

BORGATTI AND EVERETT (1992): NOTIONS OF POSITION IN SOCIAL NETWORK ANALYSIS

Main Points to Consider

- There are two definitions of position:
 - Structural Equivalence: two nodes share exactly the same set of relationships to exactly the same set of nodes;
 - Automorphic Equivalence: two nodes share the same set of relationships to subsets of nodes that are isomorphic, but are not the same set of nodes.
- These definitions of equivalence differ in nature and should be applied differently to different theoretical constructs.

INTRODUCTION

There are two distinct notions of position that this paper will describe in detail. There are myriad applications for these constructs, but they have been applied incorrectly in several instances. "In fact, published works frequently define position one way and then proceed to draw conclusions as if another definition had been used."

1. BASIC NOTIONS

Position describes structural correspondence or similarity. Actors who are connected to the network in the same way are said to be in equivalent positions. This approach is different than the proximity or cohesion approach (finding actors who are strongly or closely linked to one another).

The authors posit that the difference between the types of structural equivalence measures is similar to other clearly defined notions of equality and equivalence in other areas of mathematics. Table 1 summarizes the examples provided by the authors.

	Rigor of Equivalence	
	Exactly equal (1 to 1 correspondence)	Mostly alike in a general sense; similar...
Algebra	2 relations contain the same ordered pairs <i>and</i> the pairs are 1-to-1 relations	2 relations contain the same ordered pairs
Geometry	Lengths of sides are equal, angles are equal	Lengths of sides are proportional
Positional Analysis in SNA	Structural Equivalence	Automorphic Equivalence

2. TECHNICAL DEFINITIONS AND NOTATION

- $G\{V, E\}$ – a graph, G , containing nodes V and edges E

- $V(G)$ indicated the vertex (node) set in graph G
- $E(H)$ indicates the edge set in graph H
- $P(a)$ denotes the position of actor a
- $P(\{a, b, \dots\}) = \{P(a) \cup P(b) \cup \dots\}$ = set of distinct positions of a set of nodes
- For directed graphs: $N^i(a)$ is the set of nodes from which actor a receives ties: the in-neighborhood of a
- For directed graphs: $N^o(a)$ is the set of nodes to which a sends ties: the out-neighborhood of a
- $N(a) = (N^i(a), N^o(a))$: the neighborhood of point a
- In an undirected graph, the neighborhood of point a is the set of nodes directly connected to node a
- A structural or graph-theoretic attribute makes no mention of a named point. So the property of being no more than 3 links away from any node is a structural property, but the property of being no more than 3 links away from Bill (or any particular named node) is not.

3. POSITION AS STRUCTURAL EQUIVALENCE

Burt (1976) defined a structurally equivalent set of nodes as a set of nodes connected by the same relation to exactly the same people. Using the above notation, this definition is stated as:

If $G = \langle V, E \rangle$ is a graph and $a, b \in V$, then $P(a) = P(b)$ iff $N(a) = N(b)$

In other words, these nodes have identical ego networks. However, this definition means that two connected nodes cannot occupy the same position in networks without reflexive ties. The following removes nodes a and b from the description and thereby eradicates the problem, but fails when a single directed arc connects a and b :

$$P(a) = P(b) \text{ iff } N(a) - \{a, b\} = N(b) - \{a, b\}$$

This definition also fails to distinguish nodes with reflexive loops from those without.

Nodes that are structurally equivalent have all of the same graph theoretic properties (centrality, prestige, eccentricity, degree, etc.), but the converse is not true. The notion of structural equivalence is local and we can calculate structural measures on incomplete data so long as the missing data is not in the actors of interest's ego network. (In graph theoretic measures, any missing data causes the metric to be miscalculated.)

Nodes cannot be structurally equivalent if they are more than 2 links apart. So, structurally equivalent sets of actors form a 2-clique.

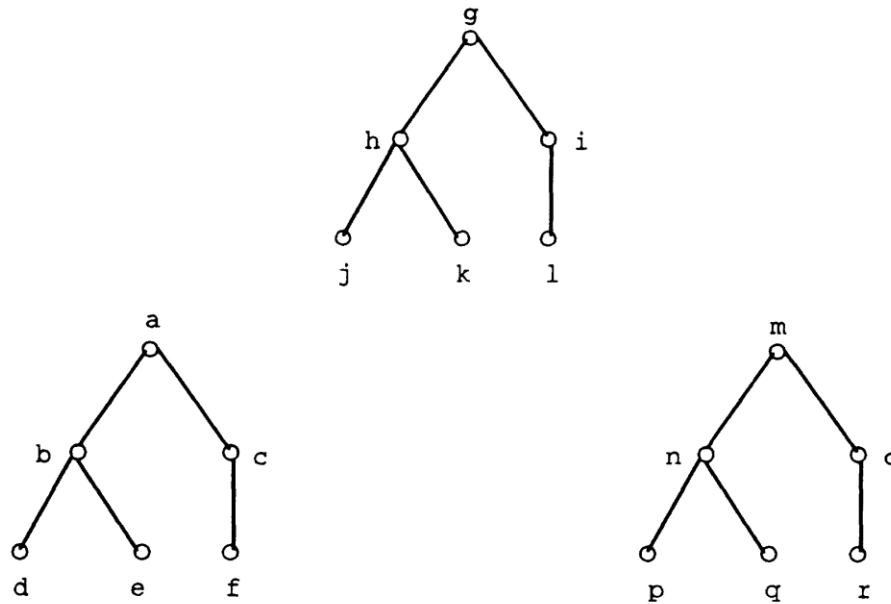


FIGURE 4. Disconnected graph.

Figure 4 illustrates how structural equivalence is about network location and illustrates the difference between proximity and similarity. Points a, g, and m are similar in their patterns of connection, but they are not proximate: not structurally equivalent. Points a, b, c, d, e, and f are all proximate, but they are not similar: not structurally equivalent. Points d and e are both proximate and similar: they are structurally equivalent.

4. POSITION AS STRUCTURAL ISOMORPHISM

Automorphic equivalence and structural isomorphism are interchangeable terms.

An isomorphism is a one-to-one mapping of the objects in one set onto another such that the relations between all objects are preserved. Isomorphic graphs are identical with respect to all graph theoretic attributes, yet the set of actors contained in each graph are different. Another way to think about isomorphism is that without naming the nodes and edges, two isomorphic graphs would be identical.

An isomorphism of a structure with itself is known as automorphism. It is often the case that a graph has an automorphism that is not the identity, for instance, rotation about the horizontal or vertical axis of a graph may be an automorphism.

Two nodes a and b in graph G are structurally isomorphic or automorphically equivalent if there exists an isomorphism π such that $\pi(a) = b$. In other words, if a and b are isomorphic, then the alters to which a is connected all occupy the same positions as the set of alters to which b is connected. Sets of isomorphic actors are called *orbits*. All isomorphic nodes are exactly identical on structural attributes like centrality, degree, prestige, eccentricity, number of cliques in which they participate, etc. Unlike structurally equivalent nodes, the converse is true: nodes with identical structural properties are necessarily isomorphic. Isomorphic nodes are independent of proximity: they may be very near or very distant from each other.

The concepts of structural equivalence and isomorphism are conceptually and applicably different from each other. One is not necessarily an approximation of the other. In the structural equivalence approach, we consider a labeled graph and examine *to whom* an actor is connected. This might be considered the surface structure of the graph. In the automorphic equivalence approach, we think of

removing the labels and identifying similar actors by examining the structure of their relations to other actors. Here we are concerned about the deep structure of the graph.

5. THE USE OF POSITION IN STRUCTURAL THEORY

Here the authors provide three examples of how the notion of position has been used in structural theory and discuss which type of position is best suited for each specific application: (1) Status/Role Systems, (2) Power in Experimental Exchange Networks, and (3) Social Homogeneity.

STATUS/ROLE SYSTEMS

Network equivalences are seen here as a formalization of status, or role, in society. So a nurse is defined as a nurse by the characteristics of the relations that he or she holds with patients, doctors, other nurses, etc. Figure 9 illustrates this concept. Figure 9(a) shows the order-giving relations among employees in a doctor's office. Figure 9(b) shows those relations collapsed into structurally similar actors (under both structural equivalence and isomorphism). Figure 9(a) describes the manifest society, while 9(b) describes the underlying social structure.

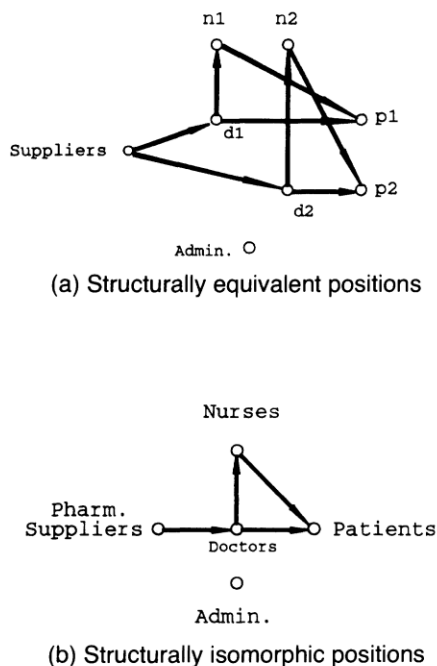
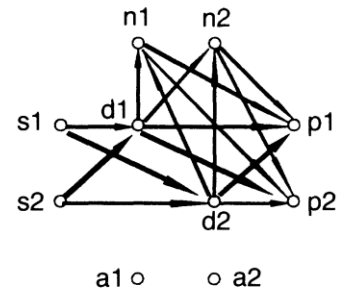
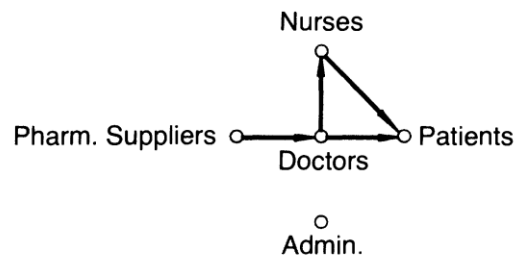


FIGURE 11. Relations among positions in a doctor's office.



(a) The "gives information about medicine" relation



(b) Relations among positions

FIGURE 9. Relations in a doctor's office.

