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Modeling and Simulation of the Operation of a Mechanical System which is Affected by Uncertainties

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Abstract

In control engineering, the mathematical model of the studied process plays an important part. In representing the mathematical model, the inherent errors due to parameter variations define the parametric (structured) uncertainties; therefore there are differences between the mathematical model used in the design process and the real plant. In most cases, the control system needs to stabilize the process and also needs to assure certain performances even in the presence of uncertainties, unmodeled dynamics, disturbance signals and measurement noise, which all make the process vulnerable.
This paper shows a method for behavioral modeling and simulation of a gear system, in which the moment of inertia and the friction coefficient represent the parameters which are affected by uncertainties, and are expressed by percentages to the nominal values. For the modeled process, by considering a classical controller, which is tuned experimentally, Matlab is used to represent the sensitivity functions in both time and frequency domains for the nominal cases, for randomly generated samples and for the worst case scenarios.

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Keywords: mathematical modeling; mechanical systems; uncertainties; control engineering; robust stability.

1. Introduction

There is a large amount of literature which deals with the problem of the mathematical modeling of systems, which also includes the mechanical systems and their control.

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The problem of uncertainties affecting real systems is discussed in [5,9,13]. The principles for the mathematical modeling of a series of systems, including mechanical ones, with examples and applications is developed in [2,4,12]. The analytical algorithms, used for quantifying the uncertainties, together with their applications in aerospace systems are investigated in [14]. General procedures for classical and robust control of the linear and nonlinear systems affected by disturbances and uncertainties are presented in [6,16] and their applications in robotics are developed in [3]. As a general approach, in order to develop robust control systems, the general algorithms, the examples and programs available in [7,11,15,17] are recommended. In [8], a mechanical system which is affected by bounded uncertainties is considered. For this system the uncertainties are time variant and an adaptive control law is proposed. Uncertainties also affect knowledge based systems, such as management systems, as discussed in [10].

The remainder of this paper is organized as follows: Section 2 shows the mathematical model of a mechanical geared system and the simulation schematic based on elementary mathematical blocks. Section 3 develops the general model of the mechanical system affected by uncertainties, including the corresponding schematic which uses elementary blocks. The graphical representations of section 4, resulted through simulation done in Matlab, using the dedicated functions of the Robust Control Toolbox [1]. The Conclusions paragraph discusses the results and the perspective this paper opens in the field of robust control.

2. The mathematical modeling of a rotary mechanical system

A driving electrical motor generally operates at rotational speeds and torques which are very different from the rotational speeds and torques needed in applications. Consequently, geared systems are used to ensure the conversion of high speed and low torque to low speed and high torque (or vice versa).

Fig. 1 shows a rotary mechanical system consisting of a gear ensemble driven by a DC motor (DCM) which generates the torque M_m. The motor's torque needs to overcome the momentum of inertia J_1, the friction momentum γ_1 and to generate the driving torque M_1 for the second gear [2,4,12].

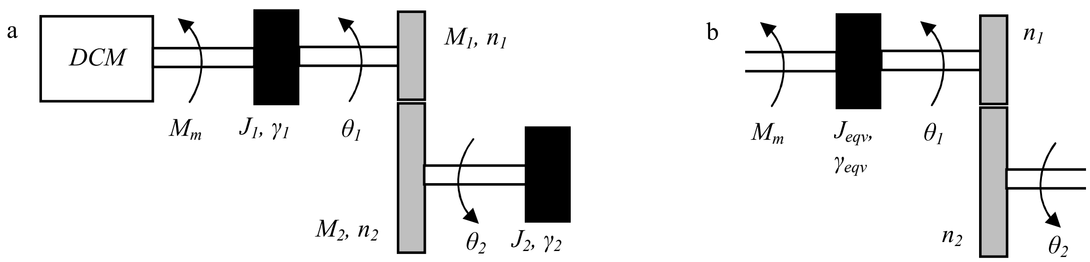

Fig. 1. Rotary mechanical system: (a) geared system; (b) equivalent system.

