Lecture 18

Week 4

Adversarial Bandits for Exp 3, Exp3P, Exp 3I:

$$\bar{R}(n, \pi) \leq O(\sqrt{nk\log k}) \leftarrow \text{Covered in Week}$$

$$\bar{R}(n, \pi) \geq \Omega(\sqrt{nk})$$

Online Convex Optimization

– Assume that: Adversary is choosing convex function in each round

Input: A convex set $S \subset \mathbb{R}^d$

For $t = 1, 2, \ldots$

predict vector $\omega_t \in S$

receive convex loss function $f_t \; S \rightarrow R$

Suffer loss $f_t(\omega_t)$

$$R(n, \pi) = \sum_{t=1}^{n} f_t(\omega_t) - \min_{u \in S} \sum_{t=1}^{n} f_t(u) \quad \text{— (1)}$$

For the full information setting, with weighted majority algorithm; S can be considered as

[These are the

$S = \{ w \quad w_t \geq 0 \; \forall i \; \& \; \sum w_i = 1 \}$ [same weights that are assigned to the experts]

In the weighted majority,

after choosing w_t, one would incur V_t from the environment (adversary). Hence loss was:

$$\text{loss} = \langle w_t, V_t \rangle = f_t(w_t)$$ ← can be written like this for convex setting, its a linear function

Now let the regret be:

$$R(n, \pi) = \sum_{t=1}^{n} f_t(w_t) - \min_{u \in U} \sum_{t=1}^{n} f_t(u) \quad \text{—— } 1a$$

s.t. $U \subset S$ containing all unit vectors

thus $U = \{ \overset{e_1}{(1, 0, 0, \ldots 0)}, \overset{e_2}{(0, 1, 0, \ldots)}, \ldots \overset{e_i}{(0, 0 \ldots 1, \ldots 0)}, \ldots \overset{e_d}{(0, 0 \ldots 0, 1)} \}$

then $f_t(u) = f_t(e_i) = \sum_{t=1}^{n} V_{ti}$

Thus eq. 1a becomes

$$R(n, \pi) = \sum_{t=1}^{n} \langle V_t, w_t \rangle - \min \sum_{t=1}^{n} V_{ti} \quad \text{—— } 1b$$

The aforementioned case was for a linear function, but convex optimization can be applied for any function, as long as its convex.

Hence reconsider eq 1

$$R(n, \pi) = \sum_{t=1}^{n} f_t(w_t) - \min_{u \in S} \sum_{t=1}^{n} f_t(u) \quad \text{—— } 1$$

For any convex function, at round t,

once w_t is chosen, f_t will be revealed by the en

The goal now is to find out an algorithm to choose ω

Follow the Leader (FTL)

Till round t, we know

$f_1, f_2, f_3, \ldots f_{t-1}$

Hence we can choose ω_t as:

$$\omega_t = \underset{\omega \in S}{\operatorname{argmin}} \sum_{i=1}^{t-1} f_i(\omega)$$