Available online at www.sciencedirect.com

ScienceDirect

Procedia Technology 22 (2016) 86 – 93

Procedia Technology

9th International Conference Interdisciplinarity in Engineering, INTER-ENG 2015, 8-9 October 2015, Tirgu-Mures, Romania

Identification of Beam Cracks by Solution of an Inverse Problem

Sara Teidj[a,]*, Abdellatif Khamlichi[a], Abdallah Driouach[a]

[a]*Systems of Communications and Detection Laboratory, Faculty of Sciences at Tetouan, BP: 2121, M'Hannech II, Tetouan 93002, Morocco*

Abstract

Detection of cracks in mechanical components as earlier as possible enables structural health monitoring and scheduling more effectively the maintenance tasks, such as replacing the critical parts just in time. Vibration analysis based techniques for crack detection have been largely considered in the literature. This methodology relies essentially on the observed changes of beam frequencies and mode shapes that are induced by the presence of damage. In the present work, using an explicit analytical model assessing the effect of a crack on beam strain energy, the beam first resonance frequencies were evaluated as function of a single crack defect characteristics. The crack equations were obtained by means of fracture mechanics approach. Variations of the first beam frequencies and modes shapes were then related explicitly to the location and depth of the crack. Measuring the beam frequency changes and monitoring their variations is shown to enable identification of the crack defect parameters by solution of an inverse problem.

Peer-review under responsibility of the "Petru Maior" University of Tirgu Mures, Faculty of Engineering

Keywords: Detection; structural health monitoring; vibration analysis; inverse problem.

1. Introduction

Detection of cracks in structural components and identification of their size for structures having the form of beams is of crucial importance in many engineering applications [1, 2]. Many works have dealt with this problem of crack detection [3-10].

Among the proposed methods, the authors in [3] have used a vibration analysis based technique and introduced a damage magnitude index. This index relies on the observed changes of beam frequencies and mode shapes.

* Corresponding author. Tel.: 0660860281.
E-mail address: sara.teidj@gmail.com

Peer-review under responsibility of the "Petru Maior" University of Tirgu Mures, Faculty of Engineering
doi:10.1016/j.protcy.2016.01.014

Vibration based methods can be used to perform non-destructive detection of cracks in beams by means of a structural model that takes into account the interaction of modal features with the crack characteristics. Gudmundson [11, 12] has developed a perturbation method based on the transfer matrix method to track variations of the modal frequencies.

Nomenclature

a	crack depth
b	with of beam
h	height of beam
ℓ	element of length
L	length total of beam
M	flexure moment
P	transverse force
E	Young's modulus
K_I	mode I stress intensity factors
K_{II}	mode II stress intensity factors
K_{III}	mode III stress intensity factors
K_{IP}	stress intensity factors for a rectangular cross-section crack opening
K_{IIP}	stress intensity factors for a rectangular cross-section crack sliding
K_{IM}	stress intensity factors for a rectangular cross-section crack tearing
W_o	strain energy of an intact beam element
W_c	strain energy resulting from the presence of a crack
K	element stiffness matrix
K^0	element stiffness matrix of an intact beam element
K^c	cracked element stiffness matrix
S_{ij}	total flexibility coefficients
S_{ij}^0	flexibility coefficients of an intact beam element
S_{ij}^c	flexibility coefficients of a beam having a crack
S	total flexibility matrix
S^0	flexibility matrix for the crack free element
S^c	flexibility matrix for the crack element
T	transformation matrix
$\tilde{a}$	predicted crack depth
f_1	first frequency
f_2	second frequency
$\tilde{f}_1$	measured first frequency are perturbed
$\tilde{f}_2$	measured second frequency are perturbed
ν	Poisson coefficient
ρ	beam material density
ε_1	first noise measurement
ε_2	second noise measurement

Dimarogonas et al. [13] have derived the flexibility matrix for a cross-section containing a crack. This matrix has enabled to solve the coupled vibration problem of a beam suffering from the presence of a crack [14].

Opening and closing of the crack occurs periodically during the deformation of the beam, the stiffness matrix and equation of motion for the cracked beam were adapted in this last work to account for this nonlinear behavior. Considering the first modal frequencies of the cracked beam, experimental evidence [15] has shown that the state of crack (open or closed) has not a significant effect on their values, yielding the possibility to calculate them by using a linear approximation and neglecting opening and closing of the crack.

