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ZPA Flank Evaluation Using Spherical Stylus

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Abstract

The article presents a solution to control worms flank ZPA using stylus with spherical top on a coordinate measuring machines. The method allows evaluation axial profile deviation of the ZPA worms to nominal profile. We consider X axis of symmetry of a ZPA worm and XY its axial plan. A control sphere has its center in the XY plane chosen. It is necessary to impose share Y center of the sphere. It requires determining the X coordinate of the center sphere to sphere is tangent to the worms flank ZPA. The contact between the sphere and the helical surface of flank ZPA worms is a point which generally not belong XY axial plane chosen. Using the method of mutual winding surfaces can be determined parametric equations ZPA worm flank. ZPA flank worms ZPA is represented by a helical surface non-ruled. Point of contact between a sphere and the helical flank can be determined by numerical iterations based on a predetermined calculation error. These iterations are based on AutoLISP program developed by the authors, which allows verification of the results in AutoCAD graphics. By running the application AutoLisp, X nominal values can be determined, which is expected tangency of spherical stylus of a 3D Coordinate measuring machines and flank of ZPA worm. By comparing the X measured values with nominal value, execution errors of ZPA worm can be evaluated on coordinate measuring machines. The contributions are numerical calculation of the tangency position determination of a sphere and flank worm ZPA, designing an AutoLisp program for evaluation the position tangency of, creation an evaluation method of executed profile deviations compared to the nominal profile of the worm ZPA.

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Keywords: ZPA worm; 3D coordinate measuring; helical surface; measurement; spherical stylus.

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1. Introduction

The purpose of this paper is to control of helical surfaces of the flanks ZPA worms using spherical stylus of radius r on a coordinate measuring machine.

It is known that helical surfaces of the worm flanks can be of two types: ruled and non-ruled. In the case of worms with ruled surfaces the generator of the helical curve is a straight line, the resulting surface is a ruled helical surface, and the curve generator of surfaces non-ruled is a very well defined curves. According to STAS 6845-82 worms with ruled helical surface is type ZA, ZE and ZN, and with non-ruled surface is type ZK [11], [12], [13], [15].

(a)

(b)

Fig.1. (a) Coordinate measuring machine (b) Spherical stylus [4]

Studied worm in the paper is cylindrical non-ruled, machined on CNC with end mills positioned inclined at an angle close to the angle of the pressure worm [1] and moved eccentric to axis of symmetry of the worm [2], [3], so that worked profile to be very close to the worm type ZA. ZPA flank worms is represented by a helical surface nonruled. Checking profile worm result can makes with coordinate measuring machines Fig.1 (a). Used measuring stylus can be in the form of a sphere Fig.1 (b) with the radius r, or may have other shapes. In this paper it determines point of tangency between a sphere and the helical surface of a worm type ZPA.

Control of the worm flank is made in axial plane XY studied but contact between stylus and a helical surface is not in this plan, but in a plane parallel to the XY, but located in the negative Z axis.

It requires knowledge of deviation introduced because of contact between the stylus and the helical surface area to correctly position the produced worm in a precision class. There are studies on error correction of surface measurement helical on coordinate measuring machines without special devices [5], [6], [9], [14].

Nomenclature

v,α	sphere parameters;
r	sphere radius;
t	plan parameter;
φ	parameter family of surfaces;
r_s	radius end mills;
γ	inclination of end mills;
u, ξ	generating tool surface parameters;
h	helical parameter;
r_i	the radius of the cylinder on which is wound the helical surface;
e	eccentricity.

2. Determination the point of tangency between a helical surface and a sphere

The method used to obtain the point of contact between the helical surface of the ZPA worm and a sphere of the radius r is:

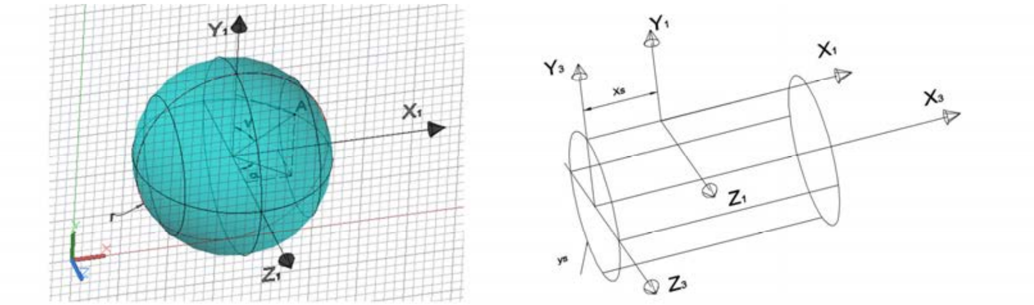
a. We place the stylus with the origin at a point P (x, y, 0) and we block the movement on the Y axis. To this position we determine:

