Optimal Versus Naive Diversification: Do Different Loss Functions Improve Portfolio Choice?

by

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Abstract

I estimate the out-of-sample performance of the equal weight, minimum variance and mean-variance model portfolios in different settings. In each setting, I vary the loss function used when estimating returns and covariances, length of the estimation window, and number of factors used in our estimation model. I find that when measuring performance by Sharpe ratio, choice of loss function strongly influences whether the mean-variance model portfolio outperforms the equal weight or minimum variance portfolio, and that the optimal loss function depends on the length of the estimation window and the dimension of the return model. It appears that we dont gain much by using more factors. The 3-factor model does a pretty good job based on Sharpe ratio, and the results are consistently the best for MVO(10). With more factors, it seems clear that we need longer estimation windows, but even then we do not gain anything in terms of Sharpe Ratio. However, when measuring performance by the certainty-equivalent return, I find that the mean-variance model portfolio does not outperform the minimum variance portfolio or the equal weight portfolio in any setting. This suggests that choosing a loss function carefully is imperative to managing estimation errors and that an investor's utility preferences and attitude towards risk should be taken into account when choosing a measure of performance.

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1. Introduction

Portfolio choice and optimization theory has developed tremendously since the results first put forth by Markowitz (1952). Markowitz gave an analytical expression for the optimal rule an investor should follow when creating a portfolio of risky assets in a static setting. The investor in question only cares about the mean and variance of the portfolio's returns. However, the performance of portfolios formed according to the optimal rule was shown to be poor out of sample, see DeMiguel et al.(2009), likely due to the fact that there is considerable estimation error when estimating the moments of returns in sample, leading to portfolio weights that fluctuate wildly over time.

Due to the core weakness of traditional portfolio choice theory to estimation errors, literature focused on managing estimation error began to develop. Methods developed can be categorized under Bayesian and Non-Bayesian approaches. Bayesian approaches include statistical approaches relying on diffuse priors (Bawa, Brown, and Klein, 1979), shrinkage estimators (Jobson and Korkie, 1980) and relying on asset-pricing models while formulating a prior (Pastor and Stambaugh, 2000). Non-Bayesian approaches include robust allocation rules (Goldfarb and Iyengar, 2003), portfolios that exploit the moment restrictions imposed by the factor structure of returns (MacKinlay and Pastor, 2000), and portfolio rules that impose shortselling constraints (Frost and Savarino, 1988) to mention just a few.

However, in recent times, there has been work done on comparing portfolios created using Markowitz's optimal allocation rule to portfolios constructed using simpler heuristics such as giving an equal weight to all the securities or constructing the minimum variance portfolio. Doing the latter does not rely on information obtained from average returns of securities which is a primary source of estimation error. DeMiguel et al. show that the sample-based mean-variance model portfolio does not consistently outperform, when measured by multiple metrics including out of sample Sharpe ratio and the certainty-equivalent ratio, an equal weighted portfolio when tested in the context of the US equity market.

Kritzman et all.(2010) criticize DiMiguel et al. approach when estimating covariance matrices stating that DiMiguel et al. use estimation windows that are too short when estimating return moments. They also provide a thorough empirical application of portfolio choice theory, in particular mean-variance optimization, to the S&P 500 and show that substantial gains can be made, as measured by the Sharpe ratio, over the equal weighted portfolio. However, in practice due to limited data and non-stationarity it is quite possible that estimation using long windows is infeasible. The objective of my paper is to study the estimation error in estimates of moments of returns and also under what design choices can the mean-variance optimized portfolio be expected to perform better than an equal weighted portfolio. In particular, I study the bias-variance trade off when estimating moments of returns through three lenses: length of the estimation window, complexity of the model used to estimate moments and risk aversion of an investor.

2. Methodology for Model Estimation

One can think of the portfolio formation process as composed of two steps. First, collecting characteristics and estimating statistics of interest related to securities in the investable universe. These could include statistics such as means and covariances and characteristics such leverage ratios, market capitalization etc. Second, using the characteristics and statistics collected above along with a heuristic to assign weights to each security. The heuristics include the simplistic equal weighted portfolio and the more traditional Markowitz optimal portfolio.

Moment Estimation

There are many models one can assume for the evolution of returns of an asset. The most well known include the CAPM, Arbitrage Pricing Theory (APT) and Fama-French's three factor model. For my analysis, I assume that the excess returns of an asset over the risk free rate follow a factor model structure given by

$$R_{it} = \beta_i^T f_t + \epsilon_{it}$$

where R_{it} denotes the excess return of risky asset i in month t, f_t represents the vector of excess returns of the different factors, β_i represents the vector of coefficients or weights on these factors for asset i and ϵ_{it} is white noise. We also assume that ϵ_{it} is cross-sectionally and serially uncorrelated. I use three different sets of factors during my cross-sectional analysis. They are:

1. The classic Fama-French 3 factor model factors - Market, Small-Big, High-Low

2. Fama-French 5 factor model factors and Momentum - Market, Small-Big, High-Low, Robust-Weak, Conservative-Aggressive, Momentum

3. 21 factor set - This set includes all the factors from the previous set as well as artificial factors created from the product of every pair of factors.

When estimating returns, one usually performs a linear regression in the above setting to back out the coefficients needed to create forecasts out of sample. However, the choice of linear regression immediately tells us that the loss function being minimized is the Mean Squared Error (MSE). It is not entirely clear why we should choose this particular loss function when the sample size is small, we are not sure whether the return model is correctly specified, and we are primarily concerned with out-of-sample performance of the portfolio choice model. In the past it was chosen due to being convenient in the sense that analytical solution could be found to the estimation problem. Now with improvements in computing power, it is easier to implement other loss functions and minimize them numerically. This is one of the main contributions of my paper: varying the loss function used when estimating moments of returns and studying how they influence out of sample performance.

Let R_{it} denote the true return of asset i in month t and $\hat{R_{it}}$ denote the estimate of the same where $\hat{R_{it}} = \beta_i^T \hat{f}_t$. The loss functions I study are as follows:

1. Mean Squared Error (MSE):

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}(R_{it} - \hat{R_{it}})^2$$

2. Mean Squared Error with L2 Regularization (Ridge):

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}(R_{it}-\hat{R_{it}})^2+\alpha\sum_{f=1}^{F}\beta_f^2$$

3. Mean Squared Error with L1 Regularization (Lasso):

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}(R_{it}-\hat{R_{it}})^2+\lambda\sum_{f=1}^{F}|\beta_f|$$

4. Mean Absolute Error (MAE):

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}|R_{it}-\hat{R_{it}}|$$

5. Huber Loss:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}\rho((R_{it}-\hat{R_{it}})^2)$$

where

$$\left\{\begin{array}{rl}\rho((R_{it}-\hat{R_{it}})^2)=(R_{it}-\hat{R_{it}})^2 & if\ (R_{it}-\hat{R_{it}})^2\leq 1\\ \rho((R_{it}-\hat{R_{it}})^2)=\sqrt{(R_{it}-\hat{R_{it}})^2}-1 & if\ (R_{it}-\hat{R_{it}})^2>1\end{array}\right.$$