In this work a finite element based method is applied in order to derive the full beam crack coupled equations for the purpose of monitoring parametrically the modal changes resulting from the presence of a crack. Applying fracture mechanics to the beam with a single crack, the modified element stiffness is determined. This enables to state the forward problem which describes the variations of the first beam frequencies and modes shapes in terms of crack characteristics that include crack location and its actual depth. In the inverse problem attempt is made to use the measured first modal properties variations as compared to crack free reference state in order to retrieve the crack characteristics. Solution of this problem can be obtained by minimizing the residue error between the actual measured values of frequencies and their theoretical predictions as provided by the forward problem.

As solution on an inverse problem is usually polluted by noise, the effect of noise is assessed in the following as function of its level when the two first frequencies are selected for the identification of crack depth. Variation of the error is expressed as function of measurement noise affecting these two frequencies.

2. Analysis and modelling

The presence of a crack in a beam affects the deformation of the region in the vicinity of the crack, while far from that location the strain field is nearly unaffected. Hence, the element stiffness matrix for an element situated enough far from the crack may be assumed to remain unchanged.

It is usually assumed in the framework of finite element approximation that the crack does not modify the mass distribution. The crack modifies essentially the strain energy and contributes with additional fracture energy. Fracture mechanics enables to express the related flexibility coefficient by means of stress intensity factors.

A homogeneous beam having a uniform rectangular cross section and undergoing only transverse displacement due to out-of-plane flexure, Figure 1, has the following mass matrix:

$$M = \frac{\rho b h \ell}{420}\begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix} \tag{1}$$

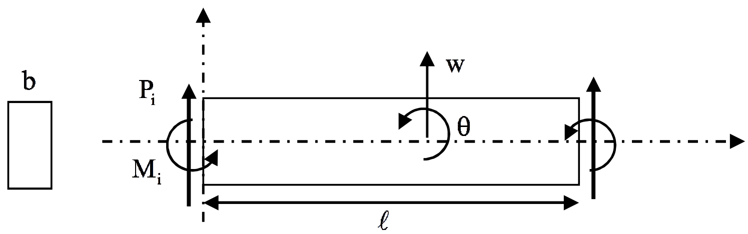

Fig . 1. Geometry of the beam element and internal efforts at the extremities.

Under the assumption that the beam element is anchored on its left extremity and subjected to a transverse force and a flexure moment on its right extremity, Figure 2, the strain energy of an intact beam element, if shear is neglected can be obtained as:

$$W_0 = \frac{2\ell}{Ebh^3}\left(3M^2 + 3MP\ell + P\ell^2\right) \tag{2}$$

The additional strain energy resulting from the presence of a crack of depth a at a given section of the beam has the following form:

$$W_c = \frac{b}{E}\int_0^a \left[(1-\nu^2)(K_I^2 + K_{II}^2) + (1+\nu)K_{III}^2\right]da \tag{3}$$

Under the action of applied external loads P and M as is shown in Figure 2, one can see that the strain energy due to crack contribution takes the following for

$$W_c = \frac{b(1-\nu^2)}{E}\int_0^a \left[(K_{IM} + K_{IP})^2 + K_{IIP}^2\right]d\xi \tag{4}$$

with K_{IM}, K_{IP}, K_{IIP} given in [15].

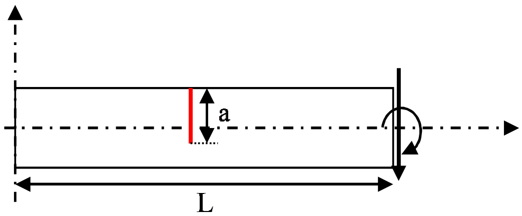

Fig . 2. Geometry of the beam element and the applied external forces

The total flexibility coefficients are then obtained as the sum of these two contributions under the following form [15]

$$S_{ij} = S_{ij}^0 + \alpha S_{ij}^c \tag{5}$$

with $\alpha = 0$ for a crack free element and $\alpha = 1$ for cracked element.

