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Procedia Technology 22 (2016) 614 – 622

Procedia Technology

9th International Conference Interdisciplinarity in Engineering, INTER-ENG 2015, 8-9 October 2015, Tirgu-Mures, Romania

Multivariable System with Level Control

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Abstract

The paper presents the mathematical modeling procedure of a multivariable process with two coupled tanks and it is determined the multivariable control algorithms, imposing a desired behavior of the closed-loop and open-loop system. They ensure the imposed performances on direct connection channels, compensate the effect of the inter-influences between channels, and the system operates decoupled. The control algorithms resulted by the design are high-order, so, based on the Hankel norm, there are determined the truncated (reduced) forms of the controllers. The computing, as well as the behavior of the system, are performed using the Matlab environment.

Peer-review under responsibility of the "Petru Maior" University of Tirgu Mures, Faculty of Engineering

Keywords: Mathematical modelling; Level control; Multivariable process; Conventional control.

1. Introduction

The level control processes cover a large part of the industrial applications. The level control systems are developed in different structures, some of them after error, with upstream or downstream valve, with free exit or through pumps. In order to control these processes, most of the time, there are used conventional controllers, because of their simple configuration and as a result, they are easy to understand and exploit.

In the present paper are mentioned a number of studies that approach the issue of mathematical modeling for multi-input-multi-output (MIMO) level control systems, and it shows a version in which it is obtained the input-state-output (ISO) representation for a process with two coupled tanks. Also, there are determined two reduced

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Peer-review under responsibility of the "Petru Maior" University of Tirgu Mures, Faculty of Engineering
doi:10.1016/j.protcy.2016.01.128

(truncated) control structures, that ensure the autonomously decoupled functioning, and that does not fundamentally change the dynamics of the system with initial calculated controllers.

Therefore, the mathematical modeling's main aspects for the level control processes and PI conventional strategies are developed in [3,14,17]. The interaction between two coupled tanks, as the mathematical modeling, are studied in [9,16], in [5] are developed the mathematical models based on state equations of the coupled tanks, and in [8] are presented the control strategies, including Zeigler-Nichols and fuzzy methods. Also, in [2] are studied the control schemes with experimental controllers, for the fluid level contained by the two coupled tanks. The coupled tanks, in different configurations of interaction on non-interaction, with associated Simulink schemes, are presented in [13], and in [12] are implemented the controllers based on predictive models, optimized through genetics algorithms. PI and MRAC control techniques, for the linearized mathematical model of the coupled tanks and the comparative studies, are presented in [15]. The design procedures of the controllers for the MIMO processes, based on imposed model and on coupled factor between interaction channels, are developed in [4]. Other studies are focused on MIMO control strategies, used in a number of applications [6].

2. The mathematical modeling of the two coupled tanks MIMO system

In scheme from Fig. 1 are presented two coupled tanks, in which: F_{a1}, F_{a2} are the fluid's input flows; F_{e1}, F_{e12}, F_{e2} – the exit flows; A_1, A_2 – the surface of the tank's base; L_1, L_2 – the fluid's level. In the control scheme, according to [1]: LT_1, LT_2 are the level transducers; LIC – the controller and level indicator; FV_1, FV_2 – the flow valves; R_1, R_{12}, R_2 – the hydraulic resistances.

The mathematical model determining, in dynamic regime, makes references to the level variations expressed as functions of introduced, extracted and accumulated quantities in tanks [3,5,17], as:

$$\begin{cases} \dfrac{d}{dt}L_1 = \dfrac{1}{A_1}\left(F_{a1} - F_{e1} - F_{e12}\right) \\ \dfrac{d}{dt}L_2 = \dfrac{1}{A_2}\left(F_{a2} + F_{e12} - F_{e2}\right) \end{cases}. \tag{1}$$

Considering:

$$F_{e1} = \frac{L_1}{R_1};\ F_{e2} = \frac{L_2}{R_2};\ F_{e12} = \frac{L_1 - L_2}{R_{12}}, \tag{2}$$