Considering the number of teeth (n_1, n_2), the angular movement (θ_1, θ_2) and the torques (M_1, M_2) produced by the two gears, the following equations stand:

$$\frac{M_1}{M_2} = \frac{n_1}{n_2} = \frac{\theta_2}{\theta_1}. \tag{1}$$

From the equilibrium of torques results:

$$J_1 \frac{d^2\theta_1(t)}{dt^2} + \gamma_1 \frac{d\theta_1(t)}{dt} + M_1(t) = M_m(t). \tag{2}$$

In a similar fashion, torque M_1 needs to covers the torques produced by the momentum of inertia J_2 and friction γ_2.

$$J_2 \frac{d^2\theta_2(t)}{dt^2} + \gamma_2 \frac{d\theta_2(t)}{dt} = M_2(t). \tag{3}$$

By replacing $\theta_2 = (n_1/n_2)\theta_1$ of (1) in equation (3), we get:

$$J_2 \frac{n_1}{n_2} \frac{d^2\theta_1(t)}{dt^2} + \gamma_2 \frac{n_1}{n_2} \frac{d\theta_1(t)}{dt} = M_2(t). \tag{4}$$

Likewise, by replacing $M_2 = (n_2/n_1)M_1$ of (1) in equation (3), and then in (2), we get:

$$J_1 \frac{d^2\theta_1(t)}{dt^2} + \gamma_1 \frac{d\theta_1(t)}{dt} + J_2\left(\frac{n_1}{n_2}\right)^2 \frac{d^2\theta_1(t)}{dt^2} + \gamma_2\left(\frac{n_1}{n_2}\right)^2 \frac{d\theta_1(t)}{dt} = M_m(t), \tag{5}$$

$$\left(J_1 + J_2\left(\frac{n_1}{n_2}\right)^2\right)\frac{d^2\theta_1(t)}{dt^2} + \left(\gamma_1 + \gamma_2\left(\frac{n_1}{n_2}\right)^2\right)\frac{d\theta_1(t)}{dt} = M_m(t), \tag{6}$$

$$J_{eqv} \frac{d^2\theta_1(t)}{dt^2} + \gamma_{eqv} \frac{d\theta_1(t)}{dt} = M_m(t), \tag{7}$$

where: J_{eqv} represents the equivalent momentum of inertia; γ_{eqv} represents the equivalent friction coefficient.
The transfer function from θ_1 to M_m is simply deduced and has the following form:

$$H_{MEC}(s) = \frac{\theta_1(s)}{M_m(s)} = \frac{\frac{1}{\gamma_{eqv}}}{s\left(\frac{J_{eqv}}{\gamma_{eqv}} s + 1\right)} = \frac{k_m}{s(T_m s + 1)}, \tag{8}$$

where: T_m represents the system's time constant, which mechanical in nature and; k_m represents the system's gain.
As a remark, the equations (1) to (7) can also be re-written by considering the angular velocities $\omega_1 = d\theta_1/dt$; $\omega_2 = d\theta_2/dt$.
The input-output mathematical model in (7) shows the dynamic behavior of the mechanical system and can be simulated using elementary mathematical operators/blocks as shown in Fig. 2. It is important to notice that the transfer function from θ_2 to θ_1 is given by the teeth ratio of the two gears and is marked in Fig. 2 by the doted block.

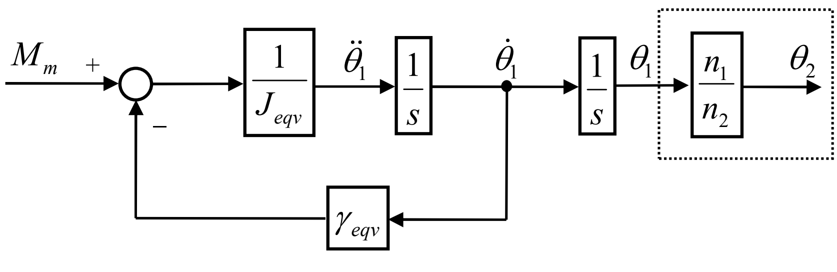

Fig. 2. Simulation schematic using elementary blocks.

3. The geared system as a system affected by uncertainties

For the mechanical system affected by uncertainties (Δ), the general (partitioned) transfer function has the general form:

$$H_{MEC}(s) = \frac{\theta_1(s)}{M_m(s)} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}, \tag{9}$$

where, the input-output relationship can be expressed in terms of the upper linear-fractional transforms $LFT_u(H_{MEC}, \Delta)$ of the form [7,9]:

$$\theta_1 = \left(H_{22} + H_{21}\Delta(I - H_{11}\Delta)^{-1} H_{12}\right) M_m = LFT_u(H_{MEC}, \Delta). \tag{10}$$