6. Cauchy Loss:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}ln(1+(R_{it}-\hat{R_{it}})^2)$$

7. Arctan Loss:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}arctan((R_{it}-\hat{R_{it}})^2)$$

8. Soft L1 Loss:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}2(\sqrt{1+(R_{it}-\hat{R_{it}})^2}-1)$$

9. Quantile Loss (for quantile τ):

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}\rho(|R_{it}-\hat{R_{it}}|)$$

where

$$\begin{cases} \rho(|R_{it} - \hat{R}_{it}|) = (1-\tau)|R_{it} - \hat{R}_{it}| & if\ R_{it} < \hat{R}_{it} \\ \rho(|R_{it} - \hat{R}_{it}|) = \tau|R_{it} - \hat{R}_{it}| & if\ R_{it} \geq \hat{R}_{it} \end{cases}$$

The loss functions I study primarily differ in how they treat outliers in the sample data, which strongly influences the bias-variance trade-off when estimating moments. There is sufficient literature discussing how even though minimizing the MSE loss function leads to the best linear unbiased estimator, commonly referred to by the acronym BLUE, the estimator is strongly affected by the presence of outliers which leads to poorer performance out of sample. To prevent this overfitting, Ridge and Lasso regressions were developed in order to perform regularization. These two types of regressions have also been widely studied in the context of portfolio choice theory. The other loss functions have not been studied as much and I will give some brief intuition on how they differ in treating outliers now. The MAE loss function is more robust to outlier than the MSE loss function in general because the penalty given to the error term, where the error term is defined by 'difference' between the true value and the predicted value, is linearly proportional under the MAE loss function and squared proportional under the MSE loss function. Essentially, the MSE loss function believes there is more information present in outliers than the MAE loss function does which makes the MSE loss function put more weight on these outliers. Minimizing the MSE leads to the mean-unbiased estimator and minimizing the MAE leads to the median-unbiased estimator. The Huber loss function is designed to combine the MSE and MAE loss functions optimally. It treats data points that result in large error terms as the MAE loss function would and data points that result in small error terms as the MSE loss function would. This makes the Huber loss function take more information from data points resulting in small error terms than the MAE loss function and less information from data points resulting in large error terms than the MSE loss function leading it be usually more robust to outliers than both of them. The Soft L1 loss is designed to be a smoother version of the MAE loss. The MAE loss is hard to use due to its gradient vanishing at time. The Soft L1 loss smooths out the MAE loss so that its gradient always exists and can be computed which makes it easier to implement. The

Arctan and Cauchy loss functions are primarily used in the context of classification problems. However, since their outputs are continuously valued, I decided to use them as loss functions and compare their results with the more traditional loss functions. Quantile-based regression aims to estimate the conditional quantile of a response variable given certain values of predictor variables. The idea is to choose the quantile value based on whether we want to overestimate or underestimate. For example, a quantile choice $\tau = 0.25$ has a stronger penalty for overestimating the dependent variable and tries to predict values below the median.

When it comes to estimating the factor model detailed above, one has to choose the length of the estimation period to use i.e how large the training sample data must be. In my analysis I use estimation windows of lengths 30 months, 60 months, 90 months and 120 months. The attentive reader would have observed above that I defined the estimate of the return on asset i in month t as $\hat{R}_{it} = \beta_i^T \hat{f}_t$ and not $\hat{R}_{it} = \beta_i^T f_t$. I also use an estimate of factor returns in month t. Specifically, an estimate of the return of factor f in month t is made by fitting an AR(p) model to lags of factor f's return in the estimation window and then using the model to make a prediction. The number of lags used is chosen according to the Bayesian Information Criterion (BIC). Lastly, the estimate of the covariance matrix of returns $\hat{\Sigma}_t$ is given by:

$$\hat{\Sigma}_t = \beta^T \hat{\Sigma}_f \beta + D$$

where the columns of β are $\beta_i, i \in \{1, 2, ...N\}$, Σ_f is the in-sample covariance matrix of the factor returns, and D is a diagonal matrix where the diagonal elements are given by the idiosyncratic variances $\sigma_i^2, i \in \{1, 2, ...N\}$ of the risky assets.

Portfolio Construction

I use μ_t to represent the vector of true expected returns on the risky assets and Σ_t to represent the true variance-covariance matrix of the risky asset returns. Their sample counterparts are denoted by $\hat{\mu}_t$ and $\hat{\Sigma}_t$. Let M denote the length of the estimation window and T the total length of the data series. Let ι represent a a vector of 1 of suitable length. Finally, let w_t represent the vector of relative weights invested in risky assets with the sum of the weights totalling 1. The subscript t refers to month t. The portfolio construction heuristics I study are the

1. Equal weight portfolio: This strategy involves building a portfolio where $w_i = \frac{1}{N}, \ \forall i \in \{1, 2, ..., N\}$. This strategy does not rely on any information from the data nor does it involve any optimization or estimation of moments.

2. In-sample minimum variance portfolio: the investor chooses the portfolio of risky assets i.e specify the weights w_t for month t that minimizes the variance of returns; that is,

$$min \ w_t^T \Sigma_t w_t$$

   s.t

$$\iota^T w_t = 1, \ w_i \geq 0 \ \forall i$$

   To implement this policy, I use only the estimate of the covariance matrix of asset returns $\hat{\Sigma}_t$ and completely ignore the estimates of the expected returns.

3. In-sample mean-variance optimal (Markowitz) portfolio: At each month t, the investor selects w_t to maximize their expected utility.

$$max \ w_t^T \mu_t - \frac{\gamma}{2} w_t^T \Sigma_t w_t$$

   such that

$$\iota^T w_t = 1, \ w_i \geq 0 \ \forall i$$

   γ represents the risk aversion of the investor. The investor optimizes the tradeoff between the mean and variance of portfolio returns. To implement this model, I follow the classic plug-in approach; that is, I solve the problem with the mean and covariance matrix of asset returns replaced by their sample counterparts $\hat{\mu}_t$ and $\hat{\Sigma}_t$, respectively. The sample covariance matrix is constructed as described above.

I also impose a short sale constraint on all three portfolios.

3. Methodology for Evaluating Performance

My goal is to study the performance of each of the aforementioned portfolios across a combination of lengths of estimation windows and model complexities. The investable universe I consider contains S&P 500 companies from 1985 to 2007. However, a particular stock does not enter the investable universe until it first enters the S&P 500 and second, it has a history of returns as long as the estimation window. For example, if the length of the estimation window is $M = 30$, then even though Amazon entered the S&P 500 in November of 2005, it would enter the investable universe only in May of 2008. This is done in order to avoid any survivorship bias in our sample.

