Lisbon School of Economics & Management
Universidade de Lisboa

Master in Applied Econometrics and Forecasting

Time Series Analysis and Forecasting

Class #7: ARIMA Models for Nonstationary Time Series

Jorge Caiado, PhD

CEMAPRE/REM/ISEG, University of Lisbon, jcaiado@iseg.ulisboa.pt

ARIMA Models for Nonstationary Time Series

Nonstationary model in the mean

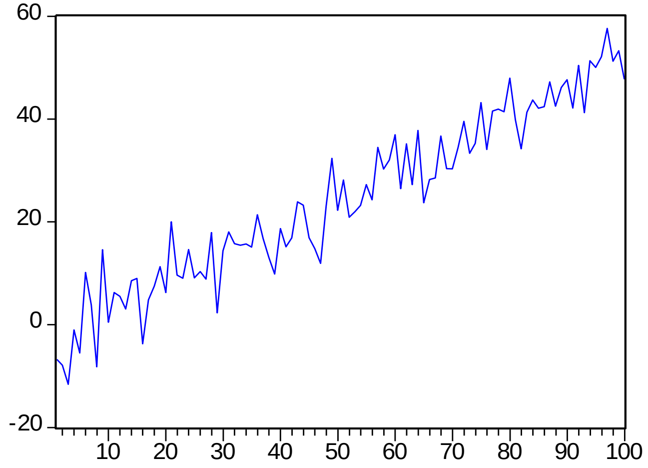
The mean function of a nonstationary model can be represented essentially by two models: **deterministic trend** models and **stochastic trend** models.

