6.8.3

Lasso.

(a) (iv) Steadily decrease,

Obviously as we approach OLS

(b) (ii) Decrease, then U shape increase

Via bias - variance tradeoff

(c) (iii) Steadily increases

Obvious $s = 0$, $\beta = 0$ ⟶ no variance

↓ increase

$s = \infty$

$\beta = \beta_{OLS}$ ⟶ high Variance

d) (iv) Steadily reduces

↓

Better fit as s ↑

e) (v) Constant ; independent of anything

6.8.5

a) General form,

$$\left[\sum_{i=1}^{n}\left(y_i - \hat{\beta}_0 - \sum_{j=1}^{p}\hat{\beta}_j x_{ij}\right)\right]^2$$

sample

$$+$$

$$\lambda \sum_{i=1}^{p} \beta_i^2$$

$\hat{\beta}_0 = 0$

$n = 2 \quad p = 2$

So,

$$\left(y_1 - \hat{\beta}_1 x_{11} - \hat{\beta}_2 x_{12}\right)^2$$

$$+$$

$$\left(y_2 - \hat{\beta}_1 x_{21} - \hat{\beta}_2 x_{22}\right)^2$$

$$+$$

$$\lambda\left(\hat{\beta}_1^2 + \hat{\beta}_2^2\right).$$

b) $x_{11} = x_{12} = x_1$
$x_{21} = x_{22} = x_2$

gives,

$$\left(y_1 - (\hat{\beta}_1 + \hat{\beta}_2)\, x_1\right)^2 + \left(y_2 - (\hat{\beta}_1 + \hat{\beta}_2)\, x_2\right)^2 + \lambda\left(\hat{\beta}_1^2 + \hat{\beta}_2^2\right) = f(\beta_1, \beta_2)$$

$$\left.\begin{aligned} \frac{\partial f}{\partial \beta_1} &= 0 \\ \frac{\partial f}{\partial \beta_2} &= 0 \end{aligned}\right\} \text{Solving gives}$$

$$\hat{\beta}_1 = \frac{x_1 y_1 + x_2 y_2 - \hat{\beta}_2 (x_1^2 + x_2^2)}{\lambda + x_1^2 + x_2^2}$$

$$\hat{\beta}_2 = \frac{x_1 y_1 + x_2 y_2 - \hat{\beta}_1 (x_1^2 + x_2^2)}{\lambda + x_1^2 + x_2^2}$$

Solve for β_1 & β_2 again to get

$$\hat{\beta}_1 = \hat{\beta}_2 = \beta$$

c) For lasso,

$$\left.\begin{aligned}&(y_1 - \hat{\beta}_1 x_{11} - \hat{\beta}_2 x_{12})^2 \\ &+ \\ &(y_2 - \hat{\beta}_1 x_{21} - \hat{\beta}_2 x_{22})^2 \\ &+ \\ &\lambda(|\hat{\beta}_1| + |\hat{\beta}_2|)\end{aligned}\right\}$$

Minimize this.

d) Alternate interpretation is

$$|\hat{\beta}_1| + |\hat{\beta}_2| < s$$

$x_{11} = x_{12}$ & $x_{21} = x_{22}$

Also $x_{11} + x_{21} = 0$

$x_{12} + x_{22} = 0$

$y_1 + y_2 = 0$

Minimize using this

$$2(y_1 - \hat{\beta}_1 + \hat{\beta}_2) x_{11})^2$$

So this obviously gives

$$\hat{\beta}_1 + \hat{\beta}_2 = \frac{y_1}{x_{11}}$$

for $\beta_1 + \beta_2 = s$

OR

$\beta_1 + \beta_2 = -s$

we have multiple solutions as can be seen.

$$\left.\begin{aligned} \beta_1 + \beta_2 &= s \\ \beta_1 &\geq 0 \\ \beta_2 &\geq 0 \end{aligned}\right\} \qquad \left.\begin{aligned} \beta_1 + \beta_2 &= -s \\ \beta_1 &\leq 0 \\ \beta_2 &\leq 0 \end{aligned}\right\}$$

general solution format

8.4.5.

p = c(0.1, 0.15, 0.2, 0.2, 0.55, 0.6, 0.6, 0.65, 0.7, 0.75)

Majority

sum(p ≥ 0.5) > sum(p < 0.5)

True → Thus RED.

no. of red predictions > no. of green based on 0.5 as class dividing threshold.

