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Reliability Analysis of Dynamic Buckling Stiffened Panels

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Abstract

The stiffened panels are often forming less thin wall. Consequently, the dynamic buckling is a major challenge to be addressed in the quest to increase the strength-to-weight ratio. Stiffened are subjected to various loading conditions can be static or dynamic, as they may suffer from degradation of the material and initial geometric distortions due to the welding assembly process used.
Buckling of structures subject to sudden dynamic impulse load can be analyzed without detour to the approach based on the equations of motion. In this approach, which can be easily adapted to the methods of calculation using the finite element method, the motion equations are solved for various values of the parameters defining the loading. The value of the load parameter for which there is a big change in the response then defines the critical load according Budiansky and Roth criterion.
In this work, dynamic buckling of stiffened panels is analyzed numerically through a nonlinear incremental formulation using explicit time integration procedure under Abaqus software package. The dynamic buckling state is recovered from the curve giving the end-shortening as function of time when the structure is subjected to a compressive rectangular pulse loading applied parallel ally to the stiffeners direction. Fixing the pulse duration and the initial distortion magnitude as well as the ratio of material degradation in the heat affected zone, the dynamic buckling load was identified for each given configuration. This process has enabled the derivation of a response surface model which was used with Monte Carlo method to determine reliability of the stiffened panel with regards to the dynamic buckling state.
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Keywords: dynamic buckling; stiffened panels; axial compression; finite element method; reliability analysis; Monte Carlo.

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1. I. Introduction

Taking the case of aeronautics, many of the loads applied to airplane structures such as those experienced during taking off, crossing turbulent zones or landing are dynamic in nature [1]. The relationship existing between the dynamic and static buckling loads can vary considerably as function of a number of factors. For example shorter duration impacts result in higher buckling loads [2] and may be substantially in excess of the static buckling loads [3] whilst longer duration impacts can result in much lower buckling loads. Standards such as those set out by the Advisory Council for Aeronautics Research in Europe related to the strategic research agenda in aeronautics [4] aim at reducing aircraft emissions by 80%. To achieve optimization of these structures with the objective to minimize weight and fuel consumption, it is essential that dynamic buckling should be taken into consideration. The consequences of lightening the weight could be in fact critical for some load patterns that should be comprehensively investigated.

Although dynamic buckling has been the subject of huge research work during the last decades [5], the crucial problem related to the relevant stability criteria to be used is still intriguing researchers in this field. Intuitively dynamic instability can be assessed by the fact that a rather petite disturbance of the applied dynamic load is susceptible to yield an abrupt different deformation pattern of the structure than that of the previous original undisturbed pattern. How big should be the jump to be viewed as instability occurring? This is still a matter of some appreciation.

Calculating the response for monotonic loading parameters by integrating the equations of motion and drawing the curve of some selected displacement as function of time, while varying the intensity of the applied load, a jump can occur from the curves drawn for the preceding neighboring values. The particular value of the load causing this change corresponds to the critical value associated to dynamic buckling according to the Budiansky-Roth criterion [6]. This criterion appears to be more convenient in the investigation of dynamic buckling of structures even if it involves in practice a large amount of work. This is due to the fact that it requires calculation of nonlinear transient response for different loads, which is quite time consuming. Bisagni [7] has carried out both numerically and experimentally an investigation on dynamic buckling of composite shells under impulsive axial compression. For the numerical studies the equations of motion were numerically solved by means of the Abaqus finite element code through the ABAQUS/Explicit procedure [8]. The investigated models of composite shells included different lay-ups with imperfection data identified from the real shell specimens that were tested earlier under static buckling conditions. The load was applied suddenly and the critical load duration has been determined for each load level. The lowest obtained DLF attained the minimum value 0.54. This study confirmed the results of a former work [9] and showed that for short loading time duration, the dynamic buckling loads are larger than the static ones. Abramovich et al. [10] have analyzed by numerical and experimental approaches the dynamic buckling of axially impacted composite cylindrical shells. They focused on the dynamic buckling criteria and initial imperfections to be used in the numerical analyses. They concluded that dynamic buckling can be monitored by using any critical response of the structure.

In this work a parametric analysis on the influence of initial geometric imperfections on the dynamic buckling of stiffened panels is performed by using Budiansky and Roth criterion [6]. The objective is to perform reliability analysis with regards to the dynamic buckling state. This will be conducted as function of the magnitude of the initial imperfections, the material degradation described by Young's modulus decrease in the heat affected zone (HAZ) [11] and the duration of the applied rectangular pulse load. A quadratic polynomial surface response model [12,13] is derived. With this predictive representation of the dynamic buckling load, reliability of the dynamic buckling state is analyzed by using Monte Carlo method, and the probability of failure associated to a given design load is calculated.

