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IMC based PID Control of a Magnetic Levitation System

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Abstract

Attraction type magnetic levitation devices are nonlinear and unstable systems with fast dynamics. If a model of such a system can be produced, it could be used in the design process of a stabilizing controller. Internal Model Control (IMC) provides a strategy that explicitly uses an existing model of the controlled process for developing a suitable controller. In this paper, a linear model that represents the nonlinear dynamics of the magnetic levitation system is first derived. Then, this model is used in the design procedure of an IMC-based PID controller, which is used for achieving stable levitation of a ferromagnetic object at predetermined distances with the help of the magnetic field produced by a coil. The results are shown by means of digital simulation, based on Simulink.

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Keywords: magnetic levitation; Internal Model Control; PID control.

1. Introduction

Magnetic levitation is the process by which a ferromagnetic object is suspended in the air against gravity with the help of a magnetic field generated by a coil. This process presents many practical applications such as: active magnetic bearings, vibration damping, suspension of wind tunnel models, transportation systems (e.g high speed passenger trains) etc.

This paper investigates the development of a control system for a single degree of freedom magnetic levitation process who aims at obtaining stable levitation of a steel ball at predetermined distances, using an IMC-based PID controller. While the principle of levitation is simple: by controlling the current through the coil an electromagnetic

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force is generated which is able to counteract the weight of a steel ball, the magnetic levitation systems (MLS) are nonlinear, unstable systems with fast dynamics. These properties make them good candidates as test beds for different control algorithms. Over the past years, various control strategies have been proposed for this type of systems, such as PID [2,10,21], phase-lead compensation [16,19], feedback linearization controllers [11,13,20], sliding mode control [1,5], fuzzy logic PID-type controllers[9,12,21], neural networks [3] etc.

In many cases, PID has proved itself to be an effective solution in controlling these systems. It is easy to implement, but setting its parameters is somewhat difficult, due to the nature of the MLS. The problems result from the plant model uncertainties and the small operating ranges of MLS.

The IMC control strategy is based on the fact that if a model of the controlled plant can be produced, even if it is an approximate one, it can be used explicitly in the design of the controller. For many plant models in industry, classical PID controllers can be viewed as equivalent parameterizations of IMC controllers [14,18]. Using the IMC specifications, the tuning process of a PID controller is simplified and it is reduced to changing just one parameter, the closed loop time constant (the IMC filter factor λ), instead of three [4,14].

In [4,14,18] PID type controllers are designed based on IMC for different types of plants (first, second order, stable, unstable, dead time). For the MLS considered in this paper, first, a linear model is deducted, which is written in the form of a second order transfer function with poles on both sides of the complex plane. Then, as shown in section 3, this model is used to design an IMC-based PID controller for the plant. The effectiveness of the resulted controller is demonstrated by means of digital simulation using Matlab/Simulink using a nonlinear model of the plant, which was validated in [8].

2. Plant model

The mathematical model of the MLS, shown in equation (1), is found by applying Newton's law for the equilibrium of forces.

$$m\frac{d^2x(t)}{dt^2} = mg + f_e(x,i,t) \quad (1)$$

where: where $f_e(x,i,t)$ is the electromagnetic force that counteracts the weight of the ball, $x(t)$ is the distance between the coil and the steel ball, $i(t)$ is the current through the coil, m is the mass of the ball and g is the gravitational constant [1,19].

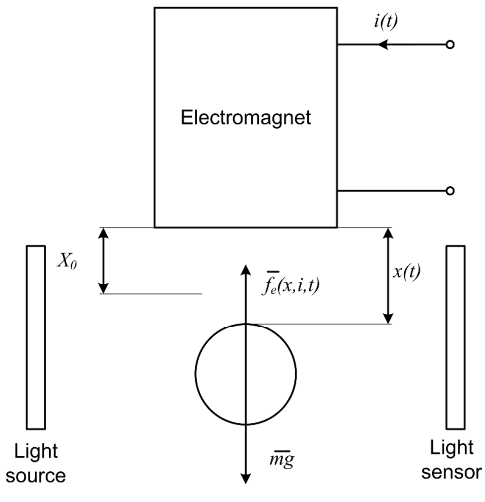

Fig. 1. The magnetic levitation system.