For a deterministic trend model, one can use the linear trend model,

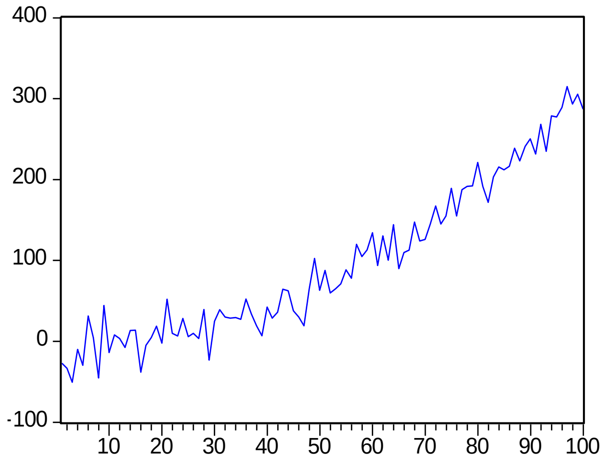
$$X_t = a + bt + a_t$$

or the quadratic trend model,

$$X_t = a + bt + ct^2 + a_t.$$

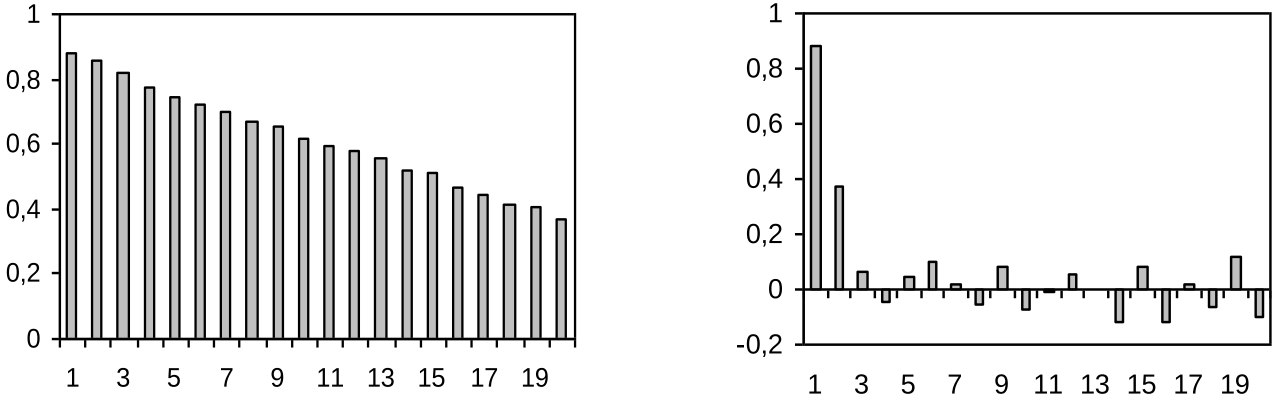


Linear trend model

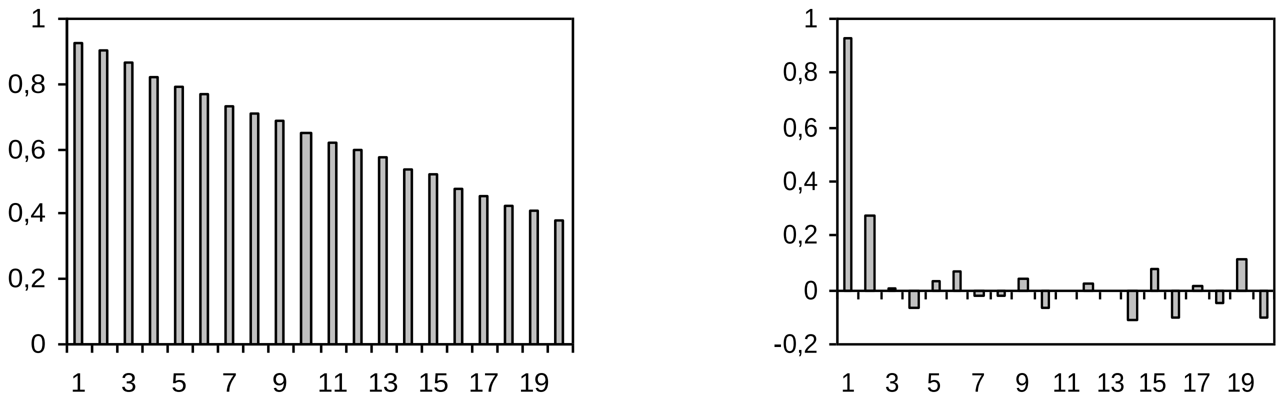


Quadratic trend model

ARIMA Models for Nonstationary Time Series



Sample ACF and PACF of a linear trend model



Sample ACF and PACF of a quadratic trend model

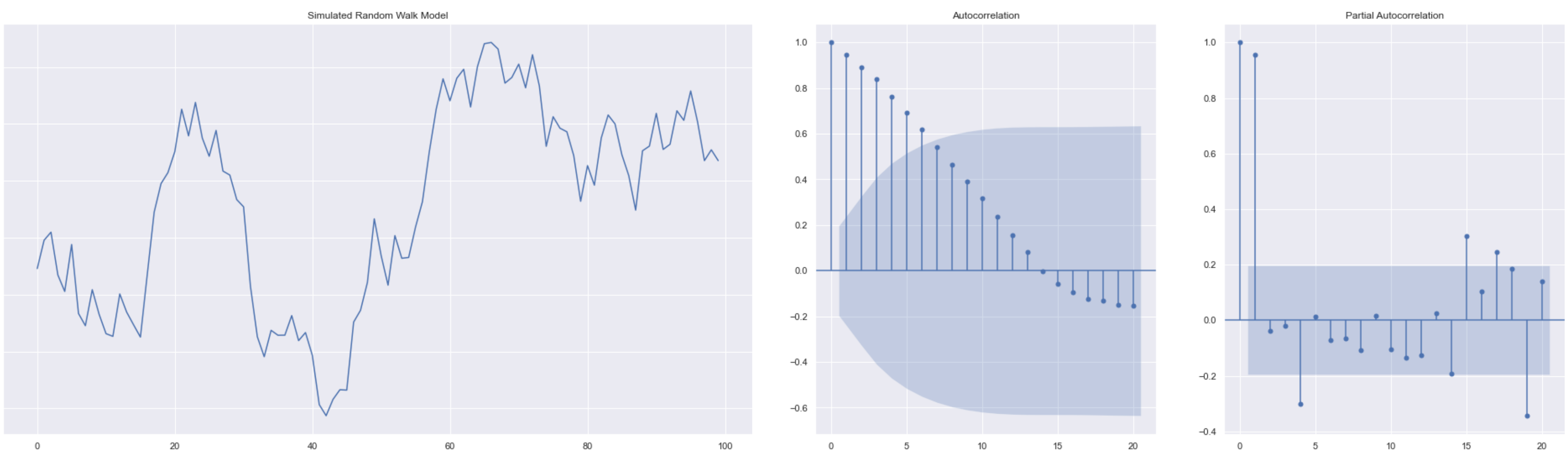
ARIMA Models for Nonstationary Time Series

Random Walk (RW) Model

A special case of the nonstationary models is the **stochastic trend model,**

$$X_t = X_{t-1} + a_t,$$

where a_t is white noise. This is the so-called “random walk without drift” model.



