Master in Applied Econometrics and Forecasting

Time Series Analysis and Forecasting

Class #2: Time Series Decomposition and Transformations

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Time Series Decomposition and Transformations

Backshift notation

A very useful notation in time series analysis is the backshift operator *B*, which is used as follows:

$$BX_t = X_{t-1}$$

In other words, *B* has the effect of shifting the data back one period. For *k* periods, the notation is

$$B^k X_t = X_{t-k}$$

For monthly data, B^{12} is used to shift attention to the same month last year,

$$B^{12} X_t = X_{t-12}$$

For quarterly data, the backshift operator is used as follows: $B^4 X_t = X_{t-4}$.

Note 1: The foward shift operator is defined as: $F^k X_t = X_{t+k}$ or $B^{-k} X_t = X_{t+k}$

Note 2: Some authors use the notation *L* instead of *B*.

Differencing

The *d*th differenced series, for some integer $d \geq 1$, is given by

$$\nabla^d X_t = (1 - B)^d X_t.$$

For $d = 1$, we have first differences

$$\nabla X_t = (1 - B) X_t = X_t - X_{t-1}.$$

For seasonal time series, we can use a *s*th seasonal differencing

$$\nabla^s X_t = (1 - B^s) X_t = X_t - X_{t-s}.$$

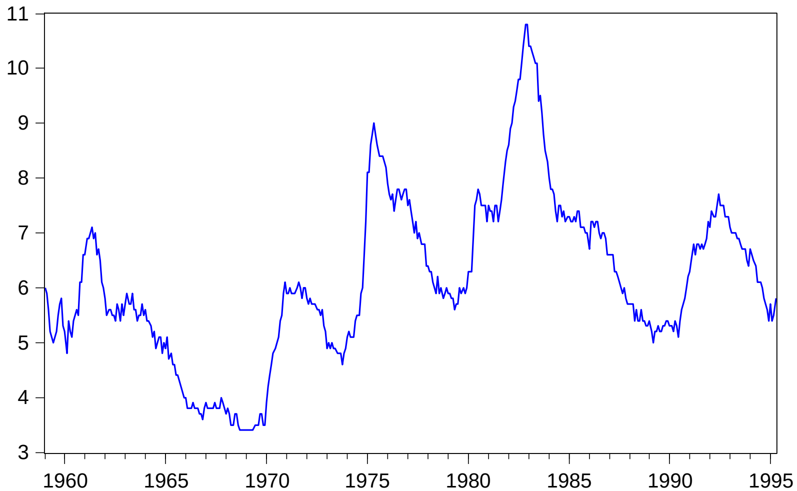
Finally, a *s*th seasonal differencing of order *D*, for some integer $D \geq 1$, is given by

$$(\nabla^s)^D X_t = (1 - B^s)^D X_t.$$

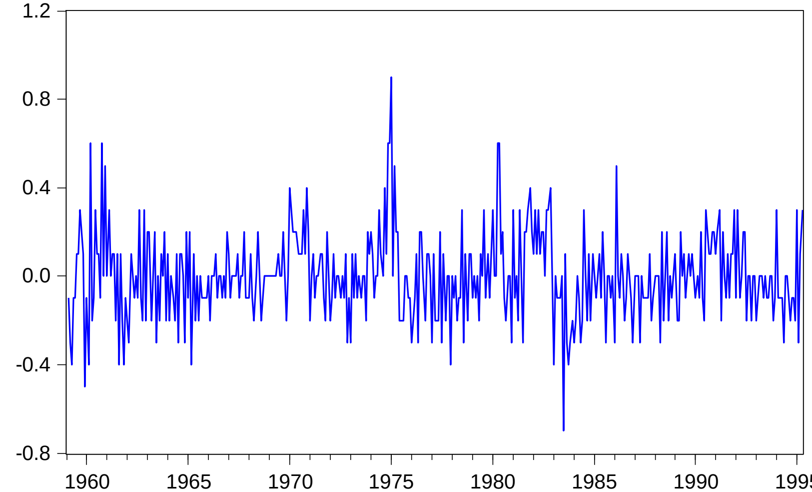
Note: Usually, *d* = 1,2 and *D* = 1,2 are sufficient to obtain ordinary (non-seasonal) and seasonal stationarity, respectively.

