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## Model Reference Adaptive Control Design for Slow Processes. A Case Study on Level Process Control

Stelian-Emilian Oltean<sup>a,\*</sup>, Mircea Dulau<sup>b</sup>, Adrian-Vasile Duka<sup>c</sup>

<sup>a,b,c</sup>Department of Electrical and Computer Engineering, "Petru Maior" University of Tirgu Mures, 1 N. Iorga st., 540088, Romania

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### Abstract

The design of controllers, using conventional techniques, for plants with nonlinear dynamics or model uncertainties can be often quite difficult. Model reference adaptive control (MRAC) is a modern alternative for classic control algorithms and a convenient method to updating the controller's parameters for slow processes with parameter variations. In this paper is presented a comparative study of two design procedures of MRAC for first order slow processes, particularly applied and tested on level process control. The first method it is based on the MIT rule and the second one it is based on the stability theory of Lyapunov. The theoretical results are obtained using Matlab simulation environment.

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### 1. Introduction

In the industrial processes many of the control applications deal with level, flow, pressure and temperature. Normally, to control such processes classical control structures can be used (e.g. PID) since they have simple configurations and are easy to implement and exploit.

In reality, not always the performances of the classical systems are the desired one because they are affected by nonlinear dynamic behavior, disturbances, modeling uncertainties and time varying parameters. In this category of systems can easily enter the level process control, which are often seen in the metallurgical industry, food processing

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\* Corresponding author. Tel.: +40-265-233212; fax: +40-265-233212.

E-mail address: [stelian.oltean@ing.upm.ro](mailto:stelian.oltean@ing.upm.ro).

industry, chemical industry, wastewater treatment industry and other. So, over the past years, a series of modern or classical controllers have been proposed.

Details of mathematical modeling of some level processes are presented in [1] for cylindrical tank, in [2] and [3] for spherical tank and in [4] and [5] for conical tank. The simulations of different conventional PI control were compared with state feedback with integral control for a spherical tank in [2] and with internal model control (IMC) for a conical tank in [5] and [6]. In [3], [6] and [7] model reference adaptive control based on MIT rule was also simulated for the three types of tanks. Several artificial intelligence methods such as neural networks, genetic algorithms and fuzzy systems have been approached in [8], [9], [10] to tune the parameters of conventional PID controllers resulting in complex adaptive systems.

Real time implementations of the level process adaptive control were made in [1], [4] and [11] and interfaced with LabVIEW using USB-based DAQ modules and Personal Computers (PC). MIT rule was used to obtain the adaptive laws in case of cylindrical and conical tanks level control and Coefficient Diagram Method design was applied for spherical tank level control.

The present paper contains the design procedures of two model reference adaptive control systems for level processes, which can be also easily extended on other slow processes. The first approach is the design of a model reference adaptive controller using the MIT rule. MIT rule is based on a simple cost function minimization with descendent gradient method. And the other procedure it is based on stability theory of Lyapunov to assure the stability of the global system. Finally the level process case study is used to compare their control performances.

## 2. Level process. Mathematical model

In the literature there are many variations of level processes, which contains: cylindrical tank, spherical tank, conical tank, mixed tank, coupled tanks, and others (Fig. 1). Whatever is the shape of the tank, level control processes are usually controlled based on the error signal, have at least an upstream or downstream control valve and are drained by freefall or using a pump. When the inlet flow  $F_a$  is used for controlling the level inside the tanks, the output flow  $F_e$ , represents the main disturbance in the control system and vice-versa [12],[13],[14].