Another example, using a slightly different doctor's office (Figure 10), will illustrate the difference between structural equivalence and structural isomorphism. The main difference here is that each doctor works with one distinct nurse and a distinct set of patients. Figure 11(a) shows these relations collapsed under structural equivalence. We see that we no longer have one node indicating the role of doctor or nurse or patient. In contrast, the graphs

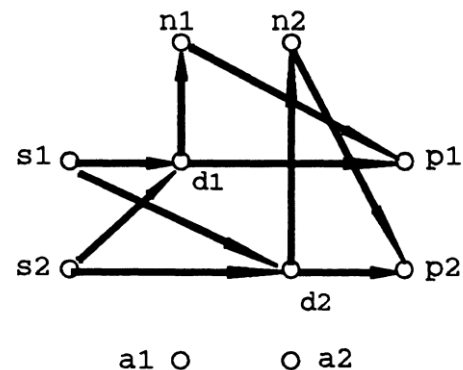


FIGURE 10. The "gives information about medicine" relation.

have the same structural isomorphism (Figure 11(b) yields the same social structure as Figure 9(b)).

We can also see from this example that structural isomorphism yields a higher level of abstraction concerning social roles than structural equivalence. General equivalence yields an even higher level of abstraction.

Borgatti and Everett go on to provide examples of published articles (Caldiera and Burt) where authors describe structural isomorphism (organizations that hold similar roles in a network) but call it structural equivalence. This is a problem not because the description of social roles was misleading or incorrect, but because a researcher attempting to verify hypotheses about the types of structure described will fail if a structural equivalence model is employed. DiMaggio makes the same mistake in his 1986 article – assigned in this week's reading! See pages 20 through 23 for more in-depth discussions of the misuses of the term structural equivalence.

POWER IN EXPERIMENTAL EXCHANGE NETWORKS

Cook and Markovsky have produced well-known work in this area, and both use the same notion of position. They explore power, which is a function of the extent to which the actors rely on each other for the provision of some unnamed good. Dependency is a function of supply and demand. Demand is internal and non-structural; supply depends on the actor's position within the structure of the network (one must be connected to those who possess demand).

Power in this context is best described under structural isomorphism: isomorphic actors have the same structural properties and thereby have the same power. Borgatti and Everett cite examples where Cook describes structural isomorphism but uses the phrase "structural equivalence" to label it. See pages 24-26 for this discussion. In this case, structural isomorphism is the appropriate model for the study because Cook's hypothesis does not rely on distance: actors that are far apart can have the same amount of power. So, for describing power exchange networks, one would use structural isomorphism because the achievement of power is unrelated to proximity. For modeling infectious processes (gossip heard or diseases contracted), however, one would use a structural equivalence model because proximity is pertinent to the outcomes.

SOCIAL HOMOGENEITY

We expect actors belonging to cohesive subgroups to exhibit similar outcomes on questions of some substance. A famous example is the Coleman, Katz, and Menzel (1953) study of adoption of the use of a new drug among physicians. The question then becomes: which approach is better for studying social homogeneity – structural equivalence or social cohesion? Since structurally equivalent actors are also members of the same cohesive subgroups, it is difficult to tell. We cannot parcel out the effects of cohesion on structurally equivalent actors – the proximity is conceptual component of structural equivalence. They are too similar to be considered separately. If we need to know about non-cohesive determinants of social homogeneity, we have to use structural isomorphism. If cohesive subsets of actors consistently exhibit greater homogeneity on some variable than isomorphic subsets, then cohesion may be the more effective predictor of homogeneity in diffusion models. If structural similarity and cohesion are important, then structural equivalence may be the better predictor.

I do not think I can do justice to this next paragraph by summarizing it, so here it is:

"...we must be careful when using structural equivalence as an independent variable. Because it necessarily confounds structural similarity with proximity, it is conceptually inelegant. Moreover, it prevents evaluation of the relative contributions of structural similarity and cohesion to predicting the outcome variable. A cleaner and more useful alternative is to use both cohesion and structural isomorphism as theoretically orthogonal independent variables, thereby separating the components of structural equivalence without losing the benefits of either."

BREIGER (1981): STRUCTURE OF ECONOMIC INTERDEPENDENCE AMONG NATIONS

Main Points to Consider

- The traditional viewpoint of the world economic system: Individual states are categorized based on their interdependent relations within the system:
 - Core states soak up the surplus;
 - Periphery states produce labor intensive goods;
 - Semi-periphery states are both exploiters and exploited.
- Can a procedure for categorizing states among their myriad import and export arrangements be found? The author posits yes, and tests his hypothesis against empirical data.
- The core/periphery structure of the system is found to have multiple competing centers rather than just one center as implied above.

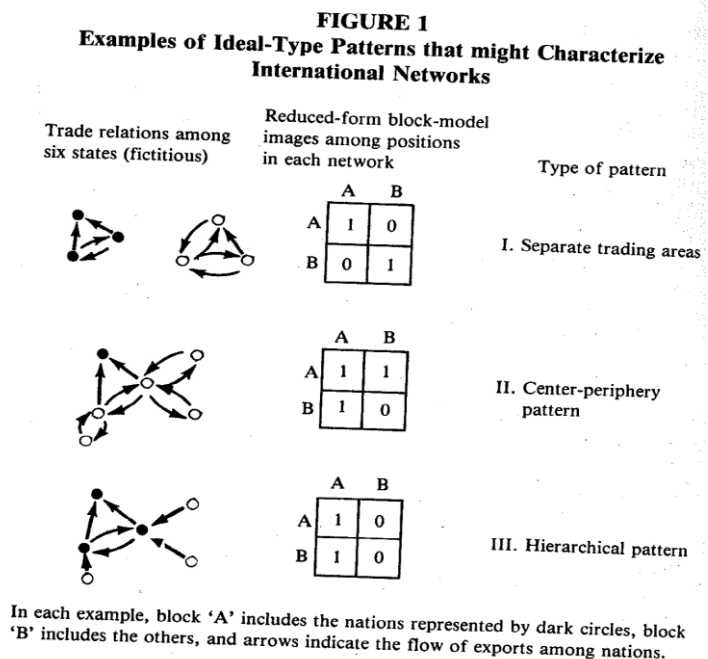
CONCEPTUAL FRAMEWORK

Three concepts from network analysis have theoretical and methodological importance in this paper: (1) structural similarity, (2) pattern, and (3) interlock. The author points out that trade arrangements among nations lends itself to analysis using matrices that show trade relations between pairs of nations over a certain commodity over a certain time period. Typically, analysis concentrates on the marginals (row and column totals) and therefore on the aggregate imports and exports. "In contrast, **a block model approach to international trade assigns states to positions according to the (1) structural similarity of the nations' imports and exports to all other states**, across various types of economic exchange, rather than on the basis of definitional aggregation."

Using this positional analysis, another goal is to discover distinctive patterns that the blocks induce on the original network data. Figure 1 (sorry that it's tilted) shows some theoretical arrangements of the blocks. **Block model analysis essentially compares the blocks that arise from the empirical data to the theorized (2) pattern of blocks, such as those shown in Figure 1.**

A third feature of block model analysis concerns the (3) interlock of these patterns across different relations (in this case, trade of commodities, agricultural goods, energy resources, etc.).

DATA



23 OECD countries, plus Israel, which together accounted for 70 percent of world trade totals for 1972, the year from which the data was taken. The trade commodities reviewed were agricultural products, raw materials, manufactured goods, and energy resources. Trade across only these four commodity classes among these 24 nations accounted for 22% of all world trade in 1972. The author notes that the year 1972 was chosen as a starting point for future study because it falls just prior to huge changes in world petroleum prices. He also notes that there is a lot of error contained in the data that he is using here and notes that it is the best available data of its type.

RESULTS: INSIDE THE CORE

FIGURE 2
Trade in Manufactured Goods among 24 Nations

1. Austral.	-----I-----I--
2. Austria	-----I-----II--
3. Belg./Lux.	-----II-----I-I--
4. Canada	-----II-----II--
5. Denmark	-----I-----I--
6. Finland	-----I-----I-I--
7. France	--I---I---I-I---I-III-
8. Germany	-IIIII----I-I-I-III
9. Greece	-----I-----I--
10. Iceland	-----I-----I--
11. Ireland	-----I-----I--
12. Israel	-----I-----I--
13. Italy	--I---II---I---IIII
14. Japan	I--I--I-----II--
15. Netherl.	--I---II---I-----II--
16. New Zea.	-----I-----I-II-
17. Norway	-----I-----I-II-
18. Portugal	-----I-----I-II-
19. Spain	-----II-----I--
20. Sweden	-----IIII-----I--II-
21. Switzer.	--I---I-----I--
22. UK	I-III-III-III-I-III-I-
23. USA	I-II-III-III-I--I--
24. Yugosla.	-----I-----I--

Rows and columns are ordered arbitrarily but identically (an alphabetical listing by country). Only the highest fifth of the interior cell values are coded as present ('I').

FIGURE 3
A Partition of the Rows and Columns of Figure 2

8. Germany	-IIIIIIIIIIIIIIII-I--
22. UK	I-I-III-III-III-III-III
23. USA	II-III-III-III-III-I--
2. Austria	II-III-III-III-III-I--
3. Belg./Lux.	III-III-III-III-III-I--
7. France	III-I-III-III-III-I--
13. Italy	III-II-I-II-III-I--
15. Netherl.	III-III-III-III-III-I--
19. Spain	I-I-I-I-I-I-I-I-I-I-I
21. Switzer.	II-I-I-I-I-I-I-I-I-I-I
24. Yugosla.	-----I-----I--
5. Denmark	-----I-----I--
6. Finland	II-III-III-III-III-I--
17. Norway	III-III-III-III-III-I--
20. Sweden	III-I-I-I-I-I-I-I-I-I
1. Austral.	-I-III-III-III-III-I--
4. Canada	-II-III-III-III-III-I--
11. Ireland	-I-III-III-III-III-I--
12. Israel	-I-III-III-III-III-I--
14. Japan	III-III-III-III-III-I--
9. Greece	-----I-----I--
10. Iceland	-----I-----I--
16. New Zea.	-----I-----I--
18. Portugal	-----I-----I--

Figure 2 shows the dichotomized¹ trade relationships among the 24 countries. Figure 2 and Figure 3 show the same data. The relation studied here is trade of manufactured commodities. Figure 2 has the countries order alphabetically; Figure 3 has the countries ordered such that countries with similar patterns of trade are grouped together and the blocks are partitioned. A strong center-periphery pattern emerges from this data (like the one shown in Figure 1). The members of the first block trade extensively with almost all other blocks. The members of the second block trade extensively with members of the first three blocks (including its own). The members of the third block trade extensively with blocks one and two, but not its own, hence it is peripheral. Blocks 4 and 5 are also peripheral. The last block seems to be outside of this system of trade. Similar structure was found across other relations (trade of other goods).

