# 1 Optimal Decision Rule (Binary)

Consider the following binary hypothesis testing problem. X follows one of two hypothesis: null hypothesis(0) or alternate hypothesis(1). We are given some observation y $\in$ Y, and asked to construct a decision rule — A function

$$\hat{X} : Y \to \{0, 1\}$$

that maximizes the probability of correct detection(PCD or $P(\hat{X} = 1|X = 1)$) given the below constraint $P(\hat{X} = 1|X = 0) \leq \alpha$ (i.e the Probability of False alarm(PFA) is upper bounded by some alpha

By Neyman-Pearson, the optimal decision rule is one of the following form:

$$\hat{X}(Y) = \begin{cases} 1, & \text{if } L(y) > t. \\ 0, & \text{if } L(y) < t. \\ Bern(\Gamma), & \text{if } L(y) = t. \end{cases} \tag{1}$$

where L(y) is the likelihood function:

$$L(y) = \frac{P(y|X = 1)}{P(y|X = 0}$$

and t is a threshold chosen such that the $PFA = \alpha$. The following program takes as input a null hypothesis and an alternative hypothesis in addition to some upper bound on the PFA $\alpha$, and returns the optimal decision rule