Time Series Decomposition and Transformations

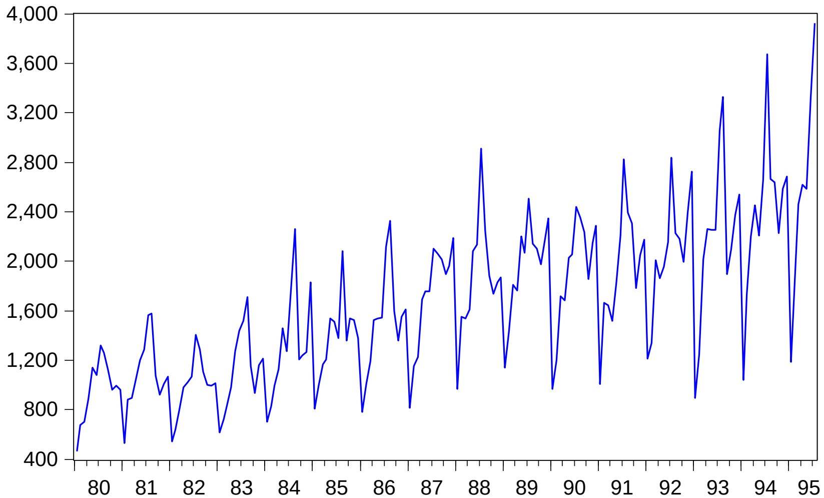
URATE



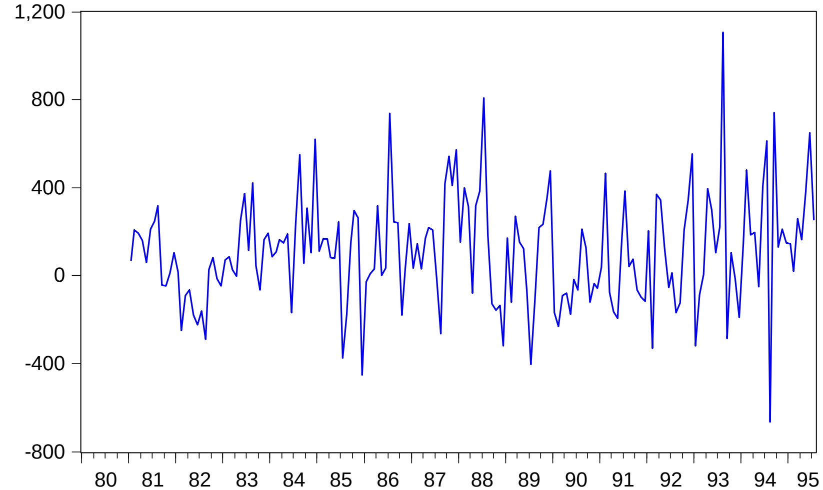
Differenced URATE



WINE



Seasonal Differenced WINE

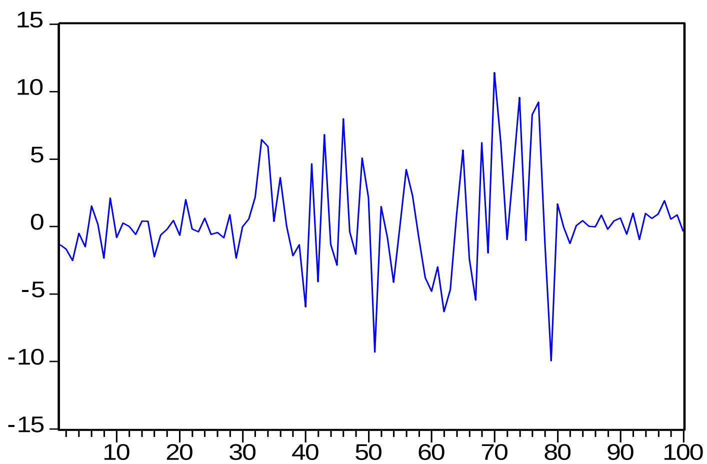


Time Series Decomposition and Transformations

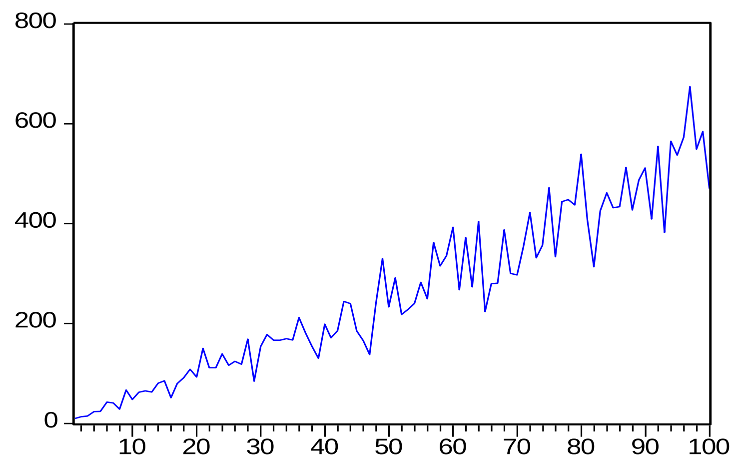
Variance transformation

Many time series are stationary in the mean but are nonstationary in the variance. To reduce this type of nonstationarity, we need variance stabilizing transformations such as the power transformation of Box-Cox (1964),