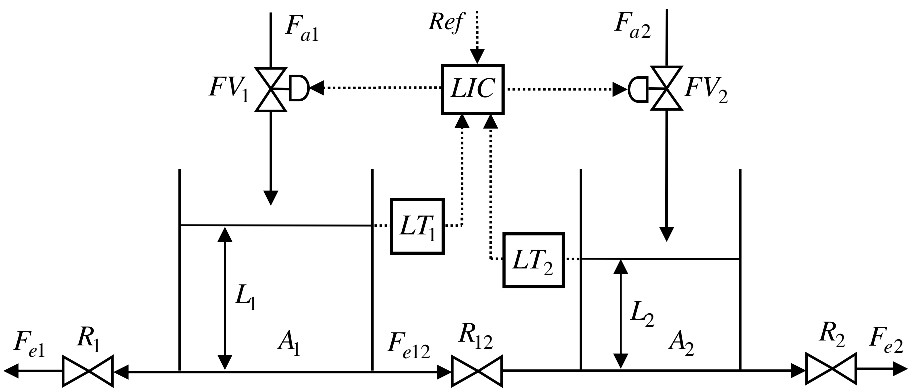

Fig. 1. The scheme of the two coupled tanks.

it results:

$$\begin{cases} \dfrac{d}{dt}L_1 = -\dfrac{R_1+R_{12}}{A_1R_1R_{12}}L_1 + \dfrac{1}{A_1R_{12}}L_2 + \dfrac{1}{A_1}F_{a1} \\ \dfrac{d}{dt}L_2 = \dfrac{1}{A_2R_{12}}L_1 - \dfrac{R_2+R_{12}}{A_2R_2R_{12}}L_2 + \dfrac{1}{A_2}F_{a2} \end{cases}, \tag{3}$$

respectively the form:

$$\begin{bmatrix} \dfrac{d}{dt}L_1 \\ \dfrac{d}{dt}L_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{R_1+R_{12}}{A_1R_1R_{12}} & \dfrac{1}{A_1R_{12}} \\ \dfrac{1}{A_2R_{12}} & -\dfrac{R_2+R_{12}}{A_2R_2R_{12}} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A_1} & 0 \\ 0 & \dfrac{1}{A_2} \end{bmatrix} \begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix}. \tag{5}$$

The relationships (4), (5) represent the MIMO-ISO mathematical model of the system with two coupled tanks, in general form [4]:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du} \end{aligned}, \tag{6}$$

in which the state variables vector is similar to the output variables vector:

$$\mathbf{x} = [x_1 \quad x_2]^T = [L_1 \quad L_2]^T = [y_1 \quad y_2]^T = \mathbf{y}, \tag{7}$$

and the command vector has the form:

$$\mathbf{u} = [u_1 \quad u_2]^T = [F_{a1} \quad F_{a2}]^T. \tag{8}$$

The process from Fig. 1, without the control block (*LIC*), may be presented as a MIMO system, with two inputs and two outputs (Fig. 2), and highlights the dependences as the following [4,17]:

$$\mathbf{Y}(s) = \mathbf{H}_F(s) \cdot \mathbf{U}(s), \tag{9}$$

in which: $\mathbf{H}_F$(s) is the transfer matrix of the MIMO technological part (including the actuators and the transducers); $\mathbf{Y}$(s) – the direct Laplace transforms' column vector of the output variables vector from (7); $\mathbf{U}$(s) – the direct Laplace transforms' column vector of the commands from (8).

Based on the relationship input-state-output (ISO) – input-output (IO):

$$\mathbf{H}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \tag{10}$$

there are obtained the transfer functions as components of the matrices $\mathbf{H}_F$(s) or $\mathbf{H}_C$(s).

The interaction between the input and the output variables is evaluated through the value of the coupled factor, calculated for the steady state (k_0) and the dynamic (k_{din}) regimes [4]:

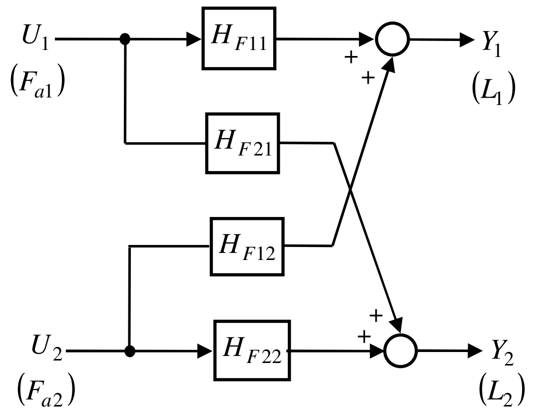

Fig. 2. The technological part of the MIMO system with two inputs and two outputs.

