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1. a) given $x \in \mathbb{R}$, prove RELU $(x) = \max(x, 0)$ is convex function.

According to convex properties.

$$f(\lambda \vec{x} + (1-\lambda)\vec{y}) \leq \lambda f(\vec{x}) + (1-\lambda) f(\vec{y}) \quad \text{--- (1)}$$

so, if RELU(x) statisfy above property it will a convex func

Let assume a function,
RELU $(x) = h(x) = \max(x, 0)$, $\forall \; x \in \mathbb{R}$.

Putting $h(x)$ in eqn 1. on LHS side, we get

$$h(\lambda x + (1-\lambda) y) = \max(\lambda x + (1-\lambda) y, 0)$$

$$\leq \lambda \max(x, 0) + (1-\lambda) \max(y, 0) \quad \text{--- (2)}$$

$$\leq \lambda h(x) + (1-\lambda) h(y)$$

Also, since at step (2), we know, $\max(r+s, t+u)$ will always less than $\max(r, s) + \max(t, u)$

So can say,
$$h(\lambda x + (1-\lambda) y) \leq \lambda h(x) + (1-\lambda) h(y)$$

Hence it proves RELU $(x) = \max(x, 0)$ is a convex function.

1 b) $f(\vec{x}) = \|A\vec{x} + \vec{b}\|_2 + \lambda\|\vec{x}\|_1$

we know norm ~~functio~~ are convex function. so splitting them and treating as separate function.

let $h(x)$ be $\|A\vec{x} + \vec{b}\|_2$ and $l(x)$ is $\lambda\|x\|_1$

To prove $h(x)$ is convex.

$$h(\lambda x + (1-\lambda)y) \leq (\|A(\lambda x + (1-\lambda)y) + \vec{b}\|_2)$$

$$\leq \lambda\|Ax\| + (1-\lambda)\|Ay\| + \lambda b + (1-\lambda)b$$

$$\leq \lambda\|Ax\| + \lambda b + (1-\lambda)(\|Ay + b\|_2)$$

$$\leq \lambda\|Ax + b\|_2 + (1-\lambda)\|Ay + b\|_2$$

$$\leq \lambda h(x) + (1-\lambda)h(y)$$

Since, it follows the convex property. similar to prove $l(x)$

$$l(\lambda x + (1-\lambda)y) \leq (\lambda\|x\| + (1-\lambda)\|y\|)$$

$$\leq \lambda l(x) + (1-\lambda)l(y)$$

, λ is a +ve constant as given $\lambda \geq 0$

similar $l(x)$ is always a convex function.

sum of two convex function is always convex function.

So we prove that l2 norm and l1 norms are convex using homogenity and triangular inequality.

c) given $x \in \mathbb{R}$,

To prove $f(x) = \frac{1}{1+e^{-x}}$ is niether convex nor concave.

$$f'(x) = \frac{d(1+e^{-x})^{-1}}{dx}$$

$$= -1 * (1+e^{-x})^{-2} * -e^{-x}$$

$$= \frac{1}{e^{+x}(1+e^{-x})^{+2}} \Rightarrow \frac{e^{-x}}{(1+e^{-x})^{+2}}$$

If we add +1 and -1, we get

$$\frac{1+e^{-x}-1}{(1+e^{-x})^{+2}} = \frac{(1+e^{-x})}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2}$$

$$\Rightarrow \frac{1}{(1+e^{-x})}\left(1-\frac{1}{(1+e^{-x})}\right)$$

$$f'(x) = f(x)\,(1-f(x))$$

$f''(x)$ or $H(x)$

$$= f'(x)\,(1-f(x)) + f(x).f'(x)$$

$$= f(x)\,(1-f(x))^2 + f(x).-(f(x)\,(1-f(x)))$$

$$\Rightarrow f(x)\,(1-f(x))^2 - f(x)^2(1-f(x))$$

inorder to prove ~~conver~~ $H(x) \geq 0$, so equating the eq^n to ≥ 0

$$f(x)\,(1-f(x))^2 - f(x)^2\,(1-f(x)) \geq 0$$

$$f(x)\,(1-f(x))[\,(1-f(x)) - f(x)) \geq 0$$

$$1 - 2f(x) \geq 0$$

$$\frac{1}{2} \leq f(x) \Rightarrow \frac{1}{2} \leq \frac{1}{1+e^{-x}}$$

$\therefore\ e^{-x} \geq 1$, [using log on both side]

$-x \geq 0$, since only for -ve value of x. This eq^n is true.

Hence $f(x)$ is not a convex function.