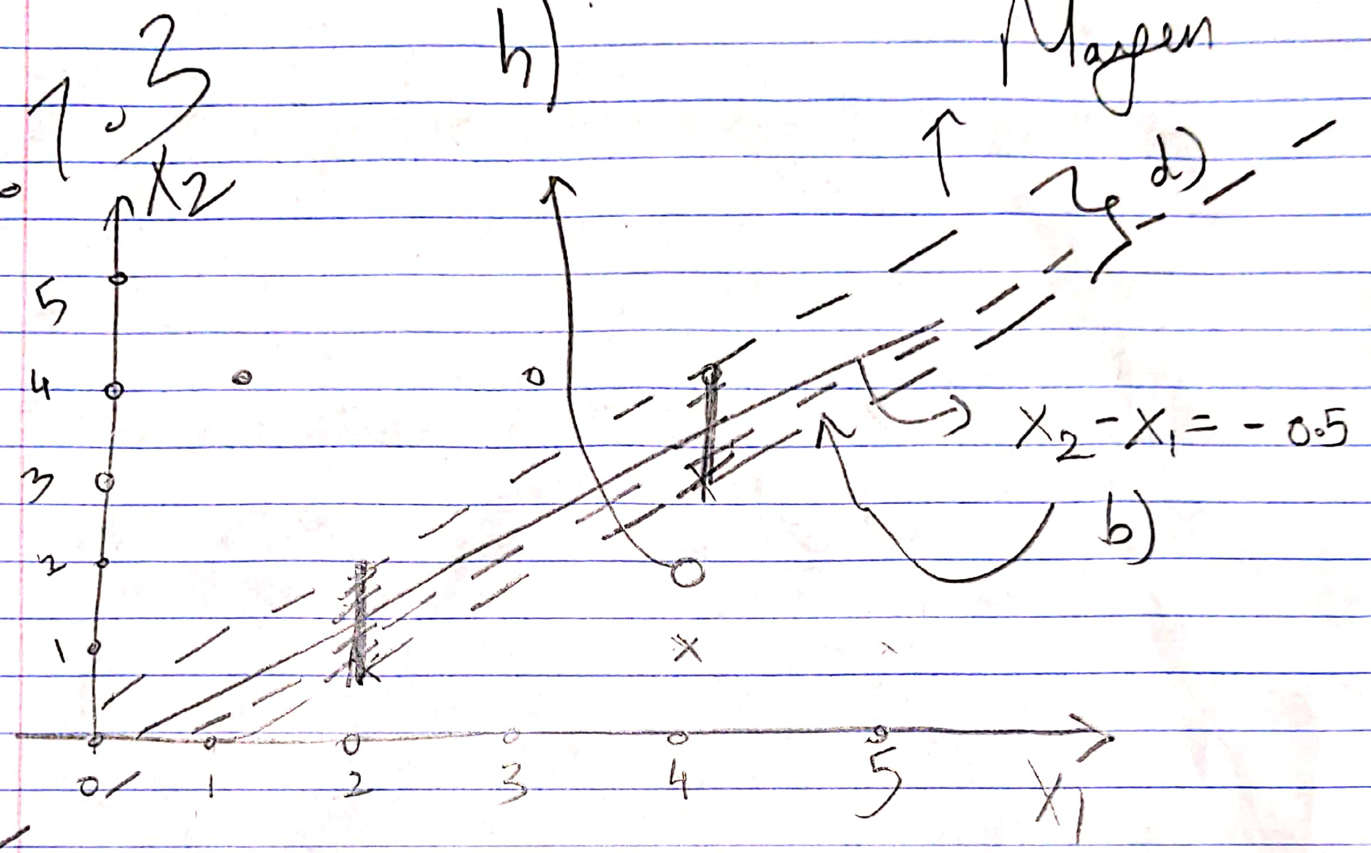
Average:

mean of p is

0.45 → < 0.5 → Green

9.7.3

a)



c) $0.5 - X_1 + X_2 > 0$

g) Not optimal

d) In diagram

e) ‖ → show support vectors

f) Slight movement in (4,1) would **not** have an effect on the maximal margin hyperplane

g) $0.8 - X_1 + X_2 > 0$

h). (4, 2) make them no longer seperable by hyperplane.