The electromagnetic force f_e generated by current $i(t)$ which flows through the coil is given by [7,19]:

$$f_e(x,i,t) = -C\left(\frac{i(t)}{x(t)}\right)^2 \tag{2}$$

where C is a nonlinear parameter which is assumed to be constant for model simplification purposes. In reality parameter C depends on the levitation point, as demonstrated in [8] and encapsulates some of the system's non-linearities which are very difficult to model. For a given levitation distance (X_0) this parameter was determined experimentally.

The linear model of the plant is determined using system linearization about an equilibrium point (I_0, X_0), where I_0 is the current through the coil when the ball is at X_0, by expanding the Taylor series of (2) and preserving the first order terms.

$$f_e(x,i,t) = -C\left(\frac{I_0}{X_0}\right)^2 - \left(\frac{2CI_0}{X_0^{\ 2}}\right)(i - I_0) + \left(\frac{2CI_0^{\ 2}}{X_0^{\ 3}}\right)(x - X_0) + \ldots \tag{3}$$

In stationary regime, when levitation is achieved ($i(t)=I_0$ and $x(t)=X_0$) the electromagnetic force cancels gravity and the acceleration $dx/dt = 0$, equation (1) takes the following form [19]:

$$mg = C\left(\frac{I_0}{X_0}\right)^2 \tag{4}$$

By combining (1),(3) and (4) we get:

$$m\frac{d^2\hat{x}}{dt^2} = -\left(\frac{2CI_0}{X_0^{\ 2}}\right)\hat{i} + \left(\frac{2CI_0^{\ 2}}{X_0^{\ 3}}\right)\hat{x} \tag{5}$$

where $\hat{x} = x - X_0$ and $\hat{i} = i - I_0$.

Equation (5) represents the linear equation which describes the dynamic of the magnetic levitation system (plant). Based on this equation, using Laplace transform, we get the transfer function of the plant shown in equation (6).

$$H(s) = \frac{\hat{X}(s)}{\hat{I}(s)} = \frac{-k_1}{ms^2 - k_2} \tag{6}$$

where k_1 and k_2 are two constants depending on parameters C, I_0 and X_0.

The displacement of the steel ball around the operating point X_0 is determined using a sensor system consisting of an infra-red LED and a phototransistor. The current through the coil is produced with the help of a current amplifier. Both the sensor and the current amplifier are linear elements which can be described by the proportional gain transfer functions K_{sens}, K_{amp}.

By considering these additional elements the following transfer function of the process is found:

$$\tilde{G}_P(s) = \frac{-k_1}{ms^2 - k_2} K_{amp} K_{sens} = \frac{-k_1^{'}}{ms^2 - k_2} \tag{7}$$

where $k_1^{'}=k_1 K_{amp} K_{sens}$
The parameters in Table 1 characterize the process linearized about the equilibrium point (X_0,I_0).

Table 1. MLS parameters.

Parameter	Value	Observation
m	0.0101 kg	Mass of the levitated steel ball
X_0	0.0076 m	Equilibrium position
I_0	0.421 A	Equilibrium current
C	$3.2084\cdot 10^{-5}$ Nm^2/A^2	Constant, corresponding to the pair $(X_0, I_0) = (7.6mm, 421mA)$.
k_1	0.4677 N/A	Parameter of the transfer function in (6)
k_2	25.9087 N/m	Parameter of the transfer function in (6)
K_{sens}	Aprox. 3333 V/m	Sensor system transfer function (gain)
K_{amp}	0.1 A/V	Current amplifier transfer function (gain)

3. IMC-based PID control of the MLS

3.1. The IMC-PID equivalence

Fig. 2.a shows the standard IMC structure. By manipulating the block diagram, an equivalent feedback control structure can be obtained (Fig. 2.b), where the feedback controller is given by:

$$G_C(s)=\frac{G_{IMC}(s)}{1-\tilde{G}_P(s)\cdot G_{IMC}(s)} \tag{8}$$

where: $G_{IMC}(s)$ is the internal model controller and $\tilde{G}_P(s)$ is the internal model. In most cases $G_C(s)$ has the form of a PID controller [18].

