Thermal Noise in Resistive Components

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Noise with unknown characteristics has been observed to occur in a resistor. The noise is quantified using the principles of statistical mechanics and the dependence on resistance and temperature is determined. Thermal agitation of charge carriers due to the applied voltage is considered to be the primary cause of the observed noise. The Boltzmann Constant was derived from the quantified noise results suggesting consistency with theoretical predictions. Future studies will investigate a larger range of resistor values and examine temperature and other factors that alter the power of noise at certain frequencies.

Voltage fluctuations resulting from thermal noise in electrical equipment that do not fit the characteristics of previously known noise types was observed in a resistor. The quantification of noise is determined in order to utilize its characteristics and determine how to optimize the performance of future technology.

Interactions between the environment and electrical components affect the performance of circuits and their components, causing alterations from the intended purpose. Electromagnetic induction [1] causes interference of external noise sources with electronic signals and also produces acoustic noise when interacting with certain materials [1]. Electromagnetic radiation is always present, such as the background radiation of the universe, Cosmic Microwave Background Radiation [2], and shielding needs to be used to isolate the intrinsic noise of a component and rule out the possibility of induction interference.

Small signals are often measured in the field of radio astronomy, which carry noise from interference of the atmosphere, cosmic sources, and instrumental noise. The original weak signal is often drowned out by the noise of the components, and original information within the signal is lost. Reducing instrumental noise and investigating atmospheric and cosmic signals from absorption and emission spectra allows for the detection of weak signals from deep space.

Similarly to the Kinetic Theory of Gases [3], we infer that the composition of solids consists of atoms in motion, however unlike gases, rotational degrees of freedom are not present due to atoms being bound in various configurations unique to that material. Motion with only translational degrees of freedom causes vibrations that are caused by charge carriers such as electrons from their movement inside materials. The release of energy through motion in the form of heat is the suspected cause of intrinsic noise present in electrical components.

In the Maxwell - Boltzmann Distribution [4], the speed of particles decreases in accordance with temperatures insinuating the presence of a relationship between these factors. Following this theory it is reasonable to predict that temperature changes will have a significant effect on the thermal noise power in a resistor. Temperature close to absolute zero [5] will result in less movement of particles thus reducing the amount of kinetic energy in the system resulting in the magnitude of thermal noise experiencing a decrease. While there exists movement of charge carriers at these low temperatures, fluctuations of the current which can be characterized as noise will not result due to quantum effects. Low temperature conditions will contribute significantly on the behavior of thermal noise in non-classical conditions [6].

Since it is difficult to produce extreme temperature in which the response of materials can be explored, a variety of resistors will be used to expose any resistance dependence. These results can help reduce noise in modern electronics despite the limitations in temperature dependent research.

Finding the sum of all the states of the system and finding the expectation value of the thermal energy for all states, energy per unit frequency is found. The relation of power, frequency, and Ohm's Law is used to derive an expression for the mean square voltage of thermal noise in a resistor, $V^2_{noise} = 4k_BTR\Delta f$ where k_B is the Boltzmann Constant [7], T is temperature in Kelvin, R is resistance in Ohms, and Δf is the range of frequencies that alters the behavior of the noise.

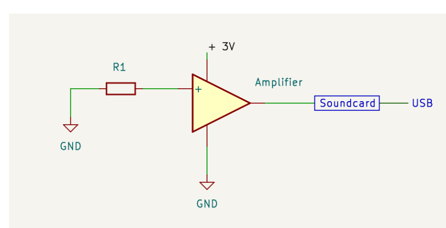

FIG. 1. Thermal noise from resistors is seen by connection to a low-noise amplifier and is analyzed using noise detecting software.

To measure the relationship between resistance and power of thermal noise, we performed measurements on a variety of resistors ranging from 18 Ω to 400 kΩ. To protect each resistor from external influences, a copper case was used as a shield. The resistor in the copper case was connected to a low-noise amplifier and a power supply. The signal was amplified by a large factor to allow for it to be detected by sensitive instruments which then could be digitized and analyzed (Fig.1).

An analog digital converter, particularly a USB sound card, was calibrated by testing the digitization of a small input voltage in the units of mV (Fig.2). To analyze the data, it is required that the digitized voltage be consistent with the input therefore a 1:1 ratio was expected and a scaling factor needed to be determined.

