Assignment 5

Q.2: For any differentiable function $R : \mathbb{R}^d \to \mathbb{R}$, for vectors w and u

Bregman Distance is defined as:

$$D_R(w \| u) = R(w) - R(u) - \langle \nabla R(u), w - u \rangle$$

To find: The Bregman Divergence of $R = \frac{1}{2}\|u\|_2^2$

$$R = \frac{1}{2}\|u\|_2^2$$

$$\|u\|_2^2 = \sum u_i^2$$

$$\nabla R(u) = u$$

$$D_R(w \| u) = \frac{1}{2}\|w\|_2^2 - \frac{1}{2}\|u\|_2^2 - \langle u, w - u \rangle$$

$$D_R(w \| u) = \frac{1}{2}\sum w_i^2 - \frac{1}{2}\sum u_i^2 - \sum (\langle w, u \rangle - \langle u, u \rangle)$$

$$= \frac{1}{2}\sum \langle w, w \rangle - \frac{1}{2}\sum \langle u, u \rangle - \sum \langle w, u \rangle + \sum \langle u, u \rangle$$

$$= \frac{1}{2}\sum \langle w, w \rangle - \sum \langle w, u \rangle + \frac{1}{2}\sum \langle u, u \rangle$$

$$D_R = \frac{1}{2}\|w - u\|_2^2$$