Gains from Markowitz Optimization: Evidence from Re-optimization of Mutual Fund Holdings*

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Abstract:

Prior studies challenge the practical usefulness of Markowitz portfolio optimization in improving the return-risk tradeoff in portfolio management. We approach this question from a unique angle by examining whether one can improve the performance of a large sample of actual mutual fund portfolios by re-optimizing the holdings using simple mean-variance optimization methods. Our analyses produce compelling evidence of benefits from Markowitz optimization. We find that simple portfolio optimization improves the risk-adjusted performance of mutual fund portfolios in spite of noisy expected return estimates inferred from mutual fund portfolio weights. Several alternative optimization strategies, including the risk-parity portfolio, minimum variance portfolio, mean-variance portfolio, and Sharpe ratio maximization portfolio, all outperform the actual holding portfolio of mutual funds in terms of Sharpe ratio and other risk-adjusted performance measures. And the results are robust to subsamples partitioned on various dimensions. In contrast to DeMiguel et al. (2009), we find that the 1/N portfolio performs the worst.

Keywords: mutual fund; risk optimization; portfolio management; expected returns.

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1. Introduction

Modern portfolio theory, with its origins traced to Markowitz (1952, 1956 and 1959), suggests that investors should construct portfolios that seek to maximize expected portfolio returns subject to certain risk constraints of the portfolio (i.e., hold a portfolio on the efficient frontier). While the mean-variance (MV) framework of Markowitz is conceptually appealing, its practical application turns out to be challenging for at least two reasons. First, the unobservable model parameters (such as security expected return and risk) have to be estimated; second, the estimated optimal portfolio is sensitive to the estimation errors such that it can deviate substantially from the true optimal portfolio. Due to these reasons, Jobson and Korkie (1980) state that "naive formation rules such as equal weight rule can outperform the Markowitz rule." A recent study by DeMiguel, Garlappi and Uppal (2009) also finds the naïve 1/N investment strategy to outperform the Markowitz rule and its various extensions. As Tu and Zhou (2011) argue, the findings that the Markowitz rule does not perform well in real datasets and can even lose money on a risk-adjusted basis in many cases raise questions about the practical utility of the modern portfolio theory for investment management.

We weigh in on this debate about the usefulness of portfolio theory from a unique angle by examining whether simple portfolio optimization can improve mutual fund performance. As noted above, the poor out-of-sample performance of alternative portfolio construction strategies stems largely from errors in estimating the model parameters, particularly the errors in expected return estimates (Merton 1980). The aforementioned studies typically estimate expected returns from securities' past performance. However, as Jagannathan and Ma (2003) suggest, "*the estimation error in the sample mean is so large* *that nothing much is lost in ignoring the mean altogether (p.1652)".* In this paper, we take a different approach. We infer the mutual fund managers' (forward-looking) return expectation from their portfolio holdings and then investigate whether several simple optimal portfolio construction strategies based on these return expectations outperform the risk-return trade-off of the managers' actual portfolios.

In theory, fund managers, as sophisticated investment professionals, should all, explicitly or implicitly, optimize the reward-to-risk ratio of their investment portfolios. If they indeed optimize their portfolio in such a way, our exercise of re-optimizing their portfolios with noisy expected returns inferred from their holdings would only add noise and therefore it would be unlikely to yield a superior risk-adjusted performance. However, a *priori*, it is unclear whether the average mutual fund manager fully optimizes on the risk dimension in constructing his or her portfolio. First, prior research suggests that overconfident investors tend to hold under-diversified portfolios at the expense of their expected utility (e.g. Odean 1998; Barber and Odean 1999; Goetzmann and Kumar 2008; and Barber and Odean 2013). Overconfidence among mutual fund managers is quite prevalent (see Puetz and Ruenzi, 2011, and also Jin, Eshraghi, Taffler, and Goya, 2015), which would likely generate under-diversified portfolios. Second, fund managers might *choose* not to optimize their portfolio due to the practical challenges in obtaining true optimal portfolios.

A challenge we face in the re-optimization exercise is that managers' return expectations are unobservable. We only observe *ex post* that managers chose to invest in certain stocks with a positive weight on each included security.[1] We assume that the investment weight for each stock in the portfolio is an indication of the portfolio manager's belief about the expected return on that stock compared to other stocks included in the portfolio. *Ceteris paribus*, we would expect fund managers to invest more in the stocks in which they expect higher returns. We impute the fund manager's return expectation for each stock in a mutual fund on the basis of the portfolio weight the manager has assigned to each stock within the portfolio. Specifically, we assign a positive expected return equal to the portfolio weight of each stock included in a portfolio.[2]

Another input into the optimization process is the covariance matrix of asset returns. We estimate the matrix using historical daily stock returns over the one- year period prior to portfolio construction. While the covariance matrix estimates are also likely to be noisy, such noise should not be too problematic for several reasons: First, the errors in estimating return covariances from historical data are likely to be considerably smaller than the errors in the expected return estimates. Second, the optimization solution is much less sensitive to the estimation errors in covariance estimates than the errors in expected return estimates (see Merton 1980). Third, as Jagannathan and Ma (2003) show, with no-short-sale

[1] We limit our analysis to mutual funds that are prohibited from short selling. Therefore, all the investments in the portfolios have only positive weights.

[2] We implicitly assume that fund managers fail to properly consider risk in their portfolio construction. Under the assumption that managers perfectly optimize their portfolio, one would have to infer managers' expected returns using inverse portfolio optimization as Zagst and Poschik (2008). There are two potential problems. First, given that fund managers have various investment constraints, inferring return expectation using inverse portfolio optimization is notoriously difficult and noisy. Second, if managers were properly optimizing their portfolio, and we were to perfectly back out their return expectations and re-optimize using the same setup and constraints, the resulting "optimal" portfolios would be identical to the actual portfolios. Under this scenario, any analysis comparing the actual versus optimized portfolio would then be meaningless. Nevertheless, to the extent that our inferred return expectations capture managers' true return expectations with noise, the presence of noise would bias against us finding that the re-optimized portfolios outperform managers' actual portfolios. In robustness analysis, we estimate expected returns as the product of portfolio weight and idiosyncratic volatility of the stock, and the results are qualitatively similar.

constraints in place, the sample covariance matrix performs as well as the other more sophisticated covariance estimates such as those estimated from factor models and shrinkage estimators.