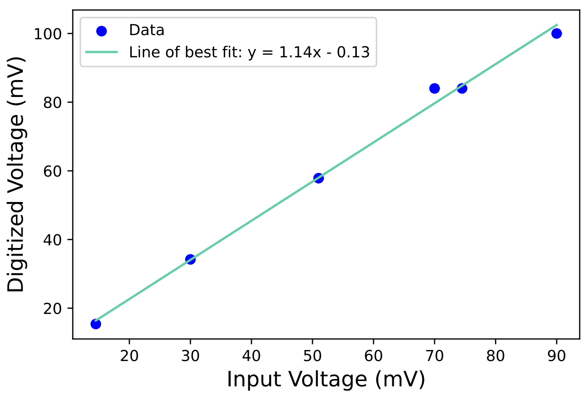

FIG. 2. Linear relationship between input voltage and digitized voltage. The digitized signal had been scaled to produce a ratio close to the expected 1:1 with the input.

The low-noise amplifier contributes its own noise to the resistor measurements, which must be quantified to isolate the thermal noise of the resistors. Connecting the amplifier to a voltage divider, a frequency dependence was explored and the amplified output was recorded (Fig. 3).

Using Welch's Method [8], the noise was represented using a power spectral density (PSD) as a function of frequency. In Fig.4, the noise is not constant and its average value cannot be determined without removing the interference from the amplifier using the theoretical amplification model. Dividing the noise data by the square of the amplification factor isolated the thermal noise in the resistor which can be clearly seen in Fig.5, and its average value can be determined.

The power spectral density in Fig. 5 indicates that the power concentration of the thermal noise lies at lower frequencies. The steady noise signal starts to exponentially decrease for frequencies higher than 19,000 Hz suggesting that a limit exists for what frequencies the noise is more prevalent at. The noise was quantified by finding the average of the noise power for each resistor from 450 to 18,500 Hz. The range of the noise found was 1.4×10^{-15}

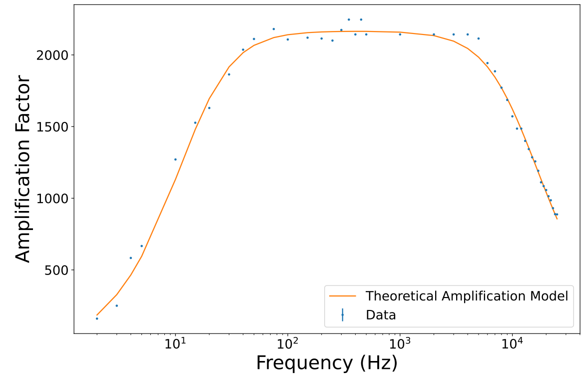

FIG. 3. Theoretical amplification model used to curve fit frequency dependent amplification data with uncertainties considered in determining the best fit.

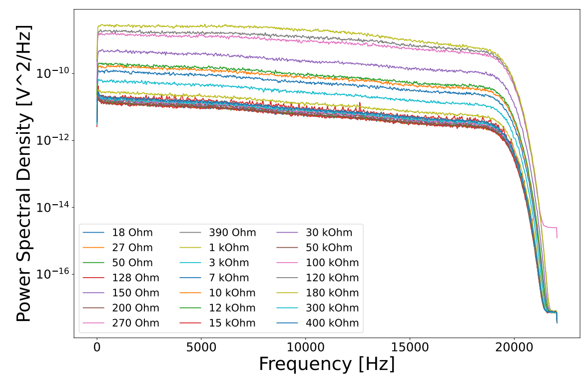

FIG. 4. Noise from resistor represented in a power spectral density with Welch's fast Fourier transform method. Interference from low-noise amplifier needs to be corrected to quantify noise.

to 9.8×10^{-18} V^2/Hz with 180 kΩ resistor having the largest noise power and 50 kΩ having the smallest value. The larger resistor has a higher noise power than the smallest which was expected, however resistors less than 50 kΩ and larger than 180 kΩ were tested. According to the theoretical formula, the smallest resistor of 18 Ω and largest resistor of 400 $kOhm$ should have the smallest and largest thermal noise power. While the general formula is still adhered to, factors previously not considered must be causing alterations from expected results.

To compare the experimentally determined data with the theory, the Boltzmann Constant was derived as seen in Fig.6. $k_B = 1.26 \times 10^{-23}$ J/K determined from the V^2_{noise} and R relation. This result is similar to the actual value of 1.38×10^{-23} J/K [9] indicating that the experimental methods designed were successful in gathering data according to theory.

