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Localization of Impact Force Occurring on Elastic Plates by Using a Particle Swarm Algorithm

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Abstract

Identifying the characteristics of the force generated by impact can be used to better determine the residual health of structures that are susceptible to undergo impacts. This can enable to reduce hugely the required experimental effort that may be needed in order to assess completely the actual state of the structure as it could be damaged by the action of forces generated during these accidental events.

This paper presents the application of a Particle Swarm Optimization technique to solve the localization problem associated to an impact that cannot be reduced to a point and occurring on homogeneous and isotropic elastic plates. This was performed through minimizing the root mean square error between the measured and the calculated responses. The problem was recognized to have the general form of parametric identification. The impacting force was assumed to result from uniformly distributed pressure acting over a rectangular patch. The Particle Swarm Optimization model has demonstrated high computational efficiency in finding the impact patch location. A parametric study was conducted finally to analyze sensitivity of the procedure to the number of sensors.

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Keywords: Inverse problem; force localization; reciprocity theorem; fitness function; Particle Swarm Optimization.

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1. Introduction

Structures can be subjected to impacts from various sources. Accidental impacts may cause considerable damage and threaten the structural integrity. During impacts, simple examination of apparent traces on the structure surface is not sufficient because the damage may be invisible and deep inside the core of the structure [1]. The inspection by experimental means to assess damage that has occurred after an impact event uses specific diagnosis techniques which are too expensive, like for example X-ray imaging, tomography, etc.

Identifying the characteristics of the force generated by impact can be used to better know the residual health of structures at risk of impacts, and thus enabling to reduce hugely the required experimental effort [2]. In case of simple linear elastic structures with homogeneous geometric and material properties such as beams or plates, identification of impact characteristics can be implemented through using a structural model [3]. This can be constructed analytically, by means of the finite element method or by experimental identification procedures. When the impact can be assumed as being punctual and the impact location is known, the impulse response functions between the impact point and the sensors placed at known positions, allows by using a regularized deconvolution to reconstruct the force signal [4-7].

When the point of impact is unknown, the inverse formulation uses a minimization technique between the measured and calculated responses to reconstruct the impact characteristics in two steps: at first point location and then force time signal.

If now the impact is not punctual, the problem is more complex because it involves identifying a distribution of pressure and not a single concentrated force. Even if the pressure can be considered to be uniform, new parameters that represent the extent of the impacted zone appear in the problem [8].

In this work, a PSO based technique is developed in order to determine the impact location characteristics for elastic rectangular plate like structures subjected to non punctual impacts. The impact zone is assumed to have the form of a rectangular patch. The objective function to be minimized and the constraints are derived as function the discrete time response functions between sensors locations where responses are measured and the conjectured impact zone center location as well as its extent parameters. A parametric study dealing with the effect of the number of sensors used is conducted.

Nomenclature

a	Length of plate
b	Width of plate
e	Thickness of plate
x	Horizontal coordinate
y	Vertical coordinate
t	Time
$w(x,y,t)$	Transverse displacement
$q(x,y,t)$	Applied loading
D	Plate flexure rigidity modulus
$\Im$	Indicative function
c	Damping coefficients
P	Impact force
E	Young's modules
ρ	The plate material density
ν	Poisson coefficient
h	Transfer function
x_0	Abscissa of impact zone centre
y_0	Ordinate of impact zone centre
u_0	Extent length of impact zone
v_0	Extent width of impact zone
ω_{mn}	Circular eigenfrequency

χ_{mn}	Damped circular eigenfrequency
ς_{mn}	Damping ratio
Δt	Sampling time
N	Total number of samples
η	bound parameter
PSO	Particle Swarm Optimization
d	Dimension of the search space (here d=4 as we have four unknowns)
c_1, c_2	Acceleration coefficients
r_1, r_2	Random numbers
$P_i = (P_{i1}, \ldots, P_{id})$	Best position by a particle having index i
g_j	Best overall position reached by all particles

2. Formulation of the direct problem

We consider a rectangular plate as shown in Figure 1 which has the dimensions a, b and e. It is assumed to be simply supported on its all ends. The plate is assumed to be made of a homogeneous and isotropic elastic material having Young's modulus, Poisson's ratio and density.