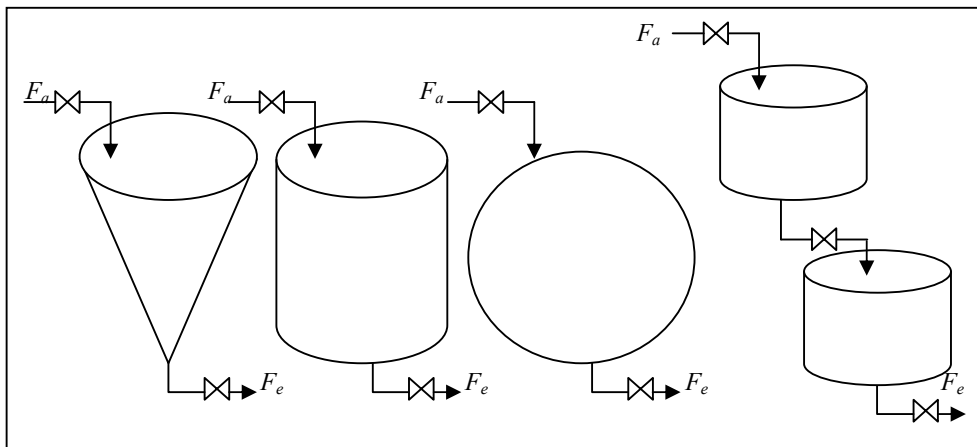


Fig. 1. Tank shapes used in level control.

In the modeling stage it was considered a level process with the experimental setup shown Fig. 2. This consists of the following equipments and devices: 20 liters feed tank (R1), 600mm water column (R2), pneumatic control valve (LV1), electro pneumatic converter (4-20)mA / (0-1.25)bar, two centrifugal feed pumps (G1,G2) with a flow rate of 4 m<sup>3</sup>/h, electronic level transmitter (LT1) with (4-20)mA output, ABB Digitric 500 Level Controller (LC1).

When a change in level, inside the process tank R2, is sensed by the level transducer LT1, the error signal is modified, and the controller LC1 takes corrective actions by generating a control signal for valve LV1 [12, 13,14].

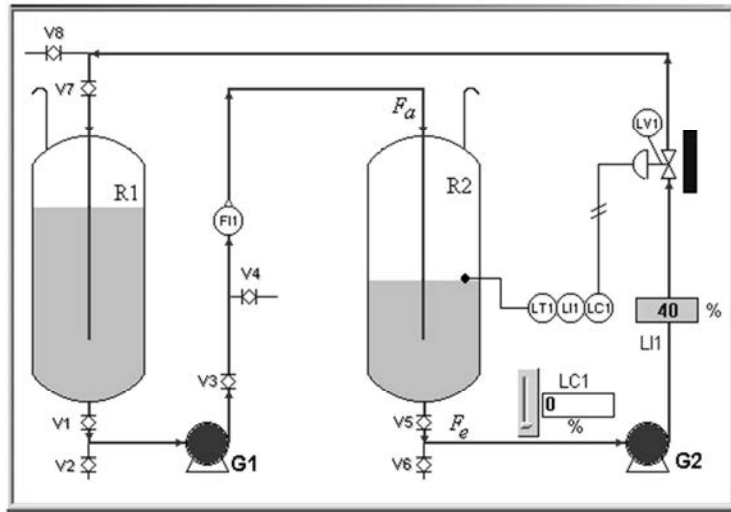


Fig. 2. Level process experimental setup.

In steady state the amount of fluid introduced in the tank is equal to the drained amount and the level inside the tank remains constant. In dynamic regime, the difference in the inlet and outlet quantities is accumulated in the system. By performing normalization to the steady state values: the controlled signal  $y(t) = \Delta L(t)/L_0$ , the control signal  $u(t) = \Delta F_a(t)/F_{a0}$ , the final general input-output mathematical model is deduced.

