Computer Vision 16720 A

HW 4

Harsh Dhruva

Andrew ID: hdhruva

Q1.1

For point P in the figure,

$$\tilde{x}_1 = \tilde{x}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad x_1 \text{ and } x_2 \text{ are origins}$$

For fundamental matrix,

$$\tilde{x}_2^T F \tilde{x}_1 = 0$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\therefore \boxed{F_{33} = 0}$$

Q1.2

Pure translation parallel to x-axis $\Rightarrow$ $t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}$

No rotation, $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E = [t]_x R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

Let $\tilde{x}_1 = [x_1 \ \ y_1 \ \ 1]^T$ $\quad \tilde{x}_2 = [x_2 \ \ y_2 \ \ 1]^T$

Epipolar line $\quad l_1 = E\tilde{x}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_x \\ t_x y_2 \end{bmatrix}$

$$l_2 = E\tilde{x}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_x \\ t_x y_1 \end{bmatrix}$$

line 1 : $-t_x y + t_x y_2 = 0$ $\quad (ax + by + c = 0)$

line 2 : $-t_x y + t_x y_1 = 0$

Both these epipolar lines are parallel to the x-axis

Q1.3 Let's say position of robot in real world is $p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

At time $t = t_1$ translation matrix not time

$$p_1 = K(R_1 p + t_1) \quad -(1)$$

At time $t = t_2$

$$p_2 = K(R_2 p + t_2) \quad -(2)$$

From eq (1) $\rightarrow$

$$K^{-1} p_1 = K^{-1} K (R_1 p + t_1)$$

$$K^{-1} p_1 = R_1 p + t_1$$

$$K^{-1} p_1 - t_1 = R_1 p$$

$$R_1^{-1}(K^{-1} p_1 - t_1) = R_1^{-1} R_1 p$$

$$p = R_1^T (K^{-1} p_1 - t_1) \qquad R_1^{-1} = R_1^T$$

Substituting In eq (2) $\rightarrow$

$$p_2 = K\left(R_2 R_1^T (K^{-1} p_1 - t_1) + t_2\right)$$

$$p_2 = K R_2 R_1^T K^{-1} p_1 - K R_2 R_1^T t_1 + K t_2$$