$$k_0 = \frac{H_{F12}(0) \cdot H_{F21}(0)}{H_{F11}(0) \cdot H_{F22}(0)}, \tag{11}$$

$$k_{din}(s) = \frac{H_{F12}(s) \cdot H_{F21}(s)}{H_{F11}(s) \cdot H_{F22}(s)}. \tag{12}$$

3. The design of the control algorithms

The need to ensure the imposed performances for each direct connection channel and to compensate the effect of the inter-influences (and of the disturbances), impose the use of MIMO controllers, resulting the MIMO control system's general scheme (Fig. 3), particularized for the case of two coupled tanks, in which the relationship (9) has the form:

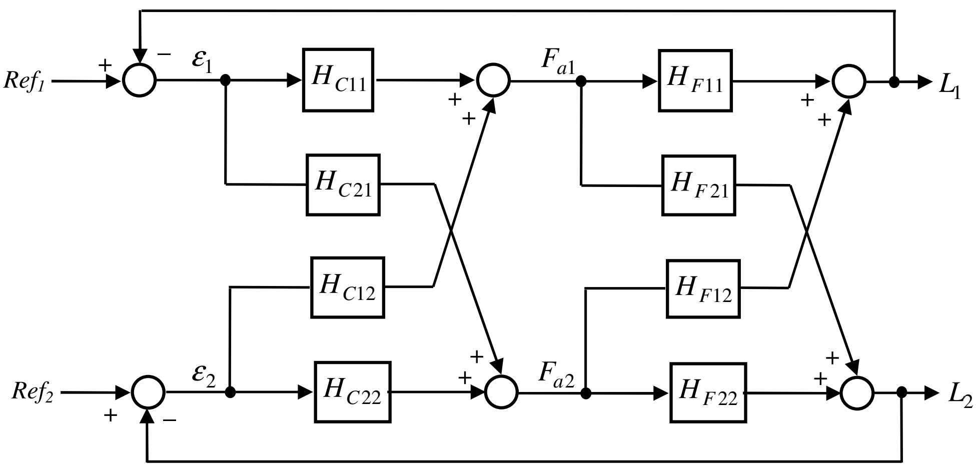

Fig. 3. The MIMO control scheme with a controller on each channel.

$$\begin{bmatrix} L_1(s) \\ L_2(s) \end{bmatrix} = \begin{vmatrix} H_{F11}(s) & H_{F12}(s) \\ H_{F21}(s) & H_{F22}(s) \end{vmatrix} \begin{bmatrix} F_{a1}(s) \\ F_{a2}(s) \end{bmatrix}, \tag{13}$$

where: $H_{F11}(s)$, $H_{F22}(s)$ are the main transfer functions (on the direct channels); $H_{F12}(s)$, $H_{F21}(s)$ – the coupling transfer functions (on the inter-influence channels).

So, the determining of the control block is based on the desired forms' description for:

- closed-loop transfer matrix,
- opened-loop transfer matrix.

3.1. The determining of the control structure based on $\mathbf{H}_0(s)$ matrix

For a known transfer matrix, $\mathbf{H}_F$(s), it is determined the transfer matrix, $\mathbf{H}_C$(s), so that it is obtained a desired form of the transfer matrix, $\mathbf{H}_0$(s).

By performing the autonomous MIMO system, where each input variable influence one single output variable, without any influence on the other output variables, it is ensured the decoupling of the channels, either in relation with output variables, either in relation with disturbances (not simultaneously).

In the case of autonomous decoupled MIMO system, in relation with the input variables, the transfer matrix, $\mathbf{H}_0$(s), is a diagonal one, having the form:

$$H_0(s) = \left.\begin{vmatrix} H_{011}(s) & 0 & \dots & 0 \\ 0 & H_{022}(s) & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & H_{0nn}(s) \end{vmatrix}\right|_{n=2} = \begin{vmatrix} H_{011}(s) & 0 \\ 0 & H_{022}(s) \end{vmatrix}, \tag{14}$$

with the number of the input and output signals are equal.