2. Buckling analysis of stiffened panels

2.1. Nonlinear static buckling analysis

Under a given pattern of applied loading, the equilibrium equations obtained by finite element discretization of the buckling problem take the following general form

$$f(u) = 0 \tag{1}$$

Where f is a nonlinear function of the nodal displacement vector u.

To solve equation (1) for a given load history on a considered time interval, Newton's method is usually used. The idea is to expand the function f around the actual approximation of the solution u^k according to Taylor expansion as

$$f(u^k) + \frac{\partial f}{\partial u_i}(u^k)\Delta u_i^k + (\Delta u_j^k)^t \frac{\partial^2 f}{\partial u_i \partial u_j}(u^k)(\Delta u_j^k) + \ldots = 0 \tag{2}$$

In which Δu^k is assumed to be small. This will be the case when the approximate solution at iteration k is enough close to the exact solution. The third term in the first half side of equation (2) can then be discarded yielding to the following linear equation

$$K^k \Delta u^k = -f(u^k) \tag{3}$$

Where $K^k = \frac{\partial f}{\partial u}(u^k)$ is the Jacobean matrix. When Δu^k is found, the next approximation to the solution is provided by $u^{k+1} = u^k + \Delta u^k$ and the process continues until convergence is reached.

2.2. Nonlinear dynamic buckling analysis

For dynamic buckling analysis, the explicit dynamic procedure enables to perform a large number of small time increments calculation to find an efficient approximation of the problem solution. This procedure is based on the explicit central-difference time integration rule. Each computational increment is inexpensive as compared to the implicit direct-integration dynamic analysis procedure because unlike this last the process does not involve any inversion of a linear system. The explicit central-difference operator satisfies the dynamic equilibrium equations at the beginning of the time increment t, the accelerations calculated at that time are used to advance the velocity solution to time $t + \Delta t / 2$ and the displacement solution to time $t + \Delta t$ the implementation of the explicit dynamics analysis procedure requires the use of an explicit integration rule together with the approximation of the mass matrix as a diagonal lumped mass matrix. Giving the kinematics state at the beginning of a given time step in terms of the acceleration $\ddot{u}_n$ and the velocity $\dot{u}_{n-1/2}$, the equations of motion are integrated using the explicit central-difference integration rule according to

$$\begin{aligned} \dot{u}_{n+1/2} &= \dot{u}_{n-1/2} + \frac{\Delta t_{n+1} + \Delta t_n}{2}\ddot{u}_n \\ u_{n+1} &= u_n + \Delta t_{n+1}\dot{u}_{n+1/2} \end{aligned} \tag{4}$$

Where n refers to the step number.

To determine the acceleration $\ddot{u}_n$ within the context of this method, the computational efficiency is enhanced by using a diagonal mass matrix M which enables to compute the accelerations at the beginning of the increment as

$$\ddot{u}_n = M^{-1}\left(P_n - I_n\right) \tag{5}$$

Where P_n the actual is applied load at step n and I_n is the internal force resulting from the disequilibrium of the equations of motion. Vector I_n is assembled from contributions of the individual elements and the global stiffness matrix does not need to be formed.

The finite element S4R of Abaqus software package can be used with both implicit and explicit solvers. This element is one of the most versatile finite elements for shell like structures and allows achieving the highest accuracy and a relatively low computing time as demonstrated by the benchmarking study performed by Eglitis [14].

3. Modelling dynamic buckling of stiffened panel under initial geometric and HAZ effect

The geometric nonlinearities are taken into account while the material is assumed to have linear elastic behaviour. The shell element S4R of the finite element Abaqus software package is used. This element has four nodes with six degrees of freedom at each node and can account for both material and geometric nonlinearities. A detailed description regarding the appropriate finite element formulation used in the following is given in [14, 15]. In the present analysis the geometrical configuration of the stiffened panel. The modelled stiffened panel has two full equal segments and two half equal edge segments. The stiffeners take the form of three equal webs.

4. Reliability analysis of welded imperfect stiffened panels

To perform reliability analysis, an analytical model giving explicit representation of the problem is constructed. This is performed by selection of pertinent trial points over the explored domain of the basic variables according to a design of experiment (DOE).