$$Y_t = T(X_t, \lambda) = \begin{cases} (X_t^{\lambda} - 1)/\lambda, & \lambda \neq 0 \\ \log X_t, & \lambda = 0 \end{cases}$$

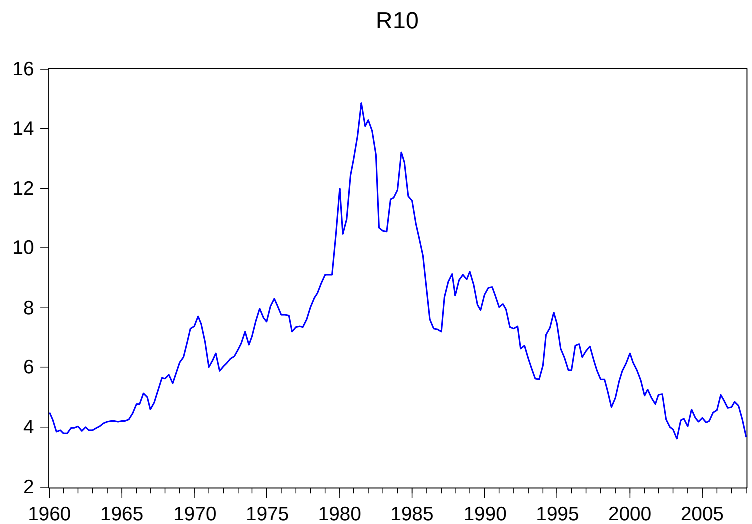


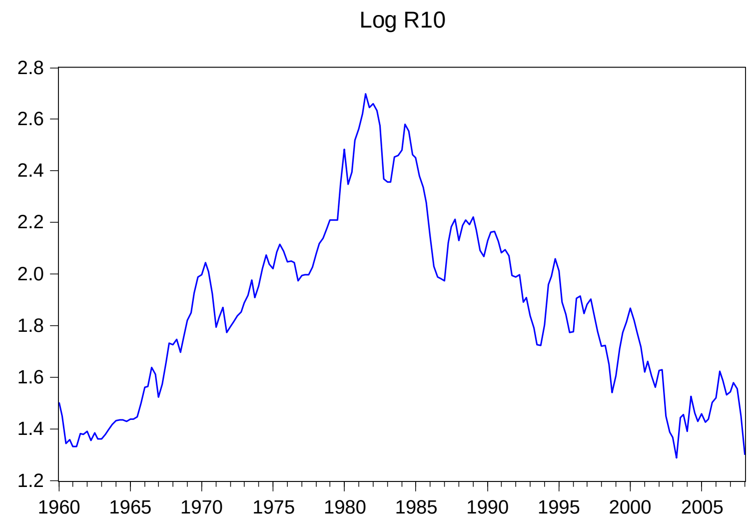
A simulated time series nonstationary in the variance but stationary in the mean