The author notes three things about the results thus far. He has identified a core-periphery structure. The countries included here are considered the 'core' of the world economy in 1972 by standard econometric measures. We have now found that this 'core' exhibits core/periphery structure itself. The structure exhibits a fair amount of symmetry and the author would like to detect if this structure exists in asymmetric data. Also, the present analysis is performed on binary data and thus does not deal with differences in trade volume among the nations listed.

RESULTS: IDENTIFYING BLOCKS OF NATIONS

The author replicated a routine for generating an index of trade intensity: he subtracted row and column averages from each matrix leaving entries that are greater than 0 (trade interactions between nations that are statistically positive) or less than 0 (and negative statistically). He is attempting to use these values to find a single partition of countries that distinguishes patterns across all of the relations (trade arrangements).

¹ An 'I' was inserted into the cells containing the top 1/5 of valued data.

The CONCOR (hierarchical clustering) algorithm was applied which resulted in identification of 4 clusters of nations. A nation-by nation correlation matrix was computed across all three trade tables (agricultural, manufactured goods, and raw materials trade), and the eigenstructure of the matrix was examined (a lot like factor, or principal components, analysis). The results of this analysis along with the clusters from the CONCOR algorithm are represented in Figure 5. The 2 dimensions account for 50% and 15% of the total variance, respectively. The first (horizontal) dimension differentiates on economic power, while the second seems to differentiate along cultural lines. When these partitions were compared to the 4th trade matrix, energy, the author found a statistically significant structure of trade.

RESULTS: STRUCTURE ACROSS MULTIPLE NETWORKS

Figure 6 shows the mean trade within each block shown in Figure 5 and across all pairs of blocks for each commodity traded. The diagrams off to the side show a reduced form block model diagram representing the structure of the positive interactions. Some interesting findings emerge using this method that were not observable when considering total trade. First, notice the segregated structure: The US block and the EEC block tend not to trade directly nor through a third party intermediary. We can see two competing cores in these two blocks. Second, the EEC block tends to trade internally more so than externally, which is probably a result of an earlier policy. Third, the UK block seems to serve as an intermediary for the 'Others' to trade with OECD countries included in the common market. An interesting question is what will develop of this relationship over time. Finally, this analysis shows that the traditional notion that flow of manufactured goods in one direction is reciprocated by flow of some other commodity in the other. We can see that the pervasive flow of manufactured goods among the blocks, so we must look to trade arrangements among other commodities to understand the network structure in detail.

FIGURE 5
A Plot of the First Two Eigenvectors of a Correlation Matrix Described in the Text.
The 4-block Partition from Figure 4 Has Been Imposed

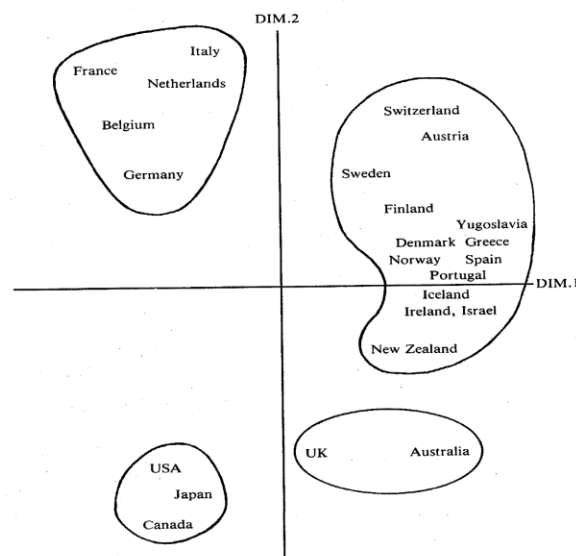


FIGURE 6
Mean Trade Within and Between the Blocks of Figure 4, and the Resulting Block-Model Images

	US	EEC	UK	Others		
I. Agricultural Products						
US	<u>296,477</u>	-37,509	-132	-28,939	US	EEC
EEC	-96,615	<u>240,867</u>	-60,740	-39,438	UK	
UK	<u>126,984</u>	-77,308	<u>106,526</u>	-7,210	Others	
Others	-25,989	-49,738	<u>14,112</u>	<u>22,956</u>		
II. Raw Materials						
US	<u>658,930</u>	-24,800	-20,996	-82,276	US	EEC
EEC	-94,758	<u>71,591</u>	-19,408	2,585	UK	
UK	<u>96,487</u>	-10,254	-5,296	-16,635	Others	
Others	-74,139	-13,675	<u>11,809</u>	<u>20,552</u>		
III. Manufactures						
US	<u>969,973</u>	-162,621	46,239	-87,094	US	EEC
EEC	-121,187	<u>453,506</u>	-116,496	-86,962	UK	
UK	<u>11,731</u>	-70,780	<u>31,513</u>	20,514	Others	
Others	-96,962	-84,614	<u>29,446</u>	<u>50,390</u>		
IV. Energy Resources						
US	<u>174,065</u>	-21,227	-18,958	-14,577	US	EEC
EEC	-26,853	<u>39,845</u>	17,028	-8,063	UK	
UK	-19,196	-6,544	-13,389	7,407	Others	
Others	-12,534	-5,901	-1,062	<u>5,325</u>		

WASSERMAN AND FAUST (1994): SOCIAL NETWORK ANALYSIS;

CHAPTER 10

Main Points to Consider

- Blockmodels are hypotheses about the structure of relations in a social network. They summarize features of an entire network in a multi-relational system.
- A discussion of rules for constructing image matrices to represent a blockmodel is presented.
- Emphasis is placed on how to interpret the results of a positional analysis when the results are presented as a blockmodel.

DEFINITION (AND SOME NOTATION)

A blockmodel is a model, or a hypothesis, about a multirelational network. It contains two things:

- (1) a partition of actors in the network into discrete subsets called positions, and
 - (2) for each pair of positions a statement of the presence or absence of a tie within or between the positions on each of the relations.
- A blockmodel is a partition of N actors onto B positions and onto mapping $\phi(i) = B_k$ if actor i is in B_k . The blockmodel also specifies the ties between and within the B positions.
 - b_{klr} indicates the presence or absence of a tie from position B_k to B_l on relation X_r . A 1 indicates a tie, 0 indicates the absence of a tie.
 - An image matrix, \mathbf{B} , is the $B \times B \times R$ array that indicates the presence or absence of ties between the blocks. Each layer of \mathbf{B} represents a hypothesized ties between and within positions on a specified relation. Each of these entries is called a block.

BUILDING BLOCKS

A block containing a 1 is called a 1-block, or a bond; a block containing a 0 is called a 0-block. These positions contain all actors that are equivalent or similar in their relations to other actors, thus a blockmodel is stated at the level of positions, and not of the individual actors.

If networks contained subsets that were perfectly structurally equivalent, then 1-blocks would be completely full of 1's and 0-blocks full of 0's, and defining the two would be simple. Defining 1-blocks and 0-blocks is not straightforward in practice. We will discuss several criteria for defining 1-blocks and 0-blocks. But first, more notation.

The values of the blocks depend on the values in the sociomatrix (which has been permuted so that actors with equivalent or similar structure are adjacent). It is common to use density of ties to determine if b_{klr} is a 1-block or a 0-block. Let g_k be the number of actors in position B_k and g_l be the number of actors in position B_l . Then the density of ties between two distinct positions is:

$$\Delta_{klr} = \frac{\sum_{i \in B_k} \sum_{j \in B_l} x_{ijr}}{g_k g_l}$$

and the density of ties within a position is

$$\Delta_{kkr} = \frac{\sum_{i \in B_k} \sum_{j \in B_k} x_{ijr}}{g_k(g_k - 1)} \text{ for } i \neq j.$$

Each section which follows presents methods for defining 1-blocks and 0-blocks.

PERFECT FIT (FAT FIT): ALL 0'S OR ALL 1'S

This criterion occurs if all actors in each position are structurally equivalent and is mostly useful for providing a baseline for assessing the goodness of fit of a blockmodel (chapter 16).

$$b_{klr} = \begin{cases} 0 & \text{if } x_{ijr} = 0 \text{ for all } i \in B_k, j \in B_l \\ 1 & \text{if } x_{ijr} = 1 \text{ for all } i \in B_k, j \in B_l \end{cases}$$

ZEROBLOCK (LEAN FIT): ALL 0'S, OTHERWISE 1

A tie between two positions is given a 0 if and only if there are no ties from actors in the rows to actors in the columns:

$$b_{klr} = \begin{cases} 0 & \text{if } x_{ijr} = 0 \text{ for all } i \in B_k, j \in B_l \\ 1 & \text{otherwise} \end{cases}$$

0-blocks may be structurally important because a single observed '1' in a position may be important. Consider military action between nations: the density of 1's is likely to be small, so each one is extremely important.

ONEBLOCK CRITERION: ALL 1'S, OTHERWISE 0

A tie between two positions, or within a single position, is given a 1 if the corresponding submatrix of the sociomatrix is completely filled with 1's. Otherwise, it is a 0-block. In practice, 1-blocks are quite rare.

$$b_{klr} = \begin{cases} 1 & \text{if } x_{ijr} = 1 \text{ for all } i \in B_k, j \in B_l \\ 0 & \text{otherwise} \end{cases}$$

α DENSITY CRITERION: IT DEPENDS

Full and complete 1-blocks or 0-blocks are very rare. The researcher can decide on a threshold density value such that the observed block value is coded according to its density in relation to the threshold, .

$$b_{klr} = \begin{cases} 0 & \text{if } \Delta_{klr} < \alpha \\ 1 & \text{if } \Delta_{klr} \geq \alpha \end{cases}$$

Some options for choosing the Δ threshold, α , that exist in the literature:

- α = the overall density computed across all relations (grand density)
- α = the density computed at each relation ($\alpha_r = \Delta_r$, so there are R separate α 's)
- α = the mean value in each row

We can view each of these options as different points on a density criterion continuum. Based on the density of the corresponding submatrix of the sociomatrix, the Zeroblock approach sets the least stringent criteria for assigning a 1 to a block, the α density method falls somewhere in the middle, and the 1-block approach is the most stringent. In the figure shown on the next page, ϵ represents the density criterion of the corresponding submatrix of the sociomatrix.

Zeroblock

α density

Oneblock

$\epsilon = \text{some really small number}$

$\epsilon = \alpha$

$\epsilon = 1$

AN EXAMPLE: KRACKHARDT'S HIGH-TECH MANAGERS

Steps in a blockmodel analysis:

1. Partition the actors in the sociomatrix into subsets such that the actors in each subset are approximately structurally equivalent.
2. Describe the ties between and within the positions.