This model depends on the tank's section  $A$ , the section of the outlet pipe  $a$ , the gravitational acceleration  $g$ , the steady state inlet flow  $F_{a0}$  and the steady state level  $L_0$ .

$$AL_0 \frac{d}{dt} y(t) + \frac{a\sqrt{2gL_0}}{2\sqrt{L_0}} y(t) = F_{a0} u(t) \quad (1)$$

Denoting  $T_p = 2A\sqrt{L_0}/(a\sqrt{2g})$ ,  $k_p = (2\sqrt{L_0}/(a\sqrt{2gL_0})) \cdot F_{a0}$  the transfer function of the process results from (1):

$$H_p(s) = \frac{k_p}{T_p s + 1} \quad (2)$$

The first order transfer function (2) can be used also for the other shapes of tanks in level process control or for other slow processes. So, the design procedures presented in the following paragraphs could be easily extended to other processes. The inherent errors resulting from modeling the process and time varying parameters define the system's uncertainties. In order to determine the way in which the variation of the parameters from the nominal process affect the closed loop behavior of the system and to test the adaptive control schemes Matlab environment is used for simulation [15],[16]. For the level control process described by (2), the uncertain process parameters are modified within a range of  $k_p = 0.5 \pm 50\%$  and  $T_p = 60 \pm 50\%$ .

### 3. Model reference adaptive control

When the plant parameters are unknown or varying slowly, or slower than the dynamic behavior of the plant, then a MRAC control scheme can be used [17], [18]. This adaptive structure offers a superior performance and robustness in time than a classical PID controller.

Figure 3 shows the general structure of the MRAC system designed for the level process control system.

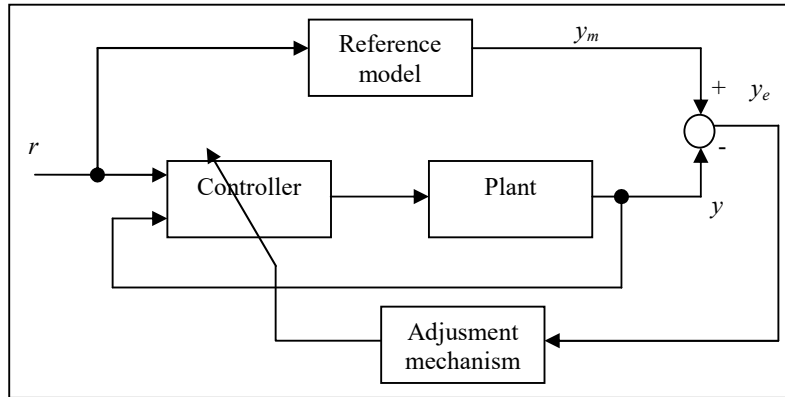


Fig. 3. Model reference adaptive control block diagram.

The MRAC structure consists of four main parts: the plant, the controller, the reference model and the adjustment mechanism. The reference model was chosen to generate the desired trajectory,  $y_m$ , which the plant output  $y$  has to follow. A standard first order system with constants  $k_m=1$  and  $T_m=10$  seconds was chosen as the reference model.

$$\begin{cases} T_m \cdot \dot{y}_m + 1 = k_m \cdot r \\ T_p \cdot \dot{y} + 1 = k_p \cdot u \end{cases} \quad \text{or} \quad \begin{cases} \dot{y}_m + a_m = b_m \cdot r \\ \dot{y} + a = b \cdot u \end{cases} \quad \text{with} \quad \begin{cases} a_m = 1/T_m, b_m = k_m/T_m \\ a = 1/T_p, b = k_p/T_p \end{cases} \quad (3)$$

Due to the specifics of the plant and the reference model (first order model), the feedforward-feedback controller of the level process control has two parameters  $k_1, k_2$  and the control law takes the following form:

$$u(t) = k_1 \cdot r(t) - k_2 \cdot y(t) \quad (4)$$

The tracking (adaptation) error  $y_e = y_m - y$  represents the deviation of the plant output from the desired trajectory. The adjustment mechanism uses this adaptation error to adjust the controller's parameters. If the level process parameters are well known and time invariant than the ideal controller parameters  $k_1, k_2$  are given by (5). In this case the adaptation error is zero and the adjustment mechanism is in a stand by state.

$$\begin{cases} k_1 = (T_p \cdot k_m) / (T_m \cdot k_p) \\ k_2 = (T_p - T_m) / (T_m \cdot k_p) \end{cases} \quad (5)$$