The applied force modeling impact is assumed to result from pressure that is uniformly distributed over a rectangular patch of the plate. The dynamic response in terms of displacement is considered at some points belonging to the set of marked points that are shown on Figure 1.

The equation of motion of a simply supported rectangular plate can be expressed under the following form

$$D\Delta\Delta w(x,y,t) + c\dot{w}(x,y,t) + \rho h\ddot{w}(x,y,t) = q(x,y,t) \quad (1)$$

with $q(x,y,t) = p(t)\Im_{[x_0-u_0/2, x_0+u_0/2]\times[y_0-v_0/2, y_0+v_0/2]}(x,y)$ and $D = Ee^3/(12(1-\upsilon^2))$

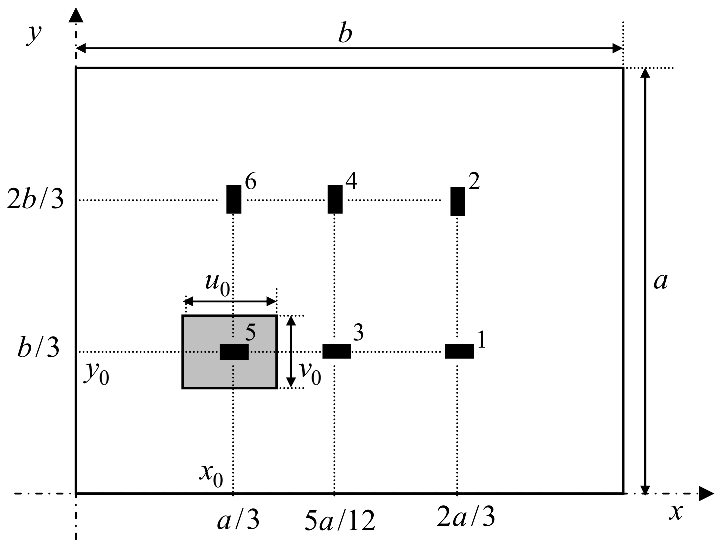

Fig. 1. Simply supported rectangular plate showing the loading patch characteristics and the considered strain gauges locations and orientations.

By applying modal superposition expansion, the displacement of a linear system can be estimated by Duhamel's convolution integral as

$$w(x,y,t)=\int_0^t h(x_0,y_0,u_0,v_0,x,y,t-\tau)\,p(\tau)d\tau \tag{2}$$

with the transfer function given by

$$\begin{gathered}h(x_0,y_0,u_0,v_0,x,y,\tau)=\frac{16}{\rho h\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mn\gamma_{mn}}\sin(\frac{m\pi x_0}{a})\sin(\frac{n\pi y_0}{b})\sin(\frac{m\pi u_0}{a})\sin(\frac{n\pi v_0}{b})\\ \sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})\sin(\gamma_{mn}\tau)e^{-\xi_{mn}\omega_{mn}\tau}\end{gathered} \tag{3}$$

Where $\gamma_{mn}=\omega_{mn}\sqrt{1-\xi_{mn}^2}$

The elastic response can be computed over the considered time interval $[0,T_c]$ by integrating explicitly equation (2).

To solve the deconvolution problem associated to equation (2), a discrete problem must be written by sampling the convolution integral. Using rectangular integration rule, this leads in the time domain to the following system of algebraic equations

$$W=HP \tag{4}$$

With $W=\begin{bmatrix}w(\Delta t) & w(2\Delta t) & \cdots & w(N\Delta t)\end{bmatrix}^t, P=\begin{bmatrix}p(\Delta t) & p(2\Delta t) & \cdots & p(N\Delta t)\end{bmatrix}^t$

and

$$H=\begin{pmatrix} H(\Delta t) & 0 & & & 0\\ H(2\Delta t) & H(\Delta t) & \ddots & & \\ H(3\Delta t) & H(2\Delta t) & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & 0\\ H(N\Delta t) & H((N-1)\Delta t) & \dots & \dots & H(\Delta t)\end{pmatrix} \tag{5}$$

which is the Toeplitz like transfer matrix.