The chapter presents the advice and friendship relations from Krackhardt's High-tech managers. The authors used the complete link clustering of Pearson product-moment correlations and performed the hierarchical clustering routine to yield three distinct positions. The resulting density tables are as follows:

Advice			
	B ₁	B ₂	B ₃
B ₁	0.429	0.714	0.81
B ₂	0.286	0.661	0.562
B ₃	0	0.292	0.133

Friendship			
	B ₁	B ₂	B ₃
B ₁	0.286	0.071	0.333
B ₂	0.071	0.196	0.229
B ₃	0.333	0.417	0.4

Reviewing the data in the density tables will inform the choice of density criterion for coding the blocks. The authors decided against the perfect fit (not feasible), 1-block (results would be all 0's) and 0-block (results would be all 1's) options. They decided to use the α density criterion with $\alpha_r = \Delta_r$. For the advice relation, $\alpha = 0.452$ and for the friendship relation $\alpha = 0.243$. The resulting image matrices are as follows:

Advice			
	0	1	1
	0	1	1
	0	0	0

Friendship			
	1	0	1
	0	0	0
	1	1	1

These matrices can be represented in a reduced graph where each node represents a block, or position. Note that the image matrices are quite different.

AN EXAMPLE: COUNTRIES TRADE NETWORK

There are 5 total relations in this data: 4 trade networks where the row countries export to the column countries, and 1 diplomatic relation matrix where the row countries have an embassy in the column country. For this analysis, the authors used three relations: manufactured goods, raw materials, and diplomatic ties.

The positional analysis (Pearson product moment and hierarchical clustering) yielded 6 positions. Although the density matrices contained some 1's and some 0's, it did not make sense again in this instance to use either the 1-block or 0-block approach. The resulting image matrices would have been

either too sparse or too dense, respectively. Again, the α criterion was employed, choosing the density of each relation as the value for α . **The collection of image matrices along with the assignment of countries to positions (not included here) constitutes a blockmodel for these data.**

VALUED RELATIONS

The ties within and between positions in the image matrix may be valued.

MAXIMUM VALUE CRITERION (ANALOGOUS TO ZEROBLOCK)

A block in which all the values in the corresponding submatrix of the sociomatrix contains only small values. Define ϵ as the highest acceptable value for a zeroblock, then:

$$b_{klr} = \begin{cases} 0 & \text{if } x_{ijr} \leq \epsilon \text{ for all } i \in \mathcal{B}_k, j \in \mathcal{B}_l \\ 1 & \text{otherwise} \end{cases}$$

MEAN VALUE CRITERION (ANALOGOUS TO DENSITY CRITERION)

When the relation is dichotomous, the mean is equal to the density. The mean is:

$$\bar{x}_r = \frac{\sum_{i=1}^g \sum_{j=1}^g x_{ijr}}{g(g-1)} \text{ for } i \neq j$$

Then the assignment rule is:

$$b_{klr} = \begin{cases} 0 & \text{if } \bar{x}_{klr} < \bar{x}_r \\ 1 & \text{if } \bar{x}_{klr} \geq \bar{x}_r \end{cases}$$

where \bar{x}_{klr} is the the value of the tie from actors in position B_k to actors in position B_l on relation X_r .

We could also choose to assign the mean value from within each block as the value assigned to each block. This is analogous to constructing a density table for dichotomous relations.

INTERPRETATION

The authors present three different ways to interpret a blockmodel:

1. Validation of a blockmodel using actor attributes;
2. Descriptions of individual positions;
3. Descriptions of the overall blockmodel.

ACTOR ATTRIBUTES

External validation of a blockmodel can occur if there is some systematic difference or similarity between the positions in terms of the characteristics of their members. For example, economic models find validation through comparisons of member nations' growth in GNP per capita; models of scientific communities have used date of degree conferred on network members, number of citations of network members by others, or amount of grant money received by network members to validate models. More examples are provided on page 409. In these cases, be aware of endogeneity: the growth in GNP may be affected by a nation's position within the network; professors who hang out together may earn similar grant monies or have their works cited by others because of the nature of their shared projects.

In Krackhardt's high tech managers, comparing attributes to positions reveals that those employees with least tenure tend to go to managers with longer tenures for advice. In the countries trade network, positions with the lowest growth rates in population have the highest average secondary school

enrollment ratio and the highest energy consumption. Positions with the highest annual growth rates in population have the lowest secondary school enrollment ratio and the lowest energy consumption.

DESCRIBING INDIVIDUAL POSITIONS

This involves describing how the positions send and receive ties. Recall from node level analysis:

- Isolates have no ties to other actors (positions);
- Senders only send ties to other actors (positions);
- Receivers only receive ties from other actors (positions);
- Carriers or ordinary points both send and receive ties from other actors (positions).

I have included the word positions here because these labels can be used in describing the relations among positions, along with some additional information.

- **Isolate positions** neither receive nor give many ties to or from other positions.
- **Sycophants** have more ties to members of other positions than to themselves and do not receive many ties.
- **Brokers** both receive ties and send ties to members of other positions.
- **The Primary Position** receives ties from members of other positions and from its own position.

Relative size of a position within the entire network is an important consideration here. We would expect relatively large positions to contain many within-block ties, and we could expect small positions to have few within-block ties. For more detailed information on the typologies of positions based on within-group and external ties, see Burt (1976) and Marsden (1989).

We can calculate an expected value of ties within a position: it is the ratio of maximum possible ties within a position $[g_k(g_k-1)]$ to maximum possible ties from within the position to all other actors outside of the position $[g_k(g-1)]$. The g_k 's cancel, and we have $(g_k-1)/(g-1)$: simply the ratio of the number of members of the position less one to the number of members of the network, less one. We can use this figure as a baseline to determine the typology of each position in an image matrix.

Proportion of ties within Position	Proportion of ties received by position	
	Close to 0	>0
$\geq \frac{g_k - 1}{g - 1}$	Isolate	Primary
$\leq \frac{g_k - 1}{g - 1}$	Sycophant	Broker

IMAGE MATRICES (DESCRIPTIONS OF THE OVERALL BLOCKMODEL)

Image Matrices for Two-position Blockmodels

A $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Null

B $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ One reflexive arc: a single cohesive subgroup and an isolate position

C $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ One arc between positions: deference directed from members of one position to members of another.

D $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Two arcs, reflexive: Pure reflexivity; two cohesive subgroups. Also separate trading areas as in Breiger (1981).

E $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Two arcs, symmetric: Pure symmetry, exogenous relations.

F $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ Two arcs, reflexive and out: One active and one passive position, in terms of choices made.

G $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ Two arcs, reflexive and in: Core periphery or hierarchy

H $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ Three arcs, two between positions: Center/periphery or hanger-on pattern.

I $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ Three arcs, two reflexive: hierarchy.

J $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ A complete image with no differentiation among positions.

Image matrices with more than two positions

A $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ *Cohesive Subgroups.* Mostly intraposition ties. The positions along the diagonal need not be cliques.

B $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ *Center-Periphery.* A core position that is internally cohesive, and one or more other positions with ties to the core but not to each other. This is similar to the structure in Breiger (1981).

C $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ *Centralized.* All ties are pointed toward or away from a single position.

D $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ *Hierarchy.* Unreciprocated ties from each position to the one immediately "above" it.

E $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ *Transitivity.* Could indicate dominance or deference between positions. If a chooses b and b chooses c, then a chooses c.

IMAGE MATRICES FOR MULTIPLE RELATIONS

One possible way to do this is to study pairs of image matrices to see if they exhibit common kinds of multirelational patterns. Do patterns occur together (multiplexity), or do they complement each other (one image matrix is the transpose of another)? Or perhaps we can find structural balance, as in when a relation of positive affect has ties within positions while a relation of negative affect has ties between positions.

Chapter 9: Structural Equivalence

9.1.1 Social Roles and Positions

Position: a collection of individuals who are similarly embedded in networks of relations, while between positions.

Note:

- 1) Position is based on the similarity of ties among subsets of actors
- 2) Actors occupying the same position need not be in direct, or even indirect, contact with one another

Role: the patterns of relations which obtain between actors or between positions.

Note:

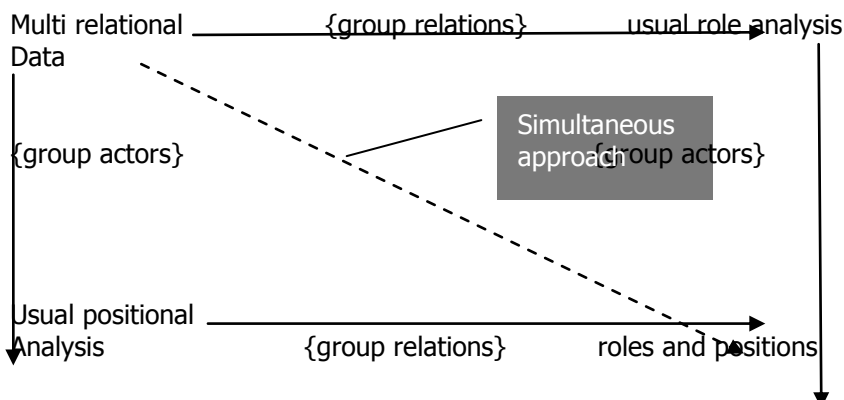
- 1) Role is defined in terms of collections of relations and the associations among relations
- 2) The notion of social role is conceptually, theoretically, and formally dependent on the notion of social position.
- 3) Network role refers to associations among relations that link social positions.
- 4) Social roles are usually based on multiple relations and the combinations of these relations. Pp349

Other approach to conceptualize a position: Burt (1976) – a position is a collection of ties in which an actor is involved. (pp350)

The assumption of positional analysis: The role structure of the group and positions of individuals in the group are apparent in the measured relations present in a set of network data. Pp350

The tasks of role and positional analysis: to provide explicit definitions of important social concepts and to identify and describe roles and positions in social networks.

9.1.2 An overview of positional and role analysis



Step by step, according to the diagram above:

Left path from top to bottom: locate subsets of actors who are similar across the collection of relations

Role analysis: either along the top or along the bottom of the diagram. A role analysis is concerned with the associations among relations. Top line: describing "global" roles; bottom line: describing "individual" or "local " roles. Pp353

Horizontal arrow on the bottom: describe the system of relations between the positions

Horizontal path along the top: describe the association among the relations.

Top to bottom along the right side: grouping actors into equivalence classes based on the description of the role system resulting from the previous step.