Based on equations (9) and (10), the mechanical system's parameters which are affected by uncertainties can be expressed as [7]:

$$\frac{1}{J_{eqv}} = \frac{1}{J_{eqvn}(1 + d_{J_{eqv}}\Delta_{J_{eqv}})} = \frac{1}{J_{eqvn}} - \frac{1}{J_{eqvn}} d_{J_{eqv}}\Delta_{J_{eqv}}(1 + d_{J_{eqv}}\Delta_{J_{eqv}})^{-1} = LFT_u(H_{J_{eqv}}, \Delta_{J_{eqv}}), \tag{11}$$

$$\gamma_{eqv} = \gamma_{eqvn}(1 + d_{\gamma_{eqv}}\Delta_{\gamma_{eqv}}) = \gamma_{eqvn} + \gamma_{eqvn} d_{\gamma_{eqv}}\Delta_{\gamma_{eqv}} = LFT_u(H_{\gamma_{eqv}}, \Delta_{\gamma_{eqv}}), \tag{12}$$

$$H_{J_{eqv}} = \begin{bmatrix} -d_{J_{eqv}} & \dfrac{1}{J_{eqvn}} \\ -d_{J_{eqv}} & \dfrac{1}{J_{eqvn}} \end{bmatrix};\ H_{\gamma_{eqv}} = \begin{bmatrix} 0 & \gamma_{eqvn} \\ d_{\gamma_{eqv}} & \gamma_{eqvn} \end{bmatrix}. \tag{13}$$

where: the index n is used to denote the nominal values of the parameters J_{eqv}, γ_{eqv} ; $d_{J_{eqv}}, d_{\gamma_{eqv}}$ represent disturbances which affect the parameters and $-1 \le \Delta_{J_{eqv}}, \Delta_{\gamma_{eqv}} \le 1$.

With the uncertainties described by (11), (12) and the matrixes (13), a simulation schematic results, as shown in Fig. 3, of which the relations (14) to (16) are extracted:

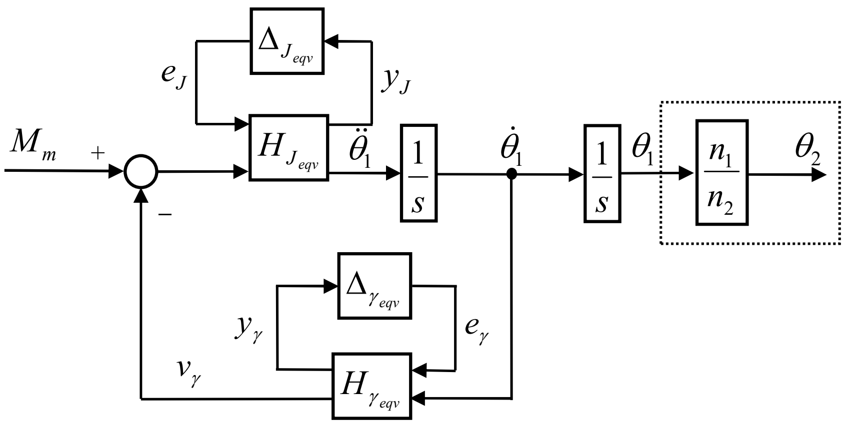

Fig. 3. Simulation schematic for the mechanical system affected by uncertainties.

$$\begin{bmatrix} y_J \\ \ddot{\theta}_1 \end{bmatrix} = \begin{bmatrix} -d_{J_{eqv}} & \dfrac{1}{J_{eqvn}} \\ -d_{J_{eqv}} & \dfrac{1}{J_{eqvn}} \end{bmatrix} \begin{bmatrix} e_J \\ M_m - v_\gamma \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} y_\gamma \\ v_\gamma \end{bmatrix} = \begin{bmatrix} 0 & \gamma_{eqvn} \\ d_{\gamma_{eqv}} & \gamma_{eqvn} \end{bmatrix} \begin{bmatrix} e_\gamma \\ \dot{\theta}_1 \end{bmatrix}, \tag{15}$$

$$e_J = \Delta_{J\,eqv} y_J;\;\; e_\gamma = \Delta_{\gamma\,eqv} y_\gamma . \tag{16}$$