The analysis relies on a rolling-sample approach. Specifically, given a T - month-long sample of asset and factor returns, I choose an estimation window of length $M, M \in \{30, 60, 90, 120\}$. In each month t, starting from $t = M + 1$, I use the data in the previous M months to estimate the parameters needed to construct one of the aforementioned portfolios. The estimated parameters are used to determine weights in a portfolio of only-risky assets. I then use these weights to compute the return in month $t+1$. This process is continued by adding the return for the next period in the sample and dropping the earliest return, until the end of the sample is reached. The outcome of this rolling-window approach is a series of $T - M$ monthly out-of-sample returns. In particular I generate monthly out of sample returns from January 1985 to June 2018.

For each set of monthly out of sample returns generated, I compute three statistics.

1. Sharpe Ratio: Defined as the sample mean of out of sample excess returns (over the risk-free asset), $\hat{\mu}$, divided by the sample standard deviation, $\hat{\sigma}$.

$$\hat{SR} = \frac{\hat{\mu}}{\hat{\sigma}}$$

2. Certainty-equivalent (CEQ) return: Defined as the risk free rate that an investor

is willing to accept rather than adopting a particular risky portfolio strategy.

$$C\hat{E}Q = \hat{\mu} - \frac{\gamma}{2}\hat{\sigma^2}$$

3. Constant Relative Risk Aversion (CRRA) utility: Measures utility of the investor while being unaffected by initial value of wealth.

$$C\hat{R}RA = \frac{\hat{\mu}^{1-\theta)}}{1-\theta}$$

In my analysis, I set $\theta = 1.5$.

The Sharpe ratio has always been a popular measure of performance for investment strategies. However, it does have some drawbacks; namely that it has underpinning assumptions including the investor being risk averse, the investor having a quadratic utility function and that the investor believes the inherent risk in a security's returns is adequately captured by the first two moments of the returns. The CEQ can be viewed to be more robust than the Sharpe ratio since it does not make any assumptions about the nature of the investors utility function and better formalizes an investor's attitude towards risk. Lastly, I look at the utility of the average excess return of a portfolio as measured by the CRRA utility function. It is important to note here that this metric might not be the most appropriate since if an investor had CRRA utility preferences, then their objective would be to perform an optimization that maximized their CRRA utility rather than performing a mean-variance optimization. The results will be interesting to note, however, since most portfolio managers in the industry look at mean-variance optimization as the standard benchmark and do not consider the functional form of an investor's utility function. So we can interpret the CRRA utility in this scenario as the utility an investor with a CRRA utility function can expect to receive.

4. Results

The results of my analysis are presented in Tables 1-12 in Appendix A.

Sharpe Ratios

Firstly, keeping the loss function the same and the number of factors in our estimation model constant, by and large increasing the length of the estimation window is associated with an increase in Sharpe ratio. This behavior is particularly noticeable amongst the Markowitz portfolios across all the risk aversion choices. It is not as apparent for the minimum variance portfolio. The equal weight portfolio's returns and statistics do not change as the number of factors in our model or the length of the estimation window changes. Secondly, keeping the loss function the same and the length of the estimation window constant, there is no definite pattern in terms of Sharpe ratio when we increase the number of factors in our estimation model. In fact, for most loss functions, using the second set of factors, Fama-French 5 factor model factors and Momentum, leads to estimating returns and covariances resulting in the highest Sharpe ratios. However, this pattern is not so strongly apparent as the previous one.

Thirdly, increasing the risk aversion of the parameter when performing the mean-variance optimization is associated with higher Sharpe ratios.

Lastly, using the Huber loss function, in a majority of case, produces substantially higher Sharpe ratios relative to those produced by other loss functions.

Digging in further, we observe a few similarities and differences to the results presented by DeMiguel et al.(2009). It is important to note that DeMiguel et al.(2009) assume a risk aversion of 1 when they construct the Markowitz portfolio. The minimum variance and equal weight portfolio outperform the Markowitz portfolios (with risk aversion 1) when estimation periods are small and/or when there are a large number of factors in our estimation model. This finding is consistent with DiMiguel et al.(2009). However, holding everything else constant, increasing the risk aversion parameter leads to the Markowitz portfolio outperforming the equal weighted and minimum variance portfolios. In fact the Markowitz portfolios outperform the minimum variance portfolio and equal weighted portfolio even when the estimation window is on the shorter side when using the Huber, Cauchy, Soft L1, Arctan loss functions.

CEQ Returns

Firstly, across all loss functions, keeping length of the estimation window the same and the number of factors in our estimation model the same, the CEQ returns of the equal weighted portfolio and the minimum variance portfolio is atleast as good if not better than the CEQ returns of the Markowitz portfolio across all risk aversion parameter choices.

Secondly, by and large, the CEQ returns of the equal weighted and minimum variance portfolios are 0.01 irrespective of the loss function, length of estimation window and number of factors in the model. For the Markowitz portfolios, the CEQ returns increases with an increase in the estimation window's length holding everything else constant.

Thirdly, focusing on the Markowitz portfolios, in contrast to the results found with Sharpe ratios, there is no clear loss function that stands out in terms of superior CEQ returns. Ridge, Lasso, Huber and the 25th Quantile loss functions produce the highest CEQ return more or less the same number of times across the different loss functions, estimation window lengths and number of factors in the model. However, the CEQ return produced is still lower than those of the equal weighted portfolio and minimum variance portfolios.

Lastly, by and large and holding all else equal, a higher risk aversion parameter is associated with a lower CEQ return. In fact, when estimation windows are shorter we observe negative values for the CEQ returns of Markowitz portfolios. Recalling the definition of the CEQ return, this means that an investor would be willing to accept a negative risk free rate in the place of the opportunity of investing in such a portfolio which is quite surprising! The results here are consistent with the findings of DeMiguel et al.(2009) and their implications are contrary to the implications of the results found bases on analysis of the Sharpe ratios. As mentioned above, the Sharpe ratio relies on assumptions of an investor having a quadratic utility and believing that the inherent riskiness of a security's returns is captured by the first two moments of the returns. It does not explicitly account for the risk aversion of the investor. The

CEQ return does not rely on these assumptions and can be considered to be a more robust measure of performance which accounts for an investor's attitude towards risk.

CRRA Utility

The comparison of CRRA utilities leads to similar conclusions to those reached when comparing Sharpe ratios. The Markowitz portfolios always outperform the equal weighted and minimum variance portfolios. This is due to the fact that the CRRA utility is a function of just the average returns achieved and that the Markowitz portfolios achieve higher average returns relative to the equal weight and minimum variance portfolios (they also have much higher variance of returns as well). Like in the Sharpe ratio case, the Huber loss function leads to substantially higher returns relative to the other loss functions which consequently leads to a higher CRRA utility.

