1 Optimal Decision Rule (Binary)

Consider the following binary hypothesis testing problem. X follows one of two hypothesis: null hypothesis(0) or alternate hypothesis(1). We are given some observation y $\in$ Y, and asked to construct a decision rule — A function

$$\hat{X} : Y \to \{0, 1\}$$

that maximizes the probability of correct detection(PCD or $P(\hat{X} = 1|X = 1)$) given the below constraint $P(\hat{X} = 1|X = 0) \leq \alpha$ (i.e the Probability of False alarm(PFA) is upper bounded by some alpha

By Neyman-Pearson, the optimal decision rule is one of the following form:

$$\hat{X}(Y) = \begin{cases} 1, & \text{if } L(y) > t. \\ 0, & \text{if } L(y) < t. \\ Bern(\Gamma), & \text{if } L(y) = t. \end{cases} \tag{1}$$

where L(y) is the likelihood function:

$$L(y) = \frac{P(y|X = 1)}{P(y|X = 0)}$$

and t is a threshold chosen such that the $PFA = \alpha$. The following program takes as input a null hypothesis and an alternative hypothesis in addition to some upper bound on the PFA α, and returns the optimal decision rule. Moreover, the program will output the ROC curve in which the points correspond to different labeled threshold points. I'm currently working on implementing the Continuous Version.