EECS 545 → Machine Learning

Lecture 6

Discriminant Functions

- Linear discriminant function is for discriminating two classes
- Specify weight vector and bias w_0, $h(x) = w^T x + w_0$
- Assign x to C_1 if $h(x) \geq 0$, assign to C_0 otherwise

- For $k > 2$ classes, each class C_k gets its own function

$$h_k(x) = w_k^T x + w_{k,0}$$

- Assign x to C_k if $h_k(x) > h_j(x)$ for all $j \neq k$

- We can select w that minimizes squared errors, but this is sensitive to outliers

- Fisher's Linear Discriminant → select w that best separates the classes
  maximize between class variances (inter-class)
  minimize within class variances (intra-class)
- Project x onto one dimension → if $w^T x \geq -w_0$ then C_1 else C_0
- Maximize distance between classes → $\underbrace{m_2 - m_1}_{\text{projected mean}} = w^T(\underbrace{\mu_2 - \mu_1}_{\text{mean}})$ where $\mu_k = \frac{1}{N_k} \sum_{n \in C_k} x_n$

- Minimize the distance within each class → $s_1^2 + s_2^2 \equiv \sum_{n \in C_1} (w^T x_n - m_1)^2 + \sum_{n \in C_2} (w^T x_n - m_2)^2$

- Objective function → $J(w) = \dfrac{(m_2 - m_1)^2}{s_1^2 + s_2^2}$