5. Conclusion

From the above discussion, I conclude that when it comes to Sharpe ratio the choice of loss function does have a strong influence on final performance. The choice of the Huber loss function led to substantially higher realized Sharpe ratios across all portfolios relative to those achieved using other loss functions. The Huber loss function also led to Markowitz portfolios outperforming the equal weighted and minimum variance portfolios even in cases where the estimation window was particularly small ($M \in \{30, 60\}$). As noted above, the Huber loss functions deals with outliers better than both the MSE and MAE loss functions and hence tackles the problem of estimation error better which is particularly pernicious in small samples. The results here contrast those found in De Miguel et al.(2009) as I do find the Markowitz portfolio consistently outperforming the equal weight and minimum variance portfolios. I also observe that the longer the estimation window, the better the performance of the portfolios in terms of Sharpe ratios. There also appear to be a sweetspot when it comes to number of factors in our estimation model in the sense that too little or too many factors are associated with lower Sharpe ratios.

In terms of CEQ returns however, the results are aligned with De Miguel et al(2009).

I find that no strategy outperforms the minimum variance and equal weighted portfolios consistently. In fact, the Markowitz portfolios achieve negative CEQ returns which is very surprising. So an investor whose attitude towards risk and utility preferences are not adequately captured by a quadratic utility function would require much longer estimation windows in order for the Markowitz portfolios to start outperforming the minimum variance and equal weighted portfolios. I also observe that the smaller the number of factors in the estimation model and the longer the estimation window, the better the performance of the Markowitz portfolios in terms of CEQ returns. The performance of the minimum variance and equal weighted portfolios do not change materially with a change in the length of the estimation window, number of factors or loss function. One particularly interesting observation to make is that the MSE loss function actually leads to the highest CEQ when the estimation window is 120 months long and the risk aversion parameter is 5 or greater for the Markowitz portfolios. We know that theoretically minimizing the MSE loss function leads to the highest Sharpe and CEQ in-sample. So this results appears to agree with De Miguel et al(2009) i.e if we were to extend our estimation window to be long enough, we would eventually see the MSE loss function outperforming the rest (Asymptotically, minimizing the Huber loss function would replicate the behavior of minimizing the MSE loss function). The analysis of results of CRRA utility closely mirrors the analysis of Sharpe ratios.

There are of course certain limitations to the analysis above. I use only a single data set of returns, I impose a short sale constraint on all portfolios being analyzed, and the combination of estimation window lengths and number of factors in the estimation model is not comprehensive to name just a few. However, there are still some informative conclusions we can draw from the analysis above. Firstly, the choice of loss function clearly matters in ones estimation methodology and if done correctly, can ameliorate the effects of estimation error. Secondly, using standard measures of performance such as Sharpe ratios might not always be appropriate or informative given the varied utility preferences and attitudes towards risk of investors.

Appendix A

Table 1: **30 Months Estimation, 3 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.17	0.14	0.11
Sharpe	0.30	0.33	0.13	0.16	0.36
CEQ	0.01	0.01	0.01	-0.03	-0.02
CRRA(1.5)	-16.58	-17.76	-13.26	-13.26	-10.00

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.05	0.13	0.10	0.08
Sharpe	0.30	0.30	0.15	0.20	0.50
CEQ	0.01	0.01	0.01	-0.01	0.01
CRRA(1.5)	-16.58	-16.97	-14.20	-14.20	-10.00

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.05	0.13	0.10	0.09
Sharpe	0.30	0.30	0.15	0.19	0.44
CEQ	0.01	0.01	0.01	-0.01	0.00
CRRA(1.5)	-16.58	-16.95	-14.37	-14.37	-10.00

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.04	0.16	0.14	0.12
Sharpe	0.30	0.31	0.06	0.07	0.25
CEQ	0.01	0.01	0.00	-0.04	-0.04

Table 1 continued from previous page

CRRA(1.5)	-16.58	-17.84	-20.35	-20.35	-11.55

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.06	0.06	0.08
Std.Dev	0.05	0.04	0.21	0.18	0.16
Sharpe	0.30	0.34	0.27	0.32	0.50
CEQ	0.01	0.01	0.04	-0.02	-0.05
CRRA(1.5)	-16.58	-17.03	-8.39	-8.39	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.03	0.05
Std.Dev	0.05	0.04	0.16	0.14	0.12
Sharpe	0.26	0.31	0.18	0.20	0.42
CEQ	0.01	0.01	0.02	-0.02	-0.02
CRRA(1.5)	-18.07	-17.73	-11.84	-11.84	-8.94

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.03	0.12	0.10	0.09
Sharpe	0.26	0.34	0.16	0.18	0.44
CEQ	0.01	0.01	0.01	-0.01	0.00
CRRA(1.5)	-18.07	-18.70	-14.72	-14.72	-10.00

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.14	0.12	0.11
Sharpe	0.26	0.31	0.14	0.16	0.36
CEQ	0.01	0.01	0.01	-0.02	-0.02
CRRA(1.5)	-18.07	-17.49	-14.38	-14.38	-10.00

Table 1 continued from previous page

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.03	0.06
Std.Dev	0.05	0.04	0.17	0.16	0.14
Sharpe	0.26	0.31	0.20	0.21	0.43
CEQ	0.01	0.01	0.02	-0.03	-0.04
CRRA(1.5)	-18.07	-17.91	-10.82	-10.82	-8.16

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.03	0.20	0.18	0.16
Sharpe	0.26	0.29	0.12	0.14	0.25
CEQ	0.01	0.01	0.00	-0.06	-0.09
CRRA(1.5)	-18.07	-19.98	-12.72	-12.72	-10.00

Table 2: **60 Months Estimation, 3 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.14	0.13	0.11
Sharpe	0.30	0.31	0.18	0.17	0.36
CEQ	0.01	0.01	0.02	-0.02	-0.02
CRRA(1.5)	-16.58	-18.52	-12.67	-13.26	-10.00

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.05	0.12	0.10	0.08
Sharpe	0.30	0.29	0.17	0.20	0.50
CEQ	0.01	0.01	0.01	-0.01	0.01
CRRA(1.5)	-16.58	-17.37	-13.96	-14.20	-10.00

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.05	0.12	0.10	0.09
Sharpe	0.30	0.29	0.17	0.19	0.44
CEQ	0.01	0.01	0.01	-0.01	0.00
CRRA(1.5)	-16.58	-17.39	-13.96	-14.37	-10.00

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.12	0.10	0.09
Sharpe	0.30	0.30	0.16	0.20	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-18.19	-14.71	-14.14	-11.55

Table 2 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.06	0.06	0.08
Std.Dev	0.05	0.04	0.17	0.14	0.14
Sharpe	0.30	0.28	0.35	0.41	0.57
CEQ	0.01	0.01	0.05	0.01	-0.02
CRRA(1.5)	-16.58	-18.56	-8.09	-8.39	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.05
Std.Dev	0.05	0.04	0.12	0.11	0.10
Sharpe	0.26	0.30	0.14	0.26	0.50
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-18.07	-18.17	-15.13	-11.84	-8.94