$$p_2 = K(R_{rel}\, p_1 + t_{rel})$$

Where $R_{rel} = R_2 R_1^T K^{-1}$

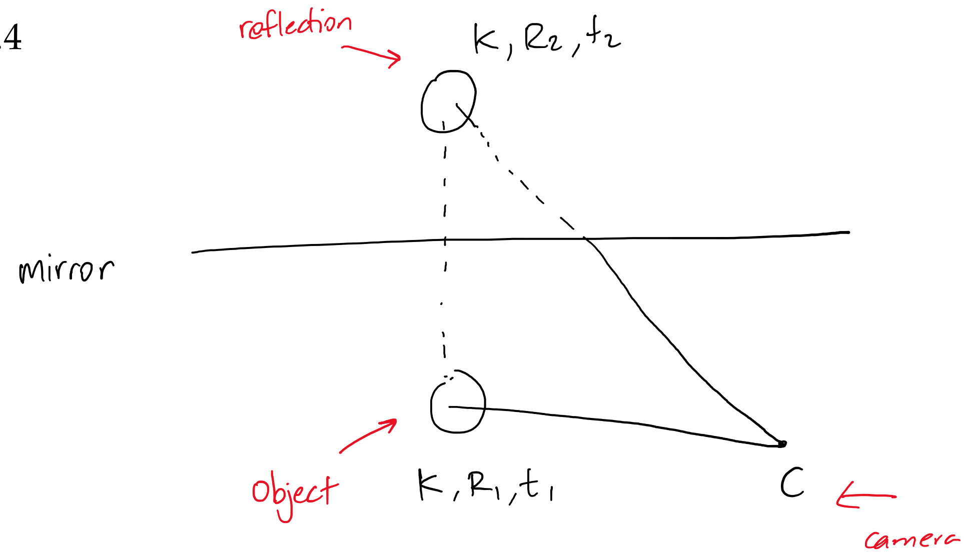
$t_{rel} = -R_2 R_1^T t_1 + t_2$

$\therefore$ Essential Matrix, $\boxed{E = t_{rel} \times R_{rel}}$

Fundamental Matrix, $F = (K^{-1})^T E K^{-1}$

$$\boxed{F = (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}}$$

Q1.4



Let the two images have positions $[R_1, t_1]$ and $[R_2, t_2]$ that the camera views as two separate images. We can think of the object to be "moved" from position 1 to position 2 in the same time as each "position" is captured by the camera separately.

Assuming all points on the object are equal distance to the mirror, the second image differs from the first by a pure translation.

$$\therefore \quad t \to t_{rel} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \longrightarrow [t_{rel}] = \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix}$$

$$\text{and} \quad R \to R_{rel} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From problem 1.3,

$$F = (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$$

$$F = (K^{-1})^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix} K^{-1}$$

Consider $F^T = \left((K^{-1})^T (t_{rel} \times R_{rel}) K^{-1} \right)^T$

$$F^T = (K^{-1})^T \left((K^{-1})^T (t_{rel} \times R_{rel}) \right)^T$$

$$= (K^{-1})^T (t_{rel} \times R_{rel})^T (K^{-1})$$

$(K^{-1})^{TT} = K^{-1}$

$$= (K^{-1})^T \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} (K^{-1})$$

$$= -(K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$$

$$\boxed{F^T = -F}$$

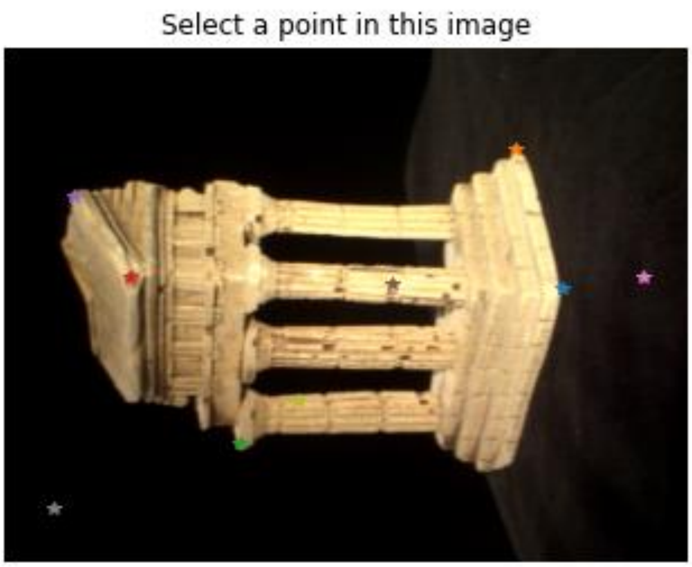
∴ The fundamental matrix relating the two Images is a skew-symmetric matrix.

