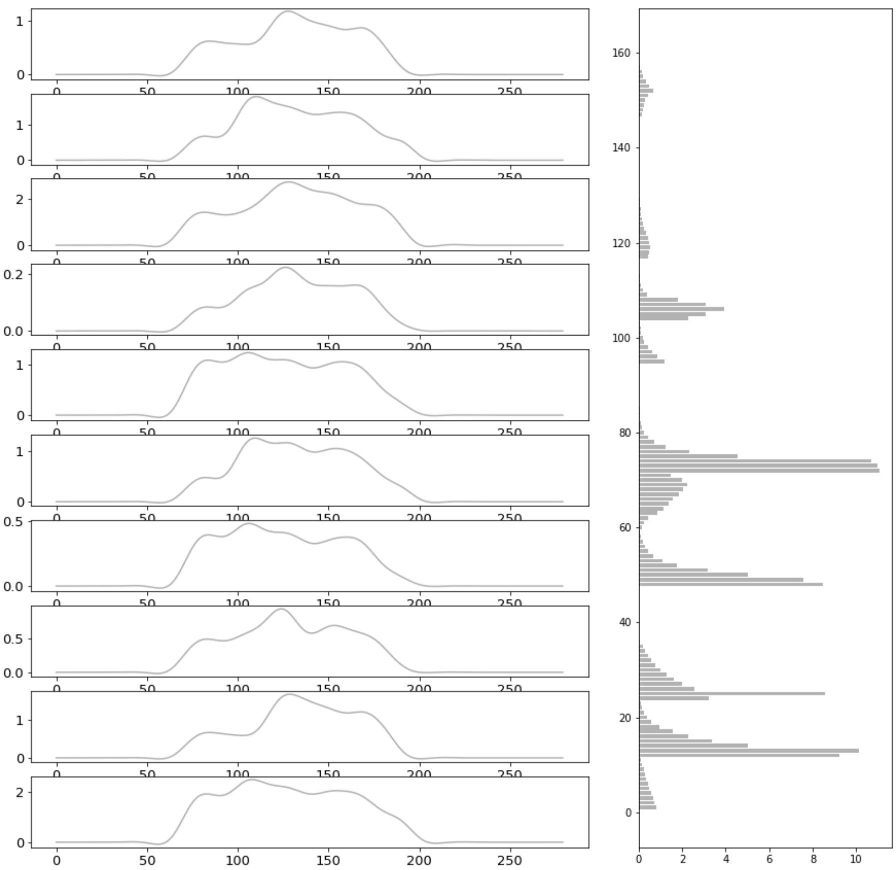


Figure 2: Modeled data features. SEEG log power of 10 sensors(left) and total power per sensor(right)

Model

The dependency structure between parameters of virtual epileptic patient(VEP) model is shown in figure 3