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.04
Std.Dev	0.05	0.03	0.09	0.09	0.09
Sharpe	0.26	0.30	0.15	0.21	0.44
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-18.07	-20.02	-17.12	-14.72	-10.00

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.14	0.12	0.11
Sharpe	0.26	0.30	0.14	0.16	0.36
CEQ	0.01	0.01	0.01	-0.02	-0.02
CRRA(1.5)	-18.07	-17.90	-14.11	-14.38	-10.00

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 2 continued from previous page

Mean	0.01	0.01	0.02	0.03	0.06
Std.Dev	0.05	0.04	0.12	0.11	0.10
Sharpe	0.26	0.30	0.18	0.31	0.55
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-18.07	-18.55	-13.75	-10.82	-8.53

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.03	0.18	0.16	0.16
Sharpe	0.26	0.23	0.09	0.15	0.25
CEQ	0.01	0.01	0.00	-0.04	-0.09
CRRA(1.5)	-18.07	-22.84	-15.92	-12.72	-10.00

Table 3: **90 Months Estimation, 3 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.11	0.09	0.10
Sharpe	0.30	0.29	0.15	0.25	0.40
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-16.58	-19.16	-15.21	-13.26	-10.00

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.04	0.09	0.09	0.09
Sharpe	0.30	0.27	0.16	0.11	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-18.16	-16.53	-20.00	-11.55

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.05	0.09	0.09	0.09
Sharpe	0.30	0.27	0.15	0.11	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-18.20	-16.58	-20.00	-11.55

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.04	0.11	0.09	0.09
Sharpe	0.30	0.28	0.14	0.11	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-18.68	-16.62	-20.00	-11.55

Table 3 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.06	0.06	0.08
Std.Dev	0.05	0.04	0.15	0.12	0.12
Sharpe	0.30	0.27	0.38	0.47	0.67
CEQ	0.01	0.01	0.05	0.02	0.01
CRRA(1.5)	-16.58	-19.01	-8.25	-8.39	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.05
Std.Dev	0.05	0.04	0.12	0.12	0.11
Sharpe	0.26	0.29	0.15	0.24	0.45
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-18.07	-18.58	-14.44	-11.84	-8.94

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.04
Std.Dev	0.05	0.03	0.09	0.09	0.09
Sharpe	0.26	0.25	0.12	0.21	0.44
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-18.07	-21.81	-19.02	-14.72	-10.00

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.12	0.09	0.10
Sharpe	0.26	0.29	0.14	0.21	0.40
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-18.07	-18.33	-15.40	-14.38	-10.00

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 3 continued from previous page

Mean	0.01	0.01	0.02	0.03	0.05
Std.Dev	0.05	0.04	0.10	0.10	0.10
Sharpe	0.26	0.29	0.17	0.34	0.55
CEQ	0.01	0.01	0.01	0.01	0.00
CRRA(1.5)	-18.07	-18.71	-15.49	-10.82	-8.55
Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.04
Std.Dev	0.05	0.04	0.13	0.13	0.11
Sharpe	0.26	0.24	0.16	0.23	0.36
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-21.67	-13.79	-11.55	-10.00

Table 4: **120 Months Estimation, 3 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.04
Std.Dev	0.05	0.04	0.11	0.09	0.08
Sharpe	0.30	0.29	0.18	0.33	0.50
CEQ	0.01	0.01	0.01	0.01	0.01
CRRA(1.5)	-16.58	-18.64	-14.28	-11.55	-10.00

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.04	0.09	0.09	0.08
Sharpe	0.30	0.27	0.18	0.28	0.38
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-16.58	-18.35	-15.92	-12.65	-11.55

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.01	0.03
Std.Dev	0.05	0.04	0.09	0.09	0.09
Sharpe	0.30	0.27	0.18	0.11	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-18.32	-15.93	-20.00	-11.55

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.01	0.03
Std.Dev	0.05	0.04	0.10	0.09	0.09
Sharpe	0.30	0.28	0.19	0.11	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-18.66	-14.39	-20.00	-11.55

Table 4 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.06	0.06	0.08
Std.Dev	0.05	0.04	0.17	0.12	0.12
Sharpe	0.30	0.24	0.36	0.47	0.67
CEQ	0.01	0.01	0.05	0.02	0.01
CRRA(1.5)	-16.58	-19.85	-8.08	-8.39	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.05
Std.Dev	0.05	0.04	0.11	0.12	0.11
Sharpe	0.26	0.28	0.16	0.24	0.45
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-18.07	-18.56	-15.20	-11.84	-8.94

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.04
Std.Dev	0.05	0.03	0.07	0.09	0.09
Sharpe	0.26	0.26	0.19	0.21	0.44
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-18.07	-21.50	-16.98	-14.72	-10.00

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.04
Std.Dev	0.05	0.04	0.11	0.09	0.10
Sharpe	0.26	0.28	0.16	0.21	0.40
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-18.07	-18.37	-14.85	-14.38	-10.00

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 4 continued from previous page

Mean	0.01	0.01	0.02	0.03	0.05
Std.Dev	0.05	0.04	0.10	0.10	0.10
Sharpe	0.26	0.28	0.22	0.34	0.55
CEQ	0.01	0.01	0.02	0.01	0.00
CRRA(1.5)	-18.07	-18.61	-13.80	-10.82	-8.55

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.04
Std.Dev	0.05	0.04	0.12	0.13	0.11
Sharpe	0.26	0.23	0.13	0.23	0.36
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-21.47	-15.87	-11.55	-10.00

Table 5: **30 Months Estimation, 6 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.19	0.17	0.16
Sharpe	0.30	0.32	0.10	0.12	0.19
CEQ	0.01	0.01	0.00	-0.05	-0.10
CRRA(1.5)	-16.58	-18.61	-14.22	-14.14	-11.55

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.13	0.11	0.10
Sharpe	0.30	0.30	0.15	0.18	0.30
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-16.58	-16.97	-14.21	-14.14	-11.55

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.13	0.12	0.11
Sharpe	0.30	0.30	0.15	0.17	0.27
CEQ	0.01	0.01	0.01	-0.02	-0.03
CRRA(1.5)	-16.58	-16.95	-14.37	-14.14	-11.55

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.03	0.03
Std.Dev	0.05	0.04	0.14	0.13	0.10
Sharpe	0.30	0.32	0.19	0.23	0.30
CEQ	0.01	0.01	0.02	-0.01	-0.02
CRRA(1.5)	-16.58	-18.34	-12.38	-11.55	-11.55