Using the hint. [$-f(x)$ is also not convex]. we can prove is not concave. so,

Assume $h(x) = -f(x)$. Now $h'(x) = \frac{-1}{1+e^{-x}} \Rightarrow$

$$\Rightarrow -\frac{1}{(1+e^{-x})^2} * -e^{-x}$$

$$\Rightarrow -\frac{e^{-x}}{(1+e^{-x})^2}$$

Now taking $h''(x)$ or $H(x)$, we get

$$\frac{d}{dx}\left(\frac{-e^{-x}}{(1+e^{-x})^2}\right) = \frac{\frac{d}{dx}(e^{-x})(1+e^{-x})^2 - \frac{d}{dx}((1+e^{-x})^2)\, e^{-x}}{((1+e^{-x})^2)^2}$$

$$= \frac{(-e^{-x})(1+e^{-x})^2 - (-2e^{-x}(1+e^{-x}))e^{-x}}{((1+e^{-x})^2)^2}$$

$$= \frac{-e^{-2x}(e^{-x}+1)}{(1+e^{-x})^3} \Rightarrow \frac{(1+e^{-x})^2}{e^{x}} + \frac{1}{e^{2x}(1+e^{-x})^3}$$

As we know inorder for it to be convex $H(x) \geq 0$.

so, $e^{x} \geq \frac{-1}{(1+e^{-x})^5}$

$$e^{x} \geq -(1+e^{-x})^{-5},$$

$$e^{x}(1+e^{-x})^5 \leq 1$$

This equality does hold for $x = 1$. Hence, it does statisfy for all values of x.

As result $h(x)$ i.e $-f(x)$ is not convex function either.

2. a) given $x \in \mathbb{R}, y \in \mathbb{R}$

$$h(x,y) = -\cos(x^2) + e^{xy} - 2y^2$$

(i) gradient of $h = \nabla h = \begin{bmatrix} \frac{\partial (h(x,y))}{\partial x} \\ \frac{\partial (h(x,y))}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x\sin(x^2) + e^{xy}y \\ e^{xy}x - 4y \end{bmatrix}$

$$H(h(x,y)) = \begin{bmatrix} \frac{\partial^2 (h(x,y))}{\partial x^2} & \frac{\partial^2 (h(x,y))}{\partial x \partial y} \\ \frac{\partial^2 (h(x,y))}{\partial y \partial x} & \frac{\partial^2 (h(x,y))}{\partial y^2} \end{bmatrix}$$

Hessian Matrix of h

$$H(h(x,y)) \Rightarrow \begin{bmatrix} 2\sin(x^2) + 4x^2\cos(x^2) + y^2e^{xy} & xye^{xy} + e^{xy} \\ xye^{xy} + e^{xy} & x^2e^{xy} - 4 \end{bmatrix}$$

(ii) second order Taylor expansion of h at $(x_0 = \overset{x_0}{\cancel{0}}, y_0 = \overset{y_0}{\cancel{0}})$

$$f(\vec{x_0}) + (\vec{x} - \vec{x_0})^T \vec{g} + \frac{1}{2}(\vec{x} - \vec{x_0})^T H (\vec{x} - \vec{x_0}) \rightarrow \text{for } 1^{st} \text{ eqn.}$$

$$f(\vec{x_0}, \vec{y_0}) + [(x_0 - \vec{x_0}) \quad (y - y_0)]^T \vec{g} + \frac{1}{2}\begin{bmatrix} x - \vec{x_0} \\ y - \vec{y_0} \end{bmatrix}^T H [x - \vec{x_0} \quad y - \vec{y_0}]$$

$$= -\cos(x_0^2) + e^{x_0y_0} - 2y_0^2 + [(x - x_0) \quad (y - y_0)]^T \begin{bmatrix} 2x_0\sin(x_0^2) + e^{x_0y_0}y_0 \\ e^{x_0y_0}x_0 - 4y_0 \end{bmatrix} +$$

$$\frac{1}{2}[(x_0 - x_0) \quad (y_0 - y_0)]^T \begin{bmatrix} 2\sin(x_0^2) + 4x_0^2\cos(x_0^2) + y_0^2e^{x_0y_0} & x_0y_0e^{x_0y_0} + e^{x_0y_0} \\ x_0y_0e^{x_0y_0} + e^{x_0y_0} & x_0^2e^{x_0y_0} - 4 \end{bmatrix}$$

$$\times [(\vec{x} - \vec{x_0}) \quad (\vec{y} - \vec{y_0})]$$

(iii) when $x_0 = 0$ and $y_0 = 0$. so substituting values in (ii) eqn. we get

$$-1 + 1 - 0 + [(\vec{x}-0)\ (\vec{y}-0)]^T \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{1}{2} [(\vec{x}-0)\ (\vec{y}-0)]^T \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix} [(\vec{x}-0)\ (\vec{y}-0)]$$

$$\Rightarrow \frac{1}{2} (2\vec{x}\vec{y} - 4\vec{y}^2)$$

$$\Rightarrow \vec{x}\vec{y} - \cancel{4}y^2 = y(x-2y)$$

(iv) we need to calculate eigen values to determine definiteness for point $(0,0)$.