Note:

- 1) A Critical decision needed to be made is how to measure similarity among actors. Similarity: here will be defined in terms of the equivalence of actors with respect to some formal mathematical property.
- 2) Simultaneous model: from top left to bottom right. This needs to define equivalence classes of actors and relational systems at the same time. Pp354

9.1.3 A brief History

9.2 Definition of Structural Equivalence

9.2.1 Definition

x_{ijr} : Indicating the presence or absence of a tie from actor i to actor j on relation \mathcal{R}_r

Actor i and actor j are structurally equivalent if:

$x_{ikr} = x_{jkr}$ and $x_{kir} = x_{kjr}$ for $k= 1,2,...,g$, and $r=1,2,..., R$.

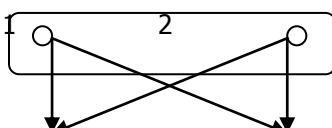
K: actors $k \neq i, j$ R: relations

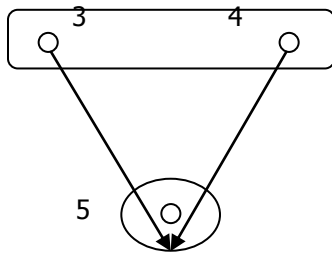
9.2.2 A example

Sociomatrix

	1	2	3	4	5
1	-	0	1	1	0
2	0	-	1	1	0
3	0	0	-	0	1
4	0	0	0	-	1
5	0	0	0	0	-

Directed graph:





1, 2 are structural equivalence, and 3, 4 are structural equivalence

9.2.3 Some issues in defining structural equivalence

Things needed to be considered:

- a) Dichotomous or valued
- b) Directional or nondirectional
- c) A relation on which self-ties (the diagonal elements of the sociomatrix) are substantively meaningful

Multiple Relations.

For two actors to be structurally equivalent in a multirelational network, they must have identical ties to and from all other actors, on all relations.

Valued Relations.

Strict definition: For two actors to be structurally equivalent on a valued relation they must have ties with identical values to and from identical other actors.

Actors are approximately structurally equivalent: if they have the same pattern of ties to and from other actors (Burt 1980) pp360.

Example: In friendship network, if the friends with whom i and j interact frequently or infrequently are exactly the same, then i and j are considered approximately structurally equivalent. And the number of frequency and infrequency would not have to be the same for i and j.

Nondirectional relations:

One only needs to consider either the rows or the columns of the symmetric sociomatrix, but not both.

Self-ties and Graph Equivalence:

- a) When a relation is reflexive (i → i for all i) and self-ties are considered substantively meaningful, then diagonal entries in the sociomatrix should be included in calculation of structural equivalence.
- b) Actors i and j are **graph equivalent** if $X_{ij} = X_{jk}$ for all actors, $k = 1, 2, \dots, g$.
- c) If i and j are graph equivalent then both the i → j and the j → i ties must be present. Since i "chooses" j, j must also "choose" i in order for i and j to be graph equivalent.

9.3 Positional Analysis

9.3.1 Simplification of Multirelational Networks

Permute → identify subgroups → simplify

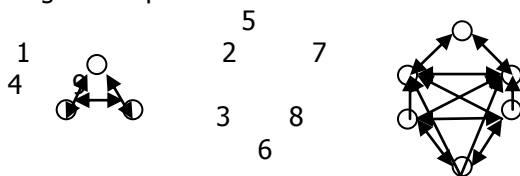
If all actors within each subset are structurally equivalent, then when the rows and columns of the original sociomatrix are permuted so that actors who are assigned to the same equivalence class occupy rows and columns that are adjacent, the submatrices corresponding to the ties between and within positions are filled with either all 0s or all 1's.

See graphs:

a. Original sociomatrix

	1	2	3	4	5	6	7	8	9
1	-	0	0	1	0	0	0	0	1
2	0	-	0	0	1	0	1	0	0
3	0	1	-	0	1	1	1	1	0
4	1	0	0	-	0	0	0	0	1
5	0	1	0	0	-	0	1	0	0
6	0	1	1	0	1	-	1	1	0
7	0	1	0	0	1	0	-	0	0
8	0	1	1	0	1	1	1	-	0
9	1	0	0	1	0	0	0	0	-

b. Original Graph



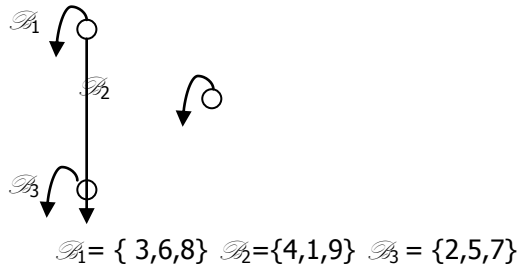
c. Permuted and partitioned sociomatrix

	6	3	8	4	1	9	2	5	7
6	-	1	1	1	0	0	0	1	1
3	1	-	1	1	0	0	0	1	1
8	1	1	-	1	0	0	0	1	1
4	0	0	0	0	-	1	1	0	0
1	0	0	0	0	1	-	1	0	0
9	0	0	0	0	1	1	-	0	0
2	0	0	0	0	0	0	0	-	1
5	0	0	0	0	0	0	0	1	-
7	0	0	0	0	0	0	0	1	-

d. Image matrix

	\mathcal{B}_1	\mathcal{B}_2	\mathcal{B}_3
\mathcal{B}_1	1	0	1
\mathcal{B}_2	0	1	0
\mathcal{B}_3	0	0	1

e. Reduced graph



9.3.2 Tasks in a Positional Analysis

Four steps of conducting positional analysis

1) A formal definition of equivalence:

In all cases the equivalence definition is stated in terms of properties of ties among actors in a network.

2) A measure of the degree to which subsets of actors approach that definition in a given set of network data

(Note: the measure of equivalence in fact measures what it is supposed to measure)

3) A representation of the equivalences

A representation of the assignments of actors to equivalence classes, and a statement of the relationships between and within the classes.

4) An assessment of the adequacy of the representation

Assessment of the adequacy of the representation (goodness-of-fit)

9.4 Measuring Structural Equivalence

9.4.1 Euclidean Distance as a Measure of Structural Equivalence

For actors i and j , this is the distance between rows i and j and columns i and j of the sociomatrix:

$$d_{ij} = \sqrt{\sum_{k=1}^g [(x_{ik} - x_{jk})^2 + (x_{ki} - x_{kj})^2]}$$

for $i \neq k, j \neq k$

Note:

- 1) if equivalent $d_{ij} = 0$
- 2) To the extent that two actors are not structurally equivalent, the Euclidean distance between the will be large.
- 3) All distances are greater than or equal to 0
- 4) The maximum possible value of d_{ij} is $\sqrt{a(g-2)}$

Multiple Relations.

$$d_{ij} = \sqrt{\sum_{r=1}^R \sum_{k=1}^g [(x_{ikr} - x_{jkr})^2 + (x_{kir} - x_{kjr})^2]}$$

9.4.2 Correlation as a measure of structural equivalence

$\bar{x}_{.i}$: the mean of the values in row i of the sociomatrix

$\bar{x}_{i.}$: the mean of the values in column i of the sociomatrix

$$r_{ij} = \frac{\sum (x_{ki} - \bar{x}_{.i})(x_{kj} - \bar{x}_{.j}) + \sum (x_{ik} - \bar{x}_{i.})(x_{jk} - \bar{x}_{j.})}{\sqrt{\sum (x_{ki} - \bar{x}_{.i})^2 + \sum (x_{ik} - \bar{x}_{i.})^2} \sqrt{\sum (x_{kj} - \bar{x}_{.j})^2 + \sum (x_{jk} - \bar{x}_{j.})^2}}$$

Multiple relations.

$$r_{ij} = \frac{\sum_{r=1}^{2R} \sum_{k=1}^g (x_{ikr} - \bar{x}_{i.})(x_{jkr} - \bar{x}_{j.})}{\sqrt{\sum_{r=1}^{2R} \sum_{k=1}^g (x_{ikr} - \bar{x}_{i.})^2} \sqrt{\sum_{r=1}^{2R} \sum_{k=1}^g (x_{jkr} - \bar{x}_{j.})^2}}$$

9.4.3 Some Considerations in Measuring Structural Equivalence

Other measures of structural equivalence

Multiple relations and multiple sociomatrices

Consider ties both to and from actors: one must calculate measures using both the sociomatrix and its transpose

Comparison of some measures of structural equivalence. Pp375

- 1) Euclidean distance reflects a smaller amount of structural equivalence than does a correlation coefficient if the actors differ in the mean and variance of their ties.
- 2) If the researcher wants to measure similarity in pattern, then the correlation coefficient is the preferred measure.
- 3) If one desires a measure of the identity of ties, then Euclidean distance may be preferable.
- 4) In sociometric data collected using rating scales, differences among people in their use of response categories would lead to different results.
- 5) In records of interaction frequencies, differential structural equivalence when equivalence is measured using Euclidean distance.
- 6) One approach to the problem is to standardize relational data to remove differences in means and variances among actors before computing structural equivalence.

9.5 Representation of Network Positions

9.5.1 Partitioning Actors

Partitioning Actors Using CONCOR

- 1) Is based on the convergence of iterated correlations:

Repeated calculation of correlations between rows (or columns) of a matrix (when this matrix contains correlations from the previous calculation) will eventually result in a correlation matrix consisting only of +1's and -1's.

- 2) All correlations between items assigned to the same subset are equal to +1 and all correlations between items in different subsets will be equal to -1.

Finally the matrix can be partitioned and simplified (blocked) to have the following form:

	+1	-1
	-1	+1

Comments: pp380

- 1) CONCOR's procedure always produces a series of bi-partitions, or a binary tree. This form is defined by the procedure, not by the structure of the network.
- 2) The resulting partition from CONCOR often has little resemblance to what are intuitively and formally understood to be social positions in the network.
- 3) The formal properties of the procedure are not well understood.

Partitioning Actors Using Hierarchical Clustering.

Deciding a threshold value α , such that pairs of actors i and j are considered nearly structurally equivalent if $d_{ij} \leq \alpha$ (when using Euclidean distance) or $r_{ij} \geq \alpha$ (when using correlation coefficient)

Complete link hierarchical clustering produces collections of entities in which all pairs are no less similar than the criterion value

Computer programs: STRUCTURE and UCINET

Extension:

- 1) Can be used to multiple relations and/or valued relations.
- 2) The input matrix contains measures of structural equivalence between all pairs of actors; for example, the matrix of Euclidean distances, or the matrix of correlations. Pp384

Some Comments:

Advantages: discrete method that gives a partition of the actors into subsets, the procedure is explicit, the interpretation is clear, and the computer programs are widely available.