The static equilibrium of an element of length ℓ having the nodes $(i, i+1)$ as shown in Figure 1 can be written under the following form

$$\begin{bmatrix} P_i \\ M_i \\ P_{i+1} \\ M_{i+1} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -\ell & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_{i+1} \\ M_{i+1} \end{bmatrix} = T \begin{bmatrix} P_{i+1} \\ M_{i+1} \end{bmatrix} \tag{6}$$

where $\left[P_i \quad M_i\right]^t$ is the vector of internal forces applied at node i and $\left[P_{i+1} \quad M_{i+1}\right]^t$ is the internal forces associated to node $i+1$.

Equations (5) and (6) enable to calculate the element stiffness matrix as

$$K = T^t S^{-1} T = T^t \left(S^0 + \alpha S^c\right)^{-1} T \tag{7}$$

Using the strain energy given in equation (2), the flexibility matrix for the crack free element writes

$$S^0 = \frac{12\ell}{Ebh^3}\begin{bmatrix} \frac{\ell^2}{3} & \frac{\ell}{2} \\ \frac{\ell}{2} & 1 \end{bmatrix} \tag{8}$$

Using the strain energy induced by a crack, as given in equation (4), the part of flexibility resulting from a crack can be calculated explicitly under the following form [16]

$$S^c = \frac{12\ell}{Ebh^3} C_I \begin{bmatrix} \frac{\ell^2(1+\gamma)}{4} & \ell \\ \ell & 2 \end{bmatrix} \tag{9}$$

where C_I and γ are given explicitly in [16].

The total flexibility matrix is then

$$S = \frac{12\ell}{Ebh^3}\begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ \overline{S}_{12} & \overline{S}_{22} \end{bmatrix} \tag{10}$$

where the terms $\overline{S}_{ij}$ are explicitly given in [16].

It was shown also in reference [16] that the stiffness matrix can be put under the following additive form

$$K = K^0 + \alpha K^c \tag{11}$$

3. Results and discussion

Let us consider a beam anchored on both ends and having a uniform rectangular cross section with the following parameters:

- $E = 67.4 GPa$
- $\nu = 0.3$
- $\rho = 2660\ kg.m^{-3}$
- $b = 20mm$
- $h = 40mm$
- $L = 1m$

Choosing a total number of 100 elements, so as the element length is $\ell = 10mm$, Figure 3 gives variations of the first two frequencies as function of the crack relative depth a/h. The crack was located in the beam section situated at the distance 0.7m from the left beam side. In general these frequencies decrease when the crack depth is increased. This is connected to the fact that the modal beam rigidity for these first modes decreases as the crack extends. The variation is rapid if the crack size is small and is slow for high crack depths. For the first frequency, this variation represents 8.8% when the crack depth reaches 50% of the initial beam height. The variation is 22.6% for the second frequency in the same conditions.

In order to monitor crack identification by solution of an inverse problem, dependencies of the first two beam frequencies on the defect characteristics are stated by using the structural dynamics eigenvalue problem solution. The inverse problem to determine crack characteristics for some measured frequencies, as they are perturbed by the presence of a crack, can then be considered. This can be achieved by using least squares regression based on the inversion of the curves given in Fig. 3.

The method of least squares is used in the following in order to solve this inverse problem. Least square means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation.