3. Formulation of the localization problem

The problem of locating non punctual impact zone, having the form of a rectangular patch, requires identifying the impact zone center position and parameters defining the extent of the impact zone, which are here the length and width of the patch. The displacement responses measured by strain sensors placed in points having respectively the coordinates (x_i,y_i) and (x_j,y_j) can be expressed under the following form

$$Y_i = H_i(x_0, y_0, u_0, v_0)P$$
$$Y_j = H_j(x_0, y_0, u_0, v_0)P \tag{6}$$

With $H_i(x_0, y_0, u_0, v_0) = H(x_0, y_0, u_0, v_0, x_i, y_i)$ and $H_j(x_0, y_0, u_0, v_0) = H(x_0, y_0, u_0, v_0, x_j, y_j)$

The commutative property that results from Maxwell-Betti theorem for elastic systems, yields as shown in [10, 11] the following relation

$$H_j(x_0, y_0, u_0, v_0)Y_i = H_i(x_0, y_0, u_0, v_0)Y_j \tag{7}$$

The interest of this relation is that it does not depend on the time history of the applied pressure. It enables then decoupling the problem of localization from that of pressure signal reconstruction. Impact characteristics are like this identified in two steps. In the first step, the impact location zone is determined. Then, in the second step pressure time signal is obtained through classical regularizing deconvolution techniques, like those based on the generalized singular value decomposition theorem. Regularization could be performed either by using Tikhonov technique or truncation technique [5].

The four parameters that define the impact location x_0, y_0, u_0 and v_0 can be found by minimizing the following loss fitness function.

The stiffness matrix under the following form

$$(x_0, y_0, u_0, v_0) = \underset{(x,y,u,v)}{A\text{rg}\min} \left\{ \phi(x, y, u, v) = \sum_{i=1}^{N_s} \sum_{\substack{j=1 \\ j \neq i}}^{N_s} \alpha_{ij} \left\| H_j(x, y, u, v)Y_i - H_i(x, y, u, v)Y_j \right\|^2 \right\} \tag{8}$$

where $N_s \geq 2$ denotes the number of used sensors and α_{ij} are weights that avoid trivial solutions to be obtained such as the points located on the borders of the plate.

The following expressions can be taken for the coefficients α_{ij} [8]

$$\alpha_{ij} = \frac{1}{\left\| H_i(x, y, u, v) \right\|^2 + \left\| H_j(x, y, u, v) \right\|^2} \tag{9}$$

The minimization of the functional ϕ is subjected to the following constraints

$$\eta \leq x_0, \eta \leq y_0, u_0 \leq \eta, v_0 \leq \eta \tag{10}$$

that are required for the patch defining the impact zone to be physically incorporated on the plate area.

4. Particle Swarm Optimization

The problem defined by equations (8), (10) and (11) takes the form of a nonlinear mathematical program for which it is not easy to explicit the objective function. To solve it, an evolutionary algorithm based on particles swarm is used [9, 10].

Particle Swarm is a heuristic optimization method that mimics the social behaviour. Its implementation needs to specify the protocol of cooperation and competition among the potential solutions. In this technique, the domain of the objective function to be minimized is chosen randomly and each particle has an index i ranging form 1 up to N_p. It occupies the position $X_i(t)$ in the space of unknowns has the velocity $V_i(t)$.

In each generation t, the value of the objective function in each position $X_i(t)$ is calculated and the following updating rules are applied

$$\begin{aligned} &V_{ij}(t+1)=wV_{ij}(t)+c_1r_{1j}(t)\left(P_{ij}(t)-X_{ij}(t)\right)+c_2r_{2j}(t)\left(g_j(t)-X_{ij}(t)\right) && i=1,\dots,N_p \\ &X_{ij}(t+1)=X_{ij}(t)+V_{ij}(t) && j=1,\dots,d \end{aligned} \tag{11}$$

The term $wV_{ij}(t)$ represents the physical component of displacement; the particle tends to follow its current direction of travel. The term $c_1r_{1j}(t)\left(P_{ij}(t)-x_{ij}(t)\right)$ is the cognitive component of displacement; the particle tends to move towards the best site in which it has already passed. The term $c_2r_{2j}(t)\left(g_j(t)-x_{ij}(t)\right)$ is the social component part of displacement; the particle tends to rely on the experience of its own and, thus, to move towards the best quality position already achieved by its neighbours.

