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# Estimation of Initial Functions for System With Delays Based On Adjoint Sensitivity Analysis

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**Abstract**

The work presents a gradient-based approach to estimation of initial functions of time delay elements appearing in models of dynamical systems. It is shown how to generate a gradient of the estimation quality index in the initial function space using adjoint sensitivity analysis. Results of gradient-based estimation of the initial function for exemplary model are presented.

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*Keywords:* Systems with dealys; estimation; gradient-based optimization; adjoint sensitivity analysis.

## 1. Introduction

Dynamical systems with delays are important class of models describing phenomena appearing in many areas, for example in industry or biology. One of the fundamental problems related to such models is a need to estimate their parameters.

There are many works dealing with this problem [1,4,8,10–14]. Unfortunately, most of proposed approaches assume that the analysed system is linear and both input and output signals for delaying elements can be measured.

More general and universal approach, for non-linear systems with delays have been proposed in [9] and [7]. Both approaches depends on gradient-based minimization of particularly defined objective function but the latter uses so-called structural adjoint sensitivity analysis which decreases significantly computational effort in case of estimation

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of many parameters. Moreover, the second approach can be applied to wider class of dynamical systems. It may be used for any non-linear dynamical system given as a block diagram with unlimited number of delays.

All mentioned approaches deal only with the problem of estimation of delay times or additionally other parameter describing the system. However the complete estimation problem for any mathematical model of dynamical system involves also estimation of initial conditions when they are unknown. In case of a delay element the initial condition (its “state” at $t = 0$ time) is a function of time, called initial function, specified for the time interval $[-\tau, 0]$, where $\tau$ is a delay time introduced by this element.

There are little works related to the problem of estimation of initial functions for systems with delays.

In paper [2] an approach a gradient based approach to estimation of initial functions for non-linear systems described by retarded type delay differential equations (RDDE). Another paper [3] deals with systems described by neutral type delay differential equations (NDDE). In both works a gradient-based estimation of initial functions is done by using adjoint sensitivity analysis.

In this work a more general approach is proposed. This approach can be applied to any non-linear dynamical system presented as a block diagram and containing unlimited number of delay elements. Therefore, it may be used for wider class than analised in [2] and [3]. For example it may be applied for systems containing delays in input (control) channel.

## 2. Problem formulation

Let us consider a model of dynamical system with one isolated delay element presented in Fig. 1.

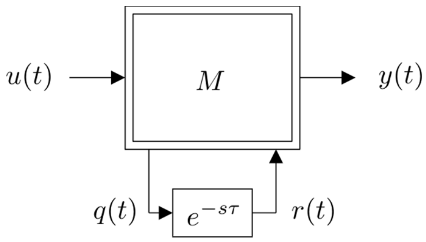


Fig. 1. Model of a dynamical system with one isolated delay element.

We do not assume any particular structure of the model *M*, for example RDDE or NDDE, but we assume that it is given in structural form — as a block diagram containing basic elements such as:

- Linear static element represented by a gain matrix *A*,
- Linear continuous-time dynamical element represented by a transfer function *K*(*s*),
- Non-linear static element described by a function $f(\cdot)$,
- Summing junction,
- Branching node.

Using such a set of elements one can build any non-linear dynamical with delays and with arbitrary structure.

For the sake of simplicity the system presented in Fig. 1 contains only one delay element but in general case there can be more delay elements with different delay times.

The delay element is described by the following input-output relationship

$$r(t) = q(t - \tau) \tag{1}$$

with the initial condition

$$q(t) = \varphi(t) \quad \text{for} \quad t \in [-\tau, 0] \tag{2}$$

The function $\varphi(t)$ is called initial function of the delay element.

Next, we define a quadratic objective function specifying the quality of the fit of the model output $y(t)$ to the measured output of the real plant $d(t)$:

$$J = \frac{1}{2}\int_0^{t_f} (y(t) - d(t))^2 \mathrm{d}t \tag{3}$$

**Problem 1.** *Find the initial function φ(t) of the delay element that minimize the objective function (3).*

This problem will be solved iteratively using gradient-based approach, therefore we need to solve following sub-task:

**Problem 2.** *Find the gradient of the objective function (3) in the space of the initial function φ(t).*

To solve Problem 2 we will use the adjoint sensitivity analysis. In works [5,6] rules for construction on the sensitivity model and the adjoint model have been presented. In addition in [7] such rules has been extended on systems with delays and it has been shown how to perform sensitivity analysis with respect to delay times. Now, we are going to show how to calculate the gradient of the objective function in the initial function space.

## 3. Problem solution

Let us present one delay element in the form which will be more suitable for further analysis. This form comes from the observation that the delay element with non-zero initial condition can be replaced by a delay element with zero initial conditions and with additional signal introduced as presented in Fig. 2.

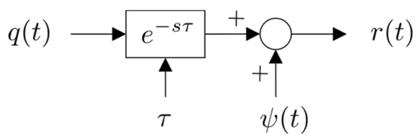


Fig. 2. Delay element with introduced additional input signal.

The function $\psi(t)$ is connected with the searched initial function $\varphi(t)$ by following relationship

$$\psi(t) = \begin{cases} \varphi(t-\tau) & \text{for } t \le \tau \\ 0 & \text{for } t > \tau \end{cases} \tag{4}$$

Thanks to this the Problem 2 may be replaced by another simpler problem of finding the gradient of the objective function in the space of the input signal $\psi(t)$ for times $t \in [0, \tau]$.