Armed with these return expectations and the risk estimates from past return history, we then construct several portfolios that represents an optimal risk-reward tradeoff under various conditions. The primary portfolio of interest is the mean-variance optimal portfolio of Markowitz (1952) where investors optimize the tradeoff between the mean and variance of portfolio returns. We construct the portfolio the same way as the sample-based mean-variance portfolio in DeMiguel et al. (2009), except several simplified constraints as described in the next section. The risk optimization requires an assumption about the degree of risk aversion of a representative investor in the portfolio. We report results using risk aversion coefficients of 1 and 3 in our main analysis. In comparison, DeMiguel et al. (2009) construct the mean-variance optimal portfolio assuming a risk aversion coefficient = 1. As a robustness check, we also perform portfolio optimization using a number of different risk aversion coefficients ranging from 5 to 50 (most risk averse) and summarize the results in Section 4.

As discussed above, the mean-variance optimal portfolio trades off portfolio risk against returns by maximizing the risk-adjusted return function (i.e., utility function) given a particular risk-aversion coefficient. An alternative is to choose a portfolio that maximizes the ratio of portfolio excess returns to risk (i.e., the Sharpe ratio). The maximum Sharpe ratio portfolio is therefore a special case of the optimal mean-variance portfolio with the largest *ex ante* Sharpe ratio (see Cornuejols and Tutuncu 2006; Kopman and Liu 2009). Because the latter analysis is independent of the assumption of a representative investor's risk aversion, we also construct maximum Sharpe ratio portfolios using the set of stocks in each mutual fund and compare the performance with that of the actual holdings (i.e., actual portfolio) of the fund.

In addition to the mean-variance optimal portfolio and maximum Sharpe ratio portfolio, we also examine several other simple portfolio construction approaches that do not require expected return estimates. Specifically, we examine the minimum variance portfolio (Jagannathan and Ma 2003; Clarke, Silva, and Thorley 2006; and DeMiguel et al., 2009), the risk parity portfolio (Asness, Frazzini and Pedersen 2012), and the 1/N or equally weighted portfolio (DeMiguel et al., 2009). We then compare the return, risk, Sharpe ratio, and turnover of these portfolios with the actual performance of each mutual fund. This procedure is repeated across all of the mutual funds for which we have data available from years 1981 to 2017. The analysis, on average, includes 1,298 mutual funds per year.

The results offer a clear indication of benefits from risk-reward optimization using a variety of simple techniques of portfolio optimization. Annual risk of the mean-variance optimized portfolio assuming a risk-aversion coefficient of 1 is lower than that of the actual portfolio for over 73.25% of the fund-year observations. In this comparison, the mean annualized risk of the optimized portfolio is 16.41% compared to 18.15% for the actual portfolio. More importantly, this risk reduction does not come at the expense of portfolio returns. The optimized portfolios earn higher average returns than the corresponding actual portfolios in over 49.94% of the fund-year observations. The mean annualized returns for the optimized portfolio is 9.38%, which is slightly higher than that of the actual portfolio (9.36%). For an average fund-year, the simple optimization procedure improves the Sharpe ratio from 0.784 to 0.817. The maximum Sharpe ratio portfolio further improves the Sharpe ratio to 0.846. We also examine the time-series variation in the improvement in Sharpe ratio. The results show that the average Sharpe ratios of the Markowitz mean-variance optimal portfolio are consistently higher than that of the actual portfolio[3].

In sharp contrast to DeMiguel et al. (2009), we find that the 1/N portfolio systematically *underperforms* the Markowitz optimal portfolios in both risk and return dimensions with annualized returns and risk of 9.12% and 18.46% respectively. In fact, it even underperforms the managers' actual portfolio on average with slightly higher risk and lower returns.

When we ignore the cross-section of expected returns, and simply minimize the portfolio risk using stocks held by mutual fund managers (i.e., the minimum variance portfolio), we find an even greater improvement in Sharpe ratio to 0.885. Risk parity portfolios, which allocate stock weights based on the inverse of asset risk, also produce significantly higher Sharpe ratios than the actual portfolios. Looking at the certainty-equivalent CEQ) returns, we find similar results. Except for the 1/N portfolio, all other portfolios generate higher CEQ returns than the actual portfolio, with the Markowitz M-V optimal portfolio generating the highest utility. Additional analyses suggest that the above findings are robust to subsamples partitioned on fund characteristics such as number of stocks in the portfolio, industry concentration, portfolio risk and portfolio covariance. Furthermore, we also compare the Fama-French three- and four-factor alphas of the actual

[3] T-tests of the time-series annualized Sharpe ratio between the actual portfolio and alternative portfolios suggest that Sharpe ratio of each alternative portfolio except the 1/N portfolio is statistically different from the Sharpe ratio of the actual portfolio. Refer to Appendix I for details.

with the optimized portfolios. A majority of the optimized portfolios also have higher alphas and information ratios than the actual portfolio.

Our paper makes two contributions to the literature. First, even though the celebrated Markowitz thesis on portfolio theory was published decades ago, its practical usefulness is not a settled issue in the profession. A notable recent study by DeMiguel et al. (2009) shows neither the sample-based mean-variance model nor its various extensions proposed in the recent literature consistently outperform the naïve 1/N rule. They conclude that the evidence suggests that '*there are still many "miles to go" before the gains promised by optimal portfolio choice can actually be realized out of sample*' (p. 1915). In our opinion, an important limitation of DeMiguel et al. (and most of the related studies) is that historical sample mean returns are used as expected return estimates.[4] Prior literature suggests such estimates suffer from a large amount of noise and therefore can lead to unreasonable "optimal" portfolios (e.g., Merton 1980; Jagannathan and Ma 2003). In the real-world investment profession, portfolio managers utilize substantially more information (than just the historical mean) to form return expectations. We provide robust evidence that significant gains can be realized from simple Markowitz optimization even if we were to use the fund managers' portfolio weights as a noisy proxy for expected returns and a simple sample-based risk model. Furthermore, our results that 1/N portfolio performs the worst suggest that the conclusion of the prior literature may not be generalizable to informed investors with reasonable return expectations.

[4] For example, historical mean return of a security can often be negative or large positive, which is economically implausible as an estimate of expected returns. Therefore, ad hoc adjustments are necessary to produce economically sensible expected return estimates.

Second, while the concept of risk-return optimization is ingrained in portfolio managers, we find that the performance of more than half of the mutual funds can be improved using simple techniques to better optimize their risk-return tradeoff.

In Section 2 we describe our portfolio construction methods. Section 3 describes the sample section procedure and performance measures. We discuss our empirical results in Section 4, and conclusions in Section 5.

2. Portfolio Construction

In this study, we compare the performance of the actual portfolio of a mutual fund with several hypothetical optimized portfolios that could have been constructed using no more information than that was available at the time of the actual portfolio construction. We describe the construction of the various risk-optimized portfolios.