ARIMA Models for Nonstationary Time Series

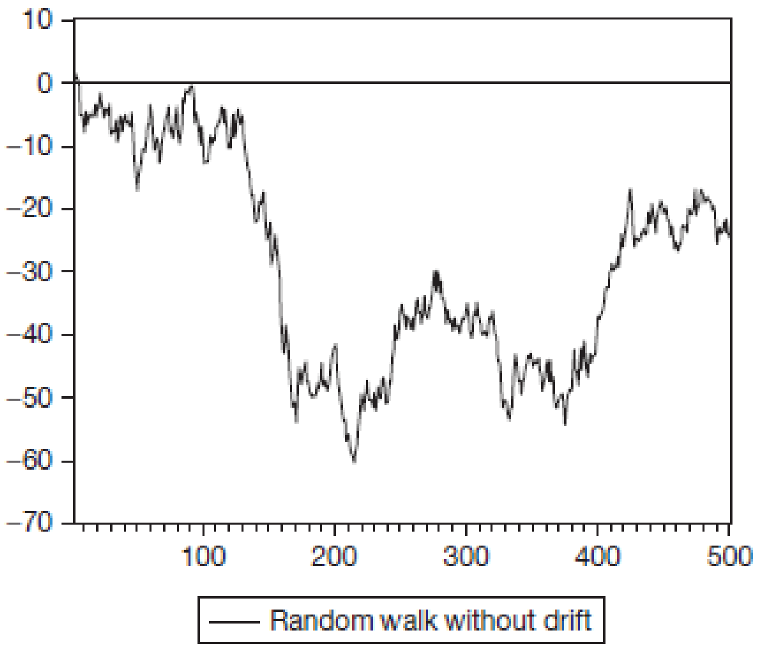
Random Walk (RW) Model with Drift

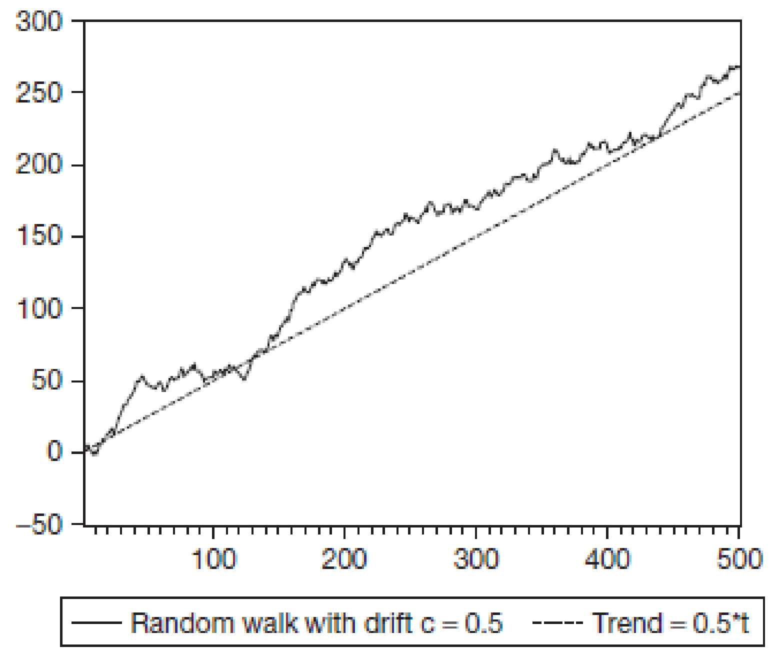
The RW model with drift is defined by

$$X_t = c + X_{t-1} + a_t,$$

where c is the drift.

What is the difference between the RW without drift ant the RW with drift?





Source: Gloria Gonzalez-Rivera (2013)

ARIMA Models for Nonstationary Time Series

Sample ACF and PACF of a RW model without drift

Sample: 2 500
Included observations: 499

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.990	0.990	492.08	0.000
		2	0.980	-0.014	975.14	0.000
		3	0.970	0.020	1449.7	0.000
		4	0.960	-0.029	1915.4	0.000
		5	0.951	0.024	2372.9	0.000
		6	0.942	0.027	2822.8	0.000
		7	0.934	0.023	3265.7	0.000
		8	0.925	0.004	3701.7	0.000
		9	0.917	0.001	4131.1	0.000
		10	0.909	-0.036	4553.2	0.000
		11	0.901	0.042	4968.9	0.000
		12	0.893	0.009	5378.5	0.000

Sample ACF and PACF of a RW model with drift

Sample: 2 500
Included observations: 499

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.993	0.993	495.20	0.000
		2	0.986	0.000	984.67	0.000
		3	0.980	-0.006	1468.4	0.000
		4	0.973	-0.004	1946.4	0.000
		5	0.966	-0.014	2418.6	0.000
		6	0.959	-0.014	2884.9	0.000
		7	0.952	-0.011	3345.1	0.000
		8	0.944	-0.019	3799.2	0.000
		9	0.937	0.015	4247.4	0.000
		10	0.930	0.000	4689.7	0.000
		11	0.923	0.006	5126.4	0.000
		12	0.916	-0.004	5557.3	0.000