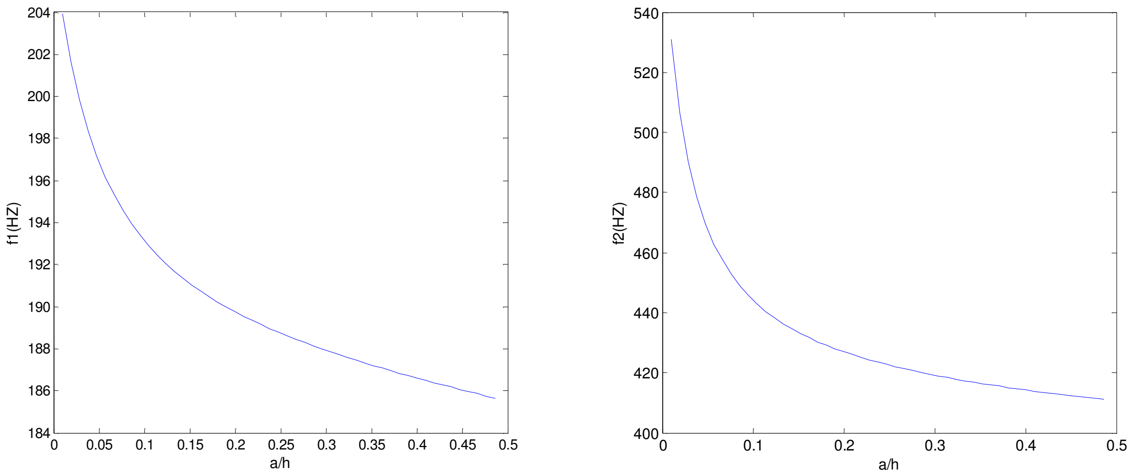

Fig. 3. Variations of the first two frequencies as function of the relative crack depth when this last is placed at the beam section $x = 0.7\,\mathrm{m}$.

In practice the measured frequencies will be with some noise. Fixing the beam height h and taking only the first two frequencies, we assume that the measured frequencies are perturbed in the following way:

$$\tilde{f}_1(a)=f_1(a)+\varepsilon_1 \tag{12}$$

$$\tilde{f}_2(a)=f_2(a)+\varepsilon_2 \tag{13}$$

with ε_1 and ε_2 are linked to noise measurement.

The forward problem consists in giving a and calculating $f_1(a)$ and $f_2(a)$ as solution of the eigenvalue problem associated to free-vibrations of the system defined by mass and rigidity matrices given respectively by equations (1) and (11). The inverse problem consists in searching the value of a by using the measured frequencies f_1 and f_2 in noise free case or the perturbed frequencies $\tilde{f}_1$ and $\tilde{f}_2$ in the presence of noise. The formulation of the inverse problem by means of the least square method takes the form of an optimization problem and writes

$$\tilde{a}=\underset{x\geq 0}{\arg\min}\left\{\left(x-f_1^{-1}\left(\tilde{f}_1(x)\right)\right)^2+\left(x-f_2^{-1}\left(\tilde{f}_2(x)\right)\right)^2\right\} \tag{14}$$

Let us assume a case study for which the exact solution of the direct problem corresponding to crack data $a=0.19\,\mathrm{mm}$ is given by the following first two frequencies:

$f_1 = 189.97\ \mathrm{Hz}$ and $f_2 = 428.03\ \mathrm{Hz}$

The error function as calculated from equation (14) gives, if the measurements are assumed to be perfect, the plot presented in Figure 4. The error function is to be minimized and the obtained minimum determines the inverse problem solution.

Figure 4 shows that the data of the forward problem $a = 0.19\,\text{mm}$ is perfectly retrieved from the measured frequencies f_1 and f_2.

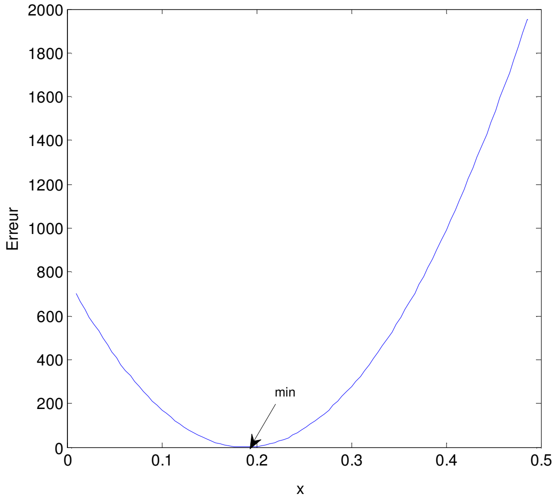

Fig. 4. Variation of the error function as function of the possible crack length x for the case with perfect measurments.

As in practice noise will affect the values of f_1 and f_2 it is required to study the effect of this noise on the inverse problem solution: $\tilde{a}(\varepsilon_1, \varepsilon_2)$. This is performed parametrically in the following.

For a fixed value of ε_2, Fig. 5 shows variations of the relative error exiting between the predicted crack depth $\tilde{a}$ and its exact value a : $(\tilde{a} - a)/a$ as function of noise level ε_1.

One can see from Figure 5 that the error is largely dependent on the measurement noise. The error increases with noise level in terms of both ε_1 and ε_2. To get a precision of 5% one should limit noise level to $5\text{x}10^{-3}$.