2.1 The mean-variance optimal portfolio

The mean-variance optimal portfolio is obtained by maximizing the following objective function:

$$max\left(\sum_{i=1}^{m}\omega_i\mu_i - \frac{\gamma}{2}\sum_{i-1}^{m}\sum_{j=1}^{m}\sigma_{i,j}\omega_i\omega_j\right)$$

Subject to

$$\sum_{i=1}^{m}\omega_i = 1$$

$$\underline{\omega} \leq \omega_i \leq \overline{\omega}$$

where ω_i is the weight of security *i*; μ_i is the excess return (over the risk-free asset) of security *i*, proxied for by the weight of the security in the actual portfolio of a mutual fund; $\sigma_{i,j}$ is the covariance between the returns of securities *i* and *j*, estimated using daily stock returns over the past year; m is the number of securities in the actual portfolio, each with at least 125 trading day returns over the past year; γ is the risk aversion coefficient; $\underline{\omega}$ and $\overline{\omega}$ are the minimum and maximum weights of all securities in the actual portfolio of a mutual fund. This optimization procedure is commonly encountered in the literature, for example, see DeMiguel et al. (2007).

2.2 Maximum Sharpe ratio portfolio

A maximum Sharpe ratio portfolio gives the highest expected return per unit of risk. We generate the maximum Sharpe ratio portfolio by maximizing the following objective function subject to the same set of constraints as above:

$$max \frac{\sum_{i=1}^{m} \omega_i \mu_i - r_f}{\sqrt{\sum_{i-1}^{m} \sum_{j=1}^{m} \sigma_{i,j} \omega_i \omega_j}}$$

2.3. Minimum variance portfolio

The minimum variance portfolio is the portfolio with the lowest possible portfolio risk. It is constructed by minimizing the variance of the portfolio subject to the same constraints as above, i.e.,

$$min \sum_{i-1}^{m} \sum_{j=1}^{m} \sigma_{i,j} \omega_i \omega_j$$

The minimum variance portfolio would be a mean-variance optimal portfolio if an investor were extremely risk averse ($\gamma \rightarrow \infty$) or if her expected excess return forecasts were the same for all securities in her opportunity set, i.e., $\mu_i = \mu_0$ for i=1, 2, …m.

We also examine two other popular portfolios: a simplified risk parity portfolio where the investment in each security makes the same contribution to the portfolio risk as every other security in the portfolio, and a 1/N portfolio, where all securities are weighted equally.

2.4. Simplified risk parity portfolio

Risk parity portfolio has become a popular asset-allocation portfolio strategy in recent years. Some high profile asset managers (e.g., Bridgewater) promote the strategy as a (superior) alternative to the traditional 60/40 stock/bond allocations (e.g. Dalio, Prince and Jensen 2015). In the risk parity portfolio, each security has equal contribution to the portfolio risk. One of the most popular risk parity portfolios ignores the correlation with other assets (e.g. Asness, Frazzini and Pedersen 2012), so the security weight is proportionate to the inverse of the security volatility, i.e.:

$$\omega_i = \frac{1/\sigma_i}{\sum_{j=1}^{m} 1/\sigma_j}$$

2.5 1/N portfolio

Using a small number of assets, DeMiguel et al. (2007) find that none of the sample-based mean-variance optimal portfolio they examine consistently beats the naïve 1/N portfolio in terms of the Sharpe ratio, certainty-equivalent return, or turnover. The 1/N portfolio is simply the portfolio in which all securities are weighted equally, i.e., $\omega_i = 1/m$. Therefore, the only decision the portfolio manager makes is about the set of securities to include in the portfolio.

3. Data and Performance Measures

3.1 Data

We obtain mutual fund data from Thomson Reuters for the period 1981 to 2017. We include all mutual funds that specify their investment objective to be aggressive growth, growth, or growth and income (i.e., investment objective code to be 2, 3, or 4 in the Thomson Reuters database)[5]. These three categories represent about 42% of fund-quarters with a non-missing investment objective code in the Thomson Reuters universe of mutual fund data. The mutual fund data includes the composition of each mutual fund portfolio, i.e., the constituent stocks in each mutual fund.

We merge the mutual fund composition data with CRSP to retrieve daily and monthly returns for the individual stocks in each mutual fund portfolio. For a stock to be included in the actual portfolio of mutual funds, we require at least 125 daily stock returns in the year prior to each holding date. We apply this filter to all the holding strategies considered in the paper, including the actual portfolio.[6] Thus, all of the performance measures are calculated based on the same set of stocks under each strategy for each fund-quarter. We use returns in excess of the risk-free rate, which are obtained from the website of Kenneth French,[7] to estimate the variance-covariance matrix for the stocks in each mutual fund portfolio. We exclude mutual funds that held more than 1,000 stocks as they are less likely to be actively managed. They are likely to be index funds, at least in substance.

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[5] Thomson Reuters classifies funds into 9 categories based on investment objective: 1. International; 2. Aggressive growth; 3. Growth; 4. Growth and income; 5. Municipal Bonds; 6. Bond & Preferred; 7. Balanced; 8. Metals; 9. Unclassified. Many fund-quarter observations (63%) have a missing investment objective code in the universe. These observations are not included in the analyses.

[6] The requirement of at least 125 daily returns would exclude recent IPO stocks from both the optimized portfolio and the actual portfolios in performance computation. Thus, it should not induce bias to the performance comparison between these portfolios.

[7] Please refer to the website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

After portfolio construction for each holding strategy as described in Section 2 above using historical returns and the weights of each security in the actual portfolio, the next step is to calculate the performance of the actual and the hypothetical optimized portfolios. Specifically, we retrieve monthly stock returns from CRSP for the subsequent 3 months between the current and next holding date to calculate the monthly excess portfolio returns (i.e., returns in excess of the risk-free rate from CRSP) for each strategy. The above process is repeated for each portfolio holding date in each year. This procedure yields a final sample comprising 48,011 fund-year observations.[8] It contains 3,799 distinct mutual funds over the period from Dec 31, 1980 to Sep 30, 2017.

Table 1 reports the number of mutual funds, the median number of stocks they hold, and the number of distinct stocks held by all funds in each year. The number of mutual funds fluctuates between 359 and 2,579 with an average of 1,298 over the 37 years sample period. The number of distinct stocks held in each year ranges from 1,985 to 5,889 with an average of 3,883. The median number of stocks held by the mutual funds each year ranges from 61 to 102 with an average of 87.