### 3.1. MRAC using MIT rule

MIT rule is used to design the adaptive law so that the controller's parameters are adjusted in a direction that minimizes a quadratic performance function with descendent gradient method. The adaptive law is driven by the partial derivative of the cost function ( $\partial y_e / \partial \theta$  is so called sensitivity function) and depends on adaptation error signal that characterizes the mismatch between the actual and desired behavior of the process.

$$J(k) = \frac{1}{2} \cdot y_e^2 \Rightarrow \frac{dk}{dt} = -\gamma \cdot \frac{dJ}{dk} = -\gamma \cdot y_e \cdot \frac{\partial y_e}{\partial k}; k = \overline{k_1, k_2} \quad (6)$$

Where  $\gamma$  is the adaptation gain (positive constant) and  $y_e = y_m - y$  is the adaptation error.

Using the equations (3) with first order linear models of the reference system and level process, respectively the MIT rule (6) the adaptation laws are:

$$y_e(t) = \frac{b_m}{p + a_m} \cdot r(t) - \frac{bk_1}{p + a + bk_2} \cdot r(t) \Rightarrow \begin{cases} \frac{dk_1}{dt} = -\gamma_1 \cdot y_e \cdot \left( -\frac{a_m}{p + a_m} \cdot r(t) \right) \\ \frac{dk_2}{dt} = -\gamma_2 \cdot y_e \cdot \left( \frac{a_m}{p + a_m} \cdot y(t) \right) \end{cases} \quad (7)$$

In (7) some terms were absorbed into the adaptation gains  $\gamma_1$  and  $\gamma_2$  and it was used also the stationary relation (8) because the controller's parameters are not known.

$$p + a_m \rightarrow p + a + b \cdot k_2 \quad (8)$$

### 3.2. MRAC using Lyapunov stability theory

This method of developing adaptive laws is based on the direct method of Lyapunov for studying the systems stability. Parks and others found a way of redesigning the MIT rule-based adaptive laws used in the MRAC schemes by applying the Lyapunov design approach. Use of the Lyapunov theory assures the stability and convergence of the adaptation error.

The differential equation that describes the adaptation error may be expressed by:

$$\begin{aligned} y_e(t) &= y_m(t) - y(t) \\ dy_e(t) &= -a_m \cdot y_e(t) + (b \cdot k_2 + a - a_m) \cdot y(t) + (b_m - b \cdot k_1) \cdot r(t) \end{aligned} \quad (9)$$

The first major problem in Lyapunov theory is to choose the proper energy function  $V$ , which in this case depends on adaptation error and controller parameters.

So, according to the stability theory of Lyapunov the equilibrium point for the adaptation error  $y_e = 0$  is asymptotically stable if function  $V$  is chosen positive definite,  $V(0)$  is 0 and the derivative  $dV/dt$  is negative definite.

$$V(y_e, k) = \frac{\gamma}{2} \cdot y_e^2 + \frac{1}{2b\gamma_2} (b \cdot k_2 + a - a_m)^2 + \frac{1}{2b\gamma_1} (b_m - b \cdot k_1)^2 \quad (10)$$

$$\frac{dV(y_e, \theta)}{dt} = -a_m \cdot y_e^2 + (b \cdot k_2 + a - a_m) \cdot \left( y_e \cdot y + \frac{1}{\gamma_2} \cdot \frac{dk_2}{dt} \right) + (b_m - b \cdot k_1) \cdot \left( y_e \cdot r - \frac{1}{\gamma_1} \cdot \frac{dk_1}{dt} \right) \leq 0$$

The derivative of the Lyapunov function  $V$  is negative definite if some of the terms in (10) are zero and the resulting adaptation laws of the controller's parameters are:

$$\begin{cases} \frac{dk_1}{dt} = \gamma_1 \cdot y_e \cdot r \\ \frac{dk_2}{dt} = -\gamma_2 \cdot y_e \cdot y \end{cases} \quad (11)$$

In the adaptation laws (11) some terms were absorbed into the adaptation gains  $\gamma_1$  and  $\gamma_2$ .