For the PSO based algorithm to converge towards the problem solution a good estimation of the bound η appearing in equation (10) is required. Here, we have used the following sequential estimation of this parameter

$$\begin{aligned} &\eta(1)=3\max\left(a,b\right)/10 \\ &\eta(t+1)=\max\left(x_0(t),y_0(t)\right)/3 \end{aligned} \tag{12}$$

5. Results and discussion

The impact zone characteristics that are a considered in this case study are given (in m) by: $x_0=0.0683$, $y_0=0.0683$, $u_0=0.0342$, $v_0=0.0342$. The pulse period considered is $T=6\,ms$. Figure 2 presents the impact pressure time signal. The maximum pressure is $2\times10^5\,Pa$.

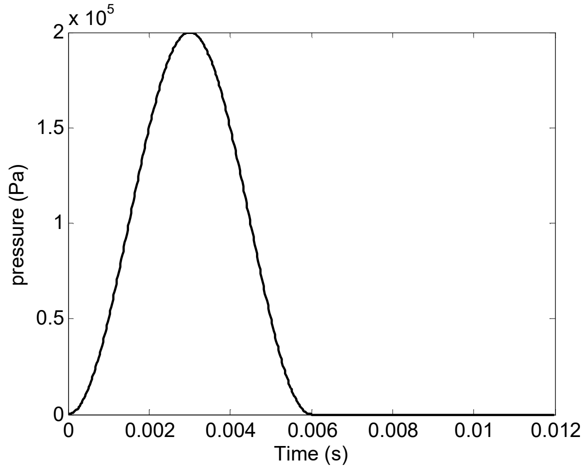

Fig. 2. Pressure signal profile having half-sine form.

The PSO based algorithm should perform search of the minimum in the fourth dimensional space containing all the unknowns of the localization problem. Rate of convergence of this algorithm is influenced by many factors: the mechanical model used for the system, the sampling rate to obtain equation (4) and the number of sensors. The model can be rendered exact by choosing a sufficient number of terms in the time transfer function defined by equation (3) before operating truncation, a hundred of terms are found to sufficient. As the sampling rate can be fixed through Shannon rule in order to preserve the frequency spectrum content of the input pressure, focus is done in the following on the crucial effect associated to the configuration of sensors that are implemented on the plate.

To study the influence of the number of implemented sensors N_s on convergence of PSO algorithm, four configurations were chosen among all the possible ones shown in Figure 1: 2 sensors (#1 and #2), 3 sensors (#1, #2 and #3) , 4 sensors (#1, #2, #3 and #4) and 5 sensors (#1, #2, #3, #4 and #5).