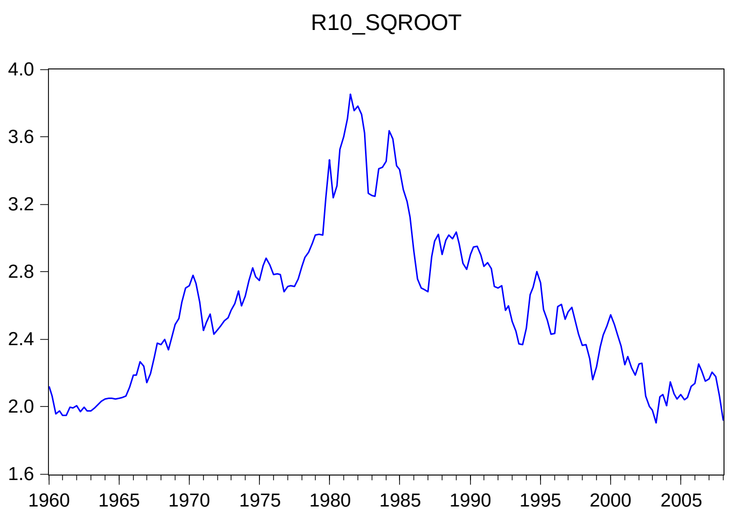


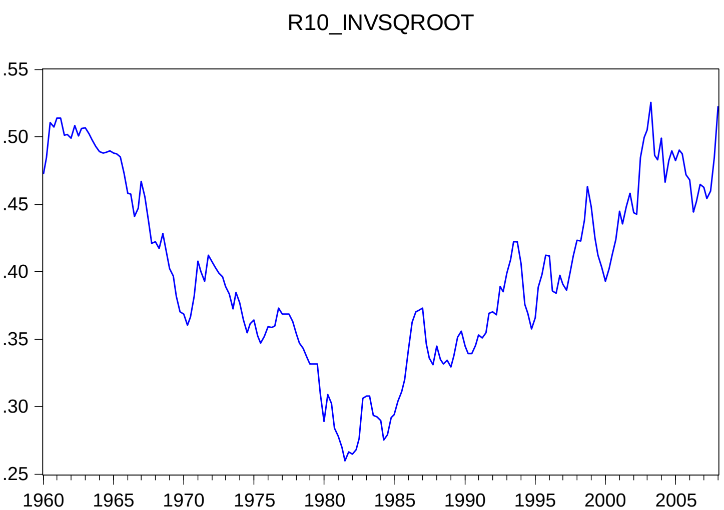
A simulated time series nonstationarity in both the mean and variance

Time Series Decomposition and Transformations









Time Series Decomposition and Transformations

Decomposing a Time Series

We can think of a time series as containing three components: trend (T_t), seasonality (S_t) and noise or irregular component (I_t).

For example, we may assume an additive model as follows:

$$X_t = T_t + S_t + I_t$$

Alternatively, we can write a multiplicative model as

$$X_t = T_t \times S_t \times I_t$$

The additive model is most appropriate if the magnitude of the seasonal fluctuations or the variation around the trend-cycle does not vary with the level of the time series.

Time Series Decomposition and Transformations

Seasonal Adjustment

Some economic time series observed at quarterly, monthly, weekly frequencies often exhibit cyclical seasonal movements that occur every quarter, month or week. For example, the monthly inflation rate in Angola reach a peak every December during Christmas period.

The **seasonally adjusted series** is obtained by removing the cyclical seasonal movements from a series, $X_t^{SA} = X_t - S_t$ (additive) or $X_t^{SA} = X_t / S_t$ (multiplicative)

Weighted moving average (WMA) methods:

- Additive decomposition
- Multiplicative decomposition

The **seasonal period** is denoted by *s* (e.g., *s*=4 for quarterly data, *s*=12 for monthly data, *s*= 7 for daily data with a weekly pattern)

Time Series Decomposition and Transformations

Multiplicative decomposition: $X_t = T_t \times S_t \times I_t$

Step1: Compute the trend-cycle component using a weighted moving average (MA):

$T_t = MA_t(12) = (0.5X_{t-6} + \cdots + X_t + \cdots + 0.5X_{t+6})/12$ if $s = 12$ (monthly)

or

$T_t = MA_t(4) = (0.5X_{t-2} + X_{t-1} + X_t + X_{t+1} + 0.5X_{t+2})/4$ if $s = 4$ (quarterly)

Step 2: Calculate the detrended series: $\frac{X_t}{T_t} = S_t \times I_t = X_t^D$

Step 3: Estimate the seasonal components for each month or quarter, averaging the detrended values for that month (m) or quarter (q):