Variability of the dynamic buckling load is assumed to from the following three main sources that include the amplitude of the initial geometric imperfection, the Young's modulus of the stiffened panel material and the rectangular pulse duration. All the other geometric and mechanical parameters are assumed to be deterministically known.

Keeping only these three active variables, the buckling load can be expressed explicitly as

$$P_{cr} = P_{cr}(w_0, E, T) \tag{6}$$

Under this hypothesis which enables to focus on the main effects governing the dynamic buckling problem of stiffened panels, derivation of a predictive model is considered in the form of a quadratic polynomial response surface that approximates the function P_{cr}.

The densities of probabilities that describe the three intervening random variables are defined then and Monte Carlo method is used to calculate the probability of failure which is associated to a given dynamic buckling design load.

5. Results and discussion

To get the maximum dynamic effect, the rectangular pulse duration is selected around the fundamental period of the stiffened panel natural vibrations. This last was computed by using Abaqus software and the obtained result was $T_0 = 9.448\,\text{ms}$ .During simulations the pulse period was varied according to the values $T = 0.5T_0, 0.75T_0$ and T_0 to assess the effect of the selected parameters on the dynamic buckling load, simulations were performed according to a full factorial DOE table $L27$.

For a given combination defined by fixing the initial geometric imperfection amplitude, Young's modulus reduction and pulse duration, finite element calculation was performed in order to determine the buckling load according to Budiansky and Roth criterion.

Figure 1 illustrates the process of determination of the buckling load by using the Budiansky and Roth criterion. In this case the dynamic buckling load is in the interval $[0.7,0.8]\times \mathrm{Pstat}$.

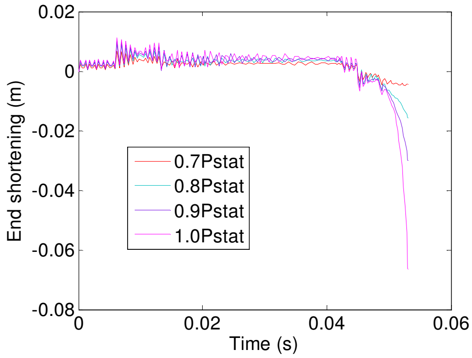

Fig. 1. End shortening as function of time and load magnitude; $\mathrm{Pstat}=0.96242\,\mathrm{MN}$

The obtained results of P_{cr}^{stat} designates the static buckling load. The dynamic buckling load can be much smaller than the static buckling load. The minimum has reached $P_{cr}^{min}=0.5967\,\mathrm{MN}$, which represents only 58.9% of the static buckling load. Some dynamic buckling loads can also be higher than the static buckling load and the maximum value is $P_{cr}^{max}=1.1068\,\mathrm{MN}$, which is 2.5% higher than the associated static buckling load.

In order to derive under a simple form a response surface model that represents the buckling load over the considered ranges of parameters as defined in Table 1, it is suitable that these should be taken nondimensionalized according to

$$\tilde{w}_0 = w_0 / w_{0,max}\,,\ \tilde{E}=E/E_{max}\,,\tilde{T}=T/T_{max} \tag{7}$$

with $w_{0,max}=6\,\mathrm{mm}$, $E_{max}=69.5\,\mathrm{GPa}$ and $T_{max}=9.448\,\mathrm{ms}$; such as all the three factors are in the interval $[0,1]$.The quadratic polynomial giving the dynamic buckling load in MN as obtained by means of the command regstats of Matlab is

$$\begin{aligned}P_{cr}(\mathrm{MN}) = &-1.6385+12.192\tilde{w}_0-5.5751\tilde{E}-2.3757\tilde{T}-3.2229\tilde{w}_0\tilde{E}+0.4716\tilde{w}_0\tilde{T}\\ &-0.79861\tilde{E}\tilde{T}-5.32072\tilde{w}_0^2+5.5248\tilde{E}^2+1.4466\tilde{T}^2\end{aligned} \tag{8}$$

The interpolation defined by equation (8) achieved an excellent determination coefficient $R^2=97.3\%$. It should be noticed that even with a linear polynomial regression model; the coefficient of determination is quite good and reaches $R^2=84.5\%$.

Analysis of variance conducted on the results shown in Table 2 by means of the command anovan of Matlab gave the results summarized in Table 1. It could be noticed that the factors and their interaction explain correctly the variations of the dynamic buckling load since the maximum relative error had not exceeded 1.1%. If one works with a linear model then the factors explain by themselves the results with an error limited to 3.4%.