Disadvantages: the decision of how many subsets to use is often arbitrary, there are many different hierarchical clustering criteria, and some procedures do not give unique solutions.

Both CONCOR and hierarchical clustering is that a "grouping" or a split that is made at one of the early stages in the analysis cannot be undone at a later stage.

9.5.2 Spatial Representations of Actor Equivalences

Studying equivalences among actors using a continuous (not mutually exclusive and exhaustive subsets like in hierarchical clustering and CONCOR) (or spatial) model.

Multidimensional Scaling (MDS): is a data analytic technique that seeks to represent similarities (or dissimilarities) among a set of entities in low-dimensional space so that entities that are more similar to each other are closer in the space, and entities that are less similar to each other are farther apart in the space.

9.5.3 Ties Between and Within Positions

The task of representing positions in a network has two parts:

- 1) Assigning actors to positions, and
- 2) Describing how the positions relate to each other.

Three ways to represent the ties between and within positions:

- 1) Density table
- 2) Image matrix
- 3) Reduced graph

All starts with permuting the rows and columns of the original sociomatrix, so that actors who are assigned to the same position are adjacent in the permuted sociomatrix.

Density Tables:

A density table is a matrix that has positions rather than individual actors as its rows and columns, and the values in the matrix are the proportion of ties that are present from the actors in the row position to the actors in column position.

To calculate the density ;

See fig. 9.10, 9.11

Density of the submatrix from \mathcal{A}_1 to \mathcal{A}_2 is $15/(4*6) = 0.625$

6: there are 6 actors in \mathcal{A}_1

4: There are 4 actors in \mathcal{A}_2

The possible ties that could be present from members of \mathcal{A}_1 to \mathcal{A}_2 are $4*6 = 24$

15: There are actually 15 ties from members of \mathcal{A}_1 to \mathcal{A}_2

Image Matrices.

An image matrix is a summary of the ties between and within positions, so that each tie is coded as either present or absent between each pair of positions.

Actual network data are seldom composed purely by oneblocks and zeroblocks, we need a guideline for deciding whether a tie exists between positions. Rules for constructing an image matrix from a density table are discussed in Chapter 10.

Example: See figure 9.11 and figure 9.12

The density of the entire sociomatrix is 0.452. If we use 0.452 as cut-off value, then we can get image matrix for the advice relation from Krackhardt's high-tech managers. See fig 9.12

Reduced Graphs.

In a reduced graph positions are represented as nodes and ties between positions in the image matrix define the arcs between nodes.

Still use Krackhardt's high-tech managers data as example, fig 9.1.3 is the reduced graph

Summary:

- 1) structural equivalence is a mathematical property that can only be met by actors who belong to the same population
- 2) structural equivalence and cohesive subgroups: Actors who are structurally equivalent must be close to each other in a graph theoretic sense.

Fig. 9.10 Advice sociaomatrix for Krackhardt's high-tech managers permuted according to positions from hierarchical clustering of correlations.

	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3	\mathcal{A}_4	
			1 2 1	1 1 1 1 1 1 1 2	
		5 9 3 5 4 0	3 9 0 8	1 7 6 2 4 2 1 6 8	1 7
\mathcal{A}_1		5 - 0 0	0 0 1 1	1 1 1 1 1 0 1 1	1 1 1 1 1 1
		9 0 - 0	0 0 0 0	1 1 1 1 1 1 1 1	1 1 1 1 1 1
		3 0 1 -	0 1 1 0	1 1 1 1 1 1 1 1	1 0 1 1 1 1
		15 1 1 1 -	1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1
		4 0 0 0	0 - 1 0	0 1 1 1 1 1 1 0	1 1 1 1 1 0
		20 0 0 0	1 0 - 0	0 0 1 1 1 1 1 1	1 1 1 1 1 0
\mathcal{A}_2		13 1 1 0	0 0 0 -	0 0 1 0 0 0 0 1	1 1 0 0 0 0
		19 1 0 1	1 0 1 0	- 1 1 1 0 0 0 1	1 1 0 0 0 1
		10 1 0 1	1 1 1 1	1 1 - 1 1 1 0 0 0	1 1 1 1 0 0
		18 1 1 1	1 1 1 1	1 1 1 - 1 0 0 0	1 1 1 1 1 1
\mathcal{A}_3		11 0 0 0	0 0 0 0	0 0 0 0 - 0 0 0 0	1 1 0 0 0 1
		17 0 0 0	0 1 0 0	0 0 0 0 0 - 0 0 0 1	1 1 0 0 1 1
		6 0 0 0	0 0 0 0	0 0 0 0 0 0 - 0 0 0 0	0 0 0 1 0 0
		12 0 0 0	0 0 0 0	0 0 0 0 0 0 - 0 0 0 0	0 0 0 1 1 1
		14 0 0 0	0 0 0 0	0 0 0 1 0 0 0 0 - 0	0 0 0 1 1 1
		2 0 0 0	0 0 0 0	0 0 0 0 0 1 0 0 - 0	0 0 0 1 1 1
		1 0 0 0	0 1 0 0	0 0 0 1 0 0 0 0 1	- 1 1 1 0
		16 0 0 0	0 0 0 0	0 0 1 1 0 0 0 0 0	1 - 0 0 0
\mathcal{A}_4		8 0 0 0	0 1 0 0	0 0 1 1 1 0 1 0 0	1 0 0 - 1 1
		21 0 0 1	0 1 1 1	0 0 0 1 0 1 1 1 1	1 0 0 1 - 1
		7 0 0 0	0 0 0 0	0 0 0 1 1 1 1 1 1	1 0 0 0 1 -

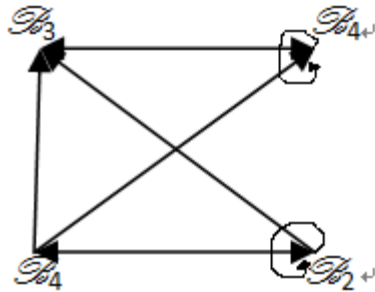
Fig 9.11 is the density table:

	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3	\mathcal{A}_4
\mathcal{A}_1	0.367	0.625	0.944	0.833
\mathcal{A}_2	0.708	0.750	0.528	0.375
\mathcal{A}_3	0.056	0.167	0.194	0.722
\mathcal{A}_4	0.250	0.250	0.667	1.000

Fig 9.12 image matrix

	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3	\mathcal{A}_4
\mathcal{A}_1	0	1	1	1
\mathcal{A}_2	1	1	1	0
\mathcal{A}_3	0	0	0	1
\mathcal{A}_4	0	0	1	1

Fig9.1.3 reduced graph



DiMaggio, Paul. (1986) .Structural analysis of organizational fields: A blockmodel approach. *Research in Organizational Behavior*, 8, 335-70.

Objective: To clarify the utility of one method for the analysis of organizational fields.

Rationale:

- 1) Identifying organizational sources of munificence or turbulence...
- 2) The effects of environmental variables may depend on the position that an organization occupies in its field.
- 3) The shift to the field level permits researchers to examine the effects of interorganizational structure on field-level variables.

Approach to partitioning organizational populations: (pp338-345)

- 1) Naturalistic approaches: on the basis of a priori commonsense descriptors, categorical definitions of organizational form.
- 2) Classification on the basis of attributes: example—using bimodalities in percentage of industry sales to partition industry leaders from other firms.
- 3) Partition on the basis of structural cohesion: Methods based on cohesion used matrices or graphs representing the presence or absence of ties between pairs of organizations in a population to partition the population into sets of organizations that interact maximally with one another and minimally with other members of the population.
- 4) Partitions based on structural equivalence: organizations in each subset share similar relations with organizations in other blocks whether or not they are connected to one another

Summary of the strategies:

1. Structural: should be based on patterns of ties
2. Capable of yielding partitions
3. Sensitive to interactional networks and cohesion...
4. Sensitive to structural equivalence
5. Capable of identifying a structure of domination
6. Open-ended in its definition of the field
7. Capable of basing partitions on simultaneous analysis of multiple networks of different kinds of relations.

Blockmodel analysis of organizational fields

Data: a survey of managing directors (chief administrative officers) of 165 American nonprofit resident theatres.

The population surveyed includes the managers of the entire membership of the Theatre Communications Group, the service or trade association of the American resident stage.

This population includes almost all U.S. permanent theater companies with both institutional stability and serious artistic aspirations.

Respondents were asked about their professional activities, their personal backgrounds, training, career experiences and expectations, and their attitudes toward a range of management and policy issues.

Main focus: three questions:

1. (advice) If you were hiring an assistant managing director (or the equivalent), whom would you consult for advice and suggestions about candidates?
2. (Association) If you were to attend a national conference of TCG, FEDAT, or LORT, which managing directors would you be most likely to join for a meal, coffee, or drinks? (unconstrained)
3. (Admiration) Most people particularly admire certain others in the field. Are there any resident-theatre managing directors whom you particularly admire?

Field Definition:

Choice of measure of interorganizational ties:

- 1) Collect data on interorganizational ties solely from managing directors.
- 2) Include questions that are at least in part affectively oriented.

Structural Analysis:

Data processing:

- 1) Data from the network questions were combined into three matrices of 84 rows and 211 columns each.
- 2) The matrices were stacked by rows and CONCOR

Result:

- 1) At the four-block level, CoNCOR yielded blocks of 26, 25, 19, 30 theatres, as well as a residual fifth block of TCG managing directors who had in common only that they were not chosen by any of the respondents to the network questions. (5 blocks)
- 2) The density matrix and image matrix from blockmodel (5-block level) of resident theatres. Each block collectively occupies a different structural position in the field.

BOOLEAN DENSITIES						IMAGE MATRIX				
A	B	C	D	E	N	A	B	C	D	E
A	18	5	5	1	0	26	1	0	0	0
B	9	8	3	3	0	25	1	1	0	0
C	16	10	4	2	0	18	1	1	0	0
D	14	6	7	2	0	30	1	1	1	0
E	7	5	3	3	0	97	1	0	0	0

- 3) Because in this network, there are both ties from and to an actor, the structure of the field can be clarified by the presentation of three derivative image matrices.