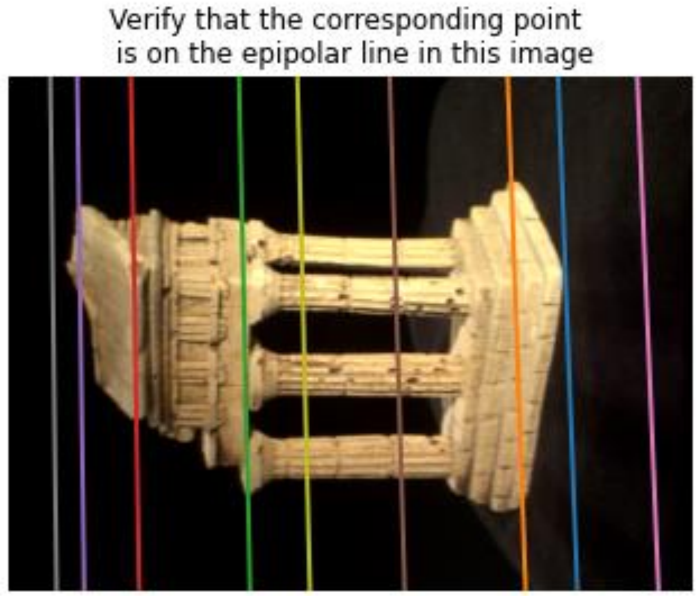
Q2.1

F = [[9.78833283e-10 -1.32135929e-07 1.12585666e-03]

[-5.73843315e-08 2.96800276e-09 -1.17611996e-05]

[-1.08269003e-03 3.04846703e-05 -4.47032655e-03]]





Q3.1

```
E =  [[ 2.26268684e-03 -3.06552495e-01  1.66260633e+00]
      [-1.33130407e-01  6.91061098e-03 -4.33003420e-02]
      [-1.66721070e+00 -1.33210351e-02 -6.72186431e-04]]
```

Q3.2 We have $w_i = [x_i \ y_i \ z_i]^T$, $\tilde{w}_i = [x_i \ y_i \ z_i \ 1]^T$

Two camera projection matrices,

$$C_1 = \begin{bmatrix} {}^1c_1^T \\ {}^1c_2^T \\ {}^1c_3^T \end{bmatrix} = \begin{bmatrix} {}^1c_{11} & {}^1c_{12} & {}^1c_{13} & {}^1c_{14} \\ {}^1c_{21} & {}^1c_{22} & {}^1c_{23} & {}^1c_{24} \\ {}^1c_{31} & {}^1c_{32} & {}^1c_{33} & {}^1c_{34} \end{bmatrix}, \quad C_2 = \begin{bmatrix} {}^2c_1^T \\ {}^2c_2^T \\ {}^2c_3^T \end{bmatrix} = \begin{bmatrix} {}^2c_{11} & {}^2c_{12} & {}^2c_{13} & {}^2c_{14} \\ {}^2c_{21} & {}^2c_{22} & {}^2c_{23} & {}^2c_{24} \\ {}^2c_{31} & {}^2c_{32} & {}^2c_{33} & {}^2c_{34} \end{bmatrix}$$

2D homogenous coordinates

$$\tilde{x}_1 \equiv \begin{bmatrix} x_{1i} \\ y_{1i} \\ 1 \end{bmatrix} \equiv C_1 \tilde{w}_i \qquad \tilde{x}_2 \equiv \begin{bmatrix} x_{2i} \\ y_{2i} \\ 1 \end{bmatrix} \equiv C_2 \tilde{w}_i$$

For Camera 1,

$$x_{1i} = \frac{{}^1c_1^T \tilde{w}_i}{{}^1c_3^T \tilde{w}_i} \Rightarrow ({}^1c_3^T \tilde{w}_i) x_{1i} - ({}^1c_1^T \tilde{w}_i) = 0$$

$$y_{1i} = \frac{{}^1c_2^T \tilde{w}_i}{{}^1c_3^T \tilde{w}_i} \Rightarrow ({}^1c_3^T \tilde{w}_i) y_{1i} - ({}^1c_2^T \tilde{w}_i) = 0$$

$$\Downarrow$$

$$[\underbrace{{}^1c_3^T x_{1i} - {}^1c_1^T}_{1\times 4}] \tilde{w}_i = 0$$

$$[\underbrace{{}^1c_3^T y_{1i} - {}^1c_2^T}_{1\times 4}] \tilde{w}_i = 0$$

$$\Downarrow$$

$$[{}^1c_{31} x_{1i} - {}^1c_{11} \quad {}^1c_{32} x_{1i} - {}^1c_{12} \quad {}^1c_{33} x_{1i} - {}^1c_{13} \quad {}^1c_{34} x_{1i} - {}^1c_{14}] \tilde{w}_i = 0$$

$$[{}^1c_{31} y_{1i} - {}^1c_{21} \quad {}^1c_{32} y_{1i} - {}^1c_{22} \quad {}^1c_{33} y_{1i} - {}^1c_{23} \quad {}^1c_{34} y_{1i} - {}^1c_{24}] \tilde{w}_i = 0$$