Table 1. Analysis of variance of the dynamic buckling load as function of the three considered factors.

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
$\tilde{w}_0$	0.01611	2	0.00805	15.08	0.0019
$\tilde{E}$	0.05103	2	0.02552	47.8	0
$\tilde{T}$	0.3084	2	0.1542	288.85	0
$\tilde{w}_0\tilde{E}$	0.00245	4	0.00061	1.15	0.4002
$\tilde{w}_0\tilde{T}$	0.00302	4	0.00075	1.41	0.3131
$\tilde{E}\tilde{T}$	0.00333	4	0.00083	1.56	0.2745
Error	0.00427	8	0.00053		
Total	0.3886	26			

From Table 1, the pulse duration most is found to be the most influential factor in the buckling problem, as indicated by the high value of the Fisher variable F in line 4. It is followed by the material Young's modulus and by the amplitude of the initial imperfection. The interactions of the factors are found to be of second order in comparison with the contributions of the factors by themselves.

Giving a design load P_{Design} , the dynamic buckling limit state can be described by the following equation

$$\Lambda(\tilde{w}_0,\tilde{E},\tilde{T}) = P_{Design} - P_{cr}(\tilde{w}_0,\tilde{E},\tilde{T}) \tag{9}$$

With $\Lambda(\tilde{w}_0,\tilde{E},\tilde{T}) > 0$ is the safe region and $\Lambda(\tilde{w}_0,\tilde{E},\tilde{T}) < 0$ is the failure region.

To perform reliability analysis, the random variables associated to the active parameters are assumed to be distributed according to the following densities of probabilities:

- Lognormal density of probabilities for $\tilde{w}_0$;
- Normal density of probabilities for $\tilde{E}$;
- Uniform density of probabilities for $\tilde{T}$ over the domain $[0.5,1]$ where the regression is established.

The mean values of these random variables were fixed respectively at $<\tilde{w}_0>=0.916$, $<\tilde{E}>=0.9209$ and $<\tilde{T}>=0.75$. Three different standard deviations were selected according to the following set of values 2%, 5% and 10%. Figure 2 gives the probability of failure as function of the design load P_{Design} for the three considered standard deviation values. The population size used in Monte Carlo process was fixed by the convergence requirement and a total number of 5×10^4 samples were found to yield sufficient accurate results.

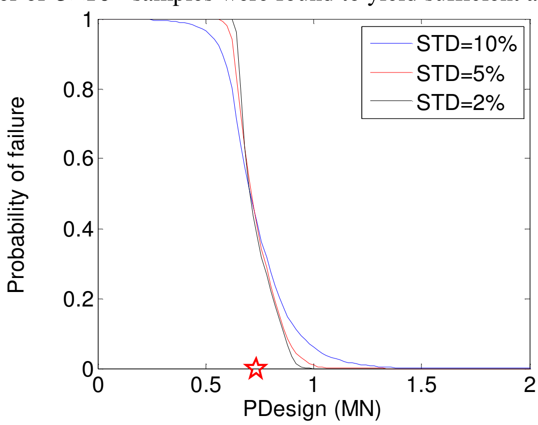

Fig. 2. Probability of failure as function of the design load for different values of the standard deviation; STD designates the standard deviation

Fig. 2 shows that the probability of failure does not vanish for the deterministic dynamic buckling design load which is according to , $P_{cr} = 0.73144\,MN$. This value is indicated by the star in figure 2. The associated probabilities of failure are: 0.4022 for $STD = 2\%$; 0.4282 for $STD = 5\%$ and 0.4339 for $STD = 10\%$.

Figure 2 shows also that the probability depend hugely on the standard deviation of the intervening random variables. In practice knowing the applied load and the standard deviation of the random variables that describe the active factors, the reliability based design load can be straightforwardly determined for a given accepted failure probability by solution of an inverse problem.

6. Conclusions

Reliability analysis of the dynamic buckling limit state of an L-shaped stiffened panel as influenced by the presence of weld induced initial distortion and material rigidity degradation in the heat affected zone was performed as function of the duration of the applied rectangular pulse load and variability of these last factors. Nonlinear finite element analysis of the dynamic buckling problem was conducted for various combinations of these parameters. This has enabled the derivation of a response surface representation of the dynamic buckling load variations as predicted by Budiansky and Roth stability criterion when applied to finite element simulation results.

The proposed methodology makes it possible to assess accurately the influence of weld induced defects and pulse duration on reliability of the dynamic buckling strength of stiffened panels.

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