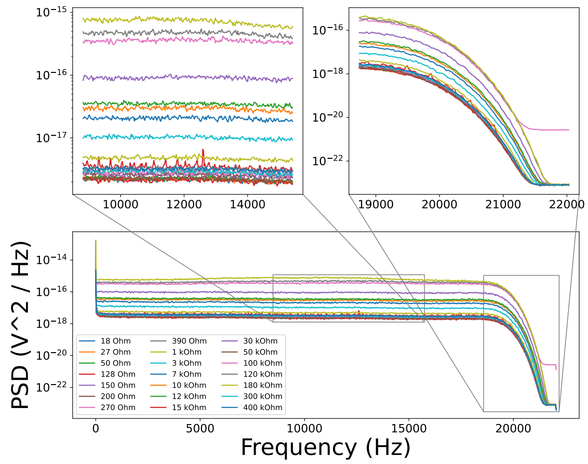

FIG. 5. Noise from resistors after correcting for gain using theoretical amplification model. The noise is stays steady until rapidly decreasing at higher frequencies.

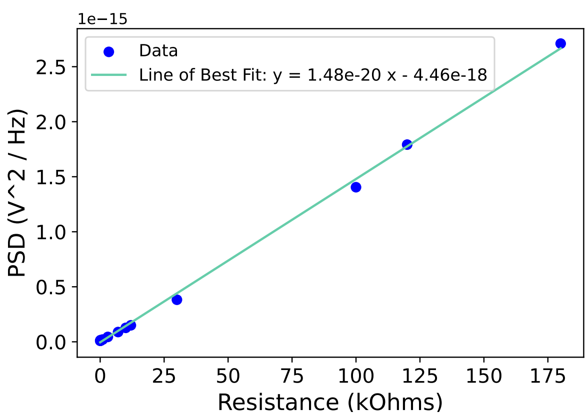

FIG. 6. Deriving the Boltzmann Constant from the relationship between resistance and PSD. From the slope of the best fit, k_B was determined.

Statistical tests were conducted and uncertainties were propagated to further analyze the results. Uncertainties in amplifier calibration were determined according to the range of acceptable output values from the noisy signal which had passed through a divider and low-noise amplifier. The amplifier calibration curve fitting resulted in a reduced chi square of $\chi >> 1$ indicating that uncertainties were underestimated and were interfering with getting the best fit. After propagating the uncertainties from sound card and amplifier calibration to the final quantified noise result, a very large uncertainty resulted. The uncertainty in the low-noise amplifier seemed to be reasonable during the attempt to calibrate the equipment at 0.02 at 0.06 V, however seeing the final uncertainty makes it obvious that there was an error in the uncertainty estimation throughout the experiment and that methods need to be revised with this in consideration.

The PSD decreases exponentially before becoming consistently low for frequencies higher than 21,500 Hz. White noise is characterized as a low frequency noise with a range of 20 - 20,000 Hz [10]. This newly measured thermal noise in resistors falls within this frequency range and could be classified as white noise.

The general trend that can be seen in Fig.5 and Fig.6 suggests that thermal noise has a greater power spectral density at higher resistances. This pattern agrees with the theoretical noise formula where at a fixed temperature, noise is solely dependent on and is linearly proportional to resistance. Temperature dependence can be explored similarly while holding resistance constant, and temperature limits can be found by searching for the occurrence of quantum effects.

From the trend of increasing resistances corresponding to a higher PSD, further recommendations are to explore noise power at higher resistances such as 1 MΩ and beyond. Conducting materials should also be tested for the presence of agitation of charge carriers resulting in thermal noise to see if this phenomenon is restricted to resistive materials.

The results of the study were consistent with the theoretical predictions made based on the principles of statistical mechanics and characteristics of electronic components. A frequency and resistance dependent relationship was highlighted which allowed for a close to the actual value derivation of the Boltzmann Constant. Limitations for calibration of equipment and quantifying uncertainties suggest refinement in the methods of future studies. These improvements will enhance our understanding of the intrinsic noise in resistors, and by extension in other electrical components allowing for the optimization of future technology and its ability to detect sensitive information. Noise minimized electronics will particularly benefit in the field of radio astronomy by allowing for more precise measurements to be made, and will have many more applications on other technologies.

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