Similarly with camera 2, we get 2 more equations

$$\therefore \quad A_i \tilde{w}_i = 0$$

$$\therefore \underbrace{\begin{bmatrix} {}^{1}c_{31}x_{1i} - {}^{1}c_{11} & {}^{1}c_{32}x_{1i} - {}^{1}c_{12} & {}^{1}c_{33}x_{1i} - {}^{1}c_{13} & {}^{1}c_{34}x_{1i} - {}^{1}c_{14} \\ {}^{1}c_{31}y_{1i} - {}^{1}c_{21} & {}^{1}c_{32}y_{1i} - {}^{1}c_{22} & {}^{1}c_{33}y_{1i} - {}^{1}c_{23} & {}^{1}c_{34}y_{1i} - {}^{1}c_{24} \\ {}^{2}c_{31}x_{2i} - {}^{2}c_{11} & {}^{2}c_{32}x_{2i} - {}^{2}c_{12} & {}^{2}c_{33}x_{2i} - {}^{2}c_{13} & {}^{2}c_{34}x_{2i} - {}^{2}c_{14} \\ {}^{2}c_{32}x_{2i} - {}^{2}c_{21} & {}^{2}c_{32}x_{2i} - {}^{2}c_{22} & {}^{2}c_{33}x_{2i} - {}^{2}c_{23} & {}^{2}c_{34}x_{2i} - {}^{2}c_{24} \end{bmatrix}}_{A_i} \tilde{w}_i = 0$$

Or $$A_i = \begin{bmatrix} {}^{1}c_3^T x_{1i} - {}^{1}c_1^T \\ {}^{1}c_3^T y_{1i} - {}^{1}c_2^T \\ {}^{2}c_3^T x_{2i} - {}^{2}c_1^T \\ {}^{2}c_3^T y_{2i} - {}^{2}c_2^T \end{bmatrix}$$