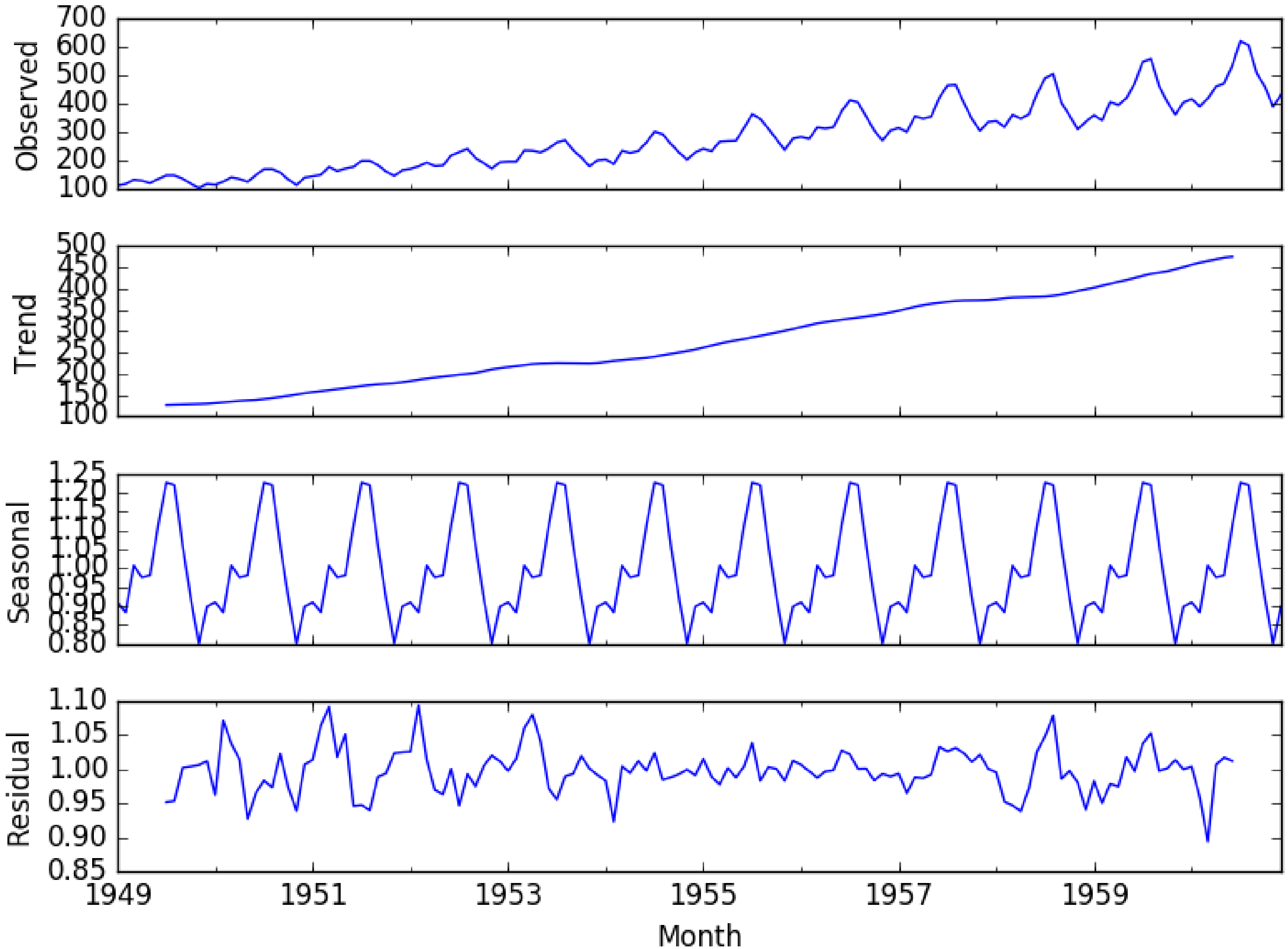
$$S_m = \frac{X_{y1,m}^D + X_{y2,m}^D + \cdots + X_{yn,m}^D}{n} \quad \text{or} \quad S_q = \frac{X_{y1,q}^D + X_{y2,q}^D + \cdots + X_{yn,q}^D}{n}$$

Then adjust the **seasonal indices so that they add to *s***:

$$\hat{S}_m = S_m / \sqrt[12]{S_1 S_2 \cdots S_{12}} \quad \text{or} \quad \hat{S}_q = S_q / \sqrt[12]{S_1 S_2 S_3 S_4}$$