Table 5 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.06	0.06	0.07
Std.Dev	0.05	0.04	0.24	0.21	0.22
Sharpe	0.30	0.34	0.26	0.27	0.32
CEQ	0.01	0.01	0.03	-0.05	-0.17
CRRA(1.5)	-16.58	-16.86	-8.04	-8.39	-7.56
Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.04
Std.Dev	0.05	0.04	0.16	0.14	0.12
Sharpe	0.26	0.33	0.15	0.20	0.33
CEQ	0.01	0.01	0.01	-0.02	-0.03
CRRA(1.5)	-18.07	-18.11	-12.83	-11.84	-10.00
Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.13	0.11	0.10
Sharpe	0.26	0.32	0.13	0.17	0.30
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-19.21	-15.55	-14.72	-11.55
Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.03
Std.Dev	0.05	0.04	0.15	0.12	0.09
Sharpe	0.26	0.32	0.08	0.16	0.33
CEQ	0.01	0.01	0.00	-0.02	-0.01
CRRA(1.5)	-18.07	-17.82	-17.90	-14.38	-11.55
Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 5 continued from previous page

Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.04	0.15	0.13	0.10
Sharpe	0.26	0.32	0.09	0.15	0.20
CEQ	0.01	0.01	0.00	-0.02	-0.03
CRRA(1.5)	-18.07	-18.35	-16.95	-14.14	-14.14

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.03	0.04
Std.Dev	0.05	0.04	0.21	0.19	0.16
Sharpe	0.26	0.31	0.12	0.16	0.25
CEQ	0.01	0.01	0.00	-0.06	-0.09
CRRA(1.5)	-18.07	-19.25	-12.64	-11.55	-10.00

Table 6: **60 Months Estimation, 6 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.02
Std.Dev	0.05	0.03	0.15	0.13	0.12
Sharpe	0.30	0.31	0.09	0.08	0.17
CEQ	0.01	0.01	0.00	-0.03	-0.05
CRRA(1.5)	-16.58	-19.43	-17.11	-20.00	-14.14

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.12	0.09	0.09
Sharpe	0.30	0.29	0.17	0.22	0.33
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-16.58	-17.37	-13.96	-14.14	-11.55

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.12	0.10	0.08
Sharpe	0.30	0.29	0.17	0.20	0.38
CEQ	0.01	0.01	0.01	-0.01	0.00
CRRA(1.5)	-16.58	-17.39	-13.96	-14.14	-11.55

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.04	0.14	0.11	0.08
Sharpe	0.30	0.30	0.13	0.27	0.38
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-16.58	-18.88	-15.22	-11.55	-11.55

Table 6 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.07	0.07	0.08
Std.Dev	0.05	0.04	0.20	0.19	0.17
Sharpe	0.30	0.28	0.33	0.37	0.47
CEQ	0.01	0.01	0.05	-0.02	-0.06
CRRA(1.5)	-16.58	-18.46	-7.78	-7.56	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.02
Std.Dev	0.05	0.04	0.13	0.11	0.09
Sharpe	0.26	0.30	0.10	0.12	0.22
CEQ	0.01	0.01	0.00	-0.02	-0.02
CRRA(1.5)	-18.07	-19.04	-17.77	-17.77	-14.14

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.10	0.10	0.08
Sharpe	0.26	0.31	0.17	0.18	0.38
CEQ	0.01	0.01	0.01	-0.01	0.00
CRRA(1.5)	-18.07	-19.80	-15.34	-14.72	-11.55

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.03	0.03
Std.Dev	0.05	0.04	0.15	0.12	0.09
Sharpe	0.26	0.30	0.21	0.25	0.33
CEQ	0.01	0.01	0.02	-0.01	-0.01
CRRA(1.5)	-18.07	-18.46	-11.17	-11.55	-11.55

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 6 continued from previous page

Mean	0.01	0.01	0.02	0.02	0.02
Std.Dev	0.05	0.04	0.15	0.12	0.10
Sharpe	0.26	0.31	0.12	0.17	0.20
CEQ	0.01	0.01	0.01	-0.02	-0.03
CRRA(1.5)	-18.07	-19.30	-14.73	-14.14	-14.14

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.03	0.21	0.18	0.15
Sharpe	0.26	0.24	0.06	0.11	0.13
CEQ	0.01	0.01	-0.01	-0.06	-0.09
CRRA(1.5)	-18.07	-22.37	-17.58	-14.14	-14.14

Table 7: **90 Months Estimation, 6 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.00	0.01	0.02
Std.Dev	0.05	0.03	0.13	0.11	0.11
Sharpe	0.30	0.28	0.02	0.09	0.18
CEQ	0.01	0.01	-0.01	-0.02	-0.04
CRRA(1.5)	-16.58	-20.30	-41.79	-20.00	-14.14

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.04	0.09	0.08	0.09
Sharpe	0.30	0.27	0.16	0.19	0.22
CEQ	0.01	0.01	0.01	0.00	-0.02
CRRA(1.5)	-16.58	-18.16	-16.53	-16.10	-14.14

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.05	0.09	0.09	0.07
Sharpe	0.30	0.27	0.15	0.11	0.43
CEQ	0.01	0.01	0.01	-0.01	0.01
CRRA(1.5)	-16.58	-18.20	-16.58	-20.00	-11.55

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.04	0.14	0.10	0.08
Sharpe	0.30	0.28	0.11	0.20	0.25
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-19.41	-16.60	-14.14	-14.14

Table 7 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.07	0.07	0.08
Std.Dev	0.05	0.01	0.17	0.15	0.14
Sharpe	0.30	0.27	0.39	0.47	0.57
CEQ	0.01	0.01	0.05	0.01	-0.02
CRRA(1.5)	-16.58	-18.79	-7.80	-7.56	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.13	0.11	0.10
Sharpe	0.26	0.28	0.12	0.18	0.30
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-19.44	-16.00	-14.14	-11.55

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.03
Std.Dev	0.05	0.03	0.12	0.11	0.07
Sharpe	0.26	0.26	0.10	0.18	0.43
CEQ	0.01	0.01	0.00	-0.01	0.01
CRRA(1.5)	-18.07	-21.83	-18.80	-14.14	-11.55

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.02
Std.Dev	0.05	0.04	0.14	0.12	0.10
Sharpe	0.26	0.29	0.11	0.17	0.23
CEQ	0.01	0.01	0.01	-0.02	-0.03
CRRA(1.5)	-18.07	-18.86	-16.06	-14.14	-13.06

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 7 continued from previous page

Mean	0.01	0.01	0.01	0.02	0.03
Std.Dev	0.05	0.04	0.13	0.12	0.10
Sharpe	0.26	0.28	0.09	0.20	0.28
CEQ	0.01	0.01	0.00	-0.01	-0.02
CRRA(1.5)	-18.07	-19.78	-18.61	-13.07	-11.85

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.04	0.04
Std.Dev	0.05	0.03	0.21	0.18	0.16
Sharpe	0.26	0.26	0.14	0.20	0.23
CEQ	0.01	0.01	0.01	-0.05	-0.09
CRRA(1.5)	-18.07	-21.36	-11.66	-10.59	-10.46