- For u and φ parameters of helical surface worms the coordinate on axis Y of a point on the surface, by which point we will consider passing a plane parallel to the XZ plane;
- Intersection points of a plane and sphere of radius r, the intersection between the two surfaces being a circle (C1);
- The intersection of helical surface and parallel plane to XZ which is a curve (C2);
- If the smallest distance d_{min} between the center of the circle (C1) to a point P, walking to the curve (C2) is in an acceptable small range, ($r_{cpl}\pm 1\mu m$), then we hold the point P considered.

b. If d_{min} is not in acceptable range then we move the stylus on the X axis into a new position and resumes calculations from the point a.

2.1. Determination of the sphere parametric equations

Determination parametric equations of the sphere in S_3 coordinate system associated worm is using coordinate transformations. First of all equations are written in the coordinate system S1 with the origin located at the center of the sphere Fig.2 (a).



(a) (b)

Fig.2. (a) Sphere positioned in S_1 system; (b) Worm system S_3

Parametric equations of the sphere written in S_1 system are:

$$\begin{cases} x_1 = r \cdot \sin v \cdot \sin\alpha \\ y_1 = r \cdot \sin v \cdot \cos\alpha \\ z_1 = r \cdot \cos v \end{cases} \tag{1}$$

To write equations sphere in coordinate system S3 of the worm Fig.2 (b), we use coordinate transformations. Coordinate transformation written in symbolic matrix form is (2):

$$r_3 = M_{31} \cdot r_1 \tag{2}$$

Written in the extended form, coordinate transformation is (3):

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x_s \\ 0 & 1 & 1 & y_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r \cdot \sin v \cdot \sin\alpha \\ r \cdot \sin v \cdot \cos\alpha \\ r \cdot \cos v \\ 1 \end{bmatrix} \tag{3}$$

After we make the calculations are obtained equations of the sphere in coordinate system of the worm, which written in parametric form are presented in the relationship (4):

$$\begin{cases} x_3 = r \cdot \sin v \cdot \sin\alpha + x_s \\ y_3 = r \cdot \sin v \cdot \cos\alpha + y_s \\ z_3 = r \cdot \cos v \end{cases} \tag{4}$$

2.2. Determination of the contact points between the plan parallel to the XZ and the sphere

We consider a sphere of radius r in coordinate system S_3 and a plane parallel to the plane X_3Z_3 located at a distance t from the axis X as shown in Fig.3. Parametric equation of the plan is written in S3 system (5):

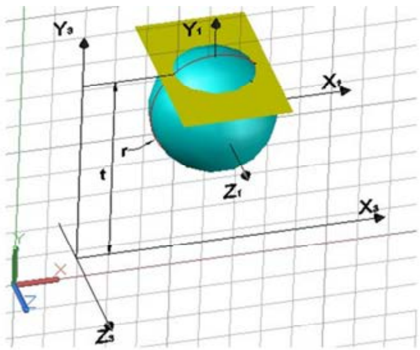


Fig.3. Contact between a plane parallel with X_3Z_3 and a sphere

$$y_3 = t \tag{5}$$

Intersection of plane and sphere is a circle whose equations are obtained by replacing the relationship (5) in (4). From this new relationship is obtained v= f (α), v parameter of the circle depending on the other parameter α (6):

$$v = \arcsin\frac{t - y_s}{r \cdot \cos\alpha} \tag{6}$$

And for to have contact the condition which must to be satisfied is relation (7), due to the fact that the sin function is defined into the range [-1,1].

$$-1 \le \frac{t - y_s}{r \cdot \cos\alpha} \le 1 \tag{7}$$

Parametric equations of the intersection resulting by replacing equation (6) in (4) and are of the form (8):

$$\begin{cases} x_3 = \dfrac{t - y_s}{\cos\alpha} \cdot \sin\alpha + x_s \\ y_3 = t \\ z_3 = \pm r \cdot \sqrt{1 - \left(\dfrac{t - y_s}{r \cdot \cos\alpha}\right)^2} \end{cases} \tag{8}$$