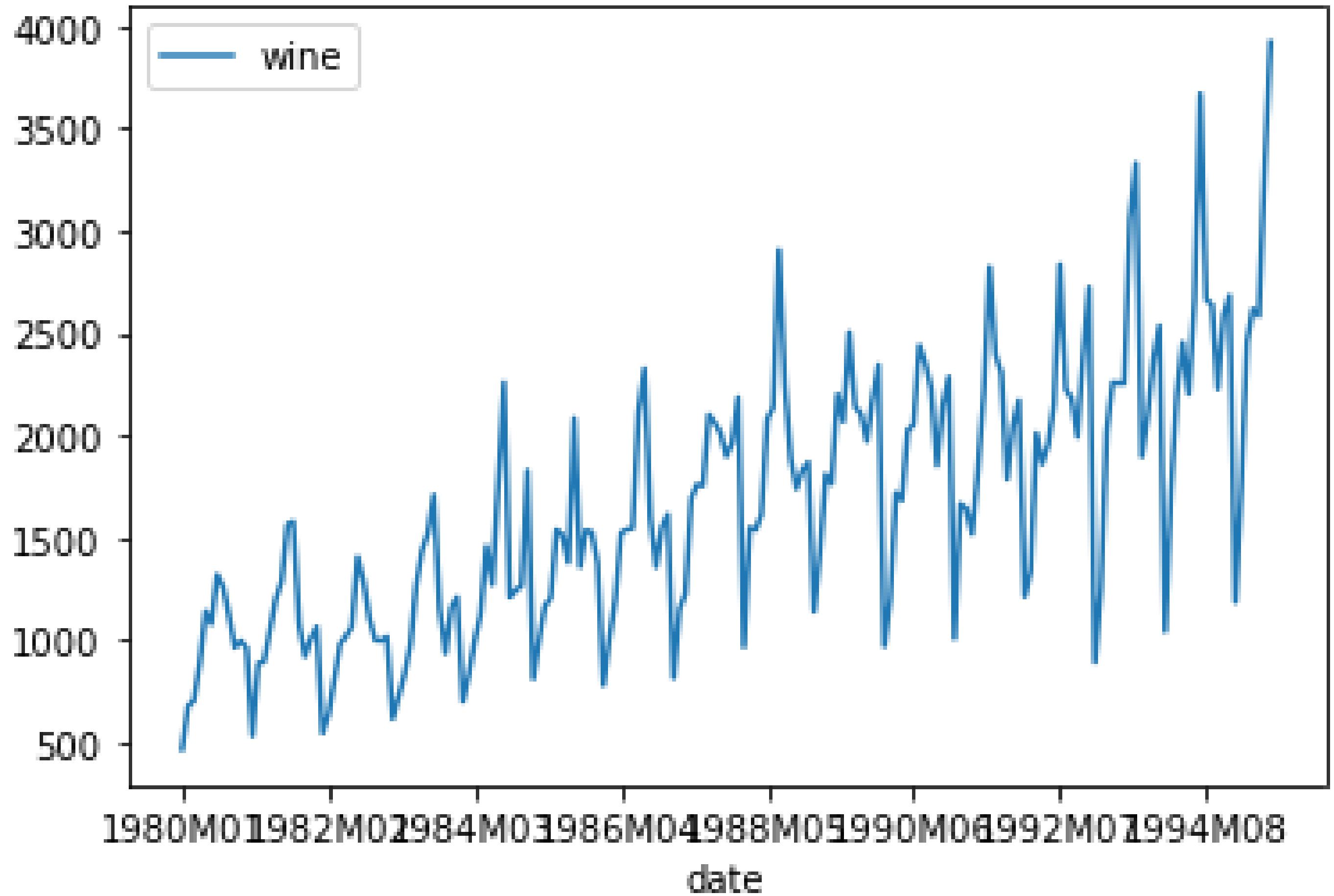
Step 4: Calculate the irregular series: $\frac{X_t^D}{\hat{S}_m} = I_t$