Table 8: **120 Months Estimation, 6 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.03	0.12	0.11	0.11
Sharpe	0.30	0.28	0.12	0.12	0.24
CEQ	0.01	0.01	0.01	-0.02	-0.03
CRRA(1.5)	-16.58	-20.13	-16.45	-17.71	-12.28

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.02
Std.Dev	0.05	0.04	0.09	0.08	0.07
Sharpe	0.30	0.27	0.18	0.19	0.29
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-16.58	-18.34	-15.94	-16.10	-14.14

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.09	0.08	0.08
Sharpe	0.30	0.27	0.18	0.29	0.43
CEQ	0.01	0.01	0.01	0.01	0.00
CRRA(1.5)	-16.58	-18.32	-15.93	-13.06	-10.81

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.04	0.12	0.10	0.08
Sharpe	0.30	0.28	0.12	0.24	0.31
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-16.58	-19.56	-16.63	-13.00	-12.70

Table 8 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.05	0.07	0.08
Std.Dev	0.05	0.04	0.15	0.15	0.14
Sharpe	0.30	0.25	0.36	0.47	0.57
CEQ	0.01	0.01	0.04	0.01	-0.02
CRRA(1.5)	-16.58	-19.61	-8.56	-7.56	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.04	0.12	0.12	0.10
Sharpe	0.26	0.28	0.15	0.23	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-19.51	-15.20	-12.23	-10.76

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.03	0.03
Std.Dev	0.05	0.03	0.09	0.09	0.08
Sharpe	0.26	0.26	0.14	0.30	0.40
CEQ	0.01	0.01	0.01	0.01	0.00
CRRA(1.5)	-18.07	-21.50	-17.54	-12.22	-11.24

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.12	0.12	0.10
Sharpe	0.26	0.28	0.18	0.21	0.30
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-19.04	-13.60	-12.69	-11.57

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 8 continued from previous page

Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.04	0.12	0.12	0.12
Sharpe	0.26	0.27	0.13	0.23	0.27
CEQ	0.01	0.01	0.01	-0.01	-0.04
CRRA(1.5)	-18.07	-20.08	-15.72	-11.98	-11.10

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.04	0.04
Std.Dev	0.05	0.03	0.14	0.12	0.12
Sharpe	0.26	0.24	0.13	0.29	0.35
CEQ	0.01	0.01	0.01	0.00	-0.03
CRRA(1.5)	-18.07	-21.73	-14.97	-10.64	-9.78

Table 9: **30 Months Estimation, 21 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.03
Std.Dev	0.05	0.03	0.17	0.15	0.14
Sharpe	0.30	0.32	0.08	0.11	0.21
CEQ	0.01	0.01	0.00	-0.04	-0.07
CRRA(1.5)	-16.58	-19.49	-16.94	-15.24	-11.75

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.13	0.10	0.10
Sharpe	0.30	0.30	0.15	0.25	0.26
CEQ	0.01	0.01	0.01	0.00	-0.02
CRRA(1.5)	-16.58	-16.97	-14.21	-12.77	-12.49

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.05	0.13	0.11	0.11
Sharpe	0.30	0.30	0.15	0.29	0.31
CEQ	0.01	0.01	0.01	0.00	-0.03
CRRA(1.5)	-16.58	-16.95	-14.37	-11.13	-10.76

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.15	0.12	0.11
Sharpe	0.30	0.32	0.12	0.21	0.29
CEQ	0.01	0.01	0.01	-0.01	-0.03
CRRA(1.5)	-16.58	-20.33	-14.71	-12.49	-11.16

Table 9 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.08	0.08	0.09
Std.Dev	0.05	0.03	0.18	0.15	0.14
Sharpe	0.30	0.32	0.12	0.54	0.63
CEQ	0.01	0.01	0.06	0.03	-0.01
CRRA(1.5)	-16.58	-19.48	-7.19	-7.00	-6.76

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.04	0.03
Std.Dev	0.05	0.03	0.16	0.14	0.15
Sharpe	0.26	0.33	0.15	0.27	0.23
CEQ	0.01	0.01	0.01	-0.01	-0.08
CRRA(1.5)	-18.07	-19.09	-12.65	-10.23	-10.83

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.13	0.10	0.09
Sharpe	0.26	0.31	0.16	0.25	0.32
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-18.07	-20.66	-13.78	-12.76	-11.84

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.17	0.16	0.14
Sharpe	0.26	0.35	0.12	0.13	0.20
CEQ	0.01	0.01	0.01	-0.04	-0.07
CRRA(1.5)	-18.07	-18.65	-13.80	-13.69	-12.04

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 9 continued from previous page

Mean	0.01	0.01	0.08	0.09	0.09
Std.Dev	0.05	0.03	0.17	0.15	0.13
Sharpe	0.26	0.31	0.12	0.58	0.70
CEQ	0.01	0.01	0.06	0.03	0.01
CRRA(1.5)	-18.07	-19.60	-7.17	-6.76	-6.62

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.02
Std.Dev	0.05	0.03	0.16	0.14	0.15
Sharpe	0.26	0.32	0.08	0.09	0.10
CEQ	0.01	0.01	0.00	-0.04	-0.10
CRRA(1.5)	-18.07	-20.61	-18.02	-17.71	-16.07

Table 10: **60 Months Estimation, 21 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.03	0.19	0.16	0.14
Sharpe	0.30	0.32	0.06	0.11	0.17
CEQ	0.01	0.01	-0.01	-0.05	-0.07
CRRA(1.5)	-16.58	-20.21	-18.04	-14.78	-12.97

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.12	0.10	0.09
Sharpe	0.30	0.29	0.17	0.15	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.01
CRRA(1.5)	-16.58	-17.37	-13.96	-16.10	-11.55

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.05	0.12	0.10	0.11
Sharpe	0.30	0.29	0.17	0.24	0.27
CEQ	0.01	0.01	0.01	0.00	-0.03
CRRA(1.5)	-16.58	-17.39	-13.96	-12.81	-11.57

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.02
Std.Dev	0.05	0.03	0.16	0.14	0.09
Sharpe	0.30	0.33	0.11	0.14	0.26
CEQ	0.01	0.01	0.00	-0.03	-0.02
CRRA(1.5)	-16.58	-19.83	-14.96	-14.38	-12.97

Table 10 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.17	0.15	0.14
Sharpe	0.30	0.32	0.10	0.23	0.21
CEQ	0.01	0.01	0.00	-0.02	-0.07
CRRA(1.5)	-16.58	-19.91	-15.30	-10.81	-11.80

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.03	0.03	0.03
Std.Dev	0.05	0.03	0.18	0.17	0.14
Sharpe	0.26	0.33	0.16	0.16	0.25
CEQ	0.01	0.01	0.01	-0.05	-0.06
CRRA(1.5)	-18.07	-19.63	-11.75	-12.23	-10.76