Source: Gloria Gonzalez-Rivera (2013)

ARIMA Models for Nonstationary Time Series

ARIMA Model

A general model for representing nonstationary nonseasonal time series is given by the autoregressive integrated moving average ARIMA(p,d,q) model

$$(1 - \emptyset_1 B - \cdots - \emptyset_p B^p)(1 - B)^d X_t = (1 - \theta_1 B - \cdots - \theta_q B^q) a_t$$

or

$$\emptyset_p(B)(1 - B)^d X_t = \theta_q(B) a_t,$$

where $(1 - B)^d$ is the differencing operator of order d, for $d \geq 1$, $\emptyset_p(B)$ is a stationary autoregressive (AR) operator, $\theta_q(B)$ is an invertible moving average (MA) operator and a_t is a zero mean white noise.

Some important special cases of the ARIMA model are ARIMA(0,1,0), ARIMA(1,1,0), ARIMA(0,1,1) and ARIMA (1,1,1) models.

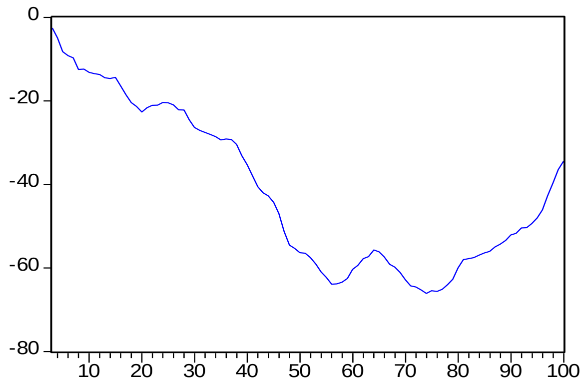
ARIMA Models for Nonstationary Time Series

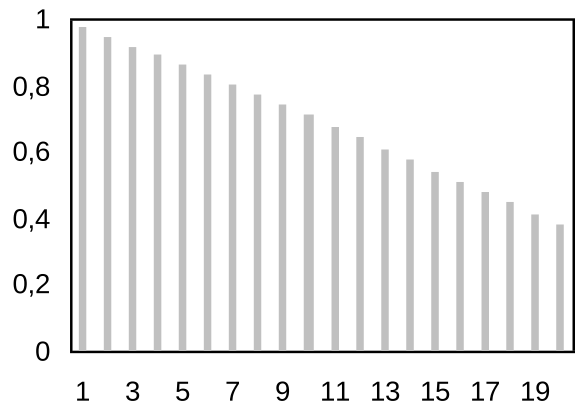
ARIMA(1,1,0) Model

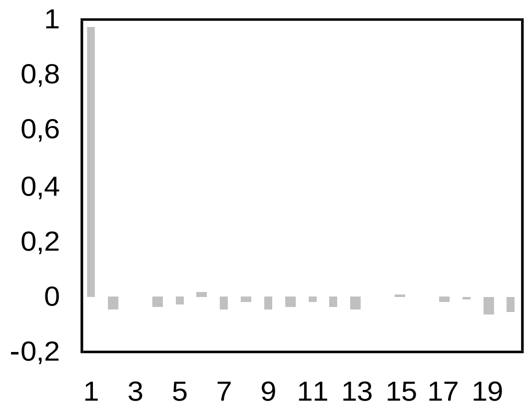
The ARIMA(1,1,0) model is defined by

$$(1 - \emptyset_1 B)(1 - B)X_t = a_t$$

where is a a_t white noise.







Simulation of an ARIMA(1,1,0) model (time series plot, SACF and SPACF):

$$(1 - 0.75B)(1 - B)X_t = a_t$$

ARIMA Models for Nonstationary Time Series

ARIMA(*p*,*d*,*q*) Modeling

Once the differentiating *d* parameter is established, then since $Y_t = (1 - B)^d X_t$ is stationary, apply the ARMA techniques previously discussed to the differenced time series Y_t.

In general, when fitting an ARIMA(*p*,*d*,*q*) to a set of time series, the following procedures are useful:

1. Plot the time series data and try to understand patterns
2. If needed, transform time series data to stabilize variance (use Box-Cox's power transformation)
3. If needed, transform time series data to make it stationary (use differencing transformations).
4. Examine the sample ACF and PACF of the differenced time series data and try to identify possible candidate models
5. Check the residuals from chosen models and, if needed, try modified models
6. Use model selection criteria (AIC, HQ, SBC) to select the "best model"
7. Use the model to compute forecasts