3.2 Performance Measures

3.2.1 Sharpe ratio

Based on the time-series of monthly excess portfolio returns, we calculate the annualized out-of-sample Sharpe ratio of each strategy for each fund-year as:

[8] Alternative holding portfolios are estimated for each fund in each quarter, based on which monthly portfolio returns are calculated for each month between two consecutive holding date (essentially between two calendar quarter ends), and then monthly data are aggregated on a fund-year level for subsequent analyses. For example, for a certain fund, year 1998 covers the holding date Dec 31 1997, Mar 31 1998, Jun 30 1998, and Sep 31 1998. Monthly returns are then calculated from Jan 1998 to Dec 1998.

$$\text{Sharpe ratio}_k = \frac{\sqrt{12}\hat{\mu}_k}{\hat{\sigma}_k}$$

where $\hat{\mu}_k$ and $\hat{\sigma}_k$ are the mean and standard deviation of the out-of-sample monthly excess portfolio returns for strategy k for each fund-year (or for each fund over its entire history).

3.2.2 Certainty-equivalent return (CEQ)

CEQ is computed as

$$\text{CEQ}_k = 12\hat{\mu}_k - \frac{\gamma}{2}(\sqrt{12}\hat{\sigma}_k)^2$$

in which $\hat{\mu}_k$ and $\hat{\sigma}_k^2$ are the mean and variance of out-of-sample monthly excess portfolio returns for strategy k, and γ is the risk aversion coefficient. Risk aversion measures units of additional expected reward an investor requires to accept one additional unit of risk. Ang (2014) suggests that most individuals have risk aversion between 1 and 10. Wetizman (2007) suggest conventional values of risk aversion coefficient roughly fall into the range of 1 to 3. The main results in DeMiguel et al. (2007) are based on a risk aversion coefficient of 1. Following Tu and Zhou (2011), we report results for $\gamma = 1$ and $\gamma = 3$.

3.2.3 Turnover

We follow DeMiguel et al. (2007) in computing the portfolio turnover ratio for each strategy for each fund-year as

$$\text{Turnover}_k = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}(|\widehat{\omega}_{k,j,t+1} - \widehat{\omega}_{k,j,t^+}|)$$

where $\widehat{\omega}_{k,j,t+1}$ is the portfolio weight of stock j at time t+1 under strategy k, and $\widehat{\omega}_{k,j,t^+}$ is the portfolio weight before rebalancing at t+1.

3.2.4 Return-loss

Return-loss is defined as the additional return needed for an alternative strategy to perform as well as the actual strategy in terms of the Sharpe ratio after considering transaction costs. As in DeMiguel et al. (2007), we compute the return-loss for each strategy k relative to the actual portfolio for each fund-year as

$$\text{Return} - \text{loss}_k = \frac{\mu_{act}}{\sigma_{act}} \times \sigma_k - \mu_k$$

in which μ_{act} and σ_{act} are the annualized monthly out-of-sample mean and standard-deviation of the net returns for the actual portfolio, and μ_k and σ_k are the counterparts for the strategy k. The net returns are the portfolio returns after transaction cost c. Specifically, the net returns are calculated as

$$\text{Net return}_k = R_k - c \times \sum_{j=1}^{N} \left(\left| \widehat{\omega}_{k,j,t+1} - \widehat{\omega}_{k,j,t^+} \right| \right)$$

in which R_k is the monthly portfolio return for strategy k, and others are defined as previously. The monthly net return is different from the raw return R_k only at each calendar quarter end, i.e., at the third month of each quarter, when portfolio turnover only occurs. In this paper, we calculate return-loss under three cases with c assumed to be 0, 20, and 50 basis points per transaction, respectively. [9]

4. Results

In this section, we first discuss the results based on the full sample, followed by results for subsamples partitioned on several fund characteristics.

4.1 Full sample results

[9] DeMiguel et al. (2007) assume a transaction cost of 50 bps in their analysis. The actual transaction costs for institutional investors are generally lower. For example, a recent Global Cost Review published by Investment Technology Group shows that the total cost transaction for US stocks averages approximately 33 bps in the first quarter of 2018 (https://www.itg.com/assets/ITG-Global-Cost-Review-2018Q1-Update.pdf).

Table 2 presents performance measures for the actual holding portfolio as well as each of the alternative holding strategies described above. Recall that we have ensured that the same set of stocks is included in the actual as well as alternative holding portfolios under various strategies.

The mean annualized return and mean annualized risk of the actual portfolio of the 48,011 fund-year observations are 9.36% and 18.15%, respectively. The mean Sharpe ratio, representing the risk-reward tradeoff, of the actual portfolio across the fund-year universe, is 0.784. The mean Sharpe ratio is calculated as the average of the Sharpe ratios for the individual holding strategies of each fund-year observation in the sample, rather than the ratio of the mean annualized return to the mean annualized risk as presented in the table. This also applies to the statistics presented in all of the remaining tables.

As seen from Table 2, the mean annualized returns for the alternative risk-optimized holding strategies are generally lower than that of the actual portfolio. However, the decrease in the return appears to be compensated for by a more than proportionate decrease in the risk associated with each of the alternative strategies except for the 1/N portfolio. The net result therefore is a greater Sharpe ratio for all of the risk-optimized strategies except the 1/N portfolio strategy. The improvement in Sharpe ratio holds for the risk-parity portfolio, the mean-variance portfolios with a risk aversion coefficient of 1 and 3[10], the minimum variance portfolio, as well as the Sharpe ratio maximization portfolio.

Consistent with the comparison based on absolute performance measures between actual portfolio and alternative holding strategies, further analyses reveal that the

[10] Similar conclusion applies to the mean-variance portfolios with risk aversion coefficient ranging from 5 to 50, as shown in the Appendix II.

annualized risk for alternative portfolios is lower than the risk for the actual portfolio for at least 70% of fund-year observations, with the exact percentage being different for each specific strategy. In contrast, the analogous percentage for the annualized return is generally close to 50%. Taken together, the percentage of fund-years yielding a Sharpe ratio for alternative portfolios greater than or equal to the Sharpe ratio for the actual portfolio ranges from 54.00% to 62.24%.

To illustrate, consider the mean-variance optimal portfolio with risk aversion coefficient of 1. The mean annual return of the optimized portfolio is 9.38%, which is slightly higher than that of the actual portfolio (9.36%). The mean annualized risk of the M-V optimal portfolio is considerably lower than that of the actual portfolio (16.41% vs. 18.15%). The higher return and lower risk of the optimal portfolio also leads to a higher Sharpe ratio for the M-V optimal portfolio than the actual mutual fund portfolios (0.784 vs. 0.817). Figure 1 plots the difference in Sharpe ratios between the optimized portfolios and the actual portfolio by year. The results show that in most of the sample years, the Sharpe ratios of the various optimized portfolios are higher than that of the actual portfolio. For instance, the Sharpe ratio of the M-V optimal portfolio (with gamma=1) has higher Sharpe ratio than the actual portfolio in 24 of the 37 years in the sample period. In fact, it significantly underperforms (in terms of Sharpe ratio) only in less than thirty percent of the 37-year period (1991, 1992, 1993, 1999, 2005, 2009, 2010, 2013, 2016 and 2017).