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.19	0.16	0.15
Sharpe	0.26	0.33	0.13	0.17	0.21
CEQ	0.01	0.01	0.01	-0.04	-0.08
CRRA(1.5)	-18.07	-20.19	-13.04	-12.22	-11.24

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.15	0.12	0.10
Sharpe	0.26	0.33	0.10	0.21	0.30
CEQ	0.01	0.01	0.00	-0.01	-0.02
CRRA(1.5)	-18.07	-19.16	-16.18	-12.69	-11.57

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 10 continued from previous page

Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.16	0.14	0.12
Sharpe	0.26	0.32	0.11	0.20	0.27
CEQ	0.01	0.01	0.00	-0.01	-0.02
CRRA(1.5)	-18.07	-19.16	-16.18	-12.69	-11.57

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.01
Std.Dev	0.05	0.03	0.18	0.16	0.15
Sharpe	0.26	0.28	0.03	0.06	0.07
CEQ	0.01	0.01	-0.01	-0.05	-0.10
CRRA(1.5)	-18.07	-21.17	-26.55	-20.00	-20.00

Table 11: **90 Months Estimation, 21 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.02
Std.Dev	0.05	0.03	0.16	0.15	0.14
Sharpe	0.30	0.29	0.07	0.09	0.13
CEQ	0.01	0.01	0.00	-0.04	-0.08
CRRA(1.5)	-16.58	-20.87	-18.75	-17.41	-14.60

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.02
Std.Dev	0.05	0.04	0.09	0.10	0.09
Sharpe	0.30	0.27	0.16	0.14	0.26
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-16.58	-18.16	-16.53	-16.92	-13.00

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.05	0.09	0.09	0.08
Sharpe	0.30	0.27	0.15	0.17	0.23
CEQ	0.01	0.01	0.01	0.00	-0.01
CRRA(1.5)	-16.58	-18.20	-16.58	-15.98	-14.61

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.15	0.12	0.10
Sharpe	0.30	0.30	0.15	0.22	0.29
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-16.58	-20.28	-13.10	-12.28	-11.81

Table 11 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.18	0.16	0.16
Sharpe	0.30	0.29	0.13	0.15	0.18
CEQ	0.01	0.01	0.01	-0.04	-0.10
CRRA(1.5)	-16.58	-20.76	-13.07	-12.81	-11.88

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.13	0.14	0.11
Sharpe	0.26	0.30	0.17	0.17	0.29
CEQ	0.01	0.01	0.01	-0.03	-0.03
CRRA(1.5)	-18.07	-20.39	-13.69	-12.97	-11.22

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.03	0.11	0.10	0.09
Sharpe	0.26	0.31	0.11	0.21	0.27
CEQ	0.01	0.01	0.01	0.00	-0.02
CRRA(1.5)	-18.07	-20.41	-18.11	-13.76	-12.79

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.14	0.12	0.11
Sharpe	0.26	0.31	0.15	0.20	0.30
CEQ	0.01	0.01	0.01	-0.01	-0.03
CRRA(1.5)	-18.07	-19.36	-13.81	-12.86	-10.95

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 11 continued from previous page

Mean	0.01	0.01	0.07	0.07	0.08
Std.Dev	0.05	0.03	0.18	0.18	0.17
Sharpe	0.26	0.29	0.13	0.38	0.46
CEQ	0.01	0.01	0.06	-0.01	-0.06
CRRA(1.5)	-18.07	-20.81	-7.33	-7.70	-7.23

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.03	0.16	0.16	0.15
Sharpe	0.26	0.30	0.12	0.15	0.20
CEQ	0.01	0.01	0.01	-0.04	-0.08
CRRA(1.5)	-18.07	-20.10	-14.41	-13.01	-11.59

Table 12: **120 Months Estimation, 21 Factors**

MSE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.01	0.03
Std.Dev	0.05	0.03	0.12	0.11	0.11
Sharpe	0.30	0.29	0.08	0.12	0.24
CEQ	0.01	0.01	0.00	-0.02	-0.03
CRRA(1.5)	-16.58	-20.62	-21.00	-17.71	-12.28

Ridge	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.02
Std.Dev	0.05	0.04	0.09	0.08	0.07
Sharpe	0.30	0.27	0.18	0.19	0.29
CEQ	0.01	0.01	0.01	0.00	0.00
CRRA(1.5)	-16.58	-18.34	-15.94	-16.10	-14.14

Lasso	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.09	0.08	0.08
Sharpe	0.30	0.27	0.18	0.29	0.43
CEQ	0.01	0.01	0.01	0.01	0.00
CRRA(1.5)	-16.58	-18.32	-15.93	-13.06	-10.81

MAE	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.02	0.02
Std.Dev	0.05	0.03	0.15	0.10	0.08
Sharpe	0.30	0.30	0.08	0.24	0.31
CEQ	0.01	0.01	0.00	0.00	-0.01
CRRA(1.5)	-16.58	-19.98	-17.69	-13.00	-12.70

Table 12 continued from previous page

Huber	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.07	0.08
Std.Dev	0.05	0.03	0.17	0.15	0.14
Sharpe	0.30	0.29	0.14	0.47	0.57
CEQ	0.01	0.01	0.01	0.01	-0.02
CRRA(1.5)	-16.58	-20.51	-12.95	-7.56	-7.07

Cauchy	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.11	0.12	0.10
Sharpe	0.26	0.29	0.17	0.23	0.33
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-20.13	-14.68	-12.23	-10.76

Quantile(25)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.01	0.03	0.03
Std.Dev	0.05	0.03	0.10	0.09	0.08
Sharpe	0.26	0.25	0.12	0.30	0.40
CEQ	0.01	0.01	0.01	0.01	0.00
CRRA(1.5)	-18.07	-21.51	-18.20	-12.22	-11.24

Arctan	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.02	0.03
Std.Dev	0.05	0.04	0.11	0.12	0.10
Sharpe	0.26	0.29	0.16	0.21	0.30
CEQ	0.01	0.01	0.01	-0.01	-0.02
CRRA(1.5)	-18.07	-19.53	-14.85	-12.69	-11.57

Soft L1	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)

Table 12 continued from previous page

Mean	0.01	0.01	0.02	0.03	0.03
Std.Dev	0.05	0.03	0.15	0.12	0.12
Sharpe	0.26	0.29	0.13	0.23	0.27
CEQ	0.01	0.01	0.01	-0.01	-0.04
CRRA(1.5)	-18.07	-20.49	-14.33	-11.98	-11.10

Quantile(75)	Equal Weight	Min Var	MVO(1)	MVO(5)	MVO(10)
Mean	0.01	0.01	0.02	0.04	0.04
Std.Dev	0.05	0.03	0.17	0.12	0.12
Sharpe	0.26	0.27	0.14	0.29	0.35
CEQ	0.01	0.01	0.01	0.00	-0.03
CRRA(1.5)	-18.07	-20.85	-12.88	-10.64	-9.78

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