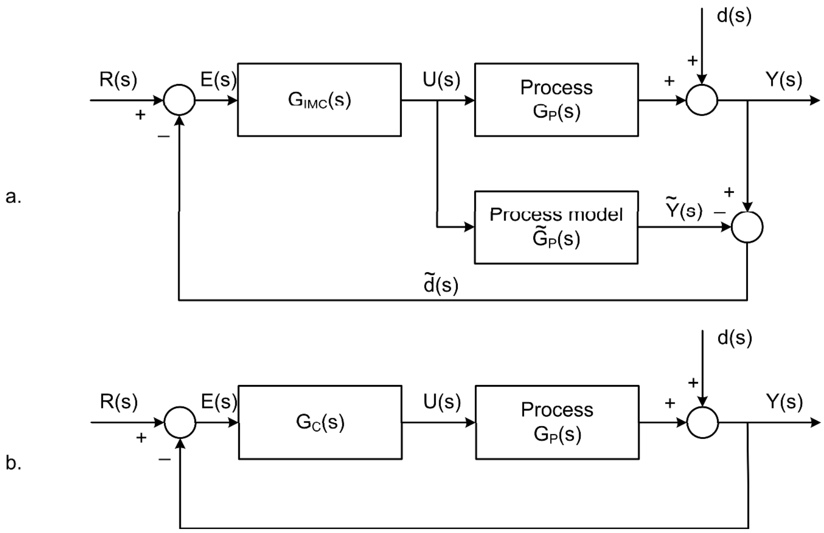

Fig. 2. (a) IMC structure; (b) Equivalent feedback structure.

3.2. IMC-based PID design for the MLS

This section shows the design procedure for a PID controller equivalent to the IMC for the MLS modeled in section 2.

For this purpose, the model of the plant of equation (7) is rewritten in the following form:

$$\tilde{G}_P(s) = \frac{k_p}{(\tau_p s + 1)(-\tau_u s + 1)} \tag{9}$$

where: $k_p = k_1^{'} / k_2$ is the process gain and $\tau_p = \tau_u = \sqrt{m / k_2}$ are two positive time constants.

As seen in (9), the process has two poles, located on both sides of the complex plane. $s=1/\tau_u$ represents the unstable pole located in the right half part of the complex plane, while $s=-1/\tau_p$ is the stable pole.

The first step in finding the PID controller is finding the internal model controller's transfer function $G_{IMC}(s)$, as shown in equation (10).

$$G_{IMC}(s) = \tilde{G}_P^{-1}(s) \cdot F(s) \tag{10}$$

As recommended in [4,15] this function includes a low-pass filter:

$$F(s) = \frac{\gamma s + 1}{(\lambda s + 1)^n} \tag{11}$$

where: n is the filter's order and is usually chosen to make $G_{IMC}(s)$ proper or semiproper, λ is the filter's time constant and γ satisfies the filter requirement of equation (12).

$$F\left(s = \frac{1}{\tau_u}\right) = 1 \tag{12}$$

Because we want to end up with an ideal PID controller, we choose $n=2$, making $G_{IMC}(s)$ improper. According to [17,18] this action is allowed since $G_{IMC}(s)$ will have a zero excess of at most 1 which is needed for the PID.