#### 4. Results

The first stage in the design of the model reference adaptive control was to choose the parameters  $T_m$  and  $k_m$  of the first order reference model, based on the experimental results obtained using the real PI control of the level process. The figure 4a shows the experimental signals (set point, process variable and command) considering [13],[19], for controlling the level inside the R2 tank, to a set point change from 65% to 25%, which empties the tank. The figure 4b shows a similar response of the reference model, which was simulated in Matlab environment.

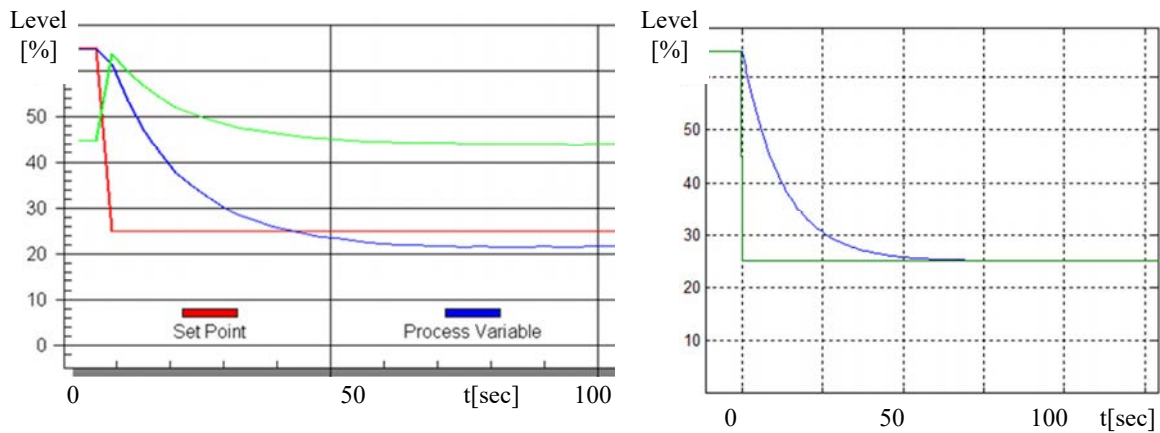


Fig. 4. (a) The step response of the experimental level process; (b) The step response of the reference model.

The MRAC scheme applies to systems with known dynamic structure (first order system considered for the level process case) and unknown or time varying constants. So, the two adaptive schemes were tested to the same scenarios. Considering a set point change from 0% to 40% and some uncertainties in the process parameters (both parameters are modified individually with  $\pm 50\%$ ) the responses after a cycle of filling and emptying the R2 tank are presented in figure 5.

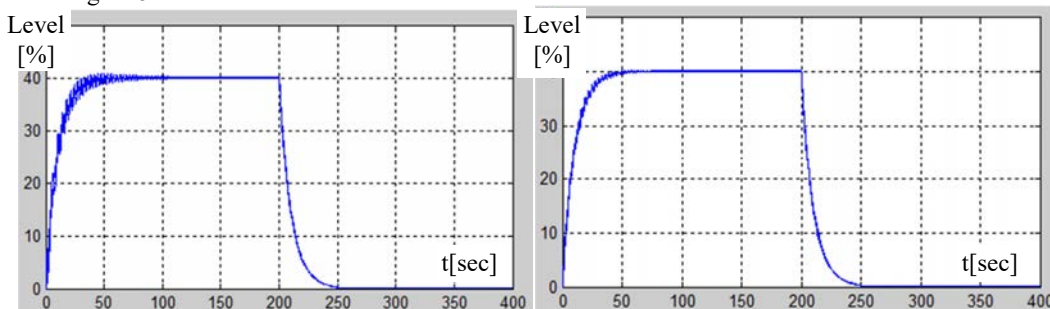


Fig. 5. The step responses of the adaptive systems (a) MIT rule; (b) Lyapunov rule.

The responses from figure 5 emphasize that both implemented methods manage to modify the controller parameters. So, the adaptation error is canceled and the plant output (the level of the R2 tank) tracks pretty quickly the imposed reference model output. Also, the MRAC which uses Lyapunov rule seems to perform better.