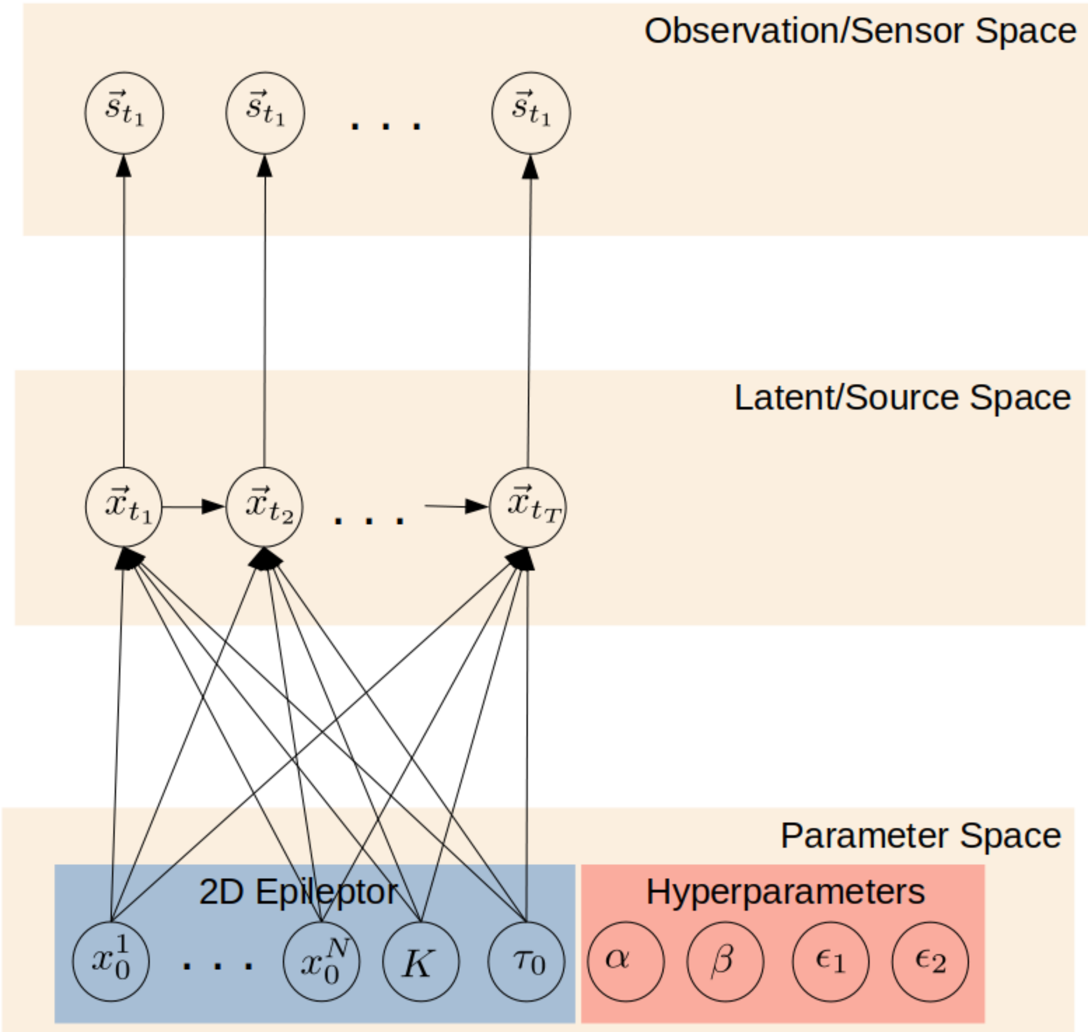


Figure 3: Probabilistic graphical model of virtual epileptic patient

$$
\begin{aligned}
&P(\theta, \mathbf{X}|\mathbf{D}) \propto P(\mathbf{D}|\theta, \mathbf{X})P(\mathbf{X}, \theta) \\
&\quad \text{where,} \\
&\qquad \mathbf{D} = (\mathbf{S}, \vec{\rho}), \mathbf{S} = \text{Log SEEG power}, \vec{\rho} = \text{Total power in each sensor} \\
&\qquad \mathbf{S} = \begin{pmatrix} s_{t_1}^1 & s_{t_1}^2 & \cdots & s_{t_1}^M \\ s_{t_2}^1 & s_{t_2}^2 & \cdots & s_{t_2}^M \\ \vdots & \vdots & \ddots & \vdots \\ s_{t_T}^1 & s_{t_T}^2 & \cdots & s_{t_T}^M \end{pmatrix}_{T \times M} \qquad \mathbf{X} = \begin{pmatrix} x_{t_1}^1 & x_{t_2}^1 & \cdots & x_{t_T}^1 \\ x_{t_1}^2 & x_{t_2}^2 & \cdots & x_{t_T}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{t_1}^N & x_{t_2}^N & \cdots & x_{t_T}^N \end{pmatrix}_{N \times T} \\
&\qquad \theta = (\vec{x}_0, k, \tau_0, \alpha, \beta)
\end{aligned}
$$