Solving equation (12) for γ we find:

$$\gamma = \lambda\left(\frac{\lambda}{\tau_u} + 2\right) \tag{13}$$

In order to find the equivalent feedback controller, we use the transformation of equation (8) as follows:

$$G_C(s) = \frac{G_{IMC}(s)}{1 - \tilde{G}_P(s) \cdot G_{IMC}(s)} = \frac{\dfrac{(\tau_p s + 1)(-\tau_u s + 1)}{k_p} \dfrac{\gamma s + 1}{(\lambda s + 1)^2}}{1 - \dfrac{k_p}{(\tau_p s + 1)(-\tau_u s + 1)} \dfrac{(\tau_p s + 1)(-\tau_u s + 1)}{k_p} \dfrac{\gamma s + 1}{(\lambda s + 1)^2}} \tag{14}$$

which leads to:

$$G_C(s)=\frac{-\tau_u(\gamma+\tau_p)}{k_p\lambda^2}\frac{\gamma\tau_p s^2+(\gamma+\tau_p)s+1}{(\gamma+\tau_p)s} \tag{15}$$

Recalling that the transfer function for a standard PID controller is:

$$G_C(s)=k_C\left(1+\frac{1}{T_i s}+T_d s\right)=k_C\frac{T_iT_d s^2+T_i s+1}{T_i s} \tag{16}$$

we find the following relationships for the PID parameters [4]:

$$\begin{aligned} k_C &= \frac{-\tau_u(\gamma+\tau_p)}{k_p\lambda^2} \\ T_i &= \gamma+\tau_p \\ T_d &= \frac{\gamma\tau_p}{\gamma+\tau_p} \end{aligned} \tag{17}$$

As seen the PID parameters in equation (17) depend on a single variable, the low-pass filter's time constant λ.

4. Simulation results

In order to test the resulted controller, Matlab/Simulink simulations were performed. These simulations were based on the following closed loop model of the magnetic levitation control system.

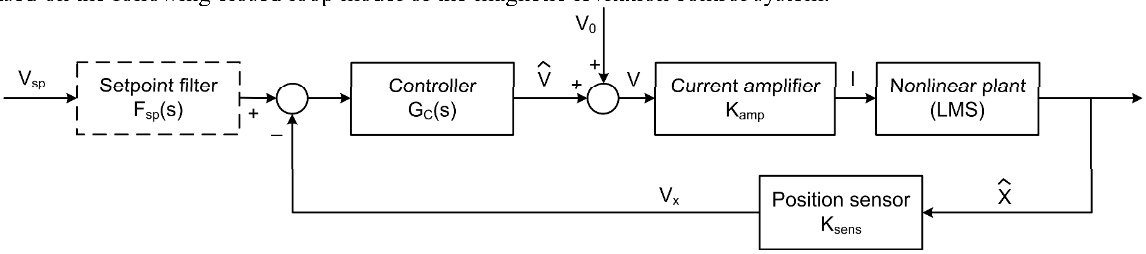

Fig. 3. Schematic diagram illustrating the model used for simulating the magnetic levitation control system using the IMC-based PID controller

The parameters of Fig. 3 are: $\hat{X}$ - displacement of the ball from the equilibrium position; V_x – sensor output voltage, proportional to $\hat{X}$; V_{sp} - setpoint voltage needed to keep the ball at the desired position $\hat{X}=0$; V_0 - supply voltage needed for I_0; $\hat{V}$ - output control voltage; I - electromagnet current.

The PID controller resulted from the procedure discussed in Section 3 is an ideal controller, which is improper. The implementation issues that it raises are addressed by considering a filter coefficient N=100 for the derivative component, that sets the location of the pole in the derivative filter far away in left hand part of the complex plane.

Equation (18) shows the transfer function of the considered PID controller.

$$G_C(s)=k_C\left(1+\frac{1}{T_i s}+\frac{T_d N s}{s+N}\right) \tag{18}$$

Since λ is the only tuning parameter of the controller (see equation (17) and (13)), Fig. 4.a shows the closed loop output responses to a step input for different values of λ. All trajectories shown in the next figures are relative to the equilibrium position X_0.