After eliminating the intermediary variable v_γ in equations (14), (15) and (16) we get the equations which describe the dynamics of the mechanical system affected by uncertainties, as shown in (17):

$$\begin{cases} \ddot{\theta}_1 = \dfrac{1}{J_{eqvn}} M_m - \dfrac{\gamma_{eqvn}}{J_{eqvn}} \dot{\theta}_1 - \dfrac{d_{\gamma_{eqv}}}{J_{eqvn}} e_\gamma - d_{J_{eqv}} e_J \\ y_J = \dfrac{1}{J_{eqvn}} M_m - \dfrac{\gamma_{eqvn}}{J_{eqvn}} \dot{\theta}_1 - \dfrac{d_{\gamma_{eqv}}}{J_{eqvn}} e_\gamma - d_{J_{eqv}} e_J \\ y_\gamma = \gamma_{eqv} e_\gamma \end{cases} \tag{17}$$

The generalized model of the mechanical system affected by uncertainties highlights the nominal component (block) and the uncertainties block as shown in Fig. 4. The structure of the control system, having the controller *K*, which can be determined using classical or robust design methods is shown in Fig. 5 [7,9,13,17].

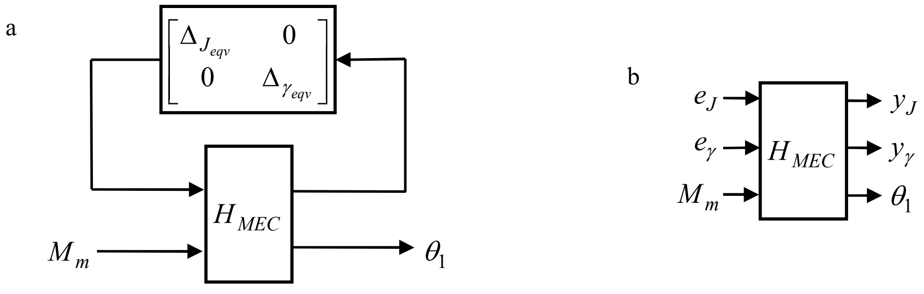

Fig. 4. The general model affected by uncertainties: (a) based on LFT; (b) input-output.

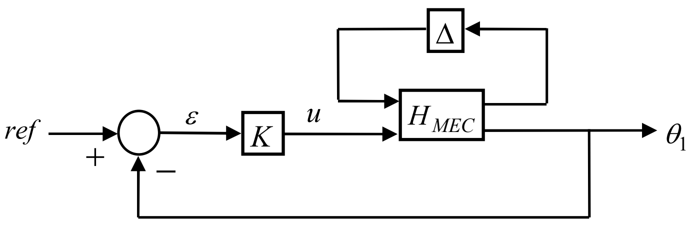

Fig. 5. The closed-loop control system.

4. Experimental results

Real systems are vulnerable to disturbance, noise, parameter variations etc., aspects that cannot be neglected in control systems engineering. The mathematical modeling and the analysis of such systems which are affected by uncertainties represents a mandatory and essential part in the design process of control systems which are able to assure the robustness of the performances.

In order to simulate the operation of the mechanical system, the following nominal values for the parameters are considered: $J_{eqvn} = 0.025$; $\gamma_{eqvn} = 0.25$. These values are affected by uncertainties in the following proportions: $d_{J_{eqv}} = 0.4$ (representing 40%), and $d_{\gamma_{eqv}} = 0.5$, (representing 50%).

Considering these variations of the parameters, expressed as a percentage, in the intervals: $J_{eqv} = [0.025 \quad 0.035]$; $\gamma_{eqv} = [0.125 \quad 0.375]$, ten randomly generated parameter sets were generated and the behavior of the system for the nominal and the unfavorable situations was shown.

The Matlab functions available in the Robust Control Toolbox allow the behavioral simulation of systems affected by uncertainties, by generating sets of characteristics around the nominal values of the parameters [1,7].

As a result, Fig. 6 shows the frequency response of the mechanical system for both the nominal case and the one which considers the uncertainties. The phase margin of the system is 71.2 degrees.

Due to the presence of the integrative term *1/s* in the transfer function of the mechanical system (8), a classical P controller ($K(s)=k_p$), tuned experimentally [6,16], is able to stabilize all responses with a null stationary error.

A measure of the closed-loop system performances is given by the sensitivity function shown in Fig. 7 for the nominal case, the uncertain case (which considers 10 randomly generated samples) and for the worst case scenario. In the time domain, the sensitivity function indicates the degree of disturbance rejection. Thus, Fig. 8 shows the step response for the same three scenarios: the nominal case, the uncertain case and the worst possible case.