We also report the certainty-equivalent return (CEQ) for each strategy. As can be seen from Table 2, statistics for CEQ of the actual portfolio and each alternative portfolio support the inferences drawn from the Sharpe ratio comparison. Specifically, the CEQ for each of the alternative risk-optimization strategy is greater than that of the actual portfolio, suggesting that an average investor would need a greater risk-free rate to compensate for not adopting the alternative holding strategy than for not adopting the actual holding strategy. Overall, the results strongly support the notion that in comparison with that actual portfolio, various alternative holding strategies that aim at optimizing the portfolio risk indeed succeed in doing so, as indicated by the significantly reduced risk, the improved Sharpe ratio, as well as the greater CEQ.

However, the construction of risk-optimized alternative portfolios is not without any cost. Turnover ratios represent the average percentage of wealth traded in each period due to portfolio rebalancing. These ratios reported in Table 2 suggest that the transaction costs associated with rebalancing the alternative holding portfolios across periods are generally greater than their counterparts for the actual portfolio. Specifically, the actual portfolio has a lowest annual turnover ratio of 147.78%, whereas the turnover ratio of the alternative holding strategies varies from 165.89% to 252.60%, suggesting that implementing these strategies would require a higher percentage of wealth to be traded in each period. Although alternative holding strategies entail a higher turnover ratio, these portfolios still perform better than the actual portfolio in terms of Sharpe ratio even after considering transaction costs, as indicated by the return-loss statistics. As can be seen, return-loss statistics are all smaller than 0 for alternative strategies except the 1/N portfolio strategy, under all three cases assuming transaction cost to be 0, 20, and 50 basis points. Since this measure is defined relative to the actual portfolio, a negative value would suggest that after taking into consideration the transaction costs, additional negative return is needed for an alternative strategy to perform as well as the actual strategy in terms of the Sharpe ratio.

4.2 Subsample results

In this subsection, we report results for subsample analyses based on a variety of fund characteristics including the investment objective of the funds, the number of holding stocks, industry concentration, portfolio risk, and portfolio covariance. We discuss each in the following.

4.2.1. The investment objectives

Table 3 presents the average portfolio return, risk, Sharpe ratio, and comparisons in these statistics between alternative holding strategies and the actual portfolio by different investment objectives. On average, the mean annual returns are declining with Aggressive Growth, Growth, and Growth and Income, whereas the portfolio risks show a similar but dominate trend, eventually resulting an increasing in Sharpe ratios. Table 3 also shows that the annualized portfolio return of 9 out of 12 alternative portfolios among fund-years with the investment objectives of Aggressive Growth and Growth is greater than the corresponding return of the actual portfolio for more than 50% times. Furthermore, all the alternative portfolios except the 1/N portfolio have a lower annualized risk than the actual portfolio for more than 60% times, regardless of the investment objectives. Finally, for each holding portfolio except the 1/N portfolio under each investment objective, more than 50% of fund-years have a higher Sharpe ratio than the actual portfolio, suggesting that the performance of optimized portfolios generally improves.

4.2.2 Number of holding stocks

We partition our sample into three subgroups based on the number of stocks held by mutual funds, and repeat all the above exercises and report the results for the lowest and higher tercile groups in Table 4. Generally, the inferences derived from the full sample analyses all apply to both of the subsamples. That is, all the alternative holding strategies designed to optimize portfolio risk succeed in doing so with greater than 50% of fund-years having a Sharpe ratio at least as large as that for the actual portfolio. For example, the minimum-variance portfolio beats the actual portfolio in terms of Sharpe ratio in 57.57% of fund-years in the lowest tercile group, whereas the corresponding figure in the highest tercile group is 62.19%. In effect, the improvement in Sharpe ratio is uniformly greater in the highest tercile group than in the lowest group for all the other alternative portfolios, and is mainly attributed to the more significant reduction in the portfolio risk. While the percentage of fund-years with a lower portfolio risk under alternative strategies than under the actual strategy in the lowest tercile group varies from 61.25% to 82.37%, it ranges from 75.18% to 91.20% in the highest tercile group.

4.2.3 Industry concentration

The second criterion we utilize to partition our sample into three subgroups is the industry concentration reflected from the stocks held by mutual funds, and we capture the characteristics following Kacperczyk et al. (2005). Specifically, the Industry Concentration Index at time t for a mutual fund is defined as the sum of the squared deviations of the value weights for each of the 10 different industries held by the mutual fund, $w_{j,t}$, relative to the industry weights of the total stock market, $\overline{w}_{j,t}$:

$$ICI_t = \sum_{j=1}^{10} (w_{j,t} - \overline{w}_{j,t})^2$$

A higher value of ICI indicates mutual funds' holdings are more concentrated in a few industries. We report the results for the lowest and higher tercile groups based on mutual funds' industry concentration illustrated above in Table 5. Likewise, all the previous inferences hold for both subsamples based on industry concentration. All the alternative holding strategies designed to optimize portfolio risk succeed in doing so with greater than 50% of fund-years having a Sharpe ratio at least as large as that for the actual portfolio. For instance, the Sharpe ratio of mean-variance portfolio with a risk aversion coefficient of 1 is superior to the actual portfolio in 53.56% of fund-years in the lowest tercile group, whereas the corresponding figure in the highest tercile group is 53.08%. The improvement in Sharpe ratio is almost identical across alternative portfolios between the highest tercile group and the lowest group. In addition, the percentage of fund-years with a lower portfolio risk under alternative strategies than under the actual strategy in the lowest tercile group can be as high as 89.42% (Sharpe ratio maximization portfolio), while the greatest percentage in the highest tercile group is 84.20% (minimum variance portfolio).

4.2.4 Portfolio risk

The third criterion we employ to partition our sample into three subgroups is the portfolio risk calculated based on the actual holding strategy. Similarly, we report the results for the lowest and higher tercile groups in Table 6. All the alternative holding strategies designed to optimize portfolio risk succeed in doing so with greater than 50% of fund-years having a Sharpe ratio at least as large as that for the actual portfolio. For instance, the Sharpe ratio of risk-parity portfolio is greater than the actual portfolio in 62.27% of fund-years in the lowest tercile group, whereas the corresponding figure in the highest tercile group is 61.16%. The improvement in Sharpe ratio is generally greater in the highest tercile group than in the lowest group for all the other alternative portfolios, and is mainly due to the larger decrease in the portfolio risk. As can be seen, the percentage of fund-years with a lower portfolio risk under alternative strategies than under the actual strategy in the lowest tercile group fluctuates from 66.82% to 81.84%, it is mostly about 10% larger in the highest tercile group.