2.3. Determination of points contact between the helical surface and a plan

We consider helical surface of the left flank Fig.4 of the worm determined in [7], [8]. Parametric equations of the left flank of generated worm were determined based on the surfaces mutual winding method [10] and are (9):

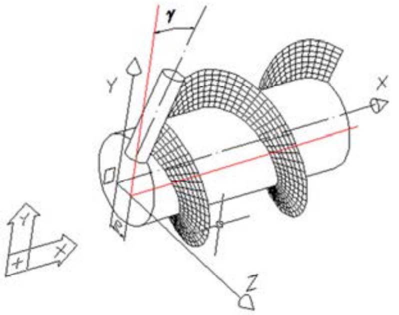


Fig.4. Generated surface of the worm flank [3]

$$\begin{cases} x_3 = r_S \cos\gamma \cos\xi + u \sin\gamma + \varphi h \\ y_3 = \cos\varphi(\sqrt{r_i^2 - e^2} + u\cos\gamma - r_S \cos\xi \sin\gamma) - \sin\varphi(r_S \sin\xi + e) \\ z_3 = \sin\varphi(\sqrt{r_i^2 - e^2} + u\cos\gamma - r_S \cos\xi \sin\gamma) + \cos\varphi(r_S \sin\xi + e) \end{cases} \tag{9}$$

Where rs =5 mm; γ=19,8°; ri=30 mm; e=1,8 mm;

$$\xi = arctg - \frac{e\sin\gamma + h\cos\gamma}{\sqrt{r_i^2 - e^2} + u\cos\gamma} \tag{10}$$

To obtain the equations of the curve representing the contact between the plane parallel to the X_3Z_3 located at a distance t from the axis X , replacing relation (5) in (9), obtain one parameter u based on the other parameter u = f (φ) relationship (11) and the contact curve equation (12):

$$u = \frac{t + \sin\varphi \cdot (r_s \cdot \sin\xi + e) + \cos\varphi\left(r_s \cdot \cos\xi \cdot \sin\gamma - \sqrt{r_i^2 - e^2}\right)}{\cos\varphi \cdot \cos\gamma} \tag{11}$$

$$\begin{cases} x_3 = r_S \cos\gamma \cos\xi + u \sin\gamma + \varphi h \\ y_3 = t \\ z_3 = \sin\varphi(\sqrt{r_i^2 - e^2} + u\cos\gamma - r_S \cos\xi \sin\gamma) + \cos\varphi(r_S \sin\xi + e) \end{cases} \tag{12}$$

In equation (11) we have expressed the parameter u according to the ξ which is a function of the parameter u (13):

$$u - f(\varphi, \xi(u)) = 0 \tag{13}$$

We have a nonlinear equation and the value of u is determined by numerical method.

Giving a value of u in the range [0,17] mm and parameter φ values successive in range [0,2π], and substituting in relation (9) we determine successively values of t . For each value of t obtained, we give values of φ again, determine the value of u with relation (11) and replaceing these parameters in (12) can be determined and represented curve contact between the helical surface and the parallel plan with X_3Z_3 plane.

2.4. Determining the distance between the center circle and the curve obtained

In Fig.5 we represented a flank of ZPA worm, a parallel plane [Pl] with the XZ plane, and a sphere. Plan intersect helical surface after a curve (C1) represented by the red color, the sphere has been sectioned and the top

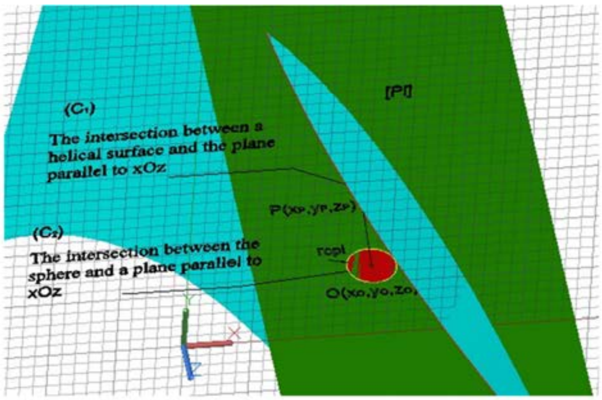


Fig.5. Distance between the center circle and the curve obtained

was removed to better observe the curve (C2) respectively the point of contact between the two curves. It is known circle center position O (x_o, y_o, z_o), his radius noted with r_{cpl} and coordinates of the point P who strolling on curve (C1). The distance between point P and O is (14):