Likelihood

$$
\begin{aligned}
P(\mathbf{S}, \rho|\mathbf{X}, \theta) &= P(\mathbf{S}|\mathbf{X}, \theta)P(\rho|\mathbf{S}, \theta) \\
P(\mathbf{S}|\mathbf{X}, \theta) &= \prod_{i=1}^{M}\prod_{j=1}^{T} P(s_{t_j}^i|\vec{x}_{t_j}, \theta) \\
P(s_t^i|\vec{x}_t, \theta) &\sim \mathcal{N}(\alpha(\log\langle G_i \, , e^{\vec{x}_t}\rangle + \beta), \epsilon_1), \text{where,} \\
&\quad G \text{ is a projection matrix from source space to sensor space} \\
P(\vec{\rho}^i|\mathbf{S}, \theta) &\sim \prod_{i=1}^{M} \mathcal{N}(\frac{1}{T}\sum_{j=1}^{T}(s_{t_j}^i)^2, \epsilon_2)
\end{aligned}
$$

Priors

Source dynamics are assumed to follow trajectories given by 2D Epileptor

$$
\begin{aligned}
\dot{x}_i &= 1 - x_i^3 - 2x_i^2 - z_i + I_1 \\
\dot{z}_i &= \frac{1}{\tau_0}\left(4(x_i - x_0) - z_i - \sum_{j=1}^{N} K_{ij}(x_j - x_i)\right)
\end{aligned}
$$

Parameter	*Prior*
x_0^i	$\mathcal{N}(\mu_{x_0}^i, 1)$
K	$\mathcal{N}(1, 10)$
τ_0	$\mathcal{N}(20, 10)$
α	$\mathcal{N}(1, 10)$
β	$\mathcal{N}(0, 10)$
ϵ_1	$\mathcal{N}(1, 10)$
ϵ_2	$\mathcal{N}(1, 10)$

Table 1: Prior distributions of all inferred parameters

$$\begin{aligned}
P(\theta, \mathbf{X}) &= P(\mathbf{X}|\theta)P(\theta) \\
P(\mathbf{X}|\theta) &= P(\vec{x}_{t_1}|\theta)\prod_{j=2}^{T} P(\vec{x}_{t_j}|\vec{x}_{t_{j-1}}, \theta) \\
P(\vec{x}_{t_j}|\vec{x}_{t_{j-1}}, \theta) &= \delta(x_{t_j} - f(x_{t_{j-1}}, \theta)) \\
f(x_{t_{j-1}}, \theta)) &= x_{t_{j-1}} + dt\dot{x} \\
P(\theta) &= P(x_0, K, \tau_0, \alpha, \beta) \\
&= P(x_0)P(K)P(\tau_0)P(\alpha)P(\beta)
\end{aligned}$$

Prior distributions of all inferred parameters are given in table 1

Results

NUTS parameters across iterations and pair plots for inferred scalar parameters from 4 chains are shown in figures 4 - 7.

sampler parameters:

No. of warmup iterations	:	200
No. of sampling iterations	:	200
Target acceptance statistic (δ)	:	0.9
Max. tree depth	:	10
Metric	:	diagonal

Problems:

- Inference works well when the observations are at source space but fails when observations are in sensor space.
- All chains hit max. tree depth
- Sampling is very slow, it takes 3-4 days to finish 200 warmup + 200 sampling itearations
- Fixing observation noise($\epsilon_1 = 1, \epsilon_2 = 1$) causes many divergent transitions in all chains