Domination: 1, if row receive ties
 Coalition: 1, if ties are reciprocate
 No contact: 1, if row send ties not receive ties

	DOMINATION	COALITION	NO CONTACT
A	0 1 1 1 1	1 0 0 0 0	0 0 0 0 0
B	0 0 1 1 0	0 1 0 0 0	0 0 0 0 1
C	0 0 0 1 0	0 0 0 0 0	0 0 1 0 1
D	0 0 0 0 0	0 0 0 0 0	0 0 0 1 1
E	0 0 0 0 0	0 0 0 0 0	0 1 1 1 1

- 4) By splitting each of the four original blocks, the CONCOR further developed 9-block level model (including the residual block)

	BOOLEAN DENSITIES	IMAGE MATRIX
A1	29 10 0 13 2 12 0 0 0	1 1 0 1 0 1 0 0 0
A2	21 10 0 12 0 1 4 0 0	1 1 0 1 0 0 0 0 0
B1	11 0 11 3 0 6 7 0 0	1 0 1 0 0 0 0 0 0
B2	14 25 0 18 0 9 0 0 0	1 1 0 1 0 1 0 0 0
C1	14 8 0 18 0 0 0 0 0	1 0 0 1 0 0 0 0 0
C2	28 5 7 16 0 9 4 1 0	1 0 0 1 0 1 0 0 0
D1	23 9 3 15 4 7 2 2 0	1 1 0 1 0 0 0 0 0
D2	14 9 0 10 4 12 3 1 0	1 1 0 1 0 1 0 0 0
E	10 4 2 9 3 3 3 3 0	1 0 0 1 0 0 0 0 0

- 5) Patterns of domination, coalition, and no contact in the resident-theater field (9-block level)

	DOMINATION	COALITION	NO CONTACT
A1	0 0 1 0 1 0 1 1 1	1 1 0 1 0 1 0 0 0	0 0 0 0 0 0 0 0 0
A2	0 0 0 0 0 0 1 1 0	1 1 0 1 0 0 0 0 0	0 0 1 0 1 1 0 0 1
B1	0 0 0 0 0 0 0 0 0	1 0 1 0 0 0 0 0 0	0 1 0 1 1 1 1 1 1
B2	0 0 0 0 1 0 1 1 1	1 1 0 1 0 1 0 0 0	0 0 1 0 0 0 0 0 0
C1	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 1 1 0 1 1 1 1 1
C2	0 0 0 0 0 0 0 1 0	1 0 0 1 0 1 0 0 0	0 1 1 0 1 0 1 0 1
D1	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 0 1 0 1 1 1 1 1
D2	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 0 1 0 1 1 1 1 1
E	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 1 1 0 1 1 1 1 1

Block-descriptions: The attributes of the occupants of these blocks.

Descriptive Statistics:

Table 3. Selected Organizational and Individual Characteristics by Block

BLOCKS	A1	A2	B	C	D1	D2	E
<i>Organizational</i>							
Mean budget in thousands and N	1,875 11	1,084 5	1,328 4	1,307 5	1,696 10	389 7	606 52
Percent Pacific and N	23.07 13	25.00 8	25.00 4	18.18 11	15.38 13	0.00 7	11.93 109
Percent Great Lakes and N	15.38 13	0.00 8	0.00 4	18.18 11	7.69 13	57.14 7	11.01 109
Percent New York and N	15.38 13	0.00 8	50.00 4	9.09 11	46.15 13	14.29 7	30.28 109
Percent founded pre-1970 and N	84.67 13	50.00 8	100.00 4	66.63 11	53.83 13	28.67 7	47.71 109
Early Ford group and N	37.69 13	25.00 8	0.00 4	9.09 11	0.00 13	0.00 7	0.00 109
<i>Individual</i>							
Mean age and N	46.67 12	43.00 5	39.50 3	40.33 6	39.36 11	28.71 7	36.73 59
More than three jobs in theatre (percent) and N	33.00 12	50.00 4	25.00 4	0.00 6	50.00 10	0.00 7	17.65 51
Salary \$27,500 or more (percent) and N	50.00 12	60.00 4	50.00 4	83.44 6	40.00 10	28.58 7	17.65 51
Year of first theatre job (mean) and N	1963 12	1965 5	1965 4	1969 6	1968 11	1977 7	1971 55

Structural of domination and stratification of rewards.

Stratification of rewards: money, power, prestige ...

Stratification of rewards should both affect and be affected by the relational hierarchy.

Table 4. Stratification System of Resident-Theatre Field—Individual and Organizational Measures by Block

BLOCKS	A1	A2	B	C	D1	D2	E
<i>Individual</i>							
Percent on TCG board and N	33.33 12	0.00 5	0.00 4	0.00 6	9.09 11	0.00 7	1.72 58
Percent LORT officers and N	50.00 12	0.00 5	0.00 4	33.33 6	9.09 11	0.00 7	0.00 58
Percent on NEA theatre panel and N	25.00 12	0.00 5	0.00 4	16.67 6	9.09 11	0.00 7	5.17 58
<i>Organizational</i>							
Percent with large NEA grants and N	53.85 13	50.00 8	50.00 4	27.27 11	46.15 13	14.29 7	8.26 109
Percent with large foundation grants and N	69.23 13	50.00 8	25.00 4	27.27 11	28.57 14	0.00 7	6.41 108

- 1) Formal position (personal rewards) and structural dominance are closed connected.
- 2) organizational rewards: receiving the national endowment for the arts and from major foundations. Block A1 (the leading block) is the only block in which more than half of the theatres received grants of \$50,000 or more from NEA or grants of \$25,000 or more from any of three leading foundation patrons in 1977, 1978, or 1979. ...

What we have learned?

- 1) The field has been partitioned into several sets or blocks of organizations and their managers.
- 2) Relations among these structurally equivalent sets of organizations have been identified.
- 3) These partitions are distinct, theoretically and empirically, from nominalist or attribute-based alternatives.
- 4) In contrast to approaches that partition on the basis of cohesion, the blockmodel analysis yields several blocks in which members seldom interact.
- 5) The social relations of managing directors are shown to define a structure of domination that is isomorphic with the stratification not just of professional but of organizational rewards.

White, Harrison C., Scott A. Boorman and Ronald L. Breiger. (1976). "Social Structure from Multiple Networks. I. Blockmodels of Roles and Positions." *American Journal of Sociology*, 81(4), 730-779.

Networks of several distinct types of social tie are aggregated by a dual model that partitions a population while simultaneously identifying patterns of relations.

Concepts and algorithms are demonstrated in five case studies involving up to 100 persons and up to eight types of tie, over as many as 15 time periods. In each case the model identifies a concrete social structure. Role and position concepts are then identified and interpreted in terms of these new models of concrete social structure.

Methods:

- 1) Create a separate matrix for each type of tie.

	MUTUAL CONTACT	ONE UNAWARE	BOTH UNAWARE
1	X		
2	X X		
3			
4	X		
5			
6	X		
7	X		
8			
9	X X		
10	X		
11			
12	X		
13			
14	X		
15			
16			
17	X		
18	X		
19			
20			
21			
22			
23	X		
24			
25	X X X X X X		
26	X X X X X X		
27			
28	X X		

FIG. 1.—Incidence of ties of three types among 28 biomedical scientists in a research network. Source: Griffith, Maier, and Miller (1973). Asymmetric and reciprocal "unawareness" reported separately. Alphabetical listing by surname; numerical row and column labels as shown in left margin.

- 2) A self-consistent search procedure is used to partition a population into sets of structurally equivalent actors – blocks. In each data matrix, arrange the row and column of each individuals, so that the members of a block are grouped together.
- 3) Attention is focused particularly on blocks which have no, or very few, instances of ties: these are termed zeroblocks. (pp739)

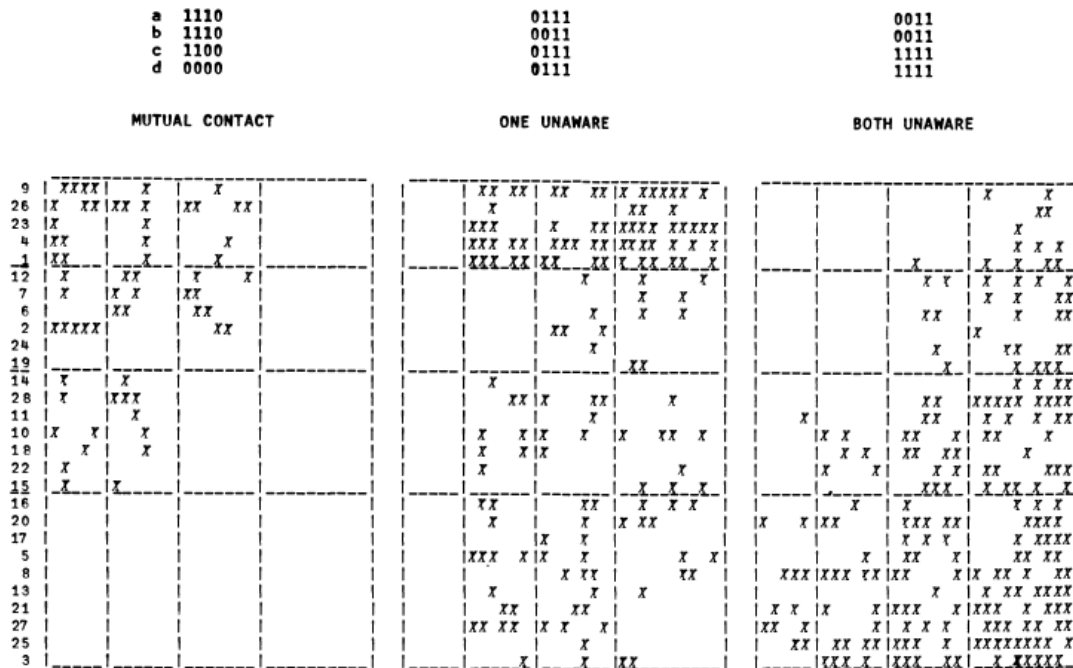


FIG. 3.—Blockmodel for biomedical network: images and blocked data matrices. In the left margin are letter labels for blocks and the rearranged numerical labels for individuals from fig. 1.

- 4) A blockmodel is a hypothesis about a set of data matrices: it specifies for each matrix which blocks will be zeroblocks when some common partition of the population is imposed on all the matrices.

Five ideas are basic to blockmodels:

- 1) Members of the population be partitioned into distinct sets: homogeneous not only to internal relations but also to external relations.
- 2) The primary indicator of a relation between sets is not the occurrence but the absence of ties between individuals in the sets.
- 3) Many different types of tie are needed to portray the social structure of a population.
- 4) The nature of a type of tie is inferred from the pattern, in a given population, of all ties of that type.
- 5) A model of social structure requires specifying, for each pair of sets on each type of tie, whether or not a zeroblock exists.

Two Algorithms:

- 1) BLOCKER.

Crystallizers: In carrying out blocker, one can identify persons whose assignment to one or more particular blocks in the blockmodel effectively determines the placement of many other persons. Such individuals may be termed crystallizers: they resemble sociometric stars in importance, not because of the number of choices they receive but because of their strategic "structural" position in the overall matrix following from the hypothesized model.