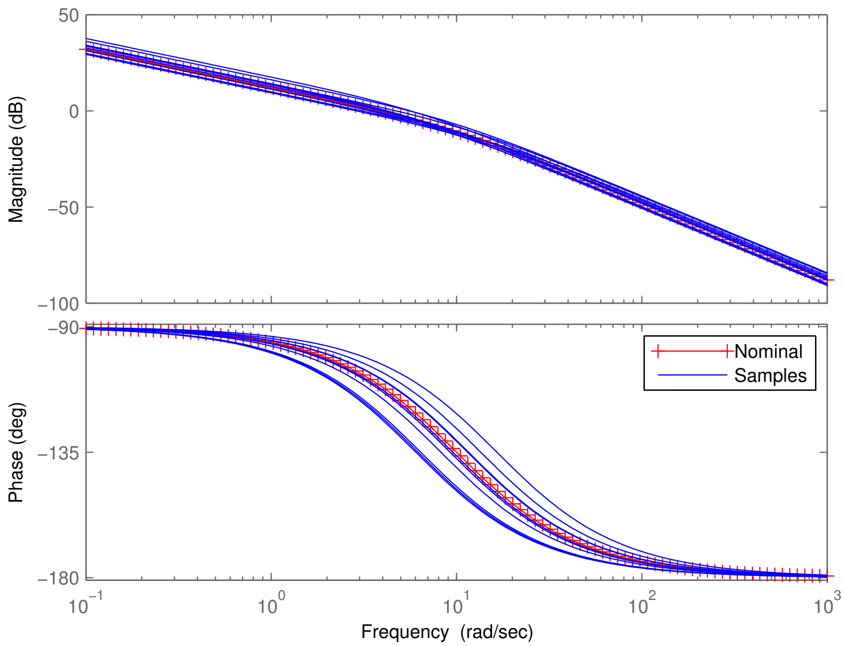

Fig. 6. Frequency domain behavior of the mechanical system. Bode diagrams.

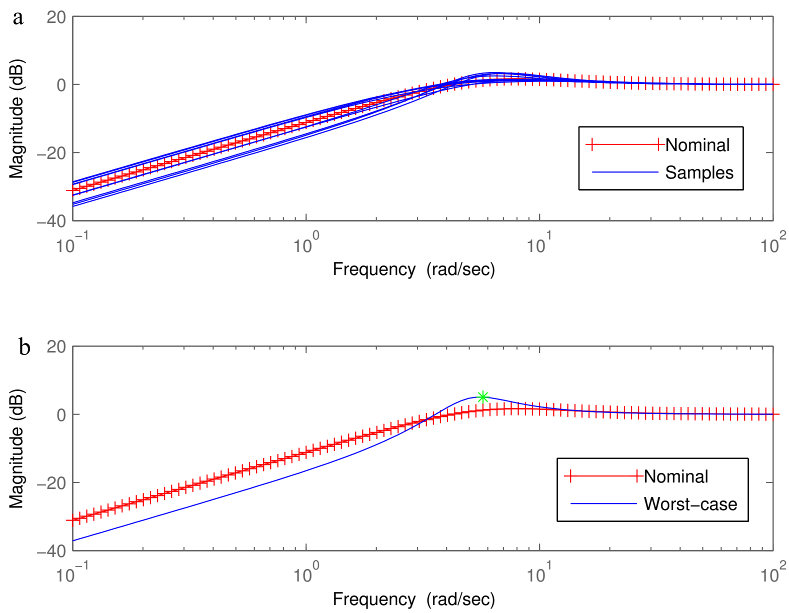

Fig. 7. Frequency domain sensitivity function: (a) nominal and uncertain cases; (b) nominal and worst-case.