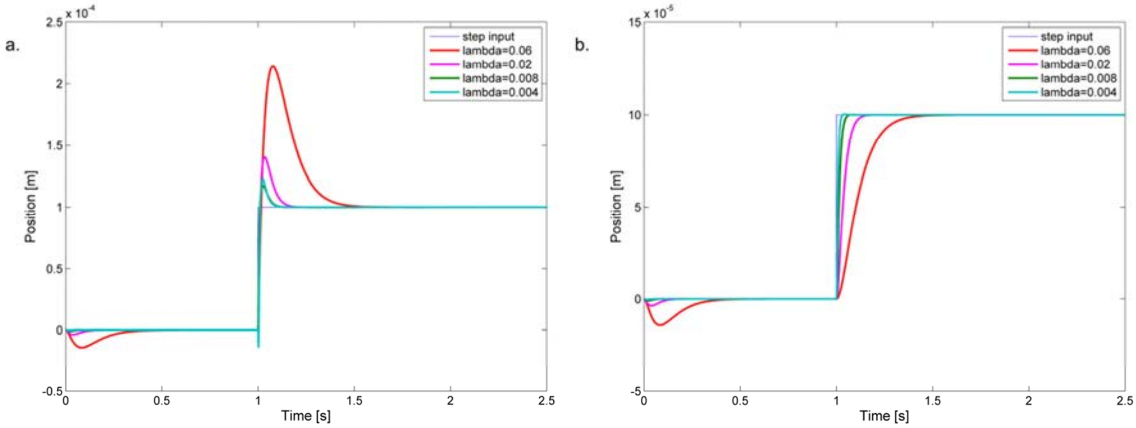

Fig. 4. (a) Closed loop output response to a step input for different values of λ; (b) Closed loop output response to a step input for different values of λ, with setpoint filter.

Fig.4 shows that the closed loop response to step input has quite significant overshoot. This is not acceptable since the levitated object could find itself outside the range of the position sensor. A step response without overshoot can be obtained by using a setpoint filter of the following form [4]:

$$F_{sp}(s) = \frac{1}{\gamma s + 1} \tag{19}$$

Fig. 4.b shows the closed loop output responses to a step input for different values of λ, considering the setpoint filter of equation (19). By choosing λ=0.008, almost perfect set-point tracking capabilities (no overshoot, fast response time) are obtained, as shown in Fig. 5.

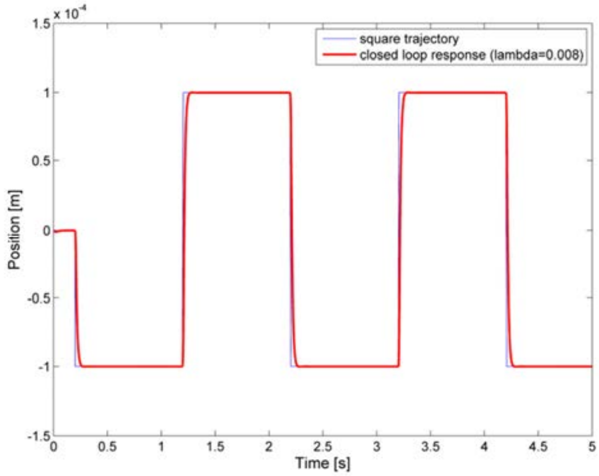

Fig. 5. Closed loop output response for square trajectory tracking (λ=0.008), with setpoint filter.

5. Conclusion

This paper showed the design procedure of an IMC-based PID controller for a nonlinear unstable process. For the MLS used in this paper, a second order linear model was found that had one unstable pole. Using this model as internal model, by following the procedure described in Section 3 and in [4], an ideal PID controller was determined for the process. The resulting controller had the advantage that its tuning parameters (k_c, T_i, T_d) had an analytical representation which depended on model's parameters and on a single variable parameter (λ), which became the controller tuning parameter. This simplified the tuning process. The results in Section 4 show the closed loop system response to a step change for various values of λ. By using a standard choice as setpoint filter the system's response is improved and the initial overshoot is eliminated. Choosing a convenient value for λ results in a damped response with short settling time, which is suitable for the limited operating range of the position sensor, but also for assuring perfect setpoint tracking capabilities.

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