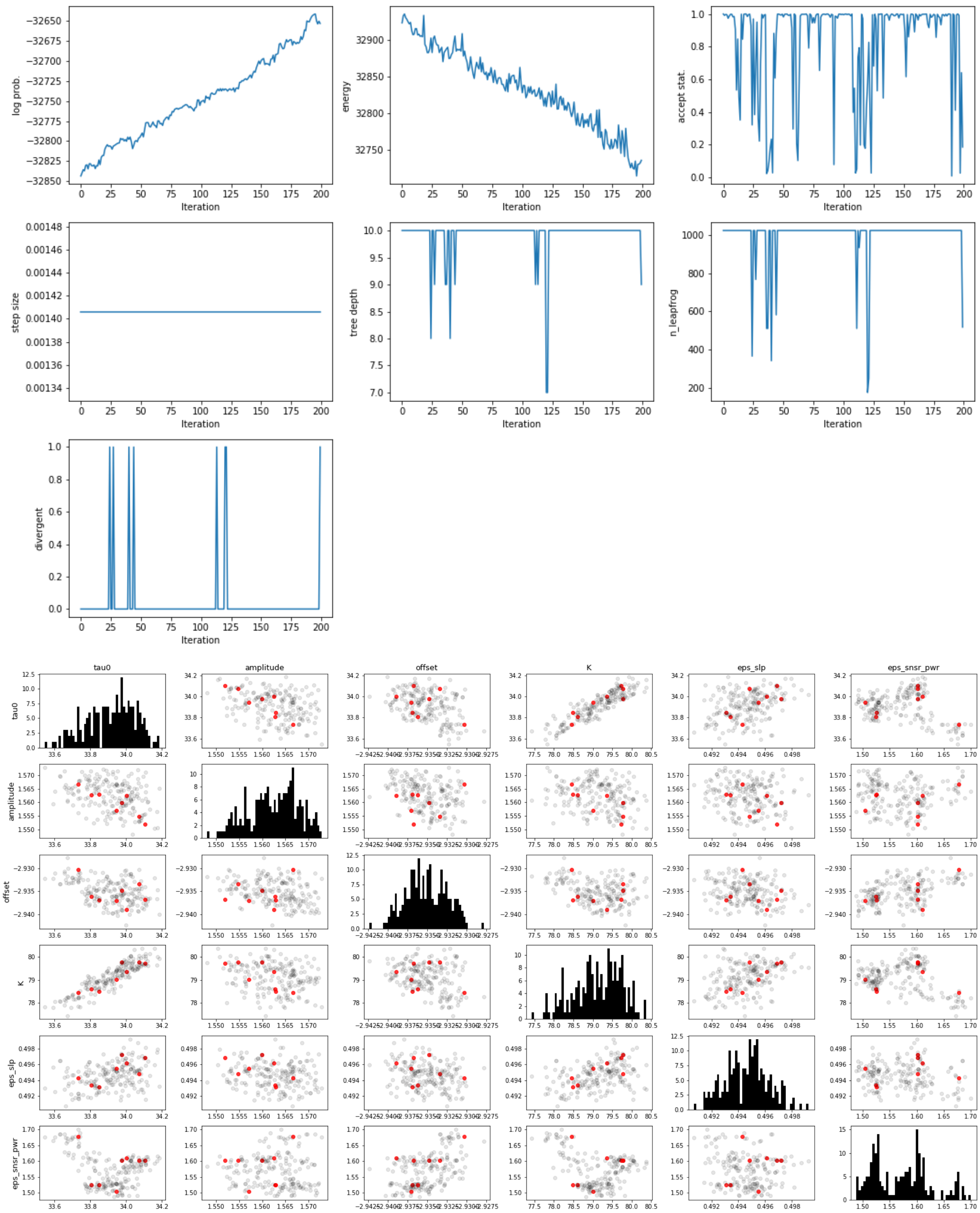


