Analysis

The two algorithms implemented here are the brute force and the graham scan algorithm

Brute Force

O(n^3) In the brute force algorithm, I take two points O(n^2) at a time to find out if all the rest of the points lie on one side of those pointsO(n). This way I can determine if the line segment formed by the two points is a hull point or not.

When all the hull forming line segments are determined this way, I run post processing to order these line segments by making sure that the end point of the previous line is the start point of the next line.

This is followed by reversing the list of points thus formed in case they are not in a counter clock wise fashion

This post processing is less than O(n^3) and thus overall the algorithm takes O(n^3)

Graham Scan

O(nlogn) Here, I sort the points in the increasing order of the x values of the points. O(nlogn)

Then I calculate the upper and lower hulls by making sure that every 3 sequence of points have the correct orientation found by the determinant of those three points. O(n)

The O(nlogn) dominates and therefore the overall time complexity is O(nlogn)

Output

The output in either case is a list of points that make up the hull and the starting point is not included twice (also as the last point) to make the hull closed.

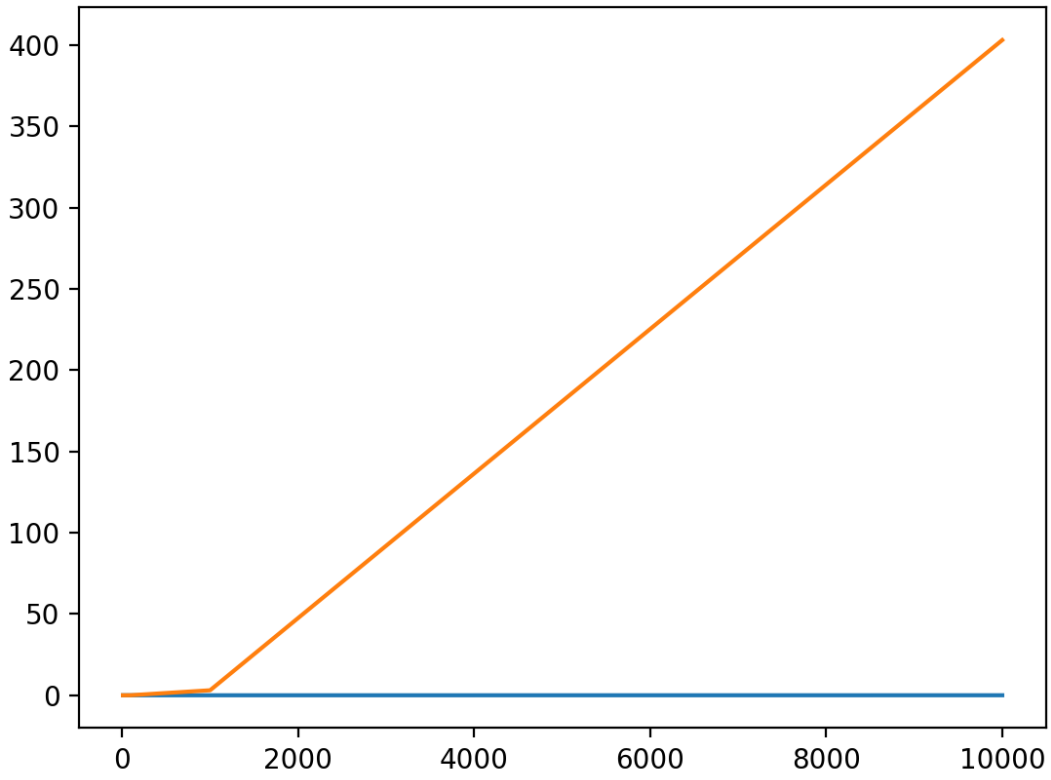
Time Comparison

Num Points	Graham Scan (seconds)	Brute Force (seconds)
10	4.9114227294921875e-05	0.0005199909210205078
100	0.0013020038604736328	0.0391690731048584
1000	0.0012481212615966797	3.076591730117798
10000	0.014687061309814453	823.682062149
100000	0.1307981014251709	> 10 min

As can be seen from the table, the time taken to compute the hull increases very quickly in case of the n^3 brute force algorithm. The graham scan algorithm on other hand easily computes the hull in a decent time limit as it does not increase a lot with the increase in the number of inputs.

The brute force additionally does not handle those cases well where there are a lot of collinear points as it has to do a post processing O(n^2) of finding all the points that are collinear and remove each of them one at a time, finding the one that is longest in terms of length.

Following is a graph representing the above tabular result.



The green line is the time increase for the brute force while the blue represent the graham scan

the time taken for 100000 points in graham scan algorithm was 0.1307981014251709 seconds while the brute force took for ever to compute. Therefore it is not given here.

The 403 seconds for 10000 points with brute force was just the time needed to compute the hull. The post processing as a whole also took up 420 seconds.