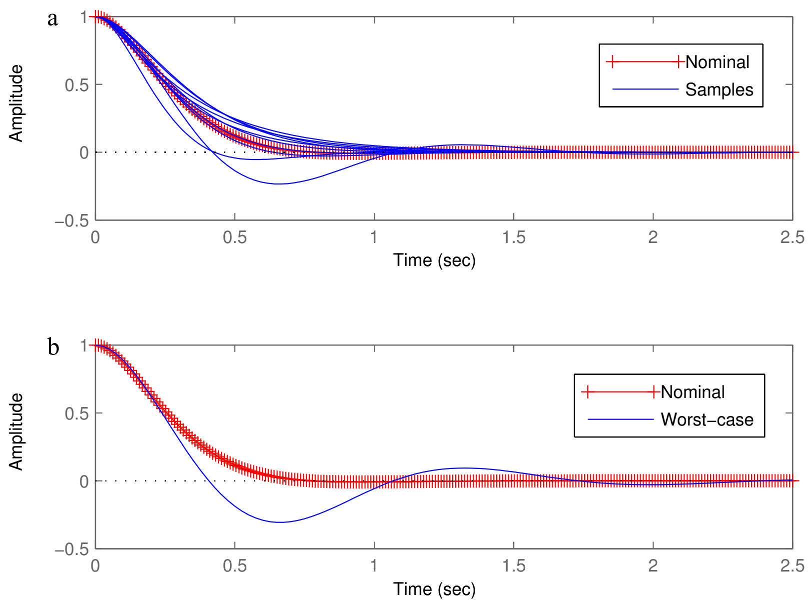

Fig. 8. Time domain sensitivity function: (a) nominal and uncertain cases; (b) nominal and worst-case.

5. Conclusions

This paper studied the behavior of a rotary mechanical system, having its parameters affected by uncertainties, by using the existing Matlab functions available in the Robust Control Toolbox. These functions facilitate the representation of the parameters affected by uncertainties and of the sensitivity functions, in both time and frequency domains, as methods of analyzing the performances and disturbance rejection of the closed loop.

Based on the classical mathematical model, taken from literature, the dynamic equations and the schematic of the rotary mechanical system affected by uncertainties were deducted.

In a future approach the effort of the authors will focus on the design of robust controllers using H-infinity, H2 and mixed synthesis, by imposing more cost functions.

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References

[1] Balas G. et al. Robust Control Toolbox. Getting Started Guide. The MathWorks Inc.; 2013.
[2] Bolton W. Mechatronics. A multidisciplinary approach. Fourth Edition. Prentice Hall; 2008.
[3] Celentano L. A General and Efficient Robust Control Method for Uncertain Nonlinear Mechanical Systems. The 44th IEEE Conference on Decision and Control and European Control Conference. Spain; 2005. p. 659-665.
[4] Chen CT. Analog and Digital Control System Design: Transfer-Function, State-Space, Algebraic Methods. Saunders College Publishing; 2004.
[5] Doyle J. et al. Feedback Control Theory. Macmillan Publishing Co.; 1990.
[6] Dumitrache I. Automation (Romanian version, Automatica). Vol. 1. Bucharest. Romanian Academy Publishing; 2009.
[7] Gu DW et al. Robust Control Design with MATLAB. Advanced textbooks in control and signal processing. Springer-Verlag London Limited; 2005.
[8] Huang QM. et al. Adaptive Robust Control for Fuzzy Mechanical Systems: Constraint-Following and Redundancy in Constraints. IEEE Transactions on Fuzzy Systems. Vol. 23. Issue 4. USA; 2015. p. 1113-1126.
[9] Ionescu V. et al. Systems theory. Robust synthesis. Numerical methods (Romanian version, Teoria sistemelor. Sinteza robustă. Metode numerice de calcul). All Publishing; 1994.
[10] Kovács L. et al. Uncertainty Management in Knowledge Modelling. The 8th International Conference Interdisciplinarity in Engineering (Inter-Eng 2014). Vol. 19. “Petru Maior” University of Tîrgu Mureş; Procedia Technology. 2015. p. 4-11.
[11] Mohammed TH. Modeling Mechanical and Electrical Uncertain Systems using Functions of Robust Control MATLAB Toolbox®3. International Journal of Advanced Computer Science and Applications (IJACSA). Vol. 6. No. 4. 2015. p. 79-84.
[12] Păstrăvanu O. et al. Bond-Graph Language in Modeling and Simulation of the Processes (Romanian version, Limbajul Bond-Graph în modelarea şi simularea proceselor fizico-tehnice), Iaşi. Gh. Asachi Publishing; 2001.
[13] Popescu D. Analysis and Synthesis of Robust Systems (Romanian version, Analiza si sinteza sistemelor robuste). Craiova Universitaria Publishing; 2006.
[14] Shankar S. et al. Analytical Algorithms to Quantify the Uncertainty in Remaining Useful Life Prediction. IEEE-AC Paper. 2013. p. 1-11.
[15] Vitecek A. et al. Robust control of mechanical systems. The 16th IEEE International Carpathian Control Conference (ICCC). Hungary; 2015. p. 575-579.
[16] Voicu M. System Theory (Romanian version,Teoria sistemelor). Bucharest. Romanian Academy Publishing; 2008.
[17] Zhou K. Essentials of Robust Control. Prentice Hall; 2009.