Figure 4: Chain 1

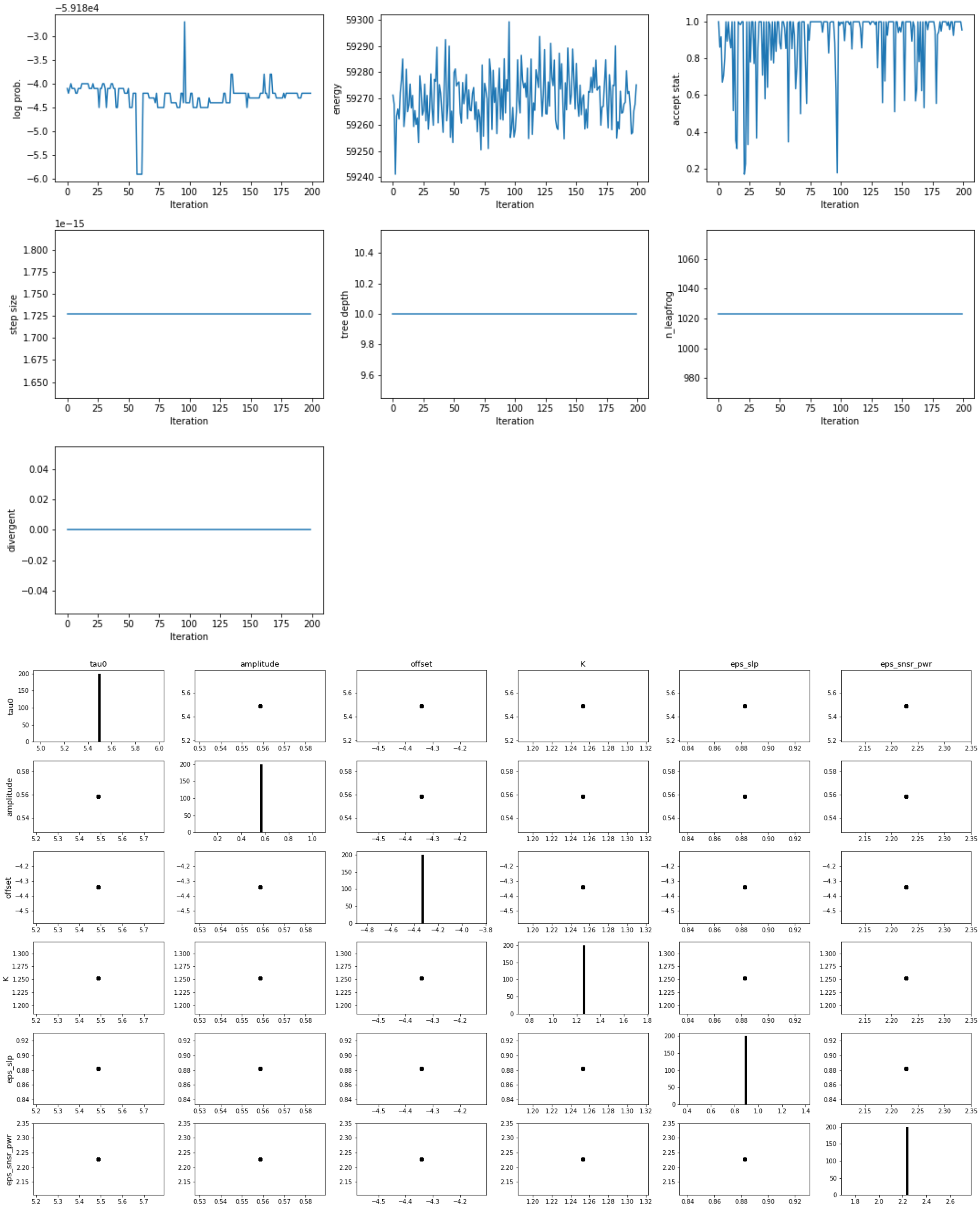
