Module 1
Differential Equations
of
First Order and First Degree

Sub-module 1.2

Applications of FOFD DE

Syllabus

Module No.	Unit No.	Details	Hrs.	CO
1	**Differential Equation of First Order and First Degree**		13	CO 1
	1.1	Differential Equation of first order and first degree- Exact differential equations, Equations reducible to exact equations by integrating factors.		
	1.2	Linear differential equations (Review), Equation reducible to linear form. Applications of Differential Equation of first order and first degree		
	1.3	Linear Differential Equation with constant coefficients: Complimentary function, particular integrals of differential equation of the type f(D)y=X, where X is e^{ax}, $\sin(ax+b)$, $\cos(ax+b)$, x^n, $e^{ax}V$		
	1.4	Cauchy's homogeneous linear differential equation		
	1.5	Method of variation of parameters		
		# **Self-learning topic:** Bernoulli's equation. Equation reducible to Bernoulli's equation.		

Example 1

- A chain coiled up near the edge of a smooth table starts to fall over the edge. The velocity v when a length x has fallen is given by $xv\frac{dv}{dx} + v^2 = gx$. Show that when $g = 32$, $v = 8\sqrt{x/3}$.
- Soln:$v\frac{dv}{dx} + v^2\frac{1}{x} = g$

Put $v^2 = u$

$$2v\frac{dv}{dx} = \frac{du}{dx}$$

$\therefore \frac{du}{dy} + 2u\frac{1}{x} = 2g$ which is in linear form.

- Here $P = \frac{1}{x}, \quad Q = 2g$

- $I.F. = e^{\int P dx} = e^{\int \frac{2}{x} dx} = x^2$
- G.S. is given by $u(I.F.) = \int Q(I.F.)dx + c$
- $ux^2 = \int 2g\, x^2 dx + c$
- $v^2 x^2 = \frac{2gx^3}{3} + c$
- $when\ x = 0, v = 0\ therefore\ c = 0$
- $v^2 x^2 = \frac{2gx^3}{3}$
- When g=32,
- $v^2 = \frac{2gx}{3} = \frac{64x}{3}$
- $v = 8\sqrt{x/3}$

Example 2

- **Solve $\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$ for the case in which the circuit has initial current i_0 at time $t=0$ and the e.m.f. impressed is given by $E=E_0e^{-kt}$**
- **Solution:** Given $\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$ [Linear D.E.]
- Here $P=\frac{R}{L},\ Q=\frac{E}{L}$
- $I.F.=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$
- G.S. is $ie^{\frac{R}{L}t}=\int\frac{E}{L}e^{\frac{R}{L}t}dt+c$
- $=\frac{E_0}{L}\int e^{-kt}e^{\frac{R}{L}t}\,dt+c$
- $=\frac{E_0}{L}\int e^{\left(\frac{R}{L}-k\right)t}dt+c$
- $=\frac{E_0}{R-kL}e^{\left(\frac{R}{L}-k\right)t}+c$

- But at $t = 0, i = i_0, and\ E = E_0$
- $\therefore\ i_0 = \frac{E_0}{R-kL} + c$
- $c = i_0 - \frac{E_0}{R-kL}$
- $\therefore\ ie^{\frac{R}{L}t} = \frac{E_0}{R-kL} e^{\left(\frac{R}{L}-k\right)t} + i_0 - \frac{E_0}{R-kL}$
- (or) $i = \frac{E_0}{R-kL} e^{-kt} + \left(i_0 - \frac{E_0}{R-kL}\right) e^{-\frac{R}{L}t}$

Example 3

- **The equation of electromotive force in terms of current i for an electrical circuit having resistance R and a condenser of a capacity c, in series is $E = R\,i + \int \frac{i}{c}\,dt$. Find the current i at any time t, when $E = E_0 \sin wt$**
- **Solution:** Given $Ri + \int \frac{i}{c}\,dt = E_0 \sin wt$
- Differentiating on both sides with respect to t, we get
- $R\frac{di}{dt} + \frac{i}{c} = wE_0 \cos wt$
- Dividing throughout by R we get,
- $\frac{di}{dt} + \frac{i}{Rc} = \frac{wE_0}{R}\cos wt \quad [Linear\ D.E.]$
- $I.F. = e^{\int \frac{1}{R\,c}dt} = e^{\frac{t}{Rc}}$

- The solution is given by $e^{\frac{t}{Rc}} = \int \frac{wE_0}{R} \cos wt \, e^{\frac{t}{Rc}} dt$
- $= \frac{wE_0}{R} \cdot \frac{e^{\frac{t}{Rc}}}{\sqrt{\left(\frac{1}{Rc}\right)}} \cos\left(wt - tan^{-1}\frac{w}{\frac{1}{Rc}}\right) + c_1$
- $= \frac{wcE_0}{\sqrt{1+R^2c^2w^2}} e^{\frac{t}{Rt}} \cos(wt - \emptyset) + c_1$
- Where $\tan \emptyset = Rc$
- (or)
- $i = \frac{wcE_0}{\sqrt{1+R^2c^2w^2}} \cos(wt - \emptyset) + c_1 e^{\frac{t}{Rt}}$
- Is the required current at time t.

Practice problems

- In a circuit containing inductance L resistance R and voltage E, the current i is given by $E = Ri + L\frac{di}{dt}$ .If $L = 640h, R = 250W$ & $E = 500\ volts$ and $i = 0\ when\ t = 0$, find the time that elapses before the current reaches 90% of its maximum value.

- The charge q on the plate of a condenser of capacity C charged through a resistance R by the steady voltage V satisfies the differential equation $R\frac{dq}{dt} + \frac{q}{c} = V$ If q=0 at t=0 , find q. Also find the current flowing into the plate.