$$d = \sqrt{(x_O - x_P)^2 + (y_O - y_P)^2 + (z_O - z_P)^2} \tag{14}$$

We retain the minimum distance d_{min} between the two points. If the difference between the minimum distance and radius (C2) is contained in an acceptable range, small enough (1 μm), then the point P (x_p,y_p,z_p) corresponding to the minimum distance is the point of tangency between the curve (C1) and (C2). The point of tangency of the curve also represents the point of contact between the stylus and helical surface.

3. Program designed to calculate the tangent point between sphere and helical surface

Program designed to give points of contact between the stylus with spherical top and the helical surface of the worm flank is designed in AutoLisp, and the representations are made in AutoCAD.

As input we need by the measured coordinate of the stylus in axial plane with the coordinate measuring machine, the radius of the sphere and the data of the worm surface. Worm position on the coordinate measuring machine must be identical to the position of generated worm. The program will return the first point of tangency met, respectively the coordinates of the center sphere which we have tangency, nominal point of contact between the stylus and helical surface, which can then be used to correct measurement errors.

To control deviations in axial plane resulting from the processing flank of the worm we compare nominal coordinate obtained using the program with the measured coordinate of the stylus.

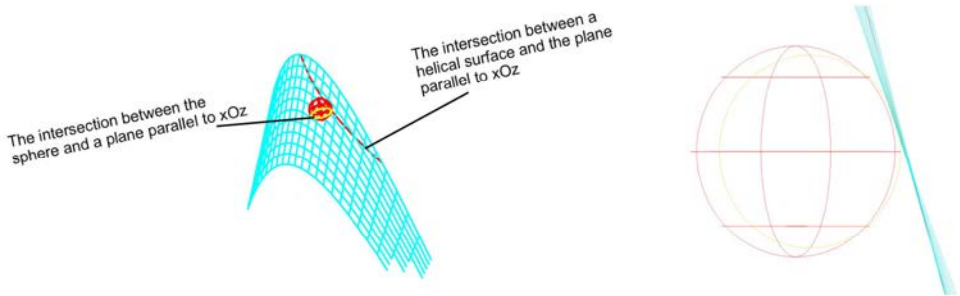
We introduce in the designed program coordinates result from the measurement of the point that we want to control, we run the program designed and we get the coordinates of the nominal value of intersection of the helical surface and stylus, respectively coordinates of the center of the sphere, Fig.6 where :

```
Command: SFERA
(5.338 "c:" 5.338 33.6363 0 "r:" 0.931526 "P" 6.23027 33.6363
-0.270928 "t:" 33.6363 "er:" 0.000972741 "u:" 7.761 "fi:" -0.020944)
```

Fig.6. The returned result by the program

- The first value represents the position of the origin stylus on the axis OX [mm],
- The second value is the coordinates of the center circle (C2) [mm],
- The third value is the radius of the circle (C2) [mm],
- follows the tangent point coordinates [mm], the parameter t, the error with that was obtained the result, parameters u [mm] and φ [rad] belonging the helical surface.

The result can be seen in Fig. 7 where curves (C1), (C2) and the worm flank surface ZPA has been represented through points.



(a) (b)

Fig.7. (a) The sphere represented in the tangency position; (b) Detail of the sphere seen from another angle

4. Conclusions

In the case of coordinate correction of the measuring point with coordinate measuring machine using a spherical stylus can be concluded that:

Measurement can be done with general coordinate measuring machine using a spherical stylus.

The contact between the stylus and the helical surface is not at the intersection of the sphere with the XY plane, so that at the worm flank measurement should be compared in the axial plane theoretical position of the stylus considering the point of contact and stylus position returned from the coordinate measuring machine,

The point of contact between the stylus and the helical surface of the worm can be obtained by evaluating the distance between the center circle determined from the points of intersection of the sphere with the plane and resulting curve of intersection considered the same plan with helical surface of the worm.

The contributions are numerical calculation of the tangency position between a sphere and flank worm ZPA, designing an AutoLISP program for assessing the position of tangency, creation an evaluation method of deviations between executed profile and the nominal profile of the worm ZPA.

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