so. taking hessian of k, i.e $k(x,y) = xy - 2y^2$

$$\nabla(k(x,y)) = \begin{bmatrix} \frac{\partial}{\partial x}(xy - 2y^2) \\ \frac{\partial}{\partial y}(xy - 2y^2) \end{bmatrix} = \begin{bmatrix} y \\ x - 4y \end{bmatrix}$$

$$H(k(x,y)) = \begin{bmatrix} \frac{\partial^2(k(x,y))}{\partial x^2} & \frac{\partial^2(k(x,y))}{\partial x\,\partial y} \\ \frac{\partial^2(k(x,y))}{\partial x\,\partial y} & \frac{\partial^2(k(x,y))}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix}$$

so eigen values of would be $\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda-4 \end{bmatrix} = 0$.

$$\lambda^2 + 4\lambda - 1 = 0$$

$$\lambda_1 = -2 + \sqrt{5} \text{ and } \lambda_2 = -2 - \sqrt{5}$$

since λ_1 is positive and λ_2 is negative, we can surely say about definiteness. So its termed as indefinite.

2. b) given $\vec{x}, \vec{b} \in \mathbb{R}^n$ and $M \in \mathbb{R}^{n \times n}$ are symmetric and invertible matrix

(i) $(\vec{x} - M^{-1}b)^T M (\vec{x} - M^{-1}\vec{b})$

$$= (\vec{x} - M^{-1}b)^T (M\vec{x} - \vec{b})$$

$$= \vec{x}^T M \vec{x} - b^T (M^{-1})^T M\vec{x} - x^T b + b^T M^{-1} \vec{b} \qquad [(M^{-1})^T = M^{-1}]$$

$$= \vec{x}^T M \vec{x} - 2\vec{b}^T \vec{x} + \vec{b}^T M^{-1} \vec{b}$$

[since inner product of symmetric matrix are same. we can say $\vec{b}^T\vec{x} = \vec{x}^T\vec{b}$]

(ii) given $\vec{x}, \vec{\mu} \in \mathbb{R}^n$, $\theta \in \mathbb{R}^n$ and $A+B$ is invertible and $A, B \in \mathbb{R}^{n \times n}$

$$f(x) = \underbrace{(\vec{x} - \vec{\mu})^T A (\vec{x} - \vec{\mu})} + \underbrace{(\vec{x} - \vec{\theta})^T B (\vec{x} - \vec{\theta})}$$

$$= \vec{x}^T A \vec{x} - 2\mu^T A \vec{x} + \mu^T A \mu + \vec{x}^T B \vec{x} - 2\theta^T B \vec{x} + \theta^T B \theta$$

~~Combing~~ terms to make $(A+B)$ Quadratic form,

$$\vec{x}^T (A+B) x - 2(\mu^T A + \theta^T B) x + \underbrace{\mu^T A \mu + \theta^T B \theta}_{\text{constant term } (c)}$$

Let M be $(A+B)$

and $b = A^T \mu + B^T \theta$

Using the hint from (i), we can say.

$$\vec{x}^T M x - 2\vec{b}^T x + \mu^T A \mu + \theta^T B \theta$$

$$= \underbrace{(\vec{x} - M^{-1}b)^T M (\vec{x} - M^{-1}\vec{b})}_{\text{single quadratic form.}} \underbrace{- \vec{b}^T M^{-1} \vec{b} + c}_{\text{new constant}}$$

3 a) $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$

$$A^T A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$$

$$\det(A^T A - \lambda I) = \begin{bmatrix} 17-\lambda & 8 \\ 8 & 17-\lambda \end{bmatrix}$$

$$= \lambda^2 - 34\lambda + 225$$

$$\lambda_1 = \sqrt{25} \,,\; \lambda_2 = \sqrt{9}$$
$$= 5 \qquad = 3$$

So finding vector corresponding to λ_1, i.e $V_1 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Eigen vector corresponding to λ_2. $\Rightarrow x_1 = x_2$.