The design of the control block is performed according to the relationship [4]:

$$H_C(s) = [H_F(s)]^{-1} \cdot H_0(s) \cdot [I_n - H_0(s)]^{-1}\Big|_{n=2} = [H_F(s)]^{-1} \cdot H_0(s) \cdot [I_2 - H_0(s)]^{-1}, \tag{15}$$

where I_2 represents the identity matrix.

3.2. The determining of the control structure based on $\mathbf{H}_d(s)$ matrix

If for the MIMO system with the two coupled tanks, it is imposed a behavior characterized by the transfer matrix on direct path:

$$H_d(s) = \begin{vmatrix} H_{d1} & 0 \\ 0 & H_{d2} \end{vmatrix}, \tag{16}$$

then it is possible to determine the controllers, based on the dynamic coupling factor (12), with the relationships [4]:

$$\begin{aligned} H_{C11}(s) &= \frac{H_{d1}(s)}{H_{F11}(s)} \cdot \frac{1}{1-k_{din}(s)}; \qquad H_{C12}(s) = -\frac{H_{d2}(s)}{H_{F22}(s)} \cdot \frac{H_{F21}(s)}{H_{F11}(s)} \frac{1}{1-k_{din}(s)} \\ H_{C21}(s) &= -\frac{H_{d1}(s)}{H_{F11}(s)} \cdot \frac{H_{F12}(s)}{H_{F22}(s)} \frac{1}{1-k_{din}(s)}; \quad H_{C22}(s) = \frac{H_{d2}(s)}{H_{F22}(s)} \cdot \frac{1}{1-k_{din}(s)} \end{aligned}. \tag{17}$$

4. Matlab experimental results

In the simulation it is considered the MIMO process with the two coupled tanks (Fig. 1), where, using the values: A_1=0.8; A_2=0.4; R_1=0.5; R_{12}=0.3; R_2=1, result the matrices:

$A=\begin{bmatrix} -6.6667 & 4.1667 \\ 8.3333 & -10.8333 \end{bmatrix}$; $B=\begin{bmatrix} 1.25 & 0 \\ 0 & 2.5 \end{bmatrix}$, respectively the representation from Fig. 4.

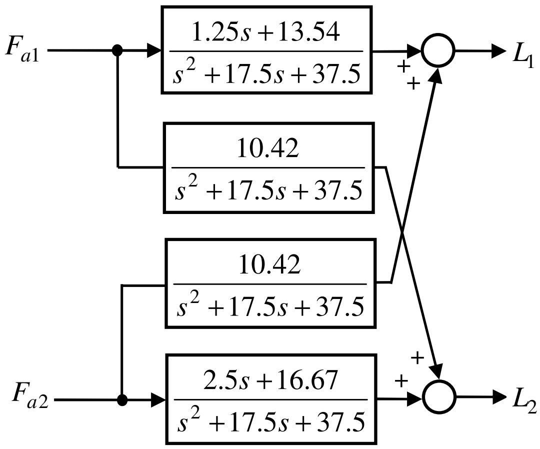

Fig. 4. The technological parts of the MIMO process with the two coupled tanks.

The value of the coupled factor, for the steady state regime (11) is k_0 = 0.48, and for the dynamic regime (12) is:

$$k_{din}(s)=\frac{108.6}{3.125s^2+54.69s+225.7}.$$

In a first approach, according to [14], for the desired transfer matrix of the closed-loop system, it is imposed a diagonal form:

$$H_0(s)=\begin{vmatrix} \frac{1}{0.4s+1} & 0 \\ 0 & \frac{1}{0.3s+1} \end{vmatrix},$$

and it is determined the transfer matrix of the controllers, according to (15).

The state matrix of the initial, calculated controller has the size [16 x 16]. Applying the techniques of reducing the order of the initial controller, based on Hankel norm [7], it is obtained a truncated controller with the state matrix having the size [4 x 4] and the approximating error, $\varepsilon=3.64e-8$. Based on the relationships (10), the transfer functions of the controllers on each channel are:

$$H_{C11}(s)=-\frac{2s^2+13.33s-3.14e-15}{s^2+9.71e-17s-7.85e-32}; \quad H_{C12}(s)=\frac{-8.33s-1.04e-15}{s^2+3.47e-16s-5.89e-32};$$

$$H_{C21}(s)=\frac{-11.11s-3.70e-15}{s^2+9.71e-17s-7.85e-32};\quad H_{C22}(s)=\frac{1.33s^2+14.44s+6.81e-15}{s^2+3.47e-16s-5.89e-32}.$$

Fig. 5 shows the responses of the MIMO system [10,11], considering the step input. They highlight the effect of the input variables on the output variables, on the direct and inter-influence channels, as well as the autonomously decoupled functioning.