4.2.5 Portfolio covariance

The last fund characteristic we use to partition our sample into three subgroups is the portfolio covariance calculated based on the actual holding strategy. We calculate the covariance between each pair of distinct stocks held by a mutual fund and then sum up all the covariances, based on which we divide fund-year observations into three groups and report the results for the lowest and higher tercile groups in Table 7. All the alternative holding strategies designed to optimize portfolio risk succeed in doing so with a larger percentage of fund-years having a Sharpe ratio beating the actual portfolio for the highest tercile group than for the lowest tercile group. Regarding the portfolio risk, the percentage of fund-years with a lower portfolio risk under alternative strategies than under the actual strategy in the lowest tercile group is significantly smaller than those in the highest tercile group.

Overall, the full-sample as well as the subsample analyses suggest that alternative holding strategies designed to optimize portfolio risk, including risk-parity portfolio, minimum variance portfolio, mean-variance portfolio, and Sharpe ratio maximization portfolio, all outperform the actual holding portfolio of mutual funds in terms of Sharpe ratio. And this improvement can be mainly ascribed to the significantly reduced portfolio risk under alternative strategies.

4.3 Information ratio

Mutual funds are often evaluated on the basis of performance relative to certain benchmarks by calculating the information ratio. The information ratio is defined as active return divided by tracking error, where active return is the security's return in excess of the return of a benchmark, and tracking error is the standard deviation of the active return. An alternative definition of the information ratio is the ratio of the Jensen's alpha or the Fama-French model alpha to a regression-adjusted tracking error for the risk in the denominator. This alternate definition is often labeled as the appraisal ratio.

If fund managers were to optimize the tradeoff between active returns and active risk (i.e., maximize the information ratio), their optimal information ratio portfolio might not necessarily have the maximum Sharpe ratio. Since we do not have the benchmark for each individual mutual fund, so we are unable to conduct portfolio optimization on the active return space. To address the above concern, we employ the alternate definition of the information ratio and use the alpha of the portfolios calculated from the Fama and French three- and four-factor models. We estimate the active risk as the standard deviation of the residuals from the Fama-French model regressions. The information ratio is calculated using annualized values of alpha and active risk.

Table 8 presents the comparison of related statistics between the actual holding portfolio and alternatives computed with the Fama-French 3-factor model (in Panel A) and the Carhart 4-factor model (in Panel B). As can be seen, the main inferences drawn using the Sharpe ratio performance measure remain unchanged. Specifically, regardless of the three- or four-factor model employed for the computation, the alternative optimized portfolios generally have higher alphas and higher information ratios than the actual portfolio. More than 50% of fund-years yield a greater information ratio under alternative strategies than under the actual portfolio with the exception of the mean-variance portfolio with a risk aversion coefficient of 1 in Panel B. Taken together, these results suggest that holding strategies designed to optimize portfolio risk outperform the actual portfolio of mutual funds in terms of not only the Sharpe ratio, but also the information/appraisal ratio.

4.4 Robustness checks

4.4.1 Subsample analysis by Assets under Management (AUM)

We conduct additional subsample analyses based on the Assets under Management (AUM) of mutual funds. We divide our full sample into three subgroups – high, medium, and low -- based on AUM in each sample year, and conduct similar exercises as we did for other subsample tests. Untabulated results suggest that our main results are not sensitive to mutual funds' AUM and analogous statistics are similar across the lowest and highest tercile of AUM.

4.4.2 Real estimated trading costs

To the extent that our assumption of trading costs do not necessarily reflect those of real-world transactions of mutual funds, we conduct a sub-period analysis during which the estimated real trading costs are available from a global financial technology company[11]. We apply the real estimated trading costs, which are time-variant and size-dependent, to a sub-period from 2009Q4 to 2017Q4, and repeat all the previous analyses in this subsample. The results, untabulated for brevity, are robust to this sub-period sample.

4.4.3 Actual Portfolios including IPO stocks

In our main test, we require each holding stock of mutual funds to have at least 125 daily stock returns in the year prior to the portfolio holding date in order to estimate the variance-covariance matrix and thereby optimized portfolio weights under different strategies. One concern of this data requirement is that firms undergoing IPOs immediately

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[11] https://www.itg.com/thinking-article/1q18-global-cost-review/.

before portfolio holding dates are excluded from the portfolio, which may bias the results against the actual portfolios. To deal with this concern, we compute all of the above statistics for an additional portfolio comprising of all the stocks in a mutual fund, but without requiring each stock to have at least 125 daily stock returns in the year prior to the holding date. We repeat the analysis comparing this actual portfolio with all of the risk-optimized portfolio strategies. Untabulated results suggest the tenor of the results is unchanged.

5. Conclusions

In this paper, we investigate whether several simple portfolio construction strategies designed to optimize portfolio risk outperform the risk-return trade-off, i.e., the Sharpe ratio of the mutual fund managers' actual portfolios. We find consistent evidence that alternative optimization strategies, including the risk-parity portfolio, minimum variance portfolio, mean-variance portfolio, and Sharpe ratio maximization portfolio, all outperform the actual holding portfolio of mutual funds in terms of Sharpe ratio. We get similar results using sub-samples generated from multiple classification criteria. Our results also provide useful insights on the debate of whether the Markowitz optimization technique can yield better performance than naïve strategies such as the 1/N rule, and suggest that gains can be achieved by optimizing the risk-reward tradeoff of portfolios held by sophisticated mutual fund managers.