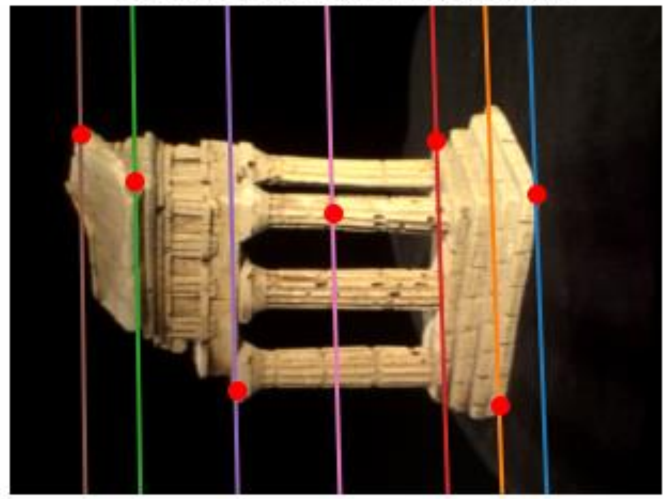
4x4

Q3.3

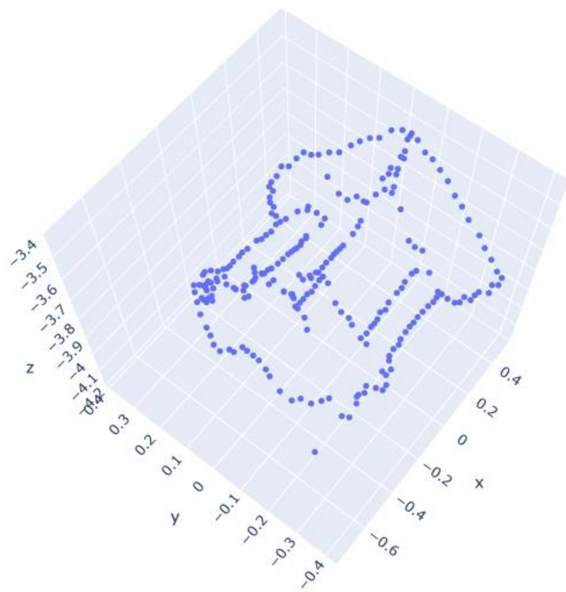
Q4.1

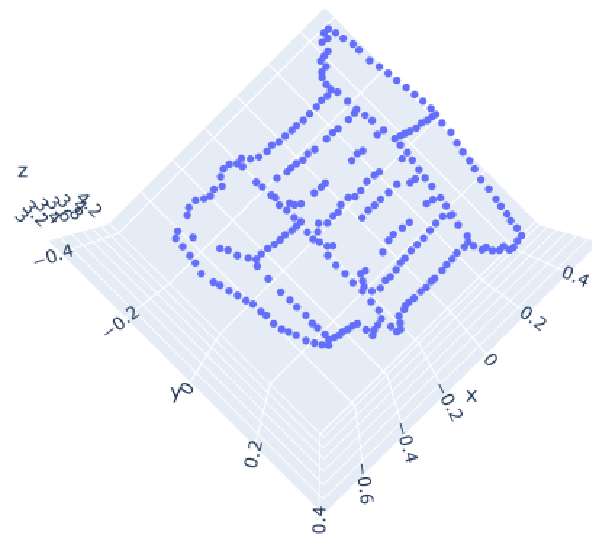
Select a point in this image

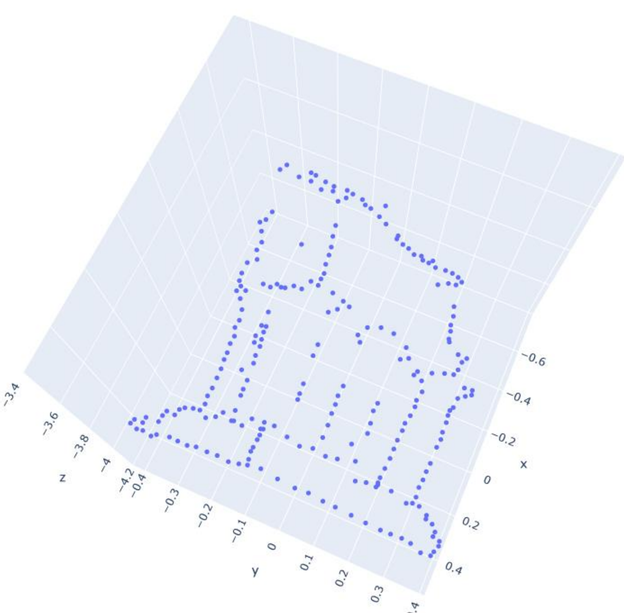
Verify that the corresponding point
is on the epipolar line in this image

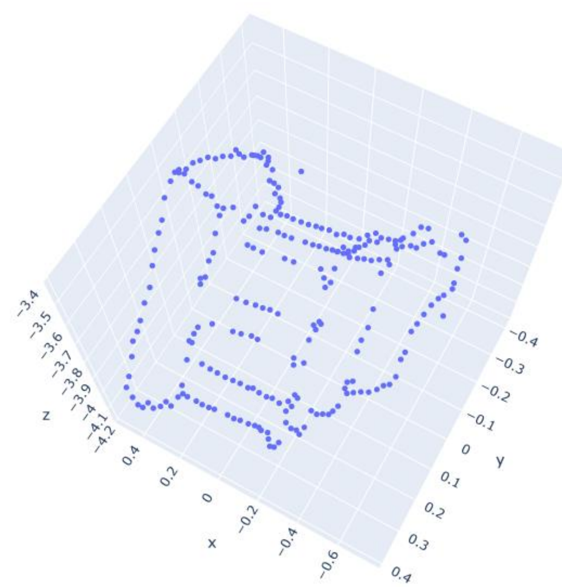


Q4.2


z
−3.4
−3.5
−3.6
−3.7
−3.8
−3.9
−4
−4.1
−4.2
y
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
x
0.4
0.2
0
−0.2
−0.4
−0.6