Time Series Decomposition and Transformations



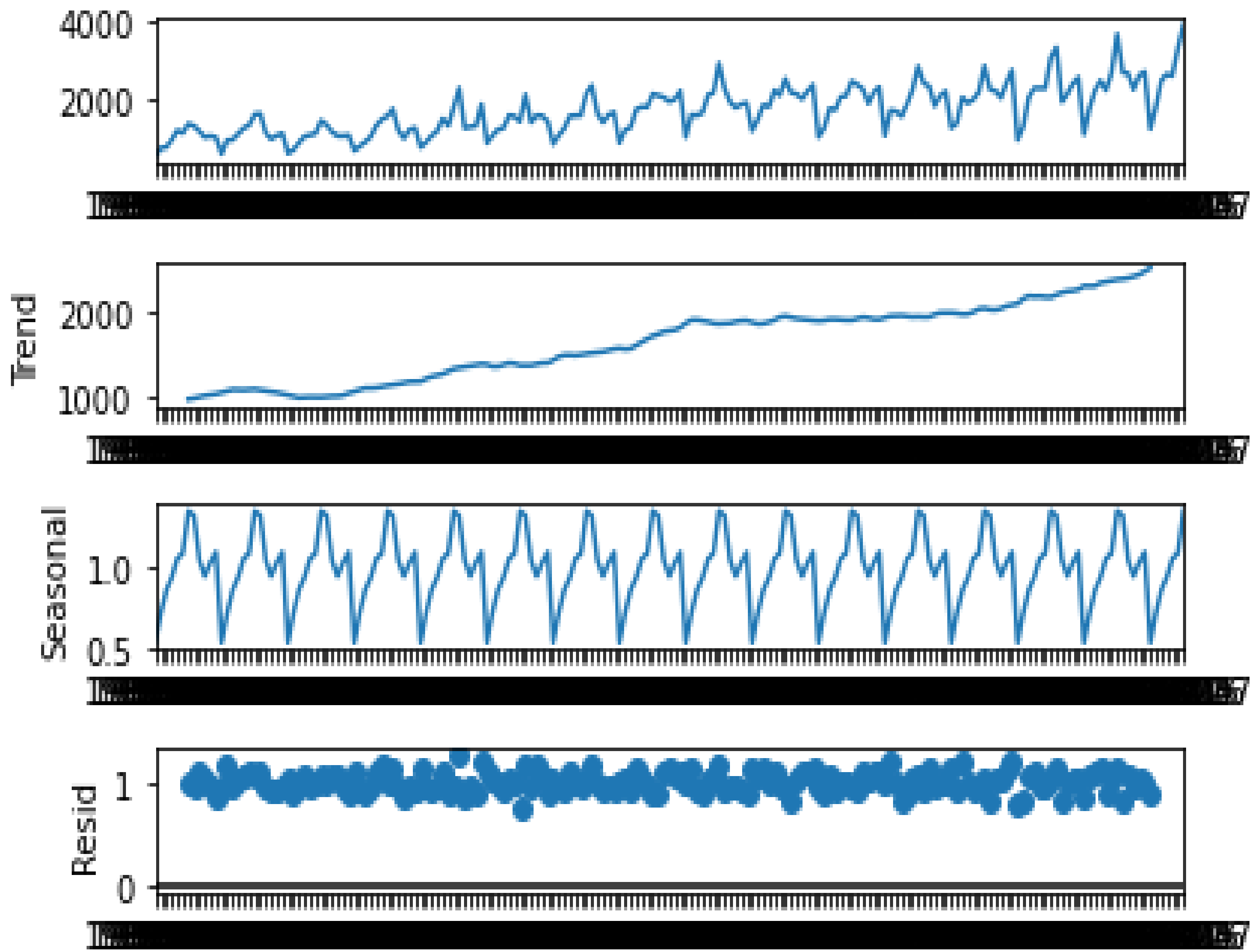
Time Series Decomposition and Transformations

Example: Figure below shows the result of the multiplicative decomposition of the wine consumption in Australia from January 1980 to July 1995



Time Series Decomposition and Transformations

Multiplicative decomposition of wine consumption in Australia



Time Series Decomposition and Transformations

Additive decomposition: $X_t = T_t + S_t + I_t$

Step1: Compute the trend-cycle component using a weighted moving average (MA):

$T_t = MA_t(12) = (0.5X_{t-6} + \cdots + X_t + \cdots + 0.5X_{t+6})/12$ if $s = 12$ (monthly)

or

$T_t = MA_t(4) = (0.5X_{t-2} + X_{t-1} + X_t + X_{t+1} + 0.5X_{t+2})/4$ if $s = 4$ (quarterly)

Step 2: Calculate the detrended series: $X_t - T_t = S_t + I_t \ = X_t^D$

Step 3: Estimate the seasonal components for each month or quarter, averaging the detrended values for that month (*m*) or quarter (*q*):