In the second approach, for the desired transfer matrix of the system on the direct path, it is imposed a diagonal form (16):

$$H_d(s)=\begin{vmatrix}\frac{2.5}{s} & 0\\ 0 & \frac{3.33}{s}\end{vmatrix},$$

and it is determined the transfer matrix of the controllers, according to (17).

The transfer matrix of the initial controller has the size [22 x 22]. Applying the techniques of reducing the order of the calculated controller, based on the Hankel norm [7], it is obtained the truncated controller with the state matrix having the size [4 x 4] and the approximating error, $\varepsilon=0.0084$.

The transfer functions of the controllers on each channel, calculated from (10) are:

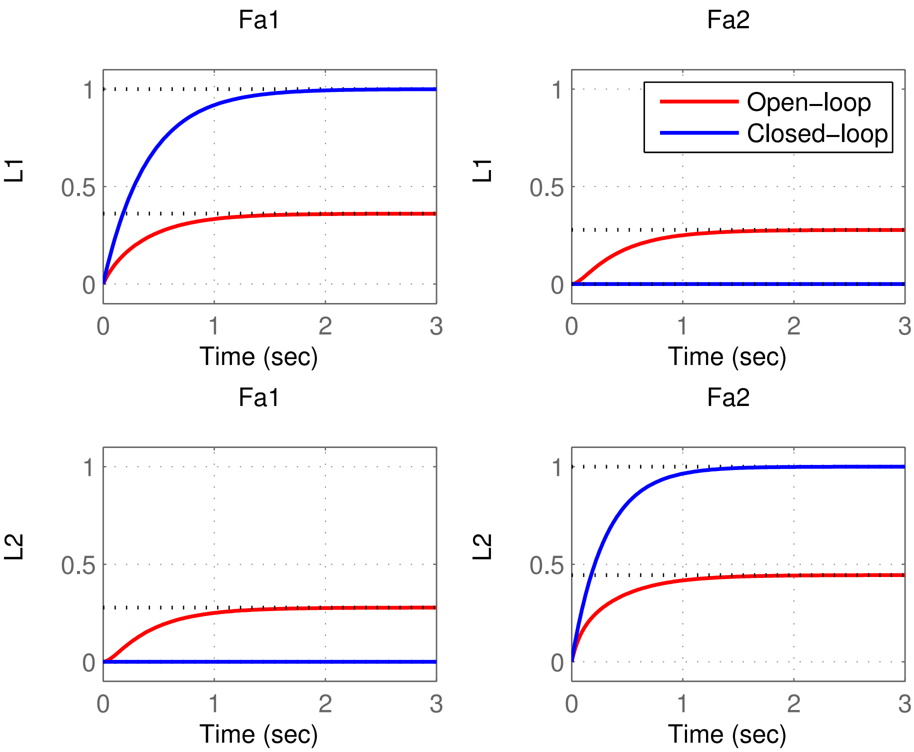

Fig. 5. The step responses of the opened-loop and the closed-loop system.

$$H_{C11}(s) = -\frac{2s^2 + 12.08s - 4.57e - 17}{s^2 + 1.70e - 16s - 6.41e - 34}; \quad H_{C12}(s) = \frac{-10.55s - 1.82e - 16}{s^2 + 2.46e - 17s - 8.01e - 35};$$

$$H_{C21}(s) = \frac{-8.23s - 1.62e - 15}{s^2 + 1.70e - 16s + 6.41e - 34}; \quad H_{C22}(s) = \frac{1.32s^2 + 19.16s + 7.37e - 17}{s^2 + 2.46e - 17s - 8.01e - 35}.$$

The autonomously decoupled functioning, compared to the open-loop functioning is presented in Fig. 6 [10,11].