Floater: Other persons are allowed multiple, alternative assignments by the blocker algorithm; these are termed floaters.

- 2) CONCOR

The differences between BLOCKER and CONCER:

CONCOR produces from raw data an assignment of individuals to blocks, and thence suggests a blockmodel hypothesis.

BLOCKER demands a blockmodel hypothesis and derives from it any assignment of men to blocks that satisfies the hypothesis for the given set of data matrices.

Five Case Studies:

1) A biomedical Research Network

Data: Griffith et al. (1973) identified 173 scientists studying the neural control of hunger and thirst. 107 responded to Griffith's question-naire. In more than half the possible instances one respondent was unaware of another.

Blockmodel:

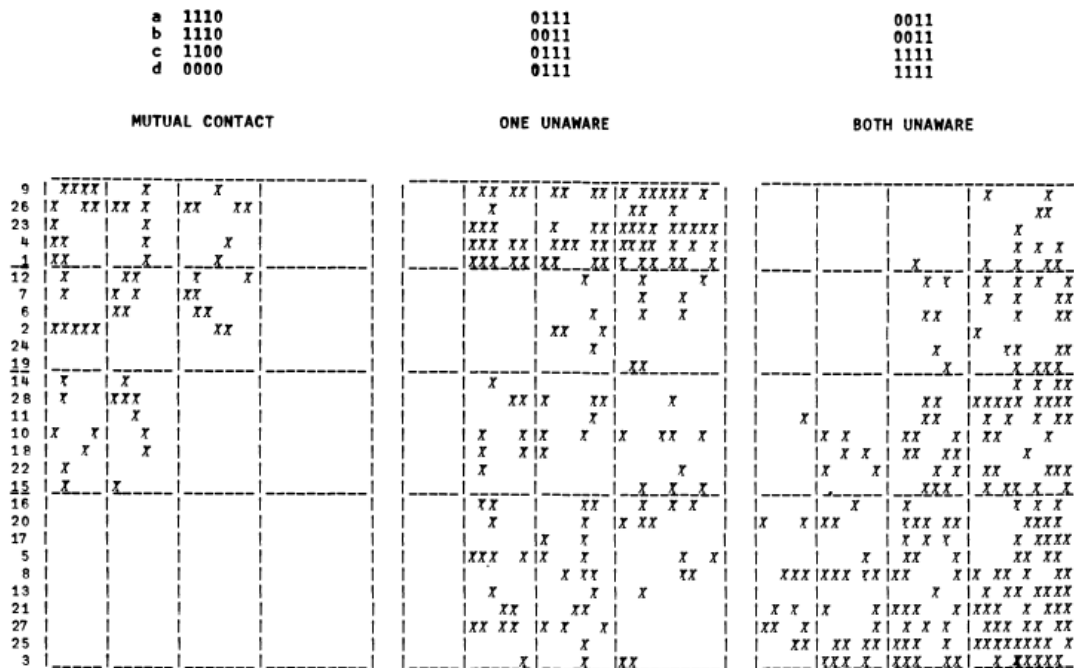


FIG. 3.—Blockmodel for biomedical network: images and blocked data matrices. In the left margin are letter labels for blocks and the rearranged numerical labels for individuals from fig. 1.

2) A monastery in Crisis

Data.: Sampson (1996). 12-months period. He developed a variety of observational, interview, and experimental information on the monastery's social structure.

He defined four sorts of relation: affect, esteem, influence, and sanction. – on which respondents were to give their first three choices, first on the positive side and then on the negative. "Like" for positive, "Antagonism" for negative side.

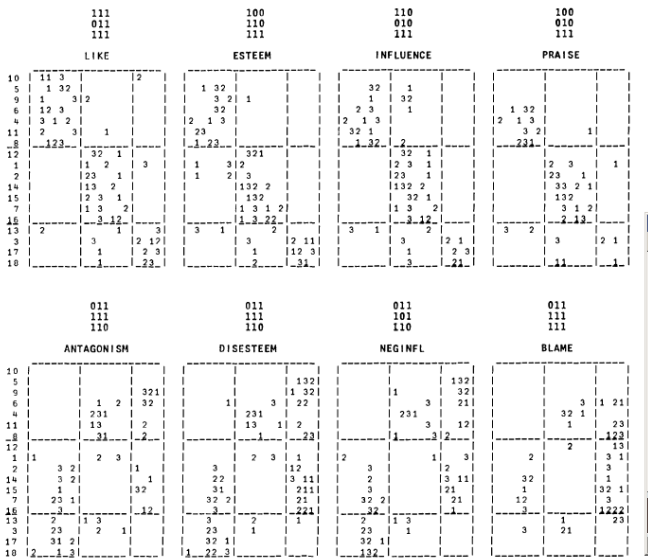


FIG. 4.—Blockmodel for the monastery, time 4: images and data matrices. In the latter, 3 stands for first choice, 2 for second, and 1 for third on each type of tie. Source: Sampson (1969).

If use CONCOR and raise the cutoff density for zeroblocks to half the average density, the resulting blockmodel is:

1 0 0	1 0 0	1 1 0	1 0 0
0 1 0	0 1 0	0 1 0	0 1 0
0 1 1	0 1 1	0 1 1	0 0 1
Like	Esteem	Influence	Praise
0 1 1	0 1 1	0 1 1	0 1 1
1 0 1	1 0 1	1 0 1	1 0 1
1 1 0	1 0 0	1 0 0	0 0 1
Antagonism	Disesteem	Neg. Infl.	Blame

Five block model produced by BLOCKER

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>a</i>	0	1	0	0	0
<i>b</i>	1	1	0	0	0
<i>c</i>	0	1	1	0	0
<i>d</i>	0	0	1	1	0
<i>e</i>	1	0	1	1	1

It is showed that for the result produced by CONCOR (3-block model) the Like image is identical with Esteem, and Disesteem with negative influence.

3) Cliques and Strata in the Bank Wiring Room

Data: Homans (1950) Bank Wiring Room, suggested a xis-blockmodel

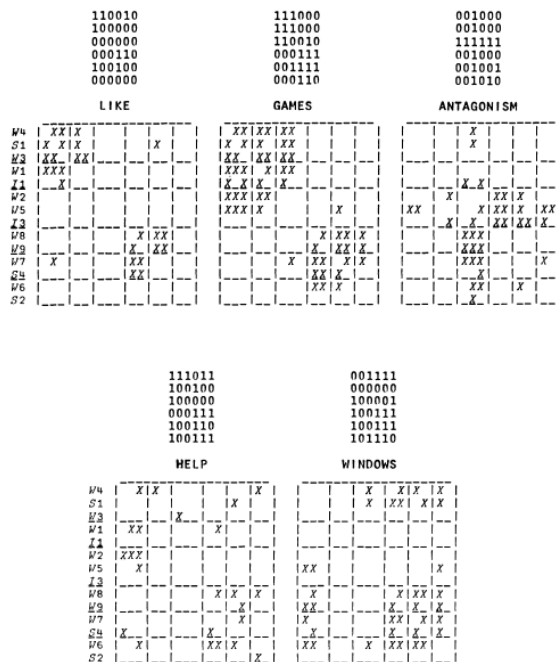


FIG. 5.—Blockmodel for the Bank Wiring Room: images and data matrices

²⁶ An additional type of tie is not reported here because it could be received only by soldermen.

4) Newcomb's second Fraternity

Data: Newcomb (1961), two experiments in which 17 previously unacquainted male undergraduates lived together in a fraternity style house, expenses paid.

Data is collected from observation and self-report.

A blockmodel:

1	0	0	0	0	1
1	1	0	1	0	1
1	1	1	1	1	1
Like			Antagonism		

5) Management Conflict in a Company

Data – eight types of tie specifically relevant to the business activities of the 16 top managers in 1958 of Firth-Sterling, a company with about 2000 employees producing specialty alloy and abrasives products for industrial use.

		110				000						011			
		110				111						101			
		101				011						000			
		SIMILAR POLICY				FRIENDSHIP				UNCOMFORTABLENESS					
11		XX	X	X							X	X	X		
9			X								X	X			
13			X	X							X	X			
2		X		XX				X	X		X			XX	
4		X				XXX			X						
6							XX	X			X			X	
7		X		X	X		X	XXX			X				
8		X		X			XX	X			X				
1			XX	X			XXX				X			XXX	
15				XX			X	XX			X				
5					XX		X		X						
10		X						X							
12		XXX													
14															
3		X													
16		XX													

FIG. 6.—Blockmodel for Firth-Sterling management: images and data matrices.
Source: White (1961).

Blockmodels over Time

The fraternity data: during the selected weeks: 0,3,5,8,13,15

WEEK	LIKE			ANTAGONISM		
	.21	.18	0	.14	.18	.31
0	.14	.25	.05	.29	.08	.12
	.10	.20	.10	.24	.12	.17
	.25	.07	.04	.02	.21	.33
3	.20	.15	0	.25	0	.20
	.20	.08	.06	.21	.12	.20
	.28	.04	.02	.02	.07	.43
5	.11	.42	0	.11	0	.37
	.21	.08	.03	.14	.04	.37
	.29	.04	.02	0	.10	.43
8	.14	.33	0	.07	0	.42
	.20	.12	.03	.07	0	.50
	.31	.04	.02	.02	.07	.43
13	.10	.42	0	.03	0	.46
	.21	.08	.03	.02	.12	.43
	.33	0	0	0	0	.50
15	.10	.42	0	.07	0	.42
	.20	.12	.03	.05	.04	.50

Monastery data: Somes for Like and Antagonism choices over three time periods. (individual choices contribute either +3, +2, or +1 to the sums, depending on tie strength.

Like:

T2					T3					T4				
2	12	0	3	1	2	10	4	0	2	4	11	2	0	2
11	5	6	2	0	10	12	1	1	0	9	14	0	1	0
3	0	8	5	2	1	3	11	4	0	0	0	11	4	3
0	2	15	9	0	1	0	16	9	0	0	0	16	8	0
2	0	10	2	10	2	0	5	1	16	2	0	5	1	18

Antagonism:

T2					T3					T4				
0	0	0	0	6	0	0	0	0	6	0	0	0	0	6
0	1	2	8	13	0	1	16	0	6	0	0	13	4	9
4	7	0	1	6	3	10	0	0	5	1	5	2	3	1
1	7	0	0	17	3	12	0	0	10	0	15	0	0	9
4	15	4	1	0	0	19	5	0	0	2	17	6	1	0