The only thing that has to be done is to build the adjoint sensitivity model for the delay element presented in Fig. 2. It can be easily done by using rules presented previously in [5,6]. The result, the sensitivity model and the corresponding adjoint system are presented in Fig. 3.

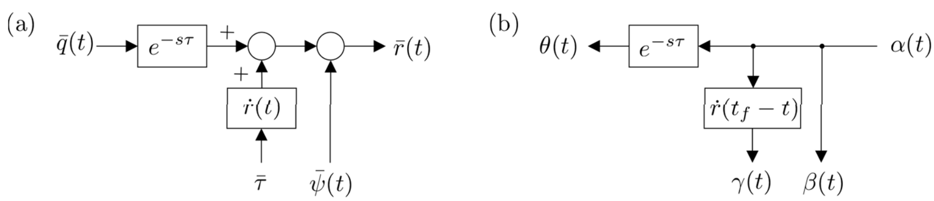


Fig. 3. The sensitivity model (a) and the adjoint system (b) for the delay element presented in Fig. 2.

The output signal $\beta(t)$ of the adjoint system is a reversed in time gradient of the objective function $J$ in the space of the input signal $\psi(t)$:

$$\beta(t_f - t) = \nabla_{\psi(t)} J \tag{5}$$

and by taking into account (4) it is also the solution to the Problem 2.

## 4. Numerical example

Let us consider that the real system and the model have the same structure presented in Fig. 4.

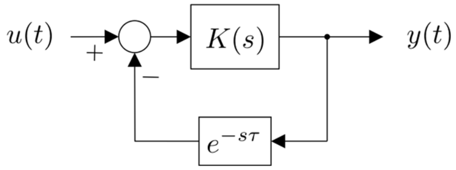


Fig. 4. An example of a system with delay.

The transfer function $K(s) = 1/(1+s)$, delay time $\tau$ = 1, final time $t_f = 5$[s], input signal $u(t)$ = const = 1. The initial function of the delay element appearing in the plant is a piecewise constant signal and is presented in Fig. 5(a) by a solid line.

The gradient-based estimation procedure has been run for 1000 iterations. In one iteration both original model and the adjoint system have been simulated, the signal β(t) has been recorded and used in the simplest gradient-descent updating rule with constant learning coefficient assuring numerical convergence. The value of the objective function J in successive iterations is presented in Fig 5(c). All simulations and the estimation procedure were done in Matlab.

The estimated initial function of the delay element in the model is presented in Fig. 5(a) by dashed line. One can see that it is convergent to the true initial function of the plant. Fig. 5(b) presents output signal of the plant $d(t)$ and the model $y(t)$. The differences between these two signals are almost invisible, which is also confirmed by small value of the objective function at the end of the estimation procedure (less than $10^{-4}$).

## 5. Conclusions

The gradient-based approach to estimation of initial functions for system with delays has been presented. In this algorithm the gradient of the objective function in the initial function space is calculated by using adjoint sensitivity analysis. The method may be applied for any non-linear system containing unrestricted number of delay elements with different delay times. The simulated adjoint system gives not only the discussed gradient with respect to the initial functions but also w.r.t all parameters of the model, therefore estimation of other parameters is possible without increase of simulation time of the adjoint system. The numerical example confirms usefulness of the proposed method. The presented approach can also be extended on systems where the output signals are measured only at discrete time points. It would require different definition of the minimized performance index. The same methodology, without major modifications, can be exploited for estimation of unknown/unmeasurable signals inside any complex non-linear system.

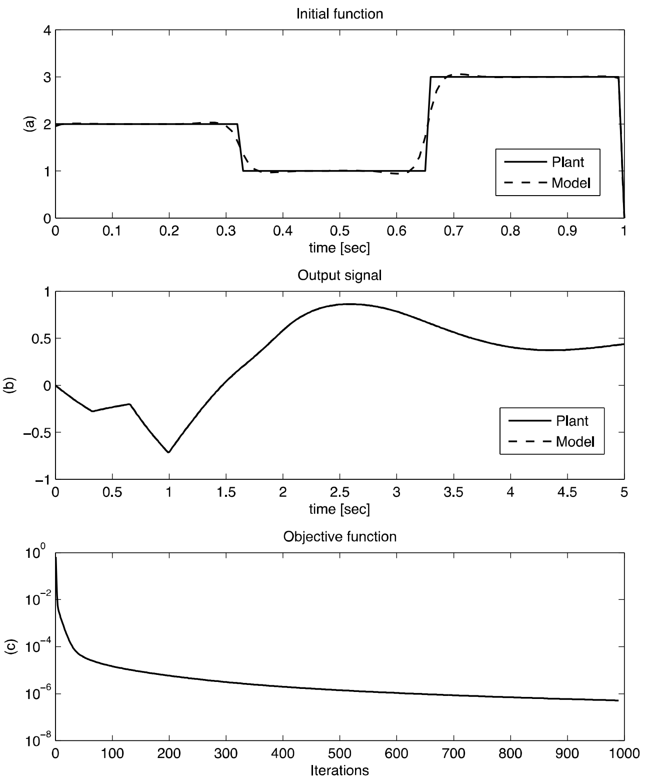


Fig. 5. Results of the initial function estimation: (a) the initial functions of the plant and the model after estimation procedure, (b) output signals of the plant and the model, (c) value of the objective function during estimation procedure.

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