The adaptation speed of the parameters could be improved by increasing the adaptation gains (e.g. from 1 to 3), but the comparative study made in Fig. 6 indicates that adaptive control scheme with MIT rule may become unstable in time for higher values (e.g. 10).

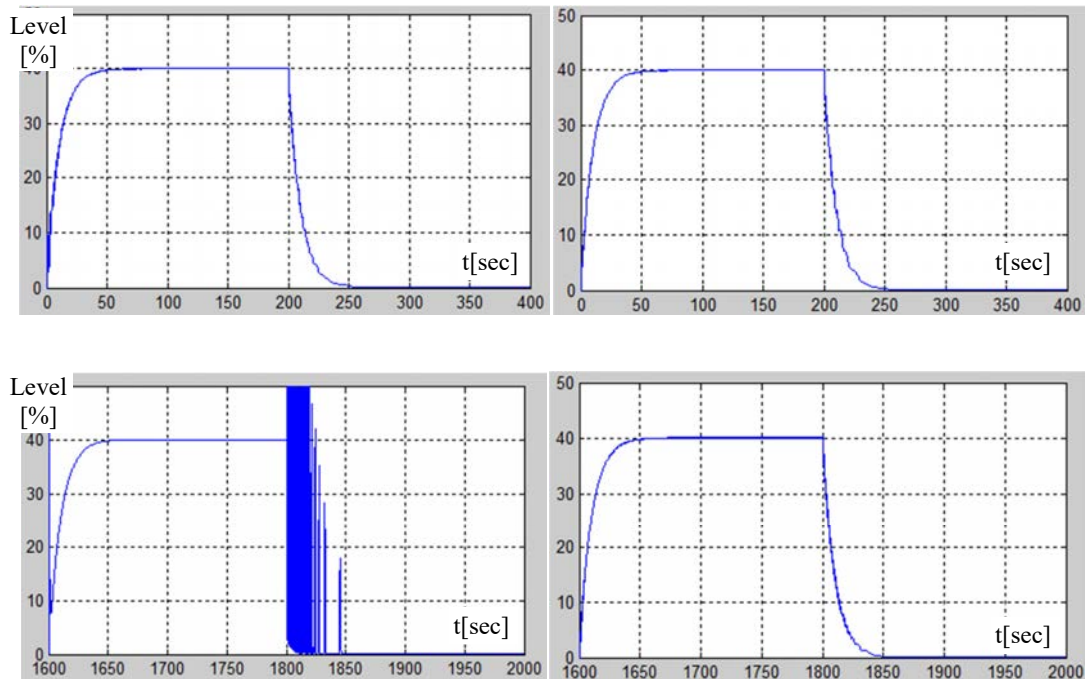


Fig. 6. Increasing the adaptation gains to 3, and than to 10 (a) MIT rule; (b) Lyapunov rule.

## 5. Conclusions

The paper presented the study of two MRAC schemes applied to a level process control. The design procedure was made by choosing a general first order reference model (based on a level process experimental setup) and so the control diagram could be easily extended to other slow processes. Both adaptive control diagrams (using MIT rule and Lyapunov rule) are inherently nonlinear.

The results shown firstly that the MRAC based on MIT rule is locally stable, depending on the adaptive gain and initial controller's parameters (close to stable parameters). For larger variations of the plant, the MIT rule may lead to instability and unbounded signal response.

The MRAC system based on Lyapunov theory can handle large variations of the plant parameters with slow varying dynamic response. Furthermore, in this case the stability of the closed-loop system and the convergence of the adaptation error are assured by the Lyapunov theory of stability.

In the next stage, a discrete adaptive structure can be determined and implemented digitally to tune the controller parameters and to control the level process. An eventual delay time or a second order system should be considered for a new mathematical model of the level process. Also, the design procedure could be tested on level processes with conical, spherical or coupled tanks or on other slow processes.

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