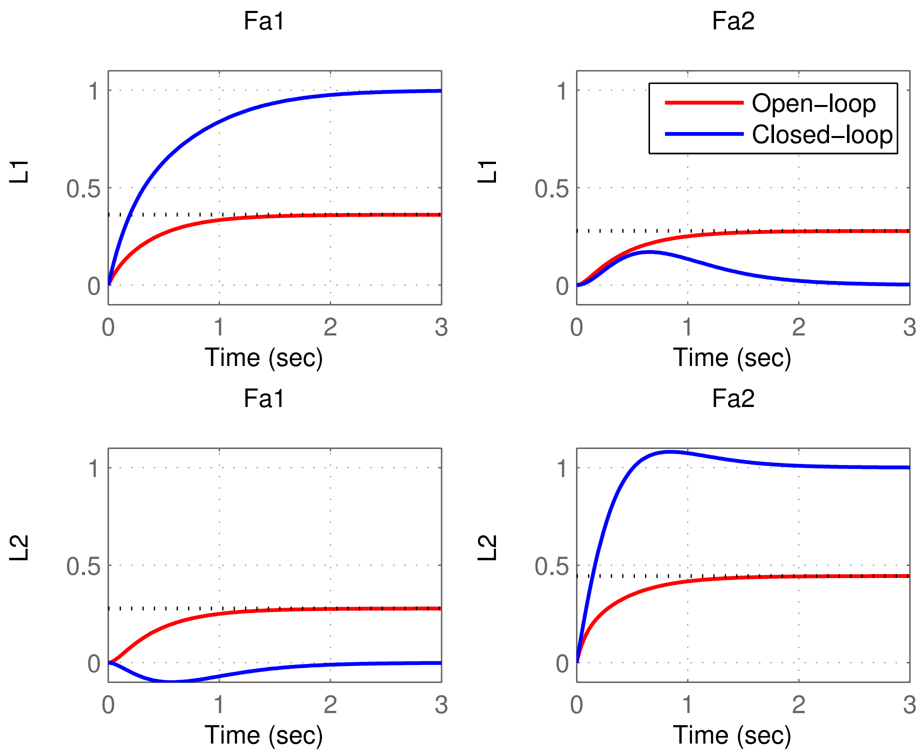

Fig. 6. The step responses of the opened-loop and the closed-loop system.

5. Conclusions

In order to control the level, in the single-input-single-output schemes, it is possible to implement conventional type P, PI, PID controllers, with the tuning parameters calculated based on the experimental criteria (e.g. Zeigler-Nichols, Kapelovici etc.). In the case of MIMO level control processes, the mathematical model becomes more complex and the inter-influences between channels can not be neglected. In each and every case, the accuracy of the tuning depends fundamentally on the accuracy of the mathematical model of the process.

Furthermore, in an accurate presentation, the mathematical model is nonlinear and finding the structures of the controllers which does not allow the rapid decrease of the performances is needed. In these conditions, the design strategies take into account a closed-loop imposed model or an opened-loop imposed model, considering the coupling factor between the channels.

So, based on the ISO mathematical model of the system with the two coupled tanks, are determined two control structures which ensure the decoupled functioning and good performances in closed-loop. Both strategies present the disadvantage that the controllers are high-order, and consequently difficult to implement. On the other hand, using techniques of reducing the order of the systems, the states which ensure a negligible energetic transfer, are eliminated. This allowed finding the truncated controllers, but these does not essentially change the dynamic of the closed-loop system. The behavior of the system with the two coupled tanks is studied through Matlab simulation, using dedicated functions.

In the future, there will be considered the PI type controllers on each channel and there will be determined the relationships for $HC_{12}(s)$ and $HC_{21}(s)$, depending on $HC_{11}(s)$, $HC_{22}(s)$ and $\mathbf{H}_F(s)$, which ensure the diagonal forms for $\mathbf{H}_0(s)$. Also, it is needed to determine the robust control laws, using H-infinite methods (e.g. loop-shaping, mixed synthesis etc.) and the highlighting of the sensitivity functions.

Acknowledgements

This paper is based upon work supported by the Energy and Electrotechnologies Management research center, Department of Electrical and Computer Engineering, “Petru Maior” University of Tîrgu-Mureş.

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