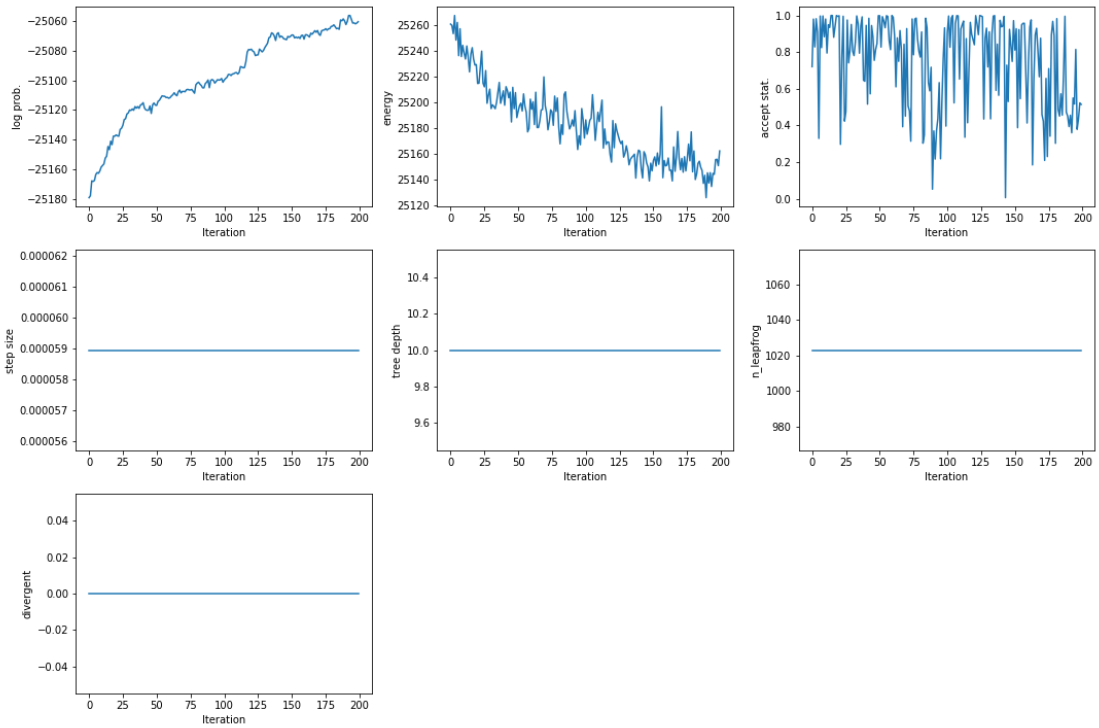