$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\lambda_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$\Rightarrow x_1 = -x_2$$

So vector $V = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, let S be invertible matrix i.e $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ of V.

$$S^{-1}AS = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & +1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$$

$A = S\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}S^{-1}$ $\Rightarrow$ Therefore for n

$$A^n = \left(S\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}S^{-1}\right)^n = S\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}^n S^{-1}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3^n & 0 \\ 0 & 5^n \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^n = \frac{1}{2}\begin{bmatrix} 3^n + 5^n & -3^n + 5^n \\ -3^n + 5^n & 3^n + 5^n \end{bmatrix}$$

3 b i) given SVD of A as $\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}^T$

$$\underset{U}{\Downarrow} \qquad \underset{\Sigma}{\Downarrow} \qquad \underset{V}{\Downarrow}$$

If U, Σ, V^T are singular value decomposition of A, then

Prop 1 ⇒ U is matrix with orthonormal columns. mean $U^TU = I$.

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 & 0 \\ 0 & \left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Prop 2 ⇒ Similarly V should a orthonormal vector. so $V^T = V^{-1}$

$$V^{-1} = \frac{\begin{bmatrix} \frac{\sqrt{3}}{2} & +\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}}{\det(V)}$$

and $\det(V) = \frac{3}{4} + \frac{1}{4} \Rightarrow 1$.

$$V^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

so $V^{-1} = V^T$.

Prop 3 ⇒ Σ should a non-negative diagonal matrix. which satisfy the condition since $\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

Prop 4 ⇒ Finally, $U\Sigma V^T$ by multiply it provides real valued matrix A.

Hence U, Σ and V^T are SVD of matrix A. Proved using properties.

(ii) U and V are rotate matrix if it follows Rotation matrix properties

→ det (U) = 1

→ $U^{-1} = U^T$

→ dot product with any of the row with itself should 1

→ dot product with any of the column/row other than itself should 0.

$$U = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \qquad \det(U) = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= 1$$

since det (U) = 1.

$$U^{-1} = \frac{U^T}{\det(U)} = \frac{U^T}{1}$$

Moreover a rotation matrix satisfy $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

we can say for $\theta = \pi/4$.

$$U = \sqrt{2}\begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{bmatrix}$$

In similar way, we can also write V in R_θ terms as

$$V^T = \begin{bmatrix} \cos\left(\frac{\pi}{6}\right) & -\sin\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix}^T \qquad \det(V) = \left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\right] = 1$$

Hence we can U and V both are rotation matrices

(iii) To find the angle θ_u. we can use [trace (U) = trace (R_θ)].

$\therefore$ $2\cos\theta_u = \sqrt{2}$

$$\cos\theta_u = \frac{1}{\sqrt{2}} \Rightarrow \theta_u \Rightarrow \frac{\pi}{4}.$$

Similarly $\theta_V = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \frac{\pi}{6}$

(iii) using SVD, we know A can be written as.

$$A = U\Sigma V^T$$

- So as in first step matrix A, is rotated by angle of θ_v by vector V^T.
- Then using diagonal matrix Σ, would scale the vector. So secondly there would be scaling operation or transformation.
- Finally, after scaling, matrix A is again rotated by θ_u from vector U.

Therefore matrix went through Rotation $\xrightarrow{(\theta_v)}$ Scaling $\rightarrow$ Rotation (θ_u) in the order.

(iv) give unit circle with $\vec{x} = (1,0)$ and $\vec{y} = (0,1)$. Performing the transformation from (iii),

$$V^T x = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{-1}{2} \end{bmatrix} \qquad x' = \left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$$

$$V^T y = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right) \Rightarrow y = \left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right)$$

Then we scale the above mention new vector.

$$\Sigma(V^T x) = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{-1}{2} \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ \frac{-1}{4} \end{bmatrix}$$

$$\Sigma(V^T y) = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ \frac{\sqrt{3}}{4} \end{bmatrix}$$

Finally we apply 3rd transformation by multiplying U vector.

$$\therefore U(\Sigma V^T x) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}\begin{bmatrix} \sqrt{3} \\ -\frac{1}{4} \end{bmatrix} = \left(\frac{4\sqrt{3}+1}{4\sqrt{2}}, \frac{4\sqrt{3}-1}{4\sqrt{2}}\right)$$

$$U(\Sigma V^T y) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}\begin{bmatrix} \sqrt{2} \\ \frac{\sqrt{3}}{4} \end{bmatrix} = \left(\frac{4-\sqrt{3}}{4}, \frac{4+\sqrt{3}}{4}\right)$$

so after all the transformation

$\vec{x}$ changed to $\left(\frac{4\sqrt{3}+1}{4\sqrt{2}}, \frac{4\sqrt{3}-1}{4\sqrt{2}}\right) \approx (1.401, 1.04)$

$\vec{y}$ changed to $\left(\frac{4-\sqrt{3}}{4}, \frac{4+\sqrt{3}}{4}\right) \approx (0.566, 1.433)$

$\therefore$ Fig (b) i.e $A_1\vec{x}$ and $A_1\vec{y}$ represents correct transformation on unit circle by A.