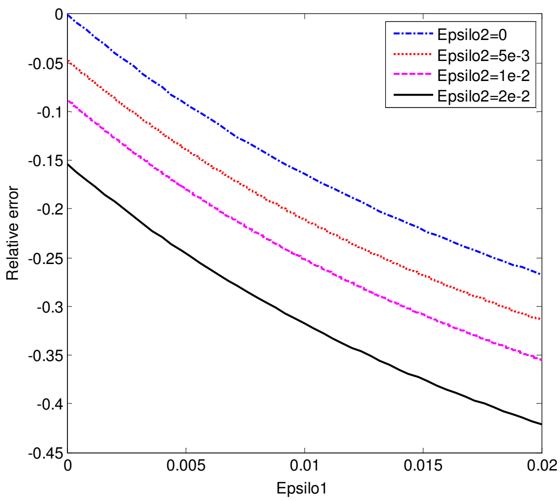

Fig. 5. Relative error of prediction as function of noise level as defined by ε_1 and ε_2

4. Conclusion

Crack additional stiffness to beam elemental stiffens matrix was used in this work to derive a homogenous linear elastic beam finite element. Assembling the obtained total stiffness matrix enabled to analyze the effect of crack characteristics on the beam first resonance frequencies. Solution of an inverse problem has been performed by using this information and enabled to assess the effect of noise on the predicted crack depth. The proposed method can be used in order to design an intelligent system for crack detection in beams. The performance of this method is high for big values of crack depth ratio and low noise.

References

[1] Owolabi GM, Swamidas ASJ, Seshadri R. Crack detection in beams using changes in frequencies and amplitudes of frequency response functions. Journal of Sound and Vibration 2003; 265:1-22.
[2] Lee J. Identification of multiple cracks in a beam using vibration amplitudes. Journal of Sound and Vibration 2009; 326: 205-212.
[3] Jassim ZA, Ali NN, Mustapha F,Abdul Jalil NA. A review on the vibration analysis for a damage occurrence of a cantilever beam. Engineering Failure Analysis 2013; 31:442-461.
[4] Moradi S, Kargozarfard M. On multiple crack detection in beam structures. Journal of Mechanical Sciences Technology 2013; 27:47-55.
[5] Khalili K, Vahidnia M. Improving the accuracy of crack length measurement using machine vision. Procedia Technology 2015; 19: 48- 55.
[6] Lele SP, Maiti SK. Modelling of transverse vibration of short beams for crack detection and measurement of crack extension. Journal of Sound and vibration 2002; 257: 559-583.
[7] Nahvi H, Jabbari M. Crack detection in beams using experimental modal data and finite element model. International Journal of Mechanical Sciences 2005; 47:1477-1497.
[8] Chatterjee A. Structural damage assessment in a cantilever beam with a breathing crack using higher order frequency response functions. Journal of Sound and Vibration 2010; 329:3325-3334.
[9] Cao M, Ye L, Zhou L, Su Z, Bai R. Sensitivity of fundamental mode shape and static deflection for damage identification in cantilever beams. Mech Syst Sig Proc 2011; 25:630–43.
[10] Maghsoodi A, Ghadami A, Mirdamadi HR. Multiple-crack damage detection in multi-step beams by a novel local flexibility-based damage index. Journal of Sound and Vibration, 2013; 332:294-305.
[11] Gudmundson P. Eigenfrequency changes of structures due to cracks, notches or other geometrical changes. Journal of the Mechanics and Physics of Solids1983; 30:339-353.
[12] Gudmundson P. The dynamic behaviour of slender structures with cross-sectional cracks. Journal of Mechanics and Physics of Solids 1983; 31:329-345.
[13] Dimarogonas AD, Paipetis SA. Vibration of cracked structures: a state of the art review. Analytical Methods in Rotor Dynamics London: Elsevier Applied Science; 1983.
[14] Papadopoulos CA. The strain energy approach for modelling cracks in rotors: a state of the art review. Mechanical Systems and signal Processing 2008; 22(4):763–789.
[15] Qian GL, Gu SN, Jiang JS. The dynamic behaviour and crack detection of a beam with a crack. Journal of Sound and Vibration 1990; 138: 233- 243
[16] Teidj S, Khamlichi A, Driouach A, Liman A. Identification of crack characteristics in beams by using modal frequencies and inverse problem. International Review of Applied Sciences and Engineering 2014; 5:145-150.