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Table 1 Summary statistics

Year	Total Number of Funds	Median number of Stocks Held (initially)	Median number of Stocks Held (excluding stocks with less than 125 daily returns during last year)	Number of Distinct Stocks Held
1981	380	64	44	1985
1982	362	61	43	2012
1983	359	71	49	2448
1984	387	75	51	2785
1985	399	74	52	2869
1986	449	80	54	3122
1987	502	85	56	2965
1988	579	80	53	3075
1989	612	74	50	3075
1990	635	73	51	2928
1991	693	76	52	2799
1992	815	80	54	3220
1993	910	86	60	3344
1994	1339	92	63	4408
1995	1782	93	61	4663
1996	2036	97	65	5232
1997	2278	102	68	5722
1998	2537	94	65	5889
1999	2579	99	68	5648
2000	2281	98	68	5228
2001	2216	99	71	4963
2002	2071	99	75	4606
2003	1965	100	78	4371
2004	1867	101	80	4327
2005	1767	100	80	4392
2006	1676	100	80	4436
2007	1606	100	78	4416
2008	1534	99	81	4298
2009	1454	100	83	4069
2010	1397	96	80	3957
2011	1355	91	80	3893
2012	1322	81	74	3695
2013	1272	83	74	3645
2014	1210	82	74	3736
2015	1159	82	74	3843
2016	1113	83	75	3800
2017	1113	79	71	3807
Average	1298	87	66	3883
Min	359	61	43	1985
Max	2579	102	83	5889

Table 2 Performance of holding strategies (48011 fund-years)

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	CEQ (%)	Turnover (annual) (%)	Return-Loss (relative to actual portfolio, 0bps) (%)	Return-Loss (relative to actual portfolio, 20bps) (%)	Return-Loss (relative to actual portfolio, 50bps) (%)	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Actual portfolio	9.36	18.15	0.784	7.24	147.78	0.00	0.00	0.00	NA	NA	NA
1/N portfolio	9.12	18.46	0.773	6.94	170.57	-0.06	-0.03	0.03	50.80	41.17	50.91
1/sigma portfolio	9.24	16.76	0.843	7.45	165.89	-0.85	-0.80	-0.71	54.18	77.19	62.24
Minimum variance portfolio	8.79	14.69	0.885	7.39	252.60	-0.98	-0.72	-0.33	49.60	86.84	59.54
Utility maximization portfolio (γ=1)	9.38	16.41	0.817	7.65	231.79	-0.43	-0.24	0.04	49.94	73.25	54.00
Utility maximization portfolio (γ=3)	9.16	15.43	0.855	7.63	236.55	-0.81	-0.59	-0.27	49.91	83.59	57.38
Sharpe ratio maximization portfolio	9.24	15.74	0.846	7.63	234.60	-0.69	-0.48	-0.17	48.56	85.85	56.11

Table 3 Performance of holding strategies (by investment objectives)

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Investment objectives	Aggressive Growth(N=4837)					
Actual portfolio	9.82	21.37	0.664	NA	NA	NA
1/N portfolio	9.48	21.62	0.652	48.98	43.77	48.65
1/sigma portfolio	9.79	19.67	0.724	54.39	79.35	61.84
Minimum variance portfolio	9.78	16.91	0.813	52.78	90.01	62.00
Utility maximization portfolio (γ=1)	10.44	18.91	0.745	54.37	78.17	59.56
Utility maximization portfolio (γ=3)	10.25	17.70	0.791	54.23	87.57	62.83
Sharpe ratio maximization portfolio	10.41	18.31	0.773	53.79	87.80	61.15
Investment objectives	Growth(N=32212)					
Actual portfolio	9.45	18.45	0.770	NA	NA	NA
1/N portfolio	9.12	18.75	0.758	50.73	41.07	50.57
1/sigma portfolio	9.24	16.95	0.829	54.12	77.99	62.00
Minimum variance portfolio	8.75	14.84	0.870	49.59	87.60	59.66
Utility maximization portfolio (γ=1)	9.47	16.61	0.808	50.04	74.18	54.34
Utility maximization portfolio (γ=3)	9.19	15.59	0.844	49.97	84.46	57.51
Sharpe ratio maximization portfolio	9.33	15.93	0.837	48.78	86.43	56.56
Investment objectives	Growth and Income(N=10062)					
Actual portfolio	8.51	15.73	0.847	NA	NA	NA
1/N portfolio	8.66	16.16	0.848	52.88	39.96	53.95
1/sigma portfolio	8.70	14.79	0.909	54.89	73.90	63.60
Minimum variance portfolio	8.08	13.13	0.931	48.25	83.52	57.94
Utility maximization portfolio (γ=1)	8.20	14.56	0.839	46.73	69.20	49.59
Utility maximization portfolio (γ=3)	8.17	13.77	0.880	47.08	79.88	54.03
Sharpe ratio maximization portfolio	8.04	13.87	0.866	44.70	83.83	51.90

Table 4 Cross-section by number of holding stocks

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Tercile	Lowest (N = 15,993)						Highest (N = 16,001)					
Actual portfolio	9.35	19.89	0.718	NA	NA	NA	9.58	16.99	0.840	NA	NA	NA
1/N portfolio	8.95	20.00	0.708	50.58	46.71	50.98	9.54	17.53	0.823	50.99	35.48	50.51
1/sigma portfolio	9.10	18.36	0.775	52.57	76.35	59.57	9.62	15.72	0.899	55.77	75.18	64.12
Minimum variance portfolio	8.77	17.03	0.793	50.79	82.37	57.57	9.05	12.89	0.980	48.67	90.15	62.19
Utility maximization portfolio (γ=1)	9.43	19.13	0.725	50.48	61.25	51.40	9.56	14.33	0.912	49.52	83.61	58.01
Utility maximization portfolio (γ=3)	9.27	18.17	0.756	51.23	74.35	55.42	9.32	13.34	0.957	49.11	90.51	60.61
Sharpe ratio maximization portfolio	9.33	18.14	0.767	50.24	79.65	56.09	9.37	13.89	0.924	46.93	91.20	56.82

Table 5 Cross-section by industry concentration

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Tercile	Lowest (N = 15,990)						Highest (N = 16,005)					
Actual portfolio	8.86	15.47	0.868	NA	NA	NA	10.08	21.32	0.706	NA	NA	NA
1/N portfolio	9.01	16.06	0.854	52.28	34.28	51.78	9.62	21.40	0.699	49.70	47.97	50.53
1/sigma portfolio	9.05	14.42	0.927	56.14	76.03	64.99	9.74	19.67	0.764	52.46	75.29	59.08
Minimum variance portfolio	8.25	12.29	0.962	47.36	87.62	59.77	9.47	17.69	0.814	51.52	84.20	58.89
Utility maximization portfolio (γ=1)	8.58	13.42	0.898	46.97	80.44	53.56	10.18	20.10	0.728	51.71	63.27	53.08
Utility maximization portfolio (γ=3)	8.40	12.61	0.937	46.85	88.47	56.58	9.98	18.94	0.768	51.86	76.38	56.90
Sharpe ratio maximization portfolio	8.36	12.95	0.912	44.40	89.42	53.29	10.14	19.15	0.771	51.27	80.53	57.62