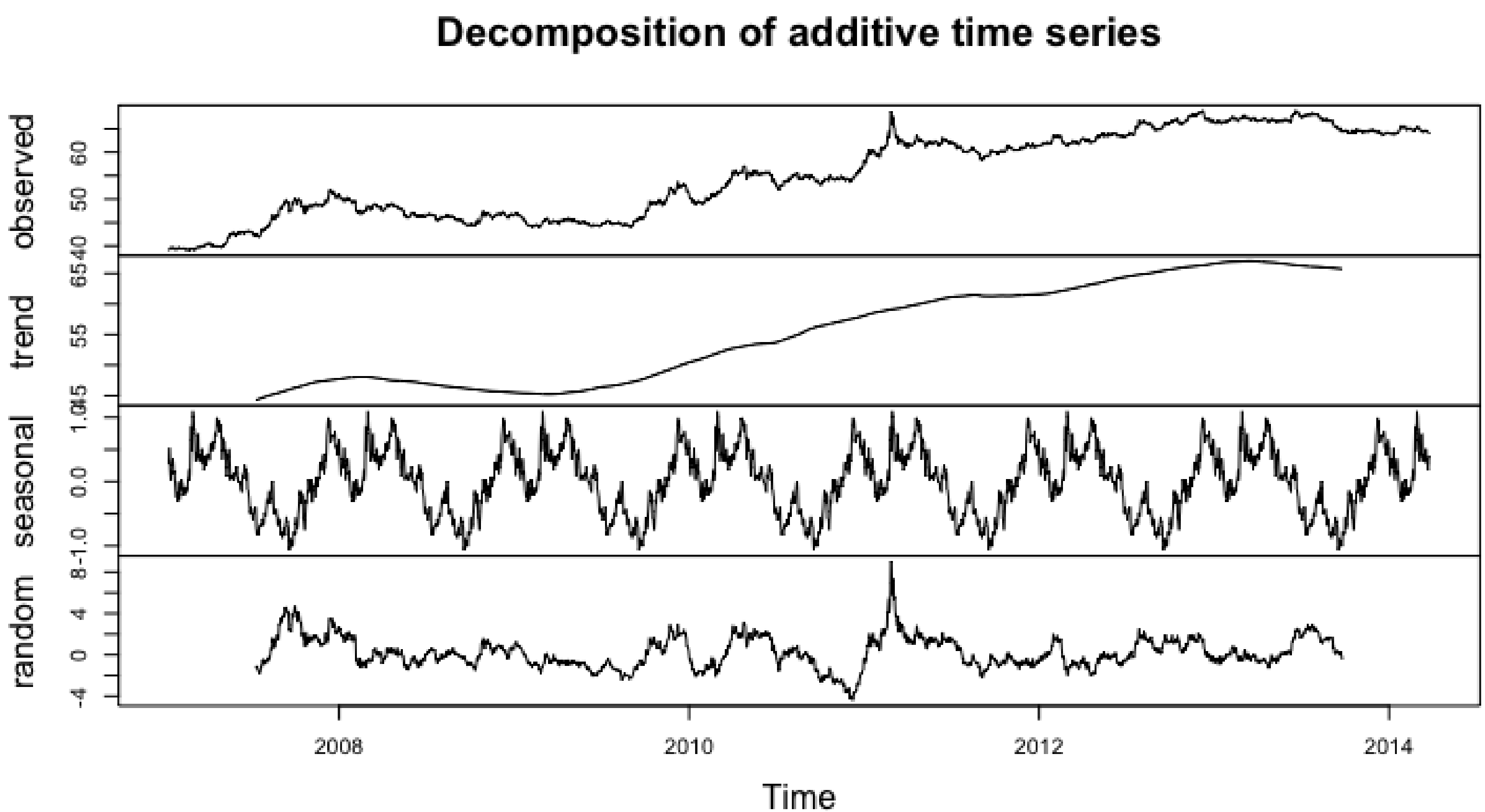
$$S_m = \frac{X_{y1,m}^D + X_{y2,m}^D + \cdots + X_{yn,m}^D}{n} \quad \text{or} \quad S_q = \frac{X_{y1,q}^D + X_{y2,q}^D + \cdots + X_{yn,q}^D}{n}$$

Then adjust the **seasonal indices so that they add up to zero:**

$$\hat{S}_m = S_m - (S_1 + S_2 + \cdots + S_{12})/12 \quad \text{or} \quad \hat{S}_q = S_q - (S_1 + S_2 + S_3 + S_4)/4$$

Step 4: Calculate the irregular series: $X_t^D - S_t = I_t$

Time Series Decomposition and Transformations



Source:
https://www.researchgate.net/publication/325359390_Forecasting_USD_to_INR_foreign_exchange_rate_using_Time_Series_Analysis_techniques_like_HoltWinters_Simple_Exponential_Smoothing_ARIMA_and_Neural_Networks/figures?lo=1

Time Series Decomposition and Transformations

Problem 1

The Airline Passengers dataset describes the total number of airline passengers in US from 1949 to 1960 (monthly observations in thousands).
Source:https://raw.githubusercontent.com/jbrownlee/Datasets/master/airline-passengers.csv

a) Import data to Python. Plot the time series. Are there any seasonal fluctuations?
b) Use multiplicative decomposition to estimate the trend-cycle, seasonal indices and irregular component.