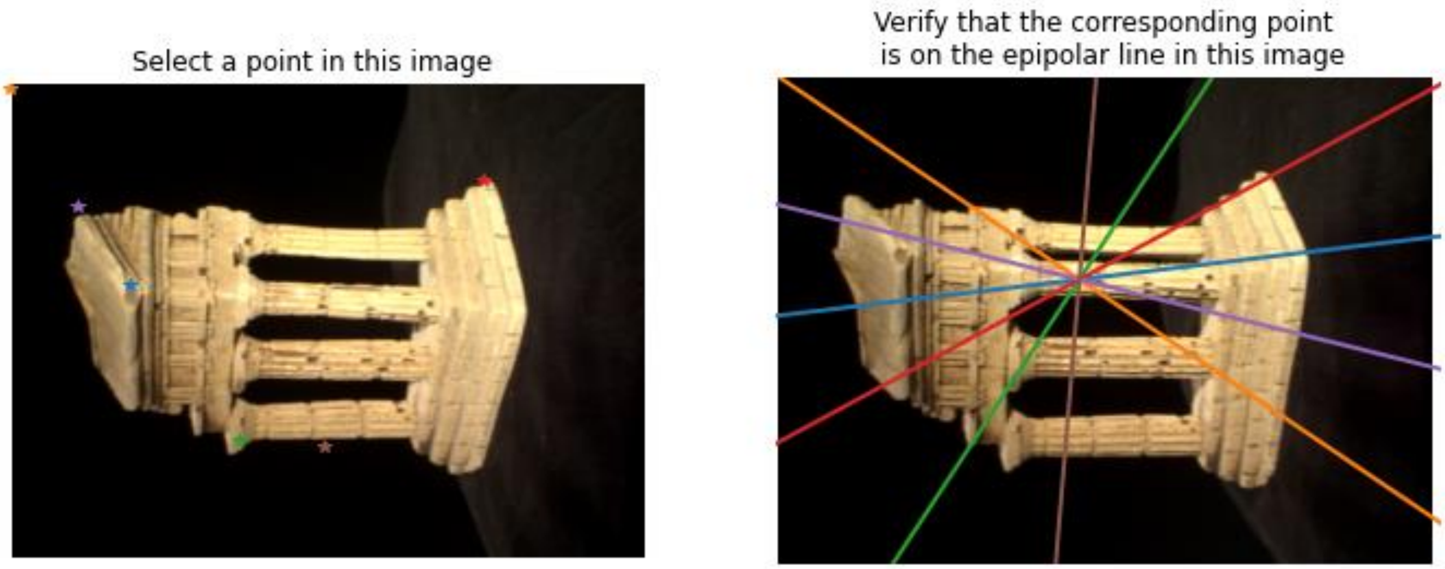

z
y
−0.4
−0.2
0
0.2
0.4
x
0.4
0.2
0
−0.2
−0.4
−0.6


z
−3.4
−3.6
−3.8
−4
−4.2
y
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
x
−0.6
−0.4
−0.2
0
0.2
0.4


z
−3.4
−3.5
−3.6
−3.7
−3.8
−3.9
−4
−4.1
−4.2
x
0.4
0.2
0
−0.2
−0.4
−0.6
y
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4

Q5.1

Using the noisy data and the regular eightpoint algorithm, the epipolar lines are seen in the following figure



Q5.2

Q5.3