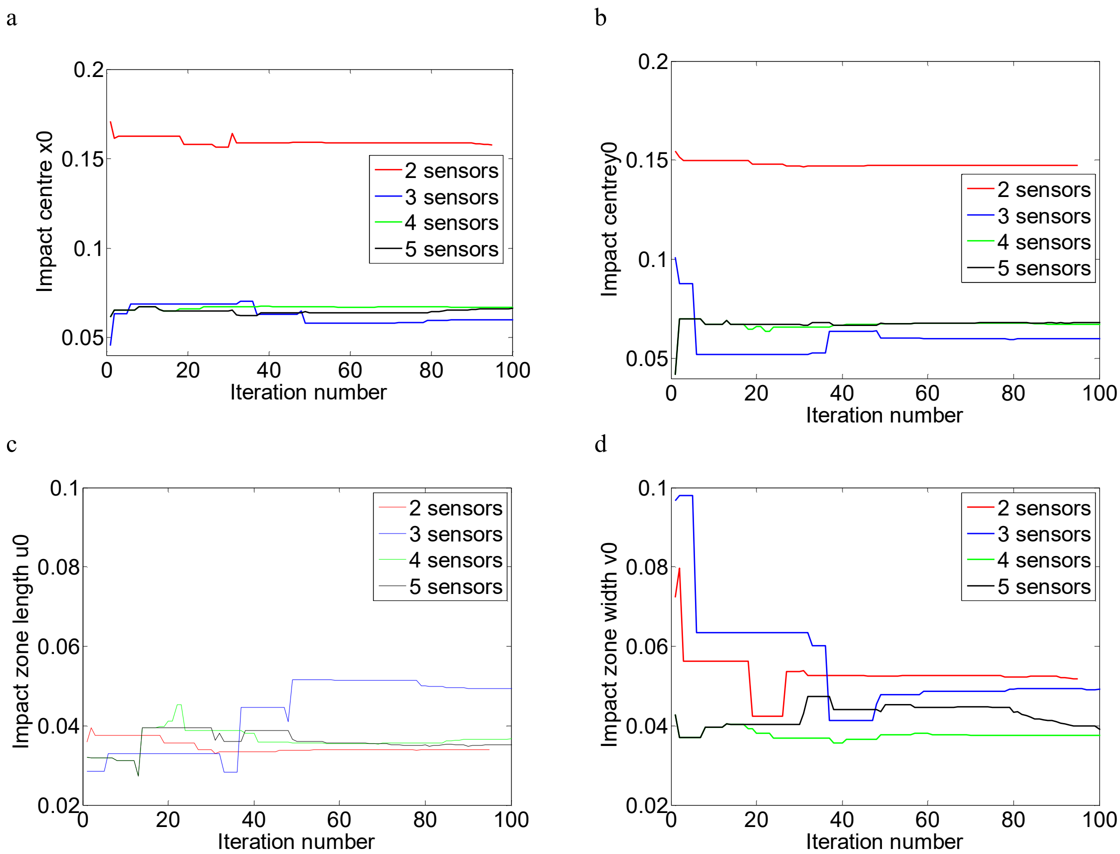

Fig. 3. Evolution of impact zone center (a) abscissa; (b) ordinate; (c) length; (d) width as function of the iteration number.

Figures 3a, 3b, 3c and 3d present evolution of the actual impact zone characteristics as function of the number of iterations (generations) of the PSO algorithm.

Obviously we noticed that two sensors are not sufficient to ensure convergence, convergence values characteristic of the impact zone remains far from exact values. Convergence even failed with three sensors; for the abscissa and ordinate of the centre of impact their optimized values are close to the actual values, whereas the values found for length and width of impact zone are still far from the exact solution. Increased number of sensors is required to ensure convergence of the algorithm.

In the case of four sensors, which corresponds to the green curve plotted in Figure 3 the convergence has been reached, but it is not accurate, there is always a relative error.

For the fourth configuration where we used five sensors, the exact solution is obtained. We need more than 100 generations PSO to ensure convergence of the algorithm. The advantage associated with the provision of a sufficient number of sensors is considerable for the smooth running of the location procedure.

6. Conclusions

Reconstruction of force characteristics induced by non punctual objects impacting homogenous elastic rectangular plates was achieved in this work. Assuming the pressure signal resulting from impact to be uniform, the obtained results demonstrate the possibility of determining the impact zone center and extent when it has the shape of a rectangular patch. The reciprocity theorem was used in order to decouple localization step from deconvolution problem. Solution of the localization problem which took the form of a constrained non linear mathematical program was performed by using a modified version of the Particle Swarm Optimization algorithm. The proposed algorithm was found to behave pretty well and parametric studies with regards to the influence of the number of sensors used on the rate of convergence were conducted. It was found that two sensors are not sufficient, we have shown in this work to ensure convergence, we will use 5 sensors, to increase the rate of convergence, the number of sensors should be as high as possible and their locations should preferably be far from the axes of symmetries of the plate.

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