Table 6 Cross-section by portfolio risk

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Tercile			Lowest (N = 15,990)						Highest (N = 16,005)			
Actual portfolio	9.00	14.17	0.936	NA	NA	NA	9.98	23.64	0.625	NA	NA	NA
1/N portfolio	8.99	14.75	0.913	50.93	35.92	50.38	9.37	23.58	0.618	49.84	47.29	50.77
1/sigma portfolio	9.14	13.53	0.989	54.35	70.29	62.27	9.51	21.30	0.682	53.40	82.14	61.16
Minimum variance portfolio	8.82	12.04	1.026	48.73	81.79	58.82	9.04	18.53	0.736	50.70	90.61	59.88
Utility maximization portfolio (γ=1)	8.89	13.27	0.945	47.78	66.82	50.91	10.35	20.94	0.692	53.25	77.93	57.46
Utility maximization portfolio (γ=3)	8.88	12.59	0.984	48.37	77.81	54.87	9.85	19.58	0.726	52.44	87.61	60.24
Sharpe ratio maximization portfolio	8.80	12.57	0.976	46.89	81.84	53.70	10.20	20.32	0.716	51.48	88.13	58.53

Table 7 Cross-section by portfolio covariance

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Tercile	Lowest (N = 15,990)						Highest (N = 16,005)					
Actual portfolio	9.25	18.68	0.750	NA	NA	NA	9.40	18.28	0.800	NA	NA	NA
1/N portfolio	8.90	18.75	0.742	51.06	46.94	51.33	9.38	18.86	0.782	50.92	35.04	50.08
1/sigma portfolio	9.07	17.27	0.811	53.13	75.97	60.16	9.48	16.88	0.856	55.91	75.45	63.82
Minimum variance portfolio	8.83	16.14	0.832	51.40	81.57	58.41	8.92	13.74	0.938	48.94	90.98	62.05
Utility maximization portfolio (γ=1)	9.29	18.17	0.752	50.23	58.97	50.82	9.54	15.19	0.881	50.57	85.94	59.23
Utility maximization portfolio (γ=3)	9.21	17.28	0.786	51.03	72.58	55.18	9.24	14.14	0.923	49.90	91.95	61.28
Sharpe ratio maximization portfolio	9.22	17.12	0.800	50.31	78.89	56.26	9.32	14.86	0.887	47.79	91.87	57.22

Table 8. Information ratio of holding strategies (48011 fund-years)

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Information Ratio	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Information Ratio>= Actual
Panel A. Fama-French 3-factor models						
Actual portfolio	0.46	6.33	0.092	NA	NA	NA
1/N portfolio	0.83	6.24	0.169	53.99	51.17	53.85
1/sigma portfolio	1.09	5.52	0.272	58.52	69.98	58.98
Minimum variance portfolio	1.34	6.33	0.265	57.07	44.29	55.95
Utility maximization portfolio (γ=1)	0.79	6.91	0.161	54.14	31.38	52.33
Utility maximization portfolio (γ=3)	1.05	6.59	0.203	55.92	38.99	54.59
Utility maximization portfolio (γ=5)	1.16	6.48	0.222	56.43	41.59	55.12
Utility maximization portfolio (γ=10)	1.26	6.39	0.241	56.75	43.52	55.71
Utility maximization portfolio (γ=20)	1.33	6.36	0.252	56.78	44.08	55.79
Utility maximization portfolio (γ=30)	1.35	6.36	0.259	57.02	44.24	55.97
Utility maximization portfolio (γ=50)	1.36	6.36	0.264	57.06	44.00	55.94
Sharpe ratio maximization portfolio	0.90	6.62	0.165	54.49	39.63	53.24
Panel B. Carhart 4-factor models						
Actual portfolio	0.87	5.74	0.207	NA	NA	NA
1/N portfolio	1.31	5.54	0.360	54.61	52.98	54.90
1/sigma portfolio	1.45	4.96	0.413	57.70	69.71	58.23
Minimum variance portfolio	1.40	5.76	0.281	55.01	43.57	53.47
Utility maximization portfolio (γ=1)	0.84	6.26	0.143	51.04	31.63	48.81
Utility maximization portfolio (γ=3)	1.01	5.99	0.175	52.69	38.67	50.68
Utility maximization portfolio (γ=5)	1.11	5.89	0.199	53.21	41.05	51.47
Utility maximization portfolio (γ=10)	1.23	5.82	0.227	54.21	42.82	52.56
Utility maximization portfolio (γ=20)	1.33	5.79	0.249	54.47	43.51	52.77
Utility maximization portfolio (γ=30)	1.37	5.79	0.263	54.69	43.53	53.06
Utility maximization portfolio (γ=50)	1.40	5.79	0.272	54.86	43.35	53.11
Sharpe ratio maximization portfolio	0.95	5.99	0.151	52.00	39.66	50.07

Appendix I T-tests on the difference in Sharpe ratio between actual portfolio and alternative portfolios

	Actual portfolio	1/N portfolio	Sigma portfolio	Minimum variance portfolio	Utility maximization portfolio (gamma=1)	Utility maximization portfolio (gamma=3)	Sharpe ratio maximization portfolio
T-statistic	NA	0.2740	-3.1619	-2.6732	-2.4307	-3.0356	-3.0709
P-value	NA	0.7857	0.0032	0.0112	0.0202	0.0044	0.0040

Appendix II Performance of holding strategies (48011 fund-years)

Holding Strategy	Mean Annual Returns (%)	Mean Annualized Risk (%)	Sharpe Ratio	CEQ (%)	Turnover (annual) (%)	Return-Loss (relative to actual portfolio, 0bps) (%)	Return-Loss (relative to actual portfolio, 20bps) (%)	Return-Loss (relative to actual portfolio, 50bps) (%)	Percentage with annual returns>= Actual	Percentage with annualized risk<= Actual	Percentage with Sharpe Ratio>= Actual
Actual portfolio	9.36	18.15	0.784	7.24	147.78	0.00	0.00	0.00	NA	NA	NA
Utility maximization portfolio (γ=5)	9.04	15.10	0.869	7.57	240.70	-0.91	-0.68	-0.34	49.78	86.27	58.50
Utility maximization portfolio (γ=10)	8.91	14.83	0.880	7.49	246.00	-0.98	-0.74	-0.37	49.47	87.70	59.39
Utility maximization portfolio (γ=20)	8.85	14.71	0.886	7.45	250.90	-1.01	-0.75	-0.37	49.53	87.80	59.69
Utility maximization portfolio (γ=30)	8.83	14.68	0.888	7.44	253.08	-1.01	-0.75	-0.36	49.67	87.71	59.66
Utility maximization portfolio (γ=50)	8.81	14.67	0.888	7.42	254.87	-1.00	-0.74	-0.34	49.68	87.39	59.65

Figure 1 Difference in Sharpe ratios between optimized portfolios and actual portfolio