Figure 5: Chain 2



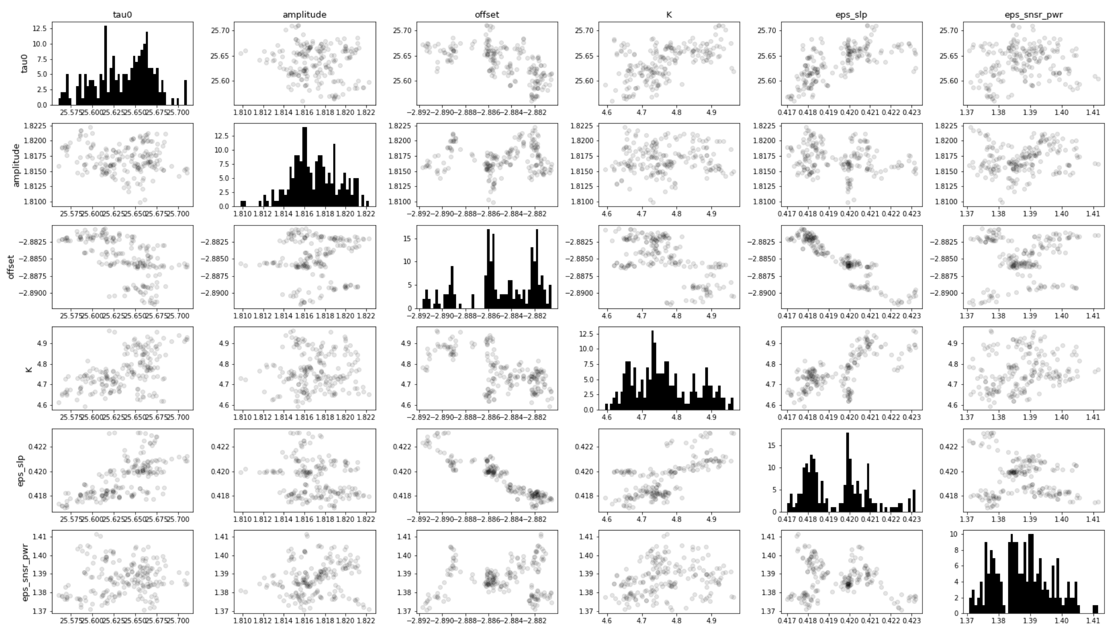


Figure 6: Chain 3

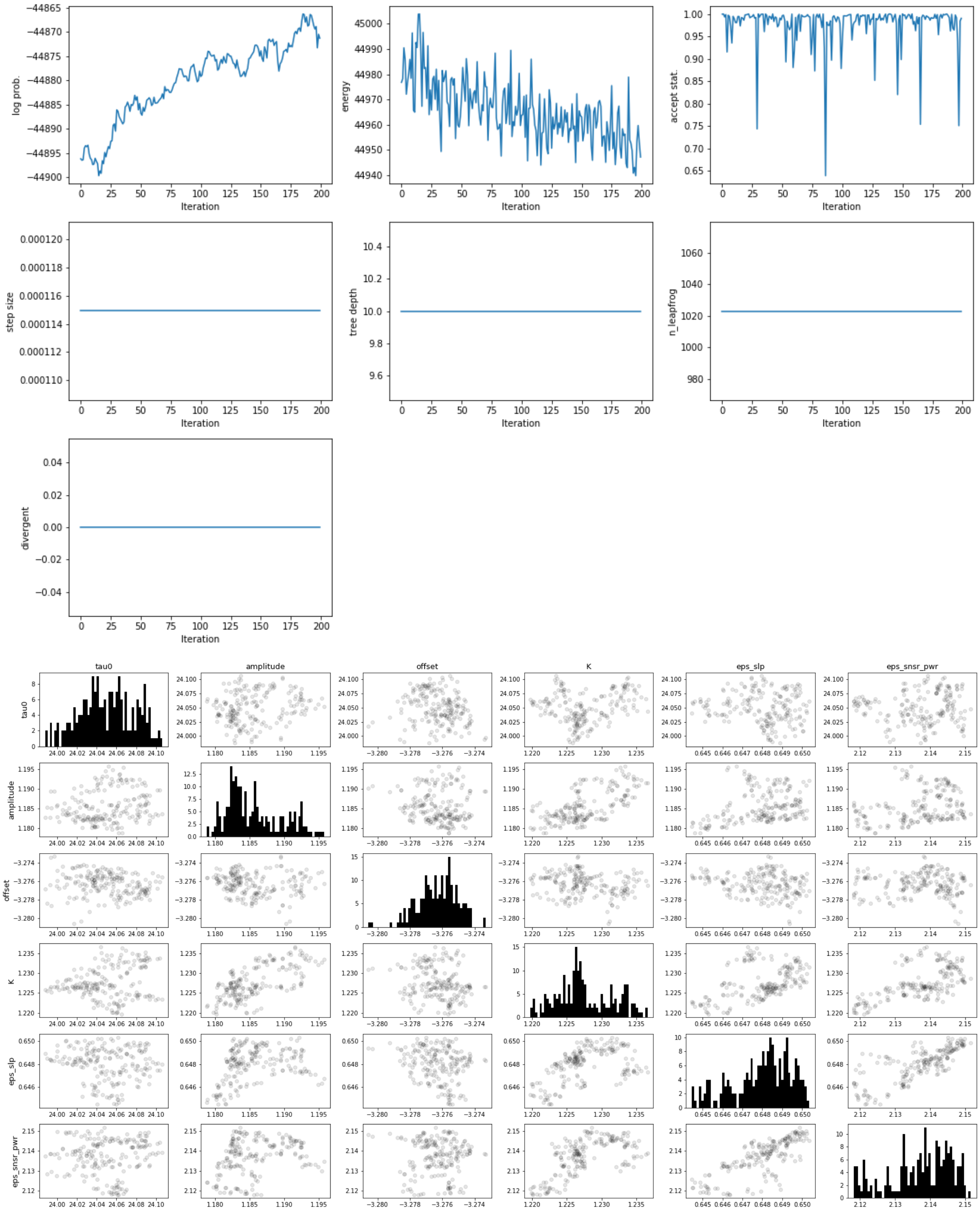


Figure 7: Chain 4