Results on the Riemann Zeta function and the Dirichlet Beta function

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Result 1

$$\zeta(2k) = \frac{\pi 4^{k-1}}{4^k - 1} \int_0^{\frac{\pi}{2}} \int_0^{\sigma_n} \int_{\sigma_{n-1}}^{\frac{\pi}{2}} \int_0^{\sigma_{n-2}} \int_{\sigma_{n-3}}^{\frac{\pi}{2}} \cdots \int_0^{\sigma_2} \left(\frac{\pi}{2} - \sigma_1 \right) \, \mathrm{d}\sigma_1 \, \mathrm{d}\sigma_2 \cdots \mathrm{d}\sigma_n$$

where, $n, k \in N$ such that $k \geq 2$ and $n = 2k - 2$ and ζ is the Riemann Zeta function.

Result 2

$$\beta(2k+1) = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \int_{\sigma_n}^{\frac{\pi}{2}} \int_0^{\sigma_{n-1}} \int_{\sigma_{n-2}}^{\frac{\pi}{2}} \int_0^{\sigma_{n-3}} \cdots \int_0^{\sigma_2} \left(\frac{\pi}{2} - \sigma_1 \right) \, \mathrm{d}\sigma_1 \, \mathrm{d}\sigma_2 \cdots \mathrm{d}\sigma_n$$

where, $n, k \in N$ such that $k \geq 1$ and $n = 